



University
of Glasgow

HIGH FREQUENCY COMMUNICATION SYSTEMS

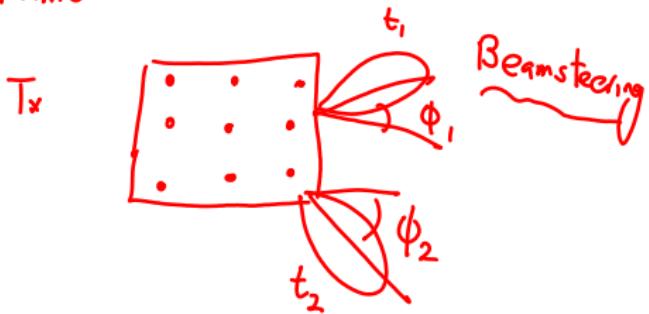
Lecture 7

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- Antenna Arrays
- Array Analysis
 - Uniform linear arrays
 - Non-uniform arrays

MIMO



ANTENNA ARRAYS

ANTENNA ARRAY MOTIVATION

- In individual antenna elements, we can't control the radiation patterns
- If we combine two antenna elements, it is possible to change the pattern significantly
 - We call the new combined structure as an antenna array.
 - We achieve higher directivity using antenna arrays

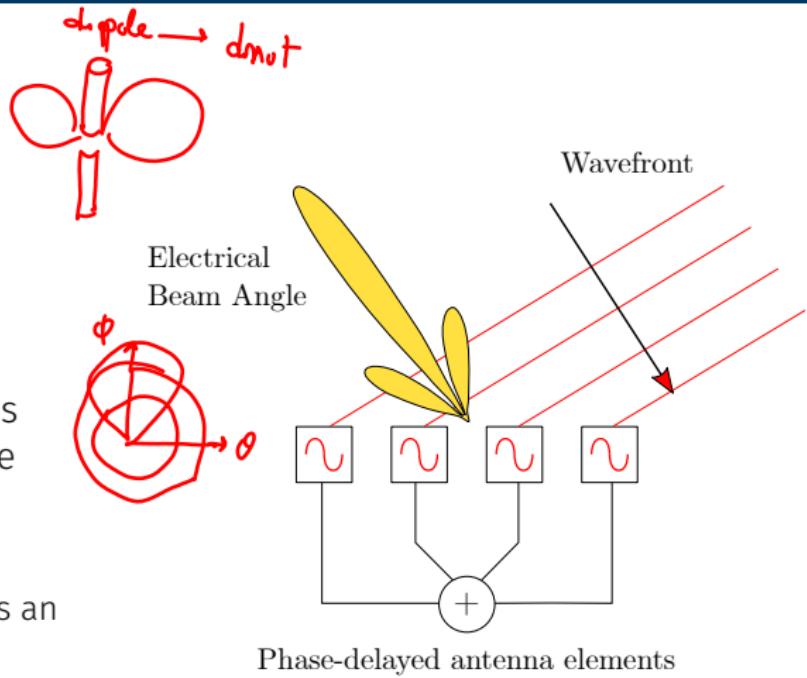


Figure 1: Typical antenna array with phased elements.

- Communications Applications
 - The goal is to focus EM energy towards the target population (cars, people, cities etc.)
 - Modern wireless communications using beamforming
- Radar - multiple target tracking
 - We would like to focus the energy on the targets as they move

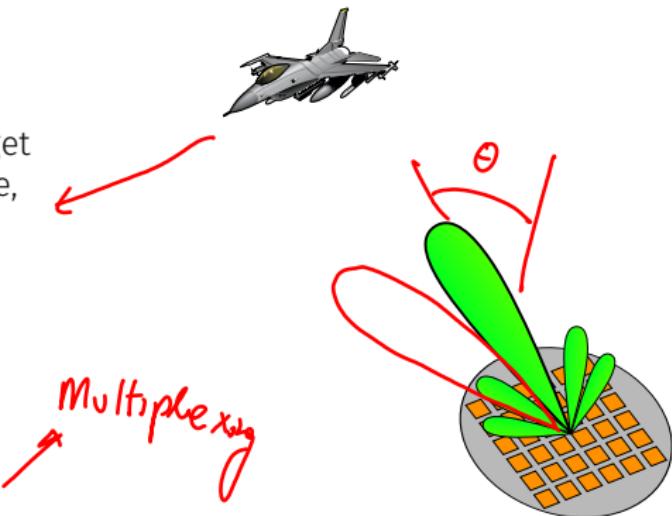


Figure 2: A Phased array antenna in RADAR based target tracking.

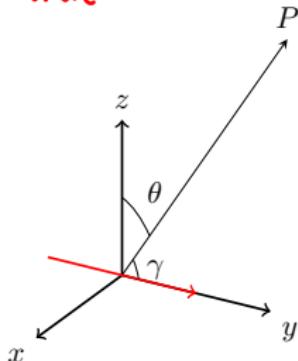
THE ARRAY ELEMENT - POINT DIPOLE

- Recall from the antenna introduction lecture, the field pattern of an infinitesimal dipole:

$$\begin{aligned}\vec{E} &= \hat{\gamma} j k \eta l_0 \left[\frac{\exp(-jkr)}{4\pi r} \right] \sin \gamma \\ &= j k \eta \vec{h} G(r)\end{aligned}$$

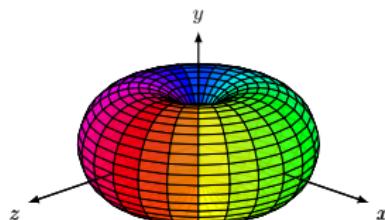
Green function

$\delta \rightarrow$ function



where $\vec{h} = \hat{\gamma} l_0 l \sin \gamma$ and $G(r)$ is the free-space Green function for a point source, $\exp(jkr)/(4\pi r)$

- We will use this as the antenna element



OBTAINING A DESIRED PATTERN

- There are some factors that determine the desired radiation pattern
 - Array Geometry
 - Element Spacing
 - Element Excitation Amplitude
 - Pattern of individual element



The total field is given by:

$$E_{total} = \underbrace{\text{Element Factor}}_{\bullet} \times \underbrace{\text{Array Factor}}_{\bullet}$$

- The simplest antenna array contains two elements
- To analyse an array, we start by using point sources as individual elements
 - The final pattern is obtained by multiplication
- First, we will ignore the mutual coupling between elements.
 - Consider an array of two point sources separated by a distance d on the z-axis.
 - Assuming that both the antenna elements are excited by the current $I_1 = I_0 \exp(j\alpha/2)$ and $I_2 = I_0 \exp(-j\alpha/2)$ where $0 \leq \alpha \leq 2\pi$.

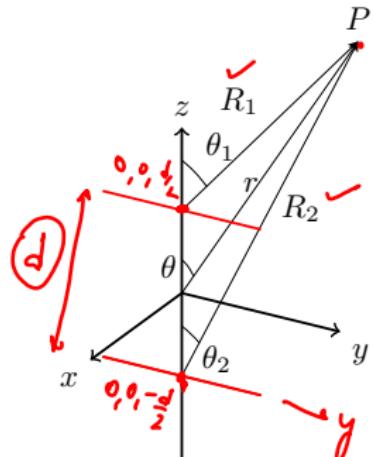


Figure 3: Two point sources forming a basic antenna array.

x ?

Neglecting any mutual coupling, we obtain the total fields by simple vector summation:

$$\vec{E}_t = \vec{E}_1 + \vec{E}_2 \\ = \hat{\gamma} j k \eta \frac{l_0 \ell}{4\pi} \left\{ \frac{e^{-jkR_1}}{R_1} e^{+j\alpha/2} \sin \gamma_1 + \frac{e^{-jkR_2}}{R_2} e^{-j\alpha/2} \sin \gamma_2 \right\}$$

We use the *far-field approximation*:

$$\gamma_1 \approx \gamma_2 \approx \gamma$$

$$\checkmark \checkmark R_1 \approx r - \frac{d}{2} \cos \theta$$

$$\checkmark \checkmark R_2 \approx r + \frac{d}{2} \cos \theta$$

$$\checkmark R_1 \approx R_2 \approx r \text{ (amplitude term)}$$

by simple
trigonometric

The total *far-field* thus becomes:

$$\begin{aligned}
 \vec{E}_t &= \hat{\gamma} j k \eta \frac{l_0 \ell}{4\pi r} \sin \gamma \left\{ e^{-jk \frac{d}{2} \cos \theta} e^{-j\alpha/2} + e^{-jk \frac{d}{2} \cos \theta} e^{+j\alpha/2} \right\} \\
 &= j k \eta \vec{h} G(r) \left\{ e^{\frac{jkd \cos \theta + \alpha}{2}} + e^{\frac{-jkd \cos \theta + \alpha}{2}} \right\} \quad \text{[Euler's]} \\
 &= \underbrace{j k \eta \vec{h} G(r)}_{\text{Element Factor}} \underbrace{2 \cos \left[\frac{1}{2} (kd \cos \theta + \alpha) \right]}_{\text{Array Factor}}
 \end{aligned}$$

We can control and change the pattern by varying d and α , which are the spacing and phase shifts.

 α α

EXAMPLE - VARYING THE PATTERN

Let's look at different cases where we consider different values of d and α .

quarter wavelength

- $\alpha = 0^\circ$ and $d = \lambda/4$, for which the array factor (AF) is

$$k = \frac{2\pi}{\lambda}$$

- $AF = 2 \cos \left[\frac{1}{2} (kd \cos \theta + \alpha) \right] = 2 \cos \left(\frac{\pi}{4} \cos \theta \right)$
- $k d \rightarrow \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} \rightarrow \frac{\pi}{2}$

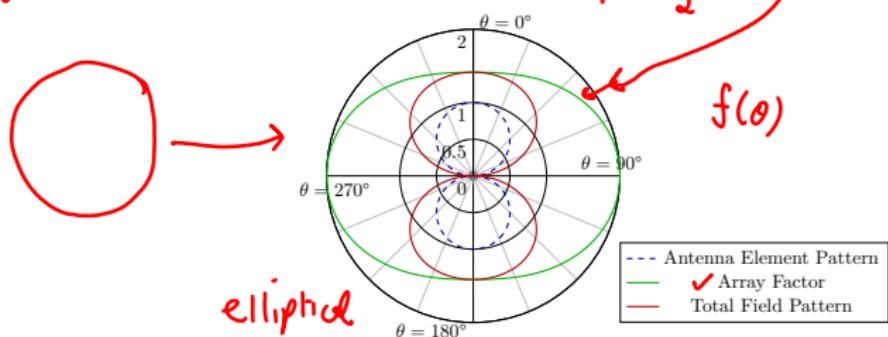


Figure 4: The total field pattern for $\alpha = 0^\circ$ and $d = \lambda/4$.

EXAMPLE - VARYING THE PATTERN

- Now looking at $\alpha = 90^\circ$ and $d = \lambda/4$, for which the array factor (AF) is

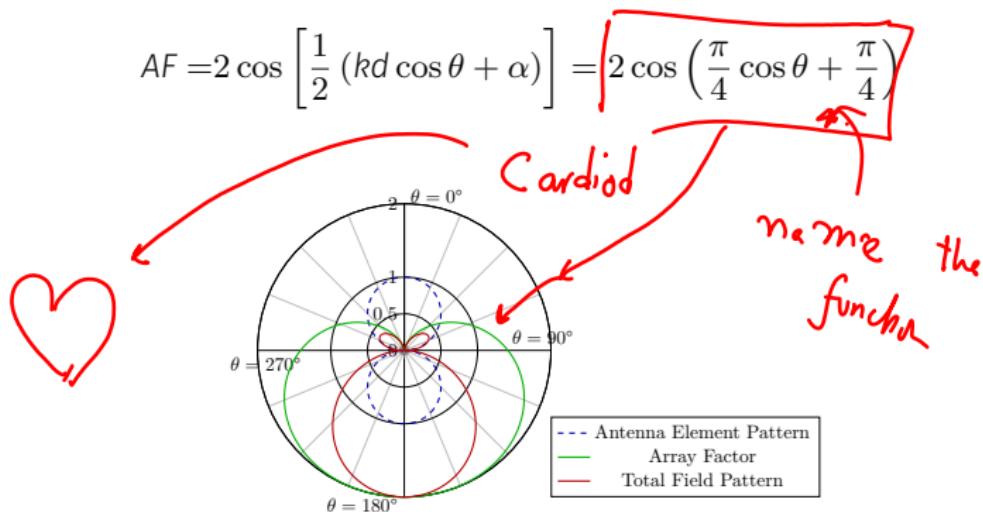


Figure 5: The total field pattern for $\alpha = 90^\circ$ and $d = \lambda/4$.

EXAMPLE - VARYING THE PATTERN

- Now looking at $\alpha = \underline{-90^\circ}$ and $d = \lambda/4$, for which the array factor (AF) is

$$\alpha = -45^\circ$$

$$AF = 2 \cos \left[\frac{1}{2} (kd \cos \theta + \alpha) \right] = 2 \cos \left(\frac{\pi}{4} \cos \theta - \frac{\pi}{4} \right)$$

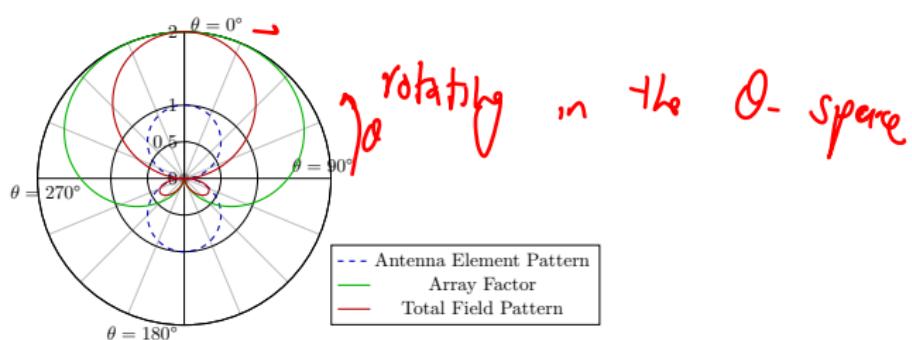


Figure 6: The total field pattern for $\alpha = -90^\circ$ and $d = \lambda/4$.

N-ELEMENT ARRAYS

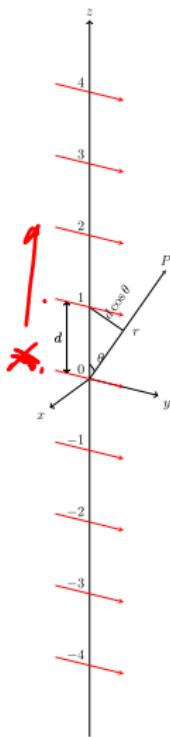
N-ELEMENT UNIFORM ARRAY

- A uniform array consists of equally spaced and identical elements
 - All elements are excited with same amplitude
 - However, the elements have a progressive phase shift.
- As before, the total field is the vector sum of the individual elements:

$$\vec{E}_T = \vec{E}_0 + \vec{E}_1 + \vec{E}_{-1} + \vec{E}_2 + \vec{E}_{-2} + \dots$$

$$= jk\eta \vec{h}G(r) \left(1 + e^{j\psi} + e^{-j\psi} + e^{2j\psi} + e^{-2j\psi} + \dots \right)$$

where $\psi = kd \cos \theta + \alpha$ → progressive phase shift



Continuing, we can write the AF as:

$$AF = \sum_{n=-N'}^{N'} e^{jn\psi} \quad \text{where } N' = \frac{(N-1)}{2} \quad (1)$$

of elements -

We can also express (1) as:

$$AF e^{jn\psi} = \sum_{n=-N'}^{N'} e^{j(n+1)\psi} \quad . \quad (2)$$

From (2) and (1) we get:

$$\begin{aligned} AF &= \frac{e^{j(N+1)\frac{\psi}{2}} - e^{-j(N-1)\frac{\psi}{2}}}{e^{j\psi} - 1} \\ &= \frac{e^{j\psi/2}}{e^{j\psi/2}} \left[\frac{e^{jN\psi/2} - e^{-jN\psi/2}}{e^{j\psi/2} - e^{-j\psi/2}} \right] \end{aligned}$$

$$AF = \frac{\sin N\psi/2}{\sin \psi/2} \quad \text{where } \psi = kd \cos \theta + \alpha$$

Noting that the above expression resembles a **sinc** function, we can extend this to find the AF of any discrete array made of uniformly spaced elements. A major difference, however, is the sidelobes don't decay with the increasing function argument.

In general, we plot the AF in the normalised form (divide by N).

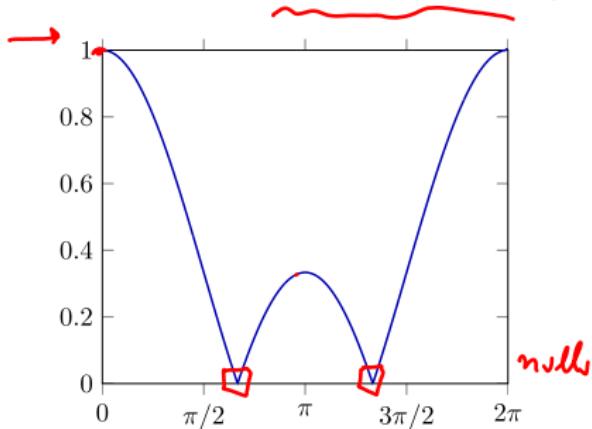
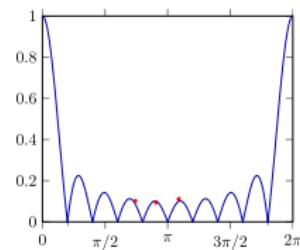
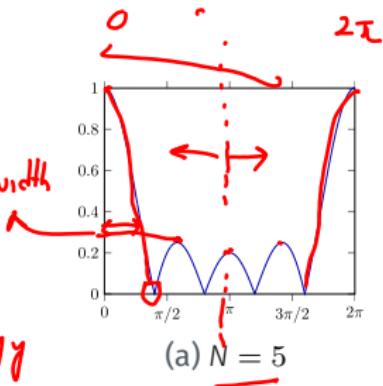


Figure 7: The normalised AF of a 3 element array.

SOME OBSERVATIONS

- As N increases, the main lobe narrows
- We get more side lobes in one period of AF as N increases
- The width of the minor/side lobes is $2\pi/N$
- The side lobe height decreases as we increase N
- AF is symmetric about π
- We also see that the peak value occurs at $\psi = \pm 2n\pi$ for $n = 0, 1, 2, \dots$
- The nulls occur at $\psi = \pm 2n\pi/N$
- The side lobe level is defined as:

$N \uparrow$ \downarrow beamwidth
 \rightarrow most of the energy is in the main lobe

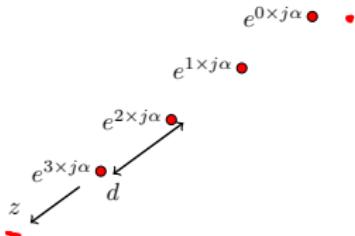


$$SLL = \frac{\text{Max side lobe value}}{\text{Max Main lobe value}} \quad | \quad 0.3$$

Figure 8: Array factors for 5 and 10 element uniform array.

EXAMPLE - A 4 ELEMENT ARRAY

A four-element ($N = 4$), uniformly excited, equally spaced array. The spacing d is $\lambda/2$ and the interelement phasing α is 90° .



$$AF: \frac{\sin N\psi/2}{\sin \psi/2}$$

Figure 9: A four element antenna array.

The normalised array factor is given by:

$$AF = 1/4 \frac{\sin 4\psi/2}{\sin \psi/2} \text{ where } \psi = \frac{2\pi}{\lambda} \frac{\lambda}{2} \cos \theta + 90^\circ$$

$$AF: \frac{1}{4} \frac{\sin 4 \left(\frac{\pi \cos \theta + \pi/2}{\lambda/2} \right)}{\sin \left(\frac{\pi \cos \theta + \pi/2}{\lambda/2} \right)^2} + c = \pi \cos \theta + \frac{\pi}{2}$$

$$\psi = kd \cos \theta + \alpha$$

4 ELEMENT ARRAY - PATTERN

- The linear plot is done as before $\rightarrow \Psi$
- We can plot the corresponding polar plot by translating peaks and main and side lobes
- We first shift the polar plot by an amount α
 - One period of the AF is considered

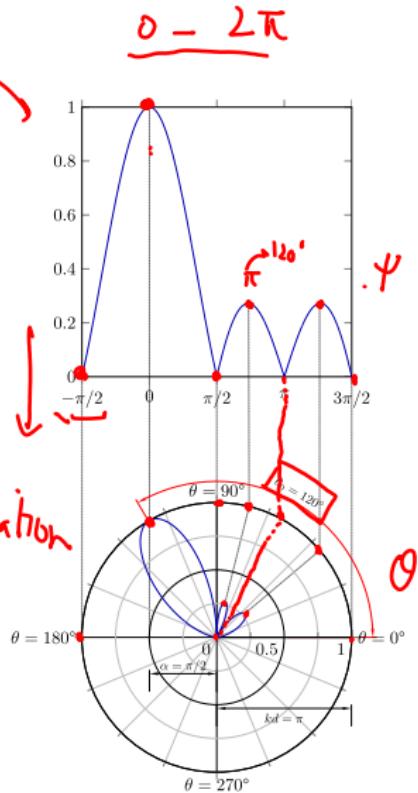
The direction of the main beam in the polar plot is found as:

$$\psi = kd \cos \theta + \alpha$$

$$\theta = \arccos \frac{\psi - \alpha}{kd}$$

$$\theta_0 = \arccos \left(\frac{0 - \pi/2}{\pi} \right) = \underline{\underline{120^\circ}}$$

Radiation pattern in polar



PATTERN BEAMWIDTH

In general there are two extreme cases which are sometimes used:

1. Broadside ($\theta_0 = 90^\circ$) when $\alpha = 0$
2. Endfire ($\theta_0 = 0^\circ$ or 180°) when $\alpha = \pm kd$

The array pattern is often characterised by *beamwidth between first nulls*.

Knowing that the nulls occur at:

$$N\psi_{FN}/2 = \pm \pi$$

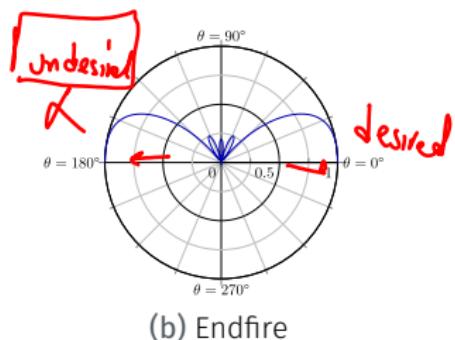
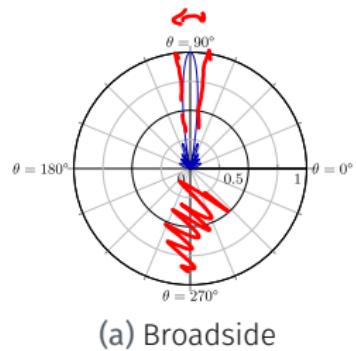
For broadside array, $\frac{N}{2} \frac{2\pi}{\lambda} d \cos \theta_{FN} = \pm \pi$

$$\theta_{FN} = \arccos \left(\pm \frac{\lambda}{Nd} \right)$$

The BWFN is:

$$BW \rightarrow 30^\circ$$

$$BWFN = |\theta_{FN, \text{left}} - \theta_{FN, \text{right}}|$$





For practical applications, we require a single pencil beam. To achieve it in the end-fire configuration, one of the ways to generate a single lobe is to use a backing ground plane. Another way to do this is to slightly decrease the element spacing below $\lambda/2$.

Some famous antennas such as the Yagi-Uda $< \lambda_2$ implements this. The Hansen-Woodyard endfire array also does it by introducing an **excess phase delay**:

$$\alpha = \pm (kd + \delta)$$

The end expressions for Hansen-Woodyard array are:

$$\begin{cases} d < \frac{\lambda}{2} \left(1 - \frac{1}{N} \right) \\ \alpha = \pm \left(kd + \frac{\pi}{N} \right) \end{cases}$$

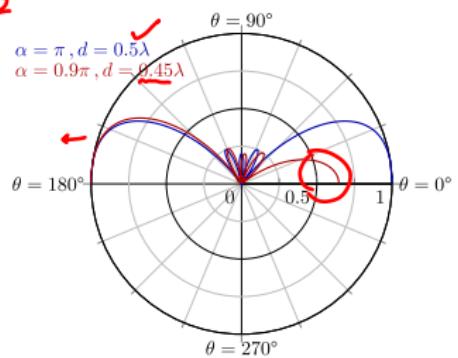


Figure 10: Slight change of the phase.

EXAMPLE - THE HANSON-WOODYARD ARRAY

Considering an example, where a five-element Hansen Woodyard has element spacing $d = 0.37\lambda$ and the element-element phase shift, $\alpha = 0.94\pi$. Let's find the radiation pattern.

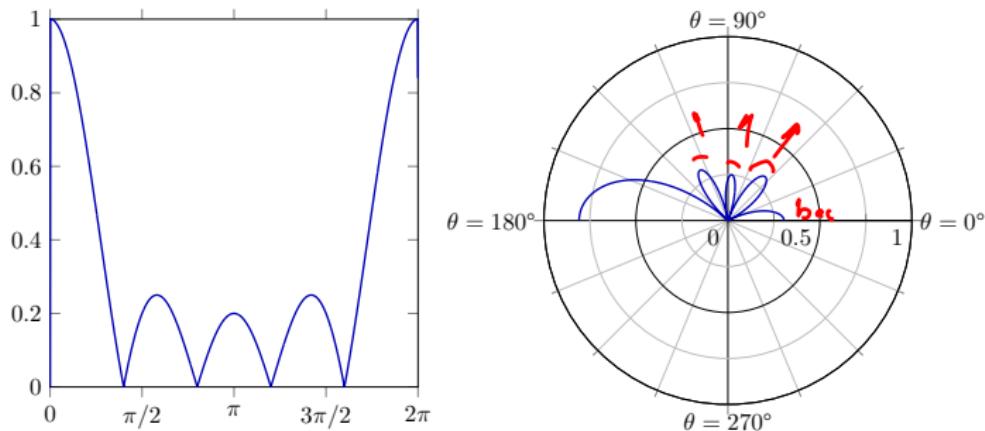


Figure 11: The side by side plots of the linear and polar patterns of a Hansen Woodyard array.

The sidelobes can be further truncated using *non-uniform* excitation on the elements. The AF can now be written as a polynomial in terms of $Z = e^{j\psi}$:

$$AF = \sum_{n=0}^{N-1} A_n e^{jn\psi} = \sum_{n=0}^{N-1} A_n Z_n$$

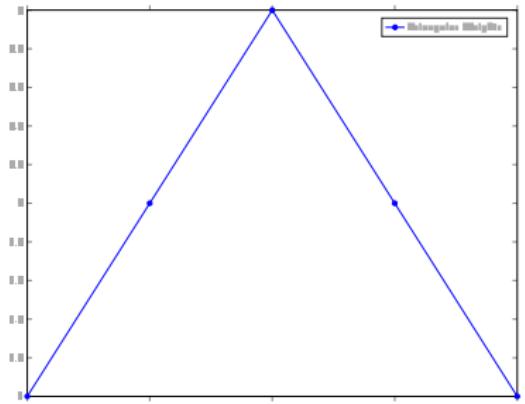
The current amplitudes A_n are real-valued and different for each n .

Let's plot the array patterns for a five-element broadside array.

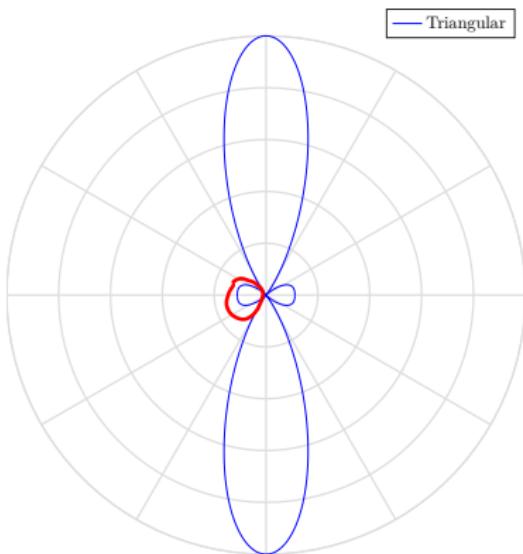
TRIANGULAR ARRAY

$$A = [1 \ 2 \ 3 \ 2 \ 1]$$

5-element



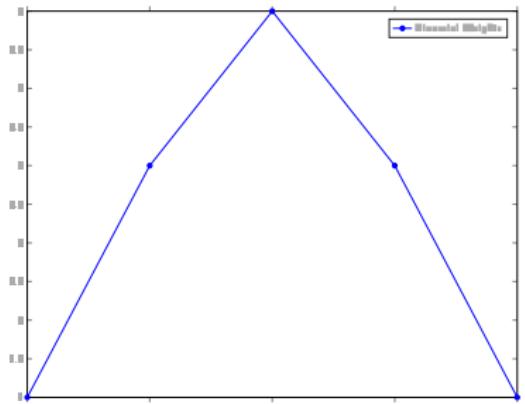
(a)



(b)

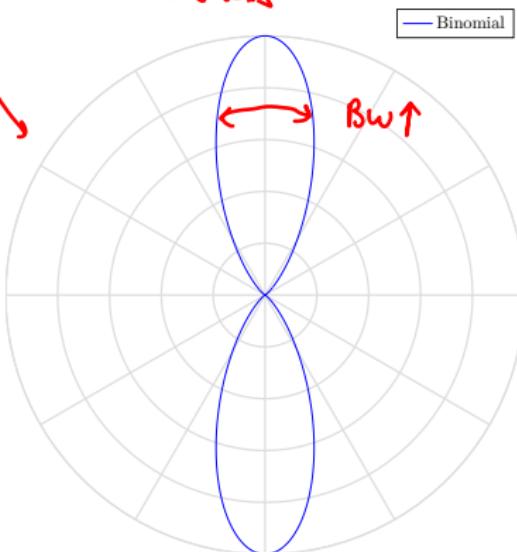
Binomial Formula

$$[1 \quad 4 \quad 6 \quad 4 \quad 1]$$



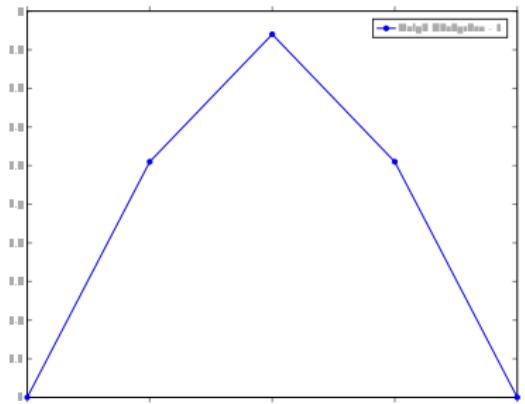
(c)

no sidelobes

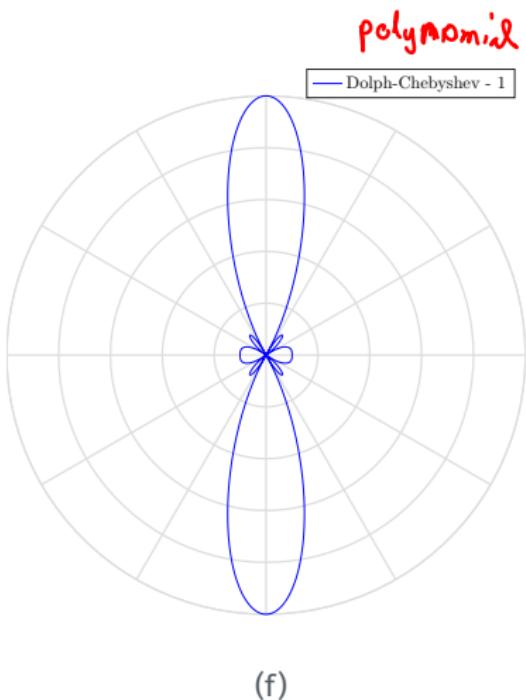


(d)

DOLPH-CHEBYSHEV ARRAY

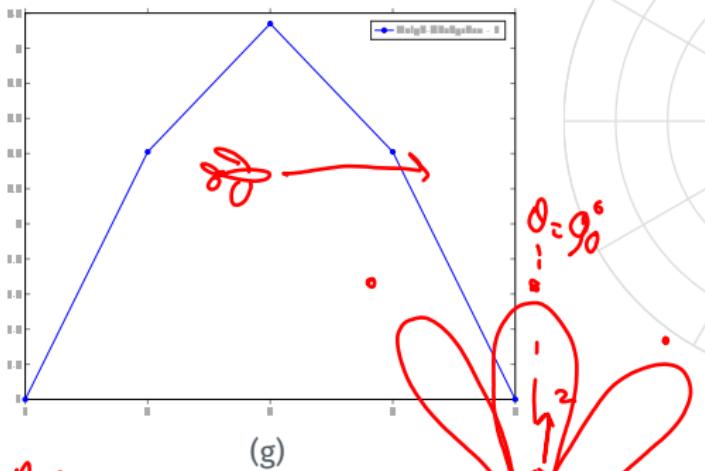
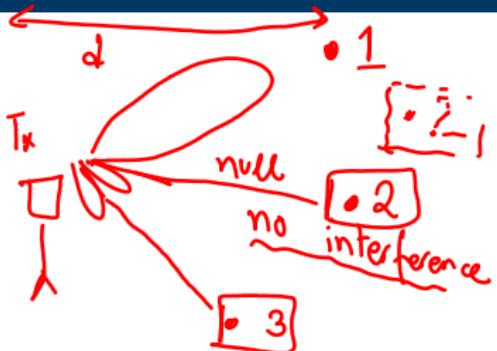


(e)



(f)

DOLPH-CHEBYSHEV-2 ARRAY



(g)

Antenna
Feeding



(h)

