Baryon Number Changing Currents*)

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An attempt is made to generalize the algebra of currents by inclusion of baryon number changing currents which connect baryons and mesons. In this algebra both baryons and mesons are grouped together in a representation, i.e. in a supermultiplet. The currents are constructed from a set of fundamental particles consisting of a spinor $SU(3)$ triplet and a scalar triplet. Most of the important hadrons are included in a supermultiplet but the baryon decimet is not. The hadrons discussed here correspond to bare hadrons without the Yukawa interactions. Mass relations, cross sections and other consequences obtained from this symmetry scheme are discussed.

§1. Introduction

The internal symmetry concept in nuclear physics was first introduced in 1932 when the proton and neutron were regarded as an isodoublet, a two component spinor of group $SU(2)$. The group was then extended to $SU(3)$ by inclusion of the lambda particle as a fundamental particle.[1] In this symmetry, both strange and non-strange particles are grouped together in a unitary multiplet. The group is further generalized to $SU(6)$[2] by inclusion of spin variables as elements of the group algebra. Particles of different spins are classified in a supermultiplet: the baryons as the 56-dimensional representation of $SU(6)$ and the mesons as the 35.

Is it possible to group together both baryons and mesons in a supermultiplet? This is possible if the algebra contains a baryon number operator and quantities which change the baryon number as its elements. The baryon number changing quantities play a role similar to the strangeness changing currents in the case of $SU(3)$. Attempts have been made in this direction, e.g., the $SU(9)$ symmetry,[3] but this scheme is only possible by ignoring the differences in the statistics of baryons and mesons.

In this paper an algebra of currents including baryon number changing currents is examined. As a guide to writing down commutation relations among currents, we introduce a scalar triplet in addition to the spinor triplet. The algebra of currents thus obtained is not a Lie algebra, so that it does not generate a unitary group $SU(9)$. The algebra is tentatively called $V(9)$ in this paper.

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Representations of $V(9)$, corresponding to particle multiplets, contain baryons and mesons. In a simple representation, called $(9, \bar{9})$ here, the baryon octet, the pseudoscalar octet, the vector octet and others are included.

In the next section, the method of constructing an algebra of currents is summarized for the sake of orientation. In §3, a discussion is given regarding the fundamental particles from which currents are constructed. There is a symmetry between isospace and ordinary space, and fundamental particles are introduced so that the scheme is consistent with this symmetry. In §4, commutation relations and anticommutation relations among these currents and representations of this algebra are investigated. In §5, we apply the method to the actual hadrons and discuss the consequences of this symmetry.

§2. Algebra, representation and multiplets

If two operators A and B commute with a Hamiltonian H, the product AB or BA also commutes with H. A set of operators commuting with H thus forms an algebra $\mathfrak{A}$. An element of $\mathfrak{A}$ operating on a state ψ gives another state ψ' with the same energy as ψ, that is, it induces a linear transformation in the vector space spanned by the degenerate eigenstates of H. The n-fold degenerate eigenstates correspond to an n-dimensional representation of the algebra $\mathfrak{A}$. Sometimes $\mathfrak{A}$ is a Lie algebra and generates a continuous transformation group. Also it may contain discrete transformations like inversion, permutation or discrete transformations in crystals.

Actually there are few cases of exact symmetries. We separate the Hamiltonian into symmetric and asymmetric terms and must consider an approximate symmetry. Matrix elements can be calculated if the representation is known.

Another method of introducing the algebra is the algebra of currents.[4] We construct currents from bilinear combinations of fundamental fields, i.e. quarks. From the particles of isospin $1/2$, u and d, we make the following combinations (which form components of isospin):

$$I_+=u^\dagger d, \quad I_-=d^\dagger u, \quad I_3=\frac{1}{2}(u^\dagger u-d^\dagger d). \tag{1}$$

To be more exact, each particle is labelled by momentum $\boldsymbol{k}$, and

$$I_+=\sum_{\boldsymbol{k}} u_{\boldsymbol{k}}^\dagger d_{\boldsymbol{k}}, \quad \text{etc.} \tag{2}$$

These are space integrals of current densities. In the following, such bilinear quantities are called currents. The isospin $\boldsymbol{I}$ satisfies the commutation relations of the generators of $SU(2)$. When third member of the fundamental field (quark), the isoscalar s, is introduced, we can make a strangeness current and strangeness changing currents which satisfy, in the standard notation, the commutation relations[5)]

$$[F_i, F_j] = i f_{ijk} F_k \tag{3}$$

and generate the $SU(3)$ group.

In order to calculate matrix elemements of F_i, we take the matrix elements of both sides of (3). If the summation over intermediate states is approximately saturated by a number of particles, say, by n particles, then the elements of algebra satisfying (3) are represented by $n \times n$ matrices.[4)] Thus a multiplet of particles which approximately saturates the sum rule corresponds to a representation of the algebra.

Each quark, being spin 1/2, has two states, spin up and down. The six particle states make 36 currents which form generators of $U(6)$. The anti-quarks, $\bar{u}\uparrow$, $\bar{u}\downarrow$, $\bar{d}\uparrow$, $\bar{d}\downarrow$, $\bar{s}\uparrow$ and $\bar{s}\downarrow$ defines another $U(6)$ and we consider $U(6) \times U(6)$. Baryons are assigned to the representation (56, 1) and mesons correspond to $(6, \bar{6})$ (static $SU(6)$ theory).

§3. Fundamental triplets

The fundamental triplet introduced by Sakata,[1)] p, n and Λ, worked very well for mesons but it did not explain the octet of baryons. The octet of baryons can be obtained by introducing the quark or ace,[6)] but we must accept fractionally charged particles and parastatistics.[7)] Although nothing is theoretically wrong for this scheme, such particles have not yet been found.

A more primitive way of explaining the octet of baryons is to introduce a scalar triplet in addition to the Sakata-like triplet. That is, besides p, n and Λ (these do not correspond to the actual proton, neutron and lambda), we introduce scaler particles having the same charge quantum numbers as K^+, K^0 and η:

$$T: \quad p,\ n,\ \Lambda,$$
$$t: \quad K^+,\ K^0,\ \eta.$$

Then the octet of baryons is given by $T\bar{t}$, and mesons by $T\bar{T}$ or $t\bar{t}$.

If the spin variables are explicitly written and fundamental particles are arranged as

$$p\uparrow,\ n\uparrow,\ \Lambda\uparrow$$
$$p\downarrow,\ n\downarrow,\ \Lambda\downarrow$$
$$K^+,\ K^0,\ \eta,$$

we notice a symmetry between isospace and ordinary space. It is invariant under the simultaneous interchange of isospin and ordinary spin and of hypercharge and baryon number. This symmetry of the hadrons,

$$\rho,\ K^*,\ \overline{K}^*,\ \phi,$$
$$\Sigma,\ N,\ \Xi,\ \Lambda,$$
$$\overline{\Sigma},\ \overline{N},\ \overline{\Xi},\ \overline{\Lambda},$$
$$\pi,\ K,\ \overline{K},\ \eta,$$

was pointed out by Lipkin, Nakanishi and others.[8] We accept this symmetry, take T and t as fundamental objects and investigate the consequences.

§4. $V(3)$ Algebra

For the moment we ignore the charge variables and consider three particles states: the spin 1/2 particle with spin up, spin down, and scalar particle. These are denoted by α, β and γ, respectively. They satisfy the following commutation relations:*)

$$\begin{aligned}&\{\alpha_k, \alpha^\dagger_{k'}\} = \{\beta_k, \beta^\dagger_{k'}\} = [\gamma_k, \gamma^\dagger_{k'}] = \delta_{kk'},\\ &[\gamma_k, \gamma_{k'}] = [\gamma^\dagger_k, \gamma^\dagger_{k'}] = 0,\\ &\{\alpha_k, \alpha_{k'}\} = \{\alpha_k, \gamma^\dagger_{k'}\} = \cdots = 0.\end{aligned} \tag{4}$$

Four currents made of α and β are three generators of the three dimensional rotation group and the baryon number:

$$\begin{aligned}&J_+ = \alpha^\dagger\beta,\ J_- = \beta^\dagger\alpha,\ J_3 = \frac{1}{2}(A-B),\ N_B = A+B,\\ &A = \alpha^\dagger\alpha, \quad B = \beta^\dagger\beta\,.\end{aligned} \tag{5}$$

With γ we can make four baryon number changing currents:

$$A_+ = \alpha^\dagger\gamma,\ A_- = \gamma^\dagger\alpha,\ B_+ = \beta^\dagger\gamma,\ B_- = \gamma^\dagger\beta, \tag{6}$$

and the scalar particle number:

$$C = \gamma^\dagger\gamma. \tag{7}$$

The baryon number changing currents satisfy commutation relations with others like

$$[A, A_+] = A_+,\ [C, B_+] = -B_+,\ [J_+, B_+] = A_+, \cdots, \tag{8}$$

but obey anticommutation relations among themselves:

$$\{A_+, B_-\} = J_+,\ \{A_+, A_-\} = A+C, \cdots. \tag{9}$$

A compact way of writing these relations is to introduce a three-component

*) γ can be made to commute or anticommute with α or β. Either case gives the same result.

quantity p and $p^\dagger$,

$$p=\begin{pmatrix}\alpha\\ \beta\\ \gamma\end{pmatrix}, \quad p^\dagger=(\alpha^\dagger, \beta^\dagger, \gamma^\dagger), \tag{10}$$

and write the ordinary currents (baryon number conserving currents) by F_i, and baryon number changing currents by G_i:

$$\begin{aligned}(p^\dagger\lambda_i p)&=F_i, \quad i=0, 1, 2, 3, 8,\\ &=G_i, \quad i=4, \cdots, 7,\end{aligned} \tag{11}$$

where λ_i is the conventional 3×3 hermitian matrix with λ_0 proportional to unity.[5] The commutation relation can be written as

$$\begin{aligned}[][F_i, F_j]&=if_{ijk}F_k,\\ [F_i, G_j]&=if_{ijk}G_k,\\ \{G_i, G_j\}&=d_{ijk}F_k.\end{aligned} \tag{12}$$

d_{ijk} is the symmetric tensor of $SU(3)$. This is not a Lie algebra (e.g., since the anticommutator does not satisfy Jacobi's identity). The relations (12) will define some algebra (or a kind of hypercomplex system), which we call $V(3)$.

This $V(3)$ has a trivial three-dimensional representation

$$F_i, G_i=\lambda_i. \tag{13}$$

General tensor representations can be obtained as follows. The vector space of states where the n particles are present defines a $3^n\times 3^n$ representation space (since currents do not change the number of particles). This is not irreducible and is decomposed according to symmetry properties. For instance, the 9-dimensional representation ($n=2$) decomposes into a quartet and a quintet:

$$3\times 3=4+5.$$

Similarly we define the currents of antiparticles $\bar{p}$: $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, by

$$\begin{aligned}-(\bar{p}^\dagger\lambda_i^T\bar{p})&=\bar{F}_i, \quad i=0, 1, 2, 3, 8,\\ &=\bar{G}_i, \quad i=4, \cdots, 7,\end{aligned} \tag{14}$$

where λ^T is the transpose of λ. These satisfy commutation relations similar to (12) except with a change of sign:

$$\begin{aligned}[][\bar{F}_i, \bar{F}_j]&=if_{ijk}\bar{F}_k,\\ [\bar{F}_i, \bar{G}_j]&=if_{ijk}\bar{F}_k,\\ \{\bar{G}_i, \bar{G}_j\}&=-d_{ijk}\bar{F}_k.\end{aligned} \tag{15}$$

This algebra is called $\overline{V}(3)$. It has a trivial three-dimensional representation

$$\overline{F}_i,\ \overline{G}_i=-\lambda_i^T, \tag{16}$$

which may be called $\overline{3}$.

Taking both, we consider $V(3)\times\overline{V}(3)$. $\overline{V}(3)$ and $V(3)$ are not quite independent of each other in the sense that baryon number changing currents G_i and $\overline{G}_i$ anticommute rather than commute, but this is a matter of sign and makes no difficulty in constructing representations. $V(3)$, $\overline{V}(3)$ and $V(3)\times\overline{V}(3)$ contain $SU(2)$ (rotation group) as a subalgebra. The fundamental representation $(3,\overline{3})$ contains, in an $SU(2)$ decomposition,

$$\begin{aligned}(3,\overline{3})&=3+2+2+1+1'\\ &=\text{spin}\,1+\text{spin}\,1/2+\text{spin}\,1/2+\text{spin}\,0+\text{spin}\,0'\\ \text{or}&=V+Sp+\overline{S}p+P+S.\end{aligned} \tag{17}$$

That is, $(3,\overline{3})$ is made of a vector, a spinor (spin 1/2 particle), its antiparicle, a pseudoscalar and a scalar.

§5. $V(9)$ symmetry

Now we include the unitary spin. The fundamental particles are $p_{a\sigma}$ having a space index a and a unitary spin index σ, each running from 1 to 3. Of 81 currents

$$C_{a\sigma}^{b\rho}=p_{b\rho}^{\dagger}p_{a\sigma}, \tag{18}$$

some have fermion character (baryon number changing currents) and obey the anticommutation relations among themselves. Others are bose-like (baryon number conserving currents) and obey the commutation relations.

$$[C_{a\sigma}^{b\rho},C_{c\pi}^{d\tau}]_{\pm}=\delta_{ad}\delta_{\sigma t}C_{c\pi}^{b\rho}\pm\delta_{bc}\delta_{\rho\pi}C_{a\sigma}^{d\tau}. \tag{19}$$

Antiparticles form $\overline{V}(9)$ and the final algebra is $V(9)\times V(\overline{9})$. As a fundamental representation we consider $(9,\overline{9})$. The decomposition of this according to the chain of subalgebras

$$V(9)\times\overline{V}(9)\supset(V(3)\times\overline{V}(3))\times SU(3)\supset SU(2)\times SU(3),$$

is

$$\begin{aligned}(9,\overline{9})&=(9,8)+(9,1)\\ &=(V_8,B_8,\overline{B}_8,P_8,S_8)+(V_1,B_1,\overline{B}_1,P_1,S_1),\end{aligned} \tag{20}$$

where V_1 means a vector meson of unitary singlet, etc. V_8, B_8, $\overline{B}_8$ and P_8 can be identified with the vector meson octet, the baryon octet, the antibaryon octet and the pseudoscalar octet respectively. V_1 and P_1 can be identified with

ω and $X_0(960)$, respectively. B_1 could be $Y_0^*(1405)$ if it has parity $(1/2)^+$. S_8 and S_1 do not correspond to any known particles: We shall see that they are much heavier than the usual resonances.

The baryon decimet is not included in this representation. This is somewhat serious from the practical point of view, since the summation over intermediate states will not saturate without the decimet.

We can say that our scheme corresponds to the situation before the switch-on of the Yukawa interaction. The decimet or the nucleon isobar is a bound state of a nucleon and a pion, just as the deuteron is a bound state of two nucleons. We must consider the mixing of these excited states with the ground state (20) in order to take proper account of the meson cloud. In this paper we shall not deal with this problem but content ourselves here with bare hadrons.

If the $V(9)$ algebra is a good symmetry, that is, if the elements of the algebra (approximately) commute with the Hamiltonian or if the summation over intermediate states is fairly well saturated by particles (20), we can get some relations for hadron phenomena.

(1) *Mass relations**)

In the case of exact symmetry, all hadrons should have the same mass. We introduce symmetry breaking terms of the form $C_{3\rho}^{3\rho}$, $C_{\alpha 3}^{\alpha 3}$ and C_{33}^{33}, meaning that there are mass differences between T and t and between isodoublets and isosinglets.

For the baryon octet we do not have the Gell-Mann Okubo mass relation but

$$m_N + m_\Xi = m_\Lambda + m_\Sigma, \quad m_{Y_0^*} = m_\Sigma. \tag{21}$$

This is because of the mixing of B_1 (identified here as Y_0^*) with Λ. The first relation is fairly well satisfied but not the second.

For vector mesons we have the same results as the $SU(6)$ case:

$$2m_{K^*} = m_\rho + m_\phi, \quad m_\rho = m_\omega. \tag{22}$$

Both are reasonably well satisfied.

Spin 0 mesons split into pseudoscalars (made of fermion pairs) and scalars (boson pairs) and we have an unfavorable situation,

$$\text{mass } (P_8) = \text{mass } (V_8), \tag{23}$$

that is, every corresponding member of P_8 and V_8 must have the same mass. We need an interaction which depends on the spin of particles to obtain the splitting of P_8 and V_8. However, the first of the relation

*) The mass relation obtained here is very similar to those obtained by the $SU(9)$ symmetry (reference 3)).

$$2m_K = m_{X_0} + m_\pi, \quad m_\pi = m_\eta \tag{24}$$

is well satisfied, although the second is badly broken.

Between baryons and mesons,[9)]

$$m_\Xi - m_\Sigma = m_{K^*} - m_\rho = m_K - m_\pi. \tag{25}$$

The first equality holds very good but not the second. For scalar mesons, S_8 and S_1, we have, for instance,

$$2m_\Sigma = m_\pi + m_{\pi'}, \quad 2m_N = m_N + m_{S_1}, \tag{26}$$

where π' is the isotriplet member of S_8'. These relations show that S_8' and S_1 should have a mass about 2 GeV.

(2) *F/D ratio for the axial vector current*

The axial vector current is given by $J_i^{\rho\sigma} = C_{a\rho}^{b\sigma}(\lambda_i)_{ba}(i=1-3)$. The matrix elements of this current for the octet baryon give

$$F/D = 1, \tag{27}$$

which is not very far from the experimental value 0.6.[10)] The baryons are almost structureless and

$$G_A/G_V = 1. \tag{28}$$

(3) *Electromagnetic structure*

Restricting our consideration to integral charges ± 1, 0, there are two possibilities for the charge assignment:

	p	n	Λ
	K^+	K^0	η
case a)	1	0	0
case b)	0	-1	-1

Either case gives correct charge for the hadrons. If we assume that the magnetic moments of hadrons come only from the charge of fundamental fermions ("p", "n", "Λ"), we have

$$\begin{aligned} &\text{a)} \quad \mu_p = \mu_{\text{“}p\text{”}}, \ \mu_n = 0, \\ &\text{b)} \quad \mu_p = 0, \ \mu_n = \mu_{\text{“}n\text{”}}. \end{aligned} \tag{29}$$

We notice that the effect of a pion cloud is not taken into account. In either case, the Coleman-Glashow formula for the electromagnetic mass splitting holds:

$$m_{\Sigma^-} - m_{\Sigma^+} = m_n - m_p + m_{\Xi^-} - m_{\Xi^0}. \tag{30}$$

(4) *Total cross sections*

Since we are dealing with bare hadrons, the symmetry may be observed at extremely high energies. We consider the high energy limit of total cross sections. Assuming the vacuum Regge trajectory (Pomeranchuon) to be a $V(9)$ singlet, we have

$$\sigma_{BA} \quad \text{all equal if} \quad B \in (9, \bar{9}). \tag{31}$$

A is any particle. Experimentally $\sigma_{pp} \approx 40$ mb, $\sigma_{\pi p} \approx 25$ mb, $\sigma_{kp} \approx 20$ mb and (31) is not very well satisfied. We introduce a symmetry breaking in the Pomeranchuon-hadron coupling, allowing it to contain terms of the form $C^{3\rho}_{3\rho}$, C^{a3}_{a3} and C^{33}_{33}. This gives relations for cross sections similar to mass formulas, e. g.,

$$2\sigma_{Kp} = \sigma_{\pi p} + \sigma_{X_0 p},$$

$$2\sigma_{pp} = \sigma_{\pi p} + \sigma_{S_1 p},$$

$$\sigma_{\Sigma p} - \sigma_{\Xi p} = \sigma_{\pi p} - \sigma_{Kp}.$$

Unfortunately, none of them can be tested experimentally, but none is inconsistent with experiment (if $\sigma_{\pi p} > 2\sigma_{Kp}$ or $2\sigma_{pp}$, our scheme would be wrong, but in practice it is not).

§6. Current densities

So far we have constructed currents in momentum representation. Local current densities can be defined in the following way. Ignoring charge variables, we consider Dirac spinor $\psi(x)$ and a complex scalar field $\phi(x)$. They satisfy equal time commutation relations

$$\{\psi^\dagger_\alpha(\boldsymbol{x}), \psi_\beta(\boldsymbol{y})\} = \delta(\boldsymbol{x}-\boldsymbol{y}),$$

$$i[\pi(\boldsymbol{x}), \phi(\boldsymbol{y})] = i[\pi^\dagger(\boldsymbol{x}), \phi^\dagger(\boldsymbol{y})] = \delta(\boldsymbol{x}-\boldsymbol{y}), \quad \text{etc.,} \tag{33}$$

where π is the canonically conjugate field of ϕ. We define the two-component spinors

$$\psi_{+} = \frac{1}{2}(1 \pm \beta)\psi, \quad \psi^\dagger_{\pm} = \frac{1}{2}\psi^\dagger(1 \pm \beta) \tag{34}$$

and the scalar fields

$$\phi_{\pm} = \frac{1}{\sqrt{2}}\left(\sqrt{m}\,\phi \pm i\frac{\pi^\dagger}{\sqrt{m}}\right), \quad \phi^\dagger_{\pm} = \frac{1}{\sqrt{2}}\left(\sqrt{m}\phi^\dagger \pm i\frac{\pi}{\sqrt{m}}\right), \tag{35}$$

where m is the mass of the scalar particle. The fields with $+$ index (anti) commute with $-$ ones. From ψ_{+}, ϕ_{+} and their conjugates, we can make nine quantities

$$\int \psi_+^\dagger \psi_+ d\boldsymbol{x},\ \int \phi_+^\dagger \phi_+ d\boldsymbol{x},\ \int \psi_+^\dagger \sigma \psi d\boldsymbol{x},\ \int \psi_+^\dagger \phi_+ d\boldsymbol{x},\ \int \phi_+^\dagger \psi_+ d\boldsymbol{x}, \tag{36}$$

which form generators of $V(3)$.

A conserved spinor current can also be defined from ψ and ϕ. In the case of no interaction and equal mass m for ψ and ϕ, it is shown that the divergence of

$$\Psi_\mu = i\phi^\dagger \gamma_\mu \psi - \frac{i}{m}\partial_\nu \phi^\dagger \gamma_\nu \gamma_\mu \psi \tag{37}$$

is zero:

$$\frac{\partial}{\partial x_\mu}\Psi_\mu = 0. \tag{38}$$

The space integral of Ψ_0 is a conserved quantity and has a character of baryon number changing current.

The physical significance of such spinor currents is not clear. From the analogy of vector and axial-vector currents, it may be conjectured that Ψ_μ plays a certain role in weak interactions. Tanikawa[11] suggests that there is an intermediate weak particle having a nonzero baryon number. If such a particle exists, the spinor currents Ψ_μ will serve as its source. Alternatively, we can eliminate intermediate particles and expect a term in the weak Hamiltonian of the form

$$H_W = g\Psi_\mu^{a\dagger}\Psi_\mu^b, \tag{39}$$

where a, b are appropriate charge indices. Existence of a term like this has not yet been investigated.

In general, spinor currents like (37) will not be strictly conserved but only partially conserved. We may put

$$\partial_\mu \Psi_\mu^a = c\psi^a, \tag{40}$$

where c is a constant and ψ^a can be identified with the baryon[12] (partially conserved spinor current hypothesis).

§7. Conclusions and discussion

The existence of a symmetry covering both baryons and mesons, that is, the existence of baryon number changing currents as operators of a good symmetry, is suggested from the almost equality of the hadron masses (except for the pion) and of the cross sections. The algebra discussed here is the simplest one including such baryon number changing elements. Although it is not a Lie algebra, representations can be obtained in the usual way.

The 81-dimensional representation, also the simplest, contains most of the important hadrons except for the baryon decimet. In order to include the decimet, thereby taking into account the effect of pion clouds, we may consider a three-particle representation rather than a particle-antiparticle configuration as was done in §5. For this purpose we must introduce fractional charges, fractional baryon numbers, etc., but the result of the $SU(6)$-56-baryon theory is reproduced.

If the fundamental particles are not ordinary particles but paraparticles obeying parastatistics,[7] the commutation relations (4) do not hold; however, commutation relations for the currents, (12) or (19) are expected to hold. For simpler representations, the situation is somewhat similar to the $SU(9)$-symmetry scheme.

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