University of Glasgow

6G WIRELESS TECHNOLOGIES

Antenna Design for 6G Applications

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- Antennas and Radiation
- Potential Functions
- Antenna Characteristics
- Antenna Arrays
- Array Analysis
  - Uniform linear arrays
  - Non-uniform arrays

- A distribution of currents and charges can generate and radiate electromagnetic fields
  - The distribution is typically localised in a region of space
  - As an example, a simple wire can act as an *antenna*
- We are interested in determining the electromagnetic fields in space, given a current distribution

- Antennas are most widely used for wireless communications
- Modern antenna invention is attributed to Heinrich Hertz (1887)
  - Radio system was developed by Guglielmo Marconi (1897)
- Due to the *duality* principle, an antenna can also act as a receiver to EM radiation
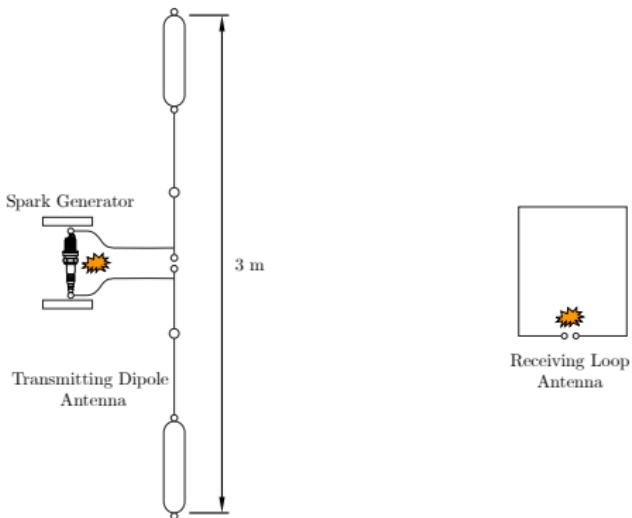

Figure 1: The Hertz's invention

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- We need a disturbance in the EM fields
  - Most commonly, this is caused by a time-varying electric current
- The disturbance also depends on the nature of the antenna
 - For a wire antenna, the discontinuities at the ends cause radiation
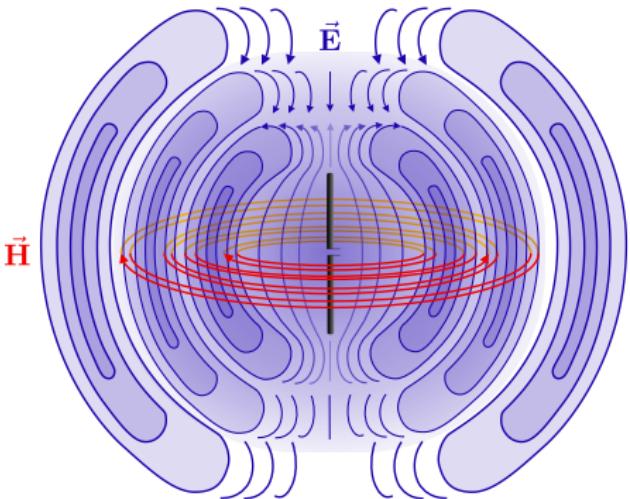
Figure 2: Antenna Radiation Mechanism

- There are mainly two ways to find the radiated fields from a given current distribution
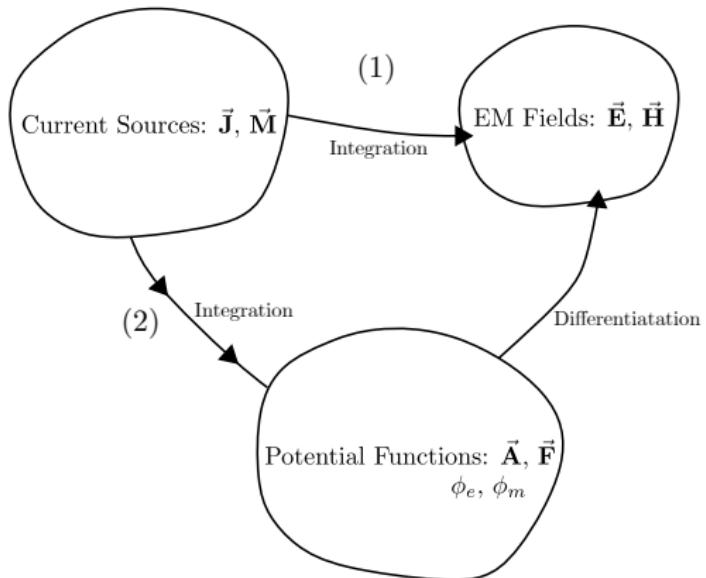

Figure 3: Two ways to find the radiated EM fields

- Solving EM fields directly using Maxwell's equations is often very difficult, especially in the spatial domain
- The introduction of scalar (ϕ) and vector $\vec{A}$ potential functions simplify the process
- We start from the fact:
  - Magnetic field is divergence-less ($\boldsymbol{\nabla} \cdot \vec{B} = 0$). We can, therefore, say that:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \vec{A} \equiv 0$$
$$\Rightarrow \vec{H} = \frac{1}{\mu}\boldsymbol{\nabla} \times \vec{A}$$

We can write Ampere's law as:

$$\boldsymbol{\nabla} \times \vec{E} = -j\omega\mu\vec{H} = j\omega\boldsymbol{\nabla} \times \vec{A}$$
$$\boldsymbol{\nabla} \times \left(\vec{E} + j\omega\vec{A}\right) = 0$$

Knowing that the $\boldsymbol{\nabla} \times (-\boldsymbol{\nabla}\phi) \equiv 0$, we set:

$$\vec{\mathbf{E}} + j\omega\vec{\mathbf{A}} = -\boldsymbol{\nabla}\phi$$
$$\vec{\mathbf{E}} = -\boldsymbol{\nabla}\phi - j\omega\vec{\mathbf{A}}$$

· ϕ is the electric scalar potential and it's a function of position.
· If we know $\vec{\mathbf{A}}$ and ϕ, we can find $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$

- We still need to figure out how to find the potentials, $\vec{\mathbf{A}}$ and ϕ for a given current density $\vec{\mathbf{J}}$.
- For this we move back to Maxwell's equations and find a relationship

$$\boldsymbol{\nabla} \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} + \vec{\mathbf{J}}$$

$$\boldsymbol{\nabla} \times \left(\boldsymbol{\nabla} \times \vec{\mathbf{A}}\right) = j\omega\mu\varepsilon\vec{\mathbf{E}} + \mu\vec{\mathbf{J}}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \vec{\mathbf{A}} = j\omega\mu\varepsilon\left(-j\omega\vec{\mathbf{A}} - \boldsymbol{\nabla}\phi\right) + \mu\vec{\mathbf{J}}$$

Continuing and using the vector identity,
$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \vec{\mathbf{A}} = \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \vec{\mathbf{A}} - \nabla^2\vec{\mathbf{A}}\right)$ and rearranging, we get,

$$\nabla^2\vec{\mathbf{A}} + \omega^2\mu\varepsilon\vec{\mathbf{A}} = -\mu\vec{\mathbf{J}} + \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \vec{\mathbf{A}} + j\omega\mu\varepsilon\phi\right)$$

The solution is complete by defining $\vec{\mathbf{A}}$ in terms of ϕ through the Lorentz gauge,

$$\boldsymbol{\nabla} \cdot \vec{\mathbf{A}} = -j\omega\mu\varepsilon\phi$$

The magnetic vector potential $\vec{\mathbf{A}}$ is finally expressed through an inhomogeneous vector wave equation:

$$\nabla^2\vec{\mathbf{A}} + k^2\vec{\mathbf{A}} = -\mu\vec{\mathbf{J}}$$

- Given an electric current density $\vec{\mathbf{J}}$
- Solve for the magnetic vector potential $\vec{\mathbf{A}}$
  - Solve for $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$

There are some assumptions in this method, namely:

- The space is homogeneous (only one material)
- The magnetic current density $\vec{\mathbf{M}}$ is zero.

- We solve $\vec{\mathbf{A}}$ individually in terms of the scalar components (A_x, A_y, A_z)
- For a forcing function p, the general solution (in terms of ψ) can be written as:

$$\nabla^2\psi + k^2\psi = -p \tag{1}$$

For a point source ($p = \delta(\vec{\mathbf{r}})$), the solution of the above equation is called the *impulse response.* And this impulse response is also called the Green function of the differential equation

- For a delta function, the solution of 1 is 0 everywhere except at the origin.
- Due to spherical symmetry, it is better to express the problem in the spherical coordinates.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = -k^2\psi$$

Substituting $\psi = G/r$, we get:

$$\frac{\partial^2 G}{\partial r^2} = k^2 G$$

$$G = C_1 e^{-jkr} + C_2 e^{+jkr}$$

In terms of ψ the solution becomes:

$$\psi = \frac{G}{r} = \frac{C_1}{r}e^{-jkr} + \frac{C_2}{r}e^{+jkr}$$

For sources displaced from the origin, we use:

- The fundamental type of antenna is the point electric dipole also known as the *The Hertzian Dipole*
- The current of a z-directed Hertzian dipole is expressed as:

$$J_z(\vec{\mathbf{r}}) = \widehat{\mathbf{z}} I \, dl \, \delta(\vec{\mathbf{r}})$$

The magnetic vector potential is given as:

$$\vec{\mathbf{A}} = \widehat{\mathbf{z}} \mu \frac{I}{4\pi} \, dl \, \frac{e^{-jkr}}{r}$$

For an arbitrary source, we have:

$$\vec{\mathbf{A}} = \frac{\mu}{4\pi} \int_V \vec{\mathbf{J}} \, dv' \, \frac{e^{-jkr}}{r}$$
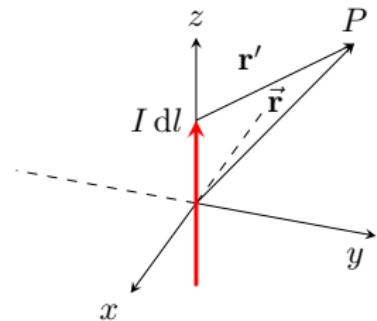
Figure 4: The Hertzian Dipole

ANTENNA CHARACTERISTICS AND PARAMETERS

- A graphical representation of the *far-field* radiation properties
- Pattern can be further described in *E-* (E_θ)and *H-* (H_ϕ) planes.

For a Hertzian dipole, the far-fields ($kr \gg 1$) are given as:

$$\vec{\mathbf{E}} = \hat{\boldsymbol{\theta}}\frac{j\omega\mu l\,\mathrm{d}l}{4\pi r}\sin(\theta)$$

$$\vec{\mathbf{H}} = \hat{\phi}\frac{jkl\,\mathrm{d}l}{4\pi r}\sin(\theta)$$
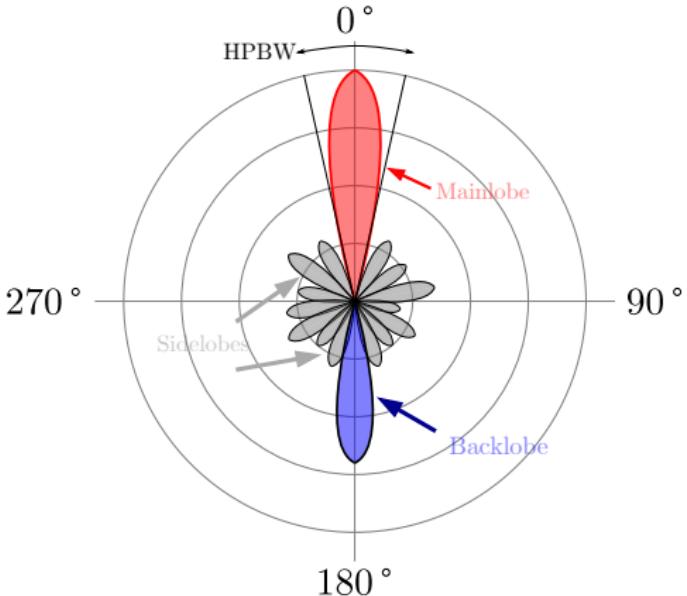
Figure 5: The Radiation Pattern

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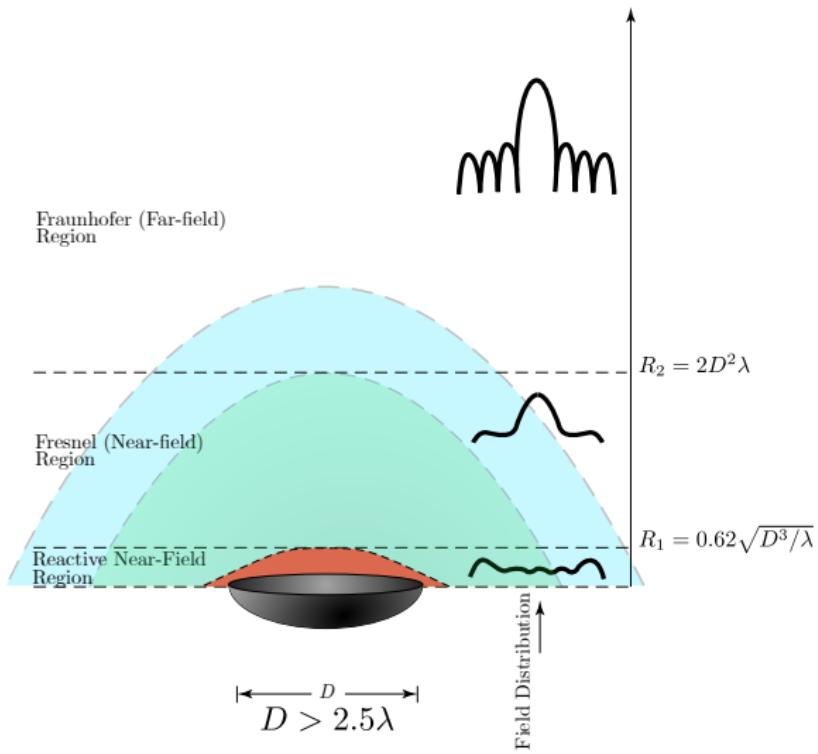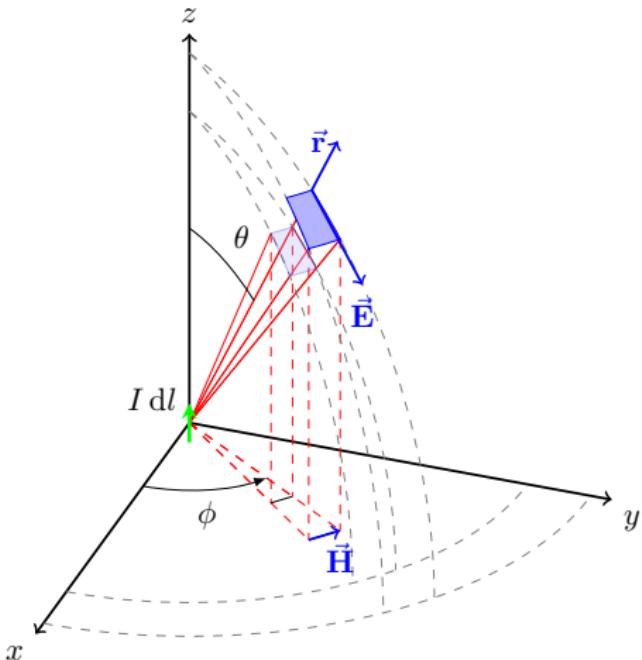
Figure 6: The Regions of Antenna Radiation
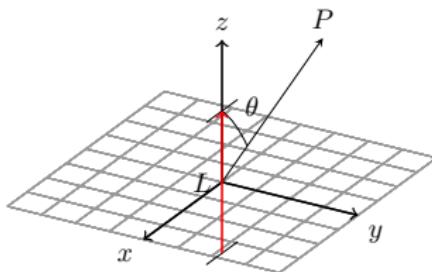
· The graphical representation is easier in the spherical coordinates
· We apply the Cartesian to Spherical coordinate transformation

- A line source with a uniform current along its extent
- Say the line is z-directed and centred on the origin
- The length of the line source is *L*

$$I(z') = \begin{cases} I_0 & x' = 0, \quad y = 0, \quad |z'| \le \frac{L}{2} \\ 0 & \text{elsewhere} \end{cases}$$

- As the current is only in the z-direction, we only find the A_z component
- For z-directed sources, $R \approx r - z' \cos\theta$ for the phase term and $R \approx r$ in the magnitude term

$$
\begin{aligned}
A_z &= \mu \int_{-L/2}^{L/2} I(z') \frac{\mathrm{e}^{-jkR}}{4\pi R} \,\mathrm{d}z' \\
&= \mu \frac{\mathrm{e}^{-jkr}}{4\pi r} \int_{-L/2}^{L/2} I_0 \mathrm{e}^{jk(z' \cos\theta)} \,\mathrm{d}z' \\
&= \mu \frac{I_0 \mathrm{e}^{-jkr}}{4\pi r} \frac{\sin\left[(kL/2)\cos\theta\right]}{(kL/2)\cos\theta}
\end{aligned}
$$

The electric field is given as:

$$\vec{\mathbf{E}} = -j\omega\vec{\mathbf{A}} - \frac{j}{\omega\mu\varepsilon}\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\vec{\mathbf{A}}\right)$$

$$= -j\omega\vec{\mathbf{A}} - (-j\omega\vec{\mathbf{A}}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}}$$

$$= j\omega\sin\theta A_z\hat{\boldsymbol{\theta}}$$

$$= \frac{j\omega\mu I_0 L e^{-jkr}}{4\pi r}\sin\theta\frac{\sin\left[(kL/2)\cos\theta\right]}{(kL/2)\cos\theta}\hat{\boldsymbol{\theta}}$$

The magnetic field can simply be found as:

$$H_\phi = \frac{E_\theta}{\eta}$$

- This describes the *complex power density* flowing out of a sphere of radius *r*
- It is real-valued and directed along the wave propagation direction

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}} \times \vec{\mathbf{H}}^*$$

- If $\vec{\mathbf{E}}$ is in the $\hat{\boldsymbol{\theta}}$ and $\vec{\mathbf{H}}$ is in the $\hat{\boldsymbol{\phi}}$ directions
- The Poynting vector will be radially directed.

- Just like voltage and current ratio gives us impedance
- The ratio of electric and magnetic field components gives us the *intrinsic impedance*

$$\frac{E_\theta}{H_\phi} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

For free space, the value is $\eta_0 = 376.7\,\Omega \approx 120\pi\,\Omega$

- The total power radiated by an antenna can be found from the Poynting vector
- We need to integrate over a surface

$$P = \iint_{S'} \vec{\mathbf{S}} \cdot \mathrm{d}S' = 1/2 \operatorname{Re} \iint_{S'} \left(\vec{\mathbf{E}} \times \vec{\mathbf{H}}^* \right) \cdot \mathrm{d}S'$$

The $\mathrm{d}S'$ in spherical coordinates refers to a solid angle and for a given radius r can be expressed as:

$$\mathrm{d}S' = r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

- Since the power varies with distance *r*, it is convenient to define the *radiation intensity*
- The radiation intensity is independent of the distance
- It is defined as the power radiated in a given direction per unit solid angle
  - It has units of watts per steradians

$$U(\theta, \phi) = \frac{1}{2} \operatorname{Re}\left(\vec{\mathbf{E}} \times \vec{\mathbf{H}^*} \right) \cdot r^2 \widehat{\mathbf{r}}$$

- For a given antenna the directivity and gain describe in what direction the radiation is, as compared to an isotropic antenna
- For the isotropic antenna, the radiation pattern is uniform (i.e.) a circle
- Directivity is defined as the ratio of radiation intensity in a certain direction to the average radiation intensity

$$D = \frac{1}{2} \frac{\max \left[\mathrm{Re}(\vec{\mathbf{E}} \times \vec{\mathbf{H}}^*) \cdot \hat{\mathbf{r}} \right]}{P/4\pi r^2}$$
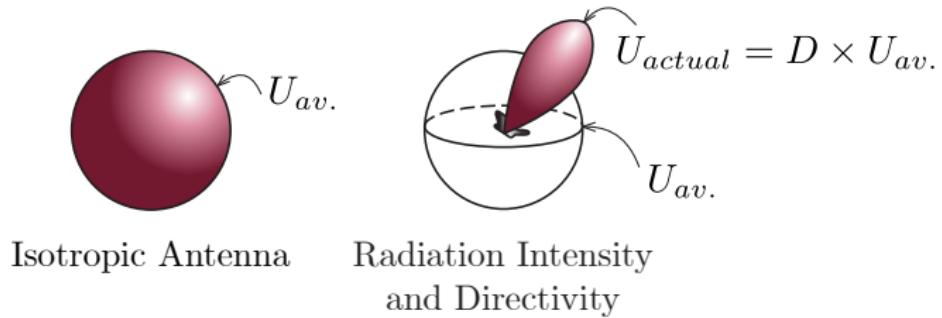
Figure 7: Relationship between Radiation Intensity and Directivity

- Although the directivity describes the radiation pattern of an antenna, we need a quantity that can be used when treating the antenna as a system
- Suppose the antenna is one component of a radio-frequency system that includes transmission lines and sources
- A parameter is helpful that determines how *efficiently* the antenna operates
  - In particular, how much input power is transferred into radiated power
- Antenna gain is defined as,

$$G = 4\pi \frac{U_m}{P_m}$$

We often describe it in terms of decibels:

$$G_{\mathrm{dB}} = 10 \log G$$

· We can treat an antenna as an impedance with real and imaginary parts
  · The real part refers to how much radiation leaves the antenna (R_r) and how much dissipates as losses (R_o)
  · The imaginary part (X_A) determines the stored power in the near field.

$$Z_A = R_A + jX_A = (R_r + R_{ohm}) + jX_A$$

- Efficiency is a metric that determines the ratio of total desired power to the total power supplied
- Radiation efficiency of antennas is a measure of how much power is radiated

$$e_{rad} = \frac{P_{rad}}{P_{in}} = \frac{P_{rad}}{P_{rad} + P_{ohm}}$$

- In individual antenna elements, we can't control the radiation patterns
- If we *combine* two antenna elements, it is possible to change the pattern significantly
  - We call the new combined structure as an antenna array.
- We achieve higher directivity using antenna arrays
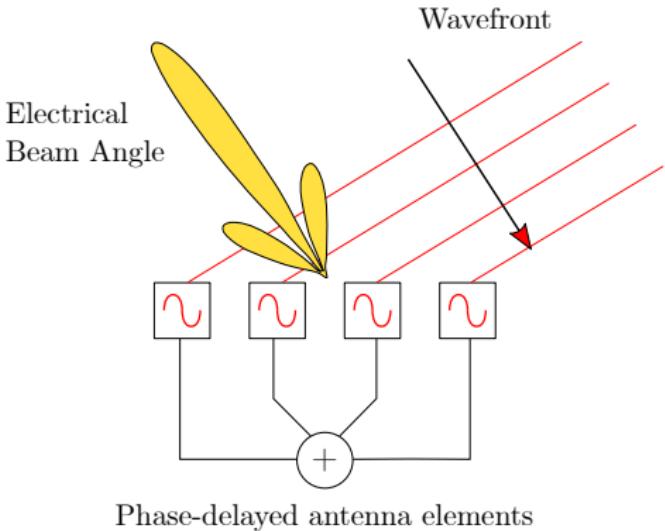
Figure 8: Typical antenna array with phased elements.

- Modern Communications Applications
  - The goal is to focus EM energy towards the target population (cars, people, cities etc.)
    - Modern wireless communications using beamforming
- Radar - multiple target tracking
  - We would like to focus the energy on the targets as they move
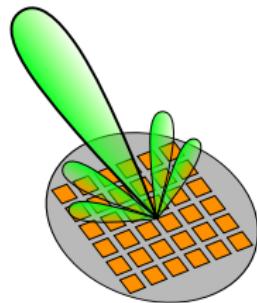
Figure 9: A Phased array antenna in RADAR based target tracking.
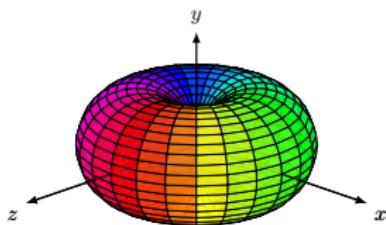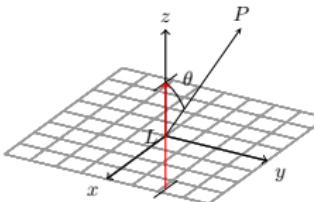
· Recall from the antenna introduction, the field pattern of an infinitesimal dipole:

$$\vec{\mathbf{E}} = \hat{\boldsymbol{\gamma}} jk\eta I_0 l \frac{\exp(-jkr)}{4\pi r} \sin\gamma$$
$$= jk\eta \vec{\mathbf{h}} G(r)$$

where $\vec{\mathbf{h}} = \hat{\boldsymbol{\gamma}} I_0 l \sin\gamma$ and $G(r)$ is the free-space Green function for a point source, $\exp(jkr)/(4\pi r)$

· We will use this as the antenna element

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· There are some factors that determine the desired radiation pattern
  · Array Geometry
  · Element Spacing
  · Element Excitation Amplitude
  · Pattern of individual element

The total field is given by:

$$E_{total} = \text{Element Factor} \times \text{Array Factor}$$

- The simplest antenna array contains two elements
- Two analyse an array, we start by using point sources as individual elements
  - The final pattern is obtained by multiplication
- First, we will ignore the mutual coupling between elements.
- Consider an array of two point sources separated by a distance d on the z-axis.
- Assuming that both the antenna elements are excited by the current $I_1 = I_0 \exp(j\alpha/2)$ and $I_2 = I_0 \exp(-j\alpha/2)$ where $0 \leq \alpha \leq 2\pi$.
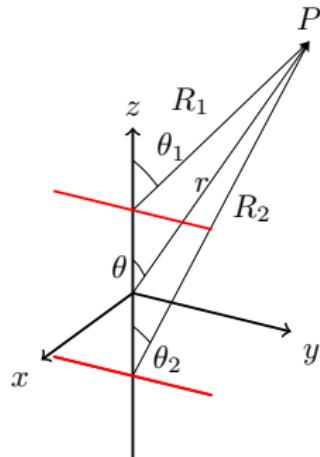
Figure 10: Two point sources forming a basic antenna array.

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Neglecting any mutual coupling, we obtain the total fields by simple vector summation:

$$\vec{E_t} = \vec{E_1} + \vec{E_2}$$
$$= \hat{\gamma} j k \eta \frac{I_0 \ell}{4\pi} \left\{ \frac{e^{-jkR_1}}{R_1} e^{+j\alpha/2} \sin\gamma_1 + \frac{e^{-jkR_2}}{R_2} e^{-j\alpha/2} \sin\gamma_2 \right\}$$

We use the *far-field* approximation:

$$\gamma_1 \approx \gamma_2 \approx \gamma$$
$$R_1 \approx r - \frac{d}{2}\cos\theta$$
$$R_2 \approx r + \frac{d}{2}\cos\theta$$
$$R_1 \approx R_2 \approx r \,(\text{amplitude term})$$

The total *far-field* thus becomes:

$$
\begin{aligned}
\vec{\mathbf{E_t}} &= \hat{\gamma} j k \eta \frac{I_0 \ell}{4\pi r} \sin \gamma \left\{ \mathrm{e}^{-jk\frac{d}{2}\cos\theta} \mathrm{e}^{-j\alpha/2} + \mathrm{e}^{-jk\frac{d}{2}\cos\theta} \mathrm{e}^{+j\alpha/2} \right\} \\
&= jk\eta \vec{\mathbf{h}} G(r) \left\{ \mathrm{e}^{\frac{kd\cos\theta+\alpha}{2}} + \mathrm{e}^{-\frac{kd\cos\theta+\alpha}{2}} \right\} \\
&= \underbrace{jk\eta \vec{\mathbf{h}} G(r)}_{\text{Element Factor}} \underbrace{2\cos\left[\frac{1}{2}\left(kd\cos\theta+\alpha\right)\right]}_{\text{Array Factor}}
\end{aligned}
$$

We can control and change the pattern by varying *d* and α, which are the spacing and phase shifts.

Let's look at different cases where we consider different values of *d* and α.

· $\alpha = 0°$ and $d = \lambda/4$, for which the array factor (AF) is
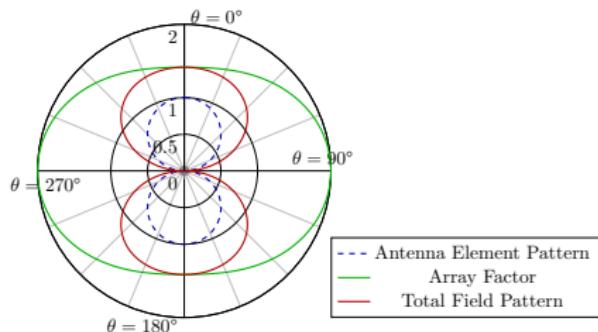
$$AF = 2\cos\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right] = 2\cos\left(\frac{\pi}{4}\cos\theta\right)$$

Figure 11: The total field pattern for $\alpha = 0°$ and $d = \lambda/4$.

· Now looking at $\alpha = 90°$ and $d = \lambda/4$, for which the array factor (AF) is
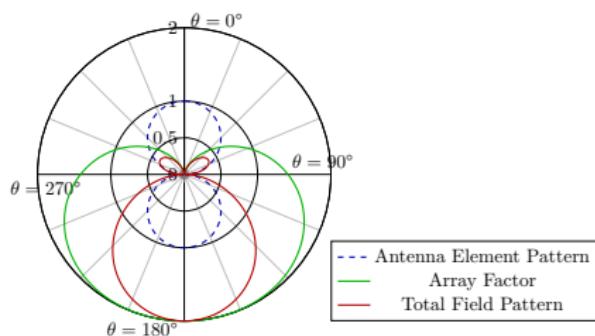
$$AF = 2\cos\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right] = 2\cos\left(\frac{\pi}{4}\cos\theta + \frac{\pi}{4}\right)$$

Figure 12: The total field pattern for $\alpha = 90°$ and $d = \lambda/4$.

· Now looking at $\alpha = -90°$ and $d = \lambda/4$, for which the array factor (AF) is

$$AF = 2\cos\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right] = 2\cos\left(\frac{\pi}{4}\cos\theta - \frac{\pi}{4}\right)$$
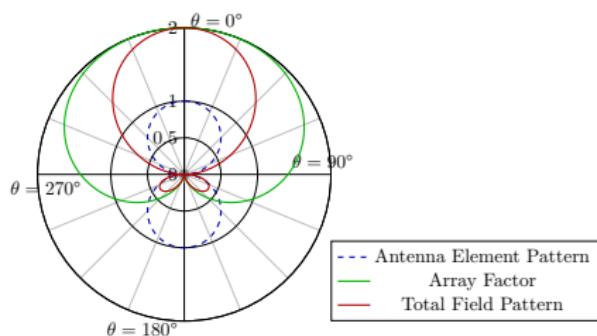
Figure 13: The total field pattern for $\alpha = -90°$ and $d = \lambda/4$.

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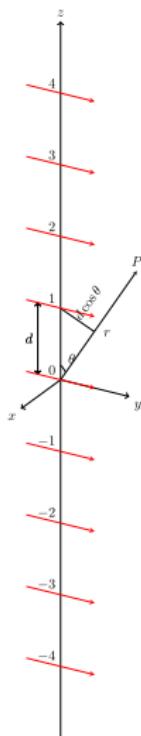
- A uniform array consists of equally spaced and identical elements
  - All elements are excited with *same* amplitude
  - However, the elements have a progressive phase shift.
- As before, the total field is the vector sum of the individual elements:

$$\vec{\mathbf{E}}_T = \vec{\mathbf{E}}_0 + \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_{-1} + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_{-2} + \ldots$$

$$= jk\eta\vec{\mathbf{h}}G(r)\left(1 + e^{j\psi} + e^{-j\psi} + e^{2j\psi} + e^{-2j\psi} + \ldots\right)$$

where $\psi = kd\cos\theta + \alpha$

Continuing, we can write the AF as:

$$AF = \sum_{n=-N'}^{N'} e^{jn\psi} \quad \text{where } N' = \frac{(N-1)}{2} \tag{2}$$

We can also express (2) as:

$$AFe^{jn\psi} = \sum_{n=-N'}^{N'} e^{j(n+1)\psi} \tag{3}$$

From (3) and (2) we get:

$$AF = \frac{e^{j(N+1)\frac{\psi}{2}} - e^{-j(N-1)\frac{\psi}{2}}}{e^{j\psi} - 1}$$

$$= \frac{e^{j\psi/2}}{e^{j\psi/2}} \left[\frac{e^{jN\psi/2} - e^{-jN\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}} \right]$$

$$AF = \frac{\sin N\psi/2}{\sin \psi/2} \text{ where } \psi = kd\cos\theta + \alpha$$

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Noting that the above expression resembles a sinc function, we can extend this to find the AF of any discrete array made of uniformly spaced elements. A major difference, however, is the sidelobes don't decay with the increasing function argument.

In general, we plot the AF in the normalised form (divide by N).
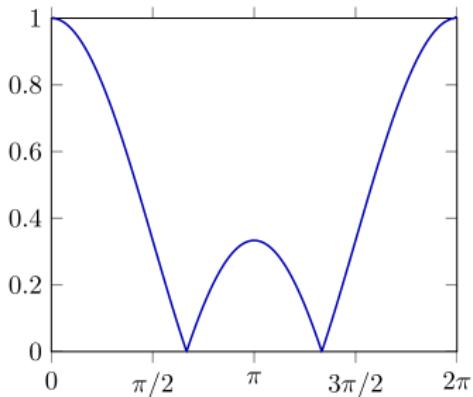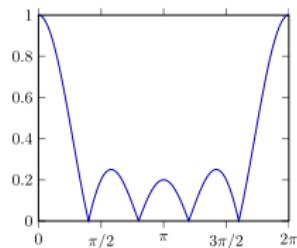
Figure 14: The normalised AF of a 3 element array.
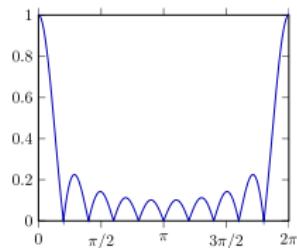
- As *N* increases, the main lobe narrows
- We get more side lobes in one period of *AF* as *N* increases
- The width of the minor/side lobes is $2\pi/N$
- The side lobe height decreases as we increase *N*
- *AF* is symmetric about π
- We also see that the peak value occurs at $\psi = \pm 2n\pi$ for $n = 0, 1, 2, \ldots$
- The nulls occur at $\psi = \pm 2n\pi/N$
- The side lobe level is defined as:

$$\text{SLL} = \frac{\text{Max side lobe value}}{\text{Max Main lobe value}}$$

(a) $N = 5$

(b) $N = 10$

Figure 15: Array factors for 5 and 10 element uniform array.

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A four-element ($N = 4$), uniformly excited, equally spaced array. The spacing d is $\lambda/2$ and the interelement phasing α is $90°$.
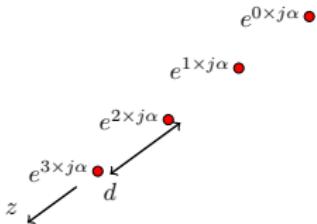
Figure 16: A four element antenna array.

The normalised array factor is given by:

$$AF = 1/4 \frac{\sin 4\psi/2}{\sin \psi/2} \text{ where } \psi = \frac{2\pi}{\lambda} \frac{\lambda}{2} \cos \theta + 90°$$
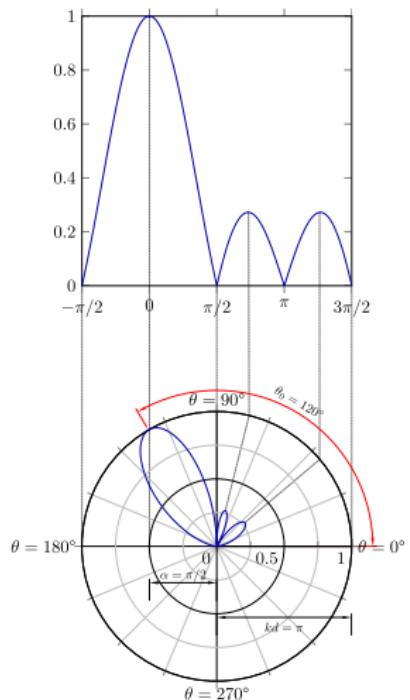
- The linear plot is done as before
- We can plot the corresponding polar plot by *translating* peaks and main and side lobes
- We first *shift* the polar plot by an amount α
  - One period of the *AF* is considered

The direction of the main beam in the polar plot is found as:

$$\psi = kd \cos\theta + \alpha$$

$$\theta = \arccos \frac{\psi - \alpha}{kd}$$

$$\theta_0 = \arccos\left(\frac{0 - \pi/2}{\pi}\right) = 120°$$

In general there are two extreme cases
which are sometimes used:

1. Broadside ($\theta_0 = 90°$) when $\alpha = 0$

2. Endfire ($\theta_0 = 0°$ or $180°$) when $\alpha = \pm kd$

The array pattern is often characterised
by *beamwidth between first nulls*.
Knowing that the nulls occur at:
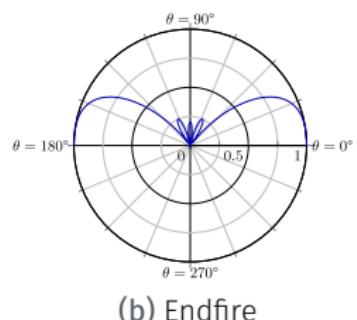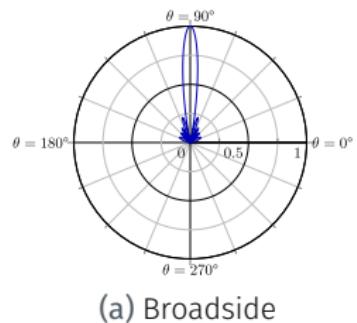
$$N\psi_{FN}/2 = \pm\pi$$

For broadside array, $\quad \dfrac{N}{2}\dfrac{2\pi}{\lambda}d\cos\theta_{FN} = \pm\pi$

$$\theta_{FN} = \arccos\left(\pm\frac{\lambda}{Nd}\right)$$

The BWFN is:

$$BWFN = |\theta_{FN,\,left} - \theta_{FN,\,right}|$$

(a) Broadside

(b) Endfire

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For practical applications, we require a single pencil beam. To achieve it in the end-fire configuration, one of the ways to generate a single lobe is to use a backing ground plane. Another way to do this is to slightly decrease the element spacing below $\lambda/2$.

Some famous antennas such as the *Yagi-Uda* implements this. The *Hansen-Woodyard* endfire array also does it by introducing an excess phase delay:

$$\alpha = \pm \left(kd + \delta \right)$$

The end expressions for *Hansen-Woodyard* array are:

$$d < \frac{\lambda}{2} \left(1 - \frac{1}{N} \right)$$

$$\alpha = \pm \left(kd + \frac{\pi}{N} \right)$$
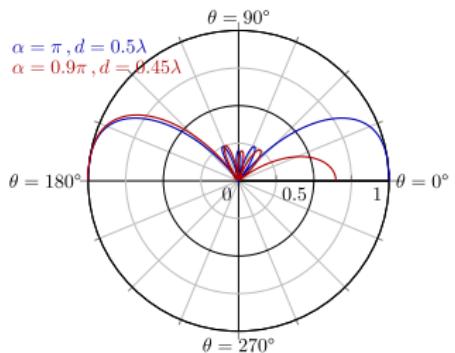
Figure 17: Slight change of the phase.

Considering an example, where a five-element Hansen Woodyard has element spacing $d = 0.37\lambda$ and the element-element phase shift, $\alpha = 0.94\pi$. Let's find the radiation pattern.
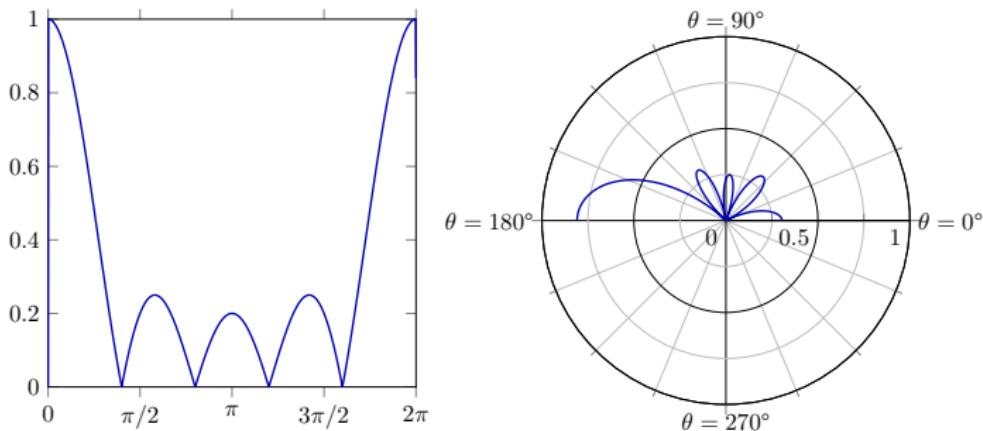
Figure 18: The side by side plots of the linear and polar patterns of a Hansen Woodyard array.
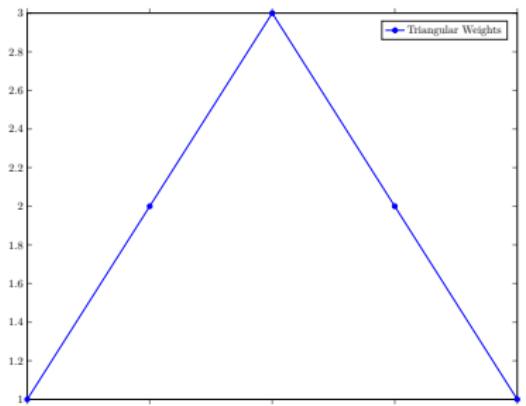
The sidelobes can be further truncated using *non-uniform* excitation on the elements. The *AF* can now be written as a polynomial in terms of $Z = e^{j\psi}$:

$$AF = \sum_{n=0}^{N-1} A_n e^{jn\psi} = \sum_{n=0}^{N-1} A_n Z_n$$
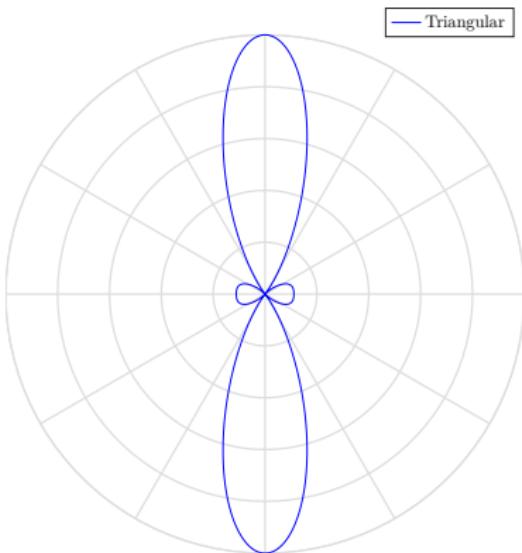
The current amplitudes A_n are real-valued and different for each *n*.

Let's plot the array patterns for a five-element broadside array.

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)