SUPERSYMMETRIC EXTENSION OF THE SU(3) × SU(2) × U(1) MODEL

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We describe a class of supersymmetric SU(3) × SU(2) × U(1) models where all quarks and leptons, as well as their scalar partners, get masses through one-loop radiative corrections.

There have been many attempts to formulate a supersymmetric extension of the standard SU(3) × SU(2) × U(1) model [1]. To get an acceptable theory supersymmetry must be broken in order to lift the mass degeneracy between the fermions and their bosonic partners. Usually, however, one obtains a mass sum rule at the tree level which is phenomenologically untenable. This sum rule is preserved by radiative corrections, to any order in the coupling constant, as long as these corrections are only first order in the supersymmetry breaking parameter [2]. A possible solution to this problem is to generate all mass splittings radiatively through graphs that are second order in the supersymmetry breaking parameter. The mass sum rule is no longer expected to hold and we will show, by explicit calculation, that it does not.

In this context, the strategy we pursue is to let all quarks, leptons, and their scalar partners be massless at the tree level. Their masses then arise through radiative corrections at the one-loop level. We will show that all scalar masses are positive. Since more symmetries must be broken in order to obtain nonvanishing fermion masses, these are naturally suppressed relative to the masses of their scalar partners. Masses for the fermionic partners of the massless gauge bosons also arise through radiative effects.

Following the approach of ref. [3], we choose our model to be of the O'Raifeartaigh type [4] in which supersymmetry is spontaneously broken in the F-terms. The superpotential is given by

$$\begin{aligned} W = {} & m_1 \bar{G}E + m_2 \bar{F}H + X(\gamma_1 \bar{G}H + \gamma_2 \bar{S}T - M^2) \\ & + m_3 \bar{S}U + m_4 \bar{V}T + k_1 A\bar{S}H \\ & + \lambda_{ij}^U \bar{U}_i H Q_j + \lambda_{ij}^D \bar{D}_i \bar{G} Q_j + \lambda_{ij}^L \bar{E}_i \bar{G} L_j \, . \end{aligned} \tag{1}$$

The gauge group is SU(3) × SU(2) × U(1). Quarks and leptons have their usual gauge quantum numbers. Quark doublet chiral superfields are denoted by Q_i (i is the generation index) and the singlets by $\bar{U}_i$, $\bar{D}_i$. Lepton doublets are denoted by L_i and the singlets by $\bar{E}_i$. In addition we introduce ten Higgs chiral superfields, each of which is a color singlet. Two fields, X and A, are SU(2) singlets. The remaining fields E, H, T, U are doublets and $\bar{F}, \bar{G}, \bar{S}, \bar{V}$ are doublets of opposite hypercharge. We have suppressed SU(3) and SU(2) indices. We further require that the above superfields be acted on by a U(1) group S (sector number) and, independently, by an R-transformation [5]. Invariance of the action under S groups. E, H, $\bar{F}$ and $\bar{G}$ together (sector 1), T, U, $\bar{S}$, and $\bar{V}$ together (sector 2), and allows Yukawa couplings to sector 1 fields only. Invariance under R puts each sector into the form of an O'Raifeartaigh model and allows Yukawa couplings to fields H and $\bar{G}$ only. Apart from the mixing through the singlet field X the two sectors communicate through the $AS̄H$ term. The quantum numbers for all superfields are given in table 1. The theory is clearly anomaly free. The above superpotential is the most general one consistent with these symmetries. We have also assumed lepton number conservation. By adjusting the phases of the superfields

Table 1
Quantum numbers of the superfields.

	SU(3)	SU(2)	U(1)	R	S
H	1	2	1	0	1
E	1	2	1	1	1
$\bar{F}$	1	$\bar{2}$	−1	1	−1
$\bar{G}$	1	$\bar{2}$	−1	0	−1
X	1	1	0	1	0
T	1	2	1	0	0
U	1	2	1	1	0
$\bar{S}$	1	$\bar{2}$	−1	0	0
$\bar{V}$	1	$\bar{2}$	−1	1	0
A	1	1	0	1	−1
$\bar{E}_i$	1	1	2	0	0
L_i	1	2	−1	1	1
$\bar{U}_i$	$\bar{3}$	1	−4/3	0	−2
$\bar{D}_i$	$\bar{3}$	1	2/3	0	0
Q_i	3	2	1/3	1	1

we can take $m_1, m_2, m_3, m_4, M, \gamma_1, \gamma_2$ and k_1 to be real and positive. Redefinition of the quark and lepton superfields allows us to take matrices λ^U_{ij} and λ^L_{ij} real and diagonal. Note that quark and lepton superfields couple directly to the Higgs superfields of sector 1. This will allow us to give quarks and leptons and their scalar partners non-zero radiative masses at the one-loop level.

Since there is no symmetry to forbid it, a Fayet–Illiopoulos D-term for the hypercharge group can appear in our model. We will asume that it is zero or negligibly small. This assumption could be justified if our theory were to arise from the spontaneous breakdown of a semisimple gauge group at very high energy [6].

For m_1, m_2 sufficiently large compared to m_3, m_4 and for $M^2 \geqslant m_3 m_4/\gamma_2$, the tree level scalar potential has a global minimum at

$$\langle \bar{S} \rangle = (m_4/m_3)^{1/2} v \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle T \rangle = (m_3/m_4)^{1/2} v \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$\langle U \rangle = \langle \bar{V} \rangle = -\gamma_2 (m_3 m_4)^{-1/2} \langle X \rangle v \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad (2)$$

where $v = (M^2/\gamma_2 - m_3 m_4/\gamma_2^2)^{1/2}$. These vacuum expectation values (vev) break SU(2) × U(1) down to $\mathrm{U(1)_{em}}$ and v sets the scale for this breaking. Moreover,

$$\langle A \rangle = (m_2/k_1)\,[(m_3^2 - m_4^2)/m_4^2]\,,$$

$$\langle \bar{F} \rangle = -(k_1/m_2) \langle A \rangle \langle \bar{S} \rangle\,. \qquad (3)$$

The vev $\langle A \rangle$ spontaneously breaks a linear combination of S and R-invariance. All other vev's vanish except for $\langle X \rangle$ which is undetermined at the tree level. The value of the potential at this minimum is $m_3 m_4 \times (2v^2 + m_3 m_4/\gamma_2^2)$. Hence, supersymmetry is spontaneously broken and the scale of this breaking is set by $m_3 m_4$.

To determine $\langle X \rangle$ we evaluate the one-loop correction to the effective potential given by [7].

$$\Delta V(\langle X \rangle) = \sum_i \frac{(-1)^F}{64\pi^2} M_i(\langle X \rangle)^4 \ln [M_i(\langle X \rangle)^2/\mu^2]\,. \qquad (4)$$

The vev that minimizes ΔV fixes $\langle X \rangle$. Since we must calculate ΔV for arbitrary values of $\langle X \rangle$ (not just very small or large values) and due to the complexity of the matrices, this function is most easily evaluated numerically. We find that for $m_3 m_4 > \mathrm{O}(v^2)$ the potential ΔV is dominated by scalar interactions and is a monotonically increasing function of $\langle X \rangle$. Hence $\langle X \rangle = 0$. For $m_3 m_4 \lesssim \mathrm{O}(v^2)$ the gauge interactions become predominant for small values of $\langle X \rangle$ and a stable minimum develops away from the origin. A numerical calculation of ΔV for typical parameters and different values of $m_3 m_4$ is shown in fig. 1. Note that a smooth, second-order phase transition takes place. For the smallest value of $m_3 m_4$ in fig. 1 the effective potential has its absolute minimum at $\langle X \rangle \approx 20$ GeV. Notice that this value of $\langle X \rangle$ is smaller than the scale of SU(2) × U(1) symmetry breaking. This calculation

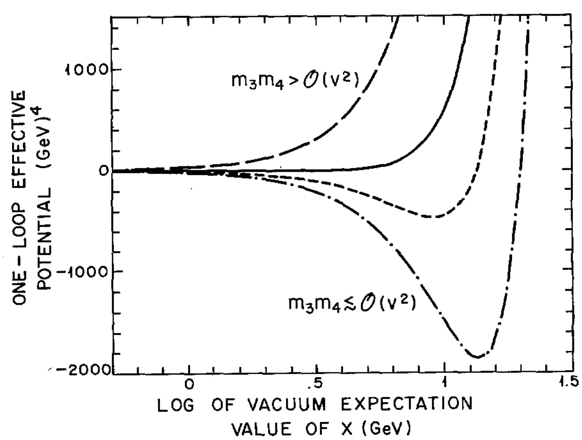

Fig. 1. A plot of the one-loop effective potential ΔV against $\langle X \rangle$ for different values of $m_3 m_4$. We have taken $m_1 = m_2 = 360$ GeV, $v = 250$ GeV, and $\lambda^2/4\pi \approx \gamma_i^2/4\pi \lesssim \mathrm{O}(1/10)$. From the top to the bottom curve we have chosen $m_3 = m_4 = 300$, 270, 240 and 210 GeV respectively.

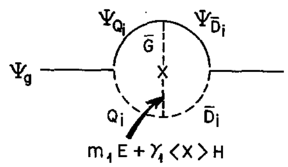

Fig. 2. Typical graph giving mass to the gluino.

does not require the use of the renormalization group, unlike [3]. We conclude that for a proper range of parameters *X develops a non-vanishing vev at the one-loop level.*

The vev $\langle X \rangle$ spontaneously breaks R-invariance. One consequence is that gluinos become massive through the two-loop diagrams of fig. 2. These masses are of order

$$m_{\text{gluino}} \propto [(\lambda^D)^2/4\pi]\, \alpha_S \langle X \rangle m_3 m_4 / m_1^2 \tag{5}$$

For the smallest value of $m_3 m_4$ in fig. 1 we find $m_{\text{gluino}} \lesssim 2$ GeV.

Since $\langle \bar{G} \rangle = \langle H \rangle = 0$ at tree level quarks, leptons and their scalar partners have vanishing tree level mass. We now analyze the radiative corrections to the scalar masses. For concreteness we focus our attention on the electron singlet scalar $\bar{E}_1$. The one-loop Feynman graphs contributing to a radiative mass for $\bar{E}_1$ are shown in figs. 3 and 4. The graphs in fig. 3 involve Yukawa coupling λ^L only. They cancel among themselves when supersymmetry is unbroken. When supersymmetry is broken the above cancellation no longer occurs and we find that graph 1a contributes to $\bar{E}_1$ a mass

$$m^2_{\bar{E},\lambda} = +[(\lambda^L)^2/16\pi^2]\,\gamma_1^2 [(m_3 m_4)^2/m_1^2]$$
$$\times [1 + O(m_3 m_4/m_1^2)]\,, \tag{6}$$

where, for simplicity, we have assumed $m_1^2 = m_2^2$. The contribution of the remaining graphs is negative, and proportional to $\langle X \rangle^2$. For a wide range of parameters, however, it turns out that $\langle X \rangle^2 \ll m_3 m_4$, so their contribution is small compared to (6). In particular, for the smallest value of $m_3 m_4$ in fig. 1 we find that $m^2_{\bar{E},\lambda} \lesssim +400\ \text{GeV}^2$. The graphs in fig. 4 involve the gauge coupling g' only. They cancel among themselves when supersymmetry is unbroken. When supersymmetry is broken they give a non-vanishing contribution to the $\bar{E}_1$ mass. However, due to the complicated mixings of the fields of section 2 with themselves and with the gauge multiplet fields, analytical evaluation of these graphs is very difficult. Instead, using expression (4), we have calculated the one-loop correction to the effective potential in the $\langle \bar{E}_1 \rangle$ direction, from which the mass can be extracted. The results are that, depending on the parameters, $m^2_{\bar{E},g}$, can be positive or negative. In particular, for $m_1 \approx m_2$ and $m_3 \approx m_4$, $m^2_{\bar{E},g}$, is *positive* and quadratic

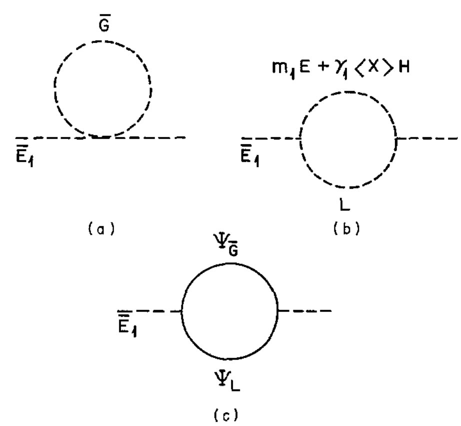

Fig. 3. Graphs, involving the Yukawa interaction only, which give mass to $\bar{E}_1$.

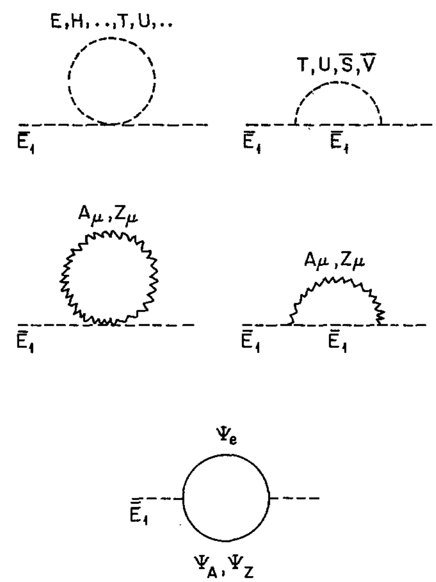

Fig. 4. Graphs, involving gauge interactions only, which contribute mass to $\bar{E}_1$.

in the hyperchange. For the smallest value of $m_3 m_4$ in fig. 1 $m^2_{\bar{E},g}$, turns out to be $\lesssim +4.0$ GeV^2. In general, for a wide range of parameters, these masses are much smaller than the positive contribution from (6).

We conclude that $\bar{E}_1$ *develops a non-vanishing mass at the one-loop level, this mass is positive, and large enough to be phenomenologically acceptable.*

Now consider the quarks and leptons. For concreteness we focus our attention on the electron. In order for the electron to get a non-vanishing radiative mass, it is necessary to break not only supersymmetry (the breaking of which suffices to give the scalar fields mass) but SU(2) × U(1) and S as well (the symmetries that would be broken by a non-zero vev $\langle \bar{G} \rangle$). The combination of vacuum expectation values which breaks all the appropriate symmetries is $\langle \bar{S} \rangle \langle A \rangle \langle X \rangle^\dagger$. In our model $\langle \bar{S} \rangle \langle A \rangle \langle X \rangle^\dagger$ vanishes when supersymmetry is unbroken. Note that this combination may vanish even when supersymmetry is broken [for example, when $m_3 m_4 > O(v^2)$ and, hence $\langle X \rangle = 0$]. In R_ξ-gauge, the graphs contributing to the electron mass to the one-loop level (and first order in $\langle \bar{S} \rangle \langle A \rangle \langle X \rangle^\dagger$) are shown in fig. 5. The vev's in graph 5a have their tree level values. However, the vev $\langle \bar{G} \rangle$ in graph 5b must be evaluated to the one-loop level. This is done by demanding that the vev's be such that all scalar field 1PI one-point functions, calculated to the one-loop level, vanish. For example, the graphs contributing to the $\bar{G}$ 1PI one-point function are shown in fig. 6. We find that the electron mass is given by

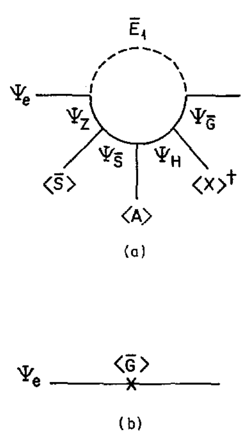

Fig. 5. Graphs giving mass to the electron.

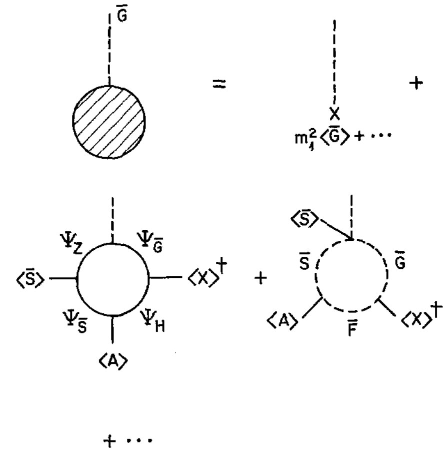

Fig. 6. Graphs contributing to the $\bar{G}$ 1PI, one-point function.

$$m_{\text{elec}} \propto (\lambda^L/4\pi)\alpha_{\text{weak}} k_1 \gamma_1 \langle \bar{S} \rangle \langle A \rangle \langle X \rangle^\dagger / m_1^2 \,. \qquad (7)$$

This mass, without any fine tuning of the parameters and with coupling constants as given in fig. 1, can easily be as small as 0.5 MeV. We conclude that, *in the range of parameters for which* $\langle \bar{S} \rangle \langle A \rangle \langle X \rangle^\dagger \neq 0$, *the electron develops a non-vanishing mass at the one-loop level.* Note that the electron mass is smaller than all the associated scalar masses and, hence, the unacceptable mass sum rule discussed earlier has been avoided.

We have shown how to give masses to a single family. However, to give masses simultaneously to all families of quarks and leptons we need to vary the λ's by a factor of 10^3–10^4. This would force us to take the supersymmetry breaking scale $(m_3 m_4)^{1/2} \gtrsim 100$ TeV, which is much larger than the scale of SU(2) × U(1) breaking. We would, therefore, no longer have spontaneous breakdown of R invariance and quarks, leptons and gluinos would be massless. One way to avoid this problem is to separate the scale of supersymmetry breaking in sector 1 from the scale of R-symmetry breaking in sector 2. To this purpose we introduce two singlet fields X_1 and X_2 with the same quantum numbers as the fields X in (1). The most general superpotential invariant under the previously described symmetries can be shown to be

$$\begin{aligned} W = {} & m_1 \bar{G} E + m_2 \bar{F} H + (\gamma_1 X_1 + \gamma_1' X_2)(\bar{G} H - M_1^2) \\ & + m_3 \bar{S} U + m_4 \bar{V} T + \gamma_2 X_2 (\bar{S} T - M_2^2) \\ & + k_1 A \bar{S} H + k_2 B \bar{G} T \,, \end{aligned} \qquad (8)$$

here we have introduced an extra singlet field B and M_1 and M_2 are now two independent masses. Quarks and leptons couple as in (1). We choose the parameters in such a way as to have the same pattern of symmetry breaking as before. For $k_1 = k_2 = \gamma_1' = 0$ the two sectors are uncorrelated. In this case, our numerical minimization of the one-loop potential shows that $\langle X_1 \rangle = 0$. In the second sector, instead, for a suitable range of parameters [namely $m_3 m_4 \lesssim O(v^2)$], $\langle X_2 \rangle$ is different than zero and breaks R-invariance. The scale of R-invariance breaking is now, of course, independent of the scale of supersymmetry breaking of the first sector. This remains approximately true for small non-zero values of k_1, k_2 and γ_1'. If $\langle A \rangle$ and $\langle B \rangle$ are both non-vanishing, then graphs similar to those of fig. 5 will give masses to all quarks and leptons. The minimization of the tree level potential gives

$$(m_3/m_4)[1 + (k_2/m_1^2)\langle B \rangle^2] = (m_4/m_3)[1 + (k_1^2/m_2^2)\langle A \rangle^2] . \quad (9)$$

At least for $m_3 \neq m_4$, (9) ensures that $\langle A \rangle$ and $\langle B \rangle$ cannot both vanish. Since there is no extra symmetry in the potential if either $\langle A \rangle$ or $\langle B \rangle$ are equal to zero, neither vev is expected to vanish.

This model, therefore, would allow both large masses for all scalar partners, and the observed masses for quarks and leptons. However, this solution relies on an unnatural adjustment of the parameters and of the mass scales of the theory.

In conclusion, we have indicated above a class of models where our program of giving all quarks and leptons and their scalar partners one-loop masses can be implemented. These models have several phenomenological difficulties; spontaneous breaking of R-invariance and of sector-number symmetry leave us with an axion and a true Goldstone boson; strangeness changing neutral currents are not sufficiently suppressed. An extension of the above ideas, in order to solve these phenomenological problems, is now under consideration.

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References

[1] P. Fayet and J. Iliopoulos, Phys. Lett. 31B (1974) 461;
P. Fayet, Phys. Lett. 69B (1977) 489; 70B (1977) 461;
S. Weinberg, HUTP-81/A047 (1981).
[2] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403;
L. Girardello and J. Iliopoulos, Phys. Lett. 88B (1979) 85.
[3] E. Witten, Phys. Lett. 105B (1981) 267.
[4] L. O'Raifeartaigh, Nucl. Phys. B96 (1975) 331.
[5] A. Salam and J. Strathdee, Nucl. Phys. B87 (1975) 85.
[6] E. Witten, Nucl. Phys. B185 (1981) 513.
[7] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 788;
S. Weinberg, Phys. Rev. D7 (1973) 2887.