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UPPER BOUNDS ON SUPERSYMMETRIC PARTICLE MASSES

R. BARBIERI

CERN, Geneva, Switzerland
and Dipartimento di Fisica, Università di Pisa, INFN, Sezione di Pisa, Italy

G.F. GIUDICE

International School for Advanced Studies, Trieste, Italy
and INFN, Sezione di Trieste, Italy

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Based on the "naturalness" criterion, upper bounds on all superparticle masses as functions of the top quark mass are derived. These bounds give an objective criterion to test (or disprove) the idea of low-energy supersymmetry, as implemented in supergravity models. These bounds strongly differentiate weakly interacting superparticles, like charginos or neutralinos, lighter than 100–200 GeV, from strongly interacting ones, like gluinos or squarks which can become heavier than 1 TeV.

1. Introduction

A seemingly embarrassing point when one starts talking about supersymmetry is the fact that, among all known particles, not even one (broken) boson-fermion supermultiplet can as yet be reconstructed. Is this not enough to discourage even the most fervent proponent of supersymmetry as a relevant extension of the standard model physics? Or – if this first question can get a natural answer – is there a sensible upper bound on superparticle masses to be used as a criterion to test (or disprove) the idea of supersymmetry at all?

The relation of these questions to the issue of the gauge symmetry breaking is manifest. Any known particle, whose mass requires the $SU(3) \times SU(2) \times U(1)$ symmetry to be broken, is paired to a superpartner which, on the contrary, can get a mass term invariant under the gauge symmetry. Accordingly, the gauge symmetry, preventing a mass term for the standard particles, naturally splits the "light" ordinary particle spectrum from the "heavy" superpartners, that may have not yet been discovered for this very reason. On the other hand, as it is well known, the effective scale of supersymmetry breaking, controlling the typical splitting inside the supermultiplets, cannot be arbitrarily separated from the electroweak breaking

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scale, if one is not willing to introduce increasingly precise tunings among parameters. In turn, the implementation of this "naturalness" criterion*, gives rise to a physical upper bound on superparticle masses in the TeV range [2].

This paper deals with a quantitative analysis of these general arguments in the context of low energy supergravity models**. Notice that these models are precisely designed to incorporate the above ideas, so as to overcome the difficulties previously met in the early attempts to extend the standard model in a supersymmetric manner.

Our strategy is quite straightforward. In the context of supergravity models (to be defined in sect. 2), we consider the electroweak symmetry breaking scale, or the Z^0-boson mass, as a function of the most general parameters a_i of the theory:

$$M_Z^2 = M_Z^2(a_i; y_t). \tag{1.1}$$

To describe the electroweak symmetry breaking, which is induced by radiative corrections [4, 5], and to get a sensible approximation, this function must include all one-loop renormalization group improved effects. In (1.1) we have made explicit the dependence on the top quark Yukawa coupling, y_t, since it plays a crucial role in determining the appropriate gauge symmetry breaking. The parameters a_i, as well as y_t, also control the masses of the various supersymmetric partners of the standard particles. By explicit calculation, eq. (1.1) exhibits the already mentioned feature that a consistent range of parameters allows arbitrarily heavy superpartners, still keeping M_Z fixed. In fact, in this limit, the theory under consideration can be thought of as a physically regularized version (with respect to quadratic divergences) of the standard model lagrangian. On the other hand, not surprisingly, this is achieved only at the price of an unnatural tuning among the physical parameters of the theory. We avoid this tuning by imposing**, for every a_i:

$$\left| \frac{a_i}{M_Z^2} \frac{\partial M_Z^2(a_i; y_t)}{\partial a_i} \right| < \Delta, \tag{1.2}$$

so that a percentage variation of any of the parameters a_i does not correspond to a percentage variation of M_Z^2 more than Δ-times larger. For example, $\Delta = 10$ amounts to tolerate in (1.1) cancellations among parameters of at most one order of magnitude. In turn, for every top quark Yukawa coupling, the inequalities (1.2) can be converted into upper bounds on all dimensional parameters of the theory and therefore on all superparticle masses. These bounds are shown in fig. 2 for $\Delta = 10$,

* The unnaturalness of light scalars has been underlined in ref. [1]. The relevance of supersymmetry to this issue has been pointed out in ref. [2].
** For a review, see ref. [3] and references therein.
*** Similar conditions were imposed on a particular supergravity model in ref. [6].

as functions of the mass

$$M_t = \frac{\sqrt{2}}{g} M_W y_t, \tag{1.3}$$

which translate the Yukawa coupling into a dimensionful parameter, and is related to the physical top quark mass by (see sect. 3):

$$\frac{M_t}{\sqrt{2}} \leqslant m_t \leqslant M_t. \tag{1.4}$$

Notice the disappearance of the bounds on the gluino and the scalar masses for special values of the top quark Yukawa coupling, $M_t \simeq 55$ GeV and $M_t \simeq 160$ GeV respectively. This feature, which will be illustrated in sect. 3, corresponds to the vanishing of some renormalization group coefficients for the mentioned values of the top Yukawa coupling. Of course, the precise determination of these values depends on neglected higher order effects, which are otherwise irrelevant to our analysis*.

The paper is organized as follows. In sect. 2, we define the general class of supergravity models that we consider and we summarize previous work relevant to our purposes. In sect. 3, we derive the upper bounds on the dimensionful parameters of the model in the special case of vanishing mixing between the two Higgs doublet superfields ($\mu = 0$). The general bounds on the same parameters and on μ itself are given in sect. 4. In sect. 5, we convert these limits into upper bounds on the masses of the various supersymmetric particles. Our conclusions are drawn in sect. 6. Appendix A gives the explicit y_t dependence of the various renormalized parameters of the theory. Appendix B relates the bounds on the dimensionful parameters of the theory with the bounds on the different supersymmetric particles.

2. The model: summary of previous work

We work in the framework of the minimal supergravity model, defined by the following lagrangian [2]:

$$L = L_{\text{susy}}\left(f = f_y + \mu H_1 H_2\right) + Amf_y + Bm\mu H_1 H_2 + m^2 \sum_a |\varphi_a|^2 + M \sum_\alpha \left(\bar{\lambda}_\alpha\right)^c \lambda_\alpha. \tag{2.1}$$

* The avoidance of the bounds for the particular values of M_t corresponds to an "unnatural" fine tuning of the Yukawa coupling y_t, which could be excluded by imposing a condition analogous to (1.2) on the derivative with respect to y_t itself. This would smooth away the singular points from figs. 1, 2.

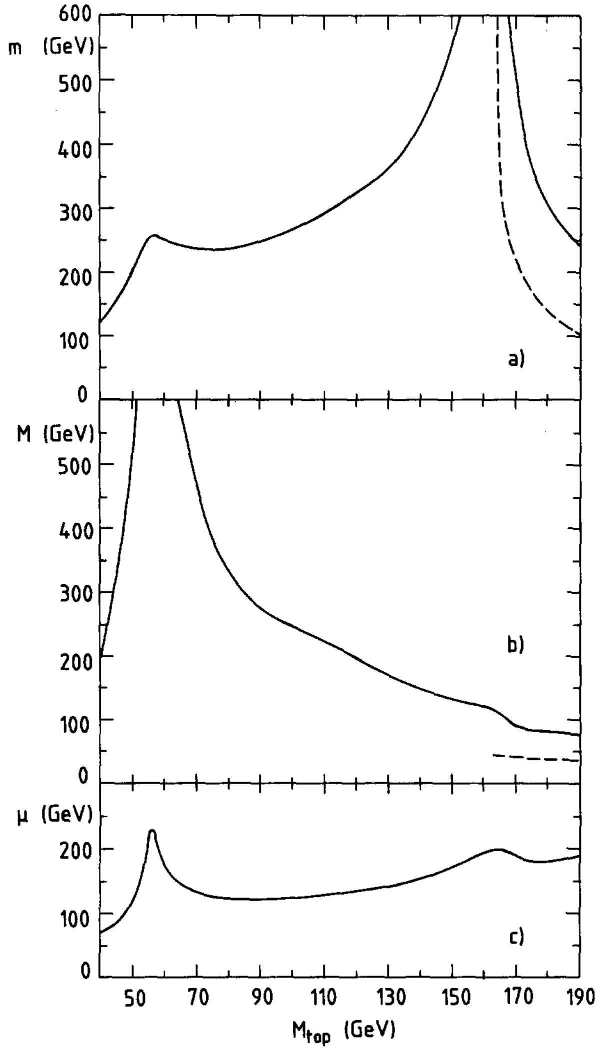

Fig. 1. Upper bounds on the parameters m (1a), M (1b) and μ (1c) for $\Delta = 10$ (fine tunings of at most one order of magnitude) as functions of $M_t = (\sqrt{2}/g) M_W y_t$. All bounds scale as $\sqrt{\Delta}$. The dotted lines in fig. 1a, b are the absolute bounds for $\mu = 0$ and $M_t > 160$ GeV (see text).

In the supersymmetric part of the lagrangian invariant under SU(3) × SU(2) × U(1), L_{susy}, the superpotential f includes the standard Yukawa terms f_y, as well as a mass coupling $\mu H_1 H_2$ between the two Higgs doublets $H_{1,2}$ of opposite hypercharge. In the remaining part of L, which softly break supersymmetry, $m^2 \Sigma_a |\varphi_a|^2$ and $M \Sigma_\alpha (\bar{\lambda}_\alpha)^c \lambda_\alpha$ are universal mass terms for all the scalar degrees of freedom φ_a and for all the SU(3) × SU(2) × U(1) gaugino fields λ_α respectively. Other than the gauge and the Yukawa couplings, L contains five more parameters: m, M, μ with

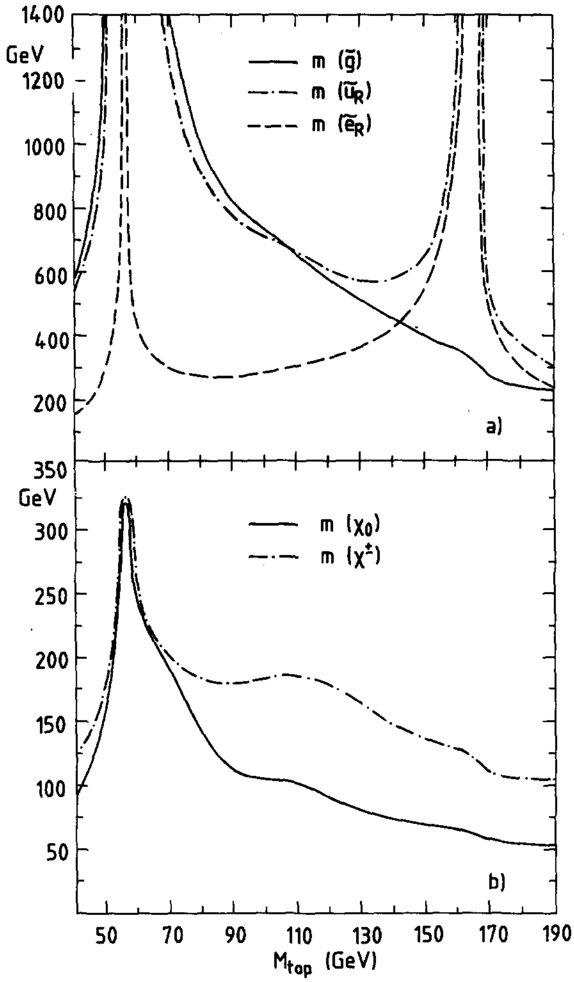

Fig. 2. Upper bounds for $\Delta = 10$ (fine tunings of at most one order of magnitude), as functions of $M_t = (\sqrt{2}/g) M_W y_t$, on the masses of: (a) gluino ($\tilde{g}$), scalar partner of the right-handed up quark ($\tilde{u}_R$), scalar partner of the right-handed electron ($\tilde{e}_R$); (b) lightest neutralino (χ^0), lightest chargino ($\chi^{\pm}$).

dimension of mass and A, B dimensionless. These parameters play the role of the a_i introduced in the previous section.

The soft breaking terms in L are the remnants of the supergravity couplings of the "observable" particles to a "hidden" sector, which determine the classical vacuum of the theory and drive the spontaneous breaking of supersymmetry [7]. For this reason, the lagrangian (2.1) and all its parameters are meant to be defined at a grand scale $M_X \simeq 10^{15} - 10^{18}$ GeV. The universality, at this grand scale, of the gaugino and the scalar mass terms is attributed to the universality of the supergravity couplings of the "hidden" to the "observable" sector [7]. Small deviations from this hypothesis may occur [8] due, e.g., to grand unification couplings, which may have interesting consequences at low energy. They would not however affect any of our conclusions. Similarly, we take the parameters m, M, μ, A, B all real, with an appropriate definition of the various fields, as it can be ascribed to a CP conserving "hidden" sector.

The one-loop renormalization down to lower energies of the above lagrangian has been intensively studied by several groups [4,5]. For the sake of completeness, we recall the results that we need for later use.

Let us confine our attention to the part of L, eq. (2.1), describing the potential along the neutral components of the Higgs fields:

$$V(H_1, H_2) = \tfrac{1}{8}(g^2 + g'^2)\left(|H_1|^2 - |H_2|^2\right)^2 + m_1^2|H_1|^2 + m_2^2|H_2|^2 - m_3^2(H_1H_2 + \text{h.c.}), \tag{2.2}$$

which, for

$$m_1^2 + m_2^2 > 2|m_3^2|, \qquad m_1^2 m_2^2 < m_3^4, \tag{2.3}$$

gives rise to the appropriate breaking in the direction

$$\langle H_1 \rangle = \sqrt{\tfrac{1}{2}}\, v \cos\beta, \qquad \langle H_2 \rangle = \sqrt{\tfrac{1}{2}}\, v \sin\beta, \tag{2.4}$$

$$v^2 \equiv \frac{8\left(m_1^2 - m_2^2 \mathrm{tg}^2\beta\right)}{(g^2 + g'^2)(\mathrm{tg}^2\beta - 1)}, \tag{2.5}$$

$$\sin 2\beta \equiv \frac{2m_3^2}{m_1^2 + m_2^2}. \tag{2.6}$$

The crucial equation is therefore:

$$M_Z^2 = \frac{2\left(m_1^2 - m_2^2 \mathrm{tg}^2\beta\right)}{\mathrm{tg}^2\beta - 1}, \tag{2.7}$$

which is equivalent to eq. (1.1), after the properly renormalized parameters m_1^2, m_2^2, m_3^2 are related to the original parameters appearing in eq. (2.1) by:

$$m_2^2 = -aM^2 - bAmM + cm^2 - dA^2m^2 + e\mu^2, \tag{2.8a}$$

$$m_1^2 = m^2 + e\mu^2 + fM^2, \tag{2.8b}$$

$$m_3^2 = g\mu M + hB\mu m + kA\mu m. \tag{2.8c}$$

In eqs. (2.8) the coefficients are functions of the gauge couplings and of the top Yukawa coupling, as obtained from the solution of the appropriate renormalization group equations, with all other Yukawa couplings neglected. Their explicit dependences on the top quark Yukawa coupling are given in appendix A.

3. The bounds for $\mu = 0$

Before going to the full parameter space, we consider the bounds on the parameters m, M for $\mu = 0$. This is done for pedagogical reasons, but also because the small μ region may have a special physical significance [3].

In this case, using eqs. (2.8), eq. (2.7) reduces to a particularly simple form

$$M_Z^2 = -2m_2^2 = 2(aM^2 + bAmM - cm^2 + dA^2m^2), \tag{3.1}$$

which makes manifest the cancellation required among the parameters m, M, A in order for m and/or M to increase arbitrarily. The cancellation depends on the coefficients a, b, c, d, or, ultimately, on the top quark Yukawa coupling. Actually, no cancellation is required if and only if a or c vanish. When a vanishes, which happens for $M_t \simeq 55$ GeV, M can be arbitrarily large, without implying any fine tunings among parameters, provided m and A are taken sufficiently small. On the other hand, when c vanishes, for $M_t \simeq 160$ GeV, it is m which can be taken large as M and A go to zero. This explains the singularities in fig. 1, which represents the upper bounds for arbitrary μ. For all other values of M_t, the cancellation required to allow an arbitrary increase of m and/or M entails a violation of the inequalities (1.2). We have chosen to plot the bounds versus M_t rather than m_t itself since, via

$$m_t = M_t \sin\beta, \tag{3.2}$$

m_t acquires an involved dependence on the original parameters a_i, unlike M_t, which is simply related to the Yukawa coupling. Since, in general, $\frac{1}{4}\pi \leqslant \beta \leqslant \frac{1}{2}\pi$, eq. (1.4) holds.

Actually, before worrying about the unnaturalness of fine tunings among the different terms in eq. (3.1), one has to see if they are possible at all. If not, one limits m and M in an absolute way, independently of (1.2). This is indeed the case for $M_t \geqslant 160$ GeV (which implies $m_t \geqslant 160$ GeV), since, from this point on, the ratio between (3.1) and M^2 or m^2 never vanishes. This is due to the fact that the coefficients in eq. (3.1) satisfy, for $M_t \geqslant 160$ GeV

$$ac < 0 \quad \text{and} \quad b^2 - 4ad < 0. \tag{3.3}$$

On the other hand, a zero of the same ratios is required in order for any of the dimensionful parameters m and/or M to become arbitrarily large, for fixed M_Z^2.

Therefore, for $\mu=0$, one gets an absolute bound on M (m) by minimizing the right-hand side of eq. (3.1) with respect to A and m (M). One obtains in this way:

$$M^2 < \frac{2d}{4ad-b^2}M_Z^2, \qquad m^2 < \frac{1}{-2c}M_Z^2 \qquad \text{for } m_t \geqslant 160\ \text{GeV}. \tag{3.4}$$

This feature is lost for arbitrary μ, even though it remains true that a bound on μ implies a bound on M and m. In fact, from the general formula (2.7), one obtains:

$$2m_2^2 > -M_Z^2, \tag{3.5}$$

so that, from eq. (2.8a):

$$M^2 < \frac{4d}{4ad-b^2}\left(e\mu^2 + \frac{M_Z^2}{2}\right), \tag{3.6a}$$

$$m^2 < -\frac{1}{c}\left(e\mu^2 + \frac{M_Z^2}{2}\right). \tag{3.6b}$$

For $M_t \leqslant 160$ GeV, the scaled right-hand side of eq. (3.1) is able to vanish. Accordingly, the bounds on M and m rely on the imposition of the inequalities (1.2). The bounds themselves are most easily obtained in the following way. Eq. (3.1) can be solved in M (m). For large M (m), neglecting terms of order $(M_Z/M)^2((M_Z/m)^2)$, one obtains:

$$M = \frac{1}{2a}\left[-bA \pm \sqrt{4ac + A^2(b^2-4ad)}\right]m. \tag{3.7}$$

Notice that M_Z^2 in eq. (3.1), and also its derivatives, eq. (1.2), only depends on the sign of A relative to the sign of M. For this reason, any of the two solutions (3.7) is enough to span the full space of parameters. If now this solution is substituted in the explicit expressions of the derivatives,

$$\frac{A}{M_Z^2}\frac{\partial M_Z^2}{\partial A} = 2\frac{Am}{M_Z^2}(bM + 2dAm), \tag{3.8a}$$

$$\frac{m}{M_Z^2}\frac{\partial M_Z^2}{\partial m} = 2\frac{m}{M_Z^2}(bAM - 2cm + 2dA^2m), \tag{3.8b}$$

$$\frac{M}{M_Z^2}\frac{\partial M_Z^2}{\partial M} = 2\frac{M}{M_Z^2}(2aM + bAm), \tag{3.8c}$$

the inequalities (1.2) acquire the form

$$m^2 < \Delta \cdot f_i(A) \qquad \left(M^2 < \Delta \cdot g_i(A)\right), \tag{3.9}$$

which is immediately converted into a bound on m (M):

$$m^2 < \Delta \cdot \max_A(\min_i f_i(A)) \qquad \left(M^2 < \Delta \cdot \max_A(\min_i g_i(A))\right). \tag{3.10}$$

The parameter A is allowed to vary only between -3 and 3 in order to avoid unwanted minima breaking color and/or electric charge [3].

For dimensional reasons, the bounds on m, M scale as $\sqrt{\Delta}$. Notice that, if we had imposed our naturalness criterion on the function M_Z rather than M_Z^2, we would have got bounds less restrictive by a factor $\sqrt{2}$ $(\Delta \to 2\Delta)$. On the other hand, $M_Z^2(a_i;\, y_t)$ is the natural combination of parameters that appear in the Higgs potential.

For $M_t \leqslant 160$ GeV, these bounds are very close to the general ones, plotted in fig. 1. For $M_t \geqslant 160$ GeV, when the switching on of μ allows a tuning of parameters that can make the scaled right-hand side of eq. (3.1) arbitrarily small, the bounds (3.4) are given in the same figure as dashed lines.

A top quark mass heavier than 190 GeV is not considered, since it would correspond to a singular behavior of the relevant Yukawa coupling before the grand scale.

4. The bounds in the general case

The study of the general case can be made along similar lines, except that the general expression for M_Z^2, eq. (2.7), is more complicated and it involves five parameters instead of three. It is therefore useful to make the following preliminary observations.

As it is clear from the previous section, we are interested in approximate zeros of the function $M_Z^2(a_i;\, y_t)$, which, in the general case, are obtained for

$$m_1^2 - m_2^2 \mathrm{tg}^2\beta \simeq 0 \tag{4.1}$$

or, equivalently,

$$m_1^2 m_2^2 \simeq m_3^4 . \tag{4.2}$$

When the parameters satisfy this relation, which is, for large m, M, μ an approximate solution of eq. (2.7), the derivatives of $M_Z^2(a_i;\, y_t)$ can be simplified to

$$\frac{a_i}{M_Z^2} \frac{\partial}{\partial a_i} M_Z^2(a_i;\, y_t) = \frac{2a_i(\partial/\partial a_i)\left(m_1^2 m_2^2 - m_3^4\right)}{M_Z^2\left(m_1^2 - m_2^2\right)} . \tag{4.3}$$

Furthermore, using eqs. (2.8) and (4.2), we replace in (4.3) the parameter B with the approximate solution:

$$B = \frac{1}{hm}\left(-kAm - gM \pm \frac{1}{\mu}\sqrt{m_1^2 m_2^2} \right) . \tag{4.4}$$

Here again we can disregard one of the two solutions corresponding to the $\pm$ sign in front of the square root, without losing any effective combination of parameters.

In this way the inequalities (1.2) acquire a form analogous to (3.9), except that now the functions f_i (g_i) depend, other than on A, on two dimensionless ratios of the three variables m, M, μ. The bounds are obtained by maximizing these functions.

5. Upper bounds on supersymmetric particles

The bounds on m, M, μ shown in fig. 1 can be easily converted into upper bounds on any kind of supersymmetric particle. This is completely straightforward for squarks, sleptons and the gluino, whose masses are directly and easily related to the above parameters (see appendix B). In fig. 2a, we show the bounds on the masses of the gluino, the scalar partner of the right-handed up quark (similar to the bounds for all other quarks) and the right-handed scalar electron (which provides among sleptons, the most stringent bound). Notice there that the scalar particle masses become unbounded for both $M_t \simeq 55$ GeV and $M_t \simeq 160$ GeV: this is because the physical squared masses receive contributions proportional to both m^2 and M^2.

The bounds on the lightest chargino and neutralino (the physical superpositions of charged and neutral gaugino and higgsino fermion fields) require a discussion of the corresponding mass matrices. The diagonalization of the 2×2 chargino mass matrix readily leads to the following bound on the squared mass of the lighter state:

$$m(\chi^{\pm})^2 < M_W^2 + \min(M_2^2, \mu_R^2), \tag{5.1}$$

where

$$M_2 = \frac{\alpha_2}{\alpha_G} M, \qquad \mu_R = \sqrt{e}\,\mu \tag{5.2}$$

and α_G is the gauge constant at the grand scale (see appendices A and B).

A bound for the neutralino is obtained by considering the corresponding 4×4 squared mass matrix, whose diagonal entries limit the lowest squared mass eigenvalue, so that

$$m(\chi^0)^2 < \min(\mu_R^2 + \tfrac{1}{2}M_Z^2, M_1^2 + \sin^2\theta_W M_Z^2), \tag{5.3}$$

where

$$M_1 = \frac{5}{3}\frac{\alpha_1}{\alpha_G} M. \tag{5.4}$$

The corresponding results are given in fig. 2b. Notice here that neither the lightest chargino nor the lightest neutralino can become arbitrarily heavy for any value of the top quark mass. This is a consequence of the bound on the parameter μ (fig. 1).

Notice that, for any given top quark mass, the limits on m, M, μ are all saturable, although not all at the same time, namely for one particular choice of the parameters. This, in turn, means that the bounds on the physical particles are also essentially saturable, again not at the same time. The limitation comes from the fact that we have simultaneously inserted in the expressions for the physical masses the maximum values for m, M, μ. Practically, this does not lead us to over-estimate significantly the real bounds.

6. Conclusions

In this paper we have derived upper bounds on the masses of the supersymmetric particles, which rely on the assumption that no cancellation takes place among the physical parameters of the minimal supergravity model by more than one order of magnitude ($\Delta = 10$). A different numerical criterion can easily be imposed by knowing that the bounds scale like $\sqrt{\Delta}$.

We feel that these results put on a sounder basis the problem of testing, or disproving, supergravity models. In our opinion, the bounds on weakly interacting particles, like charginos and neutralinos, are of particular significance. Although the knowledge of the top quark mass would be required to make a more precise statement, these results, we believe, point in the direction of e^+e^- colliders as the most efficient machines to discover supersymmetry.

Let us finally spend a word on the significance of the "naturalness" criterion that we are employing. The problem of the quadratic divergences of the Higgs squared mass is a serious one. There is no known example of cancellation between a quadratic divergence in the low energy theory and contributions from shorter distances. In the fundamental Higgs picture, supersymmetry is the only known way to avoid at all the quadratic divergences, which are replaced by squared superpartner mass terms. On the other hand, we do not know of any way to enforce a natural cancellation among the different contributions to eq. (2.7), when these masses get large. It follows that these masses must be limited. If no supersymmetric particle is found below the limits that we have given, the case for low energy supersymmetry gets, in our opinion, extremely weakened. This should certainly not be interpreted as an argument against the idea of supersymmetry at all, which has strong independent motivations. One then should however at least reconsider, theoretically even before than experimentally, the strategy to look for its signals.

Appendix A

In this appendix, we give the explicit dependence on the top quark Yukawa coupling of the coefficients appearing in eqs. (2.8), computed using the one loop renormalization group equations. We follow the same conventions of ref. [5], and we refer the reader to those papers for further details.

We first define the functions

$$E(x) = (1+\beta_3 x)^{16/3b_3}(1+\beta_2 x)^{3/b_2}(1+\beta_1 x)^{13/9b_1}, \tag{A.1}$$

$$F(x) = \int_0^x E(x')\,dx', \tag{A.2}$$

where b_1, b_2 and b_3 are the coefficients of the one-loop β-function for the gauge couplings ($b_1 = 11,\ b_2 = 1,\ b_3 = -3$) and

$$\beta_i = \frac{\alpha_i(0)}{4\pi} b_i, \qquad i = 1,\dots,3. \tag{A.3}$$

$\alpha_i(0)$ are meant to be defined at the unification scale M_X where

$$\alpha_3(0) = \alpha_2(0) = \tfrac{5}{3}\alpha_1(0) \equiv \alpha_G . \tag{A.4}$$

Then, we call

$$t \equiv \ln\frac{M_X^2}{M_W^2} , \tag{A.5}$$

$$E \equiv E(t), \qquad F \equiv F(t), \tag{A.6}$$

$$Z_i \equiv \frac{1}{1+\beta_i t}, \qquad f_i \equiv \frac{1-Z_i^2}{\beta_i}, \qquad i=1,\dots,3. \tag{A.7}$$

Next, we define the following quantities:

$$H_1 = -\frac{3\alpha_G}{8\pi}\left(\frac{1}{5}f_1 + f_2\right), \tag{A.8}$$

$$H_2 = \frac{1}{2F}\int_0^t E(t')\left[\frac{\alpha_G}{4\pi}\left(\tfrac{32}{3}f_3(t') + 6f_2(t') + \tfrac{26}{15}f_1(t')\right) + \frac{\alpha_G^2}{8\pi^2}t'^2\left(\tfrac{16}{3}Z_3(t') + 3Z_2(t') + \tfrac{13}{15}Z_1(t')\right)^2\right]\mathrm{d}t', \tag{A.9}$$

$$H_3 = -\tfrac{1}{2}H_4^2 , \tag{A.10}$$

$$H_4 = 2\left(t\frac{E}{F} - 1\right), \tag{A.11}$$

$$H_5 = Z_2^{-3/2b_2}Z_1^{-1/2b_1}, \tag{A.12}$$

$$H_6 = \tfrac{1}{2}H_4H_5 , \tag{A.13}$$

$$H_7 = -3\frac{\alpha_G}{4\pi}t\left(Z_2 + \tfrac{1}{5}Z_1\right)H_5 , \tag{A.14}$$

$$K = \frac{3FG_F}{4\sqrt{2}\pi^2 E}, \tag{A.15}$$

where G_F is the Fermi constant.

In terms of the previous quantities, we can explicitly show the dependence on the top quark Yukawa coupling, y_t, of the parameters appearing in eqs. (2.8). Parametrizing the Yukawa coupling in terms of the dimensionful parameter M_t, defined in eq. (1.3), we obtain:

$$a = H_1 + H_2 K M_t^2 + H_3 K^2 M_t^4, \tag{A.16}$$

$$b = H_4 K M_t^2 (1 - 2KM_t^2), \tag{A.17}$$

$$c = 1 - 3KM_t^2, \tag{A.18}$$

$$d = KM_t^2 (1 - 2KM_t^2), \tag{A.19}$$

$$e = H_5^2 (1 - 2KM_t^2)^{1/2}, \tag{A.20}$$

$$f = -H_1, \tag{A.21}$$

$$g = (H_6 K M_t^2 + H_7)(1 - 2KM_t^2)^{1/4}, \tag{A.22}$$

$$h = -H_5 (1 - 2KM_t^2)^{1/4}, \tag{A.23}$$

$$k = H_5 K M_t^2 (1 - 2KM_t^2)^{1/4}. \tag{A.24}$$

The choice $M_X \simeq 3 \times 10^{16}$ GeV and $\alpha_G \simeq \frac{1}{24}$ yields: $H_1 \simeq -0.53$, $H_2 \simeq 14.$, $H_3 \simeq -11.$, $H_4 \simeq 4.7$, $H_5 \simeq 1.4$, $H_6 \simeq 3.3$, $H_7 \simeq -0.84$, $K \simeq 1.25 \times 10^{-5}$ GeV^{-2}.

Appendix B

In sect. 5, the bounds on the dimensionful parameters m, M and μ are translated into bounds on the physical supersymmetric particle masses through the relations collected in this appendix (see also ref. [5]).

For the scalar partners of the right-handed up quark and electron and for the gluino:

$$m^2(\tilde{u}_R) < m^2 + \frac{2}{3\pi}\alpha_G\left(f_3 + \tfrac{1}{5}f_1\right)M^2, \tag{B.1}$$

$$m^2(\tilde{e}_R) < m^2 + \frac{3}{10\pi}\alpha_G f_1 M^2 + \sin^2\theta_W M_Z^2, \tag{B.2}$$

$$m(\tilde{g}) = Z_3 M, \tag{B.3}$$

where α_G, f_i, Z_i are defined in appendix A.

The chargino and neutralino mass matrices involve the following mass parameters, renormalized up to the low energy scale:

$$M_1 = Z_1 M, \qquad M_2 = Z_2 M, \qquad \mu_R = \sqrt{e}\,\mu. \tag{B.4}$$

The squared mass of the lightest chargino is:

$$m^2(\chi^\pm) = \tfrac{1}{2}\Big[m_2^2 + \mu_R^2 + 2M_W^2 - \sqrt{\left(M_2^2 - \mu_R^2\right)^2 + 4M_W^4\cos^2 2\beta + 4M_W^2\left(M_2^2 + \mu_R^2 + 2M_2\mu_R\sin 2\beta\right)}\,\Big] \tag{B.5}$$

and, consequently, eq. (5.1) easily follows. Since the smallest eigenvalue of a hermitian matrix is bounded by any of its diagonal entries, which in the case of the neutralino squared mass matrix are: $M_1^2 + \sin^2\theta_W M_Z^2$, $M_2^2 + \cos^2\theta_W M_Z^2$, $\mu_R^2 + \cos^2\beta M_Z^2$, $\mu_R^2 + \sin^2\beta M_Z^2$, we obtain a limit on the mass of the lightest neutralino, according to eq. (5.3).

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