In this lab session, you will learn about:

- 2 and 3D FDTD
- Stability Conditions
- Boundary Conditions
- Source Excitation

Using the discretisation approach that we learnt in the class, we can represent any point in space that is transformed from a continuous domain to a discrete domain $(i\Delta x, j\Delta y, k\Delta z)$.

After doing this, we can then express any function u in the finite difference notation in terms of space and time as:

$$u(i\Delta x, j\Delta y, k\Delta z, n\Delta t) = u^n_{i,j,k} \tag{1}$$

As shown in Fig. 1, we can programatically express the space coordinates x,y, and z with the variable i, j, and k respectively. For time t , the variable n is reserved.

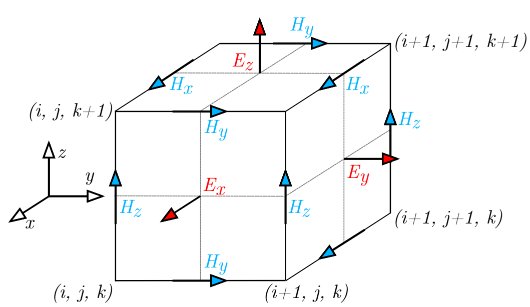

Figure 1: The Yee Lattice showing the space indices and field computation points.

The spacing chosen by Yee in his algorithm produces a 1/2 increment calculation in space:

$$\frac{\partial u(i\Delta x, j\Delta y, k\Delta z, n\Delta y)}{\partial x} = \frac{u^n_{i+1/2,j,k} - u^n_{i-1/2,j,k}}{\Delta x} + \mathcal{O}\left[(\Delta x)^2\right] \tag{2}$$

Similarly, the time derivative was also chosen to be at 1/2 time increments:

$$\frac{\partial u(i\Delta x, j\Delta y, k\Delta z, n\Delta y)}{\partial t} = \frac{u_{i,j,k}^{n+1/2} - u_{i,j,k}^{n-1/2}}{\Delta t} + \mathcal{O}\left[(\Delta t)^2\right] \tag{3}$$

1 Finite Difference Time-Domain method

For the sake of complying with duality we introduced a magnetic current term in the Maxwell's equations. Although, there is no physical basis for it, yet the term is introduced so that dual of the electric current is present in the Ampere's law:

$$\frac{\partial \vec{\mathbf{E}}}{\partial t} = \frac{1}{\varepsilon}\boldsymbol{\nabla} \times \vec{\mathbf{H}} - \frac{\sigma}{\varepsilon}\vec{\mathbf{E}} \tag{4a}$$

$$\frac{\partial \vec{\mathbf{H}}}{\partial t} = -\frac{1}{\mu}\boldsymbol{\nabla} \times \vec{\mathbf{E}} - -\frac{\rho'}{\mu}\vec{\mathbf{H}} \tag{4b}$$

where the terms ρ', and σ refer to the magnetic resistivity and electrical conductivity. In three dimensions, the magnetic fields from (4) can be written as:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} - \rho' H_x\right) \tag{5a}$$

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} - \rho' H_y\right) \tag{5b}$$

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \rho' H_z\right) \tag{5c}$$

and the electric fields as:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x\right) \tag{6a}$$
$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y\right) \tag{6b}$$
$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \tag{6c}$$

1.1 2D case

For the $2D$ case we assume ($\frac{\partial}{\partial z} = 0$), and then write the uncoupled expressions as TM and TE modes:

TM mode:

$$\frac{\partial H_x}{\partial t} = \frac{1}{\mu}\left(-\frac{\partial E_z}{\partial y} - \rho' H_x\right) \tag{7a}$$
$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_z}{\partial x} - \rho' H_y\right) \tag{7b}$$
$$\frac{\partial E_z}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z\right) \tag{7c}$$

TE mode:

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon}\left(\frac{\partial H_z}{\partial y} - \sigma E_x\right) \tag{8a}$$
$$\frac{\partial E_y}{\partial t} = \frac{1}{\varepsilon}\left(-\frac{\partial H_z}{\partial x} - \sigma E_y\right) \tag{8b}$$
$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu}\left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} - \rho' H_z\right) \tag{8c}$$

For the $2DTM$ case, the central difference equations are:

$$H_x\|_{i,j}^{n+1/2} = H_x\|_{i,j}^{n-1/2} - \frac{\Delta t}{\mu_{i,j}}\left[\frac{E_z\|_{i,j+1/2}^{n} - E_z\|_{i,j-1/2}^{n}}{\Delta y}\right] \tag{9a}$$

$$H_y\|_{i,j}^{n+1/2} = H_y\|_{i,j}^{n-1/2} - \frac{\Delta t}{\mu_{i,j}}\left[\frac{E_z\|_{i+1/2,j}^{n} - E_z\|_{i-1/2,j}^{n}}{\Delta x}\right] \tag{9b}$$

$$E_z\|_{i,j}^{n+1} = E_z\|_{i,j}^{n} - \frac{\Delta t}{\varepsilon_{i,j}}\left[\frac{H_y\|_{i+1/2,j}^{n+1/2} - H_y\|_{i-1/2,j}^{n+1/2}}{\Delta x} - \frac{H_x\|_{i,j+1/2}^{n+1/2} - H_x\|_{i,j-1/2}^{n+1/2}}{\Delta y}\right] \tag{9c}$$

$$\tag{9d}$$

Now comes the point where we need to program the set

Just as we saw the 1D expressions of the finite difference notation Maxwell's equations, let's write the 2D analogues where $\frac{\partial}{\partial z} = 0$ is assumed

As discussed in the class, we are only going to compute only one field component at a time. Moreover, the corner points are not considered when it comes to computing the fields. This is because of the *wedge condition* due to which the fields shoot to infinity. The last observation is that we need to space the computation parts uniformly across the whole region of space.

In this lab session, we look at the higher dimension cases of FDTD a bit closely. We will be using some *MATLAB* routines for this purpose. One marked difference is that we are going to define the update coefficients so that material properties can be introduced more efficiently.

Please follow the instructions below:

- In Code::Blocks, go to File and create a New Project.
- Click Console Application and press Next. At the moment, keep the remaining settings to their default. Select C as the programming language.
- Give a name, `First One` to the project name.
- Make sure you have provided a correct path to where the project will be stored in the hard disk space.
- In the Project Management Window pane, under the Workspace, you will find your project. Double click it and you will source further lists

beneath the project title. Go to source, where you will find `main.c` file. This is where you will write your C programs. Double click it and you will see the contents of the C program.

- To run the code, in the top Menu Bar, go to the Build menu, and press Build and Run. A new command line screen should pop up in which the output of the C program will be displayed.

2 EXERCISE - The Hello World!

By convention, the first thing to learn in a new programming language is to print the text "Hello, world!". Although text isn't very exciting by itself, the ability to output text is vital for debugging (fixing programs).

Most programming languages, including C, indicate text with "double quotation marks": the first " indicates the beginning of text. The computer then treats everything until the next " as text.

This causes problems if you want to print an actual " symbol in the output. For that reason, we use escape sequences to represent symbols which are difficult (or impossible) to indicate with plain text.

In addition to telling a compiler how to create an executable file, source code should also tell future programmers why the code does what it does. If anything in the code is unclear, a programmer should add comments which explain the situation. More information about comments is on the good programming style page.

```
// Name: Hello_world.c
// Purpose: Prints Hello, World! on screen
// Put your name here

// Single Line Comment

/* this type of comment can span multiple
  lines; everything is a comment until
  the computer sees the ending. */

#include <stdio.h>
```

```c
int main() {
      printf("Hello, world!");

      // wait for a keypress
      getchar();

      return 0;
}
```

3 EXERCISE - Student Population

Write a short program that displays the number of students in the different cohorts as a list:

EEE	240
CE	220
IE	100

Table 1: Student Population List

itemize

- Print a message similar to the above example.
- No line should be longer than 80 characters you will need to use multiple `printf()` statements.
- Use both types of comments (single line and multi-line).
- Use all these escape sequences:
  - `\n`
  - `\t`
  - \\
  - \’
  - \”