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CANCELLATION OF QUADRATICALLY DIVERGENT MASS CORRECTIONS IN GLOBALLY SUPERSYMMETRIC SPONTANEOUSLY BROKEN GAUGE THEORIES

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The one-loop quadratically divergent mass corrections in globally supersymmetric gauge theories with spontaneously broken abelian and non-abelian gauge symmetry are studied. Quadratically divergent mass corrections are found to persist in an abelian model with an ABJ anomaly. However, additional supermultiplets necessary to cancel the ABJ anomaly, turn out to be sufficient to eliminate the quadratic divergences as well, rendering the theory natural. Quadratic divergences are shown to vanish also in the case of an anomaly free model with spontaneously broken non-abelian gauge symmetry.

1. Introduction

Elementary scalar Higgs fields inducing spontaneous breakdown of gauge symmetries in gauge theories are known [1,2] to suffer from the malady of 'unnaturalness'. Various parameters in the Higgs sector of spontaneously broken gauge theories (SBGT) like the Glashow-Salam-Weinberg (GSW) electroweak gauge theory, receive large quantum corrections at mass scales larger than O(1 TeV) which are unstable under small perturbations of the parameters. This can be avoided by adjustments of these parameters to high degrees of precision. Such adjustments must be done anew in every order of perturbation theory, a procedure which is aesthetically undesirable and hence termed unnatural. Consider, for instance, the GSW theory embedded in a grand unified theory like SU(5) characterised by two mass scales, $M_X \sim 10^{15}$ GeV and $M_W \sim 100$ GeV. The (light) Higgs bosons of the GSW theory with mass $m_H \sim 100$ GeV, receive radiative corrections $\sim \alpha M_X^2 \ln M_X^2$ through their couplings to the heavy bosons because of primitively quadratic divergent diagrams. In order to control these corrections, one needs to adjust the mass parameter of the light Higgs bosons to an accuracy of 1 in 10^{26}.

The naturalness problem can be evaded if there exists in the theory a symmetry, albeit softly broken, that automatically constrains the radiative corrections to a reasonable size. Such a symmetry is supersymmetry. As discussed by several authors [3], and explicitly demonstrated by us [4] in the context of a model with U(1) abelian gauge symmetry, supersymmetry guarantees cancellations of primitively quadratic divergent one-loop contributions to the masses of various physical particles. Elementary scalar fields are thus endowed with a natural status in theories with supersymmetry. This result has far-reaching implications vis-a-vis the so-called gauge hierarchy problem in grand unified theories [13]. As discussed in a recent preprint by one of the authors [14], the gauge hierarchy, in a theory with two widely separated mass scales, can be made as large as desired if the theory is embedded in a supersymmetric model.

Germane to the cancellation of primitive quadratic divergences is the issue of anomalies. As we shall show in sect. 2, in models with ABJ anomaly supersymmetry by itself does not ensure vanishing of quadratic divergences, though the mass degeneracy within a supermultiplet is retained at the one loop level. In sect. 3, we review the model discussed in ref. [4] and show that the extra fermions (belonging to the additional multiplets) needed to eliminate the ABJ anomalies are also sufficient to obviate the undesirable primitive quadratically divergent contributions. To emphasize the generality of our results, we discuss in sect. 4, the primitively quadratic divergence structure of one-loop mass corrections in an anomaly-free supersymmetric *non-abelian* gauge theory with SU(2)$\times$U(1) gauge symmetry spontaneously broken through elementary Higgs. We conclude in sect. 5 with a few brief remarks.

2. U(1) model with ABJ anomaly

In this section, we discuss a globally supersymmetric gauge model with a U(1) local gauge invariance which is spontaneously broken through elementary Higgs. The U(1) gauge invariance corresponds to gauging of γ_5 transformations, and hence entails (V $-$ A) type couplings between the fermions and the gauge boson. The spectrum of the model does not have adequate fermions to cancel the resultant ABJ anomaly [5]. We show that although the tree-level mass degeneracy of the supersymmetric multiplet is intact after inclusion of one-loop corrections, these corrections are, in fact, not free of primitive quadratic divergences.

The model under consideration consists of an abelian vector multiplet $V(x,\theta,\bar{\theta})$ interacting with a left-handed chiral scalar multiplet $S(x,\theta,\bar{\theta})$; in terms of these superfield, the lagrangian density reads

$$\mathcal{L}=\mathcal{L}_0+\left[S^*e^{2gV}S+\xi V\right]_{\mathrm{D}}, \tag{2.1}$$

where $\mathcal{L}_0$ is free lagrangian for the vector multiplet V. This model is invariant under the U(1) supergauge transformations

$$V(x,\theta,\bar\theta) \to V(x,\theta,\bar\theta) + i[\Lambda(x,\theta,\bar\theta) - \Lambda^*(x,\theta,\bar\theta)] \tag{2.2}$$

and

$$S(x,\theta,\bar\theta) \to e^{-2ig\Lambda(x,\theta,\bar\theta)} S(x,\theta,\bar\theta), \tag{2.3}$$

where the supergauge parameter Λ is a chiral superfield. To obtain a polynomial lagrangian in terms of component fields, one works in the Wess-Zumino gauge whereby all but one of the auxiliary fields of V are gauged away. This remaining field together with the auxiliary fields of S are next eliminated through their field equations, leaving behind a massless gauge vector field $A_\mu(x)$, a massless Majorana fermion $\lambda(x)$, a massless complex scalar $\varphi(x)$ and a massless left-handed Weyl fermion $\psi_L(x)$; the lagrangian is*

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}\bar\lambda \not\partial \lambda - \bar\psi_L \not D \psi_L - |D_\mu\varphi|^2 \\ &- g\sqrt{2}\,[\bar\psi_L\lambda\varphi + \bar\lambda\psi_L\varphi^*] - V(\varphi),\end{aligned} \tag{2.4}$$

where the covariant derivative D_μ is defined by

$$D_\mu = \partial_\mu + igA_\mu, \tag{2.5}$$

and

$$V(\varphi) = \tfrac{1}{2}|\xi + g|\varphi|^2|^2. \tag{2.6}$$

The parameter ξ triggers spontaneous symmetry breaking; if $\xi/g < 0$, $V(\varphi)$ has a minimum at $\langle\varphi\rangle_0 = (-\xi/g)^{1/2} \neq 0$ and the U(1) gauge symmetry is spontaneously

* Notations and conventions: We use the euclidean metric $g_{\mu\nu} = \delta_{\mu\nu}$, and the Majorana representation for the γ-matrices, γ_i, $i = 1,2,3$ real, and γ_4 pure imaginary:

$$\gamma_i^2 = \gamma_4^2 = \mathbb{1}, \qquad \gamma_5 = -i\gamma_1\gamma_2\gamma_3\gamma_4, \qquad \gamma_5^2 = -\mathbb{1},$$

$$\gamma_4^T = -\gamma_4, \qquad \gamma_i^T = \gamma_i, \qquad \gamma_5^T = -\gamma_5, \qquad \gamma_\mu^\dagger = \gamma_\mu, \qquad \gamma_5^\dagger = -\gamma_5.$$

The left- and right-handed chiral spinor fields are defined by

$$\psi\binom{L}{R} = \tfrac{1}{2}(\mathbb{1} \mp i\gamma_5)\psi.$$

Charge conjugation in Majorana representation is given by

$$\psi^c = \psi^*.$$

broken. In this case $V(\varphi=\langle\varphi\rangle_0)=0$, so that supersymmetry is not broken at the tree level. If, on the other hand $\xi/g>0, V(\varphi)$ has a minimum at $\langle\varphi\rangle_0=0$, and $V(0)=\frac{1}{2}\xi^2\neq 0$ implying spontaneous breakdown of supersymmetry, with gauge symmetry remaining unbroken. Our interest focusses on the former alternative; the Higgs mechanism eliminates the Goldstone boson and generates masses for the vector boson and other particles in the usual manner. We parametrize φ as: $\varphi=\sqrt{\frac{1}{2}}\,(F+H+i\eta)$, where $\langle\varphi\rangle_0=\sqrt{\frac{1}{2}}\,F=(-\xi/g)^{1/2}$ H is the physical Higgs field and η the Goldstone ghost. The physical spectrum consists of a massive vector field A_μ, a massive Dirac spinor $E\equiv\psi_{\rm L}+\lambda_{\rm R}$, and the massive Higgs field H. All fields have the same mass $m\equiv gF$, reflecting the underlying supersymmetry.

To compute the quantum corrections to the Higgs boson mass, we quantize the theory in the R_α gauges, where α is the gauge-fixing parameter. For convenience we choose $\alpha=1$ (generalized 't Hooft-Feynman gauge). The corresponding effective lagrangian density including the gauge fixing and Faddeev-Popov ghost terms is given in appendix B. Notice that the fermion-gauge boson interaction term has the form

$$\mathcal{L}_{E\text{-}A_\mu}=-ig\bar{E}_{\rm L}\not{A}E_{\rm L}\,,$$

which is of the (V − A) type. Since there are no fermions interacting with A_μ through a (V + A) type of coupling, this model develops an ABJ anomaly. We shall remark on this towards the end of the section.

We now present the details of our computations. A supersymmetric version of dimensional regularization, called dimensional reduction [6, 7], is employed in regularizing the one-loop diagrams. In this scheme, vector and spinor indices are held fixed in four dimensions so as to maintain the equality of fermionic and bosonic degrees of freedom explicitly. The γ-algebra is also performed in four dimensions only. After performing the γ-algebra, the loop integrals are analytically continued to complex n dimensions with $n\lesssim 4$, and evaluated a la 't Hooft and Veltman [8]. Finally, after relevant renormalizations, the physical limit $n\to 4$ is taken.

Some remarks regarding tadpole diagrams are in order at this point. In the tree approximation, supersymmetry requires that the vacuum energy is zero, i.e. $V_{\rm Higgs}(H=\langle H\rangle_0=0)=0$. Further, since $V_{\rm Higgs}(H)$ has a minimum value of zero at $\langle H\rangle_0=0$, it contains no term linear in H. In general, however, the one-loop effective Higgs potential will contain terms of the form $-2m\beta H-2m^2\beta$, where $2m\beta$ is given by the sum of all tadpole graphs with H-field external line. In order to define the one-loop renormalized vacuum expectation value, one must translate the H field by an amount $2\beta/m$, for $\beta\neq 0$. This, in turn, would produce additional (β-dependent) contributions to the masses of all particles, because of their coupling to H. From the lagrangian (A.1) we observe that the physical fields H, A_μ and E, obtain the following extra contributions to their mass squared: $-6g\beta$ for H, $-4g\beta$ for A_μ and E.

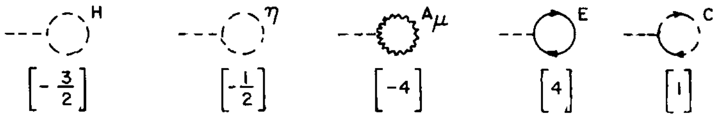

Fig. 1. H field tadpole graphs. The number in square brackets below each graph represents the coefficient of the $mgA(m^2)$ contribution from that particular graph.

The quantity β is gauge dependent and primitively quadratically divergent, being proportional to $A(m^2)$★. As the fermion self-energies are at worst logarithmically divergent in R_α gauges, the only naturalness violating quadratic divergent contributions are obtained due to the renormalization of the vacuum expectation value for $\beta \neq 0$. Hence supersymmetry implies that $\beta = 0$ is a *necessary and sufficient condition* in R_α gauges for a SBGT to be natural.

We now proceed to an explicit diagrammatic evaluation of β. The relevant tadpole graphs are drawn in fig. 1. The contribution of each tadpole has the generic form $TmgA(m)$, where T is a numerical coefficient. Below each graph we delineate its corresponding value of T. The total tadpole contribution, in generalized 't Hooft-Feynman gauge is therefore

$$\begin{aligned} 2m\beta &\equiv mgA(m^2) \sum_{\text{tadpoles}} T \\ &= -mgA(m^2) \neq 0 . \end{aligned} \tag{2.7}$$

Yet the total mass corrections due to the renormalization of the vacuum expectation value *and* other self-energy diagrams are identical for all physical fields, as demonstrated in the following.

Referring to appendix B, the contribution of the self-energy diagrams can be entirely expressed in terms of the functions $A(m^2)$ and $B_0(p, m^2, m^2)$. In fact, the generic form of these contributions to the $(mass)^2$ is

$$\Pi(m^2) = g^2\left\{aA(m^2) + bm^2\left[B_0(p, m^2, m^2)\right]_{p^2=-m^2}\right\}, \tag{2.8}$$

where a and b are numerical coefficients calculable from each graph. Following our earlier procedure, we delineate the a and b coefficients for each graph by writing

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$$A(m^2) \equiv \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2 + m^2} .$$

See appendix B.

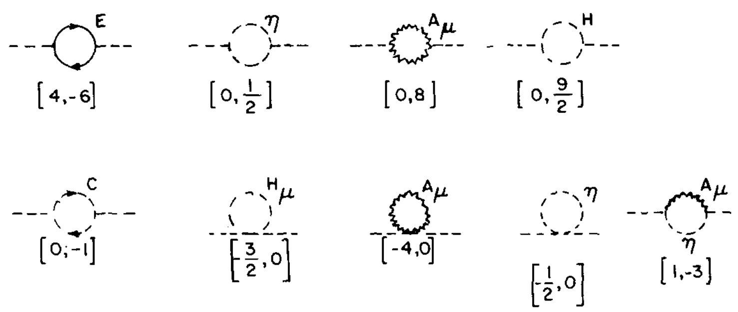

Fig. 2. H field self-energy graphs. The numbers $[a, b]$ below each graph represents the coefficients of the $g^2A(m^2)$ and $g^2m^2B_0(p, m^2, m^2)|_{p^2=-m^2}$ contributions to the mass squared respectively, from that particular graph.

$[a, b]$ below that particular graph. The following results are obtained:

(a) H field self-energy: Feynman graphs relevant for the H self-energy are drawn in fig. 2. The corrections to the mass squared are

$$\Pi_H(m^2) = g^2\{A(m^2)\Sigma a + m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\Sigma b\} \tag{2.9}$$

$$= g^2\{-A(m^2) + 3m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\}. \tag{2.10}$$

Using eq. (2.7) the total one-loop correction to the H field (mass)^2 is then

$$\delta m_H^2 = \Pi_H(m^2) - 6g\beta$$

$$= g^2\{2A(m^2) + 3m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\}. \tag{2.11}$$

(b) A_μ self-energy: The relevant Feynman graphs are drawn in fig. 3. The mass correction for A_μ from the self-energy diagram is*

$$\Pi_A(m^2) = g^2\{A(m^2)\Sigma a + m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\Sigma b\} \tag{2.12}$$

$$= 3m^2g^2[B_0(p, m^2, m^2)]_{p^2=-m^2}. \tag{2.13}$$

* In the generalized 't Hooft-Feynman gauge the vector boson two-point function has the structure (to $O(g^2)$)

$$\bar{D}_{\mu\nu}(p) = \{[1 + g^2\mathcal{P}(p^2)]p^2 + [1 + g^2\mathcal{M}(p^2)]m^2\}\delta_{\mu\nu}.$$

The one-loop correction to the (mass)^2 is derived from the pole of the propagator $\bar{\Delta}_{\mu\nu}(p) \equiv [\bar{D}_{\mu\nu}(p)]^{-1}$ and is given to $O(g^2)$

$$\Pi_A(m^2) = m^2g^2[\mathcal{M}(p^2) - \mathcal{P}(p^2)]_{p^2=-m^2}.$$

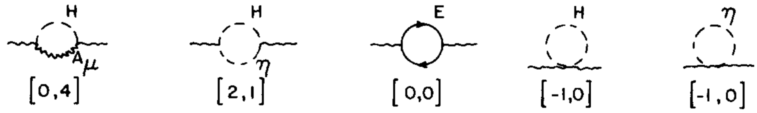

Fig. 3. A_μ field self-energy graphs. Conventions are the same as in fig. 2.

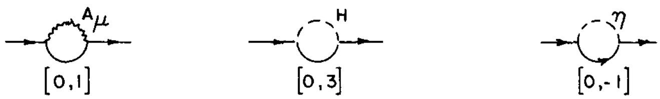

Fig. 4. E field self-energy graphs. Conventions are the same as in fig. 2.

The total one-loop correction to the A_μ field mass is

$$\delta m_A^2 = \Pi_A(m^2) - 4g\beta$$

$$= g^2\{2A(m^2) + 3m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\}. \tag{2.14}$$

(c) E field self-energy: The diagrams contributing in this case are given in fig. 4. The one-loop corrections to the mass squared from the self-energy diagrams are*

$$\Pi_E(m^2) = g^2\{A(m^2)\Sigma a + m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\Sigma b\}$$

$$= 3m^2 g^2[B_0(p, m^2, m^2)]_{p^2=-m^2}. \tag{2.15}$$

The total one-loop correction to the E field mass is

$$\delta m_E^2 = \Pi_E(m^2) - 4g\beta$$

$$= g^2\{2A(m^2) + 3m^2[B_0(p, m^2, m^2)]_{p^2=-m^2}\}. \tag{2.16}$$

Thus, to $O(g^2)$ we have verified that

$$\delta m_H^2 = \delta m_A^2 = \delta m_E^2 = g^2\{2A + 3m^2 B_0\}, \tag{2.17}$$

*To $O(g^2)$, the E field two-point function has the form

$$\bar{D}_E(p) = i\not{p}\{1 + g^2[\mathcal{P}(p^2) + \gamma_5 Q(p^2)]\} + \mathcal{M}\{1 + m(p^2)\}.$$

The one-loop correction to the (mass)^2 is given for $O(g^2)$ by

$$\Pi_E(m^2) = 2m^2 g^2\{\mathcal{M}(p^2) - \mathcal{P}(p^2)\}_{p^2=-m^2}.$$

as expected on grounds of supersymmetry. We observe that whereas the degeneracy of the masses dictated by supersymmetry is still valid at the one-loop level, the masses are not free of quadratic divergences, as already anticipated from eq. (2.7).

It may be argued that the model under consideration has an inherent pathology in the form of the ABJ anomaly which renders the theory unrenormalizable. However, the source of naturalness violation, i.e. $\beta \neq 0$, is not traceable to the ABJ anomaly in any obvious manner. In the next section we consider a model in which extra supermultiplets have been added to eliminate the ABJ anomaly. We investigate the effect of these additional supermultiplets on the quadratically divergent mass corrections.

3. U(1) model without ABJ anomaly

In this model [9], we have an abelian vector superfield $V(x,\theta,\bar{\theta})$ interacting with a left-handed chiral superfield $S(x,\theta,\bar{\theta})$ as in sect. 2, but, in addition, we have a right-handed chiral superfield $T(x,\theta,\bar{\theta})$ and a real scalar superfield $N(x,\theta,\bar{\theta})$. All multiplets are massless. The lagrangian is given in terms of these superfields by

$$\mathcal{L}=\mathcal{L}_0+\left[S^* e^{2gV}S+T^* e^{-2gV}T+\xi V\right]_D+\left[N^*N\right]_D+\left[4gT^*SN\right]_F, \qquad (3.1)$$

where $\mathcal{L}_0$ is the free lagrangian for the vector superfield, V. In addition to the supergauge invariance [eqs. (2.2), (2.3)] this model has a global SU(2) invariance for $\xi=0$. As in sect. 2, we work in the Wess-Zumino gauge, and use the field equations to eliminate the auxiliary fields of the three scalar multiplets. One is then left with the following non-vanishing component fields: a vector field A_μ and a Majorana spinor λ in V; two real scalars a, b and a Majorana spinor p in N; a complex scalar φ and a left-handed Weyl spinor ψ_L in S; a complex scalar φ' and a right-handed Weyl spinor ψ_R in T.

This model has been discussed by us in a previous work (ref. [4]) to which we refer the reader for further details. Relevant for the present discussion are the definitions of the physical Higgs fields, which are given by the parametrization of φ and φ':

$$\varphi=\sqrt{\tfrac{1}{2}}\,(F+H+i\eta), \qquad \varphi'=\sqrt{\tfrac{1}{2}}\,(H_1+iH_2), \qquad (3.2)$$

where $\langle\varphi\rangle_0=\sqrt{\frac{1}{2}}\,F=(-\xi/g)^{1/2}$ [$\xi/g<0$ is the condition for spontaneous breakdown of U(1) gauge symmetry and unbroken supersymmetry] and $\langle\varphi'\rangle_0=0$. H, H_1, H_2 are the physical Higgs fields and η is the Goldstone ghost. Further, we define the physical (Dirac) fermions $e\equiv ip_L+\psi_R$, $E\equiv\psi_L-\lambda_R$, and a neutral complex scalar $\omega\equiv-\sqrt{\frac{1}{2}}\,i(a-ib)$. In terms of physical fields, we have the following multiplet structure: (E, A_μ, H) with mass $m=gF$ as in the model of sect. 2, and in addition, another multiplet (e, H_1, H_2, ω) with the same mass m. The effective lagrangian incorporating the gauge-fixing and Faddeev-Popov ghost terms is given in appendix A. Once again, we work in the generalized 't Hooft-Feynman gauge.

Consider the gauge boson-fermion interaction term in the effective lagrangian,

$$\mathcal{L}_{\text{gb-f}} = -ig\left[\bar{e}_{\text{R}} \not{A} e_{\text{R}} + \bar{E}_{\text{L}} \not{A} E_{\text{L}}\right]. \tag{3.3}$$

Clearly, the axial vector ABJ anomaly due to the E field is equal and opposite to that due to the e field, giving a net zero contribution. This model is thus renormalizable.

To determine the effect of the additional multiplet (e, H_1, H_2, ω) on the value of β (H-field tadpoles), we calculate the extra tadpole graphs. These are drawn in fig. 5, with their respective contributions. Recall that in sect. 2, we obtained

$$2m\beta|_{(E, A_\mu, H)} = -mgA(m^2). \tag{2.7}$$

The additional contribution from e, H_1, H_2, ω tadpoles is

$$\begin{aligned} 2m\beta|_{(e, H_1, H_2, \omega)} &= mgA(m^2)\left[-\tfrac{1}{2} - \tfrac{1}{2} - 2 + 4\right] \\ &= mgA(m^2), \end{aligned} \tag{3.4}$$

implying that the total one-loop correction to the vacuum expectation value is zero. Thus, in this case the necessary and sufficient condition for naturalness is indeed satisfied. We have therefore demonstrated that the addition of the new multiplet intended to eliminate the ABJ anomaly through the fermion-gauge boson coupling, is also sufficient to remove the primitive quadratic divergences and make the theory natural.

For the sake of completeness, we now present the results of the mass corrections from the various self-energy diagrams. From figs. 6–10, we see that these corrections for the fields H_1, H_2, A_μ, ω, e and E are all equal:

$$\delta m^2 = 2m^2 g^2 B_0(p, m^2, m^2)|_{p^2 = -m^2}.$$

To close this section, we remark that supersymmetry has indeed turned out to be instrumental in rendering a SBGT with elementary scalar fields, natural, provided the ABJ anomaly is eliminated by addition of extra supermultiplets. This result is not an artifact of the model under consideration. One can presumably generalize it to any globally supersymmetric SBGT with elementary Higgs, so long as it is free of ABJ anomalies. An explicit example of validity of this statement is given in the next section, where we study the primitive quadratically divergent mass corrections in a globally supersymmetric model with spontaneously broken SU(2)$\times$U(1) gauge symmetry.

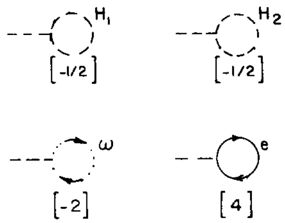

Fig. 5. Additional H-field tadpoles. Conventions are the same as in fig. 1.

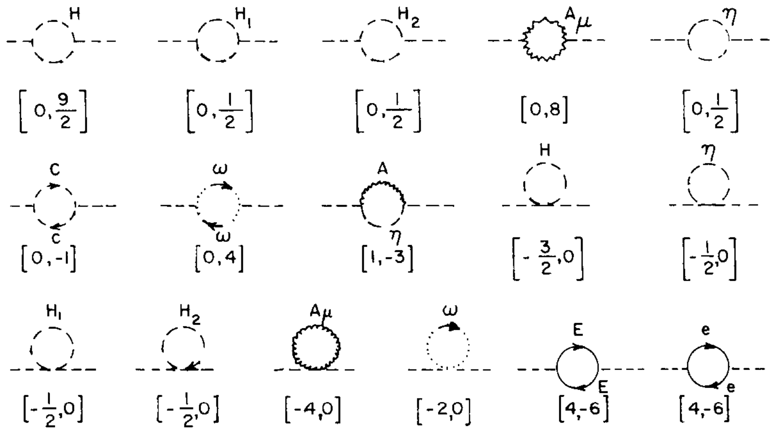

Fig. 6. H-field self-energy. Conventions are the same as in fig. 2.

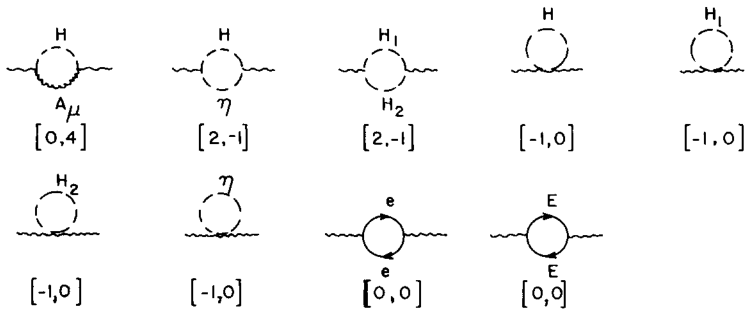

Fig. 7. A_μ field self-energy. Conventions are same as in fig. 2.

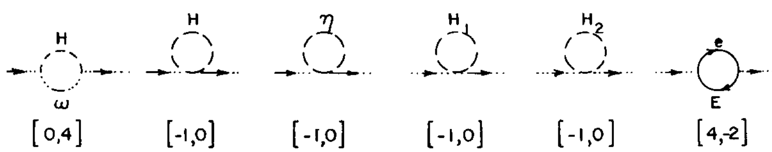

Fig. 8. ω-field self-energy. Conventions are the same as in fig. 2.

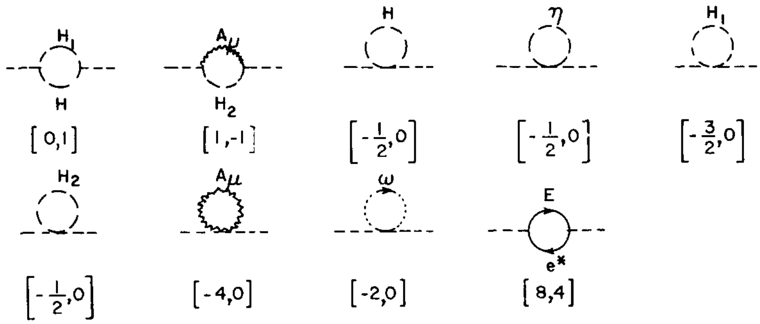

Fig. 9. H_1 field self-energy. Conventions are the same as in fig. 2.

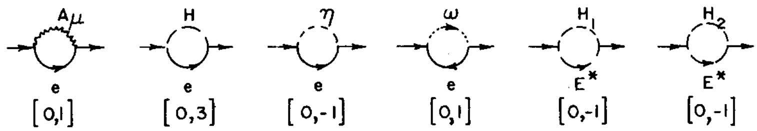

Fig. 10. e-field self-energy. Conventions are the same as in fig. 2.

4. SU(2) × U(1) model

Here we study a globally supersymmetric non-abelian gauge theory with SU(2) × U(1) gauge symmetry [10] broken spontaneously to U(1).

The model contains an SU(2) triplet vector superfield V, a singlet vector field V', a left-handed SU(2) doublet superfield S, a right-handed SU(2) doublet superfield T and a singlet real scalar superfield N. The globally supersymmetric lagrangian given by

$$\mathcal{L} = \mathcal{L}_0 + [N^*N]_{\mathrm{D}} + \left[S^\dagger \exp(g\boldsymbol{\tau}\cdot\boldsymbol{V} + g'V'\mathbb{1})S + T^\dagger \exp(-g\boldsymbol{\tau}\cdot\boldsymbol{V} - g'V'\mathbb{1})T\right]_{\mathrm{D}}$$

$$+ [2hT^\dagger SN + sN]_{\mathrm{F}}, \tag{4.1}$$

where $\mathcal{L}_0$ is the kinetic energy lagrangian for the vector multiplets V and V'. In Wess-Zumino gauge, all but one triplet, $\boldsymbol{D}$, and one singlet, D', of the auxiliary fields of the vector multiplets V, V', respectively, are gauged away. These and other auxiliary fields from the scalar multiplets, S, T, N, are eliminated by their field equations, leaving behind the following component fields: a triplet of vector fields A_μ^a and a triplet of Majorana spinors λ_a in V; a singlet vector field A'_μ and a singlet Majorana spinor λ' in V'; an SU(2) doublet of left-handed Weyl spinors $\psi_{\mathrm{L}} \equiv (\psi_0, \psi_-)_{\mathrm{L}}$ an SU(2) doublet of complex scalar fields $\Phi'' \equiv (\varphi_0'', \varphi_-'')$ in S; an SU(2) doublet of right-handed Weyl spinors $\psi_{\mathrm{R}} \equiv (\psi_0, \psi_-)_{\mathrm{R}}$ and a doublet of complex scalar fields, $\Phi' \equiv (\varphi_0', \varphi_-')$ in T; two singlet real scalars a, b and a singlet Majorana

p in N. The lagrangian in terms of these fields reads

$$\begin{aligned}\mathcal{L}= & -\tfrac{1}{2}\operatorname{tr}F_{\mu\nu}F_{\mu\nu}-\tfrac{1}{4}F'_{\mu\nu}F'_{\mu\nu}-\operatorname{tr}(\bar{\lambda}\not{D}\lambda)-\tfrac{1}{2}\lambda'\not{\partial}\lambda'\\ & -\tfrac{1}{2}\bar{p}\not{\partial}p-\tfrac{1}{2}(\partial_\mu a)^2-\tfrac{1}{2}(\partial_\mu b)^2-\bar{\psi}\not{D}\psi\\ & -(D_\mu\Phi')^\dagger(D_\mu\Phi')-(D_\mu\Phi'')^\dagger(D_\mu\Phi'')\\ & +\sqrt{2}\left[\bar{\psi}_{\mathrm{R}}(g\lambda+\tfrac{1}{2}g'\lambda'\mathbb{1})\Phi'+\bar{\psi}_{\mathrm{L}}(g\lambda+\tfrac{1}{2}g'\lambda'\mathbb{1})\Phi''+\mathrm{h.c.}\right]\\ & -\sqrt{\tfrac{1}{2}}\,ih\left[\bar{\psi}_{\mathrm{R}}p\Phi''+\bar{\psi}_{\mathrm{L}}p\Phi'-\mathrm{h.c.}\right]\\ & -\tfrac{1}{2}h\bar{\psi}\,(a-\gamma_5 b)\psi-V(\Phi',\Phi'',a,b)\,,\end{aligned}\tag{4.2}$$

where we have used matrix notation for SU(2) triplet fields, $A_\mu=\frac{1}{2}A_\mu^a\tau^a$, $\lambda=\frac{1}{2}\lambda^a\tau^a$ and $D_\mu\lambda=\partial_\mu\lambda+ig[A_\mu,\lambda]$, $D_\mu\Phi'=\partial_\mu\Phi'+\frac{1}{2}i(g\boldsymbol{\tau}\cdot A_\mu+g'A'_\mu\mathbb{1})\Phi'$, etc. The potential V looks like

$$\begin{aligned}V= & \tfrac{1}{2}g^2(\Phi'^\dagger\Phi'\Phi''^\dagger\Phi''-\Phi'^\dagger\Phi''\Phi''^\dagger\Phi')\\ & +\tfrac{1}{8}(g^2+g'^2)(\Phi''^\dagger\Phi''-\Phi'^\dagger\Phi')^2+\tfrac{1}{4}(a^2+b^2)h^2\\ & \times(\Phi'^\dagger\Phi'+\Phi''^\dagger\Phi'')+\tfrac{1}{2}|h\Phi''^\dagger\Phi'+s|^2.\end{aligned}\tag{4.3}$$

In order to break the gauge symmetry SU(2)$\times$U(1) to U(1), we choose a vacuum which preserves global supersymmetry:

$$\langle\Phi'\rangle=\langle\Phi''\rangle=\left(\sqrt{\tfrac{1}{2}}\,F,0\right),\qquad 2s+hF^2=0.\tag{4.4}$$

This would generate masses for various physical fields via the usual Higgs mechanism.

In order to see the physical mass spectrum, we adopt the following definitions:

$$\begin{aligned}&\sqrt{2}\,\varphi=(\varphi'_0+\varphi''^*_0)=\left(\sqrt{2}\,F+H_1+i\eta_1\right),\\ &\sqrt{2}\,\chi=(-\varphi'_0+\varphi''^*_0)=H_2+i\eta_2,\\ &\sqrt{2}\,\omega_-=\varphi'_--\varphi''_-,\qquad \sqrt{2}\,\omega'_-=\varphi'_-+\varphi''_-,\\ &\sqrt{2}\,\omega=-i(a-ib).\end{aligned}\tag{4.5}$$

The real fields H_1, η_1, H_2 and complex fields ω and ω_- are physical scalars, η_2 and

ω'_- are the Goldstone ghosts. The physical vector fields are

$$W_\mu^{\pm} = \sqrt{\tfrac{1}{2}}\left(A_\mu^1 \pm iA_\mu^2\right),$$

$$Z_\mu = A_\mu^3 \cos\theta + A'_\mu \sin\theta, \qquad A_\mu = -A_\mu^3 \cos\theta + A'_\mu \sin\theta,$$

$$\tan\theta = g'/g. \tag{4.6}$$

The physical fermionic fields are

$$e_- = -\lambda_{-\mathrm{L}} + \psi_{-\mathrm{R}}, \qquad E_- = \psi_{-\mathrm{L}} - \lambda_{-\mathrm{R}},$$

$$e_0 = ip_{\mathrm{L}} + \sqrt{\tfrac{1}{2}}\,(\psi_0 - \psi_0^*)_{\mathrm{R}},$$

$$E_0 = \sqrt{\tfrac{1}{2}}\,(\psi_0 + \psi_0^*)_{\mathrm{L}} - (\lambda^3 \cos\theta + \lambda' \sin\theta)_{\mathrm{R}},$$

$$\nu_{\mathrm{L}} = (-\lambda^3 \sin\theta + \lambda' \cos\theta)_{\mathrm{L}}. \tag{4.7}$$

The real scalar fields, H_1, η_1 with the complex scalar ω and the neutral fermion e_0 form a scalar supermultiplet of mass, $m = \sqrt{\frac{1}{2}}\,hF$; the complex scalar field ω_- and charged fermions e_-, E_- with vector fields $W_\mu^{\pm}$ form a vector supermultiplet of mass $M_{\mathrm{W}} = \sqrt{\frac{1}{2}}\,Fg$; real scalar field H_2 and neutral fermion E_0 with the vector field Z_μ form a vector supermultiplet with mass, $M_Z = F(\frac{1}{2}(g^2 + g'^2))^{1/2}$; finally we have a massless vector supermultiplet containing the left-handed fermion ν_{L} and the photon A_μ.

The effective lagrangian in terms of these physical fields and the gauge fixing and the Faddeev-Popov ghost lagrangians are given in appendix C.

In order to study the quadratic divergence content of the model, we have to compute the corrections to the vacuum expectation values of the various physical scalar fields due to the tadpole graphs. The relevant H_1 tadpole diagrams have been listed in fig. 11. There are three types of H_1 tadpole graphs: (i) Tadpoles with fields H_1, η_1, ω and e_0, each of mass m, going through the loop. These give quadratically divergent contribution $A(m^2)$ with *weights* $-\frac{3}{4}hm$, $-\frac{1}{4}hm$, $-hm$, and $2hm$, respectively, so that there is no net quadratic divergence. (ii) Tadpoles with fields ω_-, ω'_-, W_μ^-, e_-, E_-, C^+ and C^-, each of mass M_{W}, going around the loop. These contribute the quadratic divergence $A(M_{\mathrm{W}}^2)$ with weights $-gM_{\mathrm{W}} + \frac{1}{2}hm$, $-\frac{1}{2}hm$, $-4gM_{\mathrm{W}}$, $2gM_{\mathrm{W}}$, $2gM_{\mathrm{W}}$, $\frac{1}{2}gM_{\mathrm{W}}$ and $\frac{1}{2}gM_{\mathrm{W}}$, respectively, so that these add up to zero. (iii) Tadpoles with fields, H_2, η_2, Z_μ, E_0 and C_Z going around the loop. These yield the contribution $A(M_Z^2)$ with weights, $\frac{1}{2}hm$, $-\frac{1}{2}M_Z\sqrt{g^2 + g'^2}$, $-\frac{1}{2}hm$, $-2M_Z\sqrt{g^2 + g'^2}$, $2M_Z\sqrt{g^2 + g'^2}$ and $\frac{1}{2}M_Z\sqrt{g^2 + g'^2}$ respectively, so that their sum is also zero. Similarly, we notice that the net tadpole contributions for the other scalar

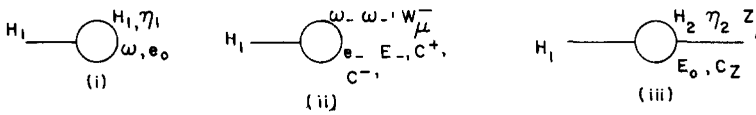

Fig. 11. H_1 field tadpole graphs.

fields vanish. Thus the vacuum expectation values of various scalar fields receive no one-loop corrections. This ensures the naturalness of the model.

Now we present the quadratic divergence content of the one loop mass corrections for one field from each of the three supermultiplets of masses m, M_W and M_Z. For this purpose we choose H_1, ω_- and H_2.

Self-energy of H_1: The only graphs that contain quadratic divergences are given in fig. 12. All other graphs have only logarithmic divergences. Underneath each diagram we have indicated the weight of the quadratically divergent integrals, $A(m^2)$ for set I, $A(M_W^2)$ for set II and $A(M_Z^2)$ for set III of the diagrams. Besides these, some of these diagrams may also have logarithmic divergences.

As can be read from the fig. 12, the weights in each one of the three sets of diagrams add up to zero; thus there are no quadratically divergent corrections to the H_1 mass from the self-energy diagrams.

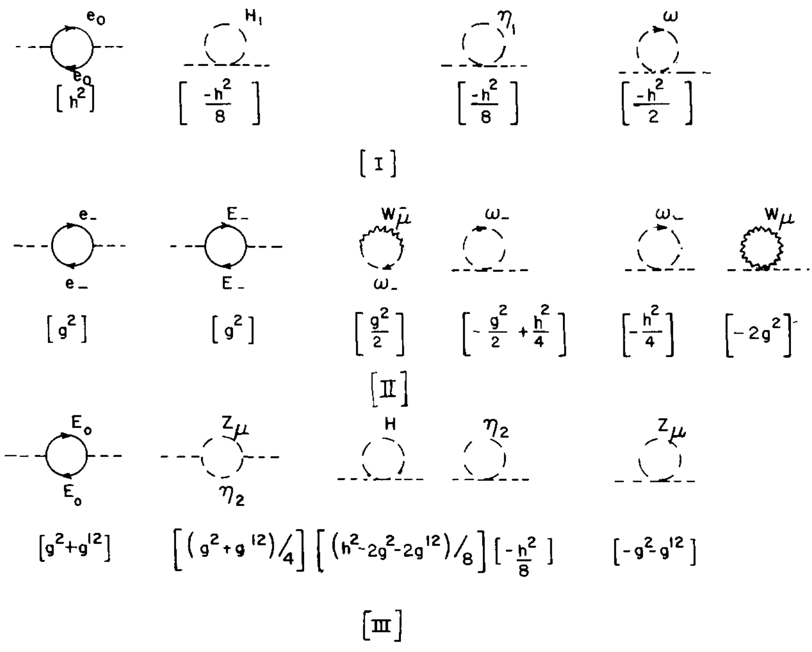

Fig. 12. Quadratically divergent contributions to H_1 field mass squared. The numbers underneath each graph have to be multiplied by $A(m^2)$ in set I, by $A(M_W^2)$ in set II and by $A(M_Z^2)$ in set III, to get the quadratically divergent contribution from the graph.

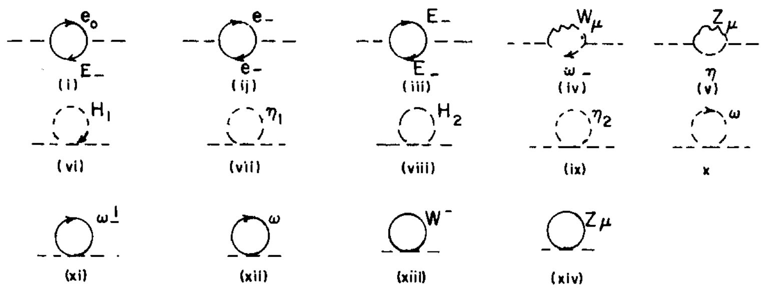

Fig. 13. Quadratically divergent contributions to H_2 self-energy.

Self-energy of H_2: The graphs which contain quadratic divergence are listed in fig. 13. The individual contributions are as below:

(i): $\frac{1}{2}(g^2+g'^2+h^2)[A(m^2)+A(M_Z^2)]$, (viii): $-\frac{3}{8}h^2A(M_Z^2)$,

(ii): $g^2A(M_W^2)$, (ix): $-\frac{1}{8}h^2A(M_Z^2)$,

(iii): $g^2A(M_W^2)$, (x): $\frac{1}{4}h^2A(M_W^2)$,

(iv): $\frac{1}{2}g^2A(M_W^2)$, (xi): $-(\frac{1}{2}g^2+\frac{1}{4}h^2)A(M_W^2)$,

(v): $\frac{1}{4}(g^2+g'^2)[2A(M_Z^2)-A(m^2)]$, (xii): $-\frac{1}{2}h^2A(m^2)$,

(vi): $(\frac{1}{8}h^2-\frac{1}{4}[g^2+g'^2])A(m^2)$, (xiii): $-2g^2A(M_W^2)$,

(vii): $-\frac{1}{8}h^2A(m^2)$, (xiv): $-(g^2+g'^2)A(M_Z^2)$.

Besides these, of course, there are logarithmic divergent contributions, which we have ignored for our present discussion. As can be seen clearly from the above contributions, the net quadratic divergent corrections to the H_2 mass at one loop level are zero.

Self-energy of ω_-: The graphs that contain quadratic divergences are given in fig. 14. The individual contributions, ignoring the logarithmically divergent part, are as follows:

(i): $\frac{1}{4}g^2[A(M_W^2)+A(M_Z^2)]$, (xii): $\dfrac{g^2g'^2}{g^2+g'^2}[2A(0)-A(M_W^2)]$,

(ii): $\frac{1}{4}(g^2+h^2)[A(M_W^2)+A(m^2)]$, (xiii): $-2g^2A(M_W^2)$,

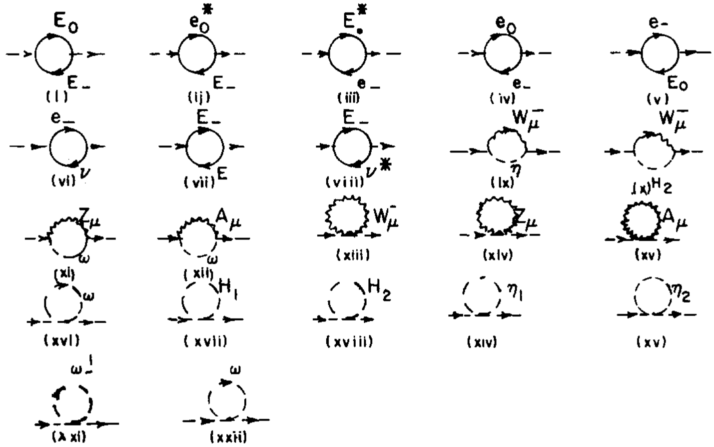

Fig. 14. Quadratically divergent contributions to ω_- self-energy.

(iii): $\frac{1}{4}g^2\left[A\left(M_W^2\right)+A\left(M_Z^2\right)\right]$, (xiv): $-\frac{\left(g^2-g'^2\right)^2}{g^2+g'^2}A\left(M_Z^2\right)$,

(iv): $\frac{1}{4}\left(g^2+h^2\right)\left[A\left(M_W^2\right)+A\left(m^2\right)\right]$, (xv): $-\frac{4g^2g'^2}{g^2+g'^2}A(0)$,

(v): $\frac{\left(g^2-g'^2\right)^2}{4\left(g^2+g'^2\right)}\left[A\left(M_W^2\right)+A\left(M_Z^2\right)\right]$, (xvi): $-\frac{1}{2}h^2A\left(M_W^2\right)$,

(vi): $\frac{g^2g'^2}{g^2+g'^2}\left[A\left(M_W^2\right)+A(0)\right]$, (xvii): $\left(\frac{1}{8}h^2-\frac{1}{4}g^2\right)A\left(m^2\right)$,

(vii): $\frac{\left(g^2-g'^2\right)^2}{4\left(g^2+g'^2\right)}\left[A\left(M_Z^2\right)+A\left(M_W^2\right)\right]$, (xix): $\frac{1}{8}h^2A\left(M_Z^2\right)$,

(viii): $\frac{g^2g'^2}{g^2+g'^2}\left[A(0)+A\left(M_W^2\right)\right]$, (xx): $-\frac{1}{8}h^2A\left(m^2\right)$,

(ix): $\frac{1}{4}g^2\left[2A\left(M_W^2\right)-A\left(m^2\right)\right]$, (xxi): $-\left(\frac{1}{8}h^2+\frac{1}{4}g^2\right)A\left(M_Z^2\right)$,

(x): $\frac{1}{4}g^2\left[2A\left(M_W^2\right)-A\left(M_Z^2\right)\right]$, (xxii): $-\frac{1}{4}\left(g^2+g'^2\right)A\left(M_W^2\right)$,

(xi): $\frac{\left(g^2-g'^2\right)^2}{4\left(g^2+g'^2\right)}\left[2A\left(M_Z^2\right)-A\left(M_W^2\right)\right]$, (xxii): $-\frac{1}{2}h^2A\left(m^2\right)$.

As can be seen from these expressions, the total quadratically divergent corrections to the mass of ω_- is zero.

To conclude this section, we have demonstrated that in a globally supersymmetric anomaly free model with spontaneously broken non-abelian gauge symmetry, quadratically divergent corrections to the masses of the various physical fields are absent, implying, thereby, that the model is natural.

5. Conclusions

We have presented our investigations on naturalness in globally supersymmetric SBGT, by evaluating one-loop quadratically divergent mass corrections for the physical fields. We observed that these corrections do not vanish in a model with an ABJ anomaly, even though the mass degeneracy within a supermultiplet is retained at the one-loop level. Addition of extra supermultiplets, to cancel the ABJ anomaly, also suffices to eliminate the quadratic divergences. One is led to conclude that anomaly-free, supersymmetric SBGT are perhaps natural in general. Cancellation of quadratic divergences in an anomaly free supersymmetric non-abelian gauge theory with spontaneoulsy broken $SU(2) \times U(1)$ symmetry lends powerful support to this idea. However, a model-independent proof is lacking at this stage.

As stated in the introduction, our results have important ramifications for grand unified theories, especially with regard to the so-called gauge hierarchy problem [14]. Further, it is worthwhile to investigate the effect, on phenomenological predictions like the proton lifetime or the value of $\sin^2 \theta_W$, of embedding a grand unified theory in a supersymmetric model. Some results in this direction have already been obtained [11]. In these models, supersymmetry is assumed to be respected (or softly broken) above mass scales of O(1 TeV). Below 1 TeV, one must break supersymmetry in order to produce mass splittings in accord with observed or predicted low energy phenomenological results. To the best of our knowledge, no satisfactory model that achieves this, has so far been proposed [12].

Thanks are due to our colleagues at the Tata Institute of Fundamental Research, Bombay and the Centre for Theoretical Studies, Indian Institute of Science, Bangalore for many helpful discussions. RKK would also like to thank Virendra Singh for hospitality at the TIFR.

Note added

After this paper was submitted for publication, M. Sohnius brought to our attention the papers of Witten [15] and Fischler et al. [16] which discuss the generation, at one-loop order, of a quadratically divergent Fayet-Illiopoulos D-term in left-right asymmetric supersymmetric U(1) gauge theories. Fischler et al., have shown that (i) this term is proportional to $\mathrm{Tr}\, Q$, where Q is the U(1) charge, and (ii) it appears only at the one-loop level. Thus $\mathrm{Tr}\,Q=0$ is a necessary and sufficient condition for naturalness. This is exactly the same as our condition for naturalness in supersymmetric theories, viz., vanishing of the ABJ anomaly. (Their result (ii) is presumably related to the Adler-Bardeen theorem.) Witten has shown that if the U(1) can be embedded in a non-abelian group G, such as in a GUT scenario, invariance under G forbids any appearance of any D-term. This is so because $\mathrm{Tr}\,Q=0$ holds in such situations. This discussion is also valid even if supersymmetry is spontaneously or softly broken [17].

Appendix A

A.1. U(1) MODEL WITH ABJ ANOMALY

We choose to work in the generalized 't Hooft-Feynman gauge, $\mathcal{L}_{\text{gauge fixing}} = -\frac{1}{2}(\partial_\mu A_\mu + m\eta)^2$. We also have a corresponding Faddeev-Popov ghost langrangian. The total effective lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & -\tfrac{1}{4}F_{\mu\nu}^2 - \bar{E}\not{\partial}E - \tfrac{1}{2}(\partial_\mu H)^2 - \tfrac{1}{2}(\partial_\mu\eta)^2 \\ & -\tfrac{1}{2}m^2A_\mu^2 - \tfrac{1}{2}m^2H^2 - m\bar{E}E \\ & + gA_\mu(\eta\partial_\mu H - H\partial_\mu\eta) - mA_\mu\partial_\mu\eta \\ & -\tfrac{1}{2}g^2A_\mu^2\left(H^2 + 2\frac{m}{g}H + \eta^2\right) - \tfrac{1}{8}g^2(H^2+\eta^2)^2 \\ & -\tfrac{1}{2}gmH(H^2+\eta^2) - g\bar{E}(H + i\gamma_5\eta)E - ig\bar{E}_{\mathrm{L}}\not{A}E_{\mathrm{L}} \\ & -\tfrac{1}{2}(\partial_\mu A_\mu + m\eta)^2 + c^*[\Box - m^2 - gmH]c\,. \end{aligned} \tag{A.1}$$

A.2. U(1) MODEL WITHOUT ABJ ANOMALY

With the same gauge-fixing term as above, the effective lagrangian is

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & -\tfrac{1}{4}F_{\mu\nu}^2 - \bar{e}\not{\partial}e - \bar{E}\not{\partial}E - |\partial_\mu\omega|^2 \\ & -\tfrac{1}{2}(\partial_\mu H)^2 - \tfrac{1}{2}(\partial_\mu\eta)^2 - \tfrac{1}{2}(\partial_\mu H_1)^2 - \tfrac{1}{2}(\partial_\mu H_2)^2 \\ & + gA_\mu[(\eta\partial_\mu H - H\partial_\mu\eta) + (H_2\partial_\mu H_1 - H_1\partial_\mu H_2)] \\ & -\tfrac{1}{2}gA_\mu^2[(m+gH)^2 + g^2(\eta^2 + H_1^2 + H_2^2)]\end{aligned}$$

$$
\begin{aligned}
& -\tfrac{1}{8}g^2\left(H_1^2+H_2^2+H^2+\eta^2+\frac{2m}{g}H\right)^2-\tfrac{1}{2}m^2\left(H_1^2+H_2^2\right) \\
& -|\omega|^2\left[g^2\left(H_1^2+H_2^2+\eta^2\right)+(m+gH)^2\right] \\
& -ig\left[\bar{e}_{\mathrm{R}}\not{A}e_{\mathrm{R}}+\bar{E}_{\mathrm{L}}\not{A}E_{\mathrm{L}}\right]-\left[(\bar{E}E+\bar{e}e)(m+gH)-g\left(\bar{E}\gamma_5E-\bar{e}\gamma_5e\right)\eta\right] \\
& +ig\sqrt{2}\left[\bar{e}_{\mathrm{R}}\omega E_{\mathrm{L}}-\bar{E}_{\mathrm{L}}\omega^*e_{\mathrm{R}}\right]+ig\left[H_1\left(\bar{E}\gamma_5e^*-\bar{e}^*\gamma_5E\right)+iH_2\left(\bar{E}\gamma_5e^*+\bar{e}^*\gamma_5E\right)\right] \\
& -\tfrac{1}{2}\left(\partial_\mu A_\mu\right)^2-\tfrac{1}{2}m^2\eta^2+c^*\Box c-m^2c^*c-gmc^*Hc. \qquad \text{(A.2)}
\end{aligned}
$$

Appendix B

Our computations typically involve the following set of integrals in n dimensions [11]:

$$A(m^2)=\int\frac{\mathrm{d}^nk}{(2\pi)^n}\frac{1}{k^2+m^2}, \qquad \text{(B.1)}$$

$$
\begin{aligned}
& B_0(p,m_1,m_2);\ B_\mu(p,m_1,m_2);\ B_{\mu\nu}(p,m_1,m_2) \\
& \quad =\int\frac{\mathrm{d}^nk}{(2\pi)^n}\frac{1;\ k_\mu;\ k_\mu k_\nu}{\left(k^2+m_1^2\right)\left[(p+k)^2+m_2^2\right]}, \qquad \text{(B.2)}
\end{aligned}
$$

$$
\begin{aligned}
& B_\mu(p,m_1,m_2)\equiv p_\mu B_1(p,m_1,m_2); \\
& B_{\mu\nu}(p,m_1,m_2)\equiv p_\mu p_\nu B_{21}(p,m_1,m_2)+\delta_{\mu\nu}B_{22}(p,m_1,m_2). \qquad \text{(B.3)}
\end{aligned}
$$

Primitive quadratic divergences are identified with poles at $n=2$. It turns out that $\mathrm{Res}[B_{22}(p,m_1,m_2)]_{n=2}=\frac{1}{2}\mathrm{Res}[A(m^2)]_{n=2}$. Also, $B_1(p,m_1,m_2)$ is related to $B_0(p,m_1,m_2)$ by

$$-p^2B_1(p,m_1,m_2)=\tfrac{1}{2}\left[A\left(m_2^2\right)-A\left(m_1^2\right)+\left(p^2+m_2^2+m_1^2\right)B_0(p,m_1,m_2)\right]. \qquad \text{(B.4)}$$

Additional identities that are useful are

$$
\begin{aligned}
& p^2B_{21}+B_{22}=\tfrac{1}{2}\left\{A\left(m_2^2\right)+\left(m_1^2-m_2^2-p^2\right)B_1\right\}, \\
& A\left(m_2^2\right)-m_1^2B_0=p^2B_{21}+4B_{22}+\tfrac{1}{2}\left(m_1^2+m_2^2+\tfrac{1}{3}p^2\right). \qquad \text{(B.5)}
\end{aligned}
$$

It is clear that with the help of these identities, the results can be expressed entirely in terms of $A(m^2)$ and $B_0(p, m_1, m_2)$ alone.

Appendix C

Here we present the lagrangian for the SU(2)×U(1) gauge model in terms of physical fields. The lagrangian with quadratic parts is

$$\begin{aligned}
\mathcal{L}_0 = & -\tfrac{1}{2}\operatorname{Tr} F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{4}F'_{\mu\nu}F'_{\mu\nu} - \bar{e}_-(\not\partial + M_W)e_- \\
& - \bar{E}_-(\not\partial + M_W)E_- - \bar{e}_0(\not\partial + m)e_0 - \bar{E}_0(\not\partial + M_Z)E_0 \\
& - \bar{\nu}_L\not\partial\nu_L - \tfrac{1}{2}(\partial_\mu H_1)^2 - \tfrac{1}{2}(\partial_\mu\eta_1)^2 - \tfrac{1}{2}(\partial_\mu H_2)^2 \\
& - \tfrac{1}{2}(\partial_\mu\eta_2)^2 - (\partial_\mu\omega^*)(\partial_\mu\omega) - \partial_\mu\omega_+\,\partial_\mu\omega_- \\
& - \partial_\mu\omega'_+\,\partial_\mu\omega'_- - \tfrac{1}{2}m^2\left(H_1^2 + \eta_1^2 + 2\omega^*\omega\right) \\
& - M_W^2\left(W_\mu^+W_\mu^- + \omega_+\omega_-\right) \\
& - \tfrac{1}{2}M_Z^2\left(H_2^2 + Z_\mu Z_\mu\right).
\end{aligned} \tag{C.1}$$

The interaction lagrangian is

$$\begin{aligned}
\mathcal{L}_{\rm int} = & \tfrac{1}{2}\sqrt{g^2+g'^2}\,Z_\mu\left(\sqrt{2}\,F\partial_\mu\eta_2 + H_1\partial_\mu\eta_2 - \eta_2\partial_\mu H_1 + H_2\partial_\mu\eta_1 - \eta_1\partial_\mu H_2\right) \\
& - \tfrac{1}{2}i\left[\frac{(g'^2-g^2)Z_\mu + 2gg'A_\mu}{\sqrt{g^2+g'^2}}\right]\left(\omega_-\partial_\mu\omega_+ - \omega_+\partial_\mu\omega_- + \omega'_-\partial_\mu\omega'_+ - \omega'_+\partial_\mu\omega'_-\right) \\
& - \tfrac{1}{2}g\Big\{W_\mu^+\Big[\omega_-\partial_\mu(\eta_1 - iH_2) - (\eta_1 - iH_2)\partial_\mu\omega - \omega'_-\partial_\mu(\eta_2 - iH_1) \\
& \qquad + (\eta_2 - iH_1)\partial_\mu\omega'_- - i\sqrt{2}\,F\partial_\mu\omega'_- + \text{c.c.}\Big]\Big\} \\
& - \tfrac{1}{8}(g^2+g'^2)\left[\left(\sqrt{2}\,F + H_1\right)^2 + \eta_1^2 + H_2^2 + \eta_2^2\right]Z_\mu Z_\mu \\
& - \frac{1}{4}\left[\frac{(g'^2-g^2)Z_\mu + 2gg'A_\mu}{\sqrt{g^2+g'^2}}\right]^2\left(\omega_+\omega_- + \omega'_+\omega'_-\right)
\end{aligned}$$

$$-\tfrac{1}{4}g^2 W_\mu^+ W_\mu^- \left[\left(\sqrt{2}\,F+H_1\right)^2+\eta_1^2+H_2^2+\eta_2^2+2(\omega_+\omega_-+\omega'_+\omega'_-)\right]$$

$$-\tfrac{1}{2}gg'\left[\frac{g'Z_\mu+gA_\mu}{\sqrt{g^2+g'^2}}\right]\left\{W_\mu^-\left[-(H_2+i\eta_1)\omega_-+\left(\sqrt{2}\,F+H_1+i\eta_2\right)\omega'_-+\text{c.c.}\right]\right\}$$

$$+\left\{\tfrac{1}{2}i\sqrt{g^2+g'^2}\,\overline{e_{0\mathrm{L}}^*}\not{Z}E_{0\mathrm{L}}-\tfrac{1}{2}ig\left[\overline{E}_{-\mathrm{L}}\not{W}^-(E_0-e_0^*)_\mathrm{L}\right.\right.$$

$$\left.\left.+\overline{e_{-\mathrm{R}}}\not{W}^-(E_0^*+e_0)_\mathrm{R}\right]+\text{h.c.}\right\}$$

$$-\sqrt{\tfrac{1}{2}}\,i\left(\frac{1}{g^2+g'^2}\right)^{1/2}\left[(g'^2-2g^2)\left(E_{-\mathrm{L}}\not{Z}E_{-\mathrm{L}}+\overline{e_{-\mathrm{R}}}\not{Z}e_{-\mathrm{R}}\right)\right.$$

$$\left.+gg'\left(\overline{E}_{-\mathrm{L}}\not{A}E_{-\mathrm{L}}+\overline{e_{-\mathrm{R}}}\not{A}e_{-\mathrm{R}}\right)\right]$$

$$-\frac{ig}{\sqrt{g^2+g'^2}}\left\{\left(g\overline{E_{0\mathrm{L}}^*}+g'\nu_\mathrm{L}\right)W_\mu^+e_{-\mathrm{L}}+(gE_{0\mathrm{R}}-g'\nu_\mathrm{R}^*)W_\mu^+E_{-\mathrm{R}}+\text{h.c.}\right\}$$

$$-\frac{ig}{\sqrt{g^2+g'^2}}\left\{\bar{e}_{-\mathrm{L}}(g\not{Z}-g'\not{A})e_{-\mathrm{L}}+\overline{E}_{-\mathrm{R}}(g\not{Z}-g'\not{A})E_{-\mathrm{R}}\right\}$$

$$-\frac{ih}{2\sqrt{2}}\left\{\overline{E_{0\mathrm{R}}^*}\omega E_{0\mathrm{L}}-\bar{e}_{0\mathrm{R}}\omega e_{0\mathrm{L}}^*+2\overline{e_{-\mathrm{R}}}\omega E_{-\mathrm{L}}-\text{h.c.}\right\}$$

$$-\tfrac{1}{2}\sqrt{g^2+g'^2}\left\{\overline{E}_0(H_1+\gamma_5\eta)E_0+\bar{e}_{0\mathrm{R}}(H_2+i\eta_2)E_{0\mathrm{L}}^*+\overline{E_{0\mathrm{L}}^*}(H_2-i\eta_2)e_{0\mathrm{R}}\right\}$$

$$-\tfrac{1}{2}g\left\{\bar{e}_-(H_1+\gamma_5\eta_1-H_2-\gamma_5\eta_2)e_-+\overline{E}_-(H_1+\gamma_5\eta_1+H_2+\gamma_5\eta_2)E_-\right\}$$

$$-\tfrac{1}{2}g\left\{\overline{E}_{-\mathrm{R}}(E_0+e_0^*)_\mathrm{L}(\omega_-+\omega'_-)+\bar{e}_{-\mathrm{L}}(E_0^*-e_0)_\mathrm{R}(-\omega_-+\omega'_-)+\text{h.c.}\right\}$$

$$+\tfrac{1}{2}\left(\frac{1}{g^2+g'^2}\right)^{1/2}\left\{\left[(g^2-g'^2)\bar{e}_{-\mathrm{R}}E_{0\mathrm{L}}^*+2gg'\bar{e}_{-\mathrm{R}}\nu_\mathrm{L}\right](\omega_-+\omega'_-)\right.$$

$$\left.+\left[(g^2-g'^2)\overline{E}_{-\mathrm{L}}E_{0\mathrm{R}}+2gg'\overline{E}_{-\mathrm{L}}\nu_\mathrm{R}^*\right](-\omega_-+\omega'_-)+\text{h.c.}\right\}$$

$$-\tfrac{1}{2}h\left\{\bar{e}_0(H_1-\gamma_5\eta_1)e_0+\overline{E_{0\mathrm{R}}^*}(H_2-i\eta_2)e_{0\mathrm{L}}+\bar{e}_{-\mathrm{R}}e_{0\mathrm{L}}(-\omega_-+\omega'_-)\right.$$

$$\left.-\overline{E_{-\mathrm{L}}}e_{0\mathrm{R}}^*(\omega_-+\omega'_-)+\text{h.c.}\right\}$$

$$
\begin{aligned}
&-\tfrac{1}{2}gM_{\mathrm{W}}[2\omega_+\omega_-H_1+\omega'_+\omega_-(H_2+i\eta_1)+\omega_+\omega'_-(H_2-i\eta_1)] \\
&-\tfrac{1}{2}(g^2+g'^2)^{1/2}M_{\mathrm{Z}}H_2(H_1H_2+\eta_1\eta_2-\omega'_+\omega_- -\omega_+\omega'_-) \\
&-\tfrac{1}{4}hm\{H_1(H_1^2+\eta_1^2-H_2^2+\eta_2^2-2\omega_+\omega_- +2\omega'_+\omega'_-) \\
&\qquad -2\eta_1(H_2\eta_2+i\omega'_+\omega_- -i\omega_+\omega'_-)\} \\
&-\tfrac{1}{4}g^2|\omega_-(H_1-i\eta_2)+\omega'_-(H_2-i\eta_1)|^2 \\
&-\tfrac{1}{4}h^2\omega^*\omega\{H_1^2+\eta_1^2+H_2^2+\eta_2^2+2\omega_+\omega_- +2\omega'_+\omega'_-\} \\
&-\tfrac{1}{32}h^2|(H_1-i\eta_1)^2-(H_2-i\eta_2)^2-2(\omega_+ +\omega'_+)(\omega_- -\omega'_-)|^2 \\
&-\tfrac{1}{8}(g^2+g'^2)[H_1H_2+\eta_1\eta_2-\omega'_+\omega_- -\omega_+\omega'_-]^2 .
\end{aligned}
\tag{C.2}
$$

To this lagrangian we add the gauge ('t Hooft-Feynman gauge) fixing terms and the Faddeev-Popov lagrangians as follows:

$$
\begin{aligned}
\mathcal{L}_{\text{gauge}} &= |\partial_\mu W_\mu^+ + iM_{\mathrm{W}}\omega'_+|^2-\tfrac{1}{2}\left(\partial_\mu A_\mu^3-M_{\mathrm{W}}\eta_2\right)^2 \\
&\quad -\left(\tfrac{1}{2}\partial_\mu A'_\mu-\frac{g'}{g}M_{\mathrm{W}}\eta_2\right)^2,
\end{aligned}
\tag{C.3}
$$

and

$$
\begin{aligned}
\mathcal{L}_{\text{F.P.}} &= C^{+*}[\Box-M_{\mathrm{W}}^2-\tfrac{1}{2}M_{\mathrm{W}}g(H_1+i\eta_2)]C^+ \\
&\quad +C^{-*}[\Box-M_{\mathrm{W}}^2-\tfrac{1}{2}gM_{\mathrm{W}}(H_1-i\eta_2)]C^- \\
&\quad +C_{\mathrm{Z}}^*\left[\Box-M_{\mathrm{Z}}^2-\tfrac{1}{2}M_{\mathrm{Z}}(g^2+g'^2)^{1/2}H_1\right]C_{\mathrm{Z}}+C_{\mathrm{A}}^*\Box C_{\mathrm{A}} \\
&\quad -ig(\partial_\mu C^{+*}C^+ -\partial_\mu C^{-*}C^-)(Z_\mu\cos\theta-A_\mu\sin\theta) \\
&\quad +ig\left[\partial_\mu C^{+*}(C_{\mathrm{Z}}\cos\theta-C_{\mathrm{A}}\sin\theta)-(\partial_\mu C_{\mathrm{Z}}^*\cos\theta-\partial_\mu C_{\mathrm{A}}^*\sin\theta)C^-\right]W_\mu^+ \\
&\quad -ig\left[\partial_\mu C^{-*}(C_{\mathrm{Z}}^*\cos\theta-C_{\mathrm{A}}^*\sin\theta)-(\partial_\mu C_{\mathrm{Z}}^*\cos\theta-\partial_\mu C_{\mathrm{A}}^*\sin\theta)C^+\right]W_\mu^- \\
&\quad +\tfrac{1}{2}gM_{\mathrm{W}}(C^{+*}\omega'_+ +C^{-*}\omega'_-)(C_{\mathrm{Z}}\cos\theta-C_{\mathrm{A}}\sin\theta) \\
&\quad -\tfrac{1}{2}gM_{\mathrm{W}}(C_{\mathrm{Z}}^*\cos\theta-C_{\mathrm{A}}^*\sin\theta)(\omega'_+C^- +\omega'_-C^+) \\
&\quad -\tfrac{1}{2}g'M_{\mathrm{W}}(C^{+*}\omega'_+ +C^{-*}\omega'_-)(C_{\mathrm{Z}}\sin\theta+C_{\mathrm{A}}\cos\theta) \\
&\quad -\tfrac{1}{2}g'M_{\mathrm{W}}(C_{\mathrm{Z}}^*\sin\theta+C_{\mathrm{A}}^*\cos\theta)(C^+\omega'_- +C^-\omega'_+) .
\end{aligned}
\tag{C.4}
$$

References

[1] K. Wilson, Phys. Rev. D3 (1971) 1818;
L. Susskind, Phys. Rev. D20 (1979) 2619
[2] G. 't Hooft, Cargese Lectures (1979)
[3] M. Veltman, Michigan preprint (1980);
S. Weinberg, Phys. Lett. 82B (1979) 387
[4] R. Kaul and P. Majumdar, CTS, IISc. Bangalore preprint (1981)
[5] P. Fayet, Nuovo Cim. 31A (1976) 626
[6] W. Siegel, Phys. Lett. 84B (1979) 193
[7] P. Majumdar, E.C. Poggio and H.J. Schnitzer, Phys. Rev. D21 (1980) 2203;
D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, Nucl. Phys. B167 (1980) 479
P. Majumdar, Ph.D dissertation, unpublished, Brandeis University (1980)
[8] G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189
[9] P. Fayet, Nucl. Phys. B113 (1976) 135
[10] P. Fayet, Nucl. Phys. B90 (1975) 104
[11] N. Sakai, Tohuku Univ. preprint TU/81/225;
S. Dimopoulos, S. Raby and F. Wilczek, preprint NSF-ITP-81-31
[12] P. Fayet and S. Ferrara, Phys. Reports 32 (1977) 249
[13] E. Gildener, Phys. Lett. 92B (1980) 111
[14] R. Kaul, CTS-TIFR preprint (1981)
[15] E. Witten, Nucl. Phys. B188 (1981) 513
[16] M.S. Fischler et al., Phys. Rev. Lett. 47 (1981) 757
[17] L. Girardello and M.T. Grisaru, Brandeis preprint (1981)