Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
B		2428217

Regulatory Pass for the Submersible of MCMS

On June 18,2023, the Titan disappeared after an hour and 45 minutes' departure. The incident has led regulators to pay more attention to safety plans for underwater tourism projects. This report aims to develop **a submersible search model** to help submersible manufacturers obtain regulatory approval. We hope to provide dive compartment manufacturers with strategies in response to the accidental loss of contact in multiple environments and emergency situations and to improve cost-benefit whenever possible. The report established three models: model 1 is the submersible position prediction model, model 2 is the cost-benefit evaluation model, and model 3 is the search mode planning model.

In Model 1, we collected Ionian Ocean flow data in June 2023 and analyzed possible paths of the submersible based on **hydrodynamics** and **Ekman transport theory**, with the results shown in Figure 4. Then, the uncertainty and probability distributions are analyzed by **Monte Carlo Simulation** to provide a basis for the determination of search and rescue patterns in Model 3.

In Model 2, we first identified the equipment that may be required for rescue operations based on the actual crash event and the rescue reports of related organizations and quantified the attribute sub-terms of the equipment. The qualified values are then transformed into a uniform standard range by normalization treatment to eliminate the dimensional influence between the different variables, making the different features comparable. Then, the weight vector of the different variables was obtained through the **AHP hierarchical analysis** (0.594,0.2764,0.1283). Finally, the **cost-benefit** of each equipment is determined according to the calculation model, and the equipment scheme of the main ship and the additional equipment of the rescue ship are given.

In Model 3, we first identify the **Archimedes spiral** with radius growth rate 3 as the basic path based on the uncertainty and probability distribution map in Model 1, and generalized it to the case of multi-objective optimization, which results in Figure 11. Next, based on the application of statistics in the field, the logistic regression function of success rate over time was determined, where k=1 and t0=6. Then, the parameters of the ARIMA model were fitted using real data and k-fold cross validation was introduced to identify the final prediction model as **ARIMA (1,2,2)**, to obtain the relationship between time and the probability of finding the submersible. The results are shown in Figure 13. Finally, using the **KDE kernel density estimation**, the relationship between the cumulative search results and the probability of finding the submersible was obtained, and the results are shown in Figure 14.

Finally, the model **robustness and sensitivity analyses** were tested. When the environmental variables are randomly generated from a uniform random distribution, the final convergent distribution of the model is little different. As for the factors that affect the model, The volume of the diving tank and the mass of the ballast water, it is found that high accuracy of these two factors will significantly increase the prediction accuracy of model 1

Keywords: ARIMA; AHP; Archimedes spiral; KDE; Monte Carlo Simulation

Contents

1 Introduction

1.1 Problem Background

To open up deep-sea exploration to the public, Greece's Maritime Cruises Mini-Submarines (MCMS) plans to offer sightseeing submersibles in the Ionian Sea. Focusing on safety, they aim to create a integrated "positioning-preparation-rescue" system to avoid incidents like the Titan disaster in June 2023 and win regulatory approval. Restatement of the Problem

Considering the background information and the conditions emphasized in the problem statement, we need to address the following issues:

- Localization Issue in the Integrated System:
  - Develop a model to track a submersible's position over time. Considering prediction uncertainties, how significant are these uncertainties? To reduce uncertainty for rescue, what data should the submersible transmit regularly before any incident? What devices are needed for detecting this data? How can this model be adapted to the Caribbean Sea?
- Preparedness Issue in the Integrated System:
  - Create a mathematical model for determining necessary additional search equipment on the host ship. How can the deployment of this equipment be optimized? How can costs for device availability, maintenance, preparation, and usage be quantified? How can the benefits of successful rescue operations be maximized? What additional search equipment might be needed for assistance?
- Rescue Issue in the Integrated System:
  - Establish a mathematical model to minimize the time to locate the submersibles. What is the initial points of deployment for the search devices? What is the search patterns based on this deployment plan? According to time and accumulated search results, what is the probability of locating the submersibles? If multiple targets exist, how to adjust the initialization deployment scheme and search mode?

1.2 Our work

The topic requires us to focus on predicting the possible locations of the submersibles over a period of time and planning search patterns based on the predicted positions. What' s more, we need to take the cost-benefit of using equipment into consideration. The following are the main aspects of our work:

- ✧ Clarify the rescue principles.
- ✧ Establish a position prediction model for the submersibles based on ocean currents, water density, and underwater geographical environment.
- ✧ Simulate a selection plan for search equipment,based on device sub-items,and develop a cost-benefit analysis model.
- ✧ Propose initial deployment points and search patterns for the equipment based on the position prediction model.

Firstly, we will clarify the rescue principles. Secondly, based on the Ekman transport principle and fluid drag, using Monte Carlo simulation, we will predict the position of the submersibles and estimate the uncertainty and probability distribution of the predicted position. Next, we will establish a cost-efficiency model that describes the relationship between the attributes of the equipment sub-items and their cost-benefit. We will use normalization and Analytic Hierarchy Process (AHP) analysis to simulate the cost-benefit under a specific selection plan. Finally, we will develop a search pattern model for the rescue vessel, expanding its search objectives from a single submersible to multiple submersibles moving in the same general vicinity. In summary, the whole modeling process can be shown as follows: At the same time, we extend our search from the Ionian Sea to the Caribbean Sea.

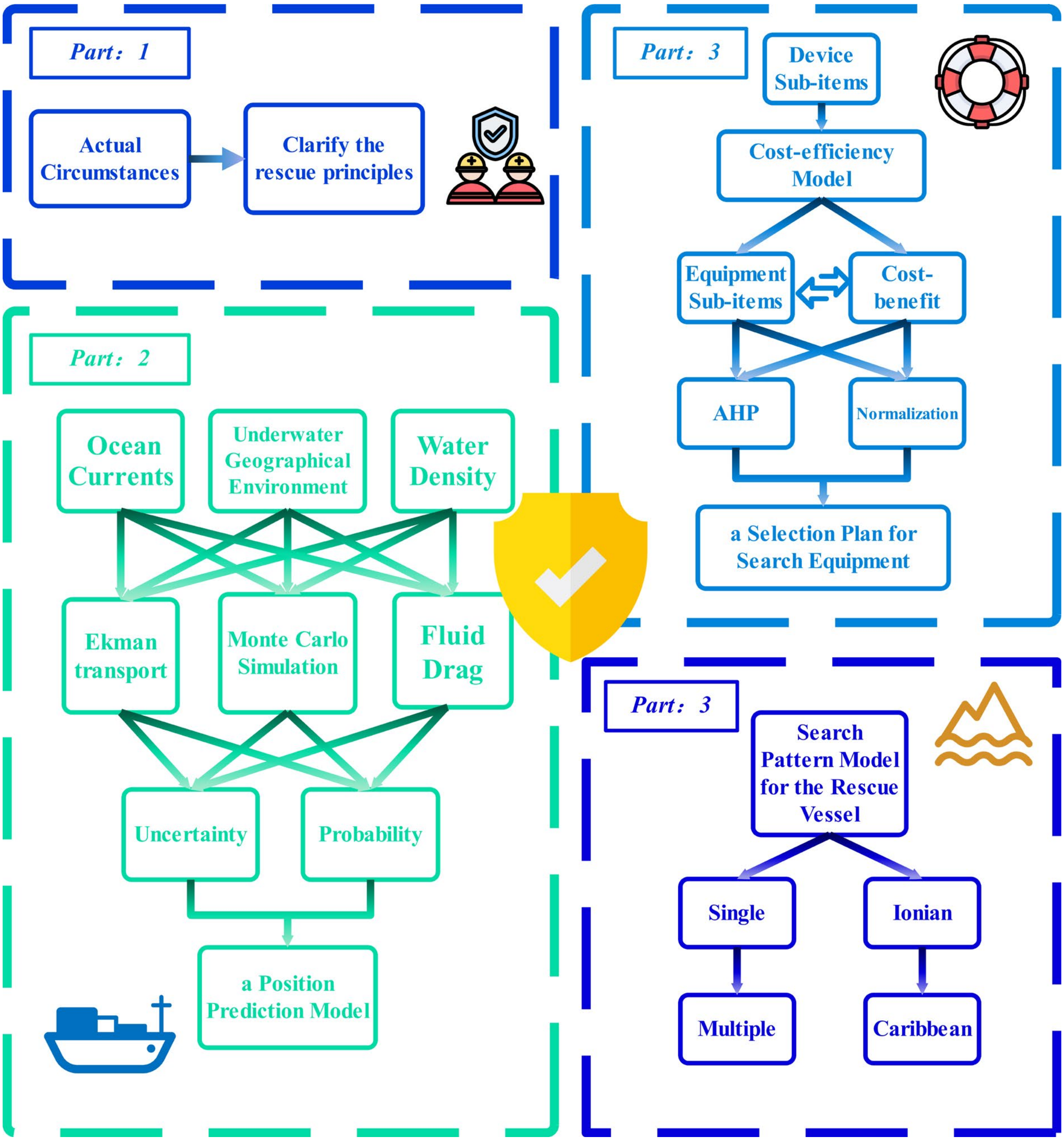

Figure 1 Model Overview

2 Assumptions and Justifications

To simplify the problem for model development and computation, we make the following basic assumptions. Each assumption is reasonable in nature.

➢ **Assumption 1: There are no environmental or technological changes during the research period, and the sea area is spacious and stable.**

Justification: Based on historical data and short-term weather reports, the environmental conditions in the Ionian Sea are relatively stable during a specific time period, with a very low possibility of significant natural disasters. Additionally, in the short term, technological advancements typically occur gradually rather than in revolutionary leaps. This assumption helps eliminate uncertainties in rescue operations and simplifies the complexity of the model.

- **Assumption 2: The data obtained during the research period is accurate and stable.**

Justification: Prior to rescue operations, preparatory work usually involves collecting and verifying the accuracy of the required data. Accurate data is crucial for guiding rescue operations, formulating strategies, and dynamically adjusting plans. By carefully selecting sources and utilizing the latest technology for data collection, the accuracy and stability of the data can be ensured within the research period. This assumption also allows researchers and rescue teams to focus on known variables and conditions, facilitating the progress of rescue operations.

- **Assumption 3: During the descent, the submersible only moves with the ocean currents.**

Justification: Considering that submarines are typically large and heavy, their self-propulsion capabilities are limited. Ocean currents are the movement of water in the ocean and generally possess significant scale and continuity. This assumption aligns with the practical situation, as other factors such as wind force and the motion of other vessels are relatively small and can be negligible in actual rescue operations. Thus, this assumption also simplifies the complexity of the model.

- **Assumption 4: The submersible is a sphere-like object**

Justification: The design of the submersible needs to consider resistance to high-pressure environments and reduction of fluid resistance. A sphere-like object allows for even distribution of external pressure and reduces the frontal area exposed to flow. Therefore, constraining the submersible as a sphere-like object is logical from an engineering and physics perspective.

- **Assumption 5: The rescue principles is clarified as following.**

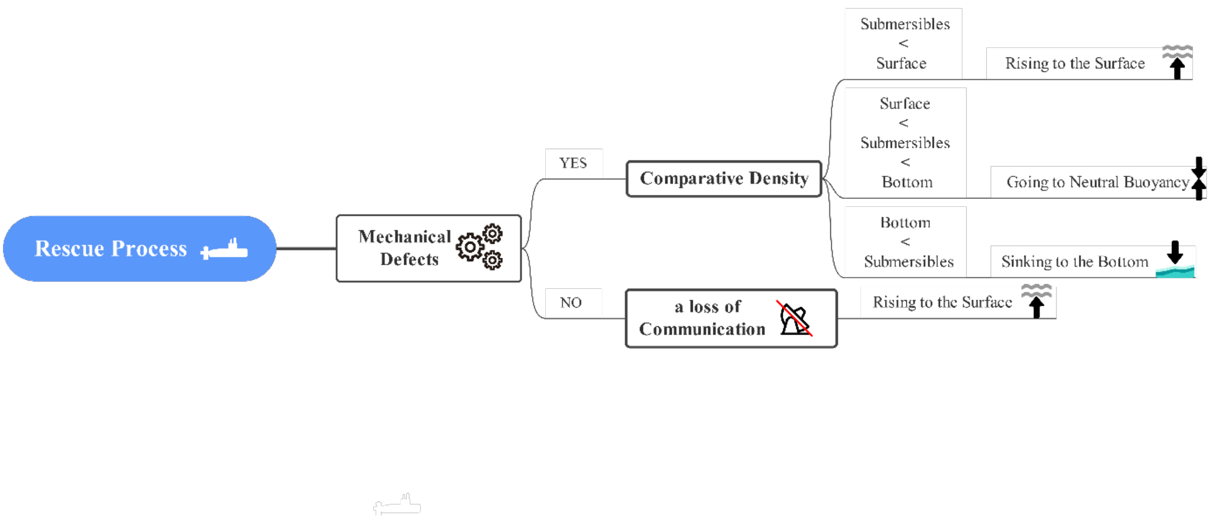

Figure 2 Graph of the Rescue Principle

3 Notations

In this article, some of the main variables used are listed below.

Table 1: Notations used in this paper

Symbol	Description	Unit
x	The location of X-axis	m
y	The location of Y-axis	m
z	The location of Z-axis	m
t	The time from now	second
m	The mass of the objection	km
n	The number of the boat	
$v(x,y,z)$	The speed on the location with Coordinates (x,y,z)	km/year
$f(\xi)$	The probability of error	%
$P(t)$	The income for moving t seconds later	%

4 Model Preparation

4.1 The Data

Due to the large amount of data and the lack of intuitiveness, we directly visualized some of the data.

Our data mainly includes the geography data of the sea floor, historical rescue situations, basic parameters of manned submersibles, current status of submersible sightseeing projects and basic parameters of rescue robots.

Table 2: Data source collation

Database Names	Database Websites Data	Type
GOFD	https://earth.nullschool.net/	Geography
DEM	https://download.gebco.net/	Geography
Seabourn	https://www.seabourn.com/	Travelling Project
Google Scholar	https://scholar.google.com/	Parameters

4.2 Geographic Coordinate System

The Cartesian coordinate system is established by taking the launch site of the submersible on the sea level as the coordinate origin O, the east-west direction as the X-axis, the north-south direction as the Y-axis, and the direction perpendicular to the sea plane as the Z-axis. Among them, the positive direction of the X-axis is due west, the positive direction of the Y-axis is due north, and the positive direction of the Z-axis is perpendicular to the sea plane.

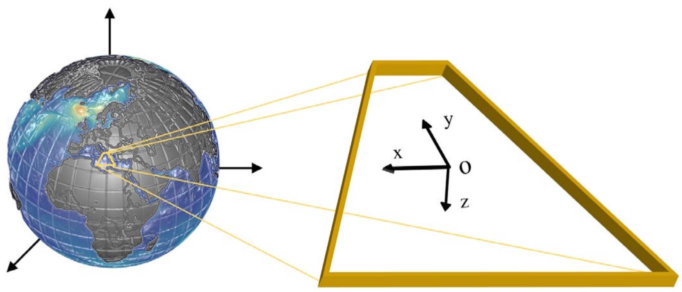

Figure 3 Geographic Coordinate System

To ensure the safety of those in the submersible, we set a safe descent speed.

$$v_{safety} = 3.7m/s \tag{1}$$

Based on the available materials and the finished model of the submersible, we set reasonable inherent physical properties of the submersible.

$$V = 23m^2 \tag{2}$$

$$m_{met} = 2.1 \times 10^4 kg \tag{3}$$

$$m_{ballast-water} = 2.35 \times 10^3 kg \tag{4}$$

5 Model I: HydroMonte Fusion Predictor(HMF Predictor)

A prediction model based on hydrodynamics and Monte Carlo method is established from three aspects: ocean current, water density at different depths and the complex influence of seabed geography. Due to the interaction of variables, these three aspects need to be analyzed in combination with the actual situation.

5.1 Kinematic Equations for Movement with Ocean Currents

In the horizontal direction, that is, the plane composed of X-axis and Y-axis, we establish the kinematic equation of the submersible moving with the ocean current.

5.1.1 Eckman Transport

Wind significantly influences ocean currents, with Ekman transport moving ocean surface water due to wind friction. However, satellite data is limited to measuring surface currents' speeds globally. This surface movement induces flow in deeper ocean layers at various depths.

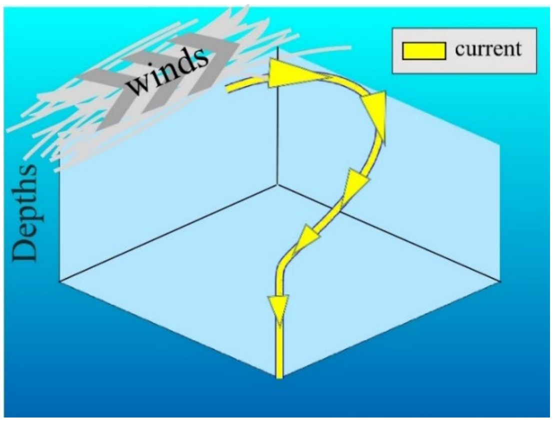

Figure 4 Diagram of Ekman Transport

Therefore, we use Ekman transport to reason about the velocity of deep ocean currents.

$$\begin{cases} v_x = v_0 \cos\left(\theta + \dfrac{\pi}{D_E} z\right) e^{\frac{\pi}{D_e} z} \\ v_y = v_0 \sin\left(\theta + \dfrac{\pi}{D_E} z\right) e^{\frac{\pi}{D_e} z} \\ v_{\min} = v_0 e^{\frac{\pi}{D_e} z} \end{cases} \tag{5}$$
,

where v_0 indicates the surface current, θ indicates the angle between the surface current and due west, D_E is the Ekman depth, it can be calculated using:

$$D_E = \pi \sqrt{\frac{2A_z}{|f|}} \tag{6}$$
,

where A_z is the vertical eddy viscosity coefficient, f is the Coriolis frequency, it can be calculated using:

$$f = 2\Omega \sin\phi \tag{7}$$
,

where Ω is the rotation rate of the Earth, ϕ is the latitude.

5.1.2 Normal Distribution of the Errors of the Collected Data

Due to immeasurable problems such as transmission errors, loss of partial data, instrument measurement errors, etc., we may have to deal with errors in data collection.

To do this, we use a normal distribution to describe the error in the collected data:

$$f(\xi) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(\xi-\mu)^2}{2\sigma^2}} \tag{8}$$
,

where μ represents the average of the upward velocities of the parties during the dive, and σ represents the variance of the measured discrete data set.

5.2 Dynkinetic Equations for Motion with Fluid Drag and Buoyancy

In the direction perpendicular to the sea plane, we establish the dynamic equation of the submersible moving with fluid drag and buoyancy.

5.2.1 Reynolds Number

In fluid mechanics, the Reynolds number is a dimensionless constant used to measure the influence of inertial and fluid forces.

$$\mathrm{Re} = \frac{\rho v L}{\mu} \tag{9}$$
,

where ρ is the density of the fluid, v is the viscosity of the fluid, L is the characteristic length of the object.

- When the Reynolds number is below 2000, it is a low Reynolds number. Currently, the fluid drag of the object mainly comes from the form drag of laminar flow.
- When the Reynolds number is higher than 4000, the Reynolds number is high. Currently, the fluid drag of the object mainly comes from the shape drag of turbulence.

In the Ionian Sea, the Reynolds number of diving chambers is approximately 10,387,281. Therefore, the fluid drag of the submersible is mainly from the shape drag of turbulence.

5.2.2 Shape Drag

We use shape drag to describe the fluid drag a submersible receives in seawater:

$$F_d = \frac{1}{2}\rho v^2 C_d A \tag{10}$$

where C_d is the drag coefficient, A is the frontal area.

5.2.3 Kinetic Equation

Taking shape drag, buoyancy and gravity into account, we establish a dynamic equation perpendicular to the sea plane:

$$(m_{met} + m_{ballast-water})\frac{dz}{dt} = Gravity - Buoyancy \mp F_d \tag{11}$$

The minus sign applies to the descent process and the plus sign applies to the ascent process.

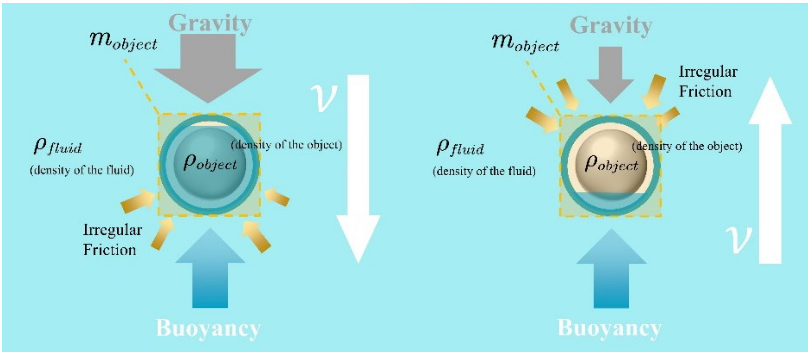

Figure 5 Force Analysis Diagram

5.2.4 Control System

1) Speed Control

In the accident of lost submersibles, the stall during diving is an important cause of disaster. Therefore, during the descent, the speed of the submersible must be lower than the set safe diving speed. This means that when the speed of the submersible is equal to the safe diving speed, the ballast water quality needs to be dynamically adjusted to ensure uniform speed or deceleration to avoid stall.

At this time, the quality of the ballast water should meet:

$$m_{ballast-water} = \frac{v}{gz}(Buoyancy \pm F_d) - m_{net} \tag{12}$$

2) Neutral Buoyancy Point

The presence of neutral buoyancy points complicates search and rescue operations, as submersibles can be located at any depth. In order to improve the success rate of rescue, we need to find all possible neutral buoyancy points based on existing data and prediction results.

During the dive, the submersible will dynamically control the ballast water quality.

- When the minimum density of the sea water > the density of the submersible, the lost submersible will surface.
- When the minimum density of sea water < the density of the submersible < maximum density of sea water, the lost submersible will surface will crash to the possible neutral buoyancy point.
- When the minimum density of the sea water < the density of the submersible, the lost submersible will sink to the bottom of the sea.

5.3 Monte Carlo Simulation Used to Calculate the Uncertainties

Due to the different forms of submersibles, the complexity of deep-sea geographical ecological environment and the possible instantaneous gradient cliffs, the actual initial parameters will be changed, such as the starting position and ocean currents. When we make a prediction of the position of the submersible, the dynamic and kinematic equations yield different results as the initial parameters such as the starting position and ocean currents change. Therefore, through Monte Carlo simulation, we can get multiple possible paths and their respective probabilities.

Algorithm 1: Monte Carlo Simulation

Input: n_{Su}: *number of submersibles*, n_{Si}: *number of simulations*

Output: *list of submersibles' locations*

Var *list*

```
for i = 1 to n_Su do
    create a submersible object submersible(i)
    initialize a random starting position for submersible(i)
    for j = 1 to n_Si do
        randomly initialize parameters for submersible(i)
        create a submersible object environment(j)
        simulate the motion path of submersible(i) using Model I
        add Model I's result into list
    end
end
```

5.4 Results

5.4.1 Uncertainty as well as the Probability Distribution

Based on the motion equation of the object, through Monte Carlo simulation, we get the uncertainty and probability distribution diagram:

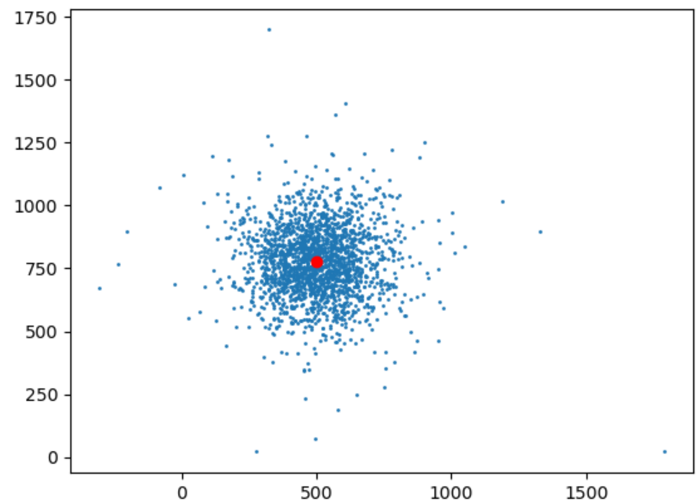

Figure 6 Uncertainty Probability Distribution

5.4.2 Measurement Equipment carried by the Submersible & Information Sent

To obtain the input variables in HMF Predictor, the submersible needs to carry some measuring equipment. The details, functions, and sent information are as follows:

- Check the submersible's horizontal coordinates.
  - GPS; A Sonardyne Homer Pro locator beacon
- Check the attitude of the submersible.
  - iXblue fiber optic gyroscope Octans
- Measure the salinity, temperature, and depth of the surrounding seawater.
  - An electrical conductivity, temperature and depth (pressure) sensor
- Assist the search vessel in locating the dive bay.
  - Sonardyne Ultra-Short Baseline (USBL) system; Auxiliary Long Baseline (LBL) acoustic positioning system
- Check the diving distance of the submersible.
  - A depth sensor; An altimeter

6 Model II: Cost-Benefit Analysis based on Normalization and AHP Hierarchy(CBA-N&AHP)

When the submersible is crashed, if the host ship can conduct some search activities before the rescue ship, then it will be very helpful to the search success. However, the host ship cannot be overarmed and must reasonably consider the cost of the equipment availability, maintenance, preparation and use, and the value generated from search success. Rescue vessels also need targeted additional equipment when we request regulatory approval.

6.1 Selection of Equipment Carried on the Host Ship

There are so many devices used for the search. Given the need for search and the economics of the host ship space, we selected the following equipment:

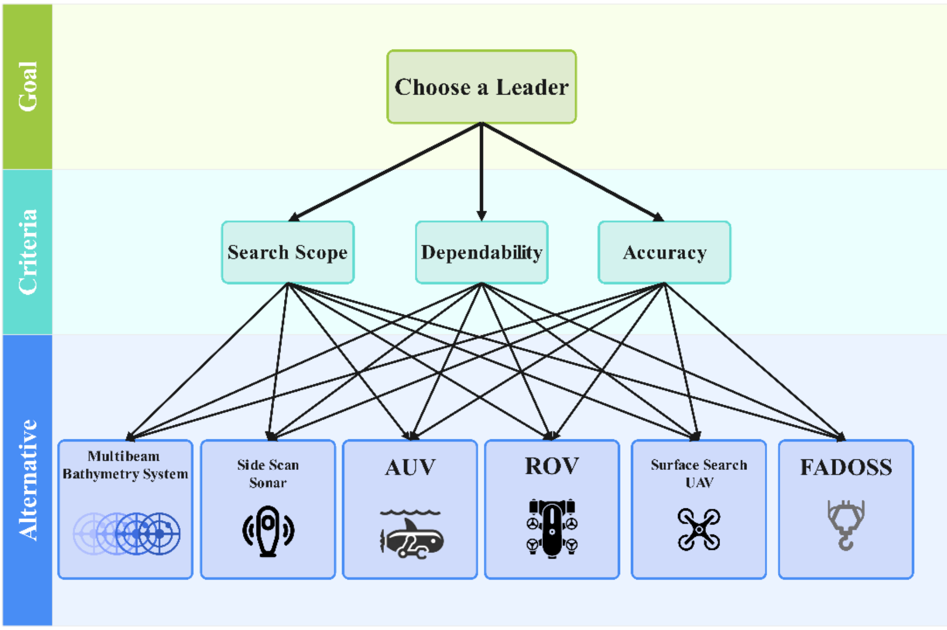

Figure 7 Schematic Diagram of AHP Algorithm

6.2 Normalization of Data

In the above table, the data varies greatly between the different children of the attribute. To facilitate computation and model building, we normalized the data:

$$\lambda^{normal} = \frac{\lambda - \lambda_{\min}^{normal}}{\lambda_{\max}^{normal} - \lambda_{\min}^{normal}}, \tag{13}$$

where λ represents an array of the same children of different devices.

6.3 Subterm Weighting Based on the AHP Hierarchical Analysis

6.3.1 Rule

In the actual search process, a good path planning can make up for the lack of search range. So, we will focus on the success of the monomer search. Under this condition, the search range is less weighted than the accuracy and reliability. Because the machine is far less reliable than the human, the reliability is less weighted than accuracy. More importantly, accurate feedback of search information is the most important task of a search device.

Therefore, we give the following weighting criteria:

	Range	Accuracy	Reliability
Range	1	1/5	1/3
Accuracy	5	1	2
Reliability	3	1/2	1

Figure 8 Matrix of Weighted Relationships

6.3.2 Hierarchical Analysis Algorithm

Based on the weighting criteria of search range, reliability and accuracy, we simulate the process of hierarchical analysis as follows:

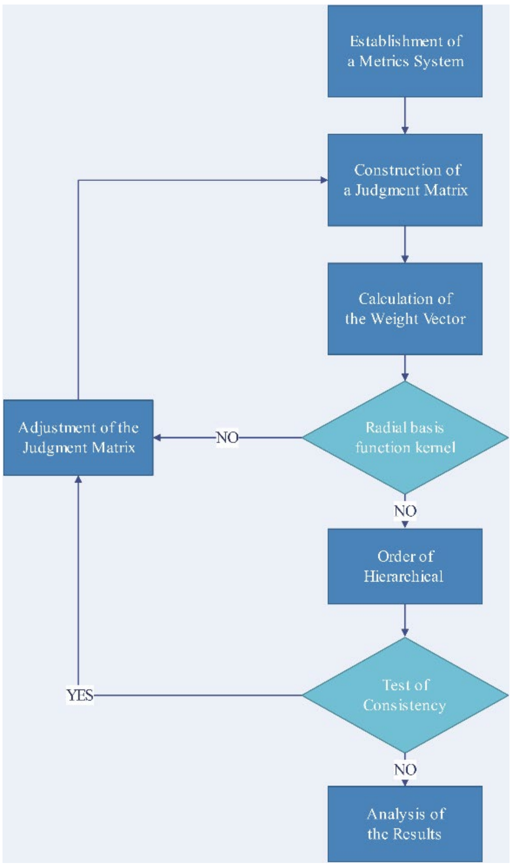

Figure 9 AHP Process Simulation

When calculating the weight matrix, there are arithmetic averaging method, set average method and eigenvalue method. Three weight vectors can be obtained from these three methods. For this, we calculate the consistency of the three using:

$$CR = \frac{1}{RI}\frac{\lambda_{\max}^{normal} - n}{n-1} \tag{14}$$

,

where RI the empirical constant.

At this point CR=0.0053222 <0.1, so the consistency is acceptable.

6.4 Cost-Benefit Analysis

In the search action, we calculate the cost-benefit of a device using:

$$CER = \frac{f(\lambda)}{\cos t} \tag{15}$$

,

where (letter) indicates the role of a search. Combined with the results weighted by the sub-terms based on the AHP hierarchical analysis, we calculate the role of the search using:

$$f(\lambda) = \omega_{range} \cdot \lambda_{range}^{normal} + \omega_{accuracies} \cdot \lambda_{accuracies}^{normal} + \omega_{reliabilities} \cdot \lambda_{reliabilities}^{normal} \tag{16}$$

.

6.5 Result

By entering the normalized data, we obtained the cost-benefit of each device:

Equipment	Costs-Benefit
Multibeam Bathymetry System	0.252944
Side Scan Sonar	0.005975
AUV	0.123684
ROV	0.162129
Surface Search UAV	0.299212
FADOSS	0.4169

7 Model III: Archimedean Spiral-Integrated ARIMA Forecasting Cumulative Distribution Framework (ASIA-CDF)

According to model 1, the uncertainty probability distribution map is circular. From the center to the circumference, the probability density decreases. In order to minimize the time to find the lost submersible, we need to choose the appropriate equipment deployment point and search mode.

7.1 Search patterns based on the Archimedes spirals

7.1.1 Reasons for Selection

To find the lost submersible more quickly, rescue ships need to search the area. At the same time, to ensure the accuracy of the search, the rescue ship should not travel too fast, but should maintain a stable speed from beginning to end. The path should also be smooth and smooth, reducing the difficulty of practical operation.

7.1.2 Device Deployment Point

The device deployment point is taken at the maximum probability density, namely the center of the uncertainty probability distribution map.

7.1.3 Archimedean spiral

The Archimedean spiral line formation originates from the movement of the point. When a moving point leaves the origin at a steady rate, its connection with the origin rotates around the origin at equal angular velocity. In this case, the trajectory of the moving point is the "Archimedes spiral".

In the search mode, the origin is the device deployment point, the moving point is the rescue ship, and the coordinates of the rescue ship can be expressed as:

$$\begin{cases} x = (a+b\theta)\cos\left(\theta + \frac{2i}{n}\pi\right) + x_{centre} \\ y = (a+b\theta)\sin\left(\theta + \frac{2i}{n}\pi\right) + y_{centre} \end{cases} , \tag{17}$$

where b is the growth rate of the spiral radius. a is the coil spacing parameter of the spiral line, which can be calculated as follows:

$$a = 2 \cdot r \cdot n \tag{18}$$

7.1.4 Search Mode

The starting position of the Archimedes spiral is influenced by the number of rescue ships and the sonar radius, which means that the straight path from the origin to the beginning of the Archimedes spiral is also the search path for the rescue ships. Taking the three rescue ships for example, there are the following search paths:

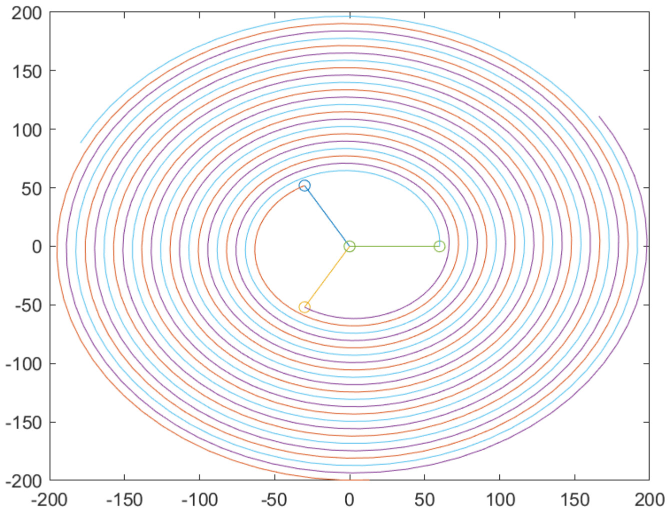

Figure 10 Single-Target Path Planning

7.1.5 Spiral Optimization under Multiple Targets

When multiple submersibles moving in the same general vicinity is lost, we only need to modify the location of the initial deployment point. We simulate the grid search of the initial deployment point as follows:

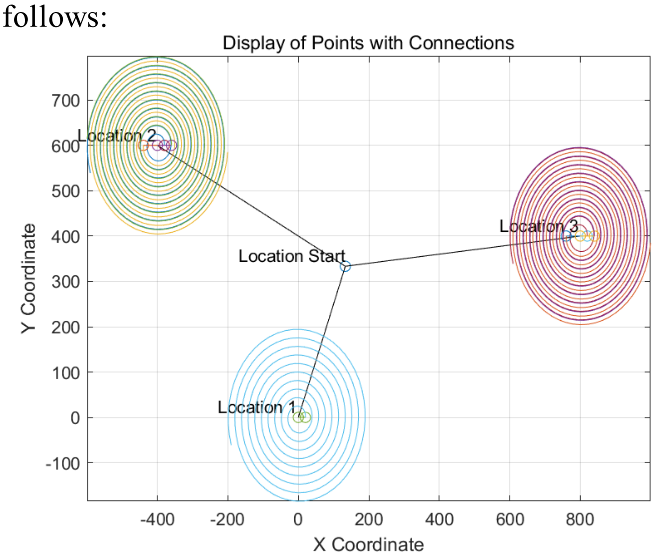

Figure 11 Multi-Objective Path Planning

7.2 Prediction of Time Series

7.2.1 Description of the Change in the Success Rate

In recent years, the statistical analysis method has played a certain role in the field of maritime search and rescue. With the increase of accidents at sea, the researchers made clear between time and search success rate to meet the classic logistic regression:

$$P(t) = \frac{1}{1+e^{k(t-t_0)}} \tag{19}$$

,

where k is the transition coefficient, and k is the fastest moment. Because the golden rescue time at sea is 12 hours, so k takes 6 hours here.

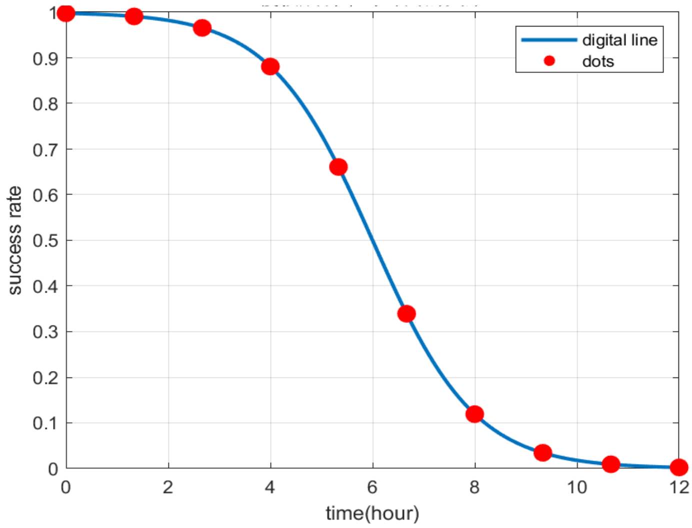

Figure 12 Simulate Curves and Discrete Lattices

7.2.2 Autoregressive Prediction Model

Despite the existence of statistics-based regression models, actual search still require scientific predictions to cope with changing unexpected conditions. It can be seen that there is a clear trend in the overall temperature change. Therefore, the ARIMA (p, d, 2) model was applied to the modeling of the time-series data

Discretize the above success rate change data. For the i-th time series, the general situation of the model is as follows:

$$\Delta^{(d)} u_{i,t} = \sum_{j=1}^{P} \beta_{i,j} \Delta^{(d)} u_{i,t-j} + \xi_{i,t} \tag{20}$$

,

where $u_{i.t}$ represents the differential unit of the order, and $\beta_{i.t}$ is the residual value of the model. Therefore, the first-order difference prediction value of the i th sequence in the post-t hour can be obtained as

$$E(\Delta^{(d)} u_{i,t}) = \sum_{j=1}^{P} \hat{\beta}_{i,j} E(\Delta^{(d)} u_{i,t-1}) \tag{21}$$

.

Note that there are lag order and difference order number in the model, so the k-fold cross-validation of the estimated results of the model is needed to find the best prediction model in the given alternative model. The results are shown in Figure.

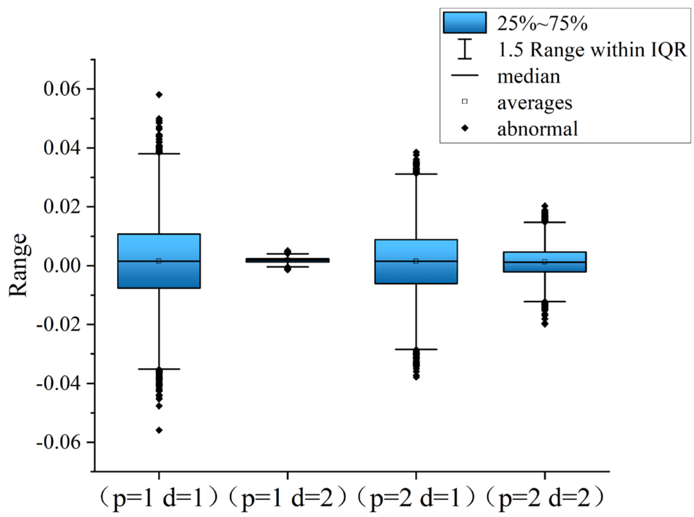

Figure 13 ARIMA Error Analysis Box Plot

As shown from the figure, the model has the best predictive power when the lag order is 1 and the difference order is 2. Therefore, we

The ARIMA (1,2,2) model is considered to have the best comprehensive prediction performance.

7.3 Cumulative Distribution of the Search Results

In the Monte Carlo simulation of Model 1, we have obtained the cumulative distribution of the search results. To calculate the probability that the dive bin might appear at each point, we used KDE kernel density estimates to count the probability density of the submersible at each position

$$\hat{f}_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K_{Gauss}(x,x')\left(\frac{x-x_i}{h}\right), \tag{22}$$

where $h > 0$ is a smoothing parameter called the bandwidth, $K_{Gauss}(x,x')$ is a Radial basis function kernel. It can be calculated using:

$$K_{Gauss}(x,x') = \exp\left(-\frac{\|x-x'\|_2^2}{2\sigma^2}\right), \tag{23}$$

where $||x-x'||_2^2$ is a square Euclidean distance.

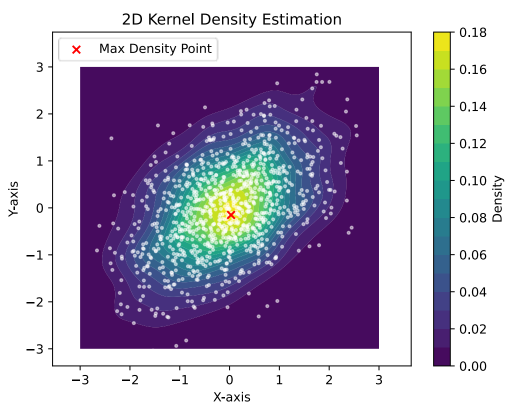

Figure 14 KDE Simulation Results

8 Test the Model

8.1 Sensitivity Analysis

In Model 1, we introduced eight environmental variables to predict the location of the submersible: ocean current speed, temperature, salinity, pressure, latitude, mass of the submersible, volume of the submersible, and mass of ballast water. Now, we will analyze the sensitivity coefficient of model 1 for each variable:

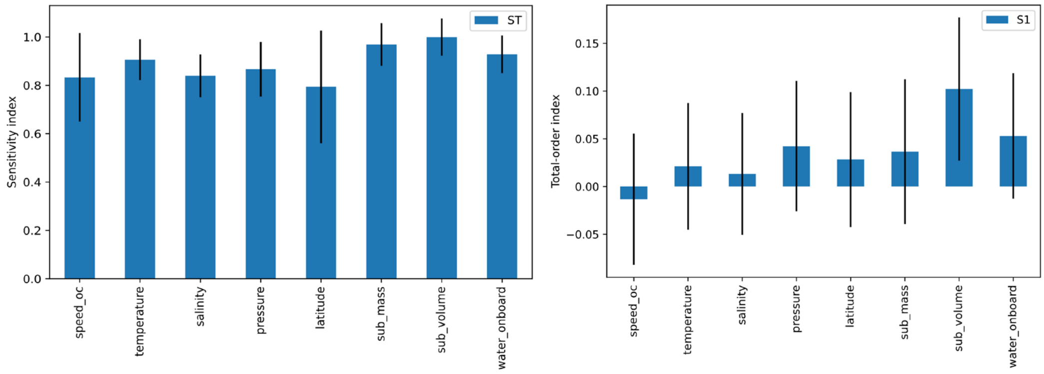

Figure 15
The overall sensitivity coefficient and the first-order differential sensitivity coefficient for each variable

According to the results, the overall effect indicators of the variables were more consistent, while the first-order effect indicators were quite different. This indicates that, this model is relatively stable overall, but varies in the influence of individual parameters.

Where, model 1 is more sensitive to the volume of the diving submersible and the mass of the ballast water. This means that if we want to improve the prediction accuracy of model 1, we need to measure the volume of the submersible and the mass of the ballast water more accurately.

8.2 Robustness Analysis

In Section 7.3, according to the location predicted from Model 1, we obtained the cumulative distribution of search results by KDE kernel density analysis. However, there may be some errors in the predicted position. These errors lead to an abnormal prediction route of the Model 1 output, which in turn can bias the KDE analysis.

In order to verify the credibility of the results of Model 1 in formulating the search scheme, we conducted a robustness analysis of Model 1:

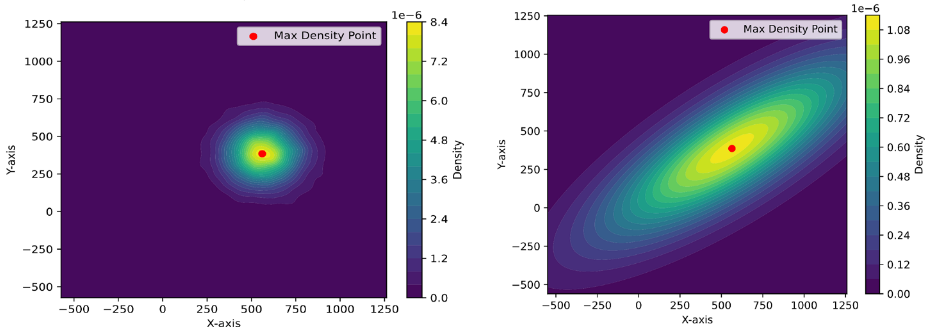

Figure 16 Variant distribution vs. normal distribution

We vary the values in the Monte Carlo simulation with a certain probability, and then compare the results of the variation value with the results of the normal value. The results are shown in the following figure:

9 Conclusion

9.1 Summary of Results

9.1.1 Result of Problem 1

According to the calculation results of model 1, the distribution of the movement trajectory and the uncertainty probability of the submersible are as follows

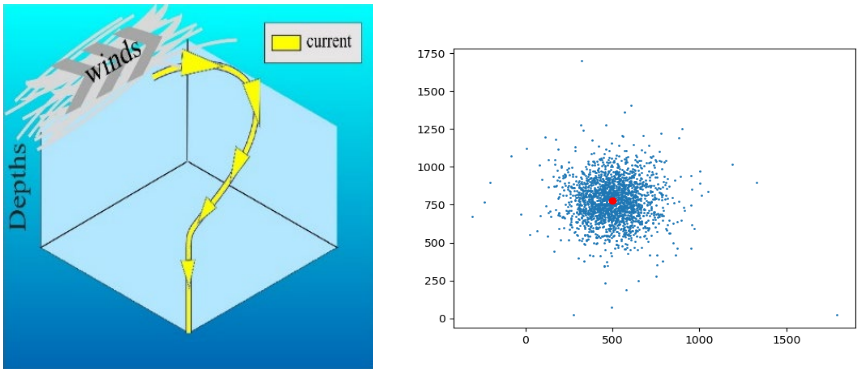

Figure 17

9.1.2 Result of Problem 2

According to the calculation results of Model 2, the main ship shall carry the following equipment:

Flyaway Deep Ocean Salvage System
Surface Search UAV
Multibeam Bathymetry System
ROV
AUV
Additionally, Rescue ships may need Side Scan Sonar

9.1.3 Result of Problem 3

According to the calculation results of Model 3, in order to minimize the time to find the diving warehouse, the initial point deployment and search mode are as follows:

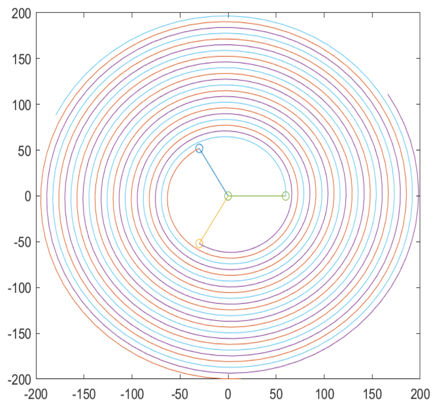

Figure 18 Search Route Map

Based on the time and cumulative search results, the success rate of finding the submersible is as follows:

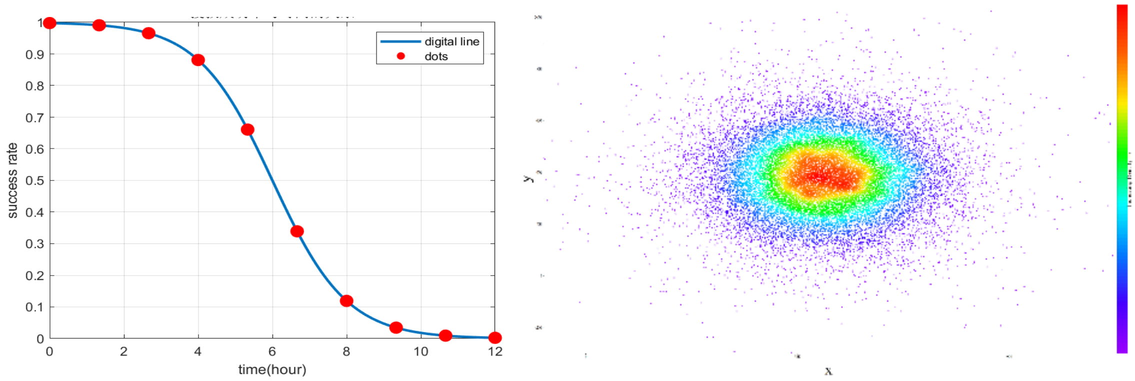

Figure 19 Survival Rate Curve and Probability Distribution

9.1.4 Result of Problem 4

According to the calculation results of model 1, in the middle of the Caribbean Sea, the movement trajectory and uncertainty of the submersibles are distributed as following:

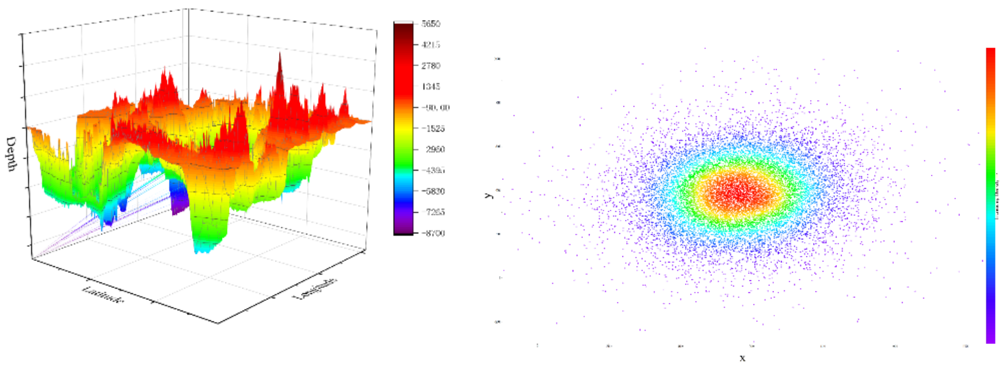

Figure 20

Based on the calculation results of Model 3, assuming that the number of the lost submersibles is 3, the deployment and search mode of the initial points are as follows:

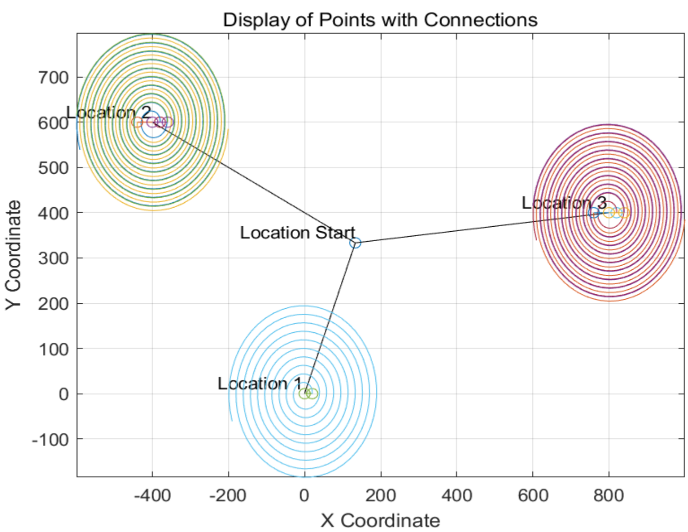

Figure 21 Multiple Submersibles

9.2 Strengths

- Model 1 comprehensively considers the influence of ocean current, seawater density, seabed geographical environment and other factors on the movement of the submersible, thus obtaining more accurate prediction results. The Monte Carlo method simulates a large combination of stochastic events and parameters, making the prediction results more comprehensive.
- The normalization method and AHP hierarchy analysis in Model 2 can quantify and compare subjective sub-items. This helps the company to determine the optimal carrying equipment and find the most economical and effective search and rescue strategy.
- The Archimedes spiral in model 3 is intuitive and easy to understand. It also fits into the case of the uncertainty probability distribution and can be directly applied to boat path planning. The ARIMA prediction method can provide the prediction of the success rate and further optimize the route selection.

9.3 Possible Improvements

- Model 1 can reduce the computational complexity and improve the computational efficiency through parallel computing. The model parameters can also be dynamically adjusted to better adapt to the actual environment and technical changes.
- Model 2 can be used to reduce subjective effects by introducing more non-stakeholder opinions and trade-offs.
- Model 3 can increase the applicability and flexibility of the model by introducing more variables, combining artificial intelligence algorithms or optimization algorithms.

References

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Appendices

The code developed for all modeling and computational aspects in this article is available on our GitHub repository. The repository contains detailed information on each code, including its functionality and corresponding results. You can access the codes and their outcomes through this repository.

MEMO
TO: Maritime Cruises Mini-Submarines (MCMS)
FROM: Team #2428217
DATE: February 5, 2024
SUBJECT: Suggestions for the Future of MCMS

Dear Sir or Madame,

To assist in the development of the submersibles your company is constructing, we have established three customized models aimed at developing safety protocols and obtaining regulatory approval.

Over the past four days, we have compiled our key results and recommendations, which we are eager to share with you.

Results

Our team conducted a series of simulations and analyses that led to the creation of several predictive and evaluative models:

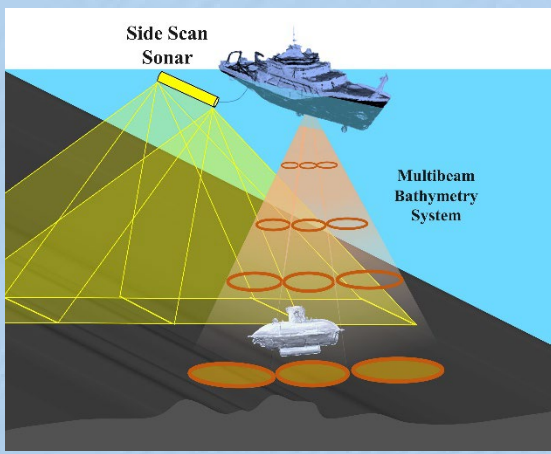
HydroMonte Fusion Predictor: This model forecasts the location of a submersible with significant accuracy. It integrates seamlessly into the submersible's safety procedures, offering real-time predictive capabilities to enhance operational security.

Cost-Benefit Analysis based on Normalization and AHP Hierarchy (CBA-N&AHP): This model assesses the cost-effectiveness of different equipment options. By applying this model, you can make informed decisions that balance regulatory approval and financial viability, preventing the risk of monetary loss.

Archimedean Spiral-Integrated ARIMA Forecasting Cumulative Distribution Framework (ASIA-CDF): Designed for planning the search vessel's path, this model enhances the probability of a successful rescue mission by optimizing early self-rescue efforts in line with first principles.

Conclutions

For the Submersible: - Location and Orientation: - Utilize GPS and a Sonardyne Homer Pro locator beacon for horizontal coordinates. - Employ an iXblue fiber optic gyroscope Octans for attitude checks. - Environmental Measurements: - An electrical conductivity, temperature, and depth (pressure) sensor for salinity, temperature, and depth. - Assistance for Search Vessel: - Sonardyne Ultra-Short Baseline (USBL) system and Auxiliary Long Baseline (LBL) for locating the dive bay. - Measuring Dive Depth: - A depth sensor and an altimeter for accurate vertical positioning.



For the Mother Ship: - Essential search equipment includes: - Multibeam Bathymetry System, AUV, ROV, Surface Search UAV, and FADOSS, ranked by priority based on their efficiency and utility in search operations.

For the Rescue Vessels: - Should be equipped with Side Scan Sonar to enhance their effectiveness during search and rescue missions.

Deployment During Loss of Communication: - A detailed strategy involving the deployment of specified equipment based on real-time assessments and the predictive models will optimize search operations.

Recommendations

To facilitate regulatory approval and ensure the safety of the submersibles, we recommend a comprehensive presentation of the model outcomes, supplemented by visual aids and datasets. These illustrations should highlight the effectiveness, efficiency, and necessity of the recommended safety procedures and equipment.

By integrating these sophisticated models into your safety protocols and decision-making processes, we are confident that your submersibles will not only meet regulatory standards but also set new benchmarks in submersible safety and operational efficiency.

We look forward to discussing these findings in further detail and assisting with any additional needs or clarifications.

Sincerely,

Team #2428217

Report on Use of AI

1. OpenAI ChatGPT (Nov 5,2023 version, ChatGPT-4)
   This article has been polished and translated by ChatGPT-4.