HIGH FREQUENCY COMMUNICATION SYSTEMS

Lecture 4

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- Transmission Lines Theory & Analysis
- Load Mismatching
- The Magic of Quarter-wave Transformer

- A technique to bridge the gap between basic circuit analysis and electromagnetic fields theory
  - Lots of similarities and analogies
- Commonly used to design microwave devices and circuits
- We use transmission lines when the electrica length of the device is greater than $\lambda/10$.

- In wave scattering problems, we place the co-ordinate system at the boundary
- In transmission lines, we use the two different lines as the boundary
- For open problems, the load is considered to be at infinity.

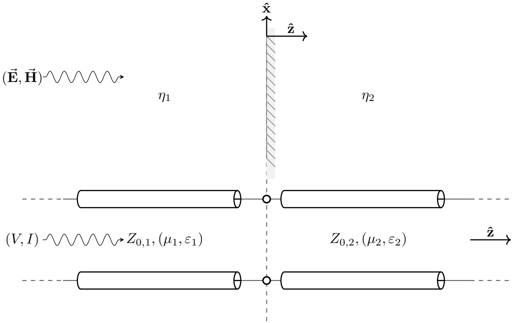
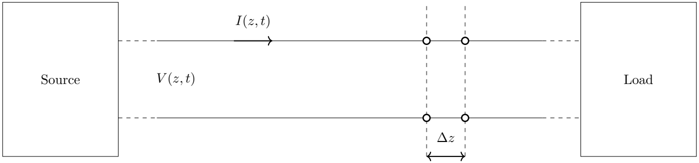
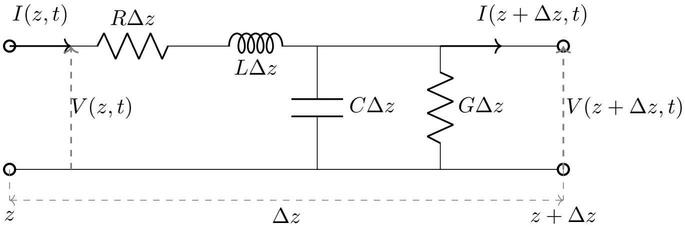


Figure 1: The analogy between wave problems and transmission lines.

TL Theory	EM Field Theory
$V_1 = V_0 \mathrm{e}^{-j\gamma_1 z}\left(1+\Gamma_L \mathrm{e}^{j2\gamma_1 z}\right)$	$E_{x,1} = E_0 \mathrm{e}^{-jk_1 z}\left(1+\Gamma \mathrm{e}^{j2k_1 z}\right)$
$I_1 = \frac{V_0}{Z_{0,1}} \mathrm{e}^{-j\gamma_1 z}\left(1+\Gamma_L \mathrm{e}^{j2\gamma_1 z}\right)$	$H_{x,1} = \frac{E_0}{\eta_0} \mathrm{e}^{-jk_1 z}\left(1+\Gamma \mathrm{e}^{j2k_1 z}\right)$
$\Gamma_L = \frac{Z_L - Z_{0,1}}{Z_L + Z_{0,1}} = \frac{Z_{0,2} - Z_{0,1}}{Z_{0,2} + Z_{0,1}}$	$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$
$V_2 = T V_0 \mathrm{e}^{-j\gamma_2 z}$	$E_{x,2} = T E_0 \mathrm{e}^{-jk_2 z}$
$I_2 = T \frac{V_0}{Z_{0,2}} \mathrm{e}^{-j\gamma_2 z}$	$H_{x,2} = T \frac{E_0}{\eta_2} \mathrm{e}^{-jk_2 z}$
$T = \frac{2 Z_{0,2}}{Z_{0,2} + Z_{0,1}}$	$T = \frac{2\eta_2}{\eta_2 + \eta_1}$

- For a transmission line, we use a distributed circuit approach
  - Energy stored in the magnetic field $\rightarrow L$
  - Energy stored in the electric field $\rightarrow C$
  - Conductive losses $\rightarrow R$
  - Dielectric losses $\rightarrow G$
- All the circuit elements are expressed per unit length
- We can express the voltage and current at any given point z and time t.


Source
$I(z,t)$
$V(z,t)$
Δz
Load


$I(z,t)$
$R\Delta z$
$L\Delta z$
$I(z+\Delta z,t)$
$V(z,t)$
$C\Delta z$
$G\Delta z$
$V(z+\Delta z,t)$
z
Δz
$z+\Delta z$

- Solving the electric circuit leads to a second order differential equation
- Analogous to the wave equation
  - Hence the analogies between voltage V and electric field $\vec{\mathbf{E}}$

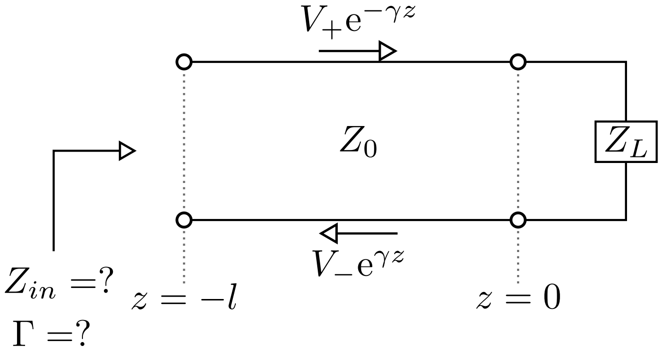
$$\left\{\frac{\partial^2}{\partial z^2} - [(R + j\omega L)(G + j\omega C)]\right\} V(z,t) = 0$$

The solution is of the type:

$$V(z) = V_+ \mathrm{e}^{-\gamma z} + V_- \mathrm{e}^{+\gamma z}$$

where the propagation constant $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$. For a lossless case $R = G = 0$.

- For a transmission line terminated with a load Z_L
  - We get reflections in the line (superposition incoming and reflected wave make up the total voltage)
- The input impedance depends on the Z_L as well as the length l of the transmission line
- For Transmission line analysis, we shift the origin to the load.



The *voltage reflection coefficient* Γ can for the line is

$$\Gamma = \frac{V_{ref}}{V_{inc}} = \frac{V_- \mathrm{e}^{\gamma z}}{V_+ \mathrm{e}^{-\gamma z}}$$
$$= \frac{V_{-,z=0}}{V_{+,z=0}} \mathrm{e}^{2\gamma z} = \Gamma_L \mathrm{e}^{2\gamma z}$$

We can express the voltage at any point on the line as:

$$V(z) = V_+ \left(\mathrm{e}^{-\gamma z} + \Gamma_L \mathrm{e}^{\gamma z} \right)$$

After finding a similar expression for the current, we can write the impedance as:

$$Z(z) = Z_0 \frac{\mathrm{e}^{-\gamma z} + \Gamma_L \mathrm{e}^{-\gamma z}}{\mathrm{e}^{-\gamma z} - \Gamma_L \mathrm{e}^{-\gamma z}}$$

At $z = 0$, $Z(0) = Z_L$, and we get:

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L} \implies \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

At $z = -l$, the input impedance $Z(-l) - Z_{in}$ is given by:

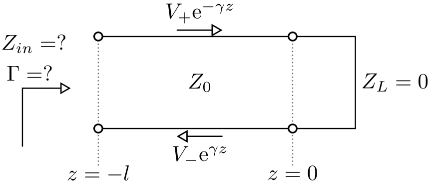
$$\begin{aligned} Z(-l) &= Z_0 \frac{\mathrm{e}^{\gamma l} + \mathrm{e}^{-\gamma l}\Gamma_L}{\mathrm{e}^{\gamma l} - \mathrm{e}^{\gamma l}\Gamma_L} \\ &= Z_0 \frac{(Z_L + Z_0)\,\mathrm{e}^{\gamma l} + (Z_L - Z_0)\,\mathrm{e}^{-\gamma l}}{(Z_L + Z_0)\,\mathrm{e}^{\gamma l} - (Z_L - Z_0)\,\mathrm{e}^{-\gamma l}} \\ &= Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \end{aligned}$$

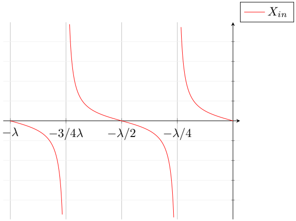
For lossless TLs $\gamma = j\beta$, therefore,

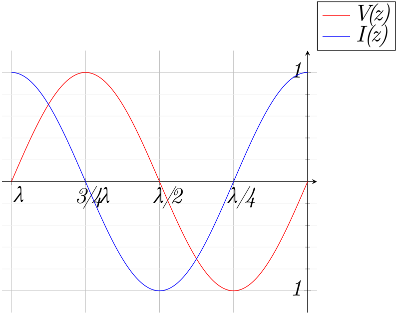
$$Z(-l) = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

For $Z_L = 0$,

$$Z_{in} = Z_0 j \tan(\beta l)$$
$$\Gamma_{in} = -1$$



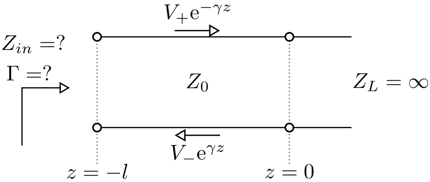


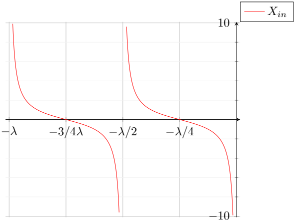


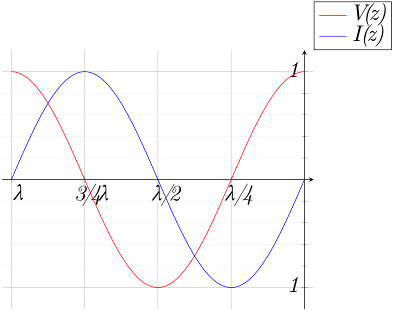
For $Z_L = \infty$,

$$Z_{in} = -Z_0 j \cot(\beta l)$$

$$\Gamma_{in} = 1$$



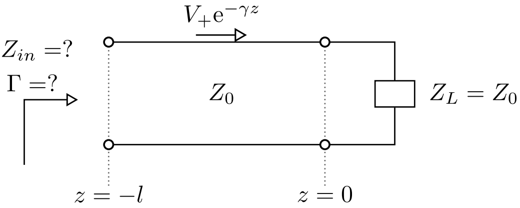




For $Z_L = Z_0$,

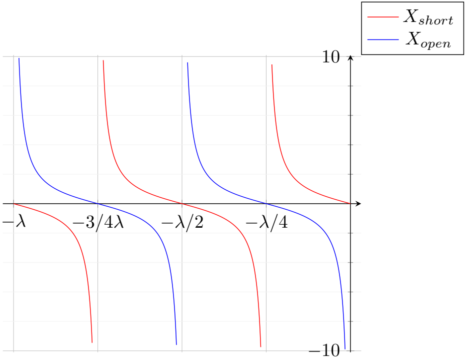
$$Z_{in} = Z_0$$

$$\Gamma_{in} = 0$$



- Ideally we desire this condition for maximum power transfer.
- No reflections take place in this case

- We see that for terminated loads with special lengths that are multiples of quarter wavelengths $(\lambda/4 + n\lambda/2)$ for $n = 1, 2, 3, \cdots$
- The load impedance is inverted
  - Open-circuit load becomes short-circuit and vice versa
- For such a line, the input impedance is $Z_{in} = \frac{Z_0^2}{Z_L}$



- We want maximum power transferred to the load from the transmission line.
    - For that to happen, we require the impedance matching of the transmission line.
- The reflections in the TL are undesirable and lead to a standing wave generation ($V = V_+ + V_-$).
- The voltage standing wave ratio (VSWR) measures the quality of impedance matching in TL.

Recall,

$$V(z) = V_+ \left(\mathrm{e}^{-\gamma z} + \Gamma_L \mathrm{e}^{\gamma z}\right)$$

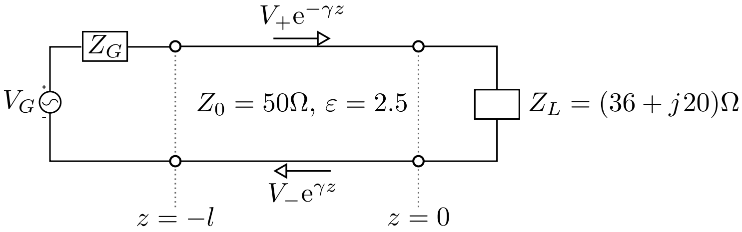
The values of the voltage reflection coefficient range from $-1 \leq |\Gamma_L| \leq 1$.

The VSWR is the ratio of the maximum and minimum absolute values of the voltage

$$VSWR = \frac{|V_{max}|}{|V_{min}|}$$

In other words,

$$|V_{max}| = V_0 \left(1 + |\Gamma_L|\right)$$
$$|V_{min}| = V_0 \left(1 - |\Gamma_L|\right)$$
$$VSWR = \frac{1 + |\Gamma_L|}{1 - \Gamma_L}$$



For the TL above with a length l of 6.330 m, find the average power delivered to the load as well as the line. The source voltage is $V_G = 100\,\mathrm{V}$ operating at a frequency of 200 MHz. The source impedance Z_G is also $50\,\Omega$. First, check whether the circuit is actually a transmission line. Also find the input impedance $Z_{in} = Z(-l)$ of the line.

First we check the electrical length of the transmission line.

$$\beta = k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$$

$$\implies \lambda = \frac{v}{f} = \frac{c}{\sqrt{\varepsilon_r} f} = \frac{3 \times 10^8}{\sqrt{2.5} \times 20 \times 10^6}$$

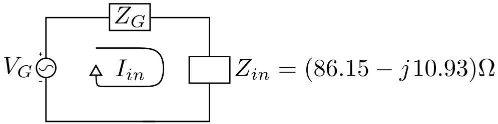
$$\lambda = 10.55\,\mathrm{m} \implies l = \frac{6.33}{10.55} = 0.6\lambda$$

As $l > 0.1\lambda$, the given circuit can be treated as a transmission line.

As Z_0 is real-valued, we treat the TL as lossless.

$$\begin{aligned} Z_{in} = Z(-l) &= Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \\ &= 50 \times \frac{(36 + j20) + j50 \tan(2\pi \times 0.6)}{50 + j(36 + j20) \tan(2\pi \times 0.6)} \\ Z_{in} &= (86.15 - \mathrm{j}10.93)\,\Omega \end{aligned}$$

The equivalent circuit now looks like:



The current in the circuit is:

$$\begin{aligned} I_{in} &= \frac{V_G}{Z_G + Z_{in}} \\ &= \frac{100}{50 + 86.15 - j10.93} = (0.7300 + \mathrm{j}0.060\,00)\,\mathrm{A} \end{aligned}$$

The average input power is:

$$P_{in} = 1/2\,\mathrm{Re}\{V_{in} I_{in}^*\} = 1/2\,\mathrm{Re}\{Z_{in} I_{in} I_{in}^*\} = 23.09\,\mathrm{W}$$

We first calculate the current $I(z)$ in the transmission line.

$$I(z) = \frac{V_+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

At $z = -l$, $I(-l) = I_{in}$ and treating $\frac{V_+}{Z_0} = I_+$

$$I(-l) = I_{in} = I_+\left(e^{+j\beta l} - \Gamma_{in} e^{-j\beta l}\right)$$

$$I_+ = I_{in} / \left(e^{+j\beta l} - \Gamma_{in} e^{-j\beta l}\right) = (-0.4700 + \mathrm{j}0.8900)\,\mathrm{A}$$

The current at the load is:

$$I_L = I_+(1 - \Gamma_L)$$

The power at the load is hence:

$$\begin{aligned} P_{load} &= 1/2\,\mathrm{Re}\{I_L I_L^* Z_L\} \\ &= 23.09\,\mathrm{W} \end{aligned}$$