HIGH-FREQUENCY COMMUNICATION SYSTEMS

Lecture 4

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Spring 2023

- Antenna Arrays
- Array Analysis
  - Uniform linear arrays
  - Non-uniform arrays

ANTENNA ARRAYS

- In individual antenna elements, we can't control the radiation patterns
- If we *combine* two antenna elements, it is possible to change the pattern significantly
  - We call the new combined structure as an antenna array.
- We achieve higher directivity using antenna arrays

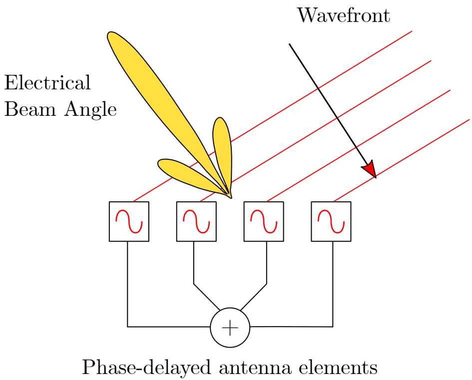


Figure 1: Typical antenna array with phased elements.

- Communications Applications
  - The goal is to focus EM energy towards the target population (cars, people, cities etc.)
    - Modern wireless communications using beamforming
- Radar — multiple target tracking
  - We would like to focus the energy on the targets as they move

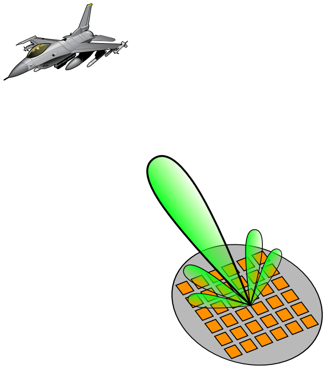


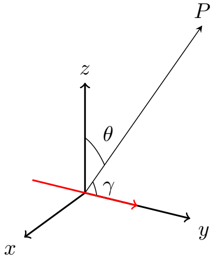
Figure 2: A Phased array antenna in RADAR-based target tracking.

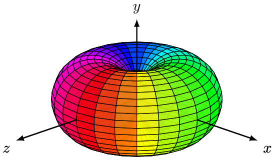
- Recall from the antenna introduction lecture, the field pattern of an infinitesimal dipole:

$$\vec{\mathbf{E}} = \hat{\boldsymbol{\gamma}} jk\eta I_0 l \frac{\exp(-jkr)}{4\pi r} \sin\gamma$$
$$= jk\eta \vec{\mathbf{h}} G(r)$$

where $\vec{\mathbf{h}} = \hat{\boldsymbol{\gamma}} I_0 l \sin\gamma$ and $G(r)$ is the free-space Green function for a point source, $\exp(jkr)/(4\pi r)$

- We will use this as the antenna element





- Some factors determine the desired radiation pattern
  - Array Geometry
  - Element Spacing
  - Element Excitation Amplitude
  - Pattern of individual element

The total field is given by:

$$E_{total} = \text{Element Factor} \times \text{Array Factor}$$

- The simplest antenna array contains two elements
- Two analyse an array, we start by using point sources as individual elements
  - The final pattern is obtained by multiplication
- First, we will ignore the mutual coupling between elements.
- Consider an array of two point sources separated by a distance *d* on the *z*-axis.
- Assuming that both the antenna elements are excited by the current $I_1 = I_0 \exp(j\alpha/2)$ and $I_2 = I_0 \exp(-j\alpha/2)$ where $0 \leq \alpha \leq 2\pi$.

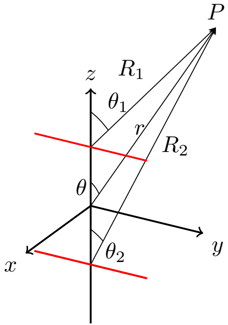


Figure 3: Two point sources forming a basic antenna array.

Neglecting any mutual coupling, we obtain the total fields by simple vector summation:

$$\begin{aligned}\vec{\mathbf{E_t}} =& \vec{\mathbf{E_1}} + \vec{\mathbf{E_2}} \\ =& \hat{\gamma} jk\eta \frac{I_0 \ell}{4\pi} \left\{ \frac{\mathrm{e}^{-jkR_1}}{R_1} \mathrm{e}^{+j\alpha/2} \sin\gamma_1 + \frac{\mathrm{e}^{-jkR_2}}{R_2} \mathrm{e}^{-j\alpha/2} \sin\gamma_2 \right\}\end{aligned}$$

We use the *far-field* approximation:

$$\begin{aligned}\gamma_1 \approx \gamma_2 \approx \gamma \\ R_1 \approx r - \frac{d}{2}\cos\theta \\ R_2 \approx r + \frac{d}{2}\cos\theta \\ R_1 \approx R_2 \approx r (\text{amplitude term})\end{aligned}$$

The total *far-field* thus becomes:

$$\begin{aligned}\vec{\mathbf{E}}_{\mathbf{t}} =& \hat{\gamma} jk\eta \frac{I_0 \ell}{4\pi r} \sin\gamma \left\{ e^{-jk\frac{d}{2}\cos\theta} e^{-j\alpha/2} + e^{-jk\frac{d}{2}\cos\theta} e^{+j\alpha/2} \right\} \\ &= jk\eta \vec{\mathbf{h}} G(r) \left\{ e^{\frac{kd\cos\theta+\alpha}{2}} + e^{-\frac{kd\cos\theta+\alpha}{2}} \right\} \\ &= \underbrace{jk\eta \vec{\mathbf{h}} G(r)}_{\text{Element Factor}} \underbrace{2\cos\left[\frac{1}{2}\left(kd\cos\theta+\alpha\right)\right]}_{\text{Array Factor}}\end{aligned}$$

We can control and change the pattern by varying d and α, which are the spacing and phase shifts.

Let's look at different cases where we consider different values of d and α.

- $\alpha = 0°$ and $d = \lambda/4$, for which the array factor (AF) is

$$AF = 2\cos\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right] = 2\cos\left(\frac{\pi}{4}\cos\theta\right)$$

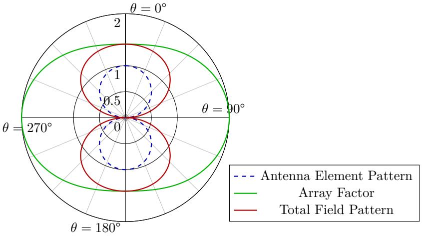


Figure 4: The total field pattern for $\alpha = 0°$ and $d = \lambda/4$.

- Now looking at $\alpha = 90°$ and $d = \lambda/4$, for which the array factor (AF) is

$$AF = 2\cos\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right] = 2\cos\left(\frac{\pi}{4}\cos\theta + \frac{\pi}{4}\right)$$

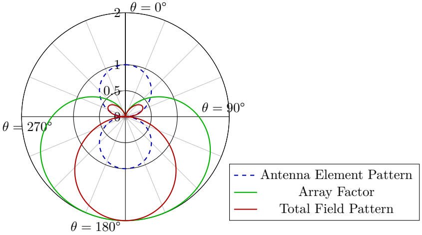


Figure 5: The total field pattern for $\alpha = 90°$ and $d = \lambda/4$.

- Now looking at $\alpha = -90^\circ$ and $d = \lambda/4$, for which the array factor (AF) is

$$AF = 2\cos\left[\frac{1}{2}\left(kd\cos\theta + \alpha\right)\right] = 2\cos\left(\frac{\pi}{4}\cos\theta - \frac{\pi}{4}\right)$$

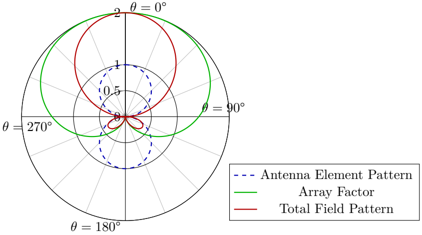
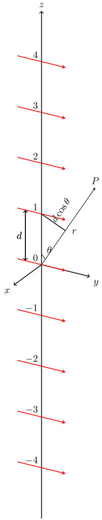


Figure 6: The total field pattern for $\alpha = -90^\circ$ and $d = \lambda/4$.

N-ELEMENT ARRAYS

- A uniform array consists of equally spaced and identical elements
  - All elements are excited with *same* amplitude
  - However, the elements have a progressive phase shift.
- As before, the total field is the vector sum of the individual elements:



$$\begin{aligned}\vec{\mathbf{E}}_T =& \vec{\mathbf{E}}_0 + \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_{-1} + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_{-2} + \dots \\ &= jk\eta\vec{\mathbf{h}}G(r)\left(1 + \mathrm{e}^{j\psi} + \mathrm{e}^{-j\psi} + \mathrm{e}^{2j\psi} + \mathrm{e}^{-2j\psi} + \dots\right)\end{aligned}$$

where $\psi = kd\cos\theta + \alpha$

Continuing, we can write the AF as:

$$AF = \sum_{n=-N'}^{N'} \mathrm{e}^{jn\psi} \quad \text{where } N' = \frac{(N-1)}{2} \tag{1}$$

We can also express (1) as:

$$AF\mathrm{e}^{jn\psi} = \sum_{n=-N'}^{N'} \mathrm{e}^{j(n+1)\psi} \tag{2}$$

From (2) and (1) we get:

$$\begin{aligned} AF &= \frac{\mathrm{e}^{j(N+1)\frac{\psi}{2}} - \mathrm{e}^{-j(N-1)\frac{\psi}{2}}}{\mathrm{e}^{j\psi} - 1} \\ &= \frac{\mathrm{e}^{j\psi/2}}{\mathrm{e}^{j\psi/2}} \left[\frac{\mathrm{e}^{jN\psi/2} - \mathrm{e}^{-jN\psi/2}}{\mathrm{e}^{j\psi/2} - \mathrm{e}^{-j\psi/2}} \right] \end{aligned}$$

$$AF = \frac{\sin N\psi/2}{\sin \psi/2} \text{ where } \psi = kd\cos\theta + \alpha$$

Noting that the above expression resembles a sinc function, we can extend this to find the AF of any discrete array made of uniformly spaced elements. A major difference, however, is the sidelobes don't decay with the increasing function argument.

In general, we plot the AF in the normalised form (divided by N).

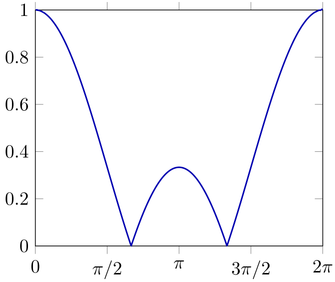
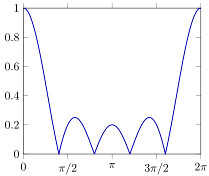


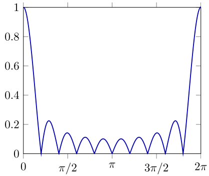
Figure 7: The normalised AF of a 3-element array.

- As N increases, the main lobe narrows
- We get more side lobes in one period of AF as N increases
- The width of the minor/side lobes is $2\pi/N$
- The side lobe height decreases as we increase N
- AF is symmetric about π
- We also see that the peak value occurs at $\psi = \pm 2n\pi$ for $n = 0, 1, 2, \ldots$
- The nulls occur at $\psi = \pm 2n\pi/N$
- The side lobe level is defined as:

$$\text{SLL} = \frac{\text{Max side lobe value}}{\text{Max Main lobe value}}$$



(a) $N = 5$



(b) $N = 10$

Figure 8: Array factors for 5 and 10 element uniform array.

A four-element ($N = 4$), uniformly excited, equally spaced array. The spacing d is $\lambda/2$ and the interelement phasing α is $90°$.

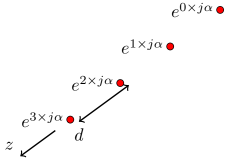


Figure 9: A four-element antenna array.

The normalised array factor is given by:

$$AF = 1/4 \frac{\sin 4\psi/2}{\sin \psi/2} \text{ where } \psi = \frac{2\pi}{\lambda}\frac{\lambda}{2}\cos\theta + 90°$$

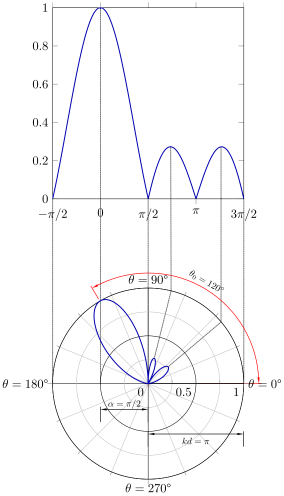
- The linear plot is done as before
- We can plot the corresponding polar plot by *translating* peaks and main and side lobes
- We first *shift* the polar plot by an amount α
  - One period of the *AF* is considered

The direction of the main beam in the polar plot is found as:

$$\psi = kd\cos\theta + \alpha$$

$$\theta = \arccos\frac{\psi - \alpha}{kd}$$

$$\theta_0 = \arccos\left(\frac{0 - \pi/2}{\pi}\right) = 120°$$



In general there are two extreme cases which are sometimes used:

1. Broadside ($\theta_0 = 90°$) when $\alpha = 0$
2. Endfire ($\theta_0 = 0°$or $180°$) when $\alpha = \pm kd$

The array pattern is often characterised by *beamwidth between first nulls*. Knowing that the nulls occur at:

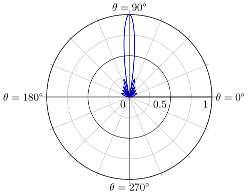
$$N\psi_{FN}/2 = \pm\,\pi$$

For broadside array, $$\frac{N}{2}\frac{2\pi}{\lambda}d\cos\theta_{FN} = \pm\,\pi$$

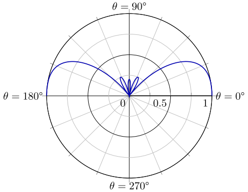
$$\theta_{FN} = \arccos\left(\pm\frac{\lambda}{Nd}\right)$$

The BWFN is:

$$BWFN = |\theta_{\text{FN, left}} - \theta_{\text{FN, right}}|$$



(a) Broadside



(b) Endfire

For practical applications, we require a single pencil beam. To achieve it in the end-fire configuration, one of the ways to generate a single lobe is to use a backing ground plane. Another way to do this is to slightly decrease the element spacing below $\lambda/2$.

Some famous antennas such as the *Yagi-Uda* implement this. The *Hansen-Woodyard* end-fire array also does it by introducing an excess phase delay:

$$\alpha = \pm\left(kd + \delta\right)$$

The end expressions for *Hansen-Woodyard* array are:

$$d < \frac{\lambda}{2}\left(1 - \frac{1}{N}\right)$$
$$\alpha = \pm\left(kd + \frac{\pi}{N}\right)$$

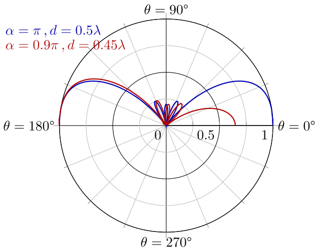


Figure 10: Slight change of the phase.

Considering an example, where a five-element Hansen Woodyard has element spacing $d = 0.37\lambda$ and the element-element phase shift, $\alpha = 0.94\pi$. Let's find the radiation pattern.

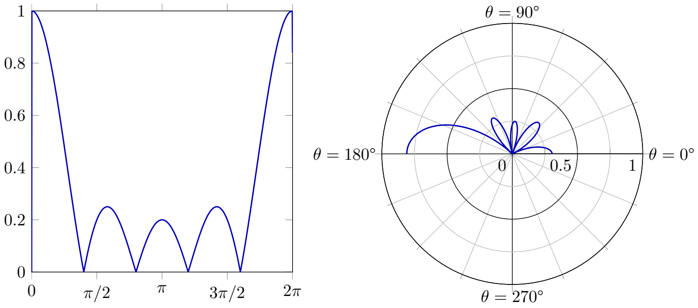


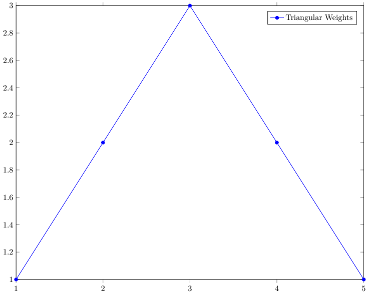
Figure 11: The side-by-side plots of the linear and polar patterns of a Hansen Woodyard array.

The sidelobes can be further truncated using *non-uniform* excitation on the elements. The AF can now be written as a polynomial in terms of $Z = e^{j\psi}$:

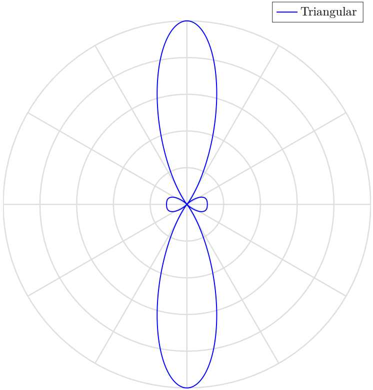
$$AF = \sum_{n=0}^{N-1} A_n e^{jn\psi} = \sum_{n=0}^{N-1} A_n Z_n$$

The current amplitudes A_n are real-valued and different for each n.

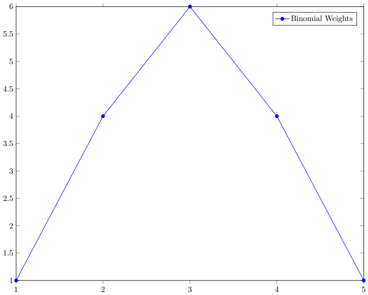
Let's plot the array patterns for a five-element broadside array.


Triangular Weights
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1
2
3
4
5

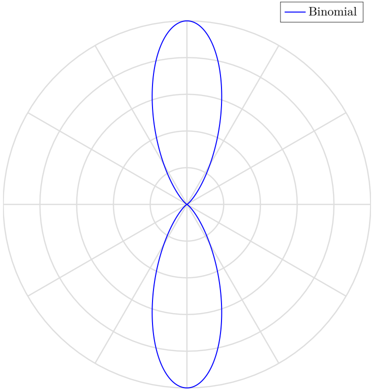
(a)


Triangular

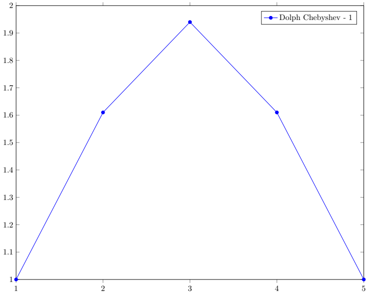
(b)



(c)

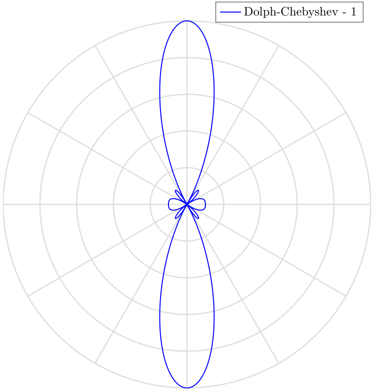


(d)



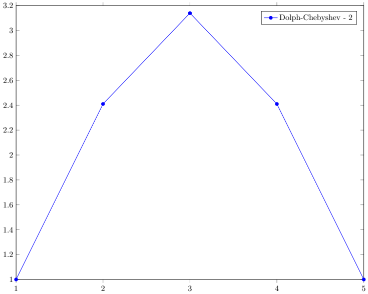
Dolph Chebyshev - 1
2
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1
1
2
3
4
5

(e)

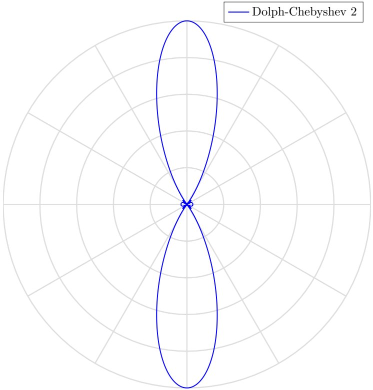


Dolph-Chebyshev - 1

(f)


Dolph-Chebyshev - 2
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
1
2
3
4
5

(g)


Dolph-Chebyshev 2

(h)