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New clustering algorithm for multijet cross sections in e^+e^- annihilation☆

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Cross sections for $e^+e^- \to n$-jets, as functions of the jet resolution parameter y_{cut}, are computed according to a new clustering algorithm. The jet multiplicity n is defined in such a way that jets i and j with energies E_i and E_j at relative angle θ_{ij} are resolved if $y_{ij}=2(1-\cos\theta_{ij})\min(E_i^2,E_j^2)/s>y_{cut}$, where s is the centre-of-mass energy squared. Using this algorithm, large higher-order corrections at small values of y_{cut} can easily be evaluated. Our calculations include resummation of leading and next-to-leading logarithms of y_{cut} to all orders in QCD perturbation theory. This enables us to predict the jet cross sections at small y_{cut} for arbitrary n. Simple analytical results for $n\leqslant 5$ are presented.

1. Introduction

One of the areas of primary experimental interest in e^+e^- annihilation at LEP energies [1] and below [2–5] has been the study of multijet cross sections, both as a test of QCD and as a good means of determining the strong coupling constant α_s. These cross sections are defined in terms of (i) a dimensionless jet resolution parameter y_{cut} and (ii) a jet recombination scheme. The jet resolution parameter was originally taken to be of the general form $y_{cut}=M_j^2/s$ where M_j is the maximum jet invariant mass and s is the centre-of-mass energy squared (the "JADE algorithm" [2]), and several different recombination schemes have been introduced (JADE, E, E_0, P,P_0 schemes [2,6,7]). Final-state particles are combined into clusters, which are in turn recombined, according to the prescribed algorithm, until any further recombination would yield clusters that exceed the resolution y_{cut}. The number of remaining clusters is defined as the jet multiplicity.

The introduction of a finite jet resolution y_{cut} makes the multijet cross sections defined in this way infrared and collinear safe, so that the experimental data should be directly comparable with perturbative QCD calculations at the parton level. So far this comparison has been performed with theoretical predictions [6–8] based on the relevant QCD matrix elements evaluated to second order in the strong coupling α_s [9].

As long as the resolution parameter y_{cut} is not small, the fixed-order perturbative calculations should be reliable. At small y_{cut} values, however, there are terms in higher order that become enhanced by powers of $\ln y_{cut}$. In this kinematical region the real expansion parameter is the large effective coupling $\alpha_s \ln^2 y_{cut}$ and therefore *any* finite-order perturbative calculation cannot give an accurate evaluation of the cross section. The logarithmic terms need to be identified and resummed to all orders in α_s before a reliable prediction can be made.

Experimentally, it is found that the two-loop theoretical calculations are not able to fit the experimental data

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for $y_{cut} < 0.05$, unless very small values of the renormalisation scale are used [10]. This may be a signal of the breakdown of the perturbative expansion in α_s at small y_{cut}.

The appearance of large double logarithmic terms is a common feature of any hard process in the semi-inclusive or Sudakov region, where emission of radiation is inhibited by the kinematics. In the case of jet cross sections at $y_{cut} \ll 1$, the jet invariant mass is constrained to be so small as to allow only emission of gluons that are soft and collinear with respect to the parton generating the jet. The double logarithmic terms $\alpha_s \ln^2 y_{cut}$ are due to such soft and collinear gluons.

In recent years, following the pioneering calculations [11] of double logarithms, the programme of resummation of Sudakov logarithms has been successfully carried out to next-to-leading order for transverse [12] and longitudinal momentum distributions [13,14] in hadron-initiated processes, as well as for quantities like the energy–energy correlation [15] and the thrust distribution [16] in e^+e^- annihilation. All these successful resummations depend on the fact that the large logarithmic corrections to the relevant quantity *exponentiate*. By this we mean that terms of the form $\alpha_s^n \ln^m y_{cut}$ with $n < m \leqslant 2n$ can be combined into an exponential function of less singular terms, in fact with $m \leqslant n+1$. Once exponentiation has been established, the resummation of leading and next-to-leading logarithms to all orders is reduced to a much simpler, finite-order calculation of the relevant exponent.

The widespread occurrence of exponentiation in perturbative QCD is a consequence of the factorization properties of multiparton amplitudes on the soft limit, which appear necessary for the cancellation of soft gluon singularities in inclusive quantities. These properties of the amplitudes will be reflected in the corresponding parton-level cross section, provided any phase-space integrations in the definition of the cross section do not induce strong kinematic correlations. Furthermore the exponentiation property and the absence of non-local phase-space correlations fit naturally with preconfinement [17] and local parton–hadron duality (LPHD) [18] in the hadronization process. According to preconfinement and LPHD, the hadronic flow in the final state follows the partonic flow quite closely, with transfers of momentum and other quantum numbers that are local in phase space. Thus the hadronization corrections needed in order to compare parton-level calculations with data at the hadron level should be relatively small when the conditions for exponentiation are satisfied.

Unfortunately, the definition of the multijet cross sections according to the original JADE (invariant mass) type of algorithm generates strong *kinematic correlations*, which have been shown [19] to lead to a breakdown of exponentiation even at the leading double-logarithmic level. These kinematic correlations strongly increase with the number of final state partons and therefore even the resummation of the leading terms to all orders appears hopeless. Furthermore, as discussed above, the hadronization corrections may not be reliably under control even if preconfinement and LPHD are valid. This seems to be the case since, using the JADE jet resolution variable, i.e. the invariant mass, jet cross sections turn out to be affected by large hadronization corrections (see the OPAL Collaboration papers in ref. [1]) for all recombination schemes except the JADE one.

Although these features certainly do not invalidate the continued use of the original JADE jet algorithm, they do suggest that a modified algorithm that preserves exponentiation of the cross section would be worth investigating.

In the next section we describe a new jet algorithm [#1] in which relative transverse momentum replaces the invariant mass of the original JADE algorithm as the jet resolution variable. To emphasize this change of variable, we call the new algorithm the *$k_\perp$-algorithm*. The use of transverse momentum to resolve jets is suggested by the coherence properties of QCD soft emission [20,21] in order to preserve exponentiation and LPHD.

The results of our calculation are presented in section 3. We study the behaviour of multijet cross sections at small values of the resolution parameter y_{cut} using the $k_\perp$-algorithm. We show that leading and next-to-leading logarithms can be resummed to all orders in α_s for any number of jets and give simple results up to five jets.

[#1] This algorithm arose from discussions at the Durham Workshop on Jet Studies at LEP and HERA, December 1990, and is sometimes referred to as the Durham algorithm.

Details of our calculations and a smooth extrapolation between the regions of small and large y_{cut} are presented elsewhere [22].

2. The $k_\perp$-jet algorithm

The $k_\perp$-algorithm is defined in a similar way to the JADE algorithm, just replacing invariant mass by transverse momentum as the jet resolution variable.

The scaled transverse momentum, defined as

$$y_{kl}=2(1-\cos\theta_{kl})\min(E_k^2, E_l^2)/s\,, \tag{1}$$

is first calculated for every pair of final-state particles (k, l). Then the two particles (i, j) with the smallest value of y_{kl} are combined together and replaced by a "pseudoparticle" with four-momentum $p_{(ij)}$ (to be defined later) if their y_{ij} is smaller than some given resolution parameter y_{cut}. This procedure is repeated until all pairs of objects (particles and/or pseudoparticles) have $y_{kl}>y_{\mathrm{cut}}$. Whatever objects remain at this stage are called jets.

In the experimental as well as theoretical analyses several different recombination schemes may be used [2,6,7]. They differ among themselves in the way in which the four-momentum $p_{(ij)}$ of a pseudoparticle is defined in terms of the momenta (p_i, p_j) of the recombined particles and/or pseudoparticles. We can define E, P, E_0 and P_0 recombination schemes as in the JADE algorithm. In the E-scheme, the four-momenta are simply added together, giving $p_{(ij)}=p_i+p_j$, whereas in the P-scheme the energy of the pseudoparticle (ij) is rescaled so that it has zero invariant mass,

$$\boldsymbol{p}_{(ij)}=\boldsymbol{p}_i+\boldsymbol{p}_j, \quad E_{(ij)}=|\boldsymbol{p}_i+\boldsymbol{p}_j|\,. \tag{2}$$

In the E_0-scheme the three-momentum rather than the energy is rescaled to give zero invariant mass. In the P_0-scheme the four-momentum p_{ij} is defined as in eq. (2), but in computing y_{kl} the centre-of-mass energy $\sqrt{s}$ is replaced by the total energy of the pseudoparticles after each combination. Although these schemes can differ significantly at finite y_{cut}, they all give the same leading and next-to-leading logarithms at small y_{cut} [22]. Therefore the formulae we shall derive in the next section can be used to resum these logarithms for any of the common recombination schemes, provided the $k_\perp$-algorithm is used to define the jets.

3. Calculation

3.1. Master equations

According to the coherent branching formalism [20,21,23], the generating function for jets originating from a parton of type a, defined according to the $k_\perp$-algorithm discussed above, satisfies the equation [22]

$$\phi_{\mathrm{a}}(Q, Q_0; u)=u+\sum_b\int_0^Q\frac{\mathrm{d}\tilde{q}}{\tilde{q}}\int_0^1\frac{\mathrm{d}z}{2\pi}\alpha_{\mathrm{s}}[z(1-z)\tilde{q}]P_{\mathrm{a}\to\mathrm{bc}}(z)\Theta(\min[z, (1-z)]\,\tilde{q}-Q_0)$$
$$\times[\phi_{\mathrm{b}}(z\tilde{q}, Q_0; u)\phi_{\mathrm{c}}((1-z)\tilde{q}, Q_0; u)-\phi_{\mathrm{a}}(\tilde{q}, Q_0; u)]\,. \tag{3}$$

$P_{\mathrm{a}\to\mathrm{bc}}(z)$ are the Altarelli–Parisi splitting functions and the coherent evolution variable is

$$\tilde{q}=\frac{q_{\mathrm{t}}}{z(1-z)}\,, \tag{4}$$

where q_{t} is the transverse momentum and z the longitudinal momentum fraction in the branching a→bc. Thus in the small-angle approximation, which is adequate for leading and next-to-leading logarithms, the Θ-function in eq. (3) enforces the resolution constraint $y_{bc} > y_{cut} = Q_0^2/Q^2$ where $Q \sim \sqrt{s}$ is the scale of the jet production process. The corresponding m-jet fraction of the total cross section is then obtained by differentiating m times at $u=0$:

$$R_m^{(a)}(y_{cut} = Q_0^2/Q^2) = \frac{1}{m!}\left(\frac{\partial}{\partial u}\right)^m \phi_a(Q, Q_0; u)\bigg|_{u=0} . \tag{5}$$

To next-to-leading order (i.e. neglecting terms that will be down by two powers of $\log y_{cut}$ for every power of α_s after integration), eq. (3) gives for quark jets

$$\phi_q(Q, Q_0; u) = u + \int_{Q_0}^{Q} \frac{d\tilde{q}}{\tilde{q}} \int_{Q_0/\tilde{q}}^{1} dz\, \alpha_s(z\tilde{q}) \frac{C_F}{\pi}\left(\frac{2}{z} - \frac{3}{2}\right)[\phi_g(z\tilde{q}, Q_0; u) - 1]\phi_q(\tilde{q}, Q_0; u) . \tag{6}$$

This equation has the solution

$$\phi_q(Q, Q_0; u) = \frac{2C_F}{\pi} \frac{\alpha_s(q)}{q}\left(\log\frac{Q}{q} - \frac{3}{4}\right). \tag{7}$$

For gluon jets the equation corresponding to (6) is

$$\phi_q(Q, Q_0; u) = u + \int_{Q_0}^{Q} \frac{d\tilde{q}}{\tilde{q}} \int_{Q_0/\tilde{q}}^{1} dz\, \alpha_s(z\tilde{q}) \frac{C_A}{\pi}\left(\frac{2}{z} - \frac{11}{6}\right)[\phi_g(z\tilde{q}, Q_0; u) - 1]\phi_g(\tilde{q}, Q_0; u)$$

$$+ \int_{Q_0}^{Q} \frac{d\tilde{q}}{\tilde{q}} \alpha_s(\tilde{q}) \frac{N_f}{3\pi} [\phi_q(\tilde{q}, Q_0; u)^2 - \phi_g(\tilde{q}, Q_0; u)] . \tag{9}$$

Substituting the solution (7) for ϕ_q and performing some manipulations, we obtain the convenient form

$$\phi_g(Q, Q_0; u) = u \exp\left(\int_{Q_0}^{Q} dq\, \{\Gamma_g(Q, q)[\phi_g(q, Q_0; u) - 1] - \Gamma_f(q)\}\right)$$

$$\times\left[1 + u \int_{Q_0}^{Q} dq\, \Gamma_f(q) \exp\left(\int_{Q_0}^{q} dq'\, \{[2\Gamma_q(q, q') - \Gamma_g(q, q')][\phi_g(q', Q_0; u) - 1] + \Gamma_f(q')\}\right)\right], \tag{10}$$

where

$$\Gamma_g(Q, q) = \frac{2C_A}{\pi} \frac{\alpha_s(q)}{q}\left(\log\frac{Q}{q} - \frac{11}{12}\right), \tag{11}$$

$$\Gamma_f(q) = \frac{N_f}{3\pi} \frac{\alpha_s(q)}{q} . \tag{12}$$

3.2. All-orders expressions for e^+e^- multijet fractions

To the precision required for a next-to-leading logarithmic calculation, the generating function for e^+e^- annihilation is simply that for a pair of quark jets. Thus the corresponding multijet fractions are

$$R_m^{(e^+e^-)}(y_{cut}) = \frac{1}{m!}\left(\frac{\partial}{\partial u}\right)^m [\phi_q(Q, Q_0; u)]^2\bigg|_{u=0} . \tag{13}$$

Introducing the quark and gluon Sudakov form factors,

$$\Delta_q(Q)=\exp\left(-\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\right), \tag{14}$$

$$\Delta_g(Q)=\exp\left(-\int_{Q_0}^{Q} dq\,[\Gamma_g(Q,q)+\Gamma_f(q)]\right), \tag{15}$$

which depend implicitly on y_{cut} via $Q_0=Q\sqrt{y_{cut}}$, we find to next-to-leading logarithmic order in y_{cut}

$$R_2^{(e^+e^-)}=[\Delta_q(Q)]^2, \tag{16}$$

$$R_3^{(e^+e^-)}=2[\Delta_q(Q)]^2\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q), \tag{17}$$

$$R_4^{(e^+e^-)}=2[\Delta_q(Q)]^2\Bigg[\left(\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\right)^2+\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(q')$$
$$+\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\int_{Q_0}^{q} dq'\,\Gamma_f(q')\Delta_f(q')\Bigg], \tag{18}$$

$$R_5^{(e^+e^-)}=[\Delta_q(Q)]^2\Bigg\{\tfrac{4}{3}\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\Bigg[\left(\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\right)^2$$
$$+3\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(g')+3\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\int_{Q_0}^{q} dq'\,\Gamma_f(q')\Delta_f(q')\Bigg]$$
$$+\int_{Q_0}^{Q} dq\,\Gamma_q(Q,q)\Delta_g(q)\Bigg[\left(\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(g')\right)^2+2\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(q')\int_{Q_0}^{q'} dq''\,\Gamma_g(q',q'')\Delta_g(q'')$$
$$+4\int_{Q_0}^{q} dq'\,\Gamma_g(q,q')\Delta_g(q')\int_{Q_0}^{q'} dq''\,\Gamma_f(q'')\Delta_f(q'')$$
$$+2\int_{Q_0}^{q} dq'\,\Gamma_f(q')\Delta_f(q')\int_{Q_0}^{q'} dq''\,[2\Gamma_q(q',q'')-\Gamma_g(q',q'')+\Gamma_g(q,q'')]\Delta_g(q'')\Bigg]\Bigg\}, \tag{19}$$

where

$$\Delta_f(q)=[\Delta_q(q)]^2/\Delta_g(q). \tag{20}$$

3.3. Comparisons with fixed-order calculations

Expanding the results of subsection 3.2 to $O(\alpha_s^2)$, we find

Table 1

m	$A_2^{(m)}$	$A_1^{(m)}$	$B_4^{(m)}$	$B_3^{(m)}$
2	$-\frac{1}{2}C_F$	$+\frac{3}{2}C_F$	$+\frac{1}{8}C_F^2$	$-\frac{3}{4}C_F^2-\frac{11}{36}C_FC_A+\frac{1}{18}C_FN_f$
3	$+\frac{1}{2}C_F$	$-\frac{3}{2}C_F$	$-\frac{1}{4}C_F^2-\frac{1}{48}C_FC_A$	$+\frac{3}{2}C_F^2+\frac{7}{12}C_FC_A-\frac{1}{12}C_FN_f$
4	0	0	$+\frac{1}{8}C_F^2+\frac{1}{48}C_FC_A$	$-\frac{3}{4}C_F^2-\frac{5}{18}C_FC_A+\frac{1}{36}C_FN_f$

$$R_m^{(e^+e^-)}=\delta_{2m}+\frac{\alpha_s}{\pi}\left[A_2^{(m)}L^2+A_1^{(m)}+O(1)\right]+\left(\frac{\alpha_s}{\pi}\right)^2\left[B_4^{(m)}L^4+B_3^{(m)}L^3+O(L^2)\right], \tag{21}$$

where $L=\ln(1/y_{\rm cut})$ and the coefficients are as given in table 1.

We have checked explicitly that all these coefficients are in agreement with those obtained by fitting the asymptotic behaviour of the corresponding fixed-order multijet cross sections, evaluated numerically. For this purpose we used the same computer program EVENT [24] as was used to calculate the multijet fractions tabulated in ref. [7], modified to use the $k_\perp$-algorithm and to extend to very small values of $y_{\rm cut}$.

The results in eqs. (16)–(18) may be combined with the fixed-order results, after subtraction of the terms that have been exponentiated, to obtain predictions for up to four jets over the full range of $y_{\rm cut}$, while eq. (19) can be used to estimate the five-jet cross section in the small-$y_{\rm cut}$ region where it is largest [22]. The five-jet prediction could be extended to higher $y_{\rm cut}$ by applying the $k_\perp$-algorithm to the five-parton matrix elements [25].

After this work was completed we received ref. [26], which also considers the resummation of large $\log y_{\rm cut}$ terms for the new $k_\perp$-algorithm.

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