HIGH FREQUENCY COMMUNICATION SYSTEMS

Lecture 3

Hasan T Abbas & Qammer H Abbasi

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LECTURE OUTLINE

- Antennas and Radiation
- Potential Functions
- Antenna Characteristics

- A distribution of currents and charges can generate and radiate electromagnetic fields
  - The distribution is typically localised in a region of space
  - As an example, a simple wire can act as an *antenna*
- We are interested in determining the electromagnetic fields in space, given a current distribution

- Antennas are most widely used for wireless communications
- Modern antenna invention is attributed to Heinrich Hertz (1887)
  - Radio system was developed by Guglielmo Marconi (1897)
- Due to the *duality* principle, an antenna can also act as a receiver to EM radiation

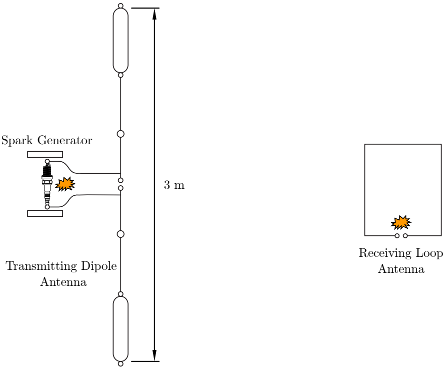


Figure 1: The Hertz's invention

- We need a disturbance in the EM fields
  - Most commonly, this is caused by a time-varying electric current
- The disturbance also depends on the nature of the antenna
  - For a wire antenna, the discontinuities at the ends cause radiation

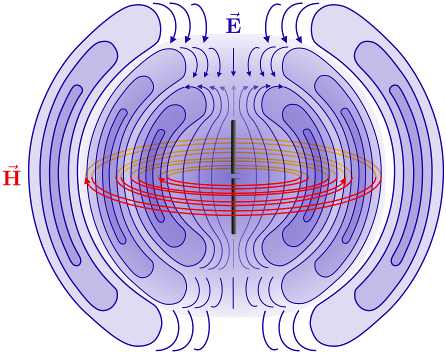


Figure 2: Antenna Radiation Mechanism

- There are mainly two ways to find the radiated fields from a given current distribution

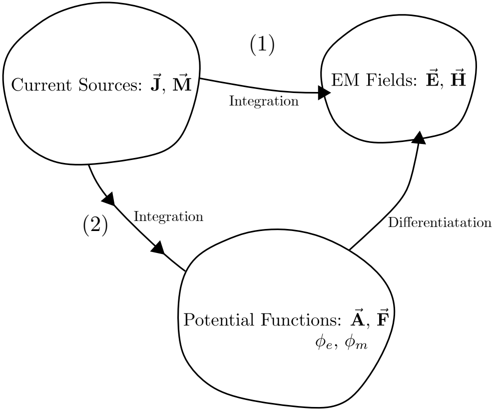


Figure 3: Two ways to find the radiated EM fields

- Solving EM fields directly using the Maxwell's equations is often very difficult, especially in the spatial domain
- The introduction of scalar (ϕ) and vector $\vec{\mathbf{A}}$ potential functions simplify the process
- We start from the fact:
  - Magnetic field is divergence-less ($\boldsymbol{\nabla} \cdot \vec{\mathbf{B}} = 0$). We can therefore, say that:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \vec{\mathbf{A}} \equiv 0$$

$$\Rightarrow \vec{\mathbf{H}} = \frac{1}{\mu} \boldsymbol{\nabla} \times \vec{\mathbf{A}}$$

We can write the Ampere's law as:

$$\boldsymbol{\nabla} \times \vec{\mathbf{E}} = -j\omega\mu\vec{\mathbf{H}} = j\omega\boldsymbol{\nabla} \times \vec{\mathbf{A}}$$

$$\boldsymbol{\nabla} \times \left(\vec{\mathbf{E}} + j\omega\vec{\mathbf{A}}\right) = 0$$

Knowing that the $\nabla \times (-\nabla\phi) \equiv 0$, we set:

$$\begin{aligned}\vec{\mathbf{E}} + j\omega\vec{\mathbf{A}} &= -\nabla\phi \\ \vec{\mathbf{E}} &= -\nabla\phi - j\omega\vec{\mathbf{A}}\end{aligned}$$

- ϕ is the electric scalar potential and its a function of position.
- If we know $\vec{\mathbf{A}}$ and ϕ, we can find $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$

- We still need to figure out how to find the potentials, $\vec{\mathbf{A}}$ and ϕ for a given current density $\vec{\mathbf{J}}$.
- For this we move back to Maxwell's equations and find a relationship

$$\boldsymbol{\nabla} \times \vec{\mathbf{H}} = j\omega\varepsilon\vec{\mathbf{E}} + \vec{\mathbf{J}}$$

$$\boldsymbol{\nabla} \times \left(\boldsymbol{\nabla} \times \vec{\mathbf{A}}\right) = j\omega\mu\varepsilon\vec{\mathbf{E}} + \mu\vec{\mathbf{J}}$$

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \vec{\mathbf{A}} = j\omega\mu\varepsilon\left(-j\omega\vec{\mathbf{A}} - \boldsymbol{\nabla}\phi\right) + \mu\vec{\mathbf{J}}$$

Continuing and using the vector identity,
$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \vec{\mathbf{A}} = \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \vec{\mathbf{A}} - \nabla^2 \vec{\mathbf{A}}\right)$ and rearranging, we get,

$$\nabla^2 \vec{\mathbf{A}} + \omega^2 \mu \varepsilon \vec{\mathbf{A}} = -\mu \vec{\mathbf{J}} + \boldsymbol{\nabla}\left(\boldsymbol{\nabla} \cdot \vec{\mathbf{A}} + j\omega\mu\varepsilon\phi\right)$$

The solution is complete by defining $\vec{\mathbf{A}}$ in terms of ϕ through the Lorentz gauge,

$$\boldsymbol{\nabla} \cdot \vec{\mathbf{A}} = -j\omega\mu\varepsilon\phi$$

The magnetic vector potential $\vec{\mathbf{A}}$ is finally expressed through an inhomogeneous vector wave equation:

$$\nabla^2 \vec{\mathbf{A}} + k^2 \vec{\mathbf{A}} = -\mu \vec{\mathbf{J}}$$

- Given an electric current density $\vec{\mathbf{J}}$
- Solve for the magnetic vector potential $\vec{\mathbf{A}}$
  - Solve for $\vec{\mathbf{E}}$ and $\vec{\mathbf{H}}$

There are some assumptions in this method, namely:

- The space is homogeneous (only one material)
- The magnetic current density $\vec{\mathbf{M}}$ is zero.

- We solve $\vec{\mathbf{A}}$ individually in terms of the scalar components (A_x, A_y, A_z)
- For a forcing function p, the general solution (in terms of ψ) can be written as:

$$\nabla^2\psi + k^2\psi = -p \tag{1}$$

For a point source ($p = \delta(\vec{\mathbf{r}})$), the solution of the above equation is called the *impulse response*. And this impulse response is also called the Green function of the differential equation

- For a delta function, the solution of 1 is 0 everywhere except at the origin.
- Due to spherical symmetry, it is better to express the problem in the spherical coordinates.

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) = -k^2\psi$$

Substituting $\psi = G/r$, we get:

$$\frac{\partial^2 G}{\partial r^2} = k^2 G$$

$$G = C_1 \mathrm{e}^{-jkr} + C_2 \mathrm{e}^{+jkr}$$

In terms of ψ the solution becomes:

$$\psi = \frac{G}{r} = \frac{C_1}{r}\mathrm{e}^{-jkr} + \frac{C_2}{r}\mathrm{e}^{+jkr}$$

For sources displaced from the origin, we use:

- The fundamental type of antenna is the point electric dipole also known as the *The Hertzian Dipole*
- The current of a z-directed Hertzian dipole is expressed as:

$$J_z(\vec{\mathbf{r}}) = \widehat{\mathbf{z}} I \, \mathrm{d}l \, \delta(\vec{\mathbf{r}})$$

The magnetic vector potential is given as:

$$\vec{\mathbf{A}} = \widehat{\mathbf{z}} \mu \frac{I}{4\pi} \, \mathrm{d}l \, \frac{\mathrm{e}^{-jkr}}{r}$$

For an arbitrary source, we have:

$$\vec{\mathbf{A}} = \frac{\mu}{4\pi} \int_V \vec{\mathbf{J}} \, \mathrm{d}v' \, \frac{\mathrm{e}^{-jkr}}{r}$$

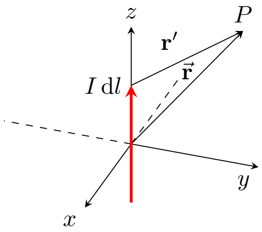


Figure 4: The Hertzian Dipole

ANTENNA CHARACTERISTICS AND PARAMETERS

- A graphical representation of the *far-field* radiation properties
- Pattern can be further described in E- (E_θ)and H- (H_ϕ) planes.

For a Hertzian dipole, the far-fields ($kr \gg 1$) are given as:

$$\vec{\mathbf{E}} = \hat{\boldsymbol{\theta}} \frac{j\omega\mu I \, \mathrm{d}l}{4\pi r} \sin(\theta)$$

$$\vec{\mathbf{H}} = \hat{\boldsymbol{\phi}} \frac{jkI \, \mathrm{d}l}{4\pi r} \sin(\theta)$$

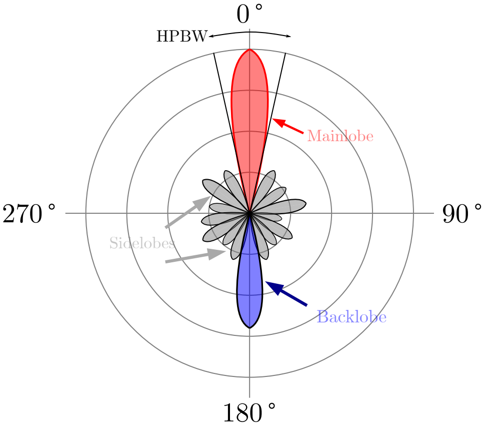


Figure 5: The Radiation Pattern

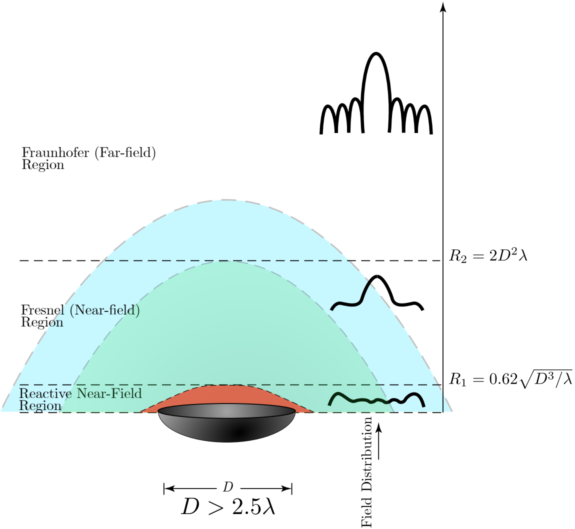
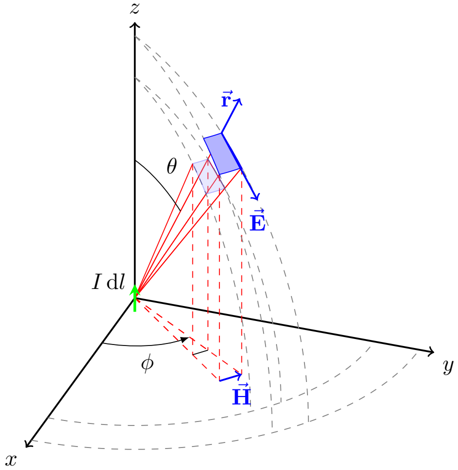


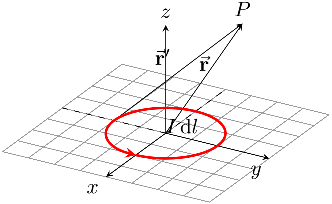
Figure 6: The Regions of Antenna Radiation

- The graphical representation is easier in the spherical coordinates
- We apply the Cartesian to Spherical coordinate transformation



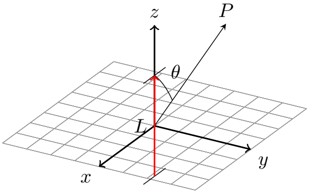
Homework

- Derive the expressions of the electric and magnetic fields for a loop antenna oriented in the $x - y$ plane as shown below
  - First derive the expression of the magnetic vector potential $\vec{\mathbf{A}}$
  - Spherical coordinate transformation needs to be done on the operators and expressions.
- Due on MS Teams on March 30.



- A line source with a uniform current along its extent
- Say the line is z-directed and centered on the origin
- The length of the line source is L

$$I(z') = \begin{cases} I_0 & x' = 0, \quad y = 0, \quad |z'| \leq \frac{L}{2} \\ 0 & \text{elsewhere} \end{cases}$$



- As the current is only in the z-direction, we only find the A_z component
- For z-directed sources, $R \approx r - z' \cos\theta$ for the phase term and $R \approx r$ in the magnitude term

$$
\begin{aligned}
A_z &= \mu \int_{-L/2}^{L/2} I(z') \frac{\mathrm{e}^{-jkR}}{4\pi R} \, \mathrm{d}z' \\
&= \mu \frac{\mathrm{e}^{-jkr}}{4\pi r} \int_{-L/2}^{L/2} I_0 \mathrm{e}^{jk\left(z' \cos\theta\right)} \, \mathrm{d}z' \\
&= \mu \frac{I_0 \mathrm{e}^{-jkr}}{4\pi r} \frac{\sin\left[(kL/2)\cos\theta\right]}{(kL/2)\cos\theta}
\end{aligned}
$$

The electric field is given as:

$$
\begin{aligned}
\vec{\mathbf{E}} &= -j\omega\vec{\mathbf{A}} - \frac{j}{\omega\mu\varepsilon}\boldsymbol{\nabla}\left(\boldsymbol{\nabla}\cdot\vec{\mathbf{A}}\right) \\
&= -j\omega\vec{\mathbf{A}} - (-j\omega\vec{\mathbf{A}}\cdot\widehat{\mathbf{r}})\widehat{\mathbf{r}} \\
&= j\omega\sin\theta A_z\hat{\boldsymbol{\theta}} \\
&= \frac{j\omega\mu I_0 L\mathrm{e}^{-jkr}}{4\pi r}\sin\theta\frac{\sin\left[(kL/2)\cos\theta\right]}{(kL/2)\cos\theta}\hat{\boldsymbol{\theta}}
\end{aligned}
$$

The magnetic field can simply be found as:

$$
H_\phi = \frac{E_\theta}{\eta}
$$

- This describes the *complex power density* flowing out of a sphere of radius r
- It is real-valued and directed along the wave propagation direction

$$\vec{\mathbf{S}} = \frac{1}{2}\vec{\mathbf{E}} \times \vec{\mathbf{H}}^{*}$$

- If $\vec{\mathbf{E}}$ is in the $\hat{\boldsymbol{\theta}}$ and $\vec{\mathbf{H}}$ is in the $\hat{\boldsymbol{\phi}}$ directions
- The Poynting vector will be radially directed.

- Just like voltage and current ratio gives us impedance
- The ratio of electric and magnetic field components give us the *intrinsic impedance*

$$\frac{E_\theta}{H_\phi} = \eta = \sqrt{\frac{\mu}{\varepsilon}}$$

For freespace, the value is $\eta_0 = 376.7\,\Omega \approx 120\pi\,\Omega$

- The total power radiated by an antenna can be found from the Poynting vector
- We need to integrate over a surface

$$P = \iint_{S'} \vec{\mathbf{S}} \cdot \mathrm{d}S' = 1/2\,\mathrm{Re} \iint_{S'} \left(\vec{\mathbf{E}} \times \vec{\mathbf{H}}^{*}\right) \cdot \mathrm{d}S'$$

The $\mathrm{d}S'$ in spherical coordinates refers to a solid angle and for a given radius r can be expressed as:

$$\mathrm{d}S' = r^2 \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

- Since the power varies with distance r, it is convenient to define the *radiation intensity*
- The radiation intensity is independent of the distance
- It is defined as the power radiated in a given direction per unit solid angle
  - It has units of watts per steradians

$$U(\theta, \phi) = \frac{1}{2}\,\mathrm{Re}\left(\vec{\mathbf{E}} \times \vec{\mathbf{H}}^{*}\right) \cdot r^2 \widehat{\mathbf{r}}$$

- For a given antenna the directivity and gain describe in what direction the radiation is, as compared to an isotropic antenna
- For the isotropic antenna, the radiation pattern is uniform (ie) a circle
- Directivity is defined as the ratio of radiation intensity in a certain direction to the average radiation intensity

$$D = \frac{1}{2} \frac{\max \left[\mathrm{Re}(\vec{\mathbf{E}} \times \vec{\mathbf{H}}^{*}) \cdot \widehat{\mathbf{r}} \right]}{P/4\pi r^2}$$

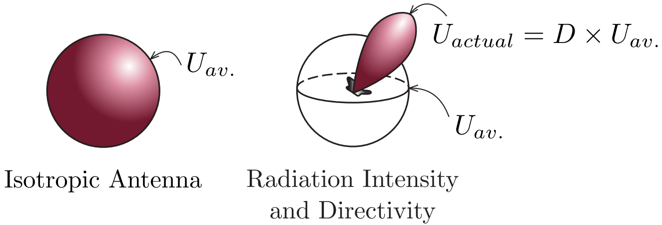


Figure 7: Relationship between Radiation Intensity and Directivity

- Although the directivity describes the radiation pattern of an antenna, we need a quantity that can be used when treating antenna as a system
- Suppose the antenna is one component of a radio-frequency system that includes transmission lines and sources
- A parameter is helpful that determines how *efficiently* the antenna operates
    - In particular, how much input power is transferred into radiated power
- Antenna gain is defined as:

$$G = 4\pi \frac{U_m}{P_m}$$

We often describe it in terms of decibels:

$$G_{\mathrm{dB}} = 10 \log G$$

- We can treat antenna as an impedance with real and imaginary parts
  - The real part refers to the how much radiation leaves the antenna (R_r) and how much dissipates as losses (R_o)
  - The imaginary part (X_A) determines the stored power in the near field.

$$Z_A = R_A + jX_A = (R_r + R_{ohm}) + jX_A$$

- Efficiency is a metric that determines the ratio of total desired power to the total power supplied
- Radiation efficiency of antennas is a measure how much power is radiated

$$e_{rad} = \frac{P_{rad}}{P_{in}} = \frac{P_{rad}}{P_{rad} + P_{ohm}}$$