A PHENOMENOLOGICAL MODEL OF PARTICLE PHYSICS BASED ON SUPERSYMMETRY

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A supersymmetric model of particle physics is developed whose phenomenology closely resembles that of supersymmetric technicolor models.

Supersymmetry [1] has attracted great attention since its discovery, both because of its intrinsic appeal, and, more recently, because it offers a context in which to solve the "naturalness" or hierarchy problems of the standard model [2–6]. It has proven difficult, however, to write down a successful phenomenological model based on supersymmetry. The problem is that when supersymmetry is broken spontaneously one usually obtains unacceptable mass relations at the tree level. Only by adding extra gauge interactions [7,8] [beyond SU(3) $\times$ SU(2) $\times$ U(1)] or by postulating some explicit soft breaking of supersymmetry [9,10] has it been possible to construct models with an acceptable mass spectrum at tree level.

Recently models of particle physics have been proposed in which supersymmetry is broken by strong gauge forces [3–5]. These "supersymmetric technicolor" models do not appear to suffer from these undesirable mass relations. In these models, masses for scalar partners of quarks and leptons and for fermionic partners of gauge bosons arise radiatively. The breaking of SU(2) $\times$ U(1) invariance either arises radiatively or through the presence of additional strong interactions. The supersymmetry partners of ordinary fields are all massive, with masses given by powers of coupling constants times a large supersymmetry-breaking scale. This suggests a strategy for building low-energy supersymmetric models. Instead of using the Fayet–Iliopoulos D-term [11] to break supersymmetry one should build models along the lines of O'Raifeartaigh models [12]. The principal contributions to scalar quark and lepton masses should arise radiatively. Similarly, masses for fermionic partners of gauge bosons should also arise through radiative effects. Below, following a suggestion of Ed Witten, we write down such a model. We compute the masses of quarks and leptons to two-loop order, and show that they are large and positive. All gauge fermions are shown to gain large mass as well. The correct pattern of SU(2) $\times$ U(1) breaking is readily obtained. No light axions or other unwanted objects appear in the spectrum. These models can thus provide a correct description of low-energy physics. We consider the possible range of values of the parameters of the theory, in order to try and bound the masses of the new particles predicted by the model. Finally, we note that the model can be unified and remark on some of the issues involved in such a unification.

The gauge group of the model is just SU(3) $\times$ SU(2) $\times$ U(1). We follow closely the notation of ref. [4], denoting the gauge fermions for color, weak isospin, and hypercharge by λ_c, w, and b, ("gluinos", "winos" and "bino"), respectively. We denote the scalar and fermion components of the chiral superfield, ϕ, by A_ϕ and ψ_ϕ. The model consists, first, of the quark and lepton superfields with their usual

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gauge quantum numbers. Quark doublets will be denoted by Q_i (i is a generation index) and singlets by $\overline{U}_i$ and $\overline{D}_i$. Similarly, lepton doublets will be denoted by L_i, and the singlets by $\overline{e}_i$. We list here the additional chiral superfields of the model, and their SU(3) $\times$ SU(2) $\times$ U(1) quantum numbers. There are four SU(2) doublets which are color singlets:

$$S, S' = (1,2,1)\,, \qquad \overline{S}, \overline{S}' = (1,2,-1)\,, \tag{1}$$

There are four SU(2) singlets which are color triplets:

$$T, T' = (3,1,-2/3)\,, \qquad \overline{T}, \overline{T}' = (\overline{3},1,2/3)\,. \tag{2}$$

Note that these multiplets of fields lend themselves readily to grand-unification in the gauge group SU(5) (they would form two 5's and two $\overline{5}$'s) [13]. Finally, there are two doublets, H_U and H_D, which play the role of Higgs fields

$$H_U = (1,2,-1)\,, \qquad H_D = (1,2,1)\,, \tag{3}$$

and two gauge-singlet fields, X and Y.

For the superpotential of the model we take

$$\begin{aligned} W = {} & m_1(\overline{S}'S + S'\overline{S}) + m_2(\overline{T}'T + T'\overline{T}) + m_3 S\overline{S} + m_4 T\overline{T} \\ & + Y(\lambda_1 S\overline{S} + \lambda_2 T\overline{T} - \mu_1^2) + \lambda_3 X(H_U H_D - \mu_2^2) \\ & + \Gamma_{ij}^U Q_i H_U \overline{U}_j + \Gamma_{ij}^D Q_i H_D \overline{D}_j + \Gamma_i^L L_i \overline{e}_i H_D\,. \end{aligned} \tag{4}$$

This model, in addition to baryon and lepton number conservation, also conserves the number of S-type and T-type fields. Both of these currents are vector-like. The model has no R-type invariance [14], so the global symmetries of the model allow Y^2, X^2, etc. type couplings. Their omission is thus not "natural" in the usual sense. However, non-renormalization theorems assure us that if these terms are not present in the tree-level lagrangian, they are not generated in perturbation theory [2,15]. The model is thus natural in this weaker sense.

Two other questions of naturalness must also be addressed. We have included in the superpotential only terms which obey a discrete invariance under which "barred" ($\overline{S}, \overline{T}, \overline{S}', \overline{T}'$) and "unbarred" (S, T, S$'$, T$'$) fields are interchanged. We will refer to this approximate symmetry as "P-invariance". P-invariance is violated by the gauge couplings. Moreover, we have omitted the Fayet–Iliopoulos D-term for hypercharge. These choices make the phenomenology of the model similar to that of supersymmetric technicolor models [4,5]. While these choices may not be strictly natural, it is not in fact unreasonable that the D-term and the coefficient of the P-violating terms in the superpotential should be small. In particular, if we omit the P-violating terms, non-renormalization theorems guarantee that such terms will not appear in perturbation theory, except for high-order effects due to wave-function renormalization. Moreover, it is possible to prove, in general, that only a finite coefficient is generated for the D-term, and that if the P-violating terms are omitted, that such a coefficient cannot appear before four-loop order. It may, in fact, be possible in this case to generalize the proof of ref. [16] to demonstrate that the D-term is not generated at all. Finally, grand-unification may forbid the D-term. This and related questions are currently under study. For the present, we will simply assume that these terms are very small, and note that this does not imply any "fine-tuning" of the bare parameters of the theory, in virtue of the arguments above.

For m_1^2 and m_2^2 sufficiently large, the minimum of the potential lies at

$$\langle A_{H_U}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}, \quad \langle A_{H_D}\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix},$$
$$v^2 = 2\mu_2^2\,. \tag{5}$$

All other scalar vacuum expectation values vanish at the minimum, except for A_Y, which is undetermined at the tree level. At one loop, it is determined [6,17] and it is in general non-zero. The calculation of the one-loop effective potential is simple in the limit

$$m_3^2, m_4^2 \ll \lambda_1\mu_1^2, \lambda_2\mu_2^2 \ll m_1^2, m_2^2\,. \tag{6}$$

In this limit, the leading term in the potential for small A_Y is

$$V(A_Y) = \frac{2|m_3'|^2(\lambda_1\mu_1^2)^2}{24\pi^2 m_1^2} + \frac{3|m_4'|^2(\lambda_2\mu_1^2)^2}{24\pi^2 m_2^2}\,, \tag{7}$$

where

$$m_3' \equiv m_3 + \lambda_1 A_Y\,, \qquad m_4' \equiv m_4 + \lambda_2 A_Y\,. \tag{8}$$

It is straightforward to minimize the potential. For general values of the parameters of the theory, m_3' and m_4' are non-zero at the minimum and the A_Y field is massive. Note, however, that if

$$m_3/m_4 = \lambda_1/\lambda_2\,, \tag{9}$$

then m_3' and m_4' vanish. This is related to the fact that, in this case, it is possible to eliminate m_3 and m_4 by a shift in Y.

The expectation values of the Higgs fields A_{H_U} and A_{H_D} give rise to quark and lepton masses in the usual way. They also contribute to masses of scalar partners of quarks and leptons. The scalar partners of quarks and leptons also get large contributions to their masses in perturbation theory. Provided the supersymmetry breaking scale is large enough, these contributions are positive and split these fields from their supersymmetry partners. We will in fact assume throughout that the scales associated with supersymmetry breaking are large enough that gauge symmetry breaking may be ignored. We will describe, later, what this assumption means for the parameters of the model.

In this limit, the gauge fermions acquire large masses in one-loop order in perturbation theory. The relevant diagrams are shown in fig. 1. Here we shall content ourselves with orders of magnitude. Detailed computations will be presented in a subsequent publication. Clearly we obtain a result of the form

$$m_\lambda = (\alpha_3/8\pi)\mathcal{M}_1 \ , \quad m_w = (\alpha_2/8\pi)\mathcal{M}_2 \ , \quad (10,11)$$

$$m_b = (\alpha_1/8\pi)(\mathcal{M}_2 + \tfrac{2}{3}\mathcal{M}_1) \ , \quad (12)$$

where α_1, α_2, and α_3 are the U(1), SU(2), and SU(3) gauge couplings, and $\mathcal{M}_1$ and $\mathcal{M}_2$ are dimensional functions of the parameters μ_1, m_1, m_2, etc. These fields also mix with the fermionic partners of the Higgs boson. The resulting mass eigenvalues clearly depend sensitively on the $\mathcal{M}_i$, but it is possible that some of the resulting fermions may be light. For reasonable values of the parameters of the model, some charged fermions might even lie in the PETRA energy range. Because their decays will involve neutral particles in the final state, some care must be taken in determining experimental limits on the masses of such charged particles. It is useful to note, however, that the masses of the lightest of these fermions go down as the parameters $\mathcal{M}_i$ go up, so it should be possible to obtain an upper limit on these parameters. This question is currently under investigation.

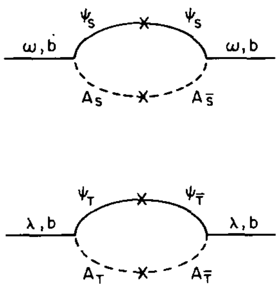

Fig. 1. Diagrams which give Majorana masses to gauge fermions.

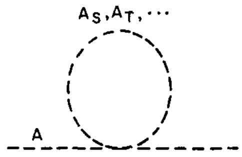

Fig. 2. Diagram which potentially contributes to scalar masses at one loop.

The scalar partners of quarks and leptons gain large masses in perturbation theory due to the breaking of supersymmetry. The one-loop diagrams of fig. 2 give no contribution as a result of the P-invariance discussed earlier. These diagrams would correspond to the appearance of the Fayet–Iliopoulos D-term. The first nonvanishing contribution to these masses comes from the diagrams shown in fig. 3 (in Wess–Zumino gauge). The calculation of these diagrams is straightforward; it is

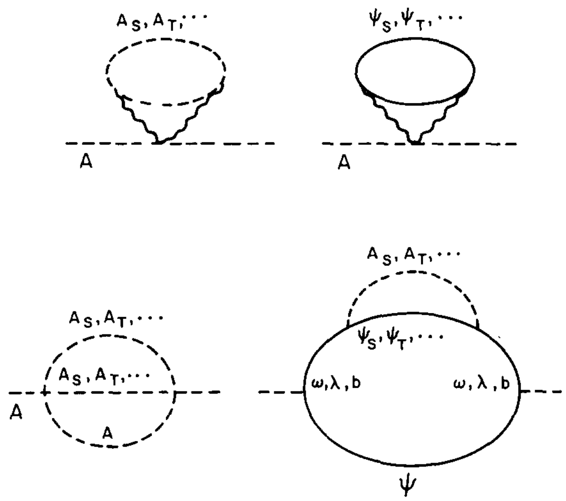

Fig. 3. Diagrams contributing to scalar masses at two-loop order.

necessary to use the dimensional reduction procedure as a regulator [18]. We have obtained a general expression for these diagrams in terms of the eigenvalues and eigenvectors of the tree-level mass matrix. This will be presented elsewhere. For a limited range of values of the parameters of the model we have obtained an explicit expression. This latter formula has been checked using supergraph techniques [15]. The details will be presented elsewhere. In general these masses are given by

$$M_A^2 = \frac{C^2}{2}\left(\frac{\alpha_3}{4\pi}\right)^2 \Lambda_T^2 + \frac{T^2}{2}\left(\frac{\alpha_2}{4\pi}\right)^2 \Lambda_S^2 + \left(\frac{\alpha_1}{4\pi}\right)^2\left(\frac{Y}{2}\right)^2\left(\tfrac{1}{2}\Lambda_S^2 + \tfrac{1}{3}\Lambda_T^2\right). \tag{13}$$

Here, $C^2 = 4/3$ for color triplets and $T^2 = 3/4$ for weak doublets. In the range

$$m_3'^2, m_4'^2 \ll \lambda_1\mu_1^2, \lambda_2\mu_1^2 \ll m_1^2, m_2^2, \tag{14}$$

the leading terms are

$$\Lambda_T^2 = 8\lambda_2^2\mu_1^4/m_2^2, \qquad \Lambda_S^2 = 8\lambda_1^2\mu_1^4/m_1^2, \tag{15, 16}$$

Note that both these quantities are positive, and that, depending on the parameters of the model, they can be arbitrarily large.

For more general values of the parameters of the model, it is necessary to know something about the tree-level mass matrices in order to evaluate Λ_S^2 and Λ_T^2. The general expression in terms of the eigenvectors and eigenvalues of these matrices will be given in a subsequent publication. Numerical investigations indicate that Λ_S^2 and Λ_T^2 are positive for a broad range of parameters of the model, though we have not proven that Λ_S^2 and Λ_T^2 are positive definite.

The lightest scalars in these models are the scalar partners of right-handed leptons. These particles decay rapidly to their fermionic partners and a Goldstone fermion (ψ_Y). The fact that these particles are not observed at PETRA sets a lower limit on their masses of order 16 GeV. This in turn implies that Λ_S must be greater than about 20 TeV, and justifies our initial assumption that these scales are large compared to the scale of gauge symmetry breakdown.

Clearly, then, this model, with appropriate choice of parameters, provides a correct description of low-energy physics. While the scales of the model are not severely constrained, the model implies a large number of mass relations. Also, if supersymmetry is to have anything to do with the hierarchy problem, Λ_T and Λ_S cannot be too large. Note, in particular, that when the supersymmetry breaking scale is sufficiently large, the positive mass-squared contributed by radiative effects to Higgs masses will be so large that only by fine-tuning μ_2^2 and λ_3 will the desired pattern of gauge-symmetry breaking be obtained. This suggests that Λ_S cannot be larger than a few hundreds of TeV. This in turn sets an upper limit on the mass of the lightest scalar lepton of at most hundreds of GeV. As mentioned earlier, limits on charged higgino masses should also constrain the parameters of the model. More detailed consideration of these limits will appear in a subsequent publication.

One particularly nice feature of the model is that, just as in supersymmetric technicolor models, strangeness-changing neutral currents are highly suppressed. Because there are only two Higgs doublets exchanges of ordinary particles give only small flavor-changing neutral currents [19]. Also, it is easy to check that the new particles implied by supersymmetry contribute negligibly to strangeness-changing neutral currents. Because the scalars of a given hypercharge and isospin are nearly degenerate in mass, the GIM mechanism [20] operates for them as well; moreover, all of these fields are quite heavy.

Grand unification [13] of the model appears straightforward. Of course, one must add color-triplet partners for the Higgs doublets. As usual in supersymmetric grand unified theories, one will have to understand why these particles, which would mediate proton decay, are superheavy [2,9]. A rough calculation gives 10^{18} GeV for the unification scale and $\sin^2\theta_w = 0.23$ [21]. The proton lifetime will depend on details of the model, but proton decay may well be observable [8,22]. Unification may imply undesirable relations among the parameters of the superpotential. In particular, one might expect $\lambda_2 m_3 = \lambda_1 m_4$. This would in turn lead to vanishing of m_3' and m_4', which would give a vanishing gluino mass. One will, of course, have to understand why some dimensional parameters of the unified model are much smaller than others. As Witten has suggested [2], supersymmetry offers unique possibilities to address such questions, but the answers are not presently available. The two-loop corrections which have been evaluated here may also be useful in exploiting schemes along the lines suggested by Witten [6] for generating large scales from small ones in supersymmetric theories. These questions are all currently under investigation. For the moment, however, we have exhibited a phenomenologically correct supersymmetric theory, which mimics in detail the spectrum of supersymmetric technicolor models.

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