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LOW-ENERGY SUPERSYMMETRY*

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We discuss the problem of constructing a model in which supersymmetry is unbroken down to low energies. It is suggested that the scalar partners of quarks and leptons may get their masses through radiative corrections and that the breaking of the weak interactions also occurs through radiative corrections. A toy model is constructed which illustrates these ideas.

1. Introduction

In supersymmetric theories [1] fermions and bosons are in the same degenerate multiplet. Since no scalar partners of the quarks and leptons have been observed, it is not clear whether a realistic model can be built with supersymmetry surviving down to low energies. For example, Dimopoulos and Georgi have shown [2] that in tree level a model based on the gauge group $SU(3)\times SU(2)\times U(1)$ with spontaneously broken supersymmetry contains either a color triplet scalar meson with charge $+\frac{2}{3}$ not heavier than the lightest charge $+\frac{2}{3}$ quark or a color triplet scalar meson with charge $-\frac{1}{3}$ not heavier than the lightest charge $-\frac{1}{3}$ quark.

Fayet [3] has suggested that the scalar partners of quarks and leptons can be given a large tree level mass by adding a new $\tilde{U}(1)$ gauge group under which the quark and lepton superfields all have a positive charge. It has been suggested that global symmetries should not be imposed [4]. However, we find that if global symmetries are not imposed, it is difficult to build such a model with the correct vacuum and with anomalies cancelled.

Consider, for example, assigning the three generations of quark and lepton superfields a common charge, +1, under the $\tilde{U}(1)$ gauge group. Weinberg [4] has noted that the $SU(3)^2\tilde{U}(1)$ anomaly is neatly cancelled by adding a ($SU(2)\times(1)$ singlet) color octet superfield, O, with $\tilde{U}(1)$ charge -2. Other color singlet superfields are added to cancel the remaining anomalies. We now show, assuming** the couplings

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** This example is for illustrative purposes and the assumptions made should not be viewed as general properties of models with an additional $\tilde{U}(1)$ gauge group. For example, SU(2) triplets with hypercharge -1 and $\tilde{U}(1)$ -2 can have large couplings to the lepton doublets if global symmetries enforcing separate electron and muon number conservation are imposed. Also the superspace potential could contain parameters with the dimensions of mass.

of lepton superfields in the superspace potential are negligibly small and that the superspace potential is cubic in the superfields (i.e. contains no mass parameters), that the vacuum of this model is supersymmetric. Suppose the scalar components of two of the lepton doublet superfields have vacuum expectation values (VEVs)

$$\langle L_1\rangle = \begin{pmatrix}\alpha\\0\end{pmatrix}, \qquad \langle L_2\rangle = \begin{pmatrix}0\\\alpha\end{pmatrix}, \tag{1}$$

the scalar component of the positron superfield has the VEV

$$\langle E_1^c\rangle = \beta, \tag{2}$$

the scalar component of the color octet superfield has the VEV

$$\langle O\rangle = \gamma\begin{bmatrix}1+\sqrt{\tfrac{1}{3}}\,i & 0 & 0\\ 0 & -1+\sqrt{\tfrac{1}{3}}\,i & 0\\ 0 & 0 & -2\sqrt{\tfrac{1}{3}}\,i\end{bmatrix}, \tag{3}$$

and all other scalar fields have no VEVs. Since the superspace potential is cubic and we are neglecting the couplings of the lepton superfields, the F terms do not contribute to the vacuum energy. Note that $\mathrm{Tr}\,\langle O\rangle^2 = 0$ so that a term $X\,\mathrm{Tr}\,O^2$ in the superspace potential [where X is a $\mathrm{SU(3)}\times\mathrm{SU(2)}\times\mathrm{U(1)}$ singlet superfield with $\tilde{\mathrm{U}}(1)+4$] does not contribute to the vacuum energy. The VEV's of L_1, L_2 and O in eqs. (1)–(3) are chosen so that the SU(2) and SU(3) D terms are zero for any values of α, β and γ.

The U(1) and $\tilde{\mathrm{U}}(1)$ D-terms give rise to a vacuum energy density

$$E_{\mathrm{VAC}} = \tfrac{1}{2}g_1^2[|\alpha|^2 - |\beta|^2 + \mu^2]^2 + \tfrac{1}{2}\tilde{g}_1^2[-8|\gamma|^2 + 2|\alpha|^2 + |\beta|^2 + \tilde{\mu}^2]^2\,. \tag{4}$$

For any values of the Fayet–Illiopoulos D-term [5] mass squared parameters, μ^2 and $\tilde{\mu}^2$, there exist values of α, β and γ for which the vacuum energy vanishes and supersymmetry is unbroken. In addition the vacuum breaks color and electromagnetism. This example illustrates the type of difficulties one encounters in attempting to build a model where the scalar partners of quarks and leptons get a large tree-level mass from a $\tilde{\mathrm{U}}(1)$ D term. For some further comments on these difficulties see the notes added in ref. [4].

If one does not want to enlarge the low-energy gauge group, the Dimopoulos–Georgi theorem implies that the scalar partners of quarks and leptons must receive a large mass through radiative corrections. The purpose of this paper is to explore this possibility. In the next section a toy model is presented where radiative corrections give a large mass to the scalar partners of the quarks and leptons and also induce the breakdown of the weak interactions. This model is similar in spirit to the supercolor model of Dine, Fishler and Srednicki [6], except that the breakdown of supersymmetry is not dynamical*, but rather occurs at tree level through an O'Raifeartaigh [8] type model. Concluding remarks are given in sect. 3.

* It is not clear whether dynamical breakdown of supersymmetry actually takes place in supercolor models. See ref. [7].

2. A toy muddle*

Our toy model contains four generations of (left-handed) quark and lepton superfields: Q_j, U_j^c, D_j^c, L_j, E_j^c, where $j=1,2,3,4$ is a generation index. Here we have suppressed SU(3) and SU(2) indices. For example, the Q_j are SU(3) triplets and SU(2) doublets, while the L_j are SU(3) singlets and SU(2) doublets. Two SU(2) doublet Higgs superfields H and H', with hypercharges $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively, are needed to Yukawa couple to the quark and lepton superfields. In addition we introduce several exotic superfields: J_1, J_2, K_1^c, K_2^c, R_1, R_2, S_1, S_2, T and X. The $\mathrm{SU(2)\times U(1)}$ singlet superfields J_i and K_i^c respectively transform under SU(3) as some representation r_3 and its conjugate $\bar{\mathrm{r}}_3$. The $\mathrm{SU(3)\times U(1)}$ singlet superfields R_i and S_i, transform as a representation r_2 of the weak SU(2) gauge group. T is also an $\mathrm{SU(3)\times U(1)}$ singlet, but transforms like the adjoint representation of SU(2) (i.e. is a triplet). Finally, X is a $\mathrm{SU(3)\times SU(2)\times U(1)}$ singlet superfield. Table 1 contains a summary of the anomaly free collection of (left-handed) chiral superfields in the model and their $\mathrm{SU(3)\times SU(2)\times U(1)}$ quantum numbers.

The superspace potential for our toy model is

$$W=X(aJ_1K_1^c+bR_1S_1-\mu^2)+M_1J_1K_2^c+M_2J_2K_1^c+\tilde{M}_1R_1S_2$$
$$+\tilde{M}_2R_2S_1+g_TH\boldsymbol{\tau}\cdot\boldsymbol{T}H'+g_U^{ij}U_i^cQ_jH'+g_D^{ij}D_i^cQ_jH+g_E^{ij}E_i^cL_jH\,, \qquad (5)$$

in which $\boldsymbol{\tau}=\frac{1}{2}\boldsymbol{\sigma}$ are the SU(2) generators and SU(2) indices are contracted with the antisymmetric tensor ε_{ab}. This superspace potential is the most general one consistent with its global symmetries. By adjusting the phases of the exotic fields, the couplings a, b and g_T and the masses μ, M_1, M_2, $\tilde{M}_1$, and $\tilde{M}_2$ can be made real and positive. Redefinition of the quark and lepton superfields allows us to make the Yukawa coupling matrices g_D^{ij} and g_E^{ij} real and diagonal.

TABLE 1

Field	SU(3)	SU(2)	U(1)
Q_i	3	2	$-\frac{1}{6}$
U_i^c	$\bar{3}$	1	$\frac{2}{3}$
D_i^c	$\bar{3}$	1	$-\frac{1}{3}$
L_i	1	2	$\frac{1}{2}$
E_i^c	1	1	-1
H	1	2	$\frac{1}{2}$
H'	1	2	$-\frac{1}{2}$
J_i	r_3	1	0
K_i^c	$\bar{\mathrm{r}}_3$	1	0
R_i	1	r_2	0
S_i	1	r_2	0
T	1	3	0
X	1	1	0

* We thank Paul Ginsparg for suggesting this title and for other not entirely useless discussions.

We assume that there is not Fayet–Illiopoulos D-term for the hypercharge group. There is no symmetry forbidding such a term. However, if this theory results from the breakdown of a semisimple gauge group at some very large mass scale, no Fayet–Illipoulos D-term is generated by perturbative corrections [7, 9].

Provided $a\mu^2 < M_1 M_2$ and $b\mu^2 < \tilde{M}_1 \tilde{M}_2$, the tree-level scalar potential of the model has a global minimum when all the scalar fields have no VEVs. Supersymmetry is spontaneously broken because the F component of the singlet X superfield has a VEV. At the three level this breaking of supersymmetry manifests itself in a splitting between the masses of scalar and fermion components of the J_i, K_i^c, R_i, and S_i superfields.

The vacuum is not uniquely determined by the three level potential, since the minimum of this potential is independent of the VEV of the singlet scalar field X*. At the one-loop level the diagrams in fig. 1 produce a mass for the X field. Assuming

$$M_1 \approx M_2 = M\,, \qquad \tilde{M}_1 \approx \tilde{M}_2 = \tilde{M}\,, \tag{6}$$

the mass-squared of the X field is

$$m_X^2 = \frac{1}{24\pi^2}\{a^2M^2D(\mathrm{r}_3)z^2 + b^2\tilde{M}^2D(\mathrm{r}_2)\tilde{z}^2\} + \mathrm{O}(z^4, \tilde{z}^4)\,, \tag{7}$$

where $D(\mathrm{r}_3)$ and $D(\mathrm{r}_2)$ denote the dimensions of representations r_3 and r_2. Also

$$z = \frac{a\mu^2}{M^2}, \qquad \tilde{z} = \frac{b\mu^2}{\tilde{M}^2}\,. \tag{8}$$

The positivity of m_X^2 establishes that there is a local minimum when the X field has no VEV**.

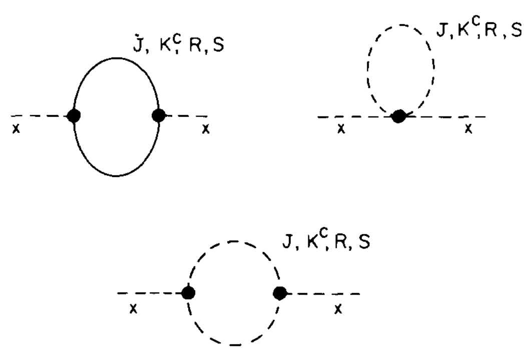

Fig. 1. Feynman diagrams contributing to the mass of the X scalar. Dashed lines represent scalars and solid lines represent fermions.

* A chiral superfield and its scalar or A component are denoted by the same symbol.

** This is a global minimum as has been checked by a computation of the complete one-loop effective potential. See refs. [10, 11].

Since supersymmetry is spontaneously broken, the scalar partners of the quarks and leptons are not protected from getting a mass. We find that in Landau gauge the leading contribution to the masses of the scalar partners of quarks and leptons, the Higgs scalars, and the SU(2) triplet T scalars comes from the two-loop graphs in fig. 2. Calculation of these Feynman diagrams reveals that scalars with SU(3)$\times$ SU(2) quantum numbers (R_3, R_2) have masses squared

$$m^2_{(R_3,R_2)} = \frac{\alpha_3^2(M^2)}{32\pi^2} M^2 z^2 [D(\mathrm{r}_3) C_2(\mathrm{r}_3) C_2(R_3)] + \frac{\alpha_2^2(\tilde{M}^2)}{12\pi^2} \tilde{M}^2 \tilde{z}^2 [D(\mathrm{r}_2) C_2(\mathrm{r}_2) C_2(R_2)] + \mathrm{O}(z^4, \tilde{z}^4) \,. \tag{9}$$

Here $C_2(r)$ is the quadratic Casimir* of a representation r, $\alpha_2 = g_2^2/4\pi$, and $\alpha_3 = g_3^2/4\pi$ [where g_2 and g_3 are the SU(2) and SU(3) gauge couplings]. The supersymmetry breaking parameters z and $\tilde{z}$ are defined in eq. (8). For simplicity we continue to use the approximation described by eq. (6). Some of the details of the two-loop calculation are presented in appendix A.

The masses squared from the two-loop diagrams in fig. 2 are positive. The reason for assigning the exotic fields J_i, K_i^c, R_i and S_i zero hypercharge is that, otherwise, the one-loop graphs involving U(1) D-term couplings would have induced masses

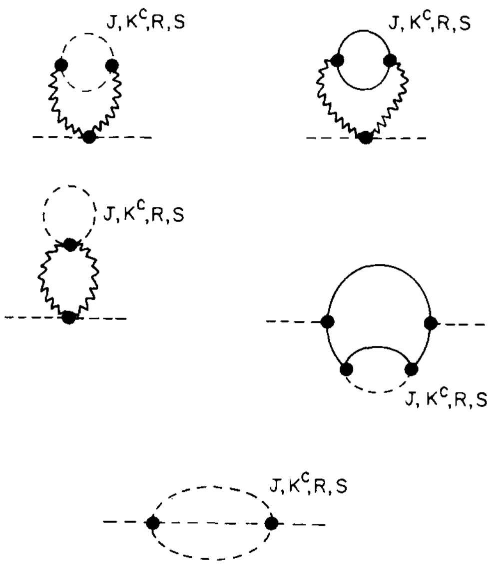

Fig. 2. Two-loop Feynman diagrams contributing to the masses of the scalars not participating in the O'Raifeartaigh model. Dashed lines represent scalars, solid lines represent fermions, and jagged lines represent gauge bosons.

* The quadratic Casimir is normalized so that for the adjoint representation of SU(N), it is N.

squared for the scalar partners of quarks and leptons that are not positive definite, but rather depend on the sign of their hypercharge.

We have just seen that integrating out the heavy J_i, K_i^c, R_i and S_i fields from the theory leaves us with an effective lagrangian that contains dimension-two operators. Explicitly

$$\mathscr{L}_{\rm eff}^{\rm DIM=2} = -m_X^2|X|^2 - m_{(3,1)}^2[|Q_i|^2 + |U_i^c|^2 + |D_i^c|^2] - m_{(1,2)}^2[|Q_i|^2 + |L_i|^2 + |H|^2 + |H'|^2 + \tfrac{8}{3}|T|^2]\,. \tag{10}$$

Here the flavor indices inside a modulus sign are understood to be summed – e.g., $|Q_i|^2 = \sum_{i=1}^4 |Q_i|^2$. The masses m_X, $m_{(3,1)}$ and $n_{(1,2)}$ are defined in eqs. (7) and (9). Dimension-four operators exists at the tree level and we neglect the radiative corrections to their coefficients. Operators with dimension greater than four are induced by radiative corrections, but their coefficients are suppressed by powers of M or $\tilde{M}$, and will be neglected*

The matrix elements of the dimension-two operators in eq. (10) depend on the subtraction point μ. However, the coefficients of these operators also depend on μ in such a way that physical quantities are subtraction point independent. The coefficients of the operators in the two square brackets of eq. (10) correspond respectively to subtraction points $\mu \simeq M$ and $\mu \simeq \tilde{M}$. At these subtraction points there are large logarithms in the matrix elements of the dimension-two operators. These can be absorbed into the coefficients of the operators by scaling down to a much smaller subtraction point using the renormalization group equations. The scaling is determined by the anomalous dimension matrix for the dimension-two operators and by the wave function renormalization of the scalar fields. In the leading logarithm approximation these are given by the "infinite" parts** of the one-loop diagrams in figs. 3 and 4. Wave function renormalization contributions depending on the gauge couplings cancel the dependence of the anomalous

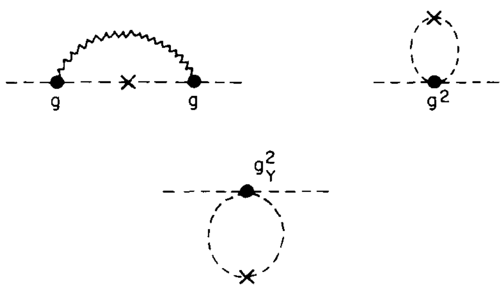

Fig. 3. Graphs that give the anomalous dimensions for the scalar mass operators. g is a gauge coupling and g_Y is a Yukawa coupling. Dashed lines represent scalars and jagged lines represent gauge bosons.

* There are non-trivial directions in field space for which the quartics vanish. Along these directions higher order terms in the two-loop effective potential are not necessarily unimportant. These do not change our results. (M. Claudson and M. Wise, to be published.)

** Our calculations are done using dimensional regularization with minimal subtraction [12].

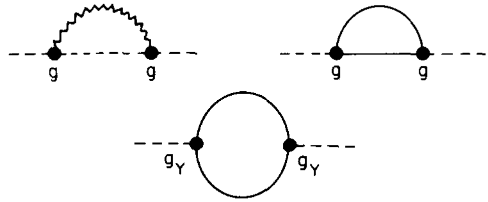

Fig. 4. Feynman diagrams that give the wave functions renormalization for scalar fields. Dashed lines represent scalars, solid lines represent fermions and jagged lines represent gauge bosons. g is a gauge coupling and g_Y a Yukawa coupling.

dimensions on the gauge couplings. However, the effects of the Yukawa couplings do not cancel.

The renormalization group equations for scaling $\mathscr{L}_{\text{eff}}^{\text{DIM}=2}$ to low energies involve several different Yukawa couplings and have no known analytic solution. For this reason, we ignore the subtraction point dependence of the Yukawa couplings and limit ourselves to extracting scaling effects of order $\alpha \ln(M^2$ or $\tilde{M}^2/\mu^2)$. In this approximation, the effective lagrangian at a subtraction point μ much less than M or $\tilde{M}$ will contain mass-squared matrices*

$$
\begin{aligned}
(\mu^2_{\tilde{Q}})_{ij} &= (m^2_{(3,1)} + m^2_{(1,2)})\delta_{ij} - \frac{g_U^{*ki} g_U^{kj} + g_D^{ki} g_D^{kj}}{8\pi^2}\left[m^2_{(3,1)} \ln\left(\frac{M^2}{\mu^2}\right) + m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right)\right], \\
(\mu^2_{\tilde{U}})_{ij} &= m^2_{(3,1)}\delta_{ij} - \frac{g_U^{*ik} g_U^{jk}}{4\pi^2}\left[m^2_{(3,1)} \ln\left(\frac{M^2}{\mu^2}\right) + m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right)\right], \\
(\mu^2_{\tilde{D}})_{ij} &= m^2_{(3,1)}\delta_{ij} - \frac{g_D^{ik} g_D^{jk}}{4\pi^2}\left[m^2_{(3,1)} \ln\left(\frac{M^2}{\mu^2}\right) + m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right)\right], \qquad (11) \\
(\mu^2_{\tilde{L}})_{ij} &= m^2_{(1,2)}\delta_{ij} - \frac{g_E^{ki} g_E^{kj}}{8\pi^2} m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right), \\
(\mu^2_{\tilde{E}})_{ij} &= -\frac{g_E^{ik} g_E^{jk}}{4\pi^2} m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right),
\end{aligned}
$$

for the scalar partners of quarks and leptons and masses squared

$$
\begin{aligned}
\mu^2_H &= m^2_{(1,2)} - \frac{3 g_D^{ij} g_D^{ij}}{8\pi^2}\left[m^2_{(3,1)} \ln\left(\frac{M^2}{\mu^2}\right) + m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right)\right] \\
&\quad - \frac{4 g_E^{ij} g_E^{ij} + 7 g_T^2}{32\pi^2} m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right), \\
\mu^2_{H'} &= m^2_{(1,2)} - \frac{3 g_U^{*ij} g_U^{ij}}{8\pi^2}\left[m^2_{(3,1)} \ln\left(\frac{M^2}{\mu^2}\right) + m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right)\right] - \frac{7 g_T^2}{32\pi^2} m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right), \qquad (12) \\
\mu^2_T &= \tfrac{8}{3} m^2_{(1,2)} - \frac{7 g_T^2}{48\pi^2} m^2_{(1,2)} \ln\left(\frac{\tilde{M}^2}{\mu^2}\right),
\end{aligned}
$$

* The heavy X scalar field has been integrated out of theory also. The X fermion is the Goldstino.

for the Higgs scalars and the weak triplet T of scalars. The leading corrections to the masses in eqs. (11) and (12) are of order $\alpha^2 \ln^2 (M^2 \text{ or } \tilde{M}^2/\mu^2)$.

For a range of parameters, $m^2_{(3,1)}$ will be roughly an order of magnitude larger than $m^2_{(1,2)}$. This is reasonable since $m^2_{(3,1)}$ is generated by strong interactions, while $m^2_{(1,2)}$ is a weak interaction effect. If this is the case, the colored scalars will have positive masses squared which are sufficiently large that they do not acquire VEVs and may be integrated out of the theory. Since the Higgs mass operators mix with those for colored scalars, the logarithmic corrections to their masses will be substantial. If the mass scale of the O'Raifeartaigh model is of order 100 TeV, the logarithms in eqs. (11) and (12) will be roughly 10 for a subtraction point μ at the weak interaction scale. Taking the Yukawa couplings for the fourth generation to be of order unity, the Higgs masses squared $\mu^2_{\rm H}$ and $\mu^2_{\rm H'}$ will be negative and, for a range of parameters, of order $-(100\ \text{GeV})^2$. Since the mass operators for L_i and T do not mix with those for colored scalars, these scalars will have positive masses squared, while E^c_i will have small negative masses squared.

We have seen that radiative corrections can provide negative masses squared of order $-(100\ \text{GeV})^2$ for the Higgs scalars. These mass terms will force H and H' to acquire VEVs, driving the breakdown of the weak interaction gauge symmetry. The VEVs for H and H' are determined by the minimum of the scalar potential

$$\begin{aligned} V = {} & |g_T H \boldsymbol{\tau} H'|^2 + |g_T H' \boldsymbol{\tau} \cdot \boldsymbol{T} + g^{ij}_E E^c_i L_j|^2 + |g_T H \boldsymbol{\tau} \cdot \boldsymbol{T}|^2 + |g^{ij}_E L_j H|^2 + |g^{ij}_E E^c_i H|^2 \\ & + \tfrac{1}{2} g_2^2 [\mathrm{H}^* \boldsymbol{\tau} H + H'^* \boldsymbol{\tau} H' + L^*_i \boldsymbol{\tau} L_i - i \boldsymbol{T}^* \times \boldsymbol{T}]^2 \\ & + \tfrac{1}{2} g_1^2 [\tfrac{1}{2}|H|^2 - \tfrac{1}{2}|H'|^2 + \tfrac{1}{2}|L_i|^2 - |E^c_i|^2]^2 \\ & + \mu^2_H |H|^2 + \mu^2_{H'} |H'|^2 + \mu^2_T |\boldsymbol{T}|^2 + L^*_i (\mu^2_L)_{ij} L_j + E^{c*}_i (\mu^2_E)_{ij} E^c_j \,. \end{aligned} \tag{13}$$

Again flavor indices not repeated within a modulus sign are summed after squaring. Here the mass squares for H, H', and E^c_i are negative, while those for L_i and T are positive. If the conditions

$$\begin{gathered} \tfrac{1}{2} g_T^2 < g_2^2 \min\{1, 4\sin^2\theta_{\rm W}\} \,, \\ -\frac{\mu^2_H + \mu^2_{H'}}{\tfrac{1}{2} g_T^2} > \frac{\mu^2_H - \mu^2_{H'}}{g_2^2 + g_1^2 - \tfrac{1}{2} g_T^2} > -\frac{\mu^2_E}{g_1^2} \,, \\ \mu^2_L > -\mu^2_{H'} \,, \qquad \mu^2_T > -2\mu^2_{H'} \end{gathered} \tag{14}$$

are satisfied, the minimum occurs for $L_i = E^c_i = \boldsymbol{T} = 0$ and

$$H = \begin{pmatrix} 0 \\ h \end{pmatrix}, \qquad H' = \begin{pmatrix} h' \\ 0 \end{pmatrix}. \tag{15}$$

The magnitudes of these VEVs are

$$|h|^2 = -\frac{\mu_H^2 + \mu_{H'}^2}{\frac{1}{2}g_T^2} - \frac{\mu_H^2 - \mu_{H'}^2}{g_2^2 + g_1^2 - \frac{1}{2}g_T^2}, \tag{16}$$

$$|h'|^2 = -\frac{\mu_H^2 + \mu_{H'}^2}{\frac{1}{2}g_T^2} + \frac{\mu_H^2 - \mu_{H'}^2}{g_2^2 + g_1^2 - \frac{1}{2}g_T^2}.$$

In eq. (14), μ_L^2 and μ_E^2 are the smallest eigenvalues of $(\mu_L^2)_{ij}$ and $(\mu_E^2)_{ij}$, respectively, and $\sin^2\theta_W = g_1^2/(g_2^2 + g_1^2)$ is the weak mixing parameter. We note that while the conditions of eq. (14) are sufficient for the minimum to be given by eqs. (15) and (16), they are not necessary for this to be the minimum. The origin of eqs. (14)–(16) is briefly discussed in appendix B. Note that although the E_i^c have negative masses squared in eq. (13), these fields receive no VEV's provided conditions (14) are satisfied. This is because the VEV's of H and H' give these fields positive masses squared.

Although the conditions of eq. (14) are satisfied for a range of parameters, they are not without phenomenological implications. In particular, to make μ_H^2 and $\mu_{H'}^2$ negative, the Yukawa constants for the fourth generation must be large. Since we need $\mu_H^2 > \mu_{H'}^2$, the magnitude of the Yukawa coupling for the charge $\frac{2}{3}$ quark of this generation must be greater than that of the charge $-\frac{1}{3}$ quark. In addition, this requires the magnitude of the VEV of H' to be greater than that of H. Together these imply that the charge $\frac{2}{3}$ quark must be heavier than the charge $-\frac{1}{3}$ quark within this generation. The remaining conditions partially restrict the masses of the particles in the fourth generation both from above and below.

Finally we note that this model has stable fractionally charged hadrons that are bound states of quarks or their scalar partners with the exotic colored fields. These bound states will have masses of order 100 TeV. The existence of fractionally charged matter is avoided only if the representation r_3 has zero triality.

3. Concluding remarks

Global $N = 1$ supersymmetry is a powerful symmetry that constrains field theories in a highly non-trivial way. It is interesting to see whether this global symmetry can be realized in nature down to low energies where experimental information will be available. In this paper we have addressed this question. Sect. 2 contained a toy model where supersymmetry is broken spontaneously at a mass scale of order 100 TeV. Once supersymmetry is broken the scalar partners of quarks and leptons are no longer protected from getting a mass. In the toy model these scalars acquired masses through radiative corrections. The breakdown of the weak interactions is also driven by radiative corrections so the masses of the scalar partners of quarks and leptons are naturally of order 100 GeV.

The model of sect. 2 is not realistic. For example, the superspace potential contains an R symmetry [13] under which the gluino transforms non-trivially. A linear combination of this anomalous "symmetry" and the usual anomalous U(1) flavor "symmetry" does not have a color anomaly. Therefore the model has a U(1) problem, i.e. it predicts an additional light neutral pseudoscalar meson [14]. In addition, the R symmetry protects some of the charged fermions from acquiring a mass. Also, this model has a Peccei–Quinn symmetry [15] and, therefore, an axion [16]. There is strong experimental evidence against the existence of an axion [17].

Much of the recent work on supersymmetry at low energies has been motivated by the hierarchy puzzle [2, 4, 6, 18, 19]. Our goal has been more modest. Assuming that a and b are of order unity and $M \simeq \tilde{M} \simeq \mu$, the fundamental mass scale in the toy model is 100 TeV. We have not attempted to explain why this mass scale is so small compared with the Planck (or perhaps unification) mass. However, it is worth noting that a weak interaction scale of 100 GeV also emerges for $\mu \simeq 10^{10}$ GeV* and $M \simeq \tilde{M} \simeq 10^{15}$ GeV with a and b still of order unity.

Similar work is being done by M. Dine and W. Fishler and by C. Nappi and B. Ovrut. We are grateful to them for telling us about their work prior to its publication. M.B.W. thanks E. Witten and P. Ginsparg for interesting discussions.

We thank H. Georgi for pointing out the existence of fractionally charged matter in our model and for stressing that a very light gluino can give rise to a U(1) problem.

Appendix A

TWO-LOOP CALCULATION OF MASSES

Here we compute the two-loop graphs contributing to the scalar masses. For simplicity we use a model where the fields in the O'Raifeartaigh part of the superspace potential,

$$W = X(aJ_1K_1^c - \mu^2) + M_1J_1K_2^c + M_2J_2K_1^c\,, \tag{A.1}$$

have hypercharge of unit magnitude, but are SU(3)×SU(2) singlets. We put $M_1 = M_2 = M$ since then there is a charge conjugation symmetry which forces the cancellation of one-loop U(1) D-term contributions to the scalar masses. The only difference between the calculation presented here and the case treated in the text is a trivial group theory factor.

Since we are calculating the leading contribution to the masses of scalars not participating in the O'Raifeartaigh model the one-particle-irreducible parts of the

* Here μ is the parameter in the O'Raifeartaigh model, not the subtraction point.

scalar two-point functions at zero external momenta give gauge-invariant masses. No wave-function renormalization is required. We work in the Landau guage where only the graphs in fig. 5 contribute.

The J_i and K_i^c superfields contain fermion fields with mass M, two scalar fields with masses squared $M_\pm^2 = M^2 \pm a\mu^2$ and two scalar fields with masses squared M^2. The breaking of supersymmetry is reflected by the splitting between the fermions and their scalar partners.

Let us first verify that the masses are zero is supersymmetry is not broken, i.e. $\mu^2 = 0$. In this case (5a) + (5b) give

$$(5\mathrm{a}) + (5\mathrm{b}) = -ig_1^2 \int \frac{\mathrm{d}^n p}{(2\pi)^n} \frac{1}{(p^2)^2} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \pi^{\mu\nu}(p) . \tag{A.2}$$

In (A.2) $\pi^{\mu\nu}(p)$ denotes the one-loop vacuum polarization of the B. This is easily computed and we obtain

$$(5\mathrm{a}) + (5\mathrm{b}) = 12ig_1^4 \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^n p}{(2\pi)^n} \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{1}{p^2[k^2 + x(1-x)p^2 - M^2]^2} . \tag{A.3}$$

The contribution from bino (b) exchange is

$$(5\mathrm{d}) = -32ig_1^4 \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^n p}{(2\pi)^n} \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{x}{p^2[k^2 + x(1-x)p^2 - M^2]^2} . \tag{A.4}$$

Finally, the U(1) D-term contribution is

$$(5\mathrm{c}) = 4ig_1^4 \int_0^1 \mathrm{d}x \int \frac{\mathrm{d}^n p}{(2\pi)^n} \int \frac{\mathrm{d}^n k}{(2\pi)^n} \frac{1}{p^2[k^2 + x(1-x)p^2 - M^2]^2} . \tag{A.5}$$

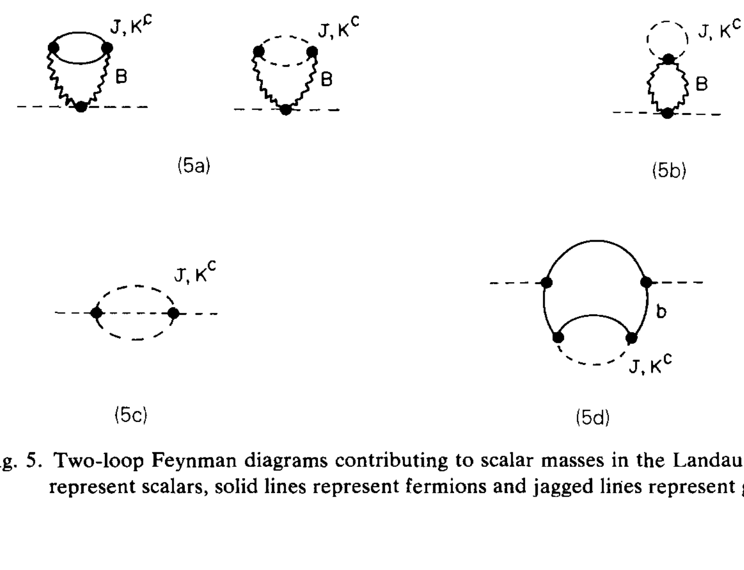

Fig. 5. Two-loop Feynman diagrams contributing to scalar masses in the Landau gauge. Dashed lines represent scalars, solid lines represent fermions and jagged lines represent gauge bosons.

Summing these

$$(5a)+(5b)+(5c)+(5d)=16ig_1^4\int_0^1 dx\,(1-2x)\int\frac{d^np}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n}\times\frac{1}{p^2[k^2+x(1-x)p^2-M^2]^2}, \tag{A.6}$$

which vanishes since the integrand is odd under the change of variables $x\to 1-x$. It is worth noting that we are using supersymmetric dimensional regularization [20], meaning that Dirac and Lorentz algebra is performed in 4-dimensions before the remaining "scalar" integrals are continued to $n=4-2\varepsilon$ dimensions.

When supersymmetry is broken, we need only take into account the contributions from the scalars with masses $M_\pm^2$ and their fermion partners. Let us start with (5c):

$$(5c)=2(ig_1^2)^2\int\frac{d^np}{(2\pi)^2}\int\frac{d^nk}{(2\pi)^n}\frac{i^3}{p^2[(k+p)^2-M_-^2][k^2-M_+^2]}. \tag{A.7}$$

Combining the denominators and Wick rotating,

$$(5c)=2ig_1^4\int_0^1 dx\int\frac{d^np}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n}\frac{1}{p^2[k^2+x(1-x)p^2+xM_-^2+(1-x)M_+^2]^2}. \tag{A.8}$$

Next we rescale p by a factor $[x(1-x)]^{1/2}$ and combine the remaining denominators introducing a new Feynman parameter y:

$$(5c)=4ig_1^4\int_0^1 dx\,[x(1-x)]^{1-n/2}\int_0^1 dy\,y\times\int\frac{d^np}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n}\frac{1}{[p^2+yk^2+y(xM_-^2+(1-x)M_+^2)]^3}. \tag{A.9}$$

Rescaling the k variable by $[y]^{1/2}$, we obtain an expression which is the same as a one-loop integral in a $2n$-dimensional space. Defining the $2n$-dimensional momentum vector $K=(k,p)$, performing the K integration and a trivial y integration gives

$$(5c)=\frac{2ig_1^4\Gamma(3-n)}{(4\pi)^n(\frac{1}{2}n-1)}\int_0^1 dx\,[x(1-x)]^{1-n/2}[xM_-^2+(1-x)M_+^2]^{n-3}. \tag{A.10}$$

Applying the same tricks to (5a)+(5b) yields

$$\begin{aligned}(5a)+(5b)&=3ig_1^4\int_0^1 dx\,x(1-x)\int\frac{d^np}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n}\frac{8}{p^2[k^2+x(1-x)p^2-M^2]^2}\\&\quad+3ig_1^4\sum_{i\in\{+,-\}}\int_0^1 dx\,(1-2x)^2\int\frac{d^np}{(2\pi)^n}\int\frac{d^nk}{(2\pi)^n}\frac{1}{p^2[k^2+x(1-x)p^2-M_i^2]^2}\\&=\frac{3ig_1^4\Gamma(3-n)}{(4\pi)^n(n/2-1)}\int_0^1 dx\left\{8x(1-x)[M^2]^{n-3}+(1-2x)^2\sum_{i\in\{+,-\}}[M_i^2]^{n-3}\right\}\\&\quad\times[x(1-x)]^{1-n/2}.\end{aligned} \tag{A.11}$$

Finally (5d) gives

$$(5\mathrm{d}) = -\frac{8i\Gamma(3-n)}{(4\pi)^n(\frac{1}{2}n-1)}g_1^4 \sum_{i\in\{+,-\}} \int_0^1 \mathrm{d}x\, x[x(1-x)]^{1\times n/2}[(1-x)M^2+xM_i^2]^{n-3}\,. \tag{A.12}$$

The remaining integrals are hypergeometric functions

$$F(a,b,c;y) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)}\int_0^1 \mathrm{d}t\, t^{b-1}(1-t)^{c-b-1}(1-yt)^{-a}\,. \tag{A.13}$$

Expanding in $\varepsilon = \frac{1}{2}(4-n)$ and then in $z = a\mu^2/M^2$ gives the mass squared for scalars with unit hypercharge

$$\frac{g_1^4}{16\pi^2}\frac{M^2z^2}{4\pi^2} + \mathrm{O}(z^4)\,. \tag{A.14}$$

Appendix B

MINIMIZATION OF THE HIGGS POTENTIAL

In this appendix, we briefly discuss the origin of eqs. (14) to (16) by minimizing the potential

$$\begin{aligned}\tilde{V} &= \tfrac{1}{4}g_T^2|H|^2|H'|^2 + \frac{1}{2g_2^2}|\boldsymbol{D}|^2 + \frac{1}{2g_1^2}D^2 \\ &\quad + \mu_H^2|H|^2 + \mu_{H'}^2|H'|^2 + \mu_T^2|\boldsymbol{T}|^2 + \mu_L^2|L_i|^2 + \mu_E^2|E_i^c|^2\,. \end{aligned}\tag{B.1}$$

Here $\boldsymbol{D}$ and D are the SU(2) and U(1) D-terms

$$\begin{aligned}\boldsymbol{D} &= g_2^2[H^*\boldsymbol{\tau}H + H'^*\boldsymbol{\tau}H' + L_i^*\boldsymbol{\tau}L_i - i\boldsymbol{T}^*\times\boldsymbol{T}]\,, \\ D &= g_1^2[\tfrac{1}{2}|H|^2 - \tfrac{1}{2}|H'|^2 + \tfrac{1}{2}|L_i|^2 - |E_i^c|^2]\,. \end{aligned}\tag{B.2}$$

Also, μ_L^2 and μ_E^2 are the smallest eigenvalues of $(\mu_L^2)_{ij}$ and $(\mu_E^2)_{ij}$, so that the difference between V [eq. (13)] and $\tilde{V}$ is positive semidefinite and vanishes for the VEVs of eqs. (15) and (16). Thus, if these VEVs minimize $\tilde{V}$, they will also minimize V. We will assume μ_E^2 to be negative, but make no assumptions regarding the signs of the other mass terms. [See eqs. (11) and (12).] It is easier to minimize $\tilde{V}$, than V, partly because $\tilde{V}$ contains a U(4)×U(4) "flavor" symmetry acting on the scalar partners of the lepton doublets and lepton singlets.

Prior to minimizing $\tilde{V}$, we must first ensure that it is bounded below*. Since the quartic terms of $\tilde{V}$ are positive semidefinite, $\tilde{V}$ is automatically bounded below in

* Of course, the scalar potential of any supersymmetric model must be bounded below for any values of the parameters. However, this need not be the case for $\tilde{V}$, since we have dropped positive semidefinite terms in passing from V to $\tilde{V}$. The potential V may also fail to be bounded below for some values of the parameters since, in computing the radiative corrections to V, we have dropped all terms of dimension greater than two. See first footnote after eq. (10).

any direction for which the quartic terms do not vanish. By separately considering each direction for which the quartic terms vanish, we find that the conditions

$$\mu_T^2 > |2\mu_H^2 + \mu_E^2|\,, \qquad \mu_L^2 > \max\{-\mu_{H'}^2, \mu_H^2, -\mu_H^2 - \mu_E^2\} \tag{B.3}$$

are sufficient for $\tilde{V}$ to be bounded in the directions for which the quartics vanish as well.

Since $\tilde{V}$ is now bounded below, we may find all stationary points of $\tilde{V}$ by solving the equations

$$\begin{aligned}
\frac{\partial \tilde{V}}{\partial \boldsymbol{T}^*} &= i\boldsymbol{D} \times \boldsymbol{T} + \mu_T^2 \boldsymbol{T} = 0\,, \\
\frac{\partial \tilde{V}}{\partial L_i^*} &= [\boldsymbol{D} \cdot \boldsymbol{\tau} + \tfrac{1}{2}D + \mu_L^2] L_i = 0\,, \\
\frac{\partial \tilde{V}}{\partial H^*} &= [\boldsymbol{D} \cdot \boldsymbol{\tau} + \tfrac{1}{2}D + \mu_H^2 + \tfrac{1}{4}g_T^2 |H'|^2] H = 0\,, \\
\frac{\partial \tilde{V}}{\partial H'^*} &= [\boldsymbol{D} \cdot \boldsymbol{\tau} - \tfrac{1}{2}D + \mu_{H'}^2 + \tfrac{1}{4}g_T^2 |H|^2] H' = 0\,, \\
\frac{\partial \tilde{V}}{\partial E_i^{c*}} &= [-D + \mu_E^2] E_i^c = 0\,.
\end{aligned} \tag{B.4}$$

Since these equations are SU(2) covariant, we may choose $\boldsymbol{D} = D_z \hat{\boldsymbol{z}}$ with $D_z \geqslant 0$. Assuming $\boldsymbol{T} \neq 0$, we must have $D_z = \mu_T^2$ and $\boldsymbol{T} = \sqrt{\tfrac{1}{2}}\,|\boldsymbol{T}|(1, i, 0)$. Since $D_z \neq 0$ in this case, it is clear that the doublets L_i, H, and H' can each have at most one non-zero component in this basis and that all of the L_i doublets must be aligned. By manipulating eqs. (B.4) for this case, one can show that

$$g_2^2 > g_1^2\,, \qquad \mu_T^2 > -2\mu_{H'}^2 \tag{B.5}$$

together with (B.3) are sufficient conditions for $\tilde{V}$ to have no stationary points with $\boldsymbol{T} \neq 0$. In a similar manner, we find that, if we impose the additional conditions

$$\tfrac{1}{2}g_T^2 < \min\{g_2^2 + g_1^2, 4g_2^2 \sin^2\theta_W\}\,, \qquad \mu_H^2 > \mu_{H'}^2\,, \tag{B.6}$$

then $\tilde{V}$ will have no stationary points with one or more of the L_i doublets non-zero.

For the remaining case ($\boldsymbol{T} = 0$ and $L_i = 0$ for all i), we may write

$$\begin{aligned}
\hat{V} = {} & \tfrac{1}{2}g_2^2 |H^* H'|^2 + \tfrac{1}{16}g_T^2 \left| |H|^2 + |H'|^2 + 2\,\frac{\mu_H^2 + \mu_{H'}^2}{\tfrac{1}{2}g_T^2} \right|^2 \\
& + \tfrac{1}{8}(g_2^2 + g_1^2 - \tfrac{1}{2}g_T^2) \left[|H|^2 - |H'|^2 - \frac{2g_1^2}{g_2^2 + g_1^2 - \tfrac{1}{2}g_T^2} |E_i^c|^2 + 2\,\frac{\mu_H^2 - \mu_{H'}^2}{g_2^2 + g_1^2 - \tfrac{1}{2}g_T^2} \right]^2 \\
& + \frac{1}{2}\,\frac{g_1^2(g_2^2 - \tfrac{1}{2}g_T^2)}{g_2^2 + g_1^2 - \tfrac{1}{2}g_T^2} (|E_i^c|^2)^2 + g_1^2 \left[\frac{\mu_E^2}{g_1^2} + \frac{\mu_H^2 - \mu_{H'}^2}{g_2^2 + g_1^2 - \tfrac{1}{2}g_T^2} \right] |E_i^c|^2 + \text{const}\,.
\end{aligned} \tag{B.7}$$

This leads to the VEVs of eqs. (15) and (16), provided that

$$\tfrac{1}{2}g_T^2 < g_2^2\,, \qquad -\frac{\mu_H^2+\mu_{H'}^2}{\frac{1}{2}g_T^2} > \frac{\mu_H^2-\mu_{H'}^2}{g_2^2+g_1^2-\frac{1}{2}g_T^2} > -\frac{\mu_E^2}{g_1^2}\,. \tag{B.8}$$

Eliminating the redundancy between eqs. (B.3), (B.5), (B.6) and (B.8) provides the conditions given in the text [eq. (14)].

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