SPONTANEOUSLY BROKEN SUPERSYMMETRIC THEORIES OF WEAK, ELECTROMAGNETIC AND STRONG INTERACTIONS

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A new class of leptons, with its own quantum number carried by the supersymmetry generator, includes a Goldstone-neutrino, a photon-neutrino, gluon-neutrinos and heavy leptons. The Goldstone-and-photon-neutrinos can be produced in low-energy processes, but only in pairs. Ordinary leptons and quarks are associated with heavy scalar bosons under supersymmetry. The latter decay quickly into the corresponding leptons or quarks, emitting a Goldstone or a photon-neutrino. Some of the phenomenological implications are discussed.

Some attempts have been made already to find realistic applications of supersymmetry to particle physics [1, 2]. Among the first problems encountered were the spontaneous breaking of both gauge invariance and supersymmetry, and the conservation of an additive quantum number; the latter, carried by the spinorial generator in the supersymmetry algebra, is associated with **R**-invariance of supersymmetric theories [1, 3]. The model of ref. [1] described weak and electromagnetic interactions for one leptonic sector (interpreted at that time as the electronic sector); the neutrino, associated with the photon under supersymmetry, was the Goldstone fermion arising from spontaneous supersymmetry breaking [4].

The construction of a theory describing both leptons and quarks appeared to be a hard problem. Electron, muon and baryon number conservation laws have to be obtained. One must find a symmetry breaking mechanism for which the desired vacuum state (i.e. the one for which some of the symmetries are conserved, and some others are spontaneously broken) is a stable one; preferentially it should be fixed at the tree-approximation, in order to avoid the appearance of non-Goldstone bosons staying massless at zeroth order [5]. In addition the resulting mass spectrum and interactions have to be realistic; one of the main difficulties, together with the vacuum stability problem, is to obtain usual charged current weak interactions mediated by vector bosons, although the theory describes a large number of scalar bosons.

A possible solution for the latter problem is to use only chiral superfields to describe known leptons, and quarks, together with scalar bosons [2]. This must be combined with an appropriate symmetry breaking mechanism; in particular we would like supersymmetry to be spontaneously broken‡1.

Superfields. We use two types of Lorentz-invariant superfields, real and chiral (often called "vector" and "scalar" respectively).

A real superfield describes a gauge-vector-boson and a gauge-Majorana-spinor-fermion. For the gauge group we shall choose here the example of SU(2) $\times$ U$'$(1) $\times$ U$''$(1) $\times$ G_{color}; the corresponding superfields are denoted by $\vec{V}$, V', V'' and V_a. The superfields V_γ and V_a, associated with the unbroken $U(1)_{\text{QED}} \times G_{\text{color}}$ gauge group, describe the photon and gluons, together with a photon-neutrino λ_γ and gluon-neutrinos λ_a.

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‡1 A limiting procedure was used in ref. [2], for which the Goldstone spinor decoupled [6]; however in the limiting theory supersymmetry is broken explicitly, although softly. This is also the case in ref. [7], where the breaking is viewed as a spontaneous one in a non-physical sector; the latter mechanism is compatible with the super-Yukawa coupling inducing zeroth-order mass terms for electrons, muon and quarks in the model of ref. [2]. Let us also indicate a misprint in that paper; one of the terms of eq. (9) should be read:

$$(\Sigma_j T_j^\dagger \exp(-(2g\vec{T}\,\vec{V} + g'FV') + g''V'')T_j)_D$$

Table 1
Transformation properties of physical components of superfields

Superfield	Vector	Majorana or two-component Dirac spinor	Complex scalar
$\vec{V}$	$\vec{V}^{\mu} \to \vec{V}^{\mu}$	$\vec{\lambda} \to e^{\gamma_5 \alpha} \vec{\lambda}$	
V'	$V'^{\mu} \to V'^{\mu}$	$\lambda' \to e^{\gamma_5 \alpha} \lambda'$	
V''	$V''^{\mu} \to V''^{\mu}$	$\lambda'' \to e^{\gamma_5 \alpha} \lambda''$	
V_a	$V_a^{\mu} \to V_a^{\mu}$	$\lambda_a \to e^{\gamma_5 \alpha} \lambda_a$	
N		$\zeta \to e^{\gamma_5 \alpha} \zeta$	$n = \frac{a - ib}{\sqrt{2}} \to e^{2i\alpha}\, n$
S		$\psi_L \to e^{-i\alpha} \psi_L$	$s \to s$
T		$\psi_R \to e^{i\alpha} \psi_R$	$t \to t$
S_i		$\psi_{iL} \to e^{i\beta_i} \psi_{iL}$	$s_i \to e^{i(\beta_i + \alpha)} s_i$
T_j		$\psi_{jR} \to e^{i\beta_j} \psi_{jR}$	$t_j \to e^{i(\beta_j - \alpha)} t_j$

A chiral superfield describes a two-component Dirac spinor and a complex scalar. N, S and S_i, N^*, T and T_j, will be left-handed and right-handed superfields, respectively. S_i and T_j will describe known leptons, and quarks, together with heavy scalar particles; electron, muon, baryon, ... numbers will be defined as external quantum numbers carried by the chiral superfields S_i and T_j.

The chiral superfields S and T (as well as N), are colourless and do not carry external quantum numbers; their scalar components (s and t) will take non vanishing vacuum expectation values, leading to a spontaneous breaking of $SU(2) \times U'(1) \times U''(1)$ to a U(1) subgroup[‡2].

Besides external quantum numbers we have also one internal quantum number, associated with R-invariance of supersymmetric theories [1, 3]. They are defined by means of the following set of transformations:

$$\vec{V}(x, \theta, \bar{\theta}) \to \vec{V}(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha})$$
$$\text{(id. for } V', V'', V_a, S, T)$$
$$N(x, \theta, \bar{\theta}) \to e^{2i\alpha} N(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha}) \quad (1)$$
$$S_i(x, \theta, \bar{\theta}) \to e^{i(\beta_i + \alpha)} S_i(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha})$$
$$T_j(x, \theta, \bar{\theta}) \to e^{i(\beta_i - \alpha)} T_j(x, \theta e^{-i\alpha}, \bar{\theta} e^{i\alpha})$$

We list in table 1 the physical components of superfields and their transformation properties[‡3].

Lagrangian density. A supersymmetric action integral is constructed from auxiliary components (D, F or G) of superfields; it is restricted by gauge invariance and conservation of internal (R) and external quantum numbers.

We denote by $\mathcal{L}_0(\vec{V}, V', V'', V_a)$ the Lagrangian density for Yang-Mills or Abelian superfields alone. These interact with chiral superfields[‡4]. We write also terms proportional to F- and G-components of left-handed superfields, if they are gauge invariant and behave like N under (1) [1, 3]. $T^\dagger SN$ will be such a superfield, but not N itself (in the present example S and T are SU(2) doublets, and N is a SU(2) singlet not invariant under $U'(1) \times U''(1)$).

Owing to external quantum numbers conservation S_i and T_j can only appear through products $T_j^\dagger S_i$. We shall denote by $\mathcal{L}_Y(S, S_i, T, T_j)$ a linear combination of auxiliary components for the left-handed superfields $ST_j^\dagger S_i$, $T^\dagger T_j^\dagger S_i$ and $T_j^\dagger S_i$, when these are gauge-invariant. The complete Lagrangian density reads:

‡2 If we consider only the superfields $\vec{V}$, V', V'', S, T and N we obtain a model of weak and electromagnetic interactions for one leptonic sector, as in ref. [1]. The difference is that the Goldstone-neutrino and photon-neutrino (which were identical in ref. [1]) are now different in general.

‡3 A Majorana spinor λ is equivalent to a two-component Dirac spinor $\lambda_L = \frac{1}{2}(1 - i\gamma_5)\lambda$ (or $\lambda_R = \frac{1}{2}(1 + i\gamma_5)\lambda = (\lambda_L)^*$).

‡4 Note that the requirement of anomaly-cancellations would lead to complicate the model by the introduction of more superfields than used here.

$$\mathcal{L} = \mathcal{L}_0(\vec{V}, V', V'', V_a) + \xi' D' + \xi'' D''$$
$$+ \left\{ \sum_{S_I = S, S_i} S_I^\dagger (\exp(2g\vec{T}\vec{V} + g'F'V' + g''F''V'' + 2g_c R_a V_a)) S_I \right.$$
$$+ \sum_{T_J = T, T_j} T_J^\dagger (\exp - (2g\vec{T}\vec{V} + g'F'V' + g''F''V'' + 2g_c R_a V_a)) T_J$$
$$\left. + N^*(\exp(g'F'V' + g''F''V''))N \right\}_D - 2h[T^\dagger SN]_F + \mathcal{L}_Y(S, S_i, T, T_j)$$

Spontaneous symmetry breaking. The detailed study of the potential of scalar fields will be given elsewhere. The result is that (if some inequalities are satisfied) the two doublets s and t acquire non-vanishing vacuum expectation values, which are proportional. Color gauge invariance is conserved, together with internal and external quantum numbers.

But $SU(2) \times U'(1) \times U''(1)$ gauge invariance is spontaneously broken to the U(1) subgroup of quantum electrodynamics; the superfield[‡5]

$$V_\gamma \sim (g'_T g''_S - g'_S g''_T)V_3 + g(g''_T - g''_S)V' + g(g'_S - g'_T)V'' \tag{3}$$

describes A^μ and λ_γ, which are the fields of the photon and photon-neutrino, respectively. The chiral superfield N is electrically neutral.

Only V_3, V', V'' and N can have auxiliary components (D_3, D', D'' and F_N) with non-vanishing vacuum expectation values. Supersymmetry is spontaneously broken, and the Goldstone spinor λ_G is a linear combination of λ_3, λ', λ'' and $\gamma_5\zeta$ (not necessarily orthogonal to the one defining the photon-neutrino):

$$\lambda_G \sim \langle D_3 \rangle \lambda_3 + \langle D' \rangle \lambda' + \langle D'' \rangle \lambda'' + \langle F_N \rangle \gamma_5 \zeta . \tag{4}$$

Table 1 shows that, if two charged heavy Dirac spinors ℓ_- and L_- can be constructed from ψ_- and λ_-, two neutral ones only, ℓ'_0 and ℓ''_0, can be constructed from ψ_0 and λ_3, λ', λ'' and ζ; as a result two linear combinations of λ_3, λ', λ'' and ζ are bound to stay massless; in general these will be the photon-neutrino λ_γ and the Goldstone neutrino λ_G, defined by (3) and (4) respectively.

The new leptonic number (R-number) is carried by spin $\frac{1}{2}$ leptons of the new class (+1 for ℓ_-, ℓ'_0, ℓ''_0, $\lambda_{\gamma L}$, λ_{GL}; −1 for L_-), but also by gluon-neutrinos (+1 for λ_{aL}) and scalar particles (+2, +1 and −1 for n, s_i and t_j respectively). The scalars s_i and t_j, which carry the same external quantum numbers as their partners ψ_{iL} and ψ_{jR}, are heavy[‡6].

The spinorial fields ψ_{iL} and ψ_{jR} would be massless in the absence of the super-Yukawa term $\mathcal{L}_Y$. This one leads to a mass matrix, with the terms

$$\langle s \rangle \, \overline{\psi_{jR}} \, \psi_{iL} , \langle t^\dagger \rangle \, \overline{\psi_{jR}} \, \psi_{iL} , \overline{\psi_{jR}} \, \psi_{iL} \tag{5}$$

when they are gauge invariant.

In a Weinberg-Salam [8] type model with the following fermion multiplets

$$\begin{pmatrix} \nu_e \\ e_- \end{pmatrix}_L \begin{pmatrix} \nu_\mu \\ \mu_- \end{pmatrix}_L \quad e_{-R} \quad \mu_{-R}$$
$$\begin{pmatrix} p \\ n_c \end{pmatrix}_L \begin{pmatrix} p' \\ \lambda_c \end{pmatrix}_L \quad \begin{matrix} p_R & p'_R \\ n_R & \lambda_R \end{matrix} \tag{6}$$

mass terms are obtained for electron, muon and quarks. Table 2 summarizes the quantum number assignments for the particles described.

A simpler situation. The structure described allows as well the embedding of right-handed doublets and left-handed singlets, and can also be extended to different situations. Models with a left-right symmetry, generalizing the notion of vector-like models [9] will be studied later. Here we indicate only that considerable simplifications occur when we choose:

$$\begin{gathered} g'_S = g'_T = g', \quad g'_N = 0 \\ g''_S = -g'', \quad g''_T = g'', \quad g''_N = 2g'' \\ \xi' \approx 0. \end{gathered} \tag{7}$$

With $\xi' = 0$, we find[‡7]

$$\langle s \rangle = \langle t \rangle \tag{8}$$

provided the conditions

$$g^2 > h^2, \quad \xi'' > 0, \quad g'' > 0, \quad F''(S_i) \geqslant 0, \quad F''(T_j) \leqslant 0 \tag{9}$$

[‡5] We use the notation $g'_S = g'F'(S)$, etc.

[‡6] The masses, in particular those of scalar particles, are not arbitrary, but fixed by the coupling constants of the theory. The main difficulty was to find a Lagrangian density for which the potential, when expanded around the desired vacuum state, leads to large *positive* mass^2 for *all* scalar particles.

[‡7] Such a model is parity-invariant, if the chiral superfields S_i and T_j are not introduced. $\vec{V}^\mu$ and V'^μ are vectors, whereas V''^μ is a pseudovector. The term $\xi''D''$ in the Lagrangian density is a scalar, whereas the term $\xi'D'$ would be a pseudoscalar.

Table 1
Spin, mass and quantum numbers of physical fields

Field	Spin	Mass	R-number	Electron number	Other quantum numbers (muon, baryon, ...)
V_a^μ	1	0	0	0	0
λ_{aL}	1/2	0	1	0	0
A^μ	1	0	0	0	0
$\lambda_{\gamma L}$	1/2	0	1	0	0
W_-^μ	1	Heavy	0	0	0
L_-	1/2	Heavy	−1	0	0
ℓ_-	1/2	Heavy	1	0	0
w_-	0	Heavy	0	0	0
Z'^μ	1	Heavy	0	0	0
ℓ'_0	1/2	Heavy	1	0	0
z' real	0	Heavy	0	0	0
Z''^μ	1	Heavy	0	0	0
ℓ''_0	1/2	Heavy	1	0	0
z'' real	0	Heavy	0	0	0
λ_{GL}	1/2	0	1	0	0
n	0	Heavy	2	0	0
ν_{eL}	1/2	0	0	1	0
s_{e_0}	0	Heavy	1	1	0
e_-	1/2	Light	0	1	0
s_{e_-}	0	Heavy	1	1	0
t_{e_-}	0	Heavy	−1	1	0
Spinors	1/2	Massless, light or heavy	0	0	1
s-type scalars	0	Heavy	1	0	1
t-type scalars	0	Heavy	−1	0	1

are satisfied; these are compatible with the construction of a gauge-invariant term $\mathcal{L}_Y(S, S_i, T, T_j)$ added in the Lagrangian density.

Defining two mixing angles θ' and θ'' by

$$\mathrm{tg}\,\theta' = g'/g, \qquad \mathrm{tg}\,\theta'' = g''/h \tag{10}$$

we find for neutral vector bosons:

$$\begin{aligned} Z'^\mu &= \cos\theta'\, W_3^\mu + \sin\theta'\, W'^\mu \\ A^\mu &= -\sin\theta'\, W_3^\mu + \cos\theta'\, W'^\mu \\ Z''^\mu &= W''^\mu \end{aligned} \tag{11}$$

and for the photon-neutrino and Goldstone neutrino:

$$\begin{aligned} \lambda_\gamma &= -\sin\theta'\,\lambda_3 + \cos\theta'\,\lambda', \\ \lambda_G &= -\sin\theta''\,\gamma_5\zeta + \cos\theta''\,\lambda''. \end{aligned} \tag{12}$$

The neutral-current phenomenology will be studied later. We shall discuss now some of the new aspects of spontaneously broken supersymmetric theories.

Exchanges of scalar particles in weak interactions processes. These models contain a lot of scalar particles, which are heavy but not superheavy. Can they contribute to known physical processes? Known fermions are described by ψ_{iL} and ψ_{jR}; only vectors and R-invariant scalars (i.e. w_-, z', z'') can be exchanged at lowest order. The corresponding Yukawa coupling constant is of order $g\, m(\text{fermion})/m_W$. As an example

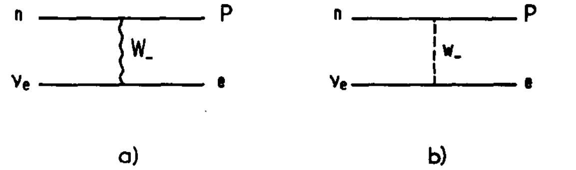

Fig. 1. Diagrams contributing to β-decay. Diagram 1b is negligible with respect to 1a since the Yukawa coupling constants vanish with fermion masses.

both a heavy vector boson W_- and a heavy scalar boson w_- contribute to β-decay (see fig. 1) but the latter amplitude, of order $G_F \times m_e m_{\text{quark}}/m_W{}^2$, is negligible. The same phenomenon occurs for μ-decay.

Search for new scalar particles. The heavy scalars s_i and t_j associated with quarks and leptons carry R-number = +1 or −1 respectively (we may call them sarks and tarks, septons and teptons). They can be produced in pairs, for example in e_+e_- scattering (see fig. 2).

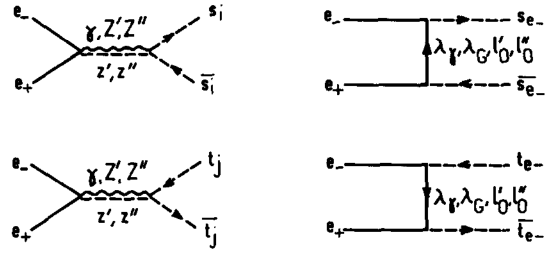

Fig. 2. Examples of production of new scalar particles in e_+e_- annihilation. The arrows show R-number conservation.

These scalars have a very short lifetime. One of the decay modes gives back the corresponding fermion, together with a Goldstone-neutrino, or a photon-neutrino (for a charged particle); see fig. 3.

If some of the sarks or tarks are not too heavy, the process $e_+e_- \to s_i\bar{s}_i$ (or $t_j\bar{t}_j$) $\to$ quark + antiquark + neutrinos might be responsible for a rise of $R = \sigma(e_+e_- \to \text{hadrons})/\sigma(e_+e_- \to \mu_+\mu_-)$.

Interactions of the Goldstone and photonic neutrinos. The Goldstone neutrino λ_G, the photon neu-

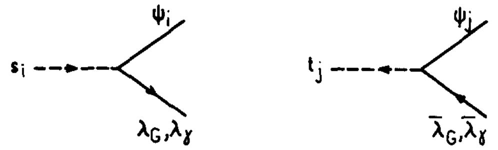

Fig. 3. Possible decay modes of the new scalar bosons.

trino λ_γ (and gluon-neutrinos λ_a) are massless and carry one unit of R-number. All other fields carrying R are heavy. Below the threshold for producing heavy new particles, we can only detect the recoil of the target scattered by such a neutrino; see fig. 4.

In the local limit these exchanges are equivalent to a four-fermion interaction simulating vector and axial neutral currents. The corresponding coupling constants are $\sim G_F$ if $m(s_{e_-})$ and $m(t_{e_-}) \sim m_{W_-}$, but may also be larger if one of the scalar particles s_{e_-} and t_{e_-} is light compared to W_-.

New processes of interest. Fig. 4 shows that a pair of new neutrinos can be produced in e_+e_- annihilation; these neutrinos have effective neutral current interactions with matter, with coupling constants $\gtrsim G_F$. Such processes are very important in the study of stellar cooling, and supernovae explosions.

We mention also that two quarks can scatter into a pair sark + tark, both R-number and baryon number being conserved (see fig. 5).

This may occur if the Fermi energy of quarks is high enough to create the heavy boson pair, i.e. for very large baryonic density. Such processes may be relevant in the study of black holes, or in the very early stages of the universe.

Conclusion. We have presented here a class of spontaneously broken supersymmetric gauge theories; similar methods can be applied for other gauge groups. Supersymmetry can be used to describe the real world

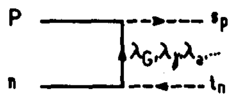

Fig. 5. Scattering quark + quark → heavy scalar bosons.

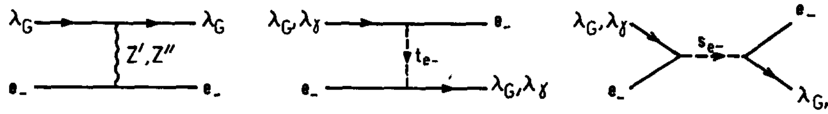

Fig. 4. Scattering of new neutrinos on electrons

as gauge (real) superfields interacting with matter (chiral) superfields.

The main interest are both theoretical (unified description of bosons and fermions based on an extension of Minkowski space-time) and experimental, by opening a large new area to investigations; we recall in particular that this framework involves a new class of leptons, with a photon-neutrino and a Goldstone-neutrino, and heavy scalar partners for usual leptons and quarks.

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