VE401 Probabilistic Methods in Eng.
RC 4

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March 25, 2020

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Definitions

Suppose A is a black box unit.

- ***Failure density*** f_A: distribution of the time T that A fails.
- ***Reliability function*** R_A: the probability that A is working at time t, $R_A(t) = 1 - F_A(t)$.
- ***Hazard rate*** ρ_A:

$$\begin{aligned}
\rho_A(t) &:= \lim_{\Delta t \to 0} \frac{P[t \leq T \leq t + \Delta t | t \leq T]}{\Delta t} \\
&= \lim_{\Delta t \to 0} \frac{P[t \leq T \leq t + \Delta t]}{P[T \geq t] \cdot \Delta t} = \frac{f_A(t)}{R_A(t)}, \\
R_A(t) &= e^{-\int_0^t \rho_A(x) \mathrm{d}x}.
\end{aligned}$$

One often has information on ρ_A, but not F_A or R_A.

Series and Parallel Systems

- Series system with k components.

$$R_s(t) = \prod_{i=1}^{k} R_i(t),$$

where R_i is the reliability of the i-th component.

- Parallel system with k components.

$$R_p(t) = 1 - \prod_{i=1}^{k} (1 - R_i(t)).$$

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- Density function. $\alpha, \beta > 0$ are parameters,

$$f(x) = \begin{cases} \alpha\beta x^{\beta-1} e^{-\alpha x^{\beta}}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

- Mean.

$$\mu = \alpha^{-1/\beta}\Gamma(1 + 1/\beta).$$

- Variance.

$$\sigma^2 = \alpha^{-2/\beta}\Gamma(1 + 2/\beta) - \mu^2.$$

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Definitions

- ***Statistics*** aims to gain information about the parameters of a distribution by conducting experiments.
- ***Population***: a large collection of instances which we want to describe probability.
- ***Random sample of size n from distribution of X***: a collection of n independent random variables $X_1, \dots, X_n$, each with the same distribution as X. ($\Leftrightarrow$ n i.i.d. random variables.)
- ***x-th percentiles***: d_x such that $x\%$ of values in sampled data are less than or equal to d_x. (***first, second, third quartile*** $\Rightarrow$ $x = 25, 50, 75$.)
- ***Interquartile range***: $\mathrm{IQR} = q_3 - q_1$, measures the dispersion of the data.
- ***Precision***: smallest decimal place of data $\{x_1, \dots, x_n\}$.
- ***Sample range***: $\max\{x_i\} - \min\{x_i\}$.

Visualization — Histograms

Choose bin width / number of bins.

1. Sturges's rule.

$$k = \lceil \log_2(n) \rceil + 1, \qquad h = \frac{\max\{x_i\} - \min\{x_i\}}{k},$$

   rounding ***up*** to the precision of the data.

2. Freedman-Diaconis rule.

$$h = \frac{2 \cdot \mathrm{IQR}}{\sqrt[3]{n}}.$$

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Thanks for your attention!