Digital Electronics
Practice for midterm exam

Outline

Below are common problems:

- Prob 1: Given the minterm expansion (standard SOP) or maxterm expansion (standard POS) for a function F: simplify F using K-map method and implement F just using NAND/NOR gates.
- Prob 2: Design a combinational logic circuit based on the requirements.
- Prob 3: Proving Boolean equations.
- Prob 4: Analyze the combinational logic circuit and manipulate the function to required form.

Problem 1: Given the minterm/maxterm expansion for a function F: simplify F using K-map method and implement F just using NAND/NOR gates.

Steps to solve:

- Step 1: Plot on a K-map (from 1 to 5 variables) (1 for minterm and 0 for maxterm)
- Step 2: Determine all prime implicants
- Step 3: Determine essential prime implicants and forming the minimum SOP/POS expression of F
- Step 4: Manipulate F to desired form (NAND/NOR..) using DeMorgan's laws, etc (2 ways)

Example:

Given the minterm expansion (standard sum of products) of logic function **f** as below:

$$f(a,b,c,d) = \Sigma m(1,5,9,10,12,15) + d(4,7,13,14)$$

a) Use K-map to simplify the function **f**
b) Implement **f** just using 2-input logic gates (except NOT gate)
c) Implement **f** using 2-input NAND gates

Prob 2: Design a combinational logic circuit based on the requirements.

Steps to solve the prob:

- B1: Determine input-output and encode them: usually using BCD_{8421}
- B2: Construct the truth table
- B3: Plot the truth table on K-map to get the minimum SOP/POS form
- B4: Manipulate the function to desired form (NAND, NOR...) and draw the circuit.

Example

A circuit that controls the shipping system of the containers operates as followings:

- Input: **Company of container** and **Type of container**. There are two companies: **A and B** and 8 types of containe numbered from **0 to 7**.

- Output: **The floor to which the container will be stored**. The building consists of 4 floors numbered from **0 to 3** .

- The containers will be moved to the floors according to the following rules:

Number of floor	Rule
0	Containers 0, 1, 2 from company A.
1	Containers 0, 2, 4, 6 from company B
2	Containers 1, 3, 5, 7 from company B
3	Containers 3, 4, 5, 6, 7 from company A

Let’s encode

- Input: 4 bits **abcd**

o **a** represents the company: a = 0 => company A, a = 1 => company B

o **bcd**: represents type of containers, bcd = 000 - 111 corresponds to 7 types of container from 0 – 7

- Output: 2 bits $\mathbf{F_1F_2}$ represents the number of floors from 0-3

=> We construct the truth table based on the rules

a	b	c	d	F1	F2
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	1	1
0	1	0	0	1	1
0	1	0	1	1	1
0	1	1	0	1	1
0	1	1	1	1	1
1	0	0	0	0	1
1	0	0	1	1	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	0	1
1	1	1	1	1	0

Prob 3: Proving Boolean equations.

Basic rules of Boolean Algebra

- 1.a: 0.0 =0
- 1.b: 1+1=1
- 2.a: 1.1=1
- 2.b: 0+0=0
- 3.a: 0.1 =1.0=0
- 3.b: 0+1=1+0=1
- 4.a: If x=0 then x'=1
- 4.b: If x=1 then x'=0

Basic rules of Boolean Algebra

- 5.a: x.0=0
- 5.b: x+1=1
- 6.a: x.1=x
- 6.b: x+0=x
- 7.a: x.x=x
- 7.b: x+x=x
- 8.a: x.x’=0
- 8.b: x+x’=1
- 9: x’’=x

Basic rules of Boolean Algebra

- 10.a: $x.y=y.x$
- 10.b: $x+y=y+x$

Tính giao hoán (commutative)

- 11.a: $x.(y.z)=(x.y).z$
- 11.b: $x+(y+z)=(x+y)+z$

Tính kết hợp (associative)

- 12.a: $x.(y+z)=x.y+x.z$
- 12.b: $x+y.z=(x+y).(x+z)$

Tính phân bố (Distributive)

- 13.a: $x+x.y=x$
- 13.b: $x.(x+y)=x$

Tính thu hút (Absorption)

Basic rules of Boolean Algebra

- 14.a: $x.y+x.y'=x$
- 14.b: $(x+y).(x+y')=x$

Tính phối hợp (combining)

- 15.a: $(x.y)'=x'+y'$
- 15.b: $(x+y)'=x'.y'$

DeMorgan

Example:

a. $b'd + a'bd + ab + ac'd = (a + d)(b + d)$

b. $[a + (bcd)'][(a'b)' + b(c' + d')(c + d')] = a + b' + d'$

c. $(a + b)(b + c)(a + c) = [(a' + b')(b' + c')(a' + c')]'$

a. $b'd + a'bd + ab + ac'd = (a + d)(b + d)$

VT $= db' + (a'bd + abd + ab) + ac'd$ (Thu hút)

$= db' + (bd(a + a') + ab) + ac'd$ (Phân phối)

$= db' + (bd + ab) + dac'$ (Bù)

$= (db' + bd + dac') + ab$ (Kết hợp)

$= d(b' + b + ac') + ab$ (Phân phối)

$= d(1 + ac') + ab$ (Bù)

$= d + ac'd + ab$ (Phân phối)

$= d + ab$ (Thu hút)

$= (a + d)(b + d)$ (Phân phối)

$=$ VP

b. $[a + (bcd)'][(a'b)' + b(c' + d')(c + d')] = a + b' + d'$

VT $= (a'bcd)'[a + b' + b(c' + d')(c + d')]$ (De Morgan)

$= (a'bcd)'(a + b' + b'd')$ (Phân phối)

$= (a'bcd)'[a + b' + b'd' + bd']$ (Thu hút)

$= (a'bcd)'[a + b' + (b'd' + bd')]$(Kết hợp)

$= (a'bcd)'[a + b' + d'(b + b')]$ (Phân phối)

$= (a'bcd)'(a + b' + d')$ (Bù)

$= (a'bcd)' (a'bd)'$ (DeMorgan)

$= (a'bcd + a'bd)'$ (DeMorgan)

$= (a'bd)'$ (Thu hút)

$= a + b' + d'$ (DeMorgan)

$=$ VP

c. $(a + b)(b + c)(a + c) = [(a' + b')(b' + c')(a' + c')]'$

VT $= (b + ac)(a + c)$ (Phân phối)

$= ab + bc + ac + ac$ (Phân phối)

$= ab + bc + ac$ (Lũy đẳng)

VP $= [(ab)'(bc)'(ac)']'$ (DeMorgan)

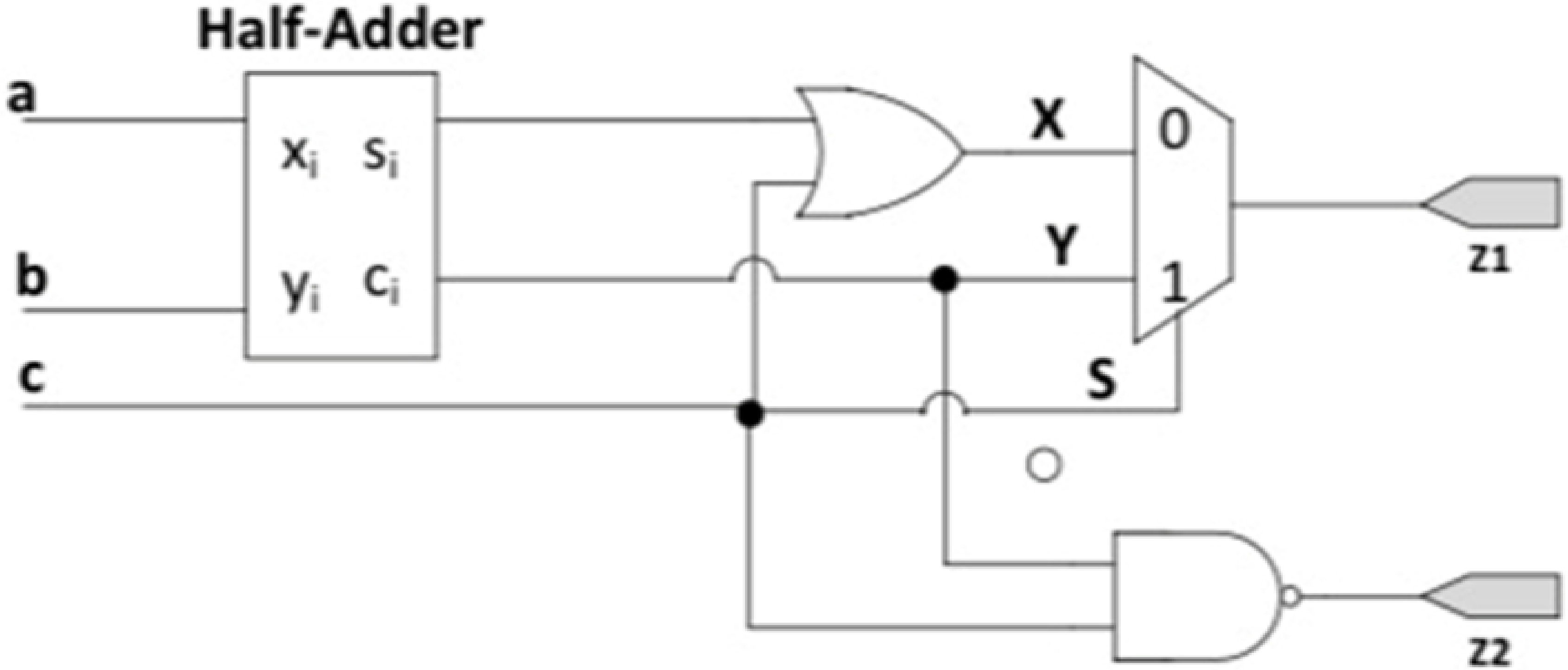
Prob 4: Analyze the combinational logic circuit and manipulate the function to required form.

Steps to solve:

- Step 1: Extract the function from the given circuit
- Step 2: Manipulate the function to required form

Example:

Analyze the following logic circuit to find out the minterm expansion (standard sum of products) **Z1(a,b,c)** and **Z2(a,b,c)**



We have:

- Z1 = S'X + SY (1)
- Z2 = (cY)' (2)
- X = S_i + c (3)
- Y = C_i (4)
- S = c (5)
- S_i = a'b + ab'(6)
- C_i= ab (7)

=> From (1) (3) (4) (5) (6) (7) we have:

Z1 = S'X + SY

= c'(S_i +c) + C_i.c = c'S_i +C_i.c

= c'(a'b + ab') + abc = a'bc' + ab'c' + abc

= $\sum m(2,4,7)$

=> From (2) (4) (7) ta có : Z2 = (cY)' = (c. C_i)' = (abc)' = a' + b' + c' =$\sum m(0,1,2,3,4,5,6)$