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SUPERSYMMETRIC RELICS FROM THE BIG BANG*

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We consider the cosmological constraints on supersymmetric theories with a new, stable particle. Circumstantial evidence points to a neutral gauge/Higgs fermion as the best candidate for this particle, and we derive bounds on the parameters in the lagrangian which govern its mass and couplings. One favored possibility is that the lightest neutral supersymmetric particle is predominantly a photino $\tilde{\gamma}$ with mass above $\frac{1}{2}$ GeV, while another is that the lightest neutral supersymmetric particle is a Higgs fermion with mass above 5 GeV or less than O(100) eV. We also point out that a gravitino mass of 10 to 100 GeV implies that the temperature after completion of an inflationary phase cannot be above 10^{14} GeV, and probably not above 3×10^{12} GeV. This imposes constraints on mechanisms for generating the baryon number of the universe.

1. Introduction

In the past few years, supersymmetric extensions of the standard model of particle physics have received a great deal of attention from theorists [1]. All these models predict an exciting variety of new particles. Alas, there are few experimental constraints on their masses and couplings, particularly on those of new neutral particles. To guide future work, both experimental and theoretical, we must learn all we can about the parameters of these theories. Here, as in many other cases, useful information can be obtained from a study of early cosmology. (For a review of this subject, see ref. [2].)

The essential feature of supersymmetric theories which makes them amenable to cosmological study is that, in many models, one of the new particles is absolutely stable. By "new," we mean the superpartners of any ordinary particles: gauge

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fermions (gluino, photino, wino, etc.), Higgs fermions (shiggs or higgsino), scalar quarks and leptons (squarks and sleptons), and the gravitino. In the very early universe, all these particles would be present in thermal equilibrium. As the temperature falls, the heavier new particles decay into the lighter ones. Eventually, only the lightest supersymmetric particle (LSP) will be left. It can disappear only by pair annihilation. We must require that this pair annihilation is efficient enough to reduce the present day number density of the LSP to an acceptable level.

In this paper we give a detailed analysis of this question for the class of models which we favor: a minimal supersymmetric extension of the standard model, with supersymmetry broken via gravitational couplings to spontaneously broken supergravity [3]. This results in mass terms for the new particles and cubic scalar couplings which violate supersymmetry [4]. The mass scale which enters is the gravitino mass $m_{3/2}$, which we take to be O(10 to 100) GeV. Some relations among the masses of the new particles can be obtained by assuming that this theory is the low-energy sector of a supersymmetric grand unified theory [5]. We will make this assumption where necessary but many of our considerations are more general than the specific supersymmetric model which we emphasize.

In sect. 2 we discuss each of the new particles, and whether or not it could be the LSP. We conclude that by far the best candidate for the LSP is a neutral gauge/Higgs fermion (in general, these particles mix so that mass eigenstates contain both gauge fermion and Higgs fermion components).

In sect. 3 we present a detailed analysis of the annihilation rates of such particles so as to provide bounds on the parameters in the lagrangian. Some preliminary results of this analysis have already been given in ref. [6]. One favoured possibility is that the LSP is mostly a photino, and has a mass $\geqslant \mathrm{O}(\frac{1}{2})$ GeV. Alternatively, the LSP may be mostly a shiggs which either has mass $\gtrsim m_{\mathrm{b}}$ or m_{t}, or else weighs less than O(100) eV. The simplest models with such a light shiggs also contain a light axion which would need to be exorcised in a completely realistic model.

2. Which is the LSP?

Many LSP candidates can be ruled out immediately by simple arguments. Let us consider each new particle in turn, and whether or not it could be the LSP.

Charged, uncolored particles: winos, charged higgsinos, charged sleptons (selectron, smuon, stau). The possibility of a stable, charged particle was studied by Wolfram [7], who concluded that there should be a number density today of about

$$n \simeq 10^{-6}\left(\frac{M}{1\ \mathrm{GeV}}\right)n_{\mathrm{B}}, \tag{2.1}$$

where M is the mass of the charged particle, and n_{B} is the baryon-number density today. These particles would be well mixed with ordinary matter; their electromagnetic interactions ensure that there is no way for them to separate from the matter which forms galaxies, stars, and planets. In particular, we should find such particles in terrestrial searches for anomalously heavy protons [7]. The results of ref. [8] tell us that for $M < 350$ GeV, there are no anomalous protons at a level $< 10^{-21} n_B$, well below that of eq. (2.1). Thus, a stable charged particle with $M < 350$ GeV can be excluded.

Gluino. A gluino should form new hadrons with strong interactions; at least one of these new hadrons must be stable if the gluino is the LSP. Calculations of the relic number density of heavy hadrons [7, 9] yield

$$n \simeq 10^{-10} n_B, \tag{2.2}$$

independent of the new hadron's mass. If there is a stable, charge one hadron, its existence can be ruled out by the anomalous proton search [8] discussed above. If the only stable new hadrons are neutral, this argument does not apply. We expect that these hadrons are today bound to ordinary nuclei, but it is not clear which nuclei. Surely the precise nuclear interaction rates inside stars which determine the present day abundance of elements are different for nuclei with new hadrons stuck to them. Thus, although we know [10] that there are no anomalous isotopes of oxygen with mass less than 44 GeV at a level $n \simeq 10^{-17} n_B$, it is not entirely clear that this means that a 20 GeV gluino could not be the LSP. We do at least know from accelerator experiments [11] that the gluino mass exceeds 2 GeV.

In the absence of firm cosmological evidence against the gluino as the LSP, we turn to theoretical arguments. Neglecting mixing with Higgs fermions, the gluino-photino mass ratio is given by [5]

$$m_{\tilde{g}}/m_{\tilde{\gamma}} = \frac{3}{8} \frac{\alpha_3}{\alpha_2 \sin^2\theta_w}, \tag{2.3}$$

to leading order in the renormalization group equations in any grand unified theory. Even after including arbitrary photino/higgsino mixing, the gluino is always heavier. Thus, in any susy GUT, the gluino is not the LSP.

Squarks. The same discussion is that for gluinos applies to searches for squarks in anomalous nuclei: they are inconclusive. Accelerator searches [12] tell us $M \gtrsim 17$ GeV.

Stable squarks might carry some of the baryon number of the universe, in which case they could be present at densities well above $10^{-10} n_B$. The reason is that, in GUTs, the cosmological baryon asymmetry (CBA) is produced at a temperature of at least 10^{10} GeV, when there was no distinction between squarks and quarks. At this time, half the CBA was stored in squarks. To decide if it is still there today, we must compute the reaction rate for the process squark + squark $\to$ quark + quark. The dominant contribution is from gluino exchange, which gives a cross section times relative velocity of order

$$\langle \sigma v_{\text{rel}} \rangle \simeq \alpha_3^2 / m_{\tilde{g}} m_s, \tag{2.4}$$

where m_s is the squark mass. Assuming that both $m_{\tilde{g}}$ and m_s are no larger than a few hundred GeV, standard estimates [7,9] tell us that the annihilation rate is large enough to reduce the excess squark density by several orders of magnitude. Thus the CBA flows from the squarks to the quarks, and the resulting relic squark density is acceptable.

Again we must turn to theoretical arguments. Models almost always predict [5] that squarks are heavier than sleptons, due to renormalization effects. Although this argument is less airtight than in the gluino case, we think it unlikely that a squark would be the LSP, and so we will not pursue this possibility.

Scalar neutrinos. There is no [13] cosmological argument against a sneutrino as the LSP.

Theoretically, we expect the sneutrino to be nearly degenerate with the scalar partner of the left-handed charged lepton in the same SU(2) weak doublet, but with the mass eigenvalues of the charged slepton $\tilde{\ell}$ split on either side of the sneutrino $\tilde{\nu}$ mass by an amount proportional to the mass of the charged lepton:

$$m_{\tilde{\ell}} = m_{\tilde{\nu}} \pm a m_{\ell}, \tag{2.5}$$

where a is a model-dependent constant. Thus, for example, we expect that there is a charged stau with a mass O(2) GeV below that of the tau sneutrino (and, indeed, all sneutrinos). This pattern of masses might be rearranged by electromagnetic radiative corrections in some models, so that it may be possible to arrange a sneutrino as the LSP. This possibility is discussed in ref. [13]; we will disregard it here.

Gravitino. The gravitino is a special case. Even if it is not the LSP, it decays so slowly that the LSPs in its decay products provide limits on the initial density of gravitinos [14].

As discussed by Weinberg [14], a 10 to 100 GeV gravitino cannot be the LSP – it annihilates much too slowly, with a cross section of order $1/M_p^2$, where $M_p = 1.2 \times 10^{19}$ GeV. A 10 to 100 GeV gravitino which is not the LSP would decay after nucleosynthesis; each gravitino would yield one LSP, but it would be much too cold for these LSPs to annihilate. The only apparent solution is inflation – this would remove the original number density of gravitinos (and all other particles); the weak coupling of gravitinos can prevent regenerating a large number density of them after inflation [15].

Weinberg has recently pointed out [16] that a process like photon + anything → gravitino + anything has a cross section of $\alpha N / M_p^2$, where N is the number of degrees of freedom. In thermal equilibrium, this means a production rate Γ of order [17]

$$\Gamma \simeq \alpha N T_R^3 / M_p^2, \tag{2.6}$$

at a temperature T_R, the temperature of the universe once it reheated after inflation. Γ is to be compared with the Hubble parameter at that time

$$H \simeq \sqrt{N}\, T_R^2/M_p \,, \tag{2.7}$$

in order to get a relative gravitino abundance

$$Y_{3/2} \simeq \Gamma/H \tag{2.8}$$

$$\simeq \alpha\sqrt{N}\, T_R/M_p \,. \tag{2.9}$$

Subsequent to the establishment of this primordial abundance, the photons in the microwave background reheated as the N degrees of freedom present at T_R annihilated. So, long after T_R but just before gravitino decay, the relative number density of gravitinos was depressed to

$$\frac{n_{3/2}}{n_\gamma} \simeq \frac{2Y_{3/2}}{N} \tag{2.10}$$

$$\simeq 2\alpha T_R/\sqrt{N}\, M_p \,. \tag{2.11}$$

Each of these gravitinos yield an LSP as it decays and these LSPs would still be around today. In order that they not dominate the mass of the universe, we must require (see also ref. [18])

$$M n_{3/2} < \rho \,, \tag{2.12}$$

where ρ is the upper limit on the mass density of the universe. This cannot be used to bound M, since we do not know T_R. But using the bounds on M from sect. 3, we can get an upper limit on T_R:

$$T_R \lesssim \frac{\sqrt{N}\, \rho M_p}{2\alpha M n_\gamma} \,. \tag{2.13}$$

Using $N = 250$, $\alpha = \frac{1}{25}$, $n_\gamma = 400\,/\mathrm{cm}^3$, and the very conservative numbers (see sect. 3) $\rho = 2 \times 10^{-29}\ \mathrm{g/cm^3}$, $M = \frac{1}{2}$ GeV, we get

$$T_R \lesssim 10^{14}\ \mathrm{GeV} \,, \tag{2.14}$$

while the more realistic numbers $\rho = 2 \times 10^{-30}\ \mathrm{g/cm^3}$ and $M = 2$ GeV give

$$T_R \lesssim 3 \times 10^{12}\ \mathrm{GeV} \,. \tag{2.15}$$

Thus, an inflationary solution to the gravitino problem cannot have a very high reheating temperature*. Since the cosmological baryon asymmetry must be produced after inflation this constraint necessitates the existence of baryon-number violating particles with masses no larger than 10^{14} GeV, and more likely less than 10^{13} GeV.

Neutral gauge and Higgs fermions. These are the only remaining LSP candidates. There are no arguments – cosmological, experimental, or theoretical – against one of these particles being the LSP, and this is the case on which we will concentrate in sect. 3.

3. Neutral gauge and Higgs fermions

3.1. MASSES AND MIXING

Here we consider the likely possibility that the lightest susy particle is a colorless, neutral fermion. We start by reviewing the structure of the charged and neutral susy fermion mass matrices [6]. We consider a minimal susy model with two light doublets of Higgs chiral superfields H_1 and H_2 of weak hypercharge ± 1 respectively. The mass matrices for the charged and neutral susy fermions – gauginos and higgsinos – are determined by the lagrangian terms

$$L \ni \varepsilon\varepsilon_{ij}\tilde{H}_1^i\tilde{H}_2^j - M_2\tilde{W}_a\tilde{W}_a - M_1\tilde{B}\tilde{B}, \tag{3.1}$$

where W_a and B denote SU(2) and U(1) gauge superfields respectively, the tildes denote fermionic components and i, j (a) are doublet (triplet) SU(2) indices. We generally expect the parameters M_1, M_2 to be $\mathrm{O}(M_\mathrm{W})$, while it is conceivable that ε might be very much smaller. We shall assume

$$M_1 = \frac{5}{3}\frac{\alpha_1}{\alpha_2}M_2, \tag{3.2}$$

where $\alpha_i = g_i^2/4\pi$, $i = 1, 2, 3$ are the gauge coupling constants, which holds to leading order in the renormalization group equations if $\mathrm{SU(2)} \times \mathrm{U(1)}$ is eventually embedded in a unifying non-abelian group. When combined with the conventional Higgs-gauge field couplings the full mass matrix for the left-handed charged fermion fields becomes

$$(\tilde{W}^+, \quad \tilde{H}_1^+)\begin{pmatrix} M_2 & g_2 v_2 \\ g_2 v_1 & -\varepsilon \end{pmatrix}\begin{pmatrix} \tilde{W}^- \\ \tilde{H}_2^- \end{pmatrix}, \tag{3.3}$$

* Eq. (2.13) does not constrain T_R if the LSP is a 100 eV higgsino (see subsect. 3.4). In this case, we can still derive a similar bound on T_R by requiring that the entropy produced by gravitino decay be acceptably small [17].

where $\langle 0|H^0_{1,2}|0\rangle = v_{1,2}$; $m_W^2 = \frac{1}{2}g_2^2(v_1^2+v_2^2)$. This matrix is diagonalized by rotations through angles $\theta_\pm$ among the positively and negatively charged fields respectively, where

$$\tan\theta_\pm = \frac{b_\pm + \sqrt{b_\pm^2 + 4a_\pm}}{2a_\pm}, \tag{3.4a}$$

with

$$a_\pm = \begin{cases} M_2 g_2 v_1 - \varepsilon g_2 v_2 \\ M_2 g_2 v_2 - \varepsilon g_2 v_1 \end{cases}, \qquad b_\pm = M_2^2 - \varepsilon^2 \pm g_2^2\left(v_2^2 - v_1^2\right). \tag{3.4b}$$

The charged fermion masses are

$$\begin{aligned} m_1 &= M_2\cos\theta_+\cos\theta_- - g_2v_2\cos\theta_+\sin\theta_- - g_2v_1\sin\theta_+\cos\theta_- - \varepsilon\sin\theta_+\sin\theta_-, \\ m_2 &= M_2\sin\theta_+\sin\theta_- + g_2v_2\sin\theta_+\cos\theta_- + g_2v_1\cos\theta_+\sin\theta_- - \varepsilon\cos\theta_+\cos\theta_-. \end{aligned} \tag{3.5}$$

Fig. 1 shows mass contours for the lightest charged fermion $\chi^\pm$ for certain ranges of the parameters (M_2, ε) and representative values of v_1/v_2. Since H_1 gives masses to the charge-$\frac{2}{3}$ quarks and $m_c \gg m_s$, $m_t \gg m_b$, it seems plausible that $v_1 \geqslant v_2$. In figs. 1a, c, e, g (1b, d, f, h) v_1/v_2 takes the values 1, 2, 4, 8 while the sign of ε is taken positive (negative). We see that in much of this parameter space there is a light $\chi^\pm$ with $m_{\chi^\pm} < m_W$ and often even $m_{\chi^\pm} < 20$ GeV. The $m_{\chi^\pm} = 20$ GeV contour serves as a constraint on the parameters M_2 and ε since no new charged fermion has been seen in e^+e^- collisions with a mass less than about 20 GeV. We shall take this constraint seriously, although it has been argued [19] that a charged susy fermion with a mass somewhat lighter than 20 GeV might still have escaped detection.

In the neutral sector there are four susy fermions which mix, namely the $\tilde{W}^3$, $\tilde{B}^0$, $\tilde{H}_1^0$ and $\tilde{H}_2^0$. Their mixing matrix is [6]

$$\left(\tilde{W}^3, \tilde{B}^0, \tilde{H}_1^0, \tilde{H}_2^0\right) \begin{pmatrix} M_2 & 0 & -\sqrt{\frac{1}{2}}\,g_2v_1 & \sqrt{\frac{1}{2}}\,g_2v_2 \\ 0 & \frac{5}{3}\frac{\alpha_1}{\alpha_2}M_2 & \sqrt{\frac{1}{2}}\,g_1v_1 & -\sqrt{\frac{1}{2}}\,g_1v_2 \\ -\sqrt{\frac{1}{2}}\,g_2v_1 & \sqrt{\frac{1}{2}}\,g_1v_1 & 0 & \varepsilon \\ \sqrt{\frac{1}{2}}\,g_2v_2 & -\sqrt{\frac{1}{2}}\,g_1v_2 & \varepsilon & 0 \end{pmatrix} \begin{pmatrix} \tilde{W}^3 \\ \tilde{B}^0 \\ \tilde{H}_1^0 \\ \tilde{H}_2^0 \end{pmatrix}, \tag{3.6}$$

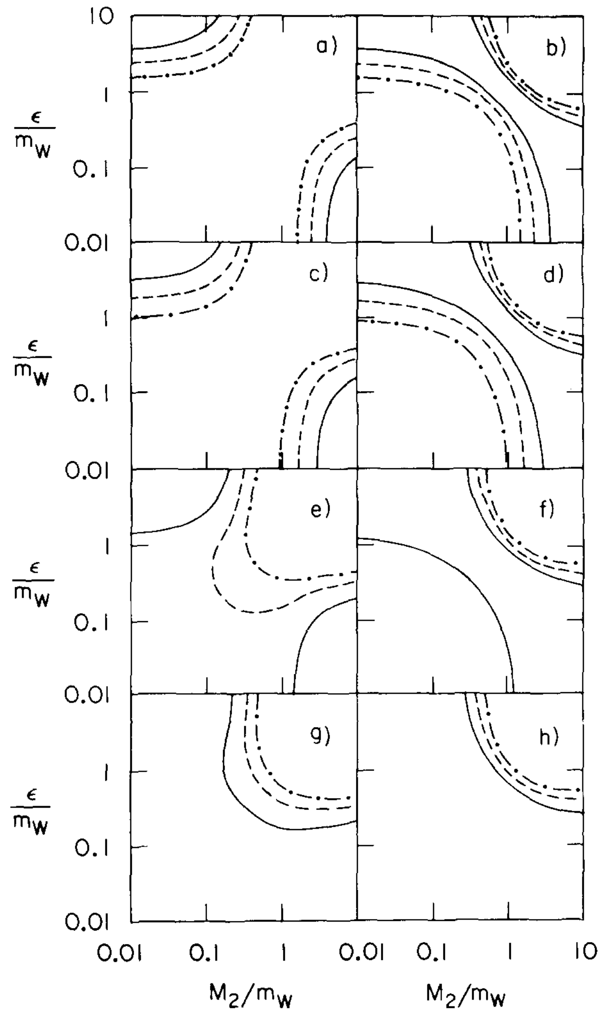

Fig. 1. Mass contours (solid = 20 GeV, dashed = 30 GeV, dot-dashed = 40 GeV) as functions of $|\varepsilon|$ and M_2 for the lightest eigenstate $\chi^{\pm}$ of the charged $\tilde{W}$, $\tilde{H}$ mass matrix (3.3), for different choices of v_1/v_2 and the sign of ε: (a) $v_1 = v_2$, $\varepsilon > 0$; (b) $v_1 = v_2$, $\varepsilon < 0$; (c) $v_1 = 2v_2$, $\varepsilon > 0$; (d) $v_1 = 2v_2$, $\varepsilon < 0$; (e) $v_1 = 4v_2$, $\varepsilon > 0$; (f) $v_1 = 4v_2$, $\varepsilon < 0$; (g) $v_1 = 8v_2$, $\varepsilon > 0$; (h) $v_1 = 8v_2$, $\varepsilon < 0$.

which in terms of the convenient linear combinations

$$\tilde{A}^0 \equiv \frac{v_1 \tilde{H}_1^0 - v_2 \tilde{H}_2^0}{v}, \qquad \tilde{S}^0 \equiv \frac{v_2 \tilde{H}_1^0 + v_1 \tilde{H}_2^0}{v}, \tag{3.7}$$

(where we have introduced $v \equiv \sqrt{v_1^2 + v_2^2}$) becomes

$$(\tilde{W}^3, \tilde{B}^0, \tilde{A}^0, \tilde{S}^0) \begin{pmatrix} M_2 & 0 & -\sqrt{\frac{1}{2}}\, g_2 v & 0 \\ 0 & \frac{5}{3}\frac{\alpha_1}{\alpha_2} M_2 & \sqrt{\frac{1}{2}}\, g_1 v & 0 \\ -\sqrt{\frac{1}{2}}\, g_2 v & \sqrt{\frac{1}{2}}\, g_1 v & \frac{-2v_1 v_2}{v^2}\varepsilon & \frac{v_1^2 - v_2^2}{v^2}\varepsilon \\ 0 & 0 & \frac{v_1^2 - v_2^2}{v^2}\varepsilon & \frac{2v_1 v_2}{v^2}\varepsilon \end{pmatrix} \begin{pmatrix} \tilde{W}^3 \\ \tilde{B}^0 \\ \tilde{A}^0 \\ \tilde{S}^0 \end{pmatrix}. \tag{3.8}$$

The general form of the orthogonal rotation that diagonalizes the matrix (3.6) is quite complicated and the Majorana mass eigenstate fields χ_i are in general complicated combinations of $\tilde{W}^3$, $\tilde{B}^0$, $\tilde{H}_1^0$ and $\tilde{H}_2^0$:

$$\chi_i \equiv \left(\alpha_i \tilde{W}^3 + \beta_i \tilde{B}^0 + \gamma_i \tilde{H}_1^0 + \delta_i \tilde{H}_2^0\right). \tag{3.9}$$

In the limit $\varepsilon \to 0$ there is a light higgsino state $\tilde{S}^0$ (as defined in eq. (3.7)) with mass equal to $2v_1 v_2 \varepsilon / v^2$. If this higgsino were extremely light with $m_{\tilde{S}_0} \leqslant \mathrm{O}(100)$ eV*, its cosmology would be similar to that of a conventional neutrino, as discussed in subsect. 3.4. In the limit of $M_2 \to 0$ there is a light photino eigenstate:

$$\tilde{\gamma} \equiv \frac{g_1 \tilde{W}^3 + g_2 \tilde{B}^0}{\sqrt{g_1^2 + g_2^2}}, \qquad m_{\tilde{\gamma}} = \frac{8}{3} \frac{g_1^2}{g_1^2 + g_2^2} M_2. \tag{3.10}$$

From a purely cosmological standpoint, the photino like the higgsino could have a mass $\lesssim \mathrm{O}(100)$ eV. However, in any conceivable model the photino and gluino masses are constrained to have roughly the same order of magnitude [20]:

$$\frac{m_{\tilde{g}}}{m_{\tilde{\gamma}}} = \frac{3}{8} \frac{\alpha_3}{\alpha_2 \sin^2\theta_W}, \tag{3.11}$$

if $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ is embedded in a unifying non-abelian group, and the gluino is already known [11] to have a mass $\gtrsim 2$ GeV. Thus we discard the

* Simple models (EHNT, ref. [5]) with such a light higgsino also contain an unacceptable axion whose mass is around the geometric mean of ε and the gravitino mass. This may be exorcised in more complicated models whose Higgs boson masses are substantially increased relative to higgsino masses.

possibility of a photino in the eV range, and then we will see in subsect. 3.3 that cosmology imposes a lower bound on $m_{\tilde{\gamma}}$ of about $\frac{1}{2}$ GeV. In the limit where M_2 and ε are both small, the remaining two mass eigenstates are

$$\tilde{Z}^0_{\pm} \equiv \frac{g_1 \tilde{B}^0 - g_2 \tilde{W}^3 \pm \sqrt{g_1^2 + g_2^2}\, \tilde{A}^0}{\sqrt{2(g_1^2 + g_2^2)}}, \qquad m_{\tilde{Z}^0_{\pm}} \simeq m_Z = \sqrt{\tfrac{1}{2}(g_1^2 + g_2^2)}\, v. \quad (3.12)$$

Mass contours for the lightest and second lightest neutral mass eigenstates χ and χ' for general M_2 and ε are plotted in figs. 2 and 3 respectively, with the same notation

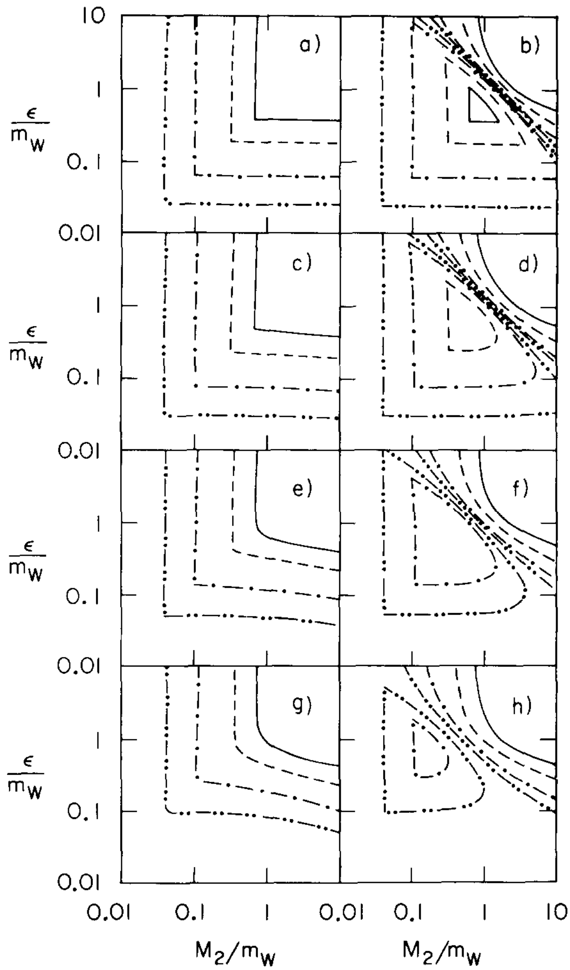

Fig. 2. Mass contours (solid = 30 GeV, dashed = 15 GeV, dot-dashed = 5 GeV, double-dot-dashed = 2 GeV) as functions of $|\varepsilon|$ and M_2 for the lightest eigenstate χ of the neutral $\tilde{\mathrm{W}}$, $\tilde{\mathrm{H}}$ mass matrix (3.6), for the same choices of v_1/v_2 and of the sign of ε as in fig. 1.

and conventions as in fig. 1. We observe that for most of parameter space there is one and often two eigenstates with mass < 30 GeV, except where M_2 and ε are both $\gtrsim m_W$.

3.2. COSMOLOGICAL MASS DENSITIES

Our primary task is to constrain the parameters M_2 and ε in the lagrangian (3.1) by requiring that the lightest neutral susy fermion χ have an acceptably low cosmological mass density ρ_χ. We know from the rate of expansion of the universe that $\rho_\chi \leqslant 2 \times 10^{-29}(\Omega h_0^2)$ g/cm^3, where Ω is the density in units of the closure

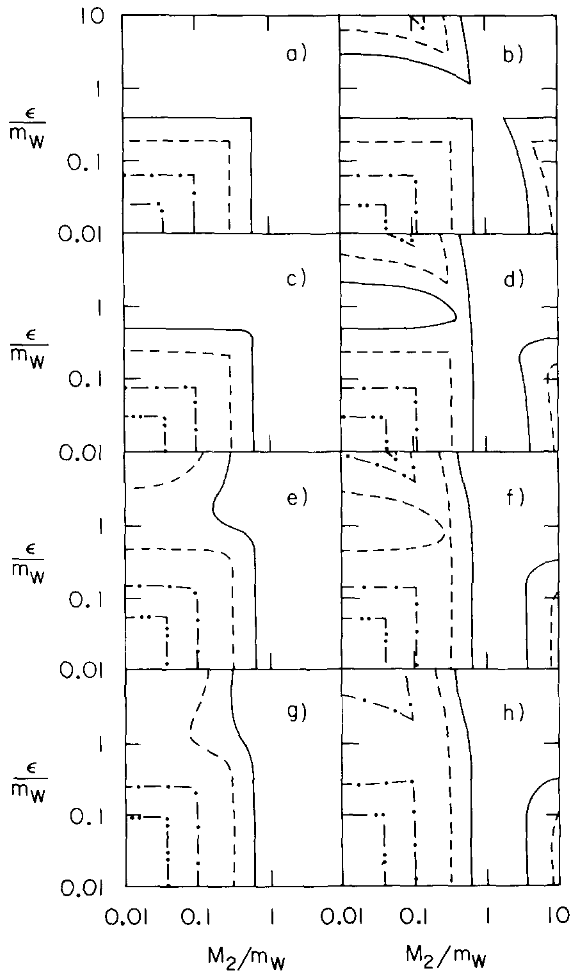

Fig. 3. Mass contours (same as fig. 2) as functions of $|\varepsilon|$ and M_2 for the next-to-lightest eigenstate in the mass matrix (3.6), for the same choices of v_1/v_2 and of the sign of ε as in fig. 1.

density, and h_0 is the Hubble parameter in units of 100 $\mathrm{km s^{-1} Mpc^{-1}}$. It is reasonable to believe that $\Omega h_0^2 \lesssim 1$ implying that $\rho_\chi \lesssim 2 \times 10^{-29}$ g/cm^3. A more restrictive constraint follows from the plausible assumption that a non-relativistic susy fermion would participate in galaxy formation, in which case the limits on "dark matter" in galaxies [21] allow one to deduce that $\rho_\chi \leqslant 2 \times 10^{-30} (\Omega h_0^2)$ g/cm^3. We will present our cosmological bounds on the susy parameters M_2 and ε and the susy particle masses for both the conservative constraint $\rho_\chi \leqslant 2 \times 10^{-29}$ g/cm^3 from the observed expansion rate of the universe, and also the plausible bound $\rho_\chi \leqslant 2 \times 10^{-30}$ g/cm^3 based on the assumption of galactic clustering. Since all but the lightest mass eigenstate can decay, these constraints pertain to the lightest eigenstate only: either χ annihilates fairly efficiently as the universe cools, reducing the number density to an acceptably low level, or else χ is very light ($<$ O(100) eV) and is as cosmologically innocuous as a conventional neutrino.

To proceed quantitatively we review and extend the analysis of refs. [22, 23] to include the general case of Majorana fermions. The rate equation for ordinary Dirac fermions [22] is

$$\frac{\mathrm{d}n}{\mathrm{d}t} = -3\frac{\dot{R}}{R}n - \langle \sigma v_{\mathrm{rel}} \rangle \left(n^2 - n_0^2 \right), \tag{3.13}$$

where n is the actual number density at time t, n_0 is the number density of heavy fermions in thermal equilibrium, R is the cosmic scale factor, σ is the annihilation cross section, and v_{rel} is the relative velocity of the annihilating particles. For identical fermions, we might expect to see a factor of two multiplying the annihilation rate in (3.13), since two Majorana neutrals are destroyed by each annihilation event. However, there is also a factor of $\frac{1}{2}$ needed to prevent double counting in the thermal average over the initial velocity distribution of the annihilating particles, and thus (3.13) is valid for Majorana as well as for Dirac fermions.

Following standard methods [22] one can rewrite (3.13) in the convenient form

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{m_\chi}{k^3} \left(\frac{8\pi^3 N_{\mathrm{F}} G}{45} \right)^{-1/2} \langle \sigma v_{\mathrm{rel}} \rangle \left(f^2 - f_0^2 \right), \tag{3.13$'$}$$

where $x \equiv kT/m_\chi$, $f(x) = n/T^3$, $f^0(x) = n_0/T^3$, k is the Boltzmann constant, $G = 1/m_{\mathrm{P}}^2$ is Newton's constant and N_{F} counts the effective number of degrees of freedom at a given temperature [22].

The annihilation $\chi\chi \to \bar{\mathrm{f}}\mathrm{f}$ occurs through conventional Z-boson exchange and through the exchange of scalar partners of conventional fermions $\tilde{\mathrm{f}}_{\mathrm{L,R}}$. These interactions give rise to a low-energy effective lagrangian

$$\mathcal{L} = \sum_{\mathrm{f}} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{f} \gamma_\mu \left(A_{\mathrm{f}} P_{\mathrm{L}} + B_{\mathrm{f}} P_{\mathrm{R}} \right) f, \tag{3.14}$$

where

$$A_{\rm f}^{Z} = \left(\gamma^2 - \delta^2\right) \frac{g_1 \sin\theta_{\rm W} + g_2 \cos\theta_{\rm W}}{4M_Z^2} \left(\frac{1}{2} Y_{\rm f_L} g_1 \sin\theta_{\rm W} - T_{\rm f_L}^3 g_2 \cos\theta_{\rm W} \right), \quad (3.15a)$$

$$B_{\rm f}^{Z} = \left(\gamma^2 - \delta^2\right) \frac{g_1 \sin\theta_{\rm W} + g_2 \cos\theta_{\rm W}}{8M_Z^2} Y_{\rm f_R} g_1 \sin\theta_{\rm W}, \quad (3.15b)$$

$$A_{\rm f}^{\tilde{\rm f}} = \frac{\left(T_{\rm f_L}^3 \alpha g_2 + \frac{1}{2} Y_{\rm f_L} \beta g_1\right)^2}{2m_{\tilde{\rm f}_L}^2} - \begin{cases} \gamma^2 m_{\rm f}^2 / 4v_1^2 m_{\tilde{\rm f}_R}^2, & \rm u, c, t \\ \delta^2 m_{\rm f}^2 / 4v_2^2 m_{\tilde{\rm f}_R}^2, & \rm e, \mu, \tau, d, s, b, \end{cases} \quad (3.15c)$$

$$B_{\rm f}^{\tilde{\rm f}} = -\frac{\left(\frac{1}{2} Y_{\rm f_R} \beta g_1\right)^2}{2m_{\tilde{\rm f}_R}^2} + \begin{cases} \gamma^2 m_{\rm f}^2 / 4v_1^2 m_{\tilde{\rm f}_L}^2, & \rm u, c, t \\ \delta^2 m_{\rm f}^2 / 4v_2^2 m_{\tilde{\rm f}_L}^2, & \rm e, \mu, \tau, d, s, b, \end{cases} \quad (3.15d)$$

with $\frac{1}{2} Y = Q + T^3$ and $\alpha, \beta, \gamma, \delta$ defined as in (3.9). The sfermion exchange terms in (3.15c, d) proportional to fermion masses result from higgsino-fermion Yukawa couplings, which can be important if the χ is a nearly pure higgsino or when heavy quark modes are kinematically accessible*.

In the non-relativistic limit $v_{\rm rel} \ll 1$, the effective interaction (3.14) leads to the $\chi\chi$ annihilation cross section

$$\langle \sigma v_{\rm rel} \rangle = \frac{1}{2\pi} \sum_{\rm f} \frac{p}{m_\chi} \left\{ \left(A_{\rm f}^2 + B_{\rm f}^2\right) \left[\tfrac{1}{6} \left(4m_\chi^2 - m_{\rm f}^2\right) v_{\rm rel}^2 + m_{\rm f}^2 \right] - 2A_{\rm f} B_{\rm f} m_{\rm f}^2 \left(1 - v_{\rm rel}^2/2\right) \right\}, \quad (3.16)$$

where $p = \sqrt{m_\chi^2 - m_{\rm f}^2}$ is a final state 3-momentum. The $v_{\rm rel}$ dependence of (3.16) results from the P-wave barrier to the annihilation of Majorana fermions, as first emphasized by Goldberg [23]. Replacing $v_{\rm rel}$ in (3.16) by its thermal average ($v_{\rm rel}^2 \simeq 6kT/m_\chi = 6x$), the cross section can be written

$$\langle \sigma v_{\rm rel} \rangle = \tilde{a} + \tilde{b} x, \quad (3.17a)$$

where

$$\tilde{a} = \sum_{\rm f} \theta(m_\chi - m_{\rm f}) \frac{1}{2\pi} \frac{p}{m_\chi} m_{\rm f}^2 (A_{\rm f} - B_{\rm f})^2,$$

$$\tilde{b} = \sum_{\rm f} \theta(m_\chi - m_{\rm f}) \frac{1}{2\pi} \frac{p}{m_\chi} \left[\left(A_{\rm f}^2 + B_{\rm f}^2\right)\left(4m_\chi^2 - m_{\rm f}^2\right) + 6A_{\rm f} B_{\rm f} m_{\rm f}^2 \right]. \quad (3.17b)$$

* There are in addition higgsino-gaugino interference terms which cannot be cast into an effective interaction of the form (3.14). We do not present these since there is no domain for which these interference contributions equal or exceed the gaugino-gaugino and/or higgsino-higgsino contributions (3.15).

The rate equation (3.13) takes the general form

$$\frac{\mathrm{d}f}{\mathrm{d}x}=\left[\frac{m_\chi}{k^3}\left(\frac{8\pi^3 N_\mathrm{F} G}{45}\right)^{-1/2}\right](\tilde{a}+\tilde{b}x)\left(f^2-f_0^2\right)$$
$$\equiv (a+bx)\left(f^2-f_0^2\right). \tag{3.18}$$

Following the approximate analytic approach of Lee and Weinberg [22], we expect the reduced number density $f(x)$ in (3.18) to remain approximately equal to its equilibrium value $f_0(x)$ until the temperature T drops to a freeze-out value T_f where the annihilation rate is equal to the rate of change in f_0:

$$\frac{\mathrm{d}f_0}{\mathrm{d}x}=(a+bx)f_0^2 \qquad \text{at} \qquad x=x_\mathrm{f}, \tag{3.19}$$

and assume that thereafter it evolves approximately according to the equation

$$\frac{\mathrm{d}f}{\mathrm{d}x}=(a+bx)f^2, \tag{3.20}$$

subject to the initial condition $f(x_\mathrm{f})=f_0(x_\mathrm{f})$. Since x_f is generally $\ll 1$, we can use the non-relativistic approximation [22]

$$f_0(x)=2k^3(2\pi x)^{-3/2}\mathrm{e}^{-1/x}, \tag{3.21}$$

to solve for the freeze-out temperature. Eqs. (3.19) and (3.21) together give

$$x_\mathrm{f}=\frac{1}{\ln\left(a x_\mathrm{f}^{1/2}+b x_\mathrm{f}^{3/2}\right)+\ln\left[2k^3/(2\pi)^{3/2}\right]} \tag{3.22a}$$

$$=\frac{1}{\ln\left(\tilde{a} x_\mathrm{f}^{1/2}+\tilde{b} x_\mathrm{f}^{3/2}\right)-\frac{1}{2}\ln\left(16\pi^6 N_\mathrm{F} G/45 m_\chi^2\right)}. \tag{3.22b}$$

We should emphasize that the non-relativistic approximation (3.16)–(3.22) is not valid when the annihilation cross section (3.16) is small enough for the scaled freeze-out temperature (3.22) to become $\gtrsim 1$, which is generally the case for a light higgsino state. We consider the case of a light higgsino separately in subsect. 3.4.

The present number density is obtained by integrating (3.20) from $x=x_\mathrm{f}$ down to $x=0$:

$$f(0)=\frac{1}{a x_\mathrm{f}+\frac{1}{2}b x_\mathrm{f}^2}. \tag{3.23}$$

The present mass density ρ_χ is given by

$$\rho_\chi = 0.8\left(\frac{T_\chi}{T_\gamma}\right)^3 T_\gamma^3 m_\chi \frac{1}{ax_f + \frac{1}{2}bx_f^2}, \tag{3.24a}$$

where $(T_\chi/T_\gamma)^3$ accounts for the subsequent reheating of the photon temperature with respect to the temperature of χ, due to the annihilation of particles with $m < x_f m_\chi$, and is tabulated [24] together with N_F in table 1. The fudge factor 0.8 is included to correct for the fact [22] that the analytic approximation (3.19) and (3.20) to the full rate equation (3.13) give a result which is approximately 25% too large. In terms of the coefficients $\tilde{a}$, $\tilde{b}$ appearing in the annihilation cross section, the mass density (3.24a) reads

$$\rho_\chi = 4.0 \times 10^{-40}\left(\frac{T_\chi}{T_\gamma}\right)^3\left(\frac{T_\gamma}{2.8°\mathrm{K}}\right)^3 N_F^{1/2}\left(\frac{\mathrm{GeV}^{-2}}{\tilde{a}x_f + \frac{1}{2}\tilde{b}x_f^2}\right)\frac{\mathrm{g}}{\mathrm{cm}^3}. \tag{3.24b}$$

Results obtained using this formula are discussed in subsect 3.3.

3.3. RESULTS

We have completed a numerical study of the two-dimensional space of parameters M_2, ε in order to distinguish the cosmologically allowed and disallowed domains. The following algorithm was used.

(i) M_2 and ε are selected and the physical mass eigenstates (3.9) and eigenmasses are obtained by diagonalizing the matrix (3.6).

TABLE 1

T_f	N_F	$(T_\chi/T_\gamma)^3$
$m_e - m_\mu$	$\frac{43}{8}$	2.75
$m_\mu - m_\pi$	$\frac{57}{8}$	3.65
$m_\pi - T_H$	$\frac{69}{8}$	4.41
$T_H - m_s$	$\frac{205}{8}$	13.1
$m_s - m_c$	$\frac{247}{8}$	15.8
$m_c - m_\tau$	$\frac{289}{8}$	18.5
$m_\tau - m_b$	$\frac{303}{8}$	19.4
$m_b - m_t$	$\frac{345}{8}$	22.1
$m_t - m_W$	$\frac{387}{8}$	24.8
$> m_W$	$\frac{423}{8}$	27.1

This table is adapted from ref. [24]. Most of the notation is described in the text, with the exception that T_H is the temperature above which it is supposed that hadrons should be described in terms of quark and gluon degrees of freedom.

(ii) The coefficients A_f and B_f in the effective lagrangian (3.14) for the lightest mass eigenstate χ are computed from eqs. (3.15).

(iii) The coefficients $\tilde{a}$ and $\tilde{b}$ in the annihilation cross section (3.17) are calculated.

(iv) The scaled freeze-out temperature x_f is determined iteratively using (3.22b), with N_F being updated using table 1 following each iteration.

(v) The cosmological mass density ρ_χ is calculated from (3.24b) and compared with the cosmological bounds $\rho_\chi \leqslant 2 \times 10^{-29}$ g cm^3 and $\rho_\chi \leqslant 2 \times 10^{-30}$ g/cm^3.

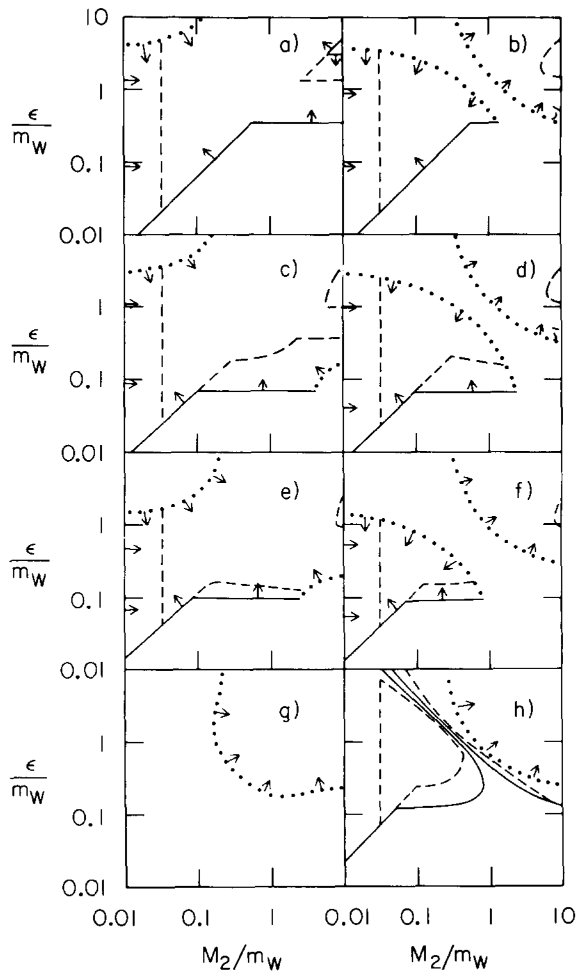

Fig. 4. Allowed domains of $|\varepsilon|$ and M_2 for the same choices of v_1/v_2 and of the sign of ε as in fig. 1. The solid curves are where $\rho_\chi = 2 \times 10^{-29}$ g/cm^3, while the dashed curves are for $\rho_\chi = 2 \times 10^{-30}$ g/cm^3. These results were obtained with $m_{\tilde{\ell}_L} = m_{\tilde{\ell}_R} = m_{\tilde{q}} = 20$ GeV or m_χ, whichever is larger.

Except in the case of a very light higgsino eigenstate ($\varepsilon < M_2 \ll m_W$) we find that $0.04 \lesssim x_f \lesssim 0.08$, which supports the non-relativistic approximation (3.16)–(3.23) based upon $x_f \ll 1$.

Our numerical results are exhibited in figs. 3–5 (preliminary early results were shown in ref. [6]). We divide the space of parameters into allowed versus disallowed regions, where the boundaries of acceptability corresponding to $\rho_\chi = 2 \times 10^{-29}$ g/cm^3 and to $\rho_\chi = 2 \times 10^{-30}$ g/cm^3 are drawn as solid and dashed lines respectively. Dotted lines represent the 20 GeV mass constraint on the lightest *charged* susy fermion mandated by PETRA and PEP. In figs. 4–6 the squark and slepton masses appearing in the $\chi\chi$ annihilation cross section (3.15b, c) are taken to be degenerate, and equal to 20, 40 and 100 GeV or m_χ, whichever is the larger, in figs. 4, 5 and 6

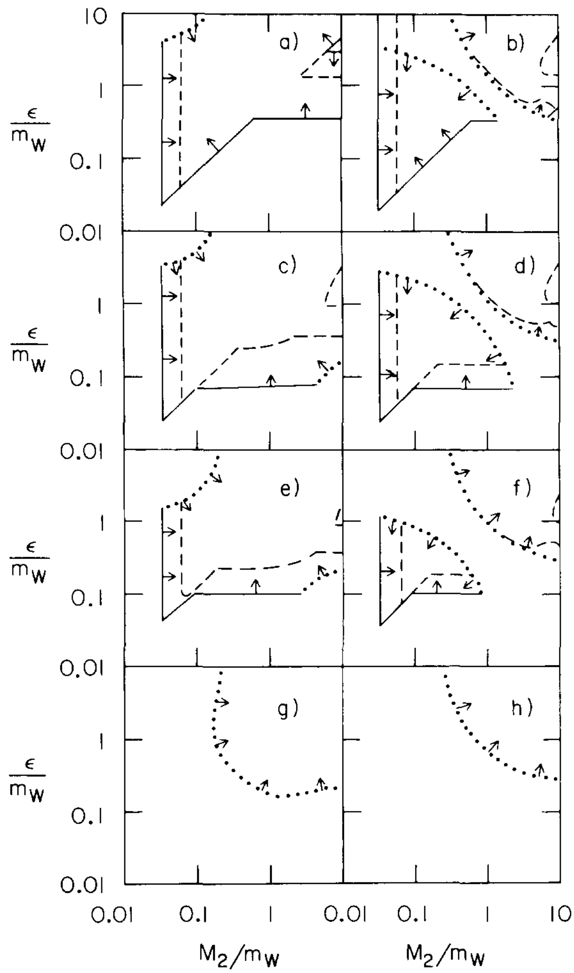

Fig. 5. As for fig. 4, except that $m_{\tilde{\ell}_L} = m_{\tilde{\ell}_R} = m_{\tilde{q}} = 40$ GeV has been used.

respectively. We begin our qualitative discussion of the results with fig. 4a, where $m_{\tilde{\ell}_L} = m_{\tilde{\ell}_R} = m_{\tilde{q}} = 20$ GeV, the vacuum expectation values v_1 and v_2 are equal, and ε is positive. There is a diagonal line traversing much of the figure from bottom left towards the top right, above which the lightest mass eigenstate χ is approximately a photino and below which χ is a pure $\tilde{S}^0$ higgsino state. Note that in the exact $v_1 = v_2$ limit, the Z-boson exchange contribution (3.15a, b) to the $\chi\chi$ annihilation cross section vanishes because $|\gamma| = |\delta|$. Therefore the $\tilde{S}^0$ can only annihilate through its Yukawa couplings to fermions, which are not large enough for the $\tilde{S}^0$ to annihilate to acceptably low levels unless $m_{\tilde{S}^0} > m_t$. Once the $\tilde{S}^0$ mass (see fig. 2a) exceeds that of the t-quark (taken to be 30 GeV on the basis of rumours [25] from UA1), its annihilation into $\bar{t}t$ pairs is sufficiently efficient that the $\tilde{S}^0$ becomes

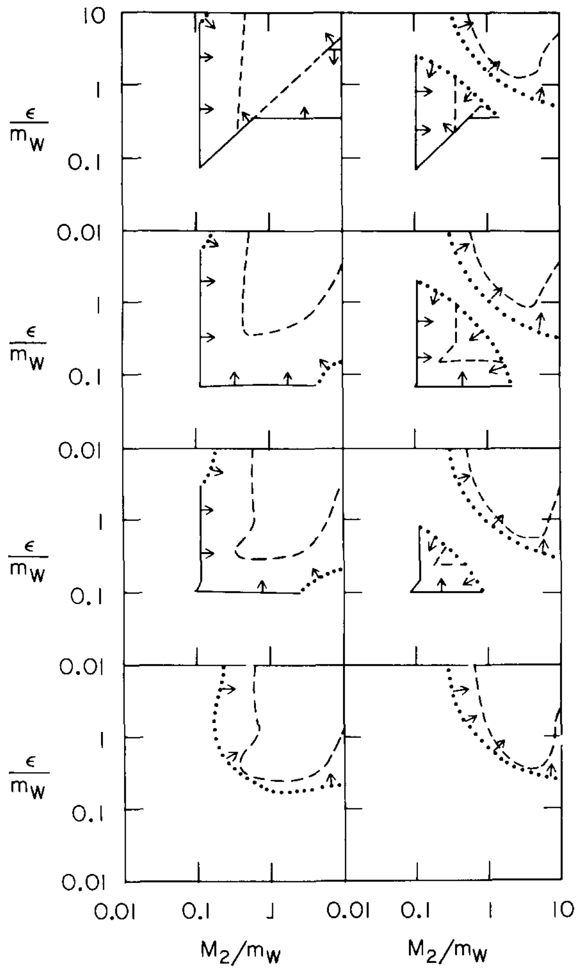

Fig. 6. As for fig. 4, except that $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L} = m_{\tilde{q}} = 100$ GeV has been used.

cosmologically allowed. This explains the solid horizontal line in fig. 4a which follows the 30 GeV contour in fig. 2a. Note that this boundary is sufficiently abrupt that the $\rho_\chi \leqslant 2 \times 10^{-29}$ g/cm^3 boundary (solid) and the $\rho_\chi \leqslant 2 \times 10^{-30}$ g/cm^3 boundary (dashed) nearly coincide.

Above the diagonal line in fig. 4a the lightest eigenstate χ is approximately a photino, which annihilates much more efficiently than the $\tilde{S}^0$ through its gauge couplings to fermions. The cosmological boundary $\rho \leqslant 2 \times 10^{-29}$ g/cm^3 occurs at a vertical line near the extreme left of the figure, corresponding to a photino mass of $m_{\tilde{\gamma}} \simeq \frac{1}{2}$ GeV (see fig. 2a). This result differs significantly from ref. [23], which excludes a photino lighter than about 2 GeV. The difference arises partly due to a factor of two discrepancy in the annihilation cross section (3.16), but primarily because we include masses for the light quarks, without which the $\chi\chi$ annihilation cross section would be purely P-wave below the charm-τ threshold. Our results were obtained using current quark masses $m_{\mathrm{u,d}} = 10$ MeV and $m_{\mathrm{s}} = 150$ MeV; the use of constituent quark masses gives a lower bound of $m_{\tilde{\gamma}}$ that is only slightly smaller. The lower bound of $m_{\tilde{\gamma}} \geqslant \frac{1}{2}$ GeV ($M_2 \geqslant 0.8$ GeV) is our most conservative bound, since it is derived using $\rho_\chi \leqslant 2 \times 10^{-29}$ g/cm^3 and sfermion masses of 20 GeV. The less conservative cosmological bound $\rho_\chi \leqslant 2 \times 10^{-30}$ g/cm^3 already forces the photino mass above the charm-τ threshold (dashed), and the use of heavier sfermion masses (figs. 5a, 6a) leads to considerably larger bounds.

If $v_1 \neq v_2$ as in figs. 4c, e, g, the constraint on the (now no longer pure $\tilde{S}^0$) higgsino mass is reduced from the t-quark mass to the b-quark mass (see figs. 2c, e, g). The reason is that there is now a significant contribution to the $\tilde{S}^0\tilde{S}^0$ annihilation cross section from Z-boson exchange. The b-quark channel is still needed to provide a significant S-wave contribution to the annihilation cross section (3.16). On the basis of figs. 4–6 we conclude generally that if the (almost pure) $\tilde{S}^0$ is the lightest susy fermion but heavier than about 100 eV, then it must be heavier than the b-quark.

The results for $\varepsilon > 0$ (figs. 4b, d, f, h) are very similar to those for $\varepsilon > 0$ except for the appearance of a narrow sheath of cosmologically forbidden territory (shown only in fig. 4h) due to an accidental zero in the mass matrix (3.6). As seen in figs. 1b, d, f, h, a similar line of zeros also appears in the charged fermion matrix (3.3), so that this tantalizing sheath is doubly excluded.

The most important change as v_1/v_2 increases (figs. 4c–h) is that the 20 GeV constraint on the lightest charged eigenstate becomes more restrictive. In figs. 4g, h we observe that if $v_1/v_2 \gtrsim 8$, much of the parameter space is excluded by the charged constraint, so that the cosmological bounds are almost irrelevant.

The constraints that result from setting the sfermion masses equal to 40, and 100 GeV or m_χ, whichever is the larger, are qualitatively similar (figs. 5, 6) except for an overall shrivelling of the cosmologically allowed regions. In particular, the lower bound on the photino mass from $\rho_\chi \leqslant 2 \times 10^{-29}$ g/cm^3 increases from $\frac{1}{2}$ GeV to 1.8 GeV for $m_{\tilde{\ell}_{\mathrm{L}}} = m_{\tilde{\ell}_{\mathrm{R}}} = m_{\tilde{\mathrm{q}}} = 40$ GeV and to 5 GeV when $m_{\tilde{\ell}_{\mathrm{L}}} = m_{\tilde{\ell}_{\mathrm{R}}} = m_{\tilde{\mathrm{q}}} = 100$ GeV. The analogous constraint from $\rho_\chi \lesssim 2 \times 10^{-30}$ g/cm^3 increases from 1.8 GeV to 3 GeV when $m_{\tilde{\mathrm{f}}} = 40$ GeV, and to approximately 15 GeV if $m_{\tilde{\mathrm{f}}} = 100$ GeV.

Figs. 7 and 8 are intended to illustrate the effects of varying the *relative* masses of right- and left-handed sleptons and of squarks. The effects of taking $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L} = m_{\tilde{q}} = 20$; $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L} = 20$, $m_{\tilde{q}} = 50$; $m_{\tilde{\ell}_R} = 20$, $m_{\tilde{\ell}_L} = 40$, $m_{\tilde{q}} = 100$ GeV (corresponding to three classes of models described in EHNT [5]) when $v_1 = 4v_2$ are shown respectively in a, b, c of figs. 7 and 8 for $\varepsilon > 0$ and $\varepsilon < 0$ respectively. We observe that the effect of changing the sfermion mass hierarchy is rather negligible, except for a slight increase in the photino mass limit with heavier squark masses (due to a suppression of the photino annihilation into light quarks). It is because these effects are small that we chose to focus our earlier presentation on the case of degenerate squarks and sleptons.

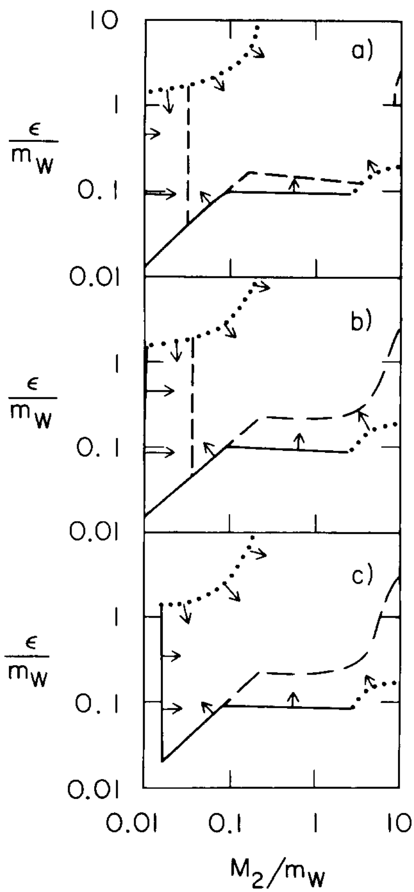

Fig. 7. The effects of varying the relative masses of the sfermions for $v_1 = 4v_2$ and $\varepsilon > 0$: (a) $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L} = m_{\tilde{q}} = 20$ GeV; (b) $m_{\tilde{\ell}_R} = m_{\tilde{\ell}_L} = 20$ GeV, $m_{\tilde{q}} = 50$ GeV; (c) $m_{\tilde{\ell}_R}20$ GeV, $m_{\tilde{\ell}_L} = 40$ GeV, $m_{\tilde{q}} = 100$ GeV.

3.4. A VERY LIGHT HIGGS FERMION?

Finally we consider the case of a very light higgsino state, with mass $\lesssim O(100)$ eV. From a cosmological standpoint, the light higgsino looks very much like a neutrino. Its annihilation into light fermions is dominated by Z-boson exchange, with a relativistic annihilation cross section

$$\langle \sigma v_{\mathrm{rel}} \rangle = \left(\gamma^2 - \delta^2\right)^2 \frac{2G_F^2 s}{3\pi}\left(g_V^{e^2} + g_A^{e^2}\right), \tag{3.25}$$

where $g_V^e = -\frac{1}{2} + 2\sin^2\theta_W$ and $g_A^e = -\frac{1}{2}$. The higgsino falls out of thermal equilibrium when the interaction rate given by (3.25) falls below the expansion rate of

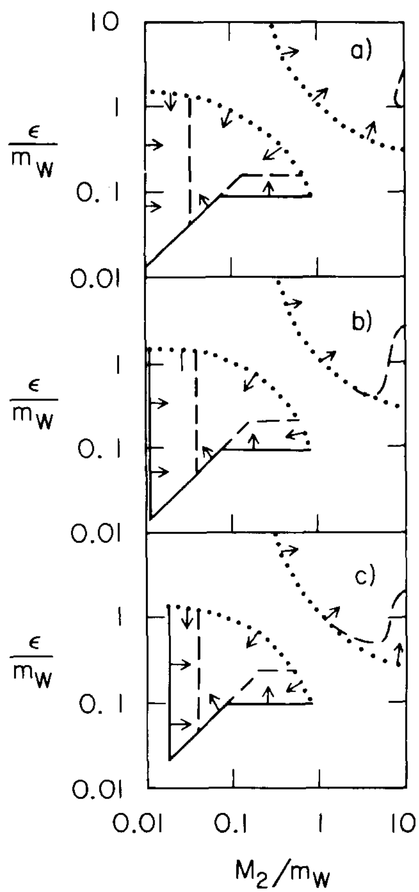

Fig. 8. As for fig. 7, but for the choices $v_1 = 4v_2$ and $\varepsilon < 0$.

the universe. This occurs at a temperature

$$T_{\mathrm{E}}=\left(\frac{1}{\left(\gamma^{2}-\delta^{2}\right)^{2}}\frac{3\sqrt{10\pi}}{G_{\mathrm{F}}^{2}N_{\mathrm{F}}^{1/4}m_{\mathrm{P}}}\right)^{1/3}. \tag{3.26}$$

Thus for higgsinos we find

$$T_{\mathrm{E}}=\frac{1.8\ \mathrm{MeV}}{\left(\gamma^{2}-\delta^{2}\right)^{2/3}}. \tag{3.27}$$

If T_{E} (3.27) becomes larger than m_{μ}, $\mu^{+}\mu^{-}$ annihilation does not reheat the higgsinos, and their number density will be less than that of the neutrinos by a factor of $\frac{43}{57}$. In this case the permissible upper bound on $m_{\mathrm{S}^{0}}$ is a factor $\frac{57}{43}$ higher than would be allowed for neutrinos. We see from eq. (3.7) that

$$|\gamma^{2}-\delta^{2}|=\left|\frac{v_{1}^{2}-v_{2}^{2}}{v_{1}^{2}+v_{2}^{2}}\right|, \tag{3.28}$$

which when combined with equation (3.27) tells us that the higgsinos avoid reheating by $\mu^{+}\mu^{-}$ annihilation if

$$\left|\frac{v_{1}^{2}-v_{2}^{2}}{v_{1}^{2}+v_{2}^{2}}\right|<2\times10^{-3}. \tag{3.29}$$

Is it plausible that v_{1}^{2} and v_{2}^{2} could have such similar values? We can only answer this question by looking at models. In most models [5] that we know v_{1} is considerably larger than v_{2}, and the condition (3.29) is not satisfied. There is one model (EHNT ref. [5]) in which v_{1} and v_{2} can be similar

$$\left|\frac{v_{1}^{2}-v_{2}^{2}}{v_{1}^{2}+v_{2}^{2}}\right|\approx\frac{g_{2}^{2}\mu^{2}}{\left(g_{2}^{2}+g_{1}^{2}\right)m_{\mathrm{W}}^{2}}, \tag{3.30}$$

but even in this case $\mu\gtrsim \mathrm{O}(20)$ GeV (EHNT, ref. [5]), so eq. (3.29) tells us that $\mu^{+}\mu^{-}$ *does* reheat the higgsinos. We conclude that in all plausible models the relevant upper mass bound on the higgsinos is the same as that for neutrinos, namely $m_{\chi}\lesssim \mathrm{O}(100)$ eV.

It is natural to suppose that as the Higgs fermion mixing parameter $\varepsilon\to0$ in eq. (3.1), so also does the corresponding Higgs boson mixing parameter. The theory would then possess a Peccei-Quinn [26] U(1) symmetry. However, the physical spectrum then contains an axion. In the EHNT model of ref. [5] the physical Higgs spectrum in this U(1) symmetry limit contains [27] an axion with a mass of around the geometric mean of ε and the gravitino mass, which is around an MeV. Such an axion would conflict with particle physics experiments and must therefore be exorcised. It is possible that this may be done in more complicated models by increasing the axion mass relative to the higgsino mass. For this reason we are reluctant to conclude that a light higgsino can be excluded by this indirect argument.

4. Summary

In this paper we have made a systematic and complete study of the cosmological constraints on supersymmetric theories extending the analysis of ref. [23] and updating ref. [6]. Our analysis is tailored preferentially, but not exclusively, to low-energy models [5] extracted from an underlying $N=1$ supergravity theory. Where necessary, we have supplemented strictly cosmological constraints by clearly stated phenomenological assumptions. We have found that the lightest supersymmetric particle (LSP) is unlikely to be a charged wino, charged higgsino, slepton, sneutrino, gluino, squark, or gravitino. Most probably the LSP is a mixture of neutral gauge and Higgs fermions. We have explored the cosmologically allowed domains of parameter space for these particles, and our results are shown in figs. 4 to 8. The cosmologically allowed domains shrivel as the assumed masses of the sfermions are increased. We find one favoured class of solutions in which there is a mass eigenstate which is approximately a photino of mass above $\frac{1}{2}$ to 2 GeV, while the lightest (approximate) higgsino is generally heavier than the photino. Lighter higgsinos do not annihilate efficiently enough to suppress their present cosmological abundance below the acceptable limits unless they are heavier than the b- or t-quark. The cosmology of a very light higgsino is similar to that of a massive neutrino, and a higgsino weighing less than O(100) eV is equally compatible with observation.

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