SUPERSYMMETRY AND WEAK, ELECTROMAGNETIC AND STRONG INTERACTIONS

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We study a possible way to construct supersymmetric theories which could be considered as realistic, excepted that the problem of mass generation for electron, muon and quarks remains. There is a new class of leptons which includes charged ones, and a "photonic neutrino". Spin 1/2-gluons and heavy spin 0- quarks are associated with ordinary vector gluons and quarks.

We have already constructed a supersymmetric theory for weak and electromagnetic interactions of the electron and its neutrino [1]; it describes the SU(2) $\times$ U(1) vector bosons W_-, Z and γ, the electron e_-, its neutrino ν, and other heavy particles, among which a charged scalar boson w_- and a heavy electron E_-. Leptonic number is conserved, and carried by the spinorial generator in the supersymmetry algebra. Charged current weak interactions are mediated both by heavy bosons W_- and w_-.

In a realistic theory we have to prevent scalar particles to be exchanged in μ- or β-decays. Moreover a Goldstone fermion verifies low-energy theorems, which make difficult to identify it with one of the known neutrinos [2]. We shall exhibit a possible way to solve these problems; however the question of mass generation for light leptons, and quarks, remains.

How to avoid scalar-particle-exchanges. In a supersymmetric theory any gauge vector boson W^μ is associated with a gauge Majorana spinor λ [3]; both are described by a vector superfield V. An SU(2) $\times$ U(1) gauge theory involves the vector superfields V and V', describing the following gauge fields:

$$\begin{cases} Z^\mu = \cos\theta\, W_3^\mu + \sin\theta\, W'^\mu \\ A^\mu = -\sin\theta\, W_3^\mu + \cos\theta\, W'^\mu \end{cases} \qquad W_-^\mu = \frac{W_1^\mu + \mathrm{i}\,W_2^\mu}{\sqrt{2}},$$
$$\begin{cases} \lambda_Z = \cos\theta\,\lambda_3 + \sin\theta\,\lambda' \\ \lambda_\gamma = -\sin\theta\,\lambda_3 + \cos\theta\,\lambda' \end{cases} \qquad \lambda_- = \frac{\lambda_1 + \mathrm{i}\lambda_2}{\sqrt{2}}. \tag{1}$$

We have noted in ref. [1] that $\theta = \mathrm{arctg}\, g'/g$ is both the SU(2) $\times$ U(1) mixing angle, like in Weinberg-Salam type models [4], and a neutrino-heavy neutrino mixing angle, like in Georgi-Glashow type models [5].

Matter is described by chiral superfields; for each of them we have a two-component (left-handed or right-handed) Dirac spinor, and a complex scalar. When SU(2) $\times$ U(1) is spontaneously broken and reduced to a U(1) subgroup, W_- and Z acquire masses, while the photon remains massless. Besides usual gauge vector interactions (due to the exchange of W_-, Z and γ), we have gauge spinor interactions: in each chiral multiplet λ_-, λ_Z and λ_γ may couple the spinor to the scalar fields (in particular λ_γ-couplings involve only charged chiral superfields). Thus scalar-particle-exchanges contribute to the scattering amplitude of the gauge fermion with a matter fermion. These exchanges have to be avoided in processes such as μ-or-β-decays.

This can be achieved if known leptons, and quarks, are described by chiral superfields only; for this purpose we shall use S_i and T_j, which are SU(2) multiplets of chiral superfields, left-handed and right-handed respectively; the covariant derivative is defined [‡1] by

$$\mathrm{i}D_\mu = \mathrm{i}\partial_\mu - \left(gT\cdot W_\mu + \frac{g'}{2}FW'_\mu\right). \tag{2}$$

For example, the chiral superfields S_e and T_e will describe the fermions $\binom{\nu_e}{e_-}_L$ and e_{-R}, together with scalar bosons denoted by $\binom{s_e^0}{s_e^-}$ and t_e^-. The scattering $\lambda_\gamma + e_- \to \lambda_\gamma + e_-$ may occur by exchange of one of the scalar bosons s_e^- and t_e^-, but the decay $\mu_- \to \nu_\mu + e_- + \bar{\nu}_e$ will be solely due to the exchange of the intermediate vector boson W_-.

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‡1 The charge matrix is $Q = T_3 - F/2$, the positron charge ($-e = -g\sin\theta$) being chosen as unit.

Symmetry breaking. Let us forget for a while about these chiral superfields S_i and T_j. In order to trigger spontaneous breaking of gauge invariance we introduce two SU(2) doublets of chiral superfields, S and T, left-handed and right-handed respectively. The Lagrangian density, already used in sections 4 and 5 of ref. [1], reads

$$\mathcal{L}_1 = \mathcal{L}_0(V, V') + [S^\dagger \exp(g\tau V + g'V')S + T^\dagger \exp -(g\tau V + g'V')T]_D + \xi' D' , \tag{3}$$

in which $\mathcal{L}_0$ denotes the part of the Lagrangian density relative to vector superfields only. SU(2) X U(1) gauge invariance is reduced to an U(1) subgroup; the neutral components of the superfields S and T acquire non vanishing vacuum expectation values[‡2]:

$$|\langle S_0 \rangle| = \frac{v_\ell}{2}, \quad |\langle T_0 \rangle| = \frac{v_r}{2}, \tag{4}$$

with

$$\tfrac{1}{4}(g^2 + g'^2)(v_\ell^2 - v_r^2) = -\xi' g' . \tag{5}$$

Expression (3) describes vectors W_-, Z and γ, Dirac spinors ℓ_-, L_-, ℓ_0, L_0, and scalars w_-, φ (complex) and z (real), with zeroth order masses [1]:

$$\begin{cases} m_Z = m_{L_0} = m_z = \tfrac{1}{2}(g^2 + g'^2)^{1/2}(v_\ell^2 + v_r^2)^{1/2} \\ m_{W_-} = m_{w_-} = \tfrac{1}{2} g (v_\ell^2 + v_r^2)^{1/2} \\ m_{L_-} = g v_\ell / \sqrt{2} \\ m_{\ell_-} = g v_r / \sqrt{2} \\ m_\gamma = m_{\ell_0} = m_\varphi = 0 \end{cases} \tag{6}$$

The gauge spinors λ and λ' combine with the Dirac spinors ψ_0 and ψ_- described by S and T; in terms of physical fields we have:

$$\begin{cases} \lambda_- = \ell_{-L} + L_{-R} \\ \lambda_{ZR} = L_{0R} \\ \lambda_{\gamma L} = \ell_{0L} \end{cases} \tag{7}$$

We can define a conserved (ℓ-type) leptonic number, which is 1 for ℓ_-, ℓ_0, and (-1) for L_-, L_0; the charged heavy leptons ℓ_- and L_- have masses related by

$$m_{L_-}^2 + m_{\ell_-}^2 = 2m_{W_-}^2 . \tag{8}$$

One of them may be relatively light[‡3].

Now we consider again the chiral superfields S_i and T_j. Their spinorial components will be lepton and quark fields. In order to obtain mass terms for their scalar components we introduce a second U(1) gauge group with coupling constant g'' (at the end we shall take $g'' = 0$). The complete Lagrangian density is $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$, with

$$\mathcal{L}_2 = \mathcal{L}_0(V'') + \Big[\sum_i S_i^\dagger \exp(2g\boldsymbol{T}\cdot\boldsymbol{V} + g'FV' + g''V'')S_i + \sum_j T_j^\dagger \exp -(2g\boldsymbol{T}\cdot\boldsymbol{V} + g'FV' + g''V'')T_j \Big]_D + \xi'' D'' . \tag{9}$$

For $\xi'' g''$ large enough the minimum of the potential is still defined by (4), together with

$$\langle S_i \rangle = \langle T_j \rangle = 0 . \tag{10}$$

We have for auxiliary components

$$\begin{cases} \langle D_Z \rangle = 0 \\ \langle D_\gamma \rangle = -\xi' \cos\theta \\ \langle D'' \rangle = -\xi'' \end{cases} \tag{11}$$

The Goldstone spinor is a linear combination of λ_γ and λ'', and mass splittings are obtained for superfields coupled to V_γ and V''.

Finally we used the limiting procedure introduced in ref. [7] and discussed in ref. [8,9], which allows to decouple the Goldstone spinor while mass splittings inside multiplets remain. We consider the limit $g'' \to 0$, $\xi'' \to \infty$, with $\xi'' g'' = 2\mu^2$ fixed; the Goldstone spinor becomes λ'', and the superfield V'' describing W''^μ and λ'' decouples. The limiting Lagrangian density $\tilde{\mathcal{L}} = \mathcal{L}_1 + \tilde{\mathcal{L}}_2$ can be obtained by using in (9) the constraint equation

$$D'' = -\xi'' = -2\mu^2/g'' . \tag{12}$$

We find:

‡2 The indices ℓ and r are relative to complex scalars fields described by left-handed and right-handed superfields, respectively: v_ℓ and v_r stand for v'' and v', used in ref. [1].

‡3 As long as we consider only the vector superfields V and V', and the chiral superfields S and T, λ_γ is the Goldstone spinor [6] arising from spontaneous supersymmetry breaking (if ξ', g and g' are not zero), and ℓ_0 is exactly massless.

$$
\begin{aligned}
\tilde{\mathcal{L}}_2 = &\Big[\sum_i S_i^\dagger \exp(2g\boldsymbol{T}\cdot\boldsymbol{V}+g'FV')S_i \\
&+ \sum_j T_j^\dagger \exp-(2g\boldsymbol{T}\cdot\boldsymbol{V}+g'FV')T_j\Big]_{\mathrm{D}} \\
&-2\mu^2\Big[\sum_i S_i^\dagger \exp(2g\boldsymbol{T}\cdot\boldsymbol{V}+g'FV')S_i \\
&+ \sum_j T_j^\dagger \exp-(2g\boldsymbol{T}\cdot\boldsymbol{V}+g'FV')T_j\Big]_{\mathrm{C}} .
\end{aligned} \tag{13}
$$

When the Wess–Zumino gauge is used, the last term in $\tilde{\mathcal{L}}_2$ is just a mass term for the scalar fields s_i and t_j (described by S_i and T_j); these have mass^2

$$\mu^2 + Q_i|e|\xi'\cos\theta\ , \quad \mu^2 - Q_j|e|\xi'\cos\theta\ , \tag{14}$$

respectively (note the proportionality between mass^2 splitting and charge).

Weak and electromagnetic interactions. The chiral superfields S_i and T_j describe known leptons, and quarks, together with heavy scalar particles. Electronic, muonic and baryonic numbers are conserved; they are attributed to superfields S_i and T_j, and carried by bosons as well as by fermion fields; for example electron number is 1 for the electron, its neutrino and scalar fields associated, and 0 for all other fields. Since all bosons carrying electronic, muonic or baryonic number are heavy, these quantum numbers appear as carried by fermions only, at low energies.

As an illustration we use the Weinberg–Salam model [4]. S_i are SU(2) doublets, and T_j SU(2) singlets, describing the spinor fields:

$$
\begin{array}{cccc}
\begin{pmatrix}\nu_e\\ e_-\end{pmatrix}_{\mathrm{L}} & \begin{pmatrix}\nu_\mu\\ \mu_-\end{pmatrix}_{\mathrm{L}} & e_{-\mathrm{R}} & \mu_{-\mathrm{R}} \\
\begin{pmatrix}\mathrm{p}\\ \mathrm{n_c}\end{pmatrix}_{\mathrm{L}} & \begin{pmatrix}\mathrm{p}'\\ \lambda_{\mathrm{c}}\end{pmatrix}_{\mathrm{L}} & \begin{matrix}\mathrm{p_R}\\ \mathrm{n_{cR}}\end{matrix} & \begin{matrix}\mathrm{p'_R}\\ \lambda_{\mathrm{cR}}\end{matrix}
\end{array} \tag{15}
$$

The quark fields exist in three colors and anomalies cancel [10]. We recover the usual weak and electromagnetic interactions, due to the sole exchange of vector bosons W_-, Z, γ.

Strong interactions. In order to obtain strong interactions we add an SU(3) color octet of vector superfields V_{a}, describing an octet of vector gluons V_{a}^μ and an octet of spinor gluons λ_{a}. They interact with some of the chiral superfields S_i and T_j: those which describe spinor quarks q_c^f together with heavy scalar quarks ("sarks") s_c^f and t_c^f (f and c are flavor and color indices, respectively).

Vector gluons interact both with spin $\frac{1}{2}$ and spin 0 quarks. Spinor gluons couple spin $\frac{1}{2}$ quarks to spin 0 quarks: the interaction is given [‡4] by

$$\mathrm{i}g_{\mathrm{s}}\sqrt{2}\,[\bar{q}_{\mathrm{L}}(\mathrm{R_a}\lambda_{\mathrm{a}})s + \bar{q}_{\mathrm{R}}(\mathrm{R_a}\lambda_{\mathrm{a}})t] + \mathrm{h.c.}\ , \tag{16}$$

in which $\mathrm{R_a}$ is a 3×3 matrix-representation of SU(3) (and summations over flavor and color indices are understood). We have also quartic interaction terms for spin-0 quarks; they are not determined by an independent parameter but by the square of the strong gauge coupling constant g_{s}. Therefore we can have as much as eight quark flavours without ruining asymptotic freedom [‡5] [12].

A third "leptonic" number. Besides electronic, muonic and baryonic numbers (which appear as external quantum numbers), there exist an extra quantum number associated with ℓ-type leptons. It is carried by the spinorial generator in the supersymmetry algebra, and defined by means of the transformation R:

$$
\begin{cases}
V(x,\theta,\bar\theta) \to V(x,\theta \mathrm{e}^{-\mathrm{i}\alpha},\bar\theta \mathrm{e}^{\mathrm{i}\alpha}) \\
(\text{id. for } V', S, T \text{ and } V_{\mathrm{a}}) \\
S_i(x,\theta,\bar\theta) \to \mathrm{e}^{\mathrm{i}\alpha} S_i(x,\theta \mathrm{e}^{-\mathrm{i}\alpha},\bar\theta \mathrm{e}^{\mathrm{i}\alpha}) \\
T_j(x,\theta,\bar\theta) \to \mathrm{e}^{-\mathrm{i}\alpha} T_j(x,\theta \mathrm{e}^{-\mathrm{i}\alpha},\bar\theta \mathrm{e}^{\mathrm{i}\alpha})
\end{cases} \tag{17}
$$

ℓ-type quantum number is carried by ℓ_- and ℓ_0 (+1), L_- and L_0 (−1), by scalar fields s_i (+1) and t_i (−1), and by the Majorana spinors λ_{a} (+1 for λ_{aL}, or as well (−1) for λ_{aR}). We have the curious feature that this new "leptonic" number is also attributed to spinor gluons. It prevents them to acquire masses.

Note that v_ℓ and v_{r}, which verify (5), are not completely determined at lowest order; higher order computations must be performed. This can be avoided by

‡4 Note the invariance of (16) under the set of transformations

$$
\begin{aligned}
\mathrm{R}: s &\to \mathrm{e}^{\mathrm{i}\alpha}s \qquad q \to q \\
t &\to \mathrm{e}^{-\mathrm{i}\alpha}t \qquad \lambda \to \mathrm{e}^{\gamma_5\alpha}\lambda
\end{aligned}
$$

Such an invariance has been used some time ago to obtain a Yukawa-type coupling similar to (16) (see § 2.1 in ref. [11]); it is associated with the conservation of a new quantum number. In order to avoid parity violations in strong interactions, it may be necessary that the scalar quarks s and t have masses large enough or nearly equal.

‡5 I thank Dr. Ferrara for a discussion on this point.

adding (as in section 5 of ref. [1]) the gauge invariant chiral superfield N, which removes the zeroth order vacuum degeneracy. The mass spectrum can be found in ref. [1]; here we only indicate that the field $\lambda_{\gamma L}$, associated with the photon field, stays massless.

An important problem remains, namely mass generation for electron, muon and quarks. If we could forget this difficulty, we would have shown that supersymmetry is applicable to the real world. Besides vector gluons and spinor quarks, strong interactions involve spinor gluons and heavy scalar quarks. Electronic, muonic and baryonic numbers are conserved, together with a third leptonic number associated with a new class of leptons, including charged ones ‡6, and a photonic neutrino.

‡6 The new lepton ℓ_- can be relatively light. When produced in pairs, it leads to apparent violations of electronic and muonic number conservations, according to $e_+e_- \to \ell_+\ell_- \to \mu_\pm e_\mp$ (+ unobserved neutrinos). Such events have already been observed [13], and are compatible with a mass of ℓ_- in the range 1.6 to 2.0 GeV/c^2.

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