7.A.1
8.A.1

Nuclear Physics B34 (1971) 632-639. North-Holland Publishing Company

FIELD THEORY INTERPRETATION OF SUPERGAUGES IN DUAL MODELS

J. -L. GERVAIS
Laboratoire de Physique Théorique et Hautes Energies, Orsay, France *

and

B. SAKITA **
Institut des Hautes Etudes Scientifiques, 91-Bures-sur-Yvette, France
and
Laboratoire de Physique Théorique et Hautes Energies, Orsay, France

Received 13 August 1971

Abstract: Possible new invariances of generalized dual models are discussed in the context of the functional integral formulation. The operators relevant to new gauges of those models, such as those obtained by Neveu and Schwarz, are derived as infinitesimal generators of new field transformations which leave the action integral invariant.

1. INTRODUCTION

In our previous work [1] we have shown that the symmetries of N-particle dual amplitudes arise from the symmetries of the Lagrangian used for their functional integral representation. We then discussed about the construction of general dual amplitudes which possess the required symmetries, especially the conformal symmetry.

It has been conjectured [3] that the conformal invariance of the integrand of the conventional Veneziano formula for $\alpha_0 = 1$ provides enough gauge conditions to disconnect the ghost states from the physical states. In terms of the functional integral representation we may understand this conjecture using the conformal invariance of the integrand of the functional integral representation as follows. Let $\mathcal{F}(\Phi)$ be the integrand. It is a functional of $\Phi_\mu(x,y)$, which is given by

* Laboratoire associé au Centre National de la Recherche Scientifique.
Postal address: Laboratoire de Physique Théorique et Hautes Energies, Bâtiment 211, Université de Paris-Sud, Centre d'Orsay, F-91-Orsay (France).
** Present address: Department of Physics, City College, City University of New York, N.Y. 10031, U.S.A.

$$\mathcal{F}(\Phi) = \lim_{\epsilon \to 0} \frac{1}{C} \int \ldots \int \prod_i \mathrm{d}\theta_i E(z_i; k_i^2)^{-1} \exp\{\iint_D \mathrm{d}x_1 \mathrm{d}x_2 (\mathcal{L}(\Phi)$$

$$+ (2\pi)^{\frac{1}{2}} \sum_j i\rho_j(x_1, x_2) k_j \Phi(x_1, x_2))\}. \tag{1.1}$$

The definitions of all the symbols are given in ref. [1]. By the conformal invariance of the integrand we mean that $\mathcal{F}(\Phi)$ is invariant under the change of function

$$\Phi(x_1, x_2) \to \Phi'(x_1, x_2), \tag{1.2}$$

such that

$$\Phi'(x_1', x_2') = \Phi(x_1, x_2), \tag{1.3}$$

where

$$z = x_1 + ix_2 \to z' = x_1' + ix_2' \tag{1.4}$$

is a conformal transformation which maps D onto itself, namely

$$\mathcal{F}(\Phi) = \mathcal{F}(\Phi'). \tag{1.5}$$

A functional integral can be regarded as an infinite multiple integral. If the integrand is invariant under continuous transformations such as (1.5) one may factor out some of the integrals, thus reducing the number of integration variables. This is presumably equivalent to removing the spurious states.

It has been shown in ref. [1] that the new dual models, such as those of Neveu and Schwarz [4] or Bardakçi and Halpern [5], also have a functional integral representation. The corresponding Lagrangian is the sum of $\mathcal{L}(\Phi)$ and $\mathcal{L}(\Psi)$, where ψ is a field of conformal spin-$\frac{1}{2}$. The two corresponding parts of the action integral are separately invariant under conformal transformations, and the functional integrands are also proven to be invariant.

The introduction of the new field ψ in these models adds new ghost states to the models. In order to show that some of these new ghost states are disconnected from the physical states, Neveu, Schwarz and Thorn [6] have used new operators G_m.

In this paper we interpret these operators as generators of infinitesimal transformations of the fields, and we show that the action integral is invariant.

2. NEW FIELD TRANSFORMATIONS

The Lagrangian density, without the source terms, should be such that the corresponding action

$$I = \int dx_1\, dx_2\, \mathcal{L}(x_1, x_2) \tag{2.1}$$

is conformal invariant. It was shown in ref. [1] that, as a result, only two forms are possible for free fields, namely ($z = x_1 + ix_2$).

$$\mathcal{L}_1 = -2\partial_z\Phi\,\partial_{\bar z}\Phi + 2i[\bar\psi_2\,\partial_z\psi_1 + \bar\psi_1\,\partial_{\bar z}\psi_2]\,, \tag{2.2}$$

$$\mathcal{L}_2 = -2\partial_z\Phi\,\partial_{\bar z}\Phi + 2i[\psi\,\partial_{\bar z}\psi + \bar\psi\,\partial_z\bar\psi]\,. \tag{2.3}$$

If we denote by d the dimension and by J the conformal spin [1], Φ has $d = J = 0$ (scalar field), ψ_1 and ψ have $-d = J = \frac{1}{2}$; ψ_2 and $\bar\psi$ have $-d = -J = \frac{1}{2}$; $\mathcal{L}_2$ is obtained from $\mathcal{L}_1$ by letting ψ_2 be equal to $\bar\psi_1$.

In formulae (2.2) and (2.3) we have not displayed the space time indices of the fields, It is well known that Φ is a space time vector whose zeroth mode corresponds to the space-time coordinates. The indices of the other fields can be chosen arbitrarily provided $\mathcal{L}$ is a scalar. If the fields ψ_1 and ψ_2 are given space time spinor indices, one obtains the fields introduced by Bardakçi and Halpern [5]. If one considers the case where ψ is a space time vector, one recovers, depending on the boundary conditions [2, 7], the meson field of Neveu and Schwarz [4] or the fermion field of Ramond [8].

We want to discuss more general transformations which mix Φ and ψ or Φ and ψ_i in such a way that, the variation of $\mathcal{L}$ being a total derivative, the action is invariant. It will be clear that they can be made Lorentz invariant by a suitable choice of space time properties of ψ and ψ_i.

There exist two types of transformations which leave the action invariant

(i) *Linear transformations*

The field Φ is quantized with commutators while, for ψ, only quantization with anticommutators is possible [1]. As far as the fields ψ_i are concerned, both quantizations are possible, but Bardakçi and Halpern have quantized with anticommutators. We will restrict ourselves to this case since the extension to commuting ψ_i is trivial. The linear transformation we have in mind will then mix commuting and anticommuting objects. A way out is to remark that for anticommuting fields functional integrals are defined by introducing anticommuting c-numbers [9]. We can therefore introduce a c-number χ which anticommutes with all ψ_i and $\bar\psi_i$ in the functional integral. We furthermore assume that $\chi^2 = 1$ and $\bar\chi = \chi$. It is then easily checked that to first order of ϵ, the action I_1 associated with $\mathcal{L}_1$ is invariant under the transformation

$$\delta\psi_1 = \epsilon\, g_1(\partial_z\Phi)\,\chi\,, \qquad \delta\bar\psi_2 = \epsilon\, g_2(\partial_{\bar z}\Phi)\,\chi\,,$$

$$\delta\Phi = -\tfrac{1}{2}i\epsilon\,\{g_2\,\psi_1 + g_1\,\bar\psi_2 + \bar g_2\,\bar\psi_1 + \bar g_1\,\psi_2\}\,\chi\,, \tag{2.4}$$

provided that

$$\partial_{\bar{z}} g_1 = \partial_{\bar{z}} g_2 = 0. \tag{2.5}$$

The transformation (2.4) is not conformal invariant since both sides of the equality do not transform in the same way. Indeed consider a conformal transformation $z \to z'$ then

$$\Phi'(z') = \Phi(z), \qquad \psi_1'(z') = \left(\frac{dz'}{dz}\right)^{-\frac{1}{2}} \psi_1(z),$$

$$\psi_2'(z') = \left(\frac{d\bar{z}'}{d\bar{z}}\right)^{-\frac{1}{2}} \psi_2(z). \tag{2.6}$$

Expressed in terms of ψ_i' and Φ', the transformation (2.4) takes the same form except that g_1, g_2 are replaced by g_1', g_2' which are given by

$$g_i'(z') = \left(\frac{dz'}{dz}\right)^{\frac{1}{2}} g_i(z), \qquad i = 1,2. \tag{2.7}$$

Making a conformal transformation is therefore equivalent to a change of the functions g_i. It is thus sufficient to study the transformation (2.4) in a particular conformal frame if one considers all possible functions $g_1 g_2$. We choose the frame where the integral (2.1) is carried on the strip $0 \leqslant x_2 = \xi < \pi$, $-\infty \leqslant x_1 = \nu \leqslant +\infty$.

The functional integral over Φ is carried out with the condition that its normal derivative vanishes at the boundary:

$$\frac{\partial}{\partial\xi} \Phi \Big|_{\xi=0} = \frac{\partial}{\partial\xi} \Phi \Big|_{\xi=\pi} = 0. \tag{2.8}$$

As far as the fields ψ_i are concerned, two types of boundary conditions are possible *, namely [2,7] $\psi_1 \pm \psi_2 = 0$ and $\partial_n(\psi_1 \mp \psi_2) = 0$. The two relevant boundary conditions on the trip are

$$\psi_1(\nu,0) = \psi_2(\nu,0), \qquad \frac{\partial}{\partial\xi}\psi_1\Big|_{\xi=0} = -\frac{\partial}{\partial\xi}\psi_2\Big|_{\xi=0},$$

$$\psi_1(\nu,\pi) = -\psi_2(\nu,\pi), \qquad \frac{\partial}{\partial\xi}\psi_1\Big|_{\xi=\pi} = \frac{\partial}{\partial\xi}\psi_2\Big|_{\xi=\pi}, \tag{2.9}$$

which corresponds to the meson model, and

$$\psi_1(\nu,0) = \psi_2(\nu,0), \qquad \frac{\partial}{\partial\xi}\psi_1\Big|_{\xi=0} = -\frac{\partial}{\partial\xi}\psi_2\Big|_{\xi=0},$$

$$\psi_1(\nu,\pi) = \psi_2(\nu,\pi), \qquad \frac{\partial}{\partial\xi}\psi_1\Big|_{\xi=\pi} = -\frac{\partial}{\partial\xi}\psi_2\Big|_{\xi=\pi}. \tag{2.10}$$

* In ref. [1] we only discussed the meson case but our analysis can be immediately extended to the boundary condition of the fermion case.

which corresponds to the Ramond fermion model [8].

It is easy to check that those boundary conditions are invariant if

$$g_1(\nu,0) = \bar{g}_2(\nu,0)\,, \qquad g_1(\nu,\pi) = \mp\bar{g}_2(\nu,\pi)\,, \tag{2.11}$$

where - (+) refers to the meson (fermion) case. One then verifies that the most general form of g_i is

$$g_1 = \sum_{n=-\infty}^{\infty} u_n\, e^{(n+\frac{1}{2})z}\,, \qquad g_2 = \sum_{n=-\infty}^{\infty} \bar{u}_n\, e^{(n+\frac{1}{2})z}\,, \tag{2.12}$$

in the case of the boundary conditions (2.9), and

$$g_1 = \sum_{n=-\infty}^{\infty} v_n\, e^{nz}\,, \qquad g_2 = \sum_{n=-\infty}^{\infty} \bar{v}_n\, e^{nz}\,, \tag{2.13}$$

in the case of the boundary conditions (2.10). In formula (2.12) and (2.13) u_n and v_n are arbitrary complex numbers.

Using the canonical commutation relations one obtains the infinitesimal generator $F_{\{g\}}$ corresponding to the transformation (2.4). It is of the form

$$F_{\{g\}} = G_{\{g\}}\,\chi\,, \tag{2.14}$$

where

$$G_{\{g\}} = \frac{1}{i}\int_0^{\pi} d\xi\, \{(\bar{\psi}_2 g_1 + \psi_1 g_2)\,\frac{\partial}{\partial z}\Phi + (\psi_2 \bar{g}_1 + \bar{\psi}_1 \bar{g}_2)\,\frac{\partial}{\partial \bar{z}}\Phi\}\,. \tag{2.15}$$

It follows from (2.12) and (2.13) that all generators are linear combinations of the operators

$$G_n^{\pm} = \frac{1}{i}\int_0^{\pi} d\xi\, \{(\bar{\psi}_2 \pm \psi_1)\, e^{i(n+\frac{1}{2})\xi}\,\frac{\partial}{\partial z}\Phi + (\bar{\psi}_1 \pm \psi_2)\, e^{-i(n+\frac{1}{2})\xi}\,\frac{\partial}{\partial \bar{z}}\Phi\} \tag{2.16}$$

in the meson case, and

$$G_n^{\pm} = \frac{1}{i}\int_0^{\pi} d\xi\, \{(\bar{\psi}_2 \pm \psi_1)\, e^{in\xi}\,\frac{\partial}{\partial z}\Phi + (\bar{\psi}_1 \pm \psi_2)\, e^{-in\xi}\,\frac{\partial}{\partial \bar{z}}\Phi\} \tag{2.17}$$

in the fermion case.

The infinitesimal generators L_f associated with the conformal transformation (2.6) with $z' = z + \epsilon f(z)$ can be put under the form

$$L_f = L_f^{\Phi} + L_f^{+} + L_f^{-}\,, \tag{2.18}$$

where

$$L_f^{\Phi} = -\int_0^{\pi} d\xi \left[f\left(\frac{\partial \Phi}{\partial z}\right)^2 + \bar{f}\left(\frac{\partial \Phi}{\partial \bar{z}}\right)^2 \right], \tag{2.19}$$

$$L_f^{\pm} = \mp \tfrac{1}{4} \int_0^{\pi} d\xi \, [f(\psi_1 \pm \bar{\psi}_2)\, \partial_\xi (\psi_1 \pm \bar{\psi}_2) - \bar{f}(\bar{\psi}_1 \pm \psi_2)\, \partial_\xi (\bar{\psi}_1 \pm \psi_2)]\,. \tag{2.20}$$

It is easy to check that

$$[L_f^+, L_{f'}^-] = 0, \tag{2.21}$$

so that one obtains a direct product of three representations of the conformal group.

We moreover introduce the operators

$$L'_f = \tfrac{1}{4} \int_0^{\pi} d\xi \, \{f(\psi_1 + \bar{\psi}_2)\, \partial_\xi (\psi_1 - \bar{\psi}_2) + \bar{\psi}_2 \psi_1 (\partial_\xi f)$$

$$- \bar{f}(\bar{\psi}_1 + \psi_2)\, \partial_\xi (\bar{\psi}_1 - \psi_2) - \psi_2 \bar{\psi}_1 \partial_\xi \bar{f}\}\,, \tag{2.22}$$

and denote by $G_g^{\pm}$ the operator $G_{\{g\}}$ with $g_2 = \pm g_1 = g$. One can then verify that the algebra of the operators $L_f^{\pm}$, L_f^{Φ}, L'_f, $G_f^{\pm}$ is closed. Indeed one has, in particular

$$\{G_g^{\pm}, G_h^{\pm}\} = -2i(L_{gh}^{\Phi} \pm L_{gh}^{\pm})\,, \tag{2.23}$$

$$\{G_g^{\pm}, G_h^{\mp}\} = -2i L'_{gh}\,. \tag{2.24}$$

This algebra may be useful to remove the ghost states from the corresponding models. However in the case of Bardakçi and Halpern [5] ψ_i is a spinor so that the transformation (2.4) does not commute with Lorentz transformations.

For the Lagrangian (2.3) one simply equates $\psi_1 = \bar{\psi}_2$ so that G^-, L^- and L' vanish. G_n^+ is then identical to the operator which has been recently introduced to construct new gauges in the fermion model [8] and in the meson model [6].

(ii) *Non-linear transformations*

In this subsection we consider ψ_i as space time spinor. The action integral of (2.2) is invariant to first order of ϵ under the transformation

$$\delta\psi_1 = i\epsilon f(z) \gamma_\mu (\partial_z \Phi_\mu)\, \psi_1\,, \qquad \delta\psi_2 = i\epsilon \bar{f}(z) \gamma_\mu (\partial_{\bar{z}} \Phi_\mu)\, \psi_2\,,$$

$$\delta\Phi_\mu = \tfrac{1}{2}\epsilon [f \bar{\psi}_2 \gamma_\mu \psi_1 + \bar{f} \bar{\psi}_1 \gamma_\mu \psi_2]\,, \tag{2.25}$$

provided

$$\frac{\partial}{\partial \bar{z}} f = 0. \tag{2.26}$$

It is easy to check that this transformation is consistent with both of the boundary conditions (2.9) and (2.10), if

$$f(\nu, 0) = \bar{f}(\nu, 0) \quad \text{and} \quad f(\nu, \pi) = \bar{f}(\nu, \pi). \tag{2.27}$$

Therefore the general form of f is

$$f = \sum_{n=-\infty}^{\infty} f_n \, e^{nz}, \tag{2.28}$$

where f_n is real.

The generator of the transformation (2.14) is given by

$$F_f = \int_0^{\pi} d\xi [f(\partial_z \Phi_\mu) \bar{\psi}_2 \gamma_\mu \psi_1 + \bar{f}(\partial_{\bar{z}} \Phi_\mu) \bar{\psi}_1 \gamma_\mu \psi_2]. \tag{2.29}$$

The Lagrangian density corresponding to (2.3) for the spinor field is deduced from (2.2) by letting

$$\psi = \psi_1 = \bar{\psi}_2 C^{-1}, \tag{2.30}$$

where C is Pauli's charge conjugation matrix. The corresponding transformation and the generator can be obtained from (2.25), (2.29) and (2.30).

3. DISCUSSIONS

In sect. 2 we obtained new transformations of fields which leave the action integrals invariant. Some of the corresponding generators have been used to eliminate spurious states, so that we believe that these transformations are really relevant to the new gauges of generalized dual models. This makes it plausible that an argument about those new transformations can be made similar to the one given in the introduction on conformal transformations.

One of the authors (B.S.) thanks Professors C. Bouchiat, B. Jancovici, L. Michel and L. Motchane for their hospitality.

REFERENCES

[1] J.-L. Gervais and B. Sakita, Phys. Rev. D4, to be published.
[2] M. A. Virasoro, Talk at the Tel-Aviv Conference, Berkeley preprint.
[3] M. A. Virasoro, Phys. Rev. D1 (1969) 2933,
A. Galli, Nuovo Cimento 69A (1970) 275,
S. Fubini and G. Veneziano, Ann. of Phys. (to be published).

[4] A. Neveu and J. H. Schwarz, Nucl. Phys. B31 (1971) 86.
[5] K. Bardakçı and M. B. Halpern, Phys. Rev. D3 (1971) 2493.
[6] A. Neveu, J. H. Schwarz and C. B. Thorn, Phys. Letters 35B (1971) 529.
[7] Y. Aharonov, A. Casher and L. Susskind, Phys. Letters 35B (1971) 512.
[8] P. Ramond, Phys. Rev. D3 (1971) 2415;
C. B. Thorn, Berkeley preprint;
A. Neveu and J. H. Schwarz, Princeton preprint, Phys. Rev., to be published
[9] J. Rzewuski, Field Theory II, Iliffe Books Ltd., London (1969).