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TAUOLA – a library of Monte Carlo programs to simulate decays of polarized τ leptons

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A library of Monte Carlo programs for leptonic and semileptonic decays of the τ lepton is presented. It provides final state with full topology including neutrinos, resonant distributions for intermediate particles and complete spin structure throughout the decay. The program is constructed in such a way that it can be easily attached to any Monte Carlo program simulating the production of τ's. It contains subprograms simulating the following decay channels: $\tau^{\pm} \to \nu\nu e^{\pm}$, $\bar{\nu}\nu\mu^{\pm}$, $\nu\pi^{\pm}$, $\nu\rho^{\pm}$, $\nu a_1^{\pm}$, $\nu K^{\pm}$, $\nu K^{*\pm}$, $\nu n\pi^{\pm,0}$, $n > 3$. The decay products are coded in the Lund format.

PROGRAM SUMMARY

Title of the program: TAUOLA, version 1.5

Catalogue number: ABZP

Program obtainable from: CPC Program Library, Queen's University of Belfast, N. Ireland (see application form in this issue); also from WASM @ CERNVM

Computer: IBM 3090; *Installation*: CERN

Operating system: VM/CMS

Programing language used: FORTRAN 77

High speed storage required: 45000 words

No. of bits in a word: 32

Peripherals used: line printer

No. of lines in combined program and test deck: 4022

CPC Program Library subprograms used: RANMAR; *catalogue number*: ABTK [1]

Keywords: heavy lepton τ, Monte Carlo simulation, spin polarization, electro-weak theory.

Nature of physical problem
The heavy lepton τ decays into two pure leptonic and many hadronic decay channels. They can be used to study the corresponding elements of the charged current or as a tool in testing electroweak theory. In particular, since the distributions of the τ decay products are sensitive to τ polarization, they are used as spin polarimeters.

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Method of solution
The Monte Carlo simulation is suited best for studying τ decays, because typically τ may decay into several particles and the decay distributions are difficult to handle analytically. Furthermore, many Monte Carlo programs for τ production exist already and they must include high quality subprograms for τ decays.

Restrictions on the complexity of the problem
For decays into four and more pions a simplified dynamical treatment has been adopted.

Typical running time
Efficiency is ca 700 events per IBM 3090 CPU seconds, in the inclusive mode.

References
[1] F. James, Comput. Phys. Commun. 60 (1990) 329.

LONG WRITE-UP

1. Introduction

The decay of the τ lepton, besides being relevant in itself, can serve as a useful tool in the investigation of some aspects of the standard electroweak theory [1]. The τ decay allows for example to verify whether the lepton universality rule still holds, i.e. whether τ differs from μ and e only by its mass. This task is not simple since due to its heavier mass, the τ has more decay modes than the lighter leptons have.

Moreover the τ produced through $Z \to \tau^+\tau^-$ in the SLC/LEP experiments plays the role of a spin polarimeter, and can provide an important information [2,3], for precision tests of the standard model. The τ lepton offers also a unique laboratory for the study of the hadronic weak currents [4], especially in the planned, new high luminosity low energy e^+e^- accelerators (so called τ-charm factories).

Finally, an accurate knowledge of the τ decay modes can help in understanding the sometimes annoying background of other processes.

In any of the above situations, the analysis of the τ decays relies rather heavily on the Monte Carlo simulation of the τ production and decay processes. The aim of this paper is to present a library of the programs for τ decays simulation which can be useful in a wide range of the above applications. Such a universal package should in principle fulfil the following requirements:

(1) It should be possible to couple easily the subprogram simulating τ decay to any MC program simulating τ production process.

(2) All the effects of τ spin polarization should be taken into account, both in the primary decay products (ρ, a_1,...) and in the distributions of the secondary decay products (if present).

(3) The parametrization of the τ decay matrix element and the MC algorithm for the decay should be flexible enough to accommodate easily for effects such as $V + A$ admixture, finite mass of ν_τ, some data on e^+e^- cross section etc.

(4) All the exclusive τ decay channels should sum up to almost 100% of the total width. In particular the decays into strange particles should be included.

(5) Matrix elements should be written in such a way, that it will be easy to replace it by any other one including, e.g., effects due to the new physics.

The presented set of programs is very close to fulfilment of all above specifications.

Our previous requirements determine the choice of the methods used in creating the MC algorithm and the other properties of the programs. The problem of interconnecting the τ decay and τ production processes in the differential cross section, in a form suitable for an implementation in the MC algorithm, is especially crucial for the structure of the simulation programs and the way of using them. A working solution already exists, see refs. [5–7]. Due to the practical importance of this problem, and also in order to introduce the notation used in the rest of the paper, we devote to this question the entire section 2.

As for point (3), the natural solution is the use of the MC simulation (integration) with the fully specified τ decay final state, including all neutrinos. This makes any possible modification of the decay matrix element quite easy to implement. In the presence of the $V+A$ admixture and the finite mass of ν_τ an attempt of integration over moments of unobserved particles may lead to cumbersome formulae, difficult to check and/or modify. Moreover, the inclusion of the neutrinos in the event record makes the check of the conservation of the overall momentum and energy for the combined τ production and decay process rather straightforward.

A more detailed description of the decay matrix elements and of the MC algorithms are given separately for each decay channel in section 3.

Section 4 gives more detailed instruction for the use of TAUOLA in practical applications.

Let us briefly comment on the other important issue concerning point (3): To what extent should the simulation of the τ decay rely on the available experimental information and to what extent on the theoretical models? Our opinion and choice is that the model dependent theoretical predictions should be used only when good enough data are lacking.

The most serious deficiency of the present version of TAUOLA is that the τ decays into more than three pions are generated according to approximate matrix elements. In particular, within the $n\pi$ system the π's momenta are generated according to the uniformly flat phase space. The library contains at present subprograms simulating the following decay channels: $\tau^\pm \to \nu\nu e^\pm$, $\nu\nu\mu^\pm$, $\nu\pi^\pm$, $\nu\rho^\pm$, $\nu a_1^\pm$, $\nu K^\pm$, $\nu K^{*\pm}$, $\nu n\pi^{\pm,0}$, $n > 3$.

To conclude, let us also note that many of the technical developments in the presented programs are almost directly applicable to decays of other heavy fermions, notably of the t quark, or of the heavy lepton from a hypothetical fourth generation etc., see for example ref. [8].

2. Interrelating the τ production and decay process in the Monte Carlo algorithm

So far the problem of interrelating the τ production and decay process in the Monte Carlo algorithm was solved in a general form in the MC program KORALB [5] which simulates the process

$$e^+e^- \to \tau^+\tau^-(\gamma), \qquad \tau^\pm \to X^\pm, \tag{2.1}$$

taking into account the spin polarization of the decaying τ's.

Not surprisingly, a crucial condition for a proper treatment of the spin degrees of freedom of the intermediate instable lepton τ is a consequent use of the spin density matrix formalism. This condition is not incompatible, as it may look, with the requirement of keeping the τ production and decay parts of the process as much separated as possible in the Monte Carlo algorithm.

Let us discuss these problems and their solution using as an example the lowest order process

$$e^+e \to \gamma, Z^0 \to \tau^+\tau^-, \qquad \tau^+ \to \bar{\nu}_\tau \nu_e e^+, \qquad \tau^- \to \nu_\tau \bar{\nu}_e e^-. \tag{2.2}$$

Using standard techniques [9], the differential cross section for the process

$$e^+(p_1)\, e^-(p_2) \to \tau^+(q_1, s_1)\, \tau^-(q_2, s_2)$$

can be written as

$$d\sigma = |A|^2\left(1 + a_\mu s_1^\mu + b_\mu s_2^\mu + c_{\mu\nu} s_1^\mu s_2^\nu\right) d\,\mathrm{Lips}(p_1 + p_2;\, q_1, q_2) \tag{2.3}$$

where a_μ, b_μ, $c_{\mu\nu}$ are functions of p_i and q_i and $|A|^2$ is the spin averaged matrix elements squared. Moreover, the differential partial width for the decay

$$\tau^-(q) \to \bar{\nu}_\tau(k_1)\, \nu_e(k_2)\, e^-(k_3)$$

is given by

$$\mathrm{d}T_e = \frac{1}{2M}|\bar{\mathcal{M}}|^2(1 + h_\mu s^\mu)\ \mathrm{d\,Lips}(q;\ k_1,\ k_2,\ k_3). \tag{2.4}$$

h_μ is here a function of q and k_i, M denotes the mass of τ and $|\mathcal{M}|^2$ denotes the mass of τ and $|\bar{\mathcal{M}}|^2$ the spin averaged squared matrix element.

We assume here, as usual, $s^2 = -1$ and $s \cdot q = 0$. The zeroth component of the vector h_μ in the τ rest frame can be defined arbitrarily (different conventions exist [6,10]). We choose this component to be equal to zero. The cross section for the combined production and decay process can therefore be written as follows [#1]:

$$\begin{aligned}\mathrm{d}\sigma = {} & |A|^2 |\bar{\mathcal{M}}|^2 |\bar{\mathcal{M}}'|^2 (1 - a_\mu h_1^\mu - b_\mu h_2^\mu + c_{\mu\nu} h_1^\mu h_2^\nu) \\ & \times \mathrm{d\,Lips}(p_1 + p_2;\ q_1,\ q_2)\ \mathrm{d\,Lips}(q_1;\ k_1,\ k_2,\ k_3)\ \mathrm{d\,Lips}(q_2;\ k_1',\ k_2',\ k_3').\end{aligned} \tag{2.5}$$

A formal derivation of eq. (2.5) may be found [#2], e.g., in refs. [6,10,12]. Eq. (2.5) was also used in refs. [13–15].

For the purpose of the Monte Carlo calculation it is quite crucial to observe that the factor

$$f_{\mathrm{spin}} = (1 - a_\mu h_1^\mu - b_\mu h_2^\mu + c_{\mu\nu} h_1^\mu h_2^\nu), \tag{2.6}$$

which is functionally the most complex part of (2.5) has a mild dependence on all momenta (production and the decay momenta in (2.2)) and obeys the inequality $0 \le f_{\mathrm{spin}} \le 4$ [14]. In the Monte Carlo algorithm [6] it is firstly replaced by 1 and later restored by means of rejection.

More precisely, at first the productions and decay subprocesses are simulated independently according to

$$\begin{aligned}&\mathrm{d}\sigma = |A|^2\ \mathrm{d\,Lips}(p_1 + p_2; q_1,\ q_2), \\ &\mathrm{d}\Gamma = \frac{1}{2M}|\bar{\mathcal{M}}|^2\ \mathrm{d\,Lips}(q_1;\ k_1,\ k_2,\ k_3), \qquad \mathrm{d}\Gamma' = \frac{1}{2M}|\bar{\mathcal{M}}'|^2\ \mathrm{d\,Lips}(q_2;\ k_1',\ k_2',\ k_3').\end{aligned} \tag{2.7}$$

At this stage each τ decay is simulated in the τ rest frame as if it was unpolarized. The *polarimeter vectors* $h_{1,2}^\mu$ have to be calculated inside the programs simulating τ decays. In the next step all spin effects, including all spin correlations, are introduced by rejecting typically $\frac{3}{4}$ of events according to the weight f_{spin} of eq. (2.6). Such a way of proceeding, although apparently unnatural or complicated, is in fact rather simple and, while it guarantees a proper quantum mechanical treatment of the spin degrees of freedom of the decaying τ, it allows at the same time for maximal modularity in the programs.

In the case of decay of the single, polarized τ (eg. from $W \to \tau\nu$ decay) the simpler weight

$$f_{\mathrm{spin}} = 1 + s^\mu h_\mu \tag{2.8}$$

is used to reject the generated τ decays. Notice that in this simple case $0 \le f_{\mathrm{spin}} \le 2$ [14] and $\frac{1}{2}$ of events are rejected.

[#1] In the ultrarelativistic limit applicable in the LEP/SLC regime, this formula can be simplified significantly. For details see ref. [11].

[#2] The only assumption necessary to derive this formula is that τ is a spin-half particle [6]. The formula is therefore also valid for any exotic and radiative decays of τ.

3. Decay matrix elements and Monte Carlo algorithms

In this section we shall list the decay matrix elements channel by channel and describe the procedures of the MC event generation. Some preliminary information on the organization of the program will also be given.

3.1. Leptonic decays

The matrix element for the leptonic decay

$$\tau^-(P, s) \rightarrow \nu_\tau(N)\, l^-(q_1)\, \bar{\nu}_l(q_2), \qquad l = \mathrm{e},\ \mu \tag{3.1}$$

is calculated from the following expressions:

$$\bar{\mathcal{M}} = \frac{G}{\sqrt{2}} \bar{u}(\nu_\tau;\ N)\ \gamma^\mu (v + \gamma_5 a)\ u(\tau^-;\ P)\ \bar{u}(l^-;\ q_1)\ \gamma_\mu (1 - \gamma_5)\ u(\nu_{l-};\ q_2). \tag{3.2}$$

Note that the pure $V-A$ coupling predicted by the standard model is used only for the light leptons e, μ and a more general coupling $v + \gamma_5 a$ is considered for the τ current. The masses of the fermions are assigned as follows: $P^2 = M^2$, $N^2 = m^2$, $q_1^2 = \mu^2$ and $q_2^2 = 0$.

The corresponding differential partial width for the leptonic decay (3.1) reads

$$\mathrm{d}\Gamma_l = \frac{1}{2M}\left(\frac{G}{\sqrt{2}}\right)^2 \times 32\big(B + H_\mu s^\mu\big)\ \mathrm{d\ Lips}(P;\ q_1,\ q_2,\ N), \tag{3.3}$$

where

$$\begin{aligned} B &= (v+a)^2 (P\cdot q_1)(N\cdot q_2) + (v-a)^2 (P\cdot q_2)(N\cdot q_1) - Mm(v^2 - a^2)(q_1\cdot q_2), \\ H^\mu &= (v+a)^2 M q_1^\mu (N\cdot q_2) - (v-a)^2 M q_2^\mu (N\cdot q_1) + (v^2 - a^2) m q_1^\mu (P\cdot q_2) - (v^2 - a^2) m q_2^\mu (P\cdot q_1) \end{aligned} \tag{3.4}$$

and

$$\mathrm{d\ Lips}(Q;\ q_1 \ldots q_n) = (2\pi)^4 \delta^4\left(P - \sum_{i=1}^{n} q_i\right) \prod_{i=1}^{n} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2q_i^0}. \tag{3.5}$$

It is easy to verify that for $v = -a = 1$, $m = 0$ this result coincides with that of standard textbooks [9,16].

The above-mentioned polarimeter vector is therefore given by

$$h_\mu = H_\mu / B \tag{3.6}$$

and is, in the program, calculated numerically from the generated momenta. Since the polarimeter vector h_μ is calculated in the rest frame of the corresponding $\tau^\pm$, its timelike component $h_0 = 0$ (see remark in section 3.2).

In the following, we shall describe in detail the MC integration over the three body phase space for this leptonic decay. It is representative example for the other decay channels. The phase space integral is parametrized in a rather standard way, as in the other MC programs [17]. Using for the moment a more compact notation with $q_3 = N$ and $q_i^2 = m_i^2$ we rewrite the phase space as follows:

$$\mathrm{d\ Lips}(P;\ q_1,\ q_2,\ q_3) = \frac{1}{2^{11}\pi^5} \int_{M_{2,\min}^2}^{M_{2,\max}^2} \mathrm{d}M_2^2 \int \mathrm{d}\Omega_3 \frac{\lambda^{1/2}(M^2,\ m_3^2,\ M_2^2)}{M^2} \int \mathrm{d}\Omega_2 \frac{\lambda^{1/2}(M_2^2,\ m_2^2,\ m_1^2)}{M_2^2}, \tag{3.7}$$

where

$$M_2^2 = (q_1 + q_2)^2, \qquad \lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz,$$
$$M_{2,\min} = m_1 + m_2, \qquad M_{2,\max} = M - m_3$$

and $d\Omega_3 = d\cos\theta_3\, d\phi_3$ is the solid angle element of $\nu_\tau(q_3)$ in the rest frame of $\tau(P)$ and $d\Omega_2 = d\cos\theta_2\, d\phi_2$ is the solid angle element of $\bar{\nu}_l(q_2)$ in the rest frame of $W^-(q_1 + q_2)$.

With the additional change of the variables

$$\cos\theta_2 = -1 + 2x_1, \quad \phi_2 = 2\pi x_2, \qquad \cos\theta_3 = -1 + 2x_3, \quad \phi_3 = 2\pi x_4,$$
$$M_2^2 = M_{2,\min}^2 + \left(M_{2,\max}^2 - M_{2,\min}^2\right)x_5$$

for the MC purpose, the integral (3.3) is transformed into a canonical form

$$d\Gamma_l = \int_0^1 \prod_{i=1}^{5} dx_i\, W(x_1, \ldots, x_5), \tag{3.8}$$

where

$$W = \frac{1}{2M}\left(\frac{G}{\sqrt{2}}\right)^2 32B(P, q_1, q_2, q_3)$$
$$\times \frac{1}{2^{11}\pi^5} \frac{\lambda^{1/2}(M^2, m_3^2, M_2^2)}{M^2} \frac{\lambda^{1/2}(M_2^2, m_2^2, m_1^2)}{M_2^2} (4\pi)^2 \left(M_{2,\max}^2 - M_{2,\min}^2\right). \tag{3.9}$$

Note that $s^\mu = 0$ was assumed.

The Monte Carlo event generation starts with calculating q_1, q_2 and $q_3 = N$, in the τ rest frame, out of uniformly distributed random numbers $x_i \in (0, 1)$. Then, the weight W is calculated. Since the weighted events are of little interest one has to turn them into unweighted events by the rejection method with the rejection weight being

$$w = W/W_{\max}. \tag{3.10}$$

The maximum weight $W_{\max}$ is found in the initialization step by inspection of the sample of 500 MC trial events. Later, in the rejection procedure about 30% of the events are accepted and such efficiency may be regarded as rather good.

The value of the integral, i.e., in this case the partial width Γ_l, is obtained as a byproduct of the event generation from

$$\Gamma_l = \langle W \rangle, \tag{3.11}$$

where the average is taken over all generated events, prior to rejection.

The Monte Carlo procedure described up to now provides events insensitive to the τ polarization. According to the description in section 2 it is an example of the low level in the MC generation chain. What remains to be done at this stage is the calculation of the polarimeter vector

$$\boldsymbol{h} = \boldsymbol{H}/B, \qquad h_0 = 0 \tag{3.12}$$

from the generated momenta (in the τ rest frame), to be stored for later use.

3.2. General formalism for semileptonic decays

The matrix element for the semileptonic decay $\tau \to \nu_\tau + X$ is written in the form

$$\bar{\mathcal{M}} = \frac{G}{\sqrt{2}} \bar{u}(N)\, \gamma^\mu (v + a\gamma_5) u(P)\, J_\mu, \tag{3.13}$$

where $J_\mu \equiv \langle X \mid V_\mu - A_\mu \mid 0 \rangle$ denotes the matrix element of the $V-A$ current relevant for the specific final state X. The current J_μ depends in general on the momenta of all hadrons. The final states under consideration [#3] are produced either through the vector current ($2n\pi$ or Kπ) or through the axial current ($(2n+1)\pi$ or K).

The squared matrix element for the decay of a τ with spin s reads

$$|\mathcal{M}|^2 G^2 \frac{v^2 + a^2}{2} \left(\omega + H_\mu s^\mu\right), \qquad \omega + P^\mu \left(\Pi_\mu - \gamma_{va} \Pi^5_\mu\right),$$

$$H_\mu = \frac{1}{M} \left(M^2 \delta^\nu_\mu - P_\mu P^\nu\right)\left(\Pi^5_\nu - \gamma_{va} \Pi_\nu\right), \tag{3.14}$$

with

$$\Pi_\mu = 2\left[(J^* \cdot N) J_\mu + (J \cdot N) J^*_\mu - (J^* \cdot J) N_\mu\right], \qquad \Pi^{5\mu} = 2\ \mathrm{Im}\ \epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho N_\sigma, \qquad \gamma_{va} = -\frac{2va}{v^2 + a^2}. \tag{3.15}$$

($\gamma_{va} = 1$ in the standard model). For massive neutrinos and $v^2 \le a^2$ one has to add the following terms:

$$\dot{\omega} = 2\frac{v^2 - a^2}{v^2 + a^2} mM (J^* \cdot J), \qquad \hat{H}^\mu = -2\frac{v^2 - a^2}{v^2 + a^2} m\ \mathrm{Im}\ \epsilon^{\mu\nu\rho\sigma} J^*_\nu J_\rho P_\sigma \tag{3.16}$$

respectively to ω and H_μ. To obtain the polarimeter vector h in the τ rest frame is again sufficient to calculate the space components of $h_\mu = (H_\mu + \hat{H}_\mu)/(\omega + \hat{\omega})$ and set $h_0 = 0$.

For τ^+ decays all momenta in J_μ are to be read as the momenta of the antiparticles and the terms proportional to γ_{va} reverse their signs. For the final states X, which receive contributions from hadronic vector and axial vector currents at the same time [#4], the relative sign of the two contributions is reversed.

The partial decay rate for the channel under consideration reads

$$2M\Gamma_X = G^2 \frac{v^2 + a^2}{2} \frac{|N|}{4\pi M} (\omega + \hat{\omega}) \tag{3.17}$$

for the decay into a single hadron (π, K) and

$$2M\Gamma_X = \tfrac{1}{2} G^2 \frac{v^2 + a^2}{2} \int \frac{\mathrm{d}Q^2}{2\pi} \frac{|N|}{4\pi M} \int \mathrm{d\ Lips}(Q;\ q_i)\ (\omega + \hat{\omega}) \tag{3.18}$$

in the general case. Relating J_μ to the spectral functions [#5] ρ_0 and ρ_1 for spin 0 and spin 1 contributions respectively

$$(2\pi)^{-1} \int \mathrm{d\ Lips}(Q;\ q_i)\ J^*_\mu J_\nu = \left(Q_\mu Q_\nu - g_{\mu\nu} Q^2\right)\rho_1 + Q_\mu Q_\nu \rho_0 \tag{3.19}$$

[#3] Not true, e.g., for K$\pi\pi$ final states.
[#4] Such states are not included in the present version of the program.
[#5] Conventions differing by a factor 2π are in use.

one obtains

$$2M\Gamma_X = G^2\frac{v^2+a^2}{2}\int dQ^2\frac{|N|}{4\pi M}$$
$$\times\left[\left((M^2-Q^2)(M^2+2Q^2)-m^2(2M^2+Q^2-m^2)-6\frac{v^2-a^2}{v^2+a^2}mMQ^2\right)\rho_1\right.$$
$$\left.+\left((M^2-Q^2)M^2-m^2(2M^2+Q^2-m^2)+2\frac{v^2-a^2}{v^2+a^2}mMQ^2\right)\rho_0\right]. \tag{3.20}$$

In the standard model ($v=-a=1$; $m=0$) this simplifies to

$$\frac{\Gamma_X}{\Gamma_e} = 12\pi^2\int\frac{dQ^2}{M^2}\left(1-\frac{Q^2}{M^2}\right)^2\left[\left(1+\frac{2Q^2}{M^2}\right)\rho_1+\rho_0\right]. \tag{3.21}$$

3.3. Decay into one pion

The simplest application of our formalism is provided by the decay into a single pion. The hadronic current reads

$$J_\mu = f_1 Q_\mu, \tag{3.22}$$

where $f_1=\sqrt{2}\,f_\pi\cos\theta_c = 128.4$ MeV. From eqs. (3.14)–(3.16) one finds

$$\Pi_\mu = 2f_1^2\left((M^2-m^2)Q_\mu - Q^2P_\mu\right), \qquad \Pi_\mu^5 = 0 \tag{3.23}$$

and thus

$$\omega = f_1^2\left[\left(M^2-m_\pi^2\right)M^2 - m^2\left(2M^2+m_\pi^2-m^2\right)\right], \qquad \hat{\omega} = f_1^2\left(\frac{v^2-a^2}{v^2+a^2}\right)2mMm_\pi^2, \qquad \hat{H}=0. \tag{3.24}$$

The polarimeter vector in the τ rest frame reads

$$\boldsymbol{h} = -2\gamma_{va}f_1^2M(M^2-m^2)\boldsymbol{Q}/(\omega+\hat{\omega}). \tag{3.25}$$

In the standard model $\boldsymbol{h}=-\boldsymbol{n}_\pi$, where $\boldsymbol{n}$ denotes the unit vector into the direction of flight of the pion.

The corresponding differential partial width reads

$$d\Gamma_\pi = \frac{1}{2M}G^2\frac{(v^2+a^2)}{2}(\omega+\hat{\omega})\ d\,\mathrm{Lips}(P;\,Q,\,N), \tag{3.26}$$

where

$$d\,\mathrm{Lips}(P;\,Q,\,N) = \frac{1}{2^5\pi^2}\int d\Omega_3\frac{\lambda^{1/2}\left(M^2,\,m_\pi^2,\,m^2\right)}{M^2}, \tag{3.27}$$

$N^2=m^2$, $Q^2=m_\pi^2$ and $d\Omega_3 = d\cos\theta\,d\phi$ denotes the solid angle element of $\nu_\tau(N)$ in the rest frame of $\tau(P)$.

With the additional change of the variables

$$\cos\theta = -1+2x_1, \qquad \phi = 2\pi x_2,$$

for the MC purpose, the integral (3.26) is transformed into a canonical form

$$d\Gamma_\pi = \int_0^1 \prod_{i=1}^{2} dx_i \, W(x_1, x_2), \tag{3.28}$$

where

$$W = \frac{1}{2M} G^2 \frac{(v^2 + a^2)}{2} (\omega + \hat{\omega}) \frac{1}{2^5 \pi^2} \frac{\lambda^{1/2}(M^2, m_1^2, m^2)}{M^2} \frac{\lambda^{1/2}(M_2^2, m_2^2, m_1^2)}{M_2^2} (4\pi). \tag{3.29}$$

We omit any further description of this generation since it is just a simpler version of the leptonic decay mode generation. The decay into a single K^- is simulated exactly in the same way, with $f_1 = \sqrt{2}\, f_K \sin\theta_c = 35.4$ MeV.

3.4. Decay into two pions

In this subsection the simulation of the τ decay into two pions will be discussed. Since it is dominated by ρ production we will call it also the ρ channel. The notation for the kinematics of this decays will be similar to the one of the leptonic decay:

$$\tau^-(P, s) \to \nu_\tau(N)\ \pi^-(q_1)\ \pi^0(q_2). \tag{3.30}$$

For convenience, we introduce the moments $Q = q_1 + q_2$, $q = q_1 - q_2$ and the masses of the various particles are denoted by $P^2 = M^2$, $q_1^2 = q_2^2 = m_\pi^2$ [#6] and $N^2 = m^2$.

On the basis of the CVC hypothesis one predicts

$$J_\mu = \sqrt{2}\ F_\pi(Q^2) \cos\theta_c\ q_\mu \equiv f_2 q_\mu, \tag{3.31}$$

where F_π denotes the pion form factor. It is related to the experimental total cross section for the reaction $e^+e^- \to \pi^+\pi^-$ through

$$R(Q^2) = \frac{3Q^2}{4\pi\alpha^2} \sigma_{e^+e^- \to \pi^+\pi^-}(Q^2) = \frac{1}{4}\left(1 - \frac{4m_\pi^2}{Q^2}\right)^{3/2} |F_\pi|^2. \tag{3.32}$$

Once the isospin zero part is subtracted from the e^+e^- data the spectral function $\rho_1(Q^2)$ is given by

$$\rho_1(Q^2) = \frac{1}{24\pi^2} \cos^2\theta_c\ F_\pi^2(Q^2)\left(1 - \frac{4m_\pi^2}{Q^2}\right)^{3/2}. \tag{3.33}$$

The experimental results can be parametrized [18] according to the following formula

$$F_\pi(Q^2) = B_\rho(Q^2) = \frac{\left[\mathrm{BW}_\rho(Q^2) + \beta \mathrm{BW}_{\rho'}(Q^2)\right]}{1 + \beta}, \tag{3.34}$$

[#6] For the sake of simplicity of notation, we have omitted here the $\pi^\pm$, π^0 mass difference. The difference is, however, kept in the program.

where

$$\mathrm{BW}_\rho(Q^2) = \frac{M_\rho^2}{M_\rho^2 - Q^2 - \mathrm{i}M_\rho\Gamma_\rho(Q^2)}, \tag{3.35}$$

$$\Gamma_\rho(Q^2) = \Gamma_\rho \frac{M_\rho}{Q}\left(\frac{p_\pi(Q^2)}{p_\pi(M_\rho^2)}\right)^3. \tag{3.36}$$

In the last formula p_π denotes momentum of the π in the (virtual) ρ rest frame and the following set of numerical constants was used [#7] $\beta = -0.145$, $M_\rho = 773$ MeV, $\Gamma_\rho = 145$ MeV, $m_{\rho'} = 1370$ MeV, $\Gamma_{\rho'} = 510$ MeV. The contribution from the ω resonance has been subtracted. From eqs. (3.14)–(3.16) one derives

$$\Pi_\mu = 2|f_2|^2\left[2(q\cdot N)q_\mu - q^2 N_\mu\right], \qquad \Pi_\mu^5 = \hat{H}_\mu = 0 \tag{3.37}$$

and thus

$$\omega = 2|f_2|^2\left[2(q\cdot N)(q\cdot P) - q^2(N\cdot P)\right], \qquad \hat{\omega} = 2\frac{v^2-a^2}{v^2+a^2}|f_2|^2 mMq^2. \tag{3.38}$$

The polarimeter vector in the τ rest frame reads

$$\boldsymbol{h} = -2\gamma_{va}M|f_2|^2\frac{\left[2(q\cdot N)\boldsymbol{q} - q^2\boldsymbol{N}\right]}{\omega+\hat{\omega}}. \tag{3.39}$$

The Monte Carlo simulation algorithm is similar to the one used for the pure leptonic decay. It differs in the mapping of the random number x_5 into $M_2^2 = Q^2$. The ρ resonance introduces a strong peak $\sim 1/[(Q^2 - M_\rho^2)^2 + (M_\rho\Gamma_\rho)^2]$ in the decay matrix element which would spoil the efficiency of the MC algorithm. This problem is cured through importance sampling and requires the following modification with respect to the previously discussed leptonic case:

$$M_2^2 = M_\rho^2 + \Gamma_\rho M_\rho \tan\alpha, \qquad \alpha = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min})x_5, \qquad M^2_{2,\min/\max} = M_\rho^2 + \Gamma_\rho M_\rho \tan\alpha_{\min/\max}. \tag{3.40}$$

In the resulting canonical weight

$$W = \frac{1}{2M}G^2\frac{v^2+a^2}{2}(\omega+\hat{\omega})\frac{1}{2^{11}\pi^5}\frac{\lambda^{1/2}(M^2, m_3^2, M_2^2)}{M^2} \times \frac{\lambda^{1/2}(M_2^2, m_2^2, m_1^2)}{M_2^2}(4\pi)^2\frac{(M_2^2 - M_\rho^2)^2 + (\Gamma_\rho M_\rho)^2}{M_\rho\Gamma_\rho}(\alpha_{\max} - \alpha_{\min}) \tag{3.41}$$

the Breit–Wigner pole in $|F_\pi|^2$ is cancelled by the corresponding Jacobian factor $(M_2^2 - M_\rho^2)^2 + (\Gamma_\rho M_\rho)^2$. This cancellation improves the efficiency of the MC algorithm by a factor of about three.

It should be stressed that there is no reference to the explicit form of the function $|F_\pi|^2$ in the MC procedure. In fact the user may easily plug in his favorite parametrization of $|F_\pi(Q^2)|^2$ or of $\sigma_{e^+e^-\to\pi^+\pi^-}(Q)^2$ by replacing one of the subprograms: FPIK in the ρ case, or BWIGS in the K* case. In the present version of the program, masses and other parameters used in the fit, are defined in the code of

[#7] For more details see table 1 of ref. [18].

corresponding routines and are independent of any input parameters. They are thus independent of the input parameters of the program.

The corresponding value of the partial width arises as a byproduct at the end of the event simulation session. The present parametrization of $|F_\pi|^2$ yields $\Gamma(\tau \to \nu\rho)/\Gamma_e = 1.33$.

Up to this point the τ spin polarization in the simulation of the τ decay into two pions was ignored. But again, according to the prescription of section 2, the polarimeter vector $h_\mu = H_\mu/B$ is calculated, stored and at the later stage all effects due to τ polarization are introduced through the rejection technique. In this way, both the distribution of the ρ and of the pions from the ρ decay will finally depend on the τ polarization in the proper way.

The generation of this decay mode can be easily extended to the $\tau \to K^* \nu$ case, provided that the corresponding adjustment of the couplings, masses and form factor is performed. Care must be taken of the mass difference between K and π. The hadronic current given by eq. (3.31) has to be multiplied by the numerator of the K* propagator (that is $q_{\mu\nu} - Q_\mu Q_\nu/Q^2$) to fulfil the current conservation. For the form factor we use eq. (3.35) with substitutions $M_\rho \to M_{K'} = 892.1$ MeV, $\gamma_\rho \to \Gamma_{K'} = 51.3$ MeV and multiplied by $\sin\theta_c$. A fraction of $\frac{1}{3}$ of the Kπ final states is assigned to the $K^-\pi^0$ and a fraction of $\frac{2}{3}$ to the $\bar{K}^0\pi^-$ mode. For the combined rate we find the MC result $\Gamma_{K\pi}/\Gamma_e = 0.070$ in a reasonable agreement with the prediction of the integral (3.21) which gives $\Gamma_{K\pi}/\Gamma_e = 0.071$.

3.5. Decay into three pions

The τ decay into three pions

$$\tau^-(P, s) \to \nu_\tau(N)\ \pi^-(q_1)\ \pi^-(q_2)\ \pi^+(q_3), \qquad \tau^-(P, s) \to \nu_\tau(N)\ \pi^0(q_1)\ \pi^0(q_2)\ \pi^-(q_3) \tag{3.42}$$

is simulated in a way analogous to the ρ mode and is called a_1 channel. The notation is as follows: $Q = q_1 + q_2 + q_3$, $P^2 = M^2$, $q_1^2 = q_2^2 = q_3^2 = m_\pi^2$ [#8] and $N^2 = m^2$.

For definiteness the mode $\pi^-\pi^-\pi^+$ is described. The $\pi^0\pi^0\pi^-$ mode is generated with the same rate and distribution. The current is assumed to be saturated through the decay chain $\tau \to \nu + a_1(\to \pi\rho(\to \pi\pi))$, resulting in an ansatz [12,18] of the form

$$J_\mu = f_3(Q^2)\left[\left(q_{1\mu} - q_{3\mu} - Q_\mu \frac{Q\cdot(q_1 - q_3)}{Q^2}\right) F_\pi(s_2) + (1 \leftrightarrow 2)\right]. \tag{3.43}$$

$F_\pi = B_\rho$ stands for the pion form factor including ρ and ρ′ as introduced in eqs. (3.34)–(3.36) The normalization f_3 is chosen such as to stay consistent with the behavior predicted for small q_i from chiral invariance and to exhibit the a_1 resonant behavior,

$$f_3 = \cos\theta_c \frac{2\sqrt{2}}{3f_\pi} B_{a_1}(Q^2). \tag{3.44}$$

B_{a_1} denotes the Breit–Wigner enhancement of the a_1,

$$B_{a_1}(Q^2) = \frac{m_{a_1}^2}{m_{a_1}^2 - Q^2 - i m_{a_1}\Gamma_{a_1}(Q^2)}.$$

[#8] For the sake of simplicity of notation, we have omitted here the $\pi^+\pi^0$ mass difference. The difference is, however, kept in the program.

The energy dependence of Γ_{a_1} is derived from the matrix element (3.43),

$$\Gamma_{a_1}(Q^2) = \Gamma_{a_1}\left(m_{a_1}^2\right)\frac{g(Q^2)}{g\left(m_{a_1}^2\right)}, \tag{3.45}$$

with

$$\begin{aligned} g(Q^2) &= \int \frac{ds_1\, ds_2}{Q^2}\,\frac{-J^\mu J_\mu^*}{|f_3(Q^2)|^2} \\ &= -\int \frac{ds_1\, ds_2}{Q^2}\left[V_1^2|B_2|^2 + V_2^2|B_1|^2 + V_1\cdot V_2(B_1B_2^* + B_1^*B_2)\right], \end{aligned} \tag{3.46}$$

The following short-hand notation has been introduced:

$$\begin{aligned} &V_1^2 = \left(4m_\pi^2 - s_2\right) - \frac{(s_3-s_1)^2}{4Q^2}, \qquad V_2^2 = \left(4m_\pi^2 - s_1\right) - \frac{(s_3-s_2)^2}{4Q^2}, \\ &V_1\cdot V_2 = -\left(\tfrac{1}{2}Q^2 - s_3 - \tfrac{1}{2}m_\pi^2\right) - \frac{(s_3-s_1)(s_3-s_2)}{4Q^2}, \\ &Q^2 = s_1 + s_2 + s_3 - 3m_\pi^2, \qquad B_{1,2} = B_\rho(s_{1,2}). \end{aligned} \tag{3.47}$$

In this approach g is directly related to the spectral function ρ_1 as given by

$$\rho_1 = \frac{1}{6}\frac{1}{(4\pi)^3}|f_3(Q^2)|^2\frac{1}{Q^2}g(Q^2) \tag{3.48}$$

and through eq. (3.20) to the $\tau \to \nu 3\pi$ decay rate. The approximate function $g(Q^2)$ of ref. [18] (see routine GFUN in the program), reads

$$g(Q^2) = \begin{cases} Q^2\left(1.623 + 10.38/Q^2 - 9.32/Q^4 + 0.65/Q^6\right), & \text{if } Q^2 > (m_\rho + m_\pi)^2, \\ 4.1\left(Q^2 - 9m_\pi^2\right)^3\left[1 - 3.3\left(Q^2 - 9m_\pi^2\right) + 5.8\left(Q^2 - 9m_\pi^2\right)^2\right], & \text{elsewhere.} \end{cases}$$

Accordingly to the description given in the section 2, the differential partial width is generated according to

$$2M\, d\Gamma = G^2\frac{v^2 + a^2}{2}\left[\omega + \hat{\omega} + \left(H_\mu + \hat{H}_\mu\right)s^\mu\right] d\,\mathrm{Lips}(P;\, q_1,\, q_2,\, q_3,\, N). \tag{3.49}$$

The quantities ω, $\hat{\omega}$, H_μ and $\hat{H}_\mu$ have been expressed in terms of the lepton momenta and the current J^μ in eqs. (3.14)–(3.16) above.

Using for the moment a more compact notation with $q_4 = N$ and $q_i^2 = m_i^2$ the phase space is rewritten as follows:

$$\begin{aligned} &d\,\mathrm{Lips}(P;\, q_1,\, q_2,\, q_3,\, q_4) \\ &\quad = \frac{1}{2^{17}\pi^8}\int_{Q^2_{\min}}^{Q^2_{\max}} dQ^2 \int_{M^2_{2,\min}}^{M^2_{2,\max}} dM_2^2 \\ &\qquad \times \int d\Omega_4 \frac{\lambda^{1/2}\left(M^2, Q^2, m_4^2\right)}{M^2}\int d\Omega_3 \frac{\lambda^{1/2}\left(Q^2, m_3^2, M_2^2\right)}{Q^2}\int d\Omega_2 \frac{\lambda^{1/2}\left(M_2^2, m_2^2, m_1^2\right)}{M_2^2}, \end{aligned} \tag{3.50}$$

where

$$M_2^2 = (q_1 + q_2)^2,$$

$$Q_{\min} = m_1 + m_2 + m_3, \qquad Q_{\max} = M - m_4, \qquad M_{2,\min} = m_1 + m_2, \qquad M_{2,\max} = Q - m_3$$

and $d\Omega_4 = d\cos\theta_4\, d\phi_4$ is the solid angle element of $\nu_\tau(q_4)$ in the rest frame of $\tau(P)$, $d\Omega_3 = d\cos\theta_3\, d\phi_3$ is the solid angle element of ρ in the a_1 rest frame and finally $d\Omega_2 = d\cos\theta_2\, d\phi_2$ is the solid angle element of π^+ in the rest frame of ρ.

With the additional change of the variables

$$\cos\theta_2 = -1 + 2x_1, \quad \phi_2 = 2\pi x_2, \qquad \cos\theta_3 = -1 + 2x_3, \quad \phi_3 = 2\pi x_4,$$

$$\cos\theta_4 = -1 + 2x_5, \quad \phi_4 = 2\pi x_6,$$

$$Q^2 = M_{a_1}^2 + \Gamma_{a_1} M_{a_1} \tan\alpha, \qquad \alpha = \alpha_{\min} + (\alpha_{\max} - \alpha_{\min}) x_7,$$

$$Q^2_{\min/\max} = M_{a_1}^2 + \Gamma_{a_1} M_{a_1} \tan\alpha_{\min/\max}, \qquad M^2 = M^2_{2,\min} + \left(M^2_{2,\max} - M^2_{2,\min}\right) x_8,$$

the integral (3.49) is transformed, as before, into a canonical form

$$d\Gamma_a = \int_0^1 \prod_{i=1}^{8} dx_i\, W(x_1, \ldots, x_8), \tag{3.51}$$

where

$$W = \frac{1}{2M} G^2 \frac{(v^2 + a^2)}{2} (\omega + \hat{\omega})$$

$$\times \frac{1}{2^{17}\pi^8} \frac{\lambda^{1/2}(M^2, m_4^2, Q^2)}{M^2} \frac{\lambda^{1/2}(Q^2, m_3^2, M_2^2)}{Q^2} \frac{\lambda^{1/2}(M_2^2, m_2^2, m_1^2)}{M_2^2}$$

$$\times (4\pi)^3 \frac{\left(Q^2 - M_{a_1}^2\right)^2 + \left(\Gamma_{a_1} M_{a_1}\right)^2}{M_{a_1}\Gamma_{a_1}} (\alpha_{\max} - \alpha_{\min})\left(M^2_{2,\max} - M^2_{2,\min}\right). \tag{3.52}$$

The sampling is peaked as described for the $\tau \to \nu\rho$ channel. At first, the hadronic mass Q^2 is generated within the given range. This generation is done with the Breit–Wigner peak for the a_1 resonance and the corresponding Jacobian factor. Second, $(q_1 + q_2)^2 = M_2^2$ is generated with the flat distribution. The two pions with momenta q_1, q_2 are then symmetrized.

Assuming no difference for form-factors masses and widths of charged and neutral ρ, the differential rates into $\pi^-\pi^-\pi^+$ and $\pi^0\pi^0\pi^-$ are identical. Under such an assumption the right charges can be set before or after the rejection is done [#9]. The events are assigned with equal probability to the two decay modes (i.e., the ratio of Clebsch–Gordan coefficients is one). The statistical factor $\frac{1}{2}$ of the phase space with two identical particles ($\pi^-\pi^-$) is thus cancelled out as we simultaneously generate $\pi^-\pi^-\pi^+$ with $\pi^0\pi^0\pi^-$ modes. Using $m_a = 1.251$ GeV, $\Gamma_a(m_a^2) = 0.599$ GeV, one finds for the combined rate $\Gamma_{a_1}/\Gamma_e = 0.71$ in good agreement with ref. [18].

[#9] The first option is taken, because it allows to take into account in the phase space in (3.52) the $\pi^\pm$, π^0 mass difference.

3.6. Multipion modes

Multipion decay modes are included in an oversimplified way. The spin dependence of the distributions is neglected. The four-momenta of the π's are generated in the center of mass of the system of all π's accordingly to the flat phase space. To this end the algorithm of ref. [17] is used. The invariant mass of the multipion system is generated according to eq. (3.20) where only the contribution from the function ρ_1 is assumed to be nonzero. Effects due to neutrino mass and eventual $V+A$ current admixture are included.

All experimentally observed multipion τ decay modes are included:

(1) $\tau^- \to \pi^-\pi^-\pi^+\pi^0$,
(2) $\tau^- \to \pi^-\pi^0\pi^0\pi^0$,
(3) $\tau^- \to \pi^-\pi^-\pi^-\pi^+\pi^+$,
(4) $\tau^- \to \pi^-\pi^-\pi^-\pi^+\pi^+\pi^0$.

The spectral function ρ_1 is calculated from the $e^+e^- \to 4\pi$ data according to formulae (3.32) and (3.33) and the prescription given in refs. [12,19,20]. The two modes are weighted as follows:

$$\rho_1(2\pi^-\pi^+\pi^0) \propto \tfrac{1}{2}\sigma(e^+e^- \to 2\pi^+2\pi^-) + \sigma(e^+e^- \to \pi^+\pi^-2\pi^0),$$

$$\rho_1(1\pi^-3\pi^0) \propto \tfrac{1}{2}\sigma(e^+e^- \to 2\pi^+2\pi^-).$$

For six pions we simply use

$$\rho_1(3\pi^-2\pi^+\pi^0) \propto \sigma(e^+e^- \to 3\pi^+3\pi^-).$$

The experimental data for $\sigma(e^+e^- \to 2\pi^+2\pi^-)$, $\sigma(e^+e^- \to \pi^+\pi^-2\pi^0)$, $\sigma(e^+e^- \to 3\pi^+3\pi^-)$ are taken from refs. [21–23].

The case (3) of odd number π's is different. To obtain the estimate on the matrix element for 5π state production we will use the soft pion technique [24] as described in ref. [25], which relates ρ_1 with the cross section for 4π production in e^+e^- annihilation. All this data are coded in the routine SIGEE [#10] which can be easily replaced by the user.

3.7. Numerical tests of the program

So far the following tests of the program have been performed:

(1) In ref. [26] the π, e, μ energy spectrum and rates have been compared with the analytical calculations.

(2) In ref. [27] the spectrum of π^- from the ρ decay mode of τ was compared in both polarized and unpolarized cases with the analytical calculation for the zero width ρ meson.

(3) The $d\Gamma/dQ^2$ distribution of the $\pi^-\pi^0$ system in the ρ decay mode has been compared with the analytical results of ref. [18].

(4) The $d\Gamma/dQ^2$ distribution of the $\pi^+2\pi^-$ system in the a_1 decay mode has been compared with the analytical results of ref. [18]. The energy distributions of the even and odd charge pairs of pions were compared with those obtained in ref. [18]. Also the distribution of the quantity A_{LR} exhibiting parity nonconservation in the a_1 system was compared with analytical results of ref. [18].

(5) For the K and K* modes we have checked that if masses, spectral functions and coupling constants are replaced by the ones of π and ρ then the distributions in these decay modes are reproduced correctly.

(6) In the multipion decay mode, we have checked that the Monte Carlo algorithm reproduces the branching ratio given by formula (3.21), provided the same spectral function ρ_1 is used.

[#10] Courtesy of the ALEPH Collaboration.

(7) In ref. [26] some comparisons of the previous version of our program with the generator of ref. [28] were also performed. We were not able, however, to compare numerical results of our program with that of ref. [29], because of nonportability of the latter program.

All these checks were performed for the small neutrino mass. Special numerical tests of the terms proportional to this mass were not done yet.

We would like to recommend any serious user of our program to start her or his work with tests verifying whether the part of TAUOLA which is relevant for her or his interests works properly. The authors would greatly appreciate any information on the errors, if they were noticed, and on any other improvements made by the users.

4. How to use the subprograms

As was already discussed in section 2 there are two ways of combining the TAUOLA package with the main program. The first one, was used in KORALB [5] whereas the second, based on the simplified treatment of the spin degrees of freedom was used in KORALZ [7]. In the second case the production and decays of $\tau^{\pm}$ are treated in an noncoherent way, but still keeping track of most important (longitudinal) spin correlations. In this case the spin polarization vector is transmitted from the τ production MC program to the TAUOLA library as an input parameter. It is used there according to formula (2.8).

Before going into details how to use the TAUOLA package let us define the requirements which the broader software environment has to fulfil. First, the user must supply an initialization routine. It has to fill input common blocks of the TAUOLA package listed at the end of this subsection (let us call this routine **INICOM**). The common blocks to be initialized contain those masses, widths and coupling constants of τ and/or its decay products which are relevant for the generation. In particular, in commons **TAUBRA** and **TAUNPI** parameters defining actual proportion of different τ decay modes in the decay sample have to be initialized. In our demonstration program we force the sample to be equally populated with all decay modes. In the actual applications we suggest either to use best available experimental data or to use an output from the special high statistics run of TAUOLA. The program, calculates branching ratios from the matrix elements actually used in the generation, by means of the Monte Carlo integration. Then, the user has to provide subroutine **TRALO4(KTO,VECTAU,VECLAB,XMASS)**. The **TRALO4** routine should transform any four-momentum from the $\tau^{\pm}$ rest frame to the laboratory system. Here **VECTAU**, **VECLAB**, **XMASS** denote respectively four-momentum of the particle to be boosted, the resulting boosted vector and the mass. An input parameter **KTO=1, 2** tells whether the boost, is performed from the τ^{+} or τ^{-} system [#11].

The main interface subprogram of the TAUOLA package is either routine **DEKAY(KTO,POL)** in the KORALB like case or **DEXAY(KTO,POL)** in the KORALZ like case. In both cases, the **DEKAY** or **DEXAY** routine has to be initialized, prior to generating the first event, by calling it with the parameter **KTO=-1**. Similarly, useful statistical information about the generated series is calculated and printed if at the end of generation **DEXAY** or **DECAY** is called on with **KTO=100**. The output includes the number of events per channel, branching ratios calculated from the matrix elements actually used in the generation and other statistical information.

In the following we show an example of interfacing the τ production and decay subprograms following an example of KORALB, preserving exactly all spin structure i.e. polarization, and correlations. First, the production and decay parts of the program are initialized. Then, **TAUGEN** routine is called. It provides an

[#11] Strictly speaking, if **KTO=1,2** boost has to be performed from the τ of the LUND [30] identifier **IDFF, -IDFF** (see the tables at the end of this subsection for definition of IDFF). There exists a possibility to interchange τ^{+} with τ^{-} in the program.

unpolarized pair of $\tau^{\pm}$. Next, the routine **DEKAY(KTO,POL)** is called on twice with **KTO=1,2** and **POL=POL1,POL2** to generate decays of the unpolarized τ^{+} and τ^{-}. Their corresponding polarimetric vectors h_1^{μ}, h_2^{μ} denoted **POL1**, **POL2** are calculated inside **DEKAY**. In the next step spin weight (eq. (2.6)) is calculated with the help of the routine **TAUSPN(POL1,POL2)** and all spin effects are introduced by means of the rejection. Finally, for an *accepted* event four-momenta of $\tau^{\pm}$ decay products are boosted to the laboratory system and the event is stored in the LUND common. This is done with calls on **DEKAY(11,POL1)** and **DEKAY(12,POL2)**.

Note, that instead of repeating the generation of unpolarized τ pairs and their decays, once an event is rejected due to the spin weight, it is possible to rotate the decay products of τ^{+}, τ^{-} (as well as **POL1**, **POL2** vectors) with two random spherical angles in the $\tau^{\pm}$ rest-frames. This may increase efficiency of the program by nearly a factor 4.

The user supplied routines are: **INICOM**, **TRALO4**, **TAUGEN**, **TAUSPN**. In practical realizations they may be included as a part of the $\tau^{\pm}$ pair production program. The following short listing show an example of the user program interfacing $\tau^{\pm}$ production and decay in the KORALB manner.

```
      PROGRAM MAIN
      DOUBLE PRECISION DUMMY(4),POL1(4),POL2(4)
C---  FILLING TAUOLA COMMON BLOCKS
      CALL INICOM
C---  INITIALIZATION OF TAUOLA MAIN SUBPROGRAM (INTERFACE)
      CALL DEKAY(-1,DUMMY)
C---  GENERATION OF EVENTS (PRODUCTION+DECAY)
      DO 10 K=1,100
C---  GENERATE TAU+- PAIR, DIFF. X-SECTION AVERAGED OVER SPIN.
      CALL TAUGEN
C---  GENERATE TAU+- DECAYS IN THEIR REST FRAMES
C---  AND CALCULATE POLARIMETRIC VECTORS POL1,POL2
 5    CALL DEKAY(1,POL1)
      CALL DEKAY(2,POL2)
C---  CALCULATE SPIN WEIGHT OF THE EVENT (RATIO OF CROSS SECTION
      CALL TAUSPN(WT,POL1,POL2)
C---  GENERATE UNIFORM RANDOM NUMBER RN IN (0,1) RANGE
C---  AND REJECT OR ACCEPT THE EVENT
      RN=RANDOM(DUM)
      IF (WT/4.0.LT.RN) GO TO 5
C---  TRANSFORM ACCEPTED EVENTS TO THE LAB SYSTEM
C---  AND STORE THEM IN LUND COMMON
      CALL DEKAY(11,DUMMY)
 10   CALL DEKAY(12,DUMMY)
C---  PRINT FINAL REPORT ON THE WHOLE M.C. SAMPLE
      CALL DEKAY(100,DUMMY)
      END
```

The second example demonstrates interfacing of the τ production and decay subprograms in the KORALZ style. Here transverse spin correlations are neglected [11] since they are unimportant for ultrarelativistic τ's. The interface routine **DEXAY(KTO,POL)** is used instead of **DEKAY**. The production routine **TAUGEN(POL1,POL2)** generates a τ pair and provides two polarization four-vectors **POL1**, **POL2** which are later used by the routine **DEXAY** to simulate decays of $\tau^{\pm}$. In the present algorithm the

Table 1
Input parameters in common block DECPAR

Parameter	Meaning
GFERMI	Fermi coupling. $G = 1.16637 \times 10^{-5}$ GeV^{-2}.
GV	τ vector coupling. GV = 1 in the standard model.
GA	τ axial coupling. GA = −1 in the standard model.
CCABIB	Cosine of Cabibbo angle, cos θ_c.
SCABIB	sin θ_c.
GAMEL	Branching ratio for $\tau \to \nu\bar{\nu}e$ ($\Gamma_e = G^2 m_\tau^5 / 192\pi^3$).

DEXAY routine simulates the decays of $\tau^{\pm}$, in addition it performs all necessary boosts to the laboratory system and also writes the event into the LUND common.

```
      PROGRAM MAIN
      REAL* DUMMY(4),POL1(4),POL2(4)
C---  FILLING COMMON BLOCKS OF TAUOLA
      CALL INICOM
C---  INITIALIZE MAIN INTERFACE OF TAUOLA
      CALL DEXAY(-1,DUMMY)
C---  GENERATION OF M.C. EVENTS (PRODUCTION+DECAY)
      DO 10 K=1,100
C---  GENERATE TAU+- PAIR AND THEIR POLARIZATION VECTORS POL1,POL2
C---  POL1, POL2 ARE CALCULATED IN THE REST-FRAMES OF TAU+-
      CALL TAUGEN(POL1,POL2)
C---  GENERATE TAU+- DECAYS IN THEIRS REST-FRAMES, MOMENTA ARE
```

Table 2
Input parameters in common block PARMAS

Parameter	Meaning
AMTAU	τ mass
AMNUTA	ν_τ mass, non-zero mass required for numerical stability
AMEL	e mass
AMNUE	dummy parameter (ν_e mass)
AMMU	μ mass
AMNUMU	dummy parameter (ν_μ mass)
AMPIZ	π^0 mass
AMPI	$\pi^{\pm}$ mass
AMRO	ρ mass, for crude MC distributions [a]
GAMRO	ρ width, for crude MC distributions [a]
AMA1	a_1 mass
GAMA1	a_1 width
AMK	$K^{\pm}$ mass
AMKZ	K^0 mass
AMKST	K* mass
GAMKST	K* width

[a] The status of the ρ mass and width in PARMAS is different than for other resonances. Mass and width of ρ in PARMAS are input parameters for importance sampling in the three-particle phase space. They are essentially dummy parameters, which influence only the speed of the generation. The actual mass and width of ρ as seen in the $\pi\pi$ mass distribution is determined by the form factor as defined in routine FPIK (formula (3.34)). For other resonances masses and widths from common block PARMAS are used for importance sampling and for parametrization of form factors as well.

Table 3
Input parameters in common block TAUBRA

Parameter	Meaning
NCHAN	Number of decay channels. At present NCHAN = 8. The channels appropriately ordered are: $e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$, $\pi\nu$, $\rho\nu$, $a_1\nu$, $K\nu$, $K^*\nu$, $n\pi\nu$.
JLIST(I)	Number of the decay channel according to the above list. If decay modes are ordered as above JLIST(I) = I.
GAMPRT(I)	Branching ratio for the JLIST(I) decay mode. Arbitrary units. These parameters define actual proportion of decays in the sample to be generated.

Table 4
Input parameters in common block TAUNPI

Parameter	Meaning
CBRNPI(I)	Individual branching ratios of different types of multipion final states relative to the total multipion branching ratio. These parameters define actual proportion of decays in the sample to be generated.
AMAS(J,I)	Mass of the J-th π for the I-th type of multipion final state.
KPI(J,I)	Type of the J-th π for the I-th type of multipion final state. KPI(J,I) = −17,17,23,0 denote respectively π of the same/opposite charge as the mother τ, π^0 and no π at all.
MULT(I)	Multiplicity of the I-th type of multipion decay mode.

```
C---  TRANSFORMED TO THE LAB, AND STORED IN THE LUND COMMON
      CALL DEXAY(1,POL1)
 10   CALL DEXAY(2,POL2)
C---  PRINT FINAL REPORT ON THE WHOLE M.C. SAMPLE
      CALL DEXAY(100,DUMMY)
      END
```

As already mentioned, the user has to supply a subroutine which fills some entries in the TAUOLA commons. In the following we list these common blocks, show which entries have to be filled and explain their meaning. In the examples above it was assumed that the subroutine **INICOM** performs these tasks.

Table 5
Input parameters in common blocks JAKI,IDFC,INOUT, IDPART

Parameter	Meaning
JAK1	Type of the τ^+ decay mode according to the list given in the previous table. JAK1 = 0 denotes inclusive τ decay, JAK1 = −1 no decay at all.
JAK	The same as JAK1 but for τ^-.
JAKP	Internal variable, does not require initialization.
JAKM	Internal variable, does not require initialization.
KTOM	Internal variable, does not require initialization.
IDFF	Lund indentifier for τ^+, should be set to IDFF = −11.
INUT	Number of the input unit (dummy).
INOUT	Number of the output unit.
IA1	Lund type identifier for a_1.

Table 6
Output parameters in common block TAUBMC

Parameter	Meaning
NEVDEC(I)	Number of generated decays for the I-th channel. The channels appropriately ordered are: $e\nu\bar{\nu}$, $\mu\nu\bar{\nu}$, $\pi\nu$, $\rho\nu$, $a_1\nu$, $K\nu$, $K^*\nu$, $n\pi\nu$.
GAMPMC(I)	Branching ratio for the I-th decay mode.
GAMPER(I)	Relative statistical error for the branching ratio calculated in the Monte Carlo out of the actual matrix element in the generator.

```
      COMMON /DECPAR/ GFERMI,GV,GA,CCABIB,SCABIB,GAMEL
      COMMON / PARMAS/AMTAU,AMNUTA,AMEL,AMNUE,AMMU,AMNUMU,
     $                AMPIZ,AMPI,AMRO,GAMRO,AMA1,GAMA1,
     $                AMK,AMKZ,AMKST,GAMKST
      COMMON /TAUBRA/ GAMPRT(30),JLIST(30),NCHAN
      COMMON /TAUNPI/ CBRNPI(4),AMAS(6,4),KPI(6,4),MULT(4)
      COMMON /JAKI  / JAK1,JAK2,JAKP,JAKM,KTOM
      COMMON /IDFC  / IDFF
      COMMON /INOUT / INUT,IOUT
      COMMON /IDPART/ IA1
```

Lists of entries which have to be filled in the user supplied routine prior to generation (all masses are in GeV units) are found in tables 1–5.

The matrices **GAMPC**, **GAMPER**, **NEVDEC** are filled in at the end of the generation when **DEXAY(100,DUMMY)** or **DECAY(100,DUMMY)** is called. They are filled respectively with the branching ratios calculated from the matrix elements used in the generation, with the corresponding relative statistical errors and the number of generated events per mode (before rejection with f_{spin}).

```
      COMMON /TAUBMC/ GAMPMC(30),GAMPER(30),NEVDEC(30)
```

For the sake of completeness we include Monte Carlo results for all branching ratios (in units of $\Gamma_e = G^2 m_\tau^5/192\pi^3$) in table 7. The input parameters were the same as in our demonstration program. For the $e\nu\bar{\nu}$ we expect branching ratio to be equal exactly one, within statistical error. For $\pi\nu$ and $K\nu$ modes statistical errors are zero (constant MC weights).

Every generated event is stored in the common block **LULIST** of the LUND program. One should have in mind two limitations of this data structure:
- Identifier for the a_1 particle is not defined.
- Decays into more than four particles are not allowed.

Table 7
Monte Carlo results for the branching ratios

Decay mode	Branching ratio	Decay mode	Branching ratio
$e\nu\bar{\nu}$	0.997 ± 0.001	$\mu\nu\bar{\nu}$	0.972 ± 0.001
$\pi\nu$	0.605 ± 0.0	$\rho\nu$	1.326 ± 0.001
$a_1\nu$	0.719 ± 0.001	$K\nu$	0.040 ± 0.0
$K^*\nu$	0.070 ± 0.0001	$\pi^-2\pi^+\pi^0\nu$	0.266 ± 0.001
$\pi^-3\pi^0\nu$	0.054 ± 0.0002	$3\pi^-2\pi^+\nu$	0.029 ± 0.0001
$3\pi^-2\pi^+\pi^0\nu$	0.004 ± 0.0001		

– To resolve the second obstacle we have introduced modified LUND routine LULIST [#12] into our package. The Lund convention governs only the most external part of the program. It will be straightforward to apply new conventions, like the one proposed in ref. [31].

5. Conclusions

The present program represents an example of the Monte Carlo generator of τ decays. It has the following properties:

(1) It is interfaced to the Monte Carlo programs KORALB and KORALZ simulating τ pair productions in e^+e^- collisions at low and high (LEP) energies. The general interface to any other program simulating τ production is also discussed.

(2) All the effects of τ spin polarization are taken into account in the primary decay products (ρ, a_1...) and in the distributions of the secondary decay products (if present). This is, however, not the case for the decays to more than 3 π's when the simplified approach is taken.

(3) The parametrization of the τ decay matrix element can accommodate any admixture of $V+A$ coupling in the $W\tau\nu$ vertex, finite mass of ν_τ, and some data on e^+e^- cross section. The user is instructed how to modify the matrix element (if necessary).

(4) The matrix elements are coded in a well defined way. They can be easily replaced by the other matrix elements to include, e.g., effects due to nonstandard couplings.

(5) The following decay modes are included: $\tau^\pm \to \nu\nu e^\pm$, $\nu\nu\mu^\pm$, $\nu\mu^\pm$, $\nu\rho^\pm$, $\nu a_1^\pm$, $\nu K^\pm$, $\nu K^{*\pm}$, $\nu n\pi^{\pm,0}$, $n > 3$.

(6) The decay products are coded into common block of the Lund, version 6.3.

The τ decay library is supplemented with the demonstration program and the output of the test run. The primary purpose of the output, also included at the end of the article, is to give a means to check whether TAUOLA works properly on a given computer installation. If the statistical sample is sufficiently increased the demonstration program should reproduce the numerical values of the branching ratios quoted earlier in this paper. The demonstration program presents both, the KORALB and KORALZ, types of the interface to TAUOLA programs.

The future developments could improve multipion decays, while the sophistication level of the simulation of other decay modes is good enough for all present and many future experiments. This is true, may be except effects due to the QED radiative corrections in the decay. To this end an approximate algorithm was developed. It was later generalized and implemented in the universal package PHOTOS [32] which can be applied to the decay of any particle or resonance. We think, however, that dedicated programs for radiative τ decays: $\tau \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to \pi\gamma$ will have to be developed [33].

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TEST RUN OUTPUT

```
                    DEMONSTRATION PROGRAM
                  TESTS OF TAU DECAY ROUTINES
                  INTERFACE OF THE KORAL-B TYPE
=====================================================================
    JAK    =      0  KEY DEFINING DECAY TYPE
    IDFF   =    -11  LUND IDENTIFIER FOR FIRST TAU
    POL(3)=  -1.00  THIRD COMPONENT OF TAU POLARIZ.
    PTAU   =   0.00  THIRD COMPONENT OF TAU MOM. GEV
=====================================================================

*********************************************************************
*                  *****TAUOLA LIBRARY: VERSION 1.5 *****           *
*                  **********AUGUST 1990*****************           *
*                  **AUTHORS: S.JADACH, J.H.KUEHN, Z.WAS**          *
*                  **AVAILABLE FROM: WASM AT CERNVM ******          *
*                  **TO BE PUBLISHED IN COMP. PHYS. COMM.*          *
*                  ****DEKAY ROUTINE: INITIALIZATION******         *
*              0       JAK1    = DECAY MODE TAU+                    *
*              0       JAK2    = DECAY MODE TAU-                    *
*********************************************************************
 RANMAR INITIALIZED:    54217137          0          0

*********************************************************************
===== EVENT NO.    1 =====
    POLARIMETRIC VECTOR:          HH(1)       HH(2)       HH(3)       HH(4)
                            0.00000000  0.00000000  0.00000000  1.00000000
                  EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)      P(I,2)      P(I,3)      P(I,4)      P(I,5)
    1        4       12   NUTAU         -0.01087    -0.19897    -0.18085    0.26928     0.01000
    2    20004       -3   W    B- D      0.01087     0.19897     0.18085     1.51491     1.49082
    3        2      -17   PI   B-        0.35886     0.34550     0.31727     0.60687     0.13957
    4        2      -17   PI   B-       -0.42023     0.00440     0.03678     0.44435     0.13957
    5        2       17   PI   +        -0.06395    -0.20221     0.06669     0.26250     0.13957
    6        2       23   PI0            0.13620     0.05127    -0.03295     0.20119     0.13496
                SUM:       -1.00000     0.00000     0.00000     0.00000     1.78419     1.78419

===== EVENT NO.    2 =====
    POLARIMETRIC VECTOR:          HH(1)       HH(2)       HH(3)       HH(4)
                           -0.08511490 -0.27466440  0.95776546  1.00000000
                  EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)      P(I,2)      P(I,3)      P(I,4)      P(I,5)
    1        4       12   NUTAU          0.07546     0.24352    -0.84916     0.88667     0.00998
    2        4      -17   PI   B-        0.07546    -0.24352     0.84916     0.89753     0.13957
                SUM:       -1.00000     0.00000     0.00000     0.00000     1.78419     1.78419

===== EVENT NO.    3 =====
    POLARIMETRIC VECTOR:          HH(1)       HH(2)       HH(3)       HH(4)
                           -0.76737750 -0.58751684  0.25440270  1.00000000
                  EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)      P(I,2)      P(I,3)      P(I,4)      P(I,5)
    1        4       12   NUTAU          0.27775    -0.01650     0.13980     0.31155     0.01018
    2    20004     -123   BU *B+ D       0.27775     0.01650    -0.13980     1.47264     1.43934
    3        2       23   PI0            0.26823    -0.58063     0.10473     0.66201     0.13496
    4        2       23   PI0            0.17749     0.46958    -0.17143     0.54737     0.13497
    5        2      -17   PI   B-       -0.16797     0.12755    -0.07310     0.26326     0.13957
                SUM:        1.00000     0.00000     0.00000     0.00000     1.78419     1.78419

===== EVENT NO.    4 =====
    POLARIMETRIC VECTOR:          HH(1)       HH(2)       HH(3)       HH(4)
                           -0.95107281  0.30896068  0.00210588  1.00000000
                  EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)      P(I,2)      P(I,3)      P(I,4)      P(I,5)
    1        4       12   NUTAU          0.21296     0.10104     0.38020     0.44745     0.01001
    2    20004       -3   W    B- D      0.21296     0.10104     0.38020     1.33674     1.25967
    3        2        7   E    -         0.47065    -0.12104    -0.38172     0.61796     0.00049
    4        2       -8   NUE B          0.68361     0.22208     0.00151     0.71878    -0.00055
                SUM:        1.00000     0.00000     0.00000     0.00000     1.78419     1.78419

*********************************************************************
*                  ******** DAMNEL FINAL REPORT ********            *
*             4457     NEVRAW = NO. OF EL  DECAYS TOTAL              *
*             1272     NEVACC = NO. OF EL   DECS. ACCEPTED           *
*                0     NEVOVR = NO. OF OVERWEIGHTED EVENTS           *
*      0.41651E-12     PARTIAL WIDTH ( ELECTRON) IN GEV UNITS        *
*      1.008067130     IN UNITS GFERMI**2*MASS**5/192/PI**3          *
*      0.012369636     RELATIVE ERROR OF PARTIAL WIDTH               *
*********************************************************************

*********************************************************************
*                  ******** DAMMU FINAL REPORT ********             *
*             3886     NEVRAW = NO. OF MU  DECAYS TOTAL              *
*             1222     NEVACC = NO. OF MU   DECS. ACCEPTED           *
*                0     NEVOVR = NO. OF OVERWEIGHTED EVENTS           *
*      0.40097E-12     PARTIAL WIDTH (MU  DECAY) IN GEV UNITS        *
*      0.970460236     IN UNITS GFERMI**2*MASS**5/192/PI**3          *
*      0.013019867     RELATIVE ERROR OF PARTIAL WIDTH               *
*********************************************************************

*********************************************************************
*                  ******** DAMPI FINAL REPORT ********             *
*             1185     NEVTOT = NO. OF PI  DECAYS TOTAL              *
*      0.25031E-12     PARTIAL WIDTH ( PI DECAY) IN GEV UNITS        *
*      0.605810344     IN UNITS GFERMI**2*MASS**5/192/PI**3          *
*       0.00000000     RELATIVE ERROR OF PARTIAL WIDTH (STAT.)       *
*********************************************************************
```

```
*****************************************************************
*                      ******** DADMRO FINAL REPORT  ********    *
*          3798        NEVRAW = NO. OF RHO DECAYS TOTAL          *
*          1226        NEVACC = NO. OF RHO  DECS. ACCEPTED       *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.55501E-12        PARTIAL WIDTH (RHO DECAY) IN GEV UNITS    *
*   1.343272210        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.00920054        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      ******** DADMAA FINAL REPORT  ********    *
*         18832        NEVRAW = NO. OF A1  DECAYS TOTAL          *
*          1299        NEVACC = NO. OF A1   DECS. ACCEPTED       *
*            16        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.29318E-12        PARTIAL WIDTH (A1  DECAY) IN GEV UNITS    *
*   0.709585011        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.01241162        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      ******** DADMKK FINAL REPORT  ********    *
*          1285        NEVTOT = NO. OF K  DECAYS TOTAL           *
*   0.16425E-13        PARTIAL WIDTH ( K DECAY) IN GEV UNITS     *
*   0.039751995        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.00000000        RELATIVE ERROR OF PARTIAL WIDTH (STAT.)   *
*****************************************************************

*****************************************************************
*                      ******** DADMKS FINAL REPORT  ********    *
*          3362        NEVRAW = NO. OF K* DECAYS TOTAL           *
*          1265        NEVACC = NO. OF K*  DECS. ACCEPTED        *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.29124E-13        PARTIAL WIDTH (K* DECAY) IN GEV UNITS     *
*   0.070486784        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.00826202        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      ******** DADNPI FINAL REPORT  ********    *
*          1019        NEVRAW = NO. OF MULTIPI DECAYS TOTAL      *
*           315        NEVACC = NO. OF MULTIPI DECS. ACCEPTED    *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.10847E-12        PART. WIDTH (MULTIPI DEC) IN GEV UNITS    *
*   0.262527943        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.02806315        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      ******** DADNPI FINAL REPORT  ********    *
*          1056        NEVRAW = NO. OF MULTIPI DECAYS TOTAL      *
*           303        NEVACC = NO. OF MULTIPI DECS. ACCEPTED    *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.22717E-13        PART. WIDTH (MULTIPI DEC) IN GEV UNITS    *
*   0.054981939        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.03026197        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      ******** DADNPI FINAL REPORT  ********    *
*           992        NEVRAW = NO. OF MULTIPI DECAYS TOTAL      *
*           312        NEVACC = NO. OF MULTIPI DECS. ACCEPTED    *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.11838E-13        PART. WIDTH (MULTIPI DEC) IN GEV UNITS    *
*   0.028650202        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.03179262        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      ******** DADNPI FINAL REPORT  ********    *
*          1291        NEVRAW = NO. OF MULTIPI DECAYS TOTAL      *
*           316        NEVACC = NO. OF MULTIPI DECS. ACCEPTED    *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS       *
*   0.16607E-14        PART. WIDTH (MULTIPI DEC) IN GEV UNITS    *
*   0.004019398        IN UNITS GFERMI**2*MASS**5/192/PI**3      *
*    0.03709928        RELATIVE ERROR OF PARTIAL WIDTH           *
*****************************************************************

*****************************************************************
*                      *****TAUOLA LIBRARY: VERSION 1.5 ******    *
*                      ************AUGUST 1990*****************    *
*                      **AUTHORS: S.JADACH, J.H.KUEHN, Z.WAS**    *
*                      **AVAILABLE FROM: WASM AT CERNVM ******    *
*                      **TO BE PUBLISHED IN COMP. PHYS. COMM.*    *
*                      *****DEKAY ROUTINE: FINAL REPORT*******    *
*             0        NEV1   = NO. OF TAU+ DECS. ACCEPTED       *
*         10000        NEV2   = NO. OF TAU- DECS. ACCEPTED       *
*         10000        NEVTOT = SUM                              *
*  NOEVTS  PART.WIDTH    ERROR      ROUTINE    DECAY MODE        *
*    1272  1.0080671  0.0123696     DADMEL     ELECTRON          *
*    1222  0.9704602  0.0130199     DADMMU     MUON              *
*    1185  0.6058103  0.0000000     DADMPI     PION              *
*    1226  1.3432722  0.0092005     DADMRO     RHO (->2PI)       *
*    1299  0.7095850  0.0124116     DADMAA     A1  (->3PI)       *
*    1285  0.0397520  0.0000000     DADMKK     KAON              *
*    1265  0.0704868  0.0082620     DADMKS     K*                *
*     315  0.2625279  0.0280632     DADNPI     PI- 2PI+ PI0      *
*     303  0.0549819  0.0302620     DADNPI     PI- 3PI0          *
*     312  0.0286502  0.0317926     DADNPI     3PI- 2PI+         *
*     316  0.0040194  0.0370993     DADNPI     3PI- 2PI+ PI0     *
*                THE ERROR IS RELATIVE AND  PART.WIDTH          *
*              IN UNITS GFERMI**2*MASS**5/192/PI**3             *
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                    DEMONSTRATION PROGRAM
                  TESTS OF TAU DECAY ROUTINES
                  INTERFACE OF THE KORAL-Z TYPE
    JAK    =       0  KEY DEFINING DECAY TYPE
    IDFF   =     -11  LUND IDENTIFIER FOR FIRST TAU
    POL(3)=   -1.00  THIRD COMPONENT OF TAU POLARIZ.
    PTAU   =    0.00  THIRD COMPONENT OF TAU MOM. GEV
*                     *****TAUOLA LIBRARY: VERSION 1.5 ******          *
*                     **********AUGUST 1990 ****************          *
*                     **AUTHORS: S.JADACH, J.H.KUEHN, Z.WAS**          *
*                     **AVAILABLE FROM: WASM AT CERNVM ******          *
*                     **TO BE PUBLISHED IN COMP. PHYS. COMM.*          *
*                     ******DEXAY ROUTINE: INITIALIZATION****          *
*                  0      JAK1    = DECAY MODE FERMION1 (TAU+)         *
*                  0      JAK2    = DECAY MODE FERMION2 (TAU-)         *

===== EVENT NO.    1 =====
                EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)       P(I,2)       P(I,3)       P(I,4)       P(I,5)
    1        4       12   NUTAU        0.30895     -0.25989      0.50916      0.64988      0.01001
    2    20004      -28   K*  B- D      0.30895      0.25989     -0.50916      1.13432      0.92975
    3        2       23   PI0          0.28812      0.36935     -0.25646      0.56444      0.13496
    4        2      -18   K   B-       0.02083     -0.12946     -0.25270      0.56988      0.49367
                SUM:      -1.00000      0.00000      0.00000      0.00000      1.78419      1.78419

===== EVENT NO.    2 =====
                EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)       P(I,2)       P(I,3)       P(I,4)       P(I,5)
    1        4       12   NUTAU       -0.64600      0.60333     -0.06899      0.88667      0.00998
    2        4      -17   PI  B-       0.64600     -0.60333      0.06899      0.89753      0.13957
                SUM:      -1.00000      0.00000      0.00000      0.00000      1.78419      1.78419

===== EVENT NO.    3 =====
                EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)       P(I,2)       P(I,3)       P(I,4)       P(I,5)
    1        4       12   NUTAU       -0.34887     -0.63980     -0.06912      0.73207      0.01001
    2    20004       -3   W   B- D     0.34887      0.63980      0.06912      1.05212      0.75573
    3        2        9   MU  -        0.13999      0.20759      0.38954      0.47497      0.10566
    4        2      -10   NUMUB        0.20889      0.43220     -0.32042      0.57715     -0.00036
                SUM:      -1.00000      0.00000      0.00000      0.00000      1.78419      1.78419

===== EVENT NO.    4 =====
                EVENT LISTING (EXTENDED)
    I  K(I,1)  K(I,2)   PART/JET        P(I,1)       P(I,2)       P(I,3)       P(I,4)       P(I,5)
    1        4       12   NUTAU        0.04710     -0.15530     -0.02819      0.16502      0.01000
    2    20004       -3   W   B- D     -0.04710      0.15530      0.02819      1.61918      1.61078
    3        2      -17   PI  B-       0.17094      0.49122      0.12855      0.55364      0.13957
    4        2      -17   PI  B-      -0.11374     -0.27547     -0.24623      0.41101      0.13957
    5        2      -17   PI  B-      -0.15129     -0.01766      0.02177      0.20773      0.13957
    6        2       17   PI  +        0.02012     -0.03796      0.07420      0.16380      0.13957
    7        2       17   PI  +        0.01511     -0.01550      0.03972      0.14671      0.13957
    8        2       23   PI0          0.01175      0.01067      0.01017      0.13627      0.13496
                SUM:      -1.00000      0.00000      0.00000      0.00000      1.78419      1.78419

*                     ******** DADMEL FINAL REPORT  ********          *
*                8803     NEVRAW = NO. OF EL  DECAYS TOTAL             *
*                2514     NEVACC = NO. OF EL   DECS. ACCEPTED          *
*                   0     NEVOVR = NO. OF OVERWEIGHTED EVENTS          *
*         0.41279E-12     PARTIAL WTDTH ( ELECTRON) IN GEV UNITS       *
*         0.999069750     IN UNITS GFERMI**2*MASS**5/192/PI**3         *
*         0.008912031     RELATIVE ERROR OF PARTIAL WIDTH              *

*                     ******** DADMMU FINAL REPORT  ********          *
*                8391     NEVRAW = NO. OF MU  DECAYS TOTAL             *
*                2573     NEVACC = NO. OF MU   DECS. ACCEPTED          *
*                   0     NEVOVR = NO. OF OVERWEIGHTED EVENTS          *
*         0.40107E-12     PARTIAL WTDTH (MU  DECAY) IN GEV UNITS       *
*         0.970697701     IN UNITS GFERMI**2*MASS**5/192/PI**3         *
*         0.008772571     RELATIVE ERROR OF PARTIAL WIDTH              *

*                     ******** DADMPI FINAL REPORT  ********          *
*                2434     NEVTOT = NO. OF PI  DECAYS TOTAL             *
*         0.25031E-12     PARTIAL WTDTH ( PI DECAY) IN GEV UNITS       *
*         0.605811238     IN UNITS GFERMI**2*MASS**5/192/PI**3         *
*          0.00000000     RELATIVE ERROR OF PARTIAL WIDTH (STAT.)      *

*                     ******** DADMRO FINAL REPORT  ********          *
*                7966     NEVRAW = NO. OF RHO DECAYS TOTAL             *
*                2414     NEVACC = NO. OF RHO  DECS. ACCEPTED          *
*                   0     NEVOVR = NO. OF OVERWEIGHTED EVENTS          *
*         0.54629E-12     PARTIAL WTDTH (RHO DECAY) IN GEV UNITS       *
*         1.322178840     IN UNITS GFERMI**2*MASS**5/192/PI**3         *
*          0.00636170     RELATIVE ERROR OF PARTIAL WIDTH              *
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*                      ******** DADMAA FINAL REPORT  ********   *
*          44109       NEVRAW = NO. OF A1  DECAYS TOTAL         *
*           2495       NEVACC = NO. OF A1   DECS. ACCEPTED      *
*             11       NEVOVR = NO. OF OVERWEIGHTED EVENTS      *
*    0.29843E-12       PARTIAL WIDTH (A1  DECAY) IN GEV UNITS   *
*    0.722271562       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.00806502       RELATIVE ERROR OF PARTIAL WIDTH          *
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*                      ******** DADMKK FINAL REPORT  ********   *
*           2502       NEVTOT = NO. OF K  DECAYS TOTAL          *
*    0.16425E-13       PARTIAL WIDTH ( K DECAY) IN GEV UNITS    *
*    0.039752048       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.00000000       RELATIVE ERROR OF PARTIAL WIDTH (STAT.)  *
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*                      ******** DADMKS FINAL REPORT  ********   *
*          7265        NEVRAW = NO. OF K* DECAYS TOTAL          *
*          2650        NEVACC = NO. OF K*  DECS. ACCEPTED       *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS      *
*    0.26907E-13       PARTIAL WIDTH (K* DECAY) IN GEV UNITS    *
*    0.069963932       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.00566075       RELATIVE ERROR OF PARTIAL WIDTH          *
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*                      ******** DADNPI FINAL REPORT  ********   *
*           933        NEVRAW = NO. OF MULTIPI DECAYS TOTAL     *
*           324        NEVACC = NO. OF MULTIPI DECS. ACCEPTED   *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS      *
*    0.11097E-12       PART. WIDTH (MULTIPI DEC) IN GEV UNITS   *
*    0.268576264       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.02947173       RELATIVE ERROR OF PARTIAL WIDTH          *
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*                      ******** DADNPI FINAL REPORT  ********   *
*          1118        NEVRAW = NO. OF MULTIPI DECAYS TOTAL     *
*           290        NEVACC = NO. OF MULTIPI DECS. ACCEPTED   *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS      *
*    0.21449E-13       PART. WIDTH (MULTIPI DEC) IN GEV UNITS   *
*    0.051911905       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.03076986       RELATIVE ERROR OF PARTIAL WIDTH          *
*****************************************************************

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*                      ******** DADNPI FINAL REPORT  ********   *
*           887        NEVRAW = NO. OF MULTIPI DECAYS TOTAL     *
*           281        NEVACC = NO. OF MULTIPI DECS. ACCEPTED   *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS      *
*    0.11987E-13       PART. WIDTH (MULTIPI DEC) IN GEV UNITS   *
*    0.029011764       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.03229468       RELATIVE ERROR OF PARTIAL WIDTH          *
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*                      ******** DADNPI FINAL REPORT  ********   *
*          1059        NEVRAW = NO. OF MULTIPI DECAYS TOTAL     *
*           280        NEVACC = NO. OF MULTIPI DECS. ACCEPTED   *
*             0        NEVOVR = NO. OF OVERWEIGHTED EVENTS      *
*    0.17989E-14       PART. WIDTH (MULTIPI DEC) IN GEV UNITS   *
*    0.004353758       IN UNITS GFERMI**2*MASS**5/192/PI**3     *
*     0.03821290       RELATIVE ERROR OF PARTIAL WIDTH          *
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*                      **AVAILABLE FROM: WASM AT CERNVM ******   *
*                      **TO BE PUBLISHED IN COMP. PHYS. COMM.*   *
*                      *****DEXAY ROUTINE: FINAL REPORT******   *
*             0        NEV1   = NO. OF TAU+ DECS. ACCEPTED      *
*         10000        NEV2   = NO. OF TAU- DECS. ACCEPTED      *
*         10000        NEVTOT = SUM                             *
*  NOEVTS  PART.WIDTH     ERROR        ROUTINE     DECAY MODE   *
*    1259  0.9990698   0.0089120      DADMEL      ELECTRON      *
*    1311  0.9706977   0.0087726      DADMMU      MUON          *
*    1245  0.6058112   0.0000000      DADMPI      PION          *
*    1235  1.3221788   0.0063617      DADMRO      RHO (->2PI)   *
*    1231  0.7222716   0.0080650      DADMAA      A1  (->3PI)   *
*    1232  0.0397520   0.0000000      DADMKK      KAON          *
*    1312  0.0699639   0.0056607      DADMKS      K*            *
*     324  0.2685763   0.0294717      DADNPI      PI- 2PI+ PI0  *
*     290  0.0519119   0.0307699      DADNPI      PI- 3PI0      *
*     281  0.0290118   0.0322947      DADNPI      3PI- 2PI+     *
*     280  0.0043538   0.0382129      DADNPI      3PI- 2PI+ PI0 *
*                  THE ERROR IS RELATIVE AND  PART.WIDTH        *
*                  IN UNITS GFERMI**2*MASS**5/192/PI**3         *
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