IS THE NEUTRINO A GOLDSTONE PARTICLE?

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Using the hypotheses, that the neutrino is a goldstone particle, a phenomenological Lagrangian is constructed, which describes an interaction of the neutrino with itself and with other particles.

Recently much attention has been paid in the elementary particle physics to the problem of spontaneously broken symmetries and the related degeneracy of the vacuum state. An immediate consequence of the vacuum degeneracy is that it gives rise to a possible existence of zero mass particles, the so-called Goldstone particles [1].

Among known elementary particles only the neutrino, the photon and the graviton have zero masses. However, the last two correspond to the gauge fields and do not require the vacuum degeneracy for their existence. Therefore the neutrino is the only elementary particle the existence of which may be immediately related to the vacuum degeneracy.

We will restrict our attention to the following. If the neutrino is regarded as a Goldstone particle then this leads to a certain type of interaction of the neutrino with itself as well as with other particles. The interaction is completely defined by phenomenological constants and in this sense is universal.

For the determination of the type of spontaneously broken symmetry that causes the degeneracy of the vacuum and the corresponding properties of the neutrino as a Goldstone particle, let us consider the equation for a free neutrino

$$i\sigma_\mu \,\partial\psi/\partial x_\mu = 0 \tag{1}$$

Eq. (1) is invariant under transformations of the Poincaré group and the chiral transformations as well as under translations in the spinor space, i.e. under the transformations of the type

$$\psi \to \psi' = \psi + \zeta \qquad x_\mu \to x'_\mu = x_\mu , \tag{2}$$

where ζ is a constant spinor, anticommuting with ψ. Leaving the transformation properties of x_μ and ψ under the Poincaré group unchanged, let us replace the transformations (2) by the transformations:

$$\psi \to \psi' = \psi + \zeta \qquad \psi^+ \to \psi'^+ = \psi^+ + \zeta^+$$

$$x_\mu \to x'_\mu = x_\mu - \frac{a}{2i}(\zeta^+\sigma_\mu\psi - \psi^+\sigma_\mu\zeta)\,. \tag{3}$$

The resulting structure is a group with ten commuting and four anticommuting parameters*. It is the only possible generalization of (2) and the Poincaré group if the dimension of the group space is not enlarged. In the transformations (3) a is an arbitrary constant. Its dimension is the fourth power of length.

Let us assume that in the presence of interaction the equations for the neutrino are invariant under the transformations (3).

In the following we also assume that the number of the derivatives of the neutrino field is a minimal one that is compatible with the invariance requirement.

To construct the phenomenological action integral that satisfies the above assumptions it is sufficient to use the following differential forms

$$\omega_\mu = dx_\mu + \frac{a}{2i}(\psi^+\sigma_\mu d\psi - d\psi^+\sigma_\mu\psi), \tag{4}$$

which are invariant under transformations (3). The action integral which is invariant under the Poincaré group and the transformations (3) has the form

$$S = \frac{1}{a}\int \omega_0 \times \omega_1 \times \omega_2 \times \omega_3 \tag{5}$$

where the sign $\times$ denotes the outer product of differential forms.

The expression under the integral corresponds to the invariant infinitesimal four-dimensional volume in the space of group parameters.

* Lie groups with commuting and anticommuting parameters were considered recently by Berezin and Kats [2].

If the four-dimensional subspace in the action integral (5) is determined by the explicit functions $\psi = \psi(x)$, the integral (5) takes the following, more conventional form

$$S = \frac{1}{a} \int |W| \, d^4x \, , \qquad (6)$$

where $|W|$ is the determinant of the matrix W^{μ}_{ν}

$$W_{\mu\nu} = \delta_{\mu\nu} + a T_{\mu\nu}$$
$$T_{\mu\nu} = \frac{1}{2i} (\psi^+ \sigma_\mu \partial_\nu \psi - \partial_\nu \psi^+ \sigma_\mu \psi). \qquad (7)$$

From (6) and (7) it follows that the action integral as a function of the tensor $T_{\mu\nu}$ has the form:

$$S = \int \Big[\frac{1}{a} + T_{\mu\mu} + \tfrac{1}{2} a \, (T_{\mu\mu} T_{\nu\nu} - T_{\mu\nu} T_{\nu\mu}) + \frac{a^2}{3!} \sum_{\mathrm{p}} (-1)^{\mathrm{p}} \, T_{\mu\mu} T_{\nu\nu} T_{\rho\rho} + \frac{a^3}{4!} \sum_{\mathrm{p}} (-1)^{\mathrm{p}} \, T_{\mu\mu} T_{\nu\nu} T_{\rho\rho} T_{\sigma\sigma} \Big] d^4x \, , \qquad (8)$$

where Σ_{p} denotes the sum of the permutations of the second tensor indices in the products of the tensors $T_{\mu\nu}$. The term with $T_{\mu\mu}$ corresponds to the kinetic term, the term with products of two, three and four tensors $T_{\mu\nu}$ form the parts of the interaction Lagrangian with four, six and eight fields correspondingly. The power of the derivatives of the fields in interaction terms is determined by the number of multipliers $T_{\mu\nu}$.

The interaction of the neutrino with other fields is unambiguously defined by the requirement of invariance under the considered group. Thus, for example, the action integral for a Dirac particle is given by the following expression

$$S = \int \Big[R_{\mu\mu} + a(R_{\mu\mu} T_{\nu\nu} - R_{\mu\nu} T_{\nu\mu}) + \frac{a^2}{2!} \sum_{\mathrm{p}} (-1)^{\mathrm{p}} \, R_{\mu\mu} T_{\nu\nu} T_{\rho\rho} + \frac{a^3}{3!} \sum (-1)^{\mathrm{p}} \, R_{\mu\mu} T_{\nu\nu} T_{\rho\rho} T_{\sigma\sigma} + m \bar{\phi} \phi \, |W| \Big] d^4x \qquad (9)$$

where

$$R_{\mu\nu} = \frac{1}{2i} (\bar{\phi} \gamma_\mu \partial_\nu \phi - \partial_\nu \bar{\phi} \gamma_\mu \phi)$$

and the tensor $T_{\mu\nu}$ and the determinant $|W|$ are the same as in (7).

The weak interaction introduced into the scheme under consideration with the help of the gauge fields for an approximate unitary symmetry group for the neutrino and other leptons. The simultaneous inclusion of the electromagnetic interaction as well is carried out by means of the well-known mechanism of the spontaneously broken unitary symmetry [3]. In the limit of zero lepton mass the unitary symmetry may be exact. In order to obtain the action integral for leptons in the unitary symmetry limit it is sufficient to consider the products of spinors in formulae, (3), (4) and (7) as the products of unitary multiplets. In the formulae (3), (4) and (7) the terms for leptons with opposite chirality may be added. The unitary groups for the different chirality states do not necessarily have to coincide.

Similarly, the gravitational interaction may be included by means of introduction the gauge fields for the Poincaré group. Note, that if the gauge field for the transformation (3) is also introduced, then as a result of the Higgs effect [4] the massive gauge field with spin three-half appears and the considered Goldstone particle with spin one half disappears.

References

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