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SUPERSYMMETRIC TECHNICOLOR

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We propose a supersymmetric model of particle physics in which supersymmetry is broken dynamically by strong gauge forces. The model, as it stands, requires that one parameter be fine tuned; a grand unified version would not require any fine tuning. The model has no strong *CP* problem, and agrees with all known particle physics experiments. A variety of new particles, many of which weigh less than 100 GeV, are predicted.

1. Introduction

The standard Weinberg-Salam [1] theory of electroweak interactions is extremely elegant and economical. The vacuum expectation value of just one Higgs doublet is enough to break $\mathrm{SU(2)_L \times U(1)_Y}$ down to electromagnetism and simultaneously give mass to quarks and leptons. This economy, unfortunately, arises at the expense of "naturalness" [2,3]. Setting scalar masses equal to zero does not increase the symmetry of the theory. Thus the natural value of these masses is some fundamental energy scale of physics, such as the Planck mass M_{pl}, or the scale of grand unification. (We will use M_{pl} generically for this scale.) It is thus difficult to understand why the Fermi constant G_{F} is not of order $1/M_{\mathrm{pl}}^2$. In addition, our current ideas of particle physics require at least one other unnatural adjustment of

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fundamental parameters, for QCD will produce large violations of CP invariance unless its θ parameter [4] is extremely finely tuned ($\theta < 10^{-8}$) [5].

In order to avoid the fine tuning of Higgs masses, Susskind and Weinberg [2, 6] proposed a dynamical symmetry breaking scheme, technicolor (TC). In this scheme, G_{F} is naturally of order (300 GeV)$^{-2}$. However, quarks and leptons remain massless due to chiral symmetries. In order to eliminate these chiral symmetries, Dimopoulos and Susskind and Eichten and Lane [7] enlarged technicolor to extended technicolor (ETC). Unfortunately, it has proven difficult to develop an ETC model with a realistic mass spectrum; moreover, it seems inevitable that any such model will contain large flavor changing neutral currents [8]. In the Weinberg-Salam model, the use of one (or at most two) Higgs doublets avoids this problem [9].

Thus, it would be nice to use Higgs fields to provide masses for quarks and leptons, while using the technicolor mechanism to break $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{Y}$. However, one needs a mechanism to keep the Higgs scalars light. It is here that supersymmetry (SS) [10] may help. (For a review with extensive references, see ref. [11].) In supersymmetric theories, scalar fields are components of chiral superfields. These chiral superfields contain, besides a complex scalar field, a chiral fermion (and an auxiliary field). Global symmetries can keep scalars massless, provided SS is exact.

Of course, the spectrum of the real world is not supersymmetric, for it shows no sign of degeneracy between Bose and Fermi degrees of freedom. If nature is supersymmetric, SS must be spontaneously broken. If SS were broken at the tree level, the natural scale of this breaking (and hence of scalar masses) would be M_{pl}. Thus if SS is to play any role at ordinary energies, it must be dynamically broken. Unfortunately, it is not known if dynamical SS breaking occurs (in four dimensions) [12]. We know, however, that unbroken non-abelian gauge theories can break certain global (bosonic) symmetries. In QCD, color forces break chiral symmetries through formation of condensates

$$\langle \psi_{\bar{\mathrm{u}}} \psi_{\mathrm{u}} \rangle = \langle \psi_{\bar{\mathrm{d}}} \psi_{\mathrm{d}} \rangle \simeq \Lambda^3_{\mathrm{QCD}} . \tag{1.1}$$

Of course, in a supersymmetric version of QCD, one does not know what pattern of symmetry breaking may emerge. However, if fermion-fermion condensates form, they will break not only chiral symmetries but also supersymmetry [12]. In this paper we will assume that this type of SS breaking occurs, and use this idea to develop a model of particle physics.

The gauge group of our model will be of the form $\mathrm{SU}(3)_{\mathrm{C}} \times \mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{Y} \times \mathrm{SU}(M)_{\mathrm{SC}} \times \mathrm{SU}(N)_{\mathrm{TC}}$, with coupling constants g_3, g_2, g_1, g_M, and g_N. The last two groups (supercolor and technicolor) will be used to break supersymmetry and $\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_{Y}$, respectively. Before exploring these models, it is essential again to address the question of naturalness. In models with a U(1) factor in the gauge group, it is necessary to include in the lagrangian a term of the form

$$\mathcal{L}_D = M^2 \int \mathrm{d}^4 x\, D(x) \tag{1.2}$$

where $D(x)$ is the auxiliary field of the U(1) gauge supermultiplet. Such a term is supersymmetric and gauge invariant. The coefficient M^2 is quadratically divergent in perturbation theory (in left-right asymmetric theories), and thus its natural order of magnitude is M_{pl}^2. However, Witten [12] has pointed out that if the U(1) is embedded, at any scale, in a non-abelian group, the effective low-energy theory has $M^2=0$ to any finite order of perturbation theory (provided SS is unbroken). We will not present a unified version of our model. We will assume, however, that such a unification is possible and we will require $M^2=0$.

One might ask why we do not use one gauge group to break both SS and $\mathrm{SU}(2)\times\mathrm{U}(1)$. The answer is that such an approach seems invariably to lead to phenomenologically unacceptable consequences. We will comment on some scenarios of this type in the conclusion of the paper.

In the model which we will discuss here, the supercolor, technicolor, and color interactions are all asymptotically free and characterized by scales Λ_{SC}, Λ_{TC} and Λ_{C} respectively. We will require

$$\Lambda_{\mathrm{SC}} \gg \Lambda_{\mathrm{TC}} \gg \Lambda_{\mathrm{C}}. \tag{1.3}$$

The model has two weak doublet chiral superfields with supercolor interactions. The component fields of these doublets form condensates at a scale Λ_{SC} of order 10 TeV. These condensates break SS but not $\mathrm{SU}(2)\times\mathrm{U}(1)$. The breaking of SS leads to masses for the scalar partners of quarks and leptons, as well as for the fermionic partners of all gauge bosons.

The model also contains a weak doublet and two weak singlet chiral superfields with technicolor. At a scale Λ_{TC}, the fields will form condensates which break $\mathrm{SU}(2)_{\mathrm{L}}\times\mathrm{U}(1)_Y$ down to $\mathrm{U}(1)_{\mathrm{EM}}$ (electromagnetism). The would-be Goldstone bosons of this symmetry breaking give mass to the W and Z bosons, just as in ordinary TC models.

The model contains two weak doublets, neutral under all strong gauge groups, which serve to give mass to quarks and leptons. We call these doublets Higgs fields. They have weak Yukawa couplings to the technicolored fields as well as the quark and lepton superfields. As a result of these couplings, the techniquark condensates induce vacuum expectation values (VEV's) for the Higgs fields. These VEV's give mass to quarks and leptons.

The model has the virtue of requiring only one fine tuning of a fundamental parameter, and Witten's theorem provides a physical mechanism for this fine tuning. Moreover, the model turns out to have no *CP* problem in any strongly interacting sector, and the resulting axion is phenomenologically harmless. Flavor-changing neutral currents are naturally suppressed, and the important relation

$$M_{\mathrm{W}}/M_{\mathrm{Z}}=\cos\theta_{\mathrm{W}}+\mathrm{O}(\alpha) \tag{1.4}$$

is maintained. The model predicts a plethora of new physics: a gluino (the fermionic partner of the gluon) with mass of order 5 GeV; a host of fundamental scalars with quark and lepton quantum numbers and masses of order 100 GeV; other bizarre fermions and bosons; and new physics at the technicolor and supercolor scales.

The paper is organized as follows. In sect. 2 we discuss a supersymmetric version of QCD in order to explain our assumptions about SS breakdown in strongly interacting gauge theories, and to introduce our notation. In sect. 3, we present our model of particle physics. We describe the breaking of SS and $SU(2)\times U(1)$, and explain the origin of quark and lepton masses. We estimate the masses of scalar partners of ordinary fermions and of fermionic partners of gauge bosons. Particular attention is paid to the properties of the fermions. In sect. 4, we demonstrate that the model has no strong *CP* problem, and consider the properties of the resulting axion. In the conclusion, we briefly discuss models involving only one strong gauge group (besides color), and prospects for unification.

2. Breaking of supersymmetry: supersymmetric QCD

As we said in the introduction, we would like to imagine that SS is broken dynamically, at a scale where certain (asymptotically free) gauge couplings become strong. In analogy to chiral symmetry breaking in ordinary QCD, we hypothesize the existence of dynamically generated, SS breaking condensates. To illustrate this idea, and to establish notation, we first consider a supersymmetric version of QCD. The theory contains a color gauge superfield, V^a, and four chiral superfields, U, $\bar{U}$, D, and $\bar{D}$. U and D transform as color triplets; $\bar{U}$ and $\bar{D}$, as antitriplets. (The bar does *not* indicate complex conjugation; it is just part of the name of the superfield.) Each of these superfields may be thought of as a multiplet of ordinary fields. The chiral superfields each contain a complex scalar field, a left-handed spinor field, and a complex scalar auxiliary field (none of its derivatives appear in the lagrangian). For example, we may write

$$U=(A_U,\psi_U,F_U),$$

$$\bar{U}=(A_{\bar{U}},\psi_{\bar{U}},F_{\bar{U}}). \tag{2.1}$$

(We will use a two-component notation for spinors.) Similarly, the gauge superfield consists (in a particular gauge known as Wess-Zumino gauge [13]) of a vector field (the usual vector potential), a chiral spinor (the gluino), and a scalar auxiliary field, all in the adjoint representation:

$$V^a=\left(A^a_\mu,\lambda^a,D^a\right). \tag{2.2}$$

The supersymmetry of the theory can be made manifest by use of the superfield formalism [11], which also simplifies the treatment of certain internal symmetries. In this formalism, one introduces anticommuting, spinorial parameters, θ_α, and treats the superfields as functions of x_μ (spacetime coordinates), the parameters θ_α, and their complex conjugates, $\theta^*{}_{\dot\alpha}$. The component fields are coefficients in an expansion in powers of θ. For a chiral superfield such as U we have

$$U(x,\theta)=\exp\left(i\theta^*\sigma^\mu\theta\partial_\mu\right)\left[A_U(x)+\sqrt{2}\,\theta\psi_U(x)+\theta\theta F_U(x)\right]. \tag{2.3}$$

Similarly, the gauge superfield is (in Wess-Zumino gauge)

$$V^a(x,\theta)=\theta^*\sigma^\mu\theta A^a_\mu+i\theta\theta\theta^*\lambda^{a*}-i\theta^*\theta^*\theta\lambda^a+\tfrac{1}{2}\theta\theta\theta^*\theta^*D^a. \tag{2.4}$$

If we require that the lagrangian be invariant under separate phase rotations of the chiral superfields, the most general lagrangian consistent with SS, gauge invariance, and renormalizability is

$$\begin{aligned}\mathcal{L}=\tfrac{1}{4}(W_aW_a+W_a^*W_a^*)&+U^*\exp(gV_aT_a)U\\&+\bar{U}^*\exp(-gV_aT_a^*)\bar{U}+D^*\exp(gV_aT_a)D+\bar{D}^*\exp(-gV_aT_a^*)\bar{D},\end{aligned} \tag{2.5}$$

where T_a are the SU(3) generators, and W is the supersymmetric analog of the field strength:

$$W^a_\alpha=-i\lambda^a_\alpha+\left[\delta^\beta_\alpha D^a-\tfrac{1}{2}i(\sigma^{\mu\nu})_\alpha^{\ \beta}F^a_{\mu\nu}\right]\theta_\beta+\theta^\gamma\theta_\gamma\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\lambda^{a*\dot\alpha}. \tag{2.6}$$

In terms of component fields, the lagrangian contains the usual gauge-invariant kinetic terms for the fields A_μ, λ, ψ_i, and A_i, where $i=U,\bar{U},D,\bar{D}$. (These terms include all the usual gauge couplings.) In addition, there are some Yukawa couplings

$$\mathcal{L}=i\sqrt{2}\,gA_U^*T_a\psi_U\lambda_a-i\sqrt{2}\,gA_{\bar U}^*T_a^*\psi_{\bar U}\lambda_a+\text{h.c.}+(U\to D), \tag{2.7}$$

and terms involving the auxiliary fields

$$\begin{aligned}\mathcal{L}=F_U^*F_U&+F_{\bar U}^*F_{\bar U}+F_D^*F_D+F_{\bar D}^*F_{\bar D}+\tfrac{1}{2}D^aD^a\\&+gD^a\big(A_U^*T^aA_U-A_{\bar U}^*T^{a*}A_{\bar U}\\&+A_D^*T^aA_D-A_{\bar D}^*T^{a*}A_{\bar D}\big).\end{aligned} \tag{2.8}$$

The total lagrangian of eq. (2.5) possesses a variety of symmetries. The SS charges, Q_α and $Q^*{}_{\dot\alpha}$, generate SS transformations. They obey the following commutation relations with the members of a chiral multiplet:

$$
\begin{aligned}
&[Q_\alpha, A] = \sqrt{2}\,\psi_\alpha, && [Q^*_{\dot\alpha}, A] = 0 \\
&\{Q_\alpha, \psi_\beta\} = \sqrt{2}\,F\varepsilon_{\alpha\beta}, && \{Q^*_{\dot\alpha}, \psi_\beta\} = \sqrt{2}\,i\partial\!\!\!/_{\dot\alpha\beta}A\,, \\
&[Q_\alpha, F] = 0, && [Q^*_{\dot\alpha}, F] = -\sqrt{2}\,i\partial\!\!\!/_{\dot\alpha\beta}\psi^\beta .
\end{aligned}
\tag{2.9}
$$

With the members of the gauge multiplet, the rules are

$$
\begin{aligned}
&[Q_\alpha, A^\mu] = \sigma^\mu_{\alpha\dot\beta}\lambda^{*\dot\beta}, && [Q^*_{\dot\alpha}, A^\mu] = -\sigma^\mu_{\dot\alpha\beta}\lambda^\beta, \\
&\{Q_\alpha, \lambda_\beta\} = D\varepsilon_{\alpha\beta} - \tfrac{1}{4}F_{\mu\nu}(\sigma^{\mu\nu})_{\alpha\beta}, && \{Q^*_{\dot\alpha}, \lambda_\beta\} = 0, \\
&[Q_\alpha, D] = -i\partial\!\!\!/_{\alpha\dot\beta}\lambda^{*\dot\beta}, && [Q^*_{\dot\alpha}, D] = i\partial\!\!\!/_{\dot\alpha\beta}\lambda^\beta .
\end{aligned}
\tag{2.10}
$$

The SS transformations must be supplemented by a supergauge transformation in order to remain in Wess-Zumino gauge.

In addition to the SS and gauge symmetries, the model possesses a variety of global symmetries. There is a $U(2)\times U(2)$ symmetry involving the chiral superfields. The axial $U(1)$ is broken by anomalies as in ordinary QCD. The other $U(1)$ is just baryon number, $U(1)_B$. There is also a non-anomalous $U(1)$ symmetry which involves the gluino fields, λ^a. We call it $U(1)_X$; it is essentially the R-invariance discussed in the literature [11]. It may be observed by inspecting the component form of the lagrangian, but can be studied more elegantly and efficiently if we note that the superfield lagrangian, eq. (2.5), is invariant under the transformation

$$
\begin{aligned}
&V^a(x,\theta) \to V^a(x, e^{-i\alpha}\theta), \\
&\Phi_i(x,\theta) \to e^{i\alpha X_i}\Phi_i(x, e^{-i\alpha}\theta),
\end{aligned}
\tag{2.11}
$$

for arbitrary X_i, where Φ_i represent the four chiral superfields U, $\bar{U}$, D, and $\bar{D}$. Under this transformation, the component fields transform as

$$
\begin{aligned}
&A_i \to e^{i\alpha X_i}A_i, \\
&\psi_i \to e^{i\alpha(X_i - 1)}\psi_i, \\
&\lambda^a \to e^{i\alpha}\lambda^a .
\end{aligned}
\tag{2.12}
$$

The corresponding current has no anomalies provided that

$$
4(X_U + X_D + X_{\bar{U}} + X_{\bar{D}} - 4) + 24 = 0,
\tag{2.13}
$$

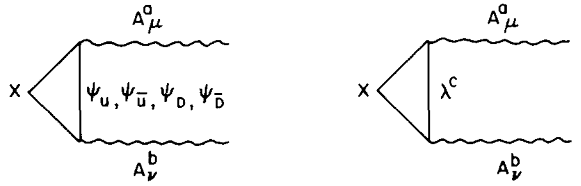

Fig. 1. Contributions to the anomaly of the X current.

as shown in fig. 1. When we come to the problem of constructing a supersymmetric model of particle physics, this type of symmetry will play a crucial role. In particular, it will insure the absence of strong CP violation.

Since our example of supersymmetric QCD is asymptotically free, it is natural to ask about possible breakdown of chiral symmetry. In particular, let us assume (without proposing a specific dynamical mechanism) that quark-antiquark condensates form, just as in ordinary QCD:

$$\langle \psi_U \psi_{\bar{U}} \rangle = \langle \psi_D \psi_{\bar{D}} \rangle = a\Lambda^3_{\mathrm{QCD}}, \tag{2.14}$$

where Λ_{QCD} is a typical hadronic scale and a is a number of order unity. In addition, we might expect the existence of scalar-scalar and gluino-gluino condensates,

$$\langle A_{\bar{U}} A_U \rangle = \langle A_{\bar{D}} A_D \rangle = b\Lambda^3_{\mathrm{QCD}}, \tag{2.15}$$

$$\langle \lambda^a \lambda^a \rangle = c\Lambda^3_{\mathrm{QCD}}. \tag{2.16}$$

In addition to breaking $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)}_B \times \mathrm{U(1)}_X$ down to $\mathrm{SU(2)}_V \times \mathrm{U(1)}_B$, the condensates of eqs. (2.14) and (2.16) also break SS. Quite generally, in fact, fermion-fermion condensates break SS. For example, consider the color-singlet composite field

$$G^U_\alpha = A_{\bar{U}} \psi_{U\alpha} + A_U \psi_{\bar{U}\alpha}. \tag{2.17}$$

We have, using eq. (2.9),

$$\left\{ Q_\alpha, G^U_\beta \right\} = 2\psi_{\bar{U}\alpha}\psi_{U\beta} + 2\psi_{U\alpha}\psi_{\bar{U}\beta} + 2\varepsilon_{\alpha\beta}\left(A_{\bar{U}} F_U + F_{\bar{U}} A_U \right). \tag{2.18}$$

The equations of motion imply $F_U = F_{\bar{U}} = 0$. Taking the vacuum expectation value of eq. (2.18), we find

$$\begin{aligned} \left\langle \left\{ Q_\alpha, G^U_\beta \right\} \right\rangle &= 2\langle \psi_{\bar{U}\alpha}\psi_{U\beta} + \psi_{U\alpha}\psi_{\bar{U}\beta} \rangle \\ &= 2\varepsilon_{\alpha\beta} a\Lambda^3_{\mathrm{QCD}}. \end{aligned} \tag{2.19}$$

Thus the existence of the fermion-fermion condensate implies Q_α does not annihilate the vacuum, i.e., supersymmetry is broken (the scalar-scalar condensate does not imply SS breaking).

Gluino-gluino condensates may also break SS, but the situation is more complicated. In particular, if we define

$$G_\alpha^V = A_\mu \sigma^\mu_{\alpha\dot\beta} \lambda^{*\dot\beta}\,, \tag{2.20}$$

we have

$$\left\langle \left\{ Q_\beta, G_\alpha^V \right\} \right\rangle = 2\varepsilon_{\alpha\beta} \langle \lambda\lambda \rangle\,. \tag{2.21}$$

Thus the gluino condensate appears to break supersymmetry. This observation is suspect, however, since G^V is not gauge invariant. Fortunately, for the model we discuss in the next section, it will not be essential to know whether such condensates in fact break supersymmetry. The question is interesting, however, and is worthy of further study. In any case, the Goldstone fermion or "goldstino" of this theory is a composite of quarks and scalars, gluinos and gluons. Its decay constant is of order Λ_{QCD}.

In addition to this massless fermion, the theory has four massless Goldstone bosons resulting from the breaking of the axial SU(2) and X symmetries; their decay constants are also expected to be of order Λ_{QCD}. We denote these particles by $\boldsymbol{\Pi}$ and X. All other states are expected to be massive; again Λ_{QCD} should be the relevant scale.

One can add electroweak interactions to this model, assigning to the quark superfields their usual $\mathrm{SU(2)_L \times U(1)}_Y$ quantum numbers. (Lepton superfields can be added to cancel hypercharge anomalies.) Since the new couplings are weak, one expects the picture developed above to remain essentially unchanged. The W and Z will thus gain mass by eating the $\boldsymbol{\Pi}$'s, as in technicolor models. The X boson will remain in the spectrum, since X symmetry remains a good symmetry (it has $\mathrm{SU(2)_L}$ anomalies, but the resulting contribution to the X boson mass will be extremely small, since $\mathrm{SU(2)_L}$ is broken).

What of the fermionic partners of the electroweak gauge bosons? These particles, which we will call the bino $[\mathrm{U}(1)_Y]$ and winos $[\mathrm{SU(2)_L}]$, gain mass from diagrams such as those shown in fig. 2. Since the intermediate states in these diagrams contain strongly interacting particles, the diagrams cannot be calculated, but we expect the various terms in the mass matrix to be of order $g_{2,1}^2 \Lambda_{\mathrm{QCD}}$.

The detailed structure of the mass matrix is complex, and is sensitive to the hypercharge assignments of the quarks. The analysis of the fermion mass matrix will in fact be simpler in the model of particle physics which we now proceed to develop.

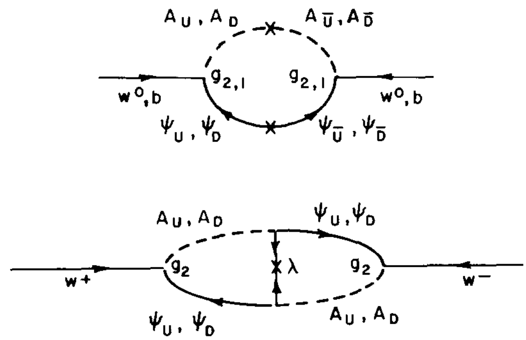

Fig. 2. Some contributions to the wino and bino mass matrices in supersymmetric QCD. The X denotes a condensate.

3. Supercolor

3.1. THE MODEL

We now proceed to develop a supersymmetric model of particle physics based on the ideas of the preceding sections. In addition to the standard gauge group, $SU(3)\times SU(2)\times U(1)$, we introduce two more unbroken gauge symmetries. The first, which we call supercolor, becomes strong at a scale Λ_{SC} of order 10 TeV. For definiteness, we take the gauge group of this interaction to be $SU(M)_{SC}$. This interaction will be responsible for supersymmetry breakdown. The second, which we call technicolor, becomes strong at a scale Λ_{TC} of order 300 GeV; we take the gauge group to be $SU(N)_{TC}$. Technicolor condensates will cause the breakdown of $SU(2)\times U(1)$.

We will also require of the model that it possess a global X symmetry (R invariance) of the type discussed in the preceding section. It will be possible to assign X quantum numbers in such a way that the corresponding current, j_μ^X, has no supercolor or technicolor anomalies. However, with this requirement, no set of X assignments will lead to a current free of color anomalies. This fact will play a crucial role in eliminating strong CP violation in the model. This will be discussed in sect. 4, after we have presented the model.

We begin by listing the chiral superfields of the model, their $SU(M)_{SC}\times SU(N)_{TC}\times SU(3)\times SU(2)\times U(1)_Y\times U(1)_X$ transformation properties, and their couplings. There are, first, two SU(2) doublets with supercolor interactions, called "superquarks".

$$S=\left(\underline{M},\underline{1},\underline{1},\underline{2},y,1-\tfrac{1}{2}M\right), \tag{3.1}$$

$$\bar{S}=\left(\underline{\overline{M}},\underline{1},\underline{1},\underline{2},-y,1-\tfrac{1}{2}M\right). \tag{3.2}$$

There are an SU(2) doublet and two singlets with technicolor interactions:

$$T=\begin{pmatrix} T_U \\ T_D \end{pmatrix}=(\underline{1},\underline{N},\underline{1},\underline{2},y',1-\tfrac{1}{2}N), \tag{3.3}$$

$$\overline{T}_U=(\underline{1},\underline{\overline{N}},\underline{1},\underline{1},-y'-1,1-\tfrac{1}{2}N), \tag{3.4}$$

$$\overline{T}_D=(\underline{1},\underline{\overline{N}},\underline{1},\underline{1},-y'+1,1-\tfrac{1}{2}N). \tag{3.5}$$

There are two weak doublets which play the role of Higgs fields

$$H_U=(\underline{1},\underline{1},\underline{1},\underline{2},-1,N), \tag{3.6}$$

$$H_D=(\underline{1},\underline{1},\underline{1},\underline{2},+1,N). \tag{3.7}$$

Finally, there are the quark and lepton superfields,

$$Q_n=\begin{pmatrix} U_n \\ D_n \end{pmatrix}=(\underline{1},\underline{1},\underline{3},\underline{2},\tfrac{1}{3},X_Q), \tag{3.8}$$

$$\overline{U}_n=(\underline{1},\underline{1},\underline{\overline{3}},\underline{1},-\tfrac{4}{3},2-N-X_Q), \tag{3.9}$$

$$\overline{D}_n=(\underline{1},\underline{1},\underline{\overline{3}},\underline{1},\tfrac{2}{3},2-N-X_Q), \tag{3.10}$$

$$L_m=\begin{pmatrix} N_m \\ E_m \end{pmatrix}=(\underline{1},\underline{1},\underline{1},\underline{2},-1,X_L), \tag{3.11}$$

$$\overline{E}_m=(\underline{1},\underline{1},\underline{1},\underline{1},2,2-N-X_L). \tag{3.12}$$

Here n and m are generation indices. The value of y' is chosen so as to avoid hypercharge anomalies; it depends on the number of quarks and leptons. The value of y is unrestricted.

The lagrangian consists, first, of the supersymmetric and gauge invariant kinetic terms of the chiral and gauge superfields. X symmetry forbids the mass terms $mS\overline{S}$ and $m'H_UH_D$ which are otherwise permitted by gauge invariance and supersymmetry. While X symmetry is violated by color anomalies, this effect should be negligible at the ultra-high energies where these couplings are determined.

We do include the following Yukawa couplings:

$$\mathcal{L}=\sum_n\left(y_{Un}Q_n\overline{U}_nH_U+y_{Dn}Q_n\overline{D}_nH_D\right)$$
$$+\sum_m y_{Lm}L_m\overline{E}_mH_D+z_+T\overline{T}_UH_U+z_-T\overline{T}_DH_D+\text{h.c.} \tag{3.13}$$

In terms of component fields, these terms generate ordinary Yukawa couplings among scalars and spinors, and certain quartic scalar couplings. For example,

$$\begin{aligned} Q\bar{U}H_U = {} & F_Q A_{\bar{U}} A_{H_U} + F_{\bar{U}} A_Q A_{H_U} + F_{H_U} A_Q A_{\bar{U}} \\ & + \psi_Q \psi_{\bar{U}} A_{H_U} + \psi_{\bar{U}} \psi_{H_U} A_Q + \psi_Q \psi_{H_U} A_{\bar{U}} . \end{aligned} \tag{3.14}$$

Other Yukawa couplings among the superfields are forbidden by requiring conservation of separate lepton numbers. Clearly this assumption can be relaxed.

As discussed in the introduction, we should also add to the lagrangian a term of the form $M^2 D^Y$, where D^Y is the auxiliary field of the hypercharge gauge multiplet. Since M^2 would be quadratically divergent in perturbation theory, it would naturally be $O(M_{\mathrm{pl}}^2)$. This would be disastrous. It would result in a spontaneous breaking of hypercharge at the Planck scale. However, as we have remarked in the introduction, Witten has shown that $M^2 = 0$ if hypercharge is unified into a non-abelian group at any scale, to all orders of perturbation theory. We will not write down a model with such a unification, but we will suppose that such a unification can be achieved. We will thus require that $M^2 = 0$ in each order of perturbation theory.

3.2. SUPERCOLOR

We will now discuss the physics of this model, starting at very high energies and working our way down to lower energies. We first discuss physics at the supercolor scale. At this scale, one can neglect all couplings other than supercolor couplings, in first approximation. It is natural to assume, following our discussion in sect. 2, that the following $SU(2)\times U(1)$ invariant condensates form:

$$\langle \psi_S \psi_{\bar{S}} \rangle = a\Lambda_{\mathrm{SC}}^3 , \qquad \langle \lambda_{\mathrm{SC}} \lambda_{\mathrm{SC}} \rangle = c\Lambda_{\mathrm{SC}}^3 , \qquad \langle A_S A_{\bar{S}} \rangle = b\Lambda_{\mathrm{SC}}^2 . \tag{3.15}$$

These condensates break supersymmetry and X symmetry. They also break the $SU(2)\times SU(2)$ invariance which exists in the absence of weak interactions. They give rise to a goldstino, G, a Goldstone boson associated with X symmetry, X, and three superpions, $\mathbf{\Pi}$. The superpions will gain masses due to electroweak interactions. These masses will be of order several hundred GeV. The fermionic partners of the $SU(2)\times U(1)$ gauge bosons, which we refer to as the "winos" and "bino", and denote by $\vec{\mathrm{w}}$ and b, gain mass through the supercolor condensates via their weak interactions. Typical diagrams contributing to these masses are shown in fig. 3. These diagrams, of course, have strongly interacting particles in intermediate states, and therefore cannot be calculated. We expect, however, for orders of magnitude,

$$\begin{aligned} m_{\mathrm{w}} & \simeq g_2^2 \Lambda_{\mathrm{SC}} \\ & \simeq 4\ \mathrm{TeV} , \end{aligned} \tag{3.16}$$

$$\begin{aligned} m_{\mathrm{b}}/m_{\mathrm{w}} & = y^2 \tan^2\theta_{\mathrm{W}} , \\ m_{\mathrm{b}} & \simeq 1.2\ \mathrm{TeV} , \qquad (y=1) . \end{aligned} \tag{3.17}$$

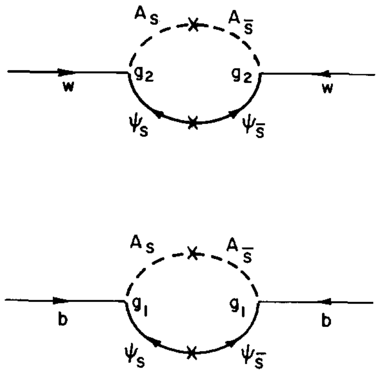

Fig. 3. Diagrams contributing to the wino and bino masses in the model of particle physics.

There is no wino-bino mixing at this stage, due to $SU(2)_L$ symmetry. These masses provide an example of how supersymmetry breakdown induced by the supercolor condensates is transmitted to lower energies via the weak interactions.

Scalar partners of quarks and leptons, as well as scalar components of Higgs fields, also gain mass at this stage. The lowest order diagrams are shown in fig. 4. These diagrams would sum to zero if SS were exact, but the wino and bino masses will create a mismatch. If we approximate the wino and bino propagators by massive free propagators, the result is

$$\begin{aligned} m_A^2 &= \frac{2}{\pi}\alpha_2 T^2 m_w^2 \ln(\Lambda_{SC}/m_w) + \frac{1}{2\pi}\alpha_1 Y^2 m_b^2 \ln(\Lambda_{SC}/m_b) \\ &\simeq T^2 (540\ \mathrm{GeV})^2 + Y^2 (70\ \mathrm{GeV})^2 . \end{aligned} \tag{3.18}$$

Here Y and T denote the hypercharge and $SU(2)_L$ assignments of the supermultiplet

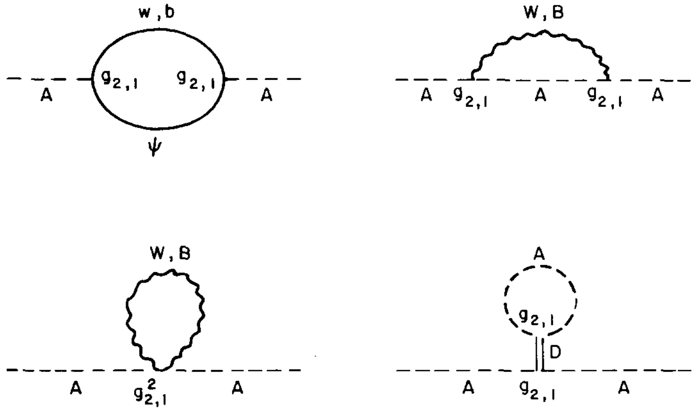

Fig. 4. Contributions to scalar masses. A and ψ are scalar and spinor components of a chiral superfield.

(T^2 is $\frac{3}{4}$ for a doublet). Note that we have cut off the integral at the scale Λ_{SC}, since the wino and bino masses are dynamical in origin, and will decrease rapidly with momentum for momenta of order Λ_{SC}. Because the sensitivity of the integral to these large scales is only logarithmic, this is probably a reasonable approximation. Estimation of higher order corrections is complicated because of the subtleties connected with the D term. This is not a problem for the order α corrections since $\mathrm{Tr}\,Y=0$; the necessary cancellation is immediate. Higher order corrections (in α) will be small provided, again, that there is no D term in the renormalized lagrangian.

The fermionic partner of the gluon, the gluino, also gains mass. The largest contribution comes from the diagrams of fig. 5. The resulting mass is of order

$$\begin{aligned} m_{\lambda_c} &\simeq \frac{1}{\pi}\alpha_3(m_w)\frac{1}{\pi}\alpha_2 n_{\text{flavors}} m_w \ln(\Lambda_{SC}/m_w) \\ &\simeq 5\ \text{GeV}. \end{aligned} \tag{3.19}$$

The gluino will decay primarily to a goldstino plus a gluon (ordinary hadrons). The gluino lifetime can be estimated using "soft goldstino theorems". If S_μ^α is the supercurrent, then

$$\langle 0|S_\mu^\alpha|G^{\dot\beta}\rangle = f_s\sigma_\mu^{\alpha\dot\beta}, \tag{3.20}$$

where f_s, the goldstino decay constant, is of order Λ_{SC}^2. One can evaluate the effective goldstino-gluino-gluon coupling by considering the matrix element

$$\frac{1}{f_s}\langle 0|\partial^\nu S_\nu^\alpha|\lambda^\beta A_\mu\rangle. \tag{3.21}$$

We can estimate this matrix element at low energies if we treat the gluino simply as a massive spinor. One obtains the effective vertex

$$\Gamma_\mu^{\alpha\beta} = (\sigma_{\mu\nu})^{\alpha\beta}k^\nu m_\lambda/f_s, \tag{3.22}$$

where k_ν is the gluon momentum. This leads to a gluino lifetime of order 10^{-10} s. This estimate must be considered rough (confining effects).

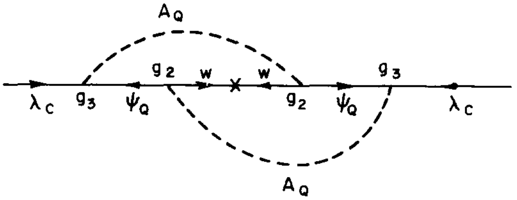

Fig. 5. Contribution to the gluino mass. The X denotes the dynamical mass of the wino.

Since the gluino carries color, it should be a "confined" object. The lightest state of this type is expected to consist of gluons bound to a gluino. It should be possible to pair produce such objects in hadron-hadron collisions, for example. They might be observed in calorimetry experiments [14].

3.3. TECHNICOLOR

As we go down in energy, technicolor forces become strong at a scale Λ_{TC} (of order 300 GeV). Neglecting electroweak and Yukawa couplings, we assume that the following condensates form:

$$\begin{aligned} \langle \psi_{T_U} \psi_{\bar{T}_U} \rangle &= \langle \psi_{T_D} \psi_{\bar{T}_D} \rangle = a' \Lambda^3_{\mathrm{TC}}, \\ \langle A_{T_U} A_{\bar{T}_U} \rangle &= \langle A_{T_D} A_{\bar{T}_D} \rangle = b' \Lambda^2_{\mathrm{TC}}, \\ \langle \lambda_{\mathrm{TC}} \lambda_{\mathrm{TC}} \rangle &= c' \Lambda^3_{\mathrm{TC}}. \end{aligned} \tag{3.23}$$

These condensates break the approximate $\mathrm{SU}(2) \times \mathrm{SU}(2)$ symmetry of the technicolor sector down to a vector SU(2); the three technipions become the longitudinal components of the W and Z bosons. These condensates also break supersymmetry and X symmetry. This will result in small $(\mathrm{O}(\Lambda_{\mathrm{TC}}/\Lambda_{\mathrm{SC}}))$ changes in the decay constants and other properties of the goldstino and X boson. Also, technicolor dynamics will give small modifications of wino-bino and scalar mass matrices.

3.4. HIGGS FIELDS

If the coupling of Higgs fields to technifields, z_+ and z_-, are sufficiently small, the interaction of eq. (3.13) may be viewed as a small perturbation on the technicolor sector, and the picture of $\mathrm{SU}(2) \times \mathrm{U}(1)$ breakdown discussed in subsect. 3.3 should remain valid. In particular, we still expect the formation of fermion-fermion and scalar-scalar condensates.

In terms of component fields, the lagrangian of eq. (3.13) yields Yukawa couplings of the form

$$\mathcal{L} = z_+ \psi_T \psi_{\bar{T}_U} A_{H_U} + z_- \psi_T \psi_{\bar{T}_D} A_{H_D} + \mathrm{h.c.} \tag{3.24}$$

If we replace the products of Fermi fields here by their VEV's, we see that the effective potential of these scalars includes linear terms of the form

$$z_+ \Lambda^3_{\mathrm{TC}} A_{H^0_U} + z_- \Lambda^3_{\mathrm{TC}} A_{H^0_D} + \mathrm{h.c.} \tag{3.25}$$

Such terms will necessarily give $A_{H^0_U}$ and $A_{H^0_D}$ vacuum expectation values; choosing

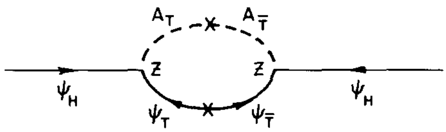

Fig. 6. Contributions to the higgino mass.

$z_+ \simeq z_- \sim 0.3$ we find

$$\langle A_{H_U^0} \rangle = \langle A_{H_D^0} \rangle \simeq 2z\Lambda_{\mathrm{TC}}^3/m_{\mathrm{A}}^2 \tag{3.26}$$

$$\simeq 75 \text{ GeV}. \tag{3.27}$$

The scalar masses appearing here are those arising from eq. (3.18) as described in subsect. 3.2. These VEV's will contribute to the mass of the W and Z bosons; for example, the W mass is given by

$$M_{\mathrm{W}}^2 = \tfrac{1}{4}g_2^2\left[F_{\mathrm{T}}^2 + \langle A_{H_U^0} \rangle^2 + \langle A_{H_D^0} \rangle^2\right], \tag{3.28}$$

where F_{T} is the technipion decay constant.

The fermionic components of the Higgs multiplets ("higginos") will also gain mass at this stage. These masses will arise through diagrams of the type shown in fig. 6. (Again these diagrams contain strongly interacting particles and are only used to obtain order of magnitude estimates). These masses are expected to be of order

$$m_{\psi_H} \simeq z^2\Lambda_{\mathrm{TC}} \tag{3.29}$$

$$\simeq 30 \text{ GeV}. \tag{3.30}$$

There will also be mixing between higginos, winos and bino, but these effects will be small due to the large mass of the winos and the bino.

3.5. QUARK AND LEPTON MASSES AND CONSTRAINTS ON Λ_{SC}

As explained in the introduction, quark and lepton mass arise in the usual way through $\langle A_{H_U^0} \rangle$ and $\langle A_{H_D^0} \rangle$ via Yukawa couplings. The u, c, t,... quarks receive their masses from $\langle A_{H_U^0} \rangle$, and d, s, b quarks and charged leptons receive theirs from $\langle A_{H_D^0} \rangle$ [see eq. (3.13)].

Up to this stage, the supercolor scale, Λ_{SC}, has appeared to be an arbitrary parameter of the model. Since the masses of winos, binos, gluinos, and scalars, as well as the coupling of the axion (see sect. 4) are controlled by this scale, it would seem difficult to set limits on these quantities. There is, however, at least one constraint on this scale. Consistency of the picture of particle physics we have developed in the previous sections requires that all coupling constants be small at energies ranging from slightly above the supercolor scale to the Planck mass. In particular, the Yukawa couplings of the model are not asymptotically free, and thus should be small at low energies. Thus the mass of the heaviest quark or lepton places a lower limit on the VEV of the Higgs field A_{H_U} and A_{H_D}. This in turn puts an upper limit on the supercolored scale Λ_{SC}. If we require $y, z_+, z_- \lesssim e = 0.3$ and assume $m_t \sim 30$ GeV then we find that Λ_{SC} is, at most, a few tens of TeV.

3.6. NEUTRAL FLAVOR-CHANGING CURRENTS

This model has no problem with flavor-changing neutral currents. Since quarks have their standard electroweak assignments, flavor-changing processes due to gauge boson exchanges are suppressed. Also, since quarks of charge $\frac{2}{3}$ all get mass from $A_{H_U^0}$, and quarks of charge $-\frac{1}{3}$ get mass from $A_{H_D^0}$, Higgs exchanges are harmless. One can readily convince oneself that wino, bino and higgino exchange give extremely small contributions to these processes. Therefore, this model avoids the difficulties encountered by extended technicolor theories.

4. *CP* violation and axions

The model we have discussed in the previous section has no strong *CP* problem, in any of its strongly interacting sectors; it has an axion, but as we will show now, this axion is phenomenologically harmless.

First, consider the supercolor sector. This sector has an anomalous U(1) symmetry. This anomaly may be used to rotate away any *CP*-violating θ parameter in the supercolor world. There is no light particle associated with breaking this anomalous U(1).

In the case of the technicolor world, there is a similar U(1) symmetry, under which the Higgs doublets, quarks and leptons also transform. This symmetry may be used to rotate away any θ parameter in the technicolor sector. There is a pseudo-Goldstone boson, analogous to the axion, which results when technicolor condensates break this symmetry. However, since the Yukawa couplings, z_+ and z_-, are not so small, this axion will be heavy (tens of GeV).

Finally there is another anomalous symmetry, the X symmetry. Under these transformations, chiral fields transform as

$$\Phi_i \to e^{i\alpha X_i}\Phi_i. \tag{3.31}$$

One can find a set of assignments of the X_i such that the corresponding current has only a color anomaly. Requiring cancellation of $SU(N)_{TC}$ and $SU(M)_{SC}$ anomalies yields the conditions

$$(2X_S + 2X_{\bar{S}} - 4)M\frac{M^2-1}{2M} + M(M^2-1) = 0,$$

$$(2X_T + X_{\bar{T}_U} + X_{\bar{T}_D} - 4)N\frac{N^2-1}{2N} + N(N^2-1) = 0 \tag{3.32}$$

(see fig. 1). Our assignments [eqs. (3.1)–(3.12)] satisfy these conditions. However, no choice of X_i leads to a current which is also free of color anomalies. Thus we may use this current to rotate away any θ angles in the color sector.

Since $X_S + X_{\bar{S}}$ is non-zero, this symmetry is broken by the supercolor condensate. The resulting pseudo-Goldstone boson gets a mass from QCD effects associated with the anomaly. This axion-like object is similar to the Weinberg-Wilczek axion [15] except that its decay constant is of order the supercolor scale, Λ_{SC}. As a result its mass is reduced, relative to that of the Weinberg-Wilczek axion, by a factor $\sqrt{\Lambda_{\mathrm{SC}}^2 G_{\mathrm{F}}} \sim 30$, so the mass is of order 1 keV. The axion couplings to ordinary matter are reduced by the same amount. No experiment designed to search for axions would have seen this one. This axion possesses two competing decay modes: decays to two photons and decay to two goldstinos. The decay to two photons may be estimated using standard current algebra techniques. The lifetime for this decay is about one year. We have not yet obtained a convincing estimate of the two-goldstino decay rate, but it may well be the dominant decay mode.

Thus this model provides a solution of the strong CP problem. Clearly this solution can be realized in other contexts [16]. It requires only that the model possess an anomalous U(1) symmetry, and that this symmetry be broken at some very large scale, much larger than the scale of weak isospin breaking. There may be astrophysical consequences of this axion. This question can be studied by using the techniques of Dicus, et al. [17].

5. Discussion and conclusions

We have seen, in the previous sections, that one can construct a reasonable model of particle physics if one assumes that strong interactions can break supersymmetry. Such a model can explain the existence of large mass hierarchies in physics. We have seen, in particular, that no unnatural adjustments of parameters are required as long as hypercharge is unified at some scale into a non-abelian group. While we have not explicitly exhibited an example of such a unification, it is certainly plausible, and we are currently attempting to construct a reasonable example.

In the model we have described here, $\mathrm{SU(2)_L}$ is not asymptotically free above the supercolor scale. The $\mathrm{SU(2)_L}$ coupling, however, will not become strong (with $\mathrm{SU(4)_{TC}} \times \mathrm{SU(5)_{SC}}$ as a gauge group, for example) until well beyond the Planck scale. In the model as it stands, the SU(3), SU(2) and U(1) couplings will not meet at a single energy. Thus grand unification of the model may not be a simple matter.

Before closing, it is worth commenting on some alternative models. It would be nice, in particular, to be able to eliminate one strongly interacting sector, either technicolor or supercolor. Eliminating supercolor leads to difficulties. First, the resulting model has an axion of precisely the Weinberg-Wilczek type, and this is phenomenologically unacceptable. Second, it is not clear that the scalar mass matrix which results is positive definite, i.e. the desired form of symmetry breakdown may not be obtained. These issues, however, are worthy of further study.

An alternative possibility is to leave out the technicolor interaction, and complicate the supercolor sector. For example, one can construct a model which allows Yukawa couplings of Higgs fields to supercolor fields, by introducing weak singlet supercolor particles. Moreover, one can arrange the model so that supercolor condensates do not themselves break $SU(2)\times U(1)$. In such models, supersymmetry breakdown will generate an effective potential for Higgs scalars of the form

$$\mu_1^2|A_{H_U}|^2+\mu_2^2|A_{H_D}|^2+\mu_3^2 A_{H_U}A_{H_D}+\text{quartic terms}. \tag{5.1}$$

In the absence of a general argument about the relative magnitudes of these terms, appearance of vacuum expectation values for $A_{H_U^0}$ and $A_{H_D^0}$ is a logical possibility. However, such models generally possess extra U(1) symmetries which yield real Goldstone bosons and undesirable axions. The viability of such scenarios, however, also appears worthy of further study.

We have seen that the model described in this paper has many attractive features. However, it does not provide any real understanding of the quark and lepton masses. The solution to this problem may lie in physics at the Planck scale.

We would like to thank Ed Witten for many enlightening discussions, and for explaining his work on dynamical breakdown of supersymmetry prior to publication. We would also like to thank Michael Peskin for relaying to us results of his discussions with S. Raby, S. Dimopoulos, and L. Susskind concerning the naturalness problems associated with the D term. We thank J.D. Bjorken for discussions of axion phenomenology.

Upon completion of this work, we learned that the idea of supersymmetric technicolor has also been considered by Dimopoulos and Raby, who discuss the D term at length [18]. These authors consider models with one strong group. Realistic models of this type would suffer from the problems discussed in our concluding remarks.

Note added in proof

We are now convinced that astrophysical arguments rule out our 1 keV axion. This axion can be avoided if we postpone resolving the strong *CP* problem until the scale of unification [16]. This and other phenomenological issues will be discussed in a subsequent publication.

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