

Luminosity Scans at the LHC

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Abstract

INTRODUCTION

For particle colliders, the most important performance parameters are the beam energy and the luminosity. High energies allow the particle physics experiments to study and observe new effects and the luminosity is used as a measure of the number of collisions. It is defined as the proportionality factor between the event rate, measured by the experiments, and the cross section of the process observed.

The Large Hadron Collider (LHC) is designed to produce proton proton collisions at a center of mass energy of 14 TeV. This energy will be the highest ever reached in a particle accelerator. The knowledge and understanding of particle physics at such high energy is based on simulations and theoretical predictions. As opposed to e^+e^- colliders, for which the Bhabba scattering cross section can be accurately calculated and used for luminosity calibration, there are no processes with well known cross sections and sufficiently high production rate to be directly used for the purpose of luminosity calibration in the early operation of the LHC.

The luminosity for colliding beams can be directly obtained from geometry and numbers of particles flowing per time unit, as pioneered by S. Van Der Meer at the ISR [1]. For the LHC, it was proposed to use this method to provide a first luminosity calibration based on machine parameters for the physics experiments [2, 3].

Later, dedicated operation of the LHC using special high- β^* optics should allow to independently obtain an accurate cross section and luminosity calibration close to the 1% level, by measurements of the very forward proton proton scattering with the TOTEM and ATLAS experiments [4, 5].

THE VAN DER MEER METHOD

We consider two bunches of N_1 and N_2 particles colliding in an interaction region as shown in Figure 1. For bunches crossing head-on at a frequency f (revolution frequency in the case of a circular collider) the luminosity is expressed as:

$$\mathcal{L}_0 = \frac{N_1 N_2 f}{A_{\text{eff}}}, \quad (1)$$

where A_{eff} is the effective transverse area in which the collisions take place. The revolution frequency in a collider is accurately known and the number of particles or

beam intensity is continuously measured with beam current transformers which should reach an accuracy of 1 % for LHC nominal beam parameters [6]. The only unknown parameter that needs to be measured is the effective transverse area which depends on the density distribution ρ_1 and ρ_2 of the two beams.

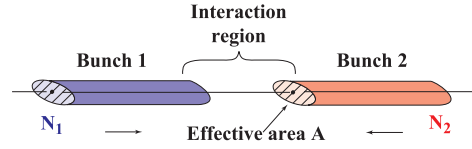


Figure 1: Luminosity from particles flux and geometry.

It was shown by S. Van Der Meer in [1], that if the density distributions in the horizontal and the vertical plane are uncorrelated and stable, the effective transverse beam size can be measured by performing scans in separation and integrating the resulting curve of the interaction rate versus the separation δu (where u stands for x or y). Independently of the beam shape, the effective area is then given by:

$$A_{\text{eff}} = \frac{\int R_x(\delta x) d\delta x}{R_x(0)} \frac{\int R_y(\delta y) d\delta y}{R_y(0)}, \quad (2)$$

where $R(\delta x, \delta y) = R_x(\delta x) R_y(\delta y)$ if the horizontal and vertical density distributions are uncorrelated, describes the evolution of the interaction rate as a function of the transverse offsets δx and δy measured during the separation scans.

For Gaussian distributions, the luminosity \mathcal{L} as a function of the transverse offsets is also a Gaussian

$$\mathcal{L} = \mathcal{L}_0 \exp \left[-\frac{\delta x^2}{2(\sigma_{1x}^2 + \sigma_{2x}^2)} - \frac{\delta y^2}{2(\sigma_{1y}^2 + \sigma_{2y}^2)} \right], \quad (3)$$

where σ_{1u} and σ_{2u} are the individual r.m.s. beam widths. Applying Equation 2 to compute the effective area we get:

$$A_{\text{eff}} = 2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}, \quad (4)$$

and

$$\mathcal{L}_0 = \frac{N_1 N_2 f N_b}{2\pi \sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}, \quad (5)$$

which is the standard formulae of the luminosity for elliptical beams colliding head-on [7]. To be noted that the

functions R_u reach a maximum for zero separation. Separation scans can therefore also be used to optimize the collision rate.

For completeness other potential systematic effects in the absolute luminosity determination from beam parameters are discussed, and it will be shown that they are very small for the relevant LHC beam conditions.

CROSSING ANGLE

The LHC beams share the same beam chamber from the interaction point up to the first separation dipole 60 m from the IP which deflects them into two separate rings. For high luminosity operation, the LHC is filled with many bunches and a crossing angle introduced to avoid extra collisions in the common beam pipe section.

A crossing angle between the two beams ϕ_u in one plane $u = x, y$, decreases the luminosity as follows:

$$\mathcal{L} = \mathcal{L}_0 \cdot S_u = \mathcal{L}_0 \frac{1}{\sqrt{1 + \frac{\sigma_{1s}^2 + \sigma_{2s}^2}{\sigma_{1u}^2 + \sigma_{2u}^2} \left(\tan \frac{\phi_u}{2} \right)^2}}. \quad (6)$$

where \mathcal{L}_0 is the luminosity without crossing angle defined in Equation 5. $\sigma_{1,2s}$ are the bunch lengths of the beams. ϕ_u is the full crossing angle between the beams. The decrease in luminosity is equivalent to an increase of the effective beam size in the crossing plane.

The extension of Eq. 6 to describe the luminosity reduction by the combination of vertical and horizontal crossing angles is:

$$\mathcal{L} = \mathcal{L}_0 \cdot S \quad (7)$$

where

$$S = \frac{1}{\sqrt{1 + \frac{\sigma_{1s}^2 + \sigma_{2s}^2}{\sigma_{1y}^2 + \sigma_{2y}^2} \left(\tan \frac{\phi_y}{2} \right)^2 + \frac{\sigma_{1s}^2 + \sigma_{2s}^2}{\sigma_{1x}^2 + \sigma_{2x}^2} \left(\tan \frac{\phi_x}{2} \right)^2}}, \quad (8)$$

where ϕ_x and ϕ_y are the projections of the crossing angle in the transverse planes.

The luminosity in the presence of transverse offsets and crossing angles can be factorized as follows:

$$\mathcal{L} = \mathcal{L}_0 \cdot S \cdot T \cdot U, \quad (9)$$

where S is the reduction factor from the crossing angles at zero separation given in Eq. 8, T the reduction factor from transverse offsets in the luminosity scans from Eq. 3, and U a cross term that can be written as:

$$U = \exp \left[S^2 \frac{\sigma_{1s}^2 + \sigma_{2s}^2}{2} \left(\frac{\delta x \tan \frac{\phi_x}{2}}{\sigma_{1x}^2 + \sigma_{2x}^2} - \frac{\delta y \tan \frac{\phi_y}{2}}{\sigma_{1y}^2 + \sigma_{2y}^2} \right)^2 \right]. \quad (10)$$

Applying Equation 2 to compute the effective area we get:

$$A_{\text{eff}} = 2\pi \frac{\sqrt{\sigma_{1x}^2 + \sigma_{2x}^2} \sqrt{\sigma_{1y}^2 + \sigma_{2y}^2}}{S} \frac{S_x S_y}{S} \quad (11)$$

where

$$S_u = \frac{1}{\sqrt{1 + \frac{\sigma_{1s}^2 + \sigma_{2s}^2}{\sigma_{1u}^2 + \sigma_{2u}^2} \left(\tan \frac{\phi}{2} \right)^2}}. \quad (12)$$

Compared to Eq. 7 we have the extra factor

$$X_{\text{corr}} = \frac{S_x S_y}{S}. \quad (13)$$

For scans performed exactly in the crossing plane ($\phi_x = 0$, $S_x = 1$ and $S = S_y$ or $\phi_y = 0$, $S_y = 1$ and $S = S_x$), the Van Der Meer scan method directly measures the correct effective beam-size including the effect of the crossing angle ($X_{\text{corr}} = 1$).

This remains approximately true also in case of a crossing angle in both planes. The correction factor X_{corr} is shown in Figure 2 for the 2010 LHC beam parameters and the nominal 7 TeV beam parameters and remains very close to 1 in all practical cases for the LHC.

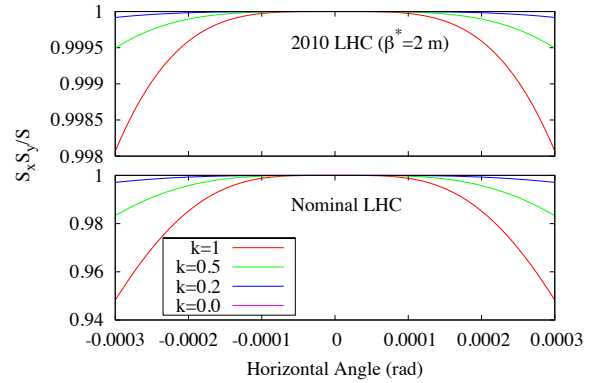


Figure 2: Ratio $S_x S_y / S$ for 7 TeV nominal LHC (bottom) and the 2010 LHC parameters (top) with ($E = 3.5$ TeV, $\beta^* = 2$ m). k represents the ratio between the projected angles, in this case ϕ_y / ϕ_x . The error becomes significant for the nominal LHC beam parameters when the angles are of equivalent amplitudes of several $100 \mu\text{rad}$, which represents a very unlikely situation for the LHC.

HOURLASS EFFECT

The β -function in a drift space varies with the distance to the minimum like:

$$\beta(s) = \beta^* \left(1 + \frac{s^2}{\beta^{*2}} \right), \quad (14)$$

and therefore the beam size $\sigma = \sqrt{\beta(s)} \varepsilon$

$$\sigma(s) = \sigma^* \sqrt{1 + \frac{s^2}{\beta^{*2}}}. \quad (15)$$

The β -function around the interaction point has the shape of a parabola. When the ratio β^* / σ_s , where σ_s is

the rms bunch length becomes ≤ 1 , a correction factor is required. The factor can be calculated according to [8]

$$H(t) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{e^{-t^2}}{\sqrt{(1+t^2/t_x^2)(1+t^2/t_y^2)}} dt, \quad (16)$$

where

$$t_u^2 = \frac{2(\sigma_{1u}^{*2} + \sigma_{2u}^{*2})}{(\sigma_{1s}^2 + \sigma_{2s}^2)(\sigma_{1u}^{*2}/\beta_{1u}^{*2} + \sigma_{2u}^{*2}/\beta_{2u}^{*2})}, \quad (17)$$

where $u = x, y$. For round beams we have

$$t_x^2 = t_y^2 = t_r^2 = \frac{2\beta^{*2}}{\sigma_{1s}^2 + \sigma_{2s}^2}, \quad (18)$$

and

$$H(t_r) = \sqrt{\pi} t_r e^{t_r^2} \operatorname{erfc}(t_r), \quad (19)$$

Table 1: Hourglass luminosity reduction factor as function of β^* calculated with $\sigma_s = 0.0755$ m for round beams.

β^*	t_r	$H(t_r)$
2.0	26.5	0.9992
1.0	13.2	0.9972
0.55	7.3	0.9908

Table 1 shows, that the luminosity reduction due to the hourglass effect reaches a level of 1 % for the nominal $\beta^* = 0.55$ m and becomes negligible for larger values like $\beta^* = 2$ m as relevant for the first years of LHC operation. The effect is more significant in the case of the RHIC collider. A description of the analysis of separation scans in presence of a non-negligible hourglass effect can be found in [9, 10].

BEAM-BEAM EFFECTS

Beam-beam effects are relevant at high intensities. Ideally all the calibration scans should be performed at low intensity to minimize this effect. However, the LHC will be running at high intensities and it could be interesting to perform scans in these beam conditions. In this case, the beam-beam force will introduce a non-linear behavior as a function of the separation which can affect the orbit or emittance and will couple the transverse distributions. The emittance growth due to the beam-beam interactions in the presence of transverse offsets was estimated in [11] and proved to be very small. The beam-beam effect will perturb the lattice as a function of the separation resulting in a dynamic change of the tunes and β -functions [13] and therefore beam size. The β -function relates to a tune shift ΔQ as follows:

$$\frac{\beta^*}{\beta_0^*} = \frac{\sin(2\pi Q)}{\sin(2\pi Q + 2\pi \Delta Q)}, \quad (20)$$

where Q is the unperturbed tune of the machine and β^* and β_0^* are the perturbed and unperturbed β -functions at the IP. The tune shift introduced by the head-on beam-beam interaction is largest for the central particles and reaches a value (for round beams) of

$$\xi = \frac{N r_0}{4\pi \varepsilon_N}, \quad (21)$$

which is referred to as the beam-beam parameter; r_0 is the classical particle radius and ε_N is the normalized emittance. The variation of β^* as a function of the separation was estimated to be of the order of 1 % in the case of nominal LHC beam parameters.

Table 2: Linear beam-beam parameter without crossing angle and β^* maximum variations for nominal LHC tunes (64.31, 59.32).

N [p/bunch]	ξ	$\Delta\beta/\beta$
5×10^9	0.0002	0.0005
4×10^{10}	0.0013	0.0040
1.15×10^{11}	0.0037	0.0097

Table 2 shows the amplitude of the β^* variations as a function of intensity for the nominal LHC tunes. We see that this effect can be reduced to a negligible level by reducing the bunch intensity. For a finite beam separation, the beam-beam interaction will also result in a change of the closed orbit. As illustrated in Figure 3, the orbit offset introduced by collisions at finite separation at the interaction points of the LHC is also very small. When the LHC is filled with many closely spaced bunches, we have to add to this the long-range beam-beam kicks from several bunch passages left and right of each IP and get effects of up to 0.3σ for the nominal LHC filling scheme. Luminosity calibration measurements in the LHC are best performed with a limited number of bunches (≤ 156) to avoid any complication by long range beam-beam interactions.

LINEAR COUPLING

By design, the betatron oscillation in the transverse x, y planes of the LHC are fully decoupled. In practice, there will be a small residual coupling which corresponds to a tilt of the beam ellipse by an angle ϕ . In addition, this effect can be slightly different for both beams present in the LHC. As shown in [14], the luminosity in the presence of asymmetric coupling can be written as:

$$\mathcal{L} = \mathcal{L}_0 \frac{1}{\sqrt{1 + \frac{(\sigma_{1\xi}^2 - \sigma_{1\eta}^2)(\sigma_{2\xi}^2 - \sigma_{2\eta}^2)}{(\sigma_{1\xi}^2 + \sigma_{2\xi}^2)(\sigma_{1\eta}^2 + \sigma_{2\eta}^2)} \sin^2(\phi_2 - \phi_1)}}. \quad (22)$$

where $\sigma_{i\eta}$, $\sigma_{i\xi}$ and ϕ_i ($i = 1, 2$) are the beam sizes along the ellipse main axes and the tilt angles of the two beams.

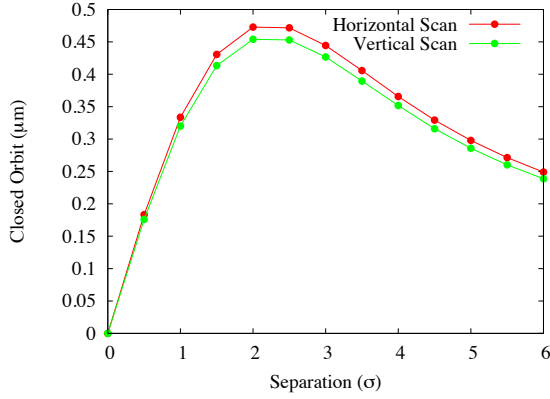


Figure 3: Self-consistent closed orbit as a function of the separation simulated with TRAIN[12] for $\xi = 0.003$ and LHC nominal tunes (64.31, 59.32).

In the case of round beams, coupling has no effect on luminosity and separation scans along the horizontal and vertical axes, and the x, y coordinates shown in Figure 4 provide the correct measurement of the effective area. In the case of elliptical beams, the coupling will result in a luminosity reduction and produce a tilt of the overlap region [14]. The effective area is then determined by the product of the effective beam sizes along the main axes of the overlap ellipse, noted as ξ and η in Figure 4. Scanning in the horizontal and vertical planes would introduce a bias. The correct result for the effective area could still be obtained using a raster scan along several parallel horizontal and vertical lines.

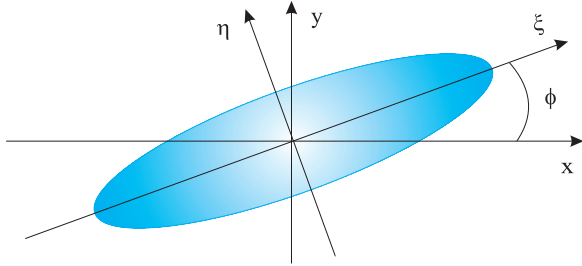


Figure 4: Tilted beam ellipse.

The bias can be computed from the C -matrix, β^* and emittance measurements [15]. An estimate of this effect is shown in Figure 5 based on optics and coupling measurements done at the LHC in 2010 at IP8 [16, 17]. For these measurements, an uncertainty of 1% is reached at a ratio of four between the horizontal and vertical emittances. In the LHC the beams are round by design. In practice, there will be some differences between the horizontal and vertical emittances but it is straightforward to keep these well below a factor two, such that the uncertainty becomes negligible.

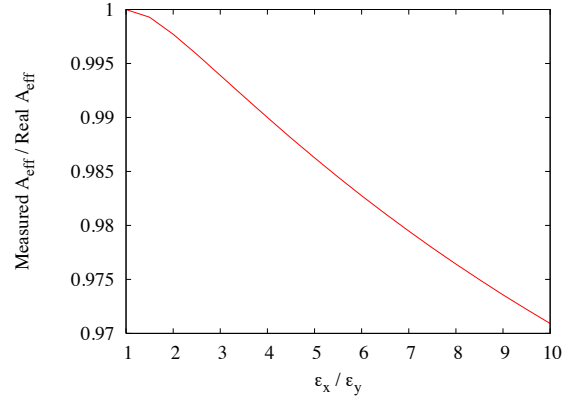


Figure 5: Effect of coupling on the measurement of the effective area derived from optics measurements performed at IP8.

OPTIMAL CONDITION FOR CALIBRATION MEASUREMENTS

Table 3: Optimal beam parameters for calibration scans. Luminosity and events per bunch crossing are calculated for an energy of 3.5 TeV.

β^* [m]	>1.0
Intensity [10^{10} p/bunch]	<4.0
Number of bunch crossing	156
Emittance [μm]	3.75
Crossing angle [μrad]	0.0
Beam-Beam Parameter	0.0013
\mathcal{L} per bunch [$10^{28}\text{cm}^{-2}\text{s}^{-1}$]	3.52
Events per bunch crossing	0.226
Event Rate [Hz]	2541

Table 3 lists a possible set of beam parameter which would minimize the non-linear effects (beam-beam, pile-up, hourglass) as well as the uncertainty on the beam current measurement, and allow to reach a statistical accuracy of the order of 1% within a few seconds. In addition it is desirable to have beams as round as possible to keep the uncertainty from linear coupling negligible. Another advantage of this proposal is that the setup time is minimized as it uses all the default LHC parameters, especially optics, and would not cost too much in terms of integrated luminosity.

MACHINE PROTECTION

The beams are displaced at the IP via a closed orbit bump that consists of four magnets and allows to control the beams independently.

One can see in Figure 6 that a four magnet separation bump extends over a large fraction of the straight section around the IP. More specifically, displacing the beams at

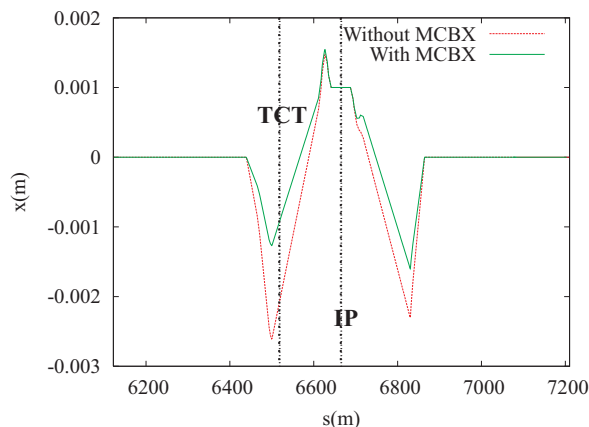


Figure 6: Example of closed orbit bumps using different orbit correctors at IP5. Displacing the beam at the IP also changes the orbit at the tertiary collimator location.

the IP will result in a change of orbit at the tertiary collimators (TCT). Given the non-negligible offset at the TCT introduced by the bumps, one has to ensure that while performing a separation scan the beams remain far enough from the aperture set by the collimators and that the displacement does not compromise the machine protection. In 2010, the displacement at the TCT was minimized by splitting the amplitude of the corrections required to find the optimum collision point between the two beams. In addition a re-qualification of the collimation system was done with the TCTs closed by 2σ with respect to their nominal settings to ensure that the margins were sufficient to safely perform the full scan.

In 2011, it was decided to move the TCTs together with the beam [18] in order to provide more flexibility and operation efficiency and ensure that the aperture margin between the dump protection and the TCTs remains large enough. It does not guarantee, that the safety margin between the TCTs and the triplet magnets is always respected. A complete assessment of the aperture reduction due the scans, especially in the crossing angle plane, should therefore be performed in order to ensure safe operation.

SUMMARY

Calibration scans were performed in the four LHC interaction points [19]. The experiments first published results and latest offline analysis can be found in [20, 21], [22], [24, 23] and [26, 25]. Two sets of measurements were performed. The first measurements, in spring 2010, were performed early in the commissioning phase of the LHC and the beam conditions and instrumentation were not optimal. The final uncertainty on the measurement was found to be of 11% from which 10% came from the preliminary determination of the beam intensity [27]. A more detailed offline analysis of the beam current data improved the overall uncertainty to about 6%. The earlier measurements also

suffered from a larger emittance blow-up due related to instrumental noise picked-up by the beam [28]. The second set of measurements was performed in October 2010. Significant progress in the calibration of the LHC instrumentation and better beams conditions and stability helped reducing the overall systematic uncertainty to about 5% for these measurements.

The precision obtained during the first year of operation of a machine as complex as the LHC represents a significant achievement and could be improved based on the arguments presented above.

Table 4: Uncertainties on the absolute luminosity for optimal beam conditions.

Beam-beam effects	negligible
Hourglass effect	negligible
Fit Model	1%
Emittance blow-up	<1%
Beam current	1-2%
Beam displacement	1-2%
Total	3-4%

Table 4 lists the known uncertainties for optimal beam parameters. The contributions from coupling and hysteresis are considered to be negligible. A total 3–4% systematic uncertainty seems to be within reach with the current equipment and procedures. This assumes perfectly stable beam conditions, low background and well working and calibrated instruments. The dominant uncertainty in these conditions originates in intensity measurement and the determination of the beam displacement.

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