GAUGE MODELS WITH SPONTANEOUSLY BROKEN LOCAL SUPERSYMMETRY

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An effective low-energy lagrangian for gauge theories based on local supersymmetry spontaneously broken at an intermediate energy between the weak interaction and the Planck scale is obtained. The derivation uses the general coupling of the Yang–Mills matter system to $n = 1$ supergravity. As illustrative examples of this framework we exhibit realistic models of supersymmetric QED and of the electroweak theory with supersymmetry breaking induced by purely gravitational effects.

The possibility that supersymmetry be broken at relatively high energies, intermediate between the weak interaction ($\simeq M_W$) and the Planck scale (M_p)[‡1], has been recently realized in connection with realistic model building [1]. In turn, gravitational effects become relevant at low energy for a large supersymmetry breaking order parameter $d \approx M_W M_p$ [2,3].

In the present work we establish a framework to discuss a class of models in which the spontaneous breaking of supersymmetry is purely induced by the supergravity couplings. This approach allows the construction of gauge models for particle interactions with a realistic tree level mass spectrum. In particular we show the coexistence of the Higgs effect for the gauge symmetry breaking and the super-Higgs phenomenon [4] and the subsequent gravitino mass generation.

It is known that the super-Higgs effect with vanishing cosmological constant requires the existence of a complex scalar field z, the scalar partner of the would-be Goldstone fermion, acquiring a vev of order $O(M_p)$ [5,3]. We consider the situation in which this field interacts only through gravitational couplings. The scale of supersymmetry breaking is taken to be $d \simeq \mu M_p$ with $\mu \ll M_p$. The interaction of the chiral multiplets y_a [‡2] associated to ordinary matter is described by a superpotential containing dimensionful parameters, generally denoted by m, with $m \ll M_p$. After the shift of the field z by its vev the classical lagrangian, neglecting interactions which vanish as $M_p \to \infty$, reduces to a massive free lagrangian for the z-field plus a softly broken globally supersymmetric lagrangian for the chiral multiplets y_a with supersymmetric Yang–Mills interactions [‡3]. The soft breaking, in the case of "minimal" coupling to supergravity [3], has a universal structure with two main properties:

(i) It preserves the ultraviolet properties of globally supersymmetric renormalizable models.

‡1 We define M_p to be the inverse of the gravitational coupling defined by the Einstein–Hilbert lagrangian $\mathcal{L} = -R\sqrt{-g}/2K^2$.

‡2 We denote multiplets by their first scalar component. We also do not consider a Fayet–Iliopoulos terms for possible U(1) factors of the gauge group G [2].

‡3 Explicit supersymmetry breaking induced by supergravity couplings has been discussed in ref. [6].

(ii) It makes manifest the mass formulae of spontaneously broken local supersymmetry [3].
As explicit examples we shall discuss a simple QED model as well as a model for electroweak interactions.

We consider the general lagrangian for an arbitrary Yang–Mills system coupled to supergravity as described by Cremmer et al. [3].

In the "minimal" locally supersymmetric coupling of the matter-Yang–Mills system the lagrangian is entirely determined by the superpotential $f(z, y_a)$ and the gauge group G. Under our previous assumption $f(z, y_a)$ has the general form

$$f(z, y_a) = \bar{g}(z) + g(y_a), \tag{1}$$

where $\bar{g}(z)$ is the z-field potential term introduced in ref. [5]:

$$\bar{g}(z) = \mu\, M_{\mathrm{p}}(z + \beta), \tag{2}$$

with

$$\beta = (2 - \sqrt{3}) M_{\mathrm{p}}. \tag{3}$$

Under our assumption, with $g(y_a)$ containing only dimensional parameters $m \ll M_{\mathrm{p}}$, the value for β given by eq. (3) is obtained by the vanishing of the cosmological constant.

This together with the extremum condition for the scalar potential uniquely determines the vacuum expectation value of the z-field to be

$$\langle z \rangle = (\sqrt{3} - 1) M_{\mathrm{p}}. \tag{4}$$

Eqs. (3) and (4) imply that the z-field corresponds to two mass eigenvalues $m_{\mathrm{A}}^2, m_{\mathrm{B}}^2$ given by [5]

$$m_{\mathrm{A}}^2 = 2\sqrt{3}\, m_{3/2}^2, \qquad m_{\mathrm{B}}^2 = 2(2 - \sqrt{3})\, m_{3/2}^2, \tag{5, 6}$$

in terms of the gravitino mass $m_{3/2}$

$$m_{3/2} = \mu \exp[\tfrac{1}{2} (\sqrt{3} - 1)^2]. \tag{7}$$

Note the mass relation of ref. [5]:

$$m_{\mathrm{A}}^2 + m_{\mathrm{B}}^2 = 4 m_{3/2}^2. \tag{8}$$

We recall that the overall scalar potential [‡4] in the "minimal" coupling of matter to supergravity is given by [3]

$$\exp[(|z|^2 + |y_a|^2)/M_{\mathrm{p}}^2]\, (|\tilde{f}^z|^2 + |\tilde{f}^a|^2 - 3|f|^2/M_{\mathrm{p}}^2) + \tfrac{1}{2} (D^\alpha)^2, \tag{9}$$

where

$$\tilde{f}^z = \partial f/\partial z + z^* f/M_{\mathrm{p}}^2 = M_{\mathrm{p}}\mu + z^* f/M_{\mathrm{p}}^2, \qquad \tilde{f}^a = \partial f/\partial y_a + y^{*a} f/M_{\mathrm{p}}^2 = \partial g(y)/\partial y_a + y^{*a} f/M_{\mathrm{p}}^2, \qquad D^\alpha = e_\alpha y^{*a} T^{\alpha}{}_{a}{}^{b} y_b, \tag{10, 11, 12}$$

T^α are the generators of the gauge group G and e_α are the gauge coupling constants.

If we now shift the z-field by its vev and neglect terms of order $1/M_{\mathrm{p}}$ or higher the effective low-energy scalar potential becomes

$$V(z, z^*, y_a, y^{*a}) = \bar{V}(z, z^*) + V(y_a, y^{*a}), \tag{13}$$

where

$$\bar{V} = m_{3/2}^2 [\sqrt{3} A^2 + (2 - \sqrt{3}) B^2], \qquad V = |\tilde{g}^a|^2 + m_{3/2}(h + h^*) + m_{3/2}^2 |y_a|^2 + \tfrac{1}{2} D^\alpha D_\alpha, \tag{14, 15}$$

with

‡4 The scalar potential of the coupled Yang–Mills-matter-supergravity system has also been given in particular cases in refs. [2,7,8].

$$\tilde{g}(y) = \exp[\tfrac{1}{2}(\sqrt{3}-1)^2]\, g(y), \quad h(y) = \tilde{g}^a(y)\, y_a - \sqrt{3}\,\tilde{g}(y). \tag{16}$$

Eqs. (13)–(15) show that in the low-energy approximation the supergravity scalar potential becomes a pure mass term for the z-field and a globally supersymmetric chiral potential with two model independent soft breaking terms:

(a) An "analytic" superrenormalizable interaction $m_{3/2}(h + h^*)$.

(b) A diagonal mass term $m_{3/2}^2 |y_a|^2$ for all matter fields.

The structure of the fermion part of the matter lagrangian is the same as in global supersymmetry with the superpotential $g(y)$ replaced by $\tilde{g}(y)$.

The term given in (a) does not contribute to the quadratic mass relations [9] while from the term (b) we recover, in this approximation, the general quadratic mass sum rule of Cremmer et al. [3].

$$\text{Supertrace } \mathcal{M}^2 = 2N m_{3/2}^2, \tag{17}$$

where N is the number of matter multiplets y_a. In deriving eq. (17) we have assumed that tr $T^\alpha = 0$ for the U(1) factors of G.

Inspection of the soft breaking terms in eq. (15) shows that they are also soft in a supersymmetric sense according to the analysis of Girardello and Grisaru [10].

The extremum condition for the matter fields has the model independent form

$$\tilde{g}^{ab}(\tilde{g}_a^* + m_{3/2} y_a) + m_{3/2}\,[(1-\sqrt{3})\tilde{g}^b + m_{3/2} y^{*b}] + D^{\alpha b} D^\alpha = 0. \tag{18}$$

The scalar mass matrix is:

$$\mathcal{M}_s^2 = \begin{pmatrix} \tilde{g}_{ac}^* \tilde{g}^{cb} + D_a^\alpha D^{\alpha b} + m_{3/2}^2 \delta_a^b & \tilde{g}_{abc}^* \tilde{g}^c + D_a^\alpha D_b^\alpha + m_{3/2} h_{ab}^* \\ \tilde{g}^{abc} g_c^* + D^{\alpha a} D^{\alpha b} + m_{3/2} h^{ab} & \tilde{g}^{ac} \tilde{g}_{cb}^* + D^{\alpha a} D_b^\alpha + m_{3/2}^2 \delta_b^a \end{pmatrix}. \tag{19}$$

The fermion mass matrix for the matter fermions is the same as in global supersymmetry [10] with $g(y)$ replaced by $\tilde{g}(y)$.

We are now ready to apply our general formalism to models for particle physics. As illustrative examples we consider first a locally supersymmetric extension of QED and then an extension of the Glashow–Weinberg–Salam model.

(1) Super QED. The model is defined by the z-field and a doublet of chiral multiplets (E, E^c), with a vector-like U(1) gauge invariance. The matter superpotential is given by the mass term

$$g(E, E^c) = mEE^c, \tag{20}$$

while the D-term is given by

$$D = e^2(|E|^2 - |E^c|^2). \tag{21}$$

From eq. (18) we get

$$\langle E \rangle = \langle E^c \rangle = 0, \tag{22}$$

i.e., unbroken U(1) gauge invariance. From eq. (19) we obtain the following four mass eigenvalues for the scalar partners of the electron and positron,

$$m_{1,2}^2 = (m_{3/2} + m_\psi)^2 - \sqrt{3}\, m_{3/2} m_\psi, \quad m_{3,4}^2 = (m_{3/2} - m_\psi)^2 + \sqrt{3}\, m_{3/2} m_\psi, \tag{23}$$

where m_ψ is the electron mass given by

$$m_\psi = \exp[\tfrac{1}{2}(\sqrt{3}-1)^2]\, m. \tag{24}$$

This theory is a realistic QED model with spontaneously broken local supersymmetry provided $m_{3/2} \gtrsim 20$ GeV. The photino – the spin-1/2 partner of the photon – has vanishing mass at the tree level.

(ii) Super electroweak theory. The superpotential for the Higgs sector is given by [11]

$$g(\mathrm{Y}, \mathrm{H}, \mathrm{H}^c) = \lambda \mathrm{Y}(\mathrm{HH}^c - m^2), \tag{25}$$

where H, H^c, are two Higgs doublets with opposite hypercharge and Y is an SU(2) × U(1) singlet. For simplicity we do not write the Higgs-couplings to matter, their inclusion being straightforward.

From eq. (18) we get, in a given range of parameters $m \gtrsim \mathrm{O}(\mu/\lambda)$, a physically acceptable minimum given by

$$\langle \mathrm{H} \rangle = \begin{pmatrix} \mu p/\lambda \\ 0 \end{pmatrix}, \quad \langle \mathrm{H}^c \rangle = \begin{pmatrix} 0 \\ \mu p/\lambda \end{pmatrix}, \quad \langle \mathrm{Y} \rangle = \mu q/\lambda \tag{26}$$

where p, q are solutions of the equations

$$(2p^2+1)q + (\sqrt{3}-1)(\sqrt{3}p^2 + m^2\lambda^2/\mu^2) = 0, \quad q^2 + (3-\sqrt{3})q + 1 + p^2 - m^2\lambda^2/\mu^2 = 0. \tag{27}$$

The seven scalar mass eigenstates correspond to two charged and five neutral particles with masses

charged scalars: $m_\pm^2 = 2(1+q^2)m_{3/2}^2 + M_\mathrm{W}^2$,

neutral scalars: $m_0^2 = 2(1+q^2)m_{3/2}^2 + M_z^2$, $\quad m_{1,2}^2 = \{2p^2 + \frac{1}{2} \pm [\frac{1}{4} + 2p^2(3-\sqrt{3}+2q)^2]^{1/2}\}\, m_{3/2}^2$,

$m_{3,4}^2 = \{2p^2 + \frac{3}{2} + q^2 \pm [(\frac{1}{2}+q^2)^2 + 2(3-\sqrt{3})^2p^2]^{1/2}\}\, m_{3/2}^2$,

vector bosons: $M_\mathrm{W}^2 = 2g_2^2\mu^2p^2/\lambda^2$, $\quad M_z^2 = 2(g_2^2 + g_y^2)\,\mu^2p^2/\lambda^2$, $\quad M_\mu^2 = 0$,

where g_2, g_y are the SU(2) and the U(1) gauge couplings, respectively.

The masses of the scalar partners of leptons and quarks are all equal to the gravitino mass up to corrections proportional to the Yukawa couplings to the Higgs.

Because of the bound $m \gtrsim \mathrm{O}(\mu/\lambda)$ and the fact that m sets the scale of the Higgs doublets vev's, the model gives a realistic theory of electroweak interactions provided $\mu/\lambda \gtrsim \mathrm{O}(1\ \mathrm{TeV})$. In turn the scalar masses require [12] $m_{3/2} \gtrsim 20$ GeV so that the supersymmetry breaking parameter d is predicted to be $d \simeq \mathrm{O}(10^{21\pm1}\ \mathrm{GeV}^2)$ ‡5.

We would like to conclude with a few comments. We have shown that the super-Higgs and the normal Higgs effects can coexist provided the scale of the gauge breaking is at least of the same order of the gravitino mass $m_{3/2} \sim d/M_\mathrm{p}$. In particular, this approach can be extended to supersymmetric grand unified theories. The emerging general feature of supersymmetry breaking induced by supergravity is the existence of a new complex scalar field beyond ordinary matter that decouples from the low energy theory and which is responsible for the spontaneous breaking of supersymmetry. It is in principle possible to complicate the situation described in this paper, for instance to relax the condition on the starting superpotential given in eq. (1) or even to consider models in which the interaction between matter and supergravity is "non-minimal" [3] ‡6. In this case the approach looses somewhat its predictive power. Nevertheless it could be interesting to consider this more general possibility.

‡5 This point has also been emphasized in ref. [13].

‡6 A model based on a "non-minimal" supergravity coupling in connection to the hierarchy problem, has been considered in ref. [14].

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