HIGH FREQUENCY COMMUNICATION SYSTEMS

Lecture 5

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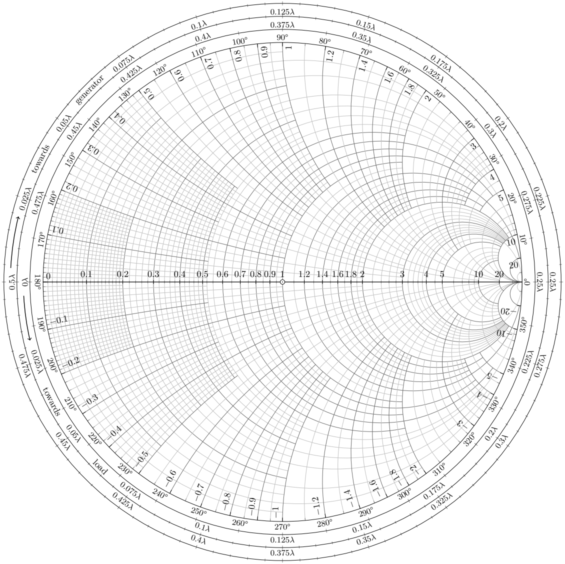
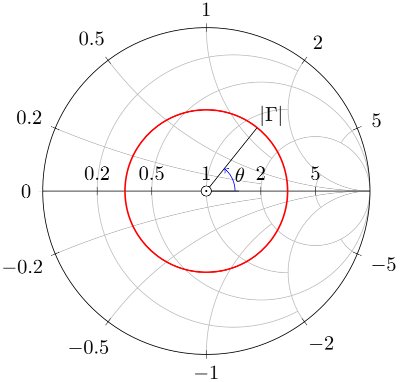
- The Smith Chart
- The Magic of Quarter-wave Transformer
- Load matching through stubs

SMITH CHART



- Developed by P Smith in 1939
- To this day, it is an integral part of microwave circuit design
- Provides a tool to visualise the transmission line phenomena such as impedance matching
- It is simply a polar plot of the reflection coefficient, Γ

- In polar coordinates, $\Gamma = |\Gamma| e^{j\theta}$
- We plot the magnitude as a radius ($|\Gamma| \leq 1$) from the centre
- The angle θ ranges from $-180°$ to $180°$
- The origin or the centre of the Smith chart is the ideal, matched point.



For lossles TLs, the *normalised* load impedance at $l = 0$ is a complex number:

$$z_L = \frac{Z_L}{Z_0} = \frac{1 + |\Gamma| e^{j\theta}}{1 - |\Gamma| e^{j\theta}}$$

Treating $\Gamma = \Gamma_r + j\Gamma_i$, the real and imaginary parts of z_L are:

$$r_L = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

$$x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2}$$

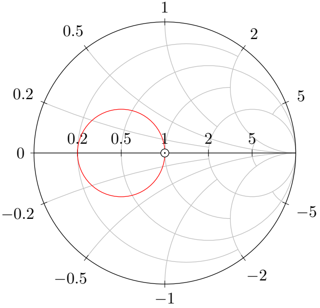
which can be written as two equations of circles:

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2 \qquad \text{(Resistance Circle)}$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2 \qquad \text{(Reactance Circle)}$$

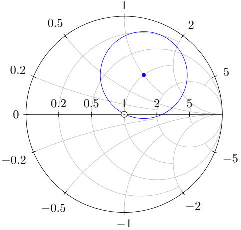
- Lets look at some examples
  - Taking $r_L = 1$ and lets plot in the Γ_r, Γ_i plane
  - But first, the equation for the resistance circle is:

$$\left(\Gamma_r - \frac{1}{1+1}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+1}\right)^2$$



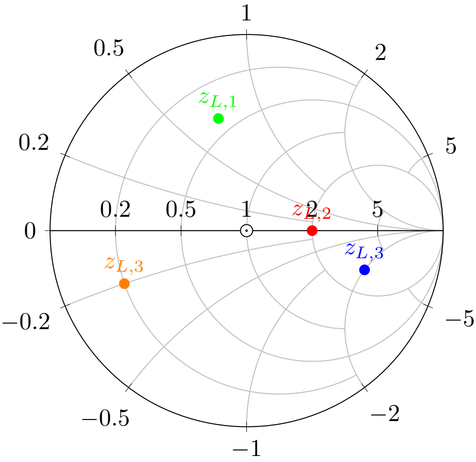
- Now the reactance circle where we take $z_L = j1 \implies x_L = 1$
- The reactance circle equation becomes:

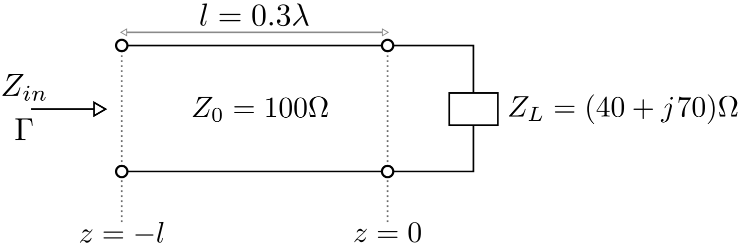
$$(\Gamma_r - 1)^2 + (\Gamma_i - 1)^2 = 1$$



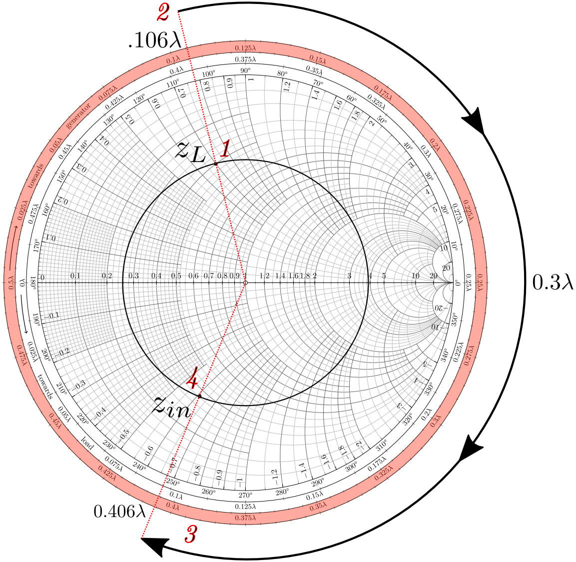
The top half is the *inductive* region and the bottom half is the *capacitive* region.

- We normally normalise the impedance to 50 Ω.
- However, the chart can be used for any value.
- $z_{L,1} = 0.4 + j0.7$
- $z_{L,2} = 2$
- $z_{L,3} = 3 - j2$
- $z_{L,4} = 0.2 - j0.2$

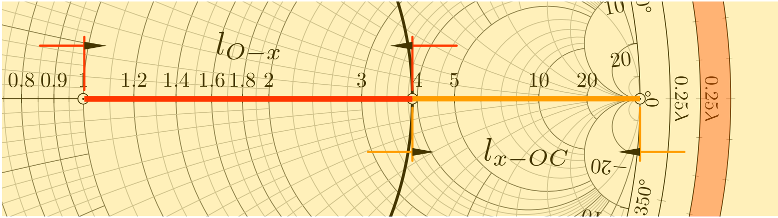




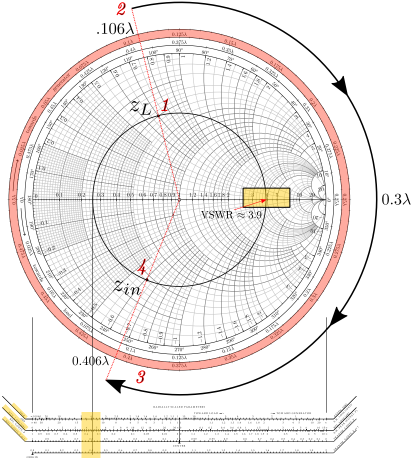
- Lets find the input impedance $Z_{in} = Z(-l)$ of the line.
- Also the reflection coefficient, Γ and the VSWR


2
.106λ
1
z_L
0.3λ
4
z_{in}
0.406λ
3

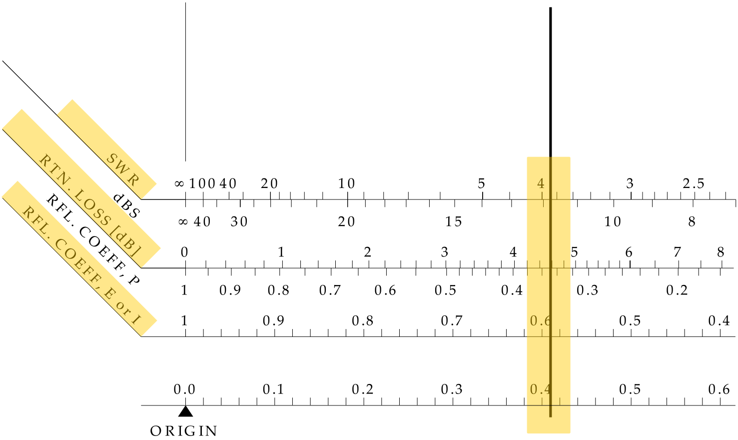
- From the Smith chart, we first plot the normalised load impedance z_L
- We then draw a circle centred on the origin with a radius such that z_L lies on the circle
- Draw a line from the origin passing through z_L to the outer circle of the Smith chart
- Move $l = 0.3\lambda$ towards the generator
- Draw a line from the origin to the new rotated point.
- The intersection point with the circle and the line drawn gives us the normalised input impedance $z_{in} \approx 0.365 - j0.61$.



- The ratio of the length of the line segments l_{O-x} and l_{O-OC} gives us VSWR
- The point on the right of the Smith chart is the open-circuit point ($r = \infty, x = \infty$)
- For this example, we get VSWR ≈ 3.9.

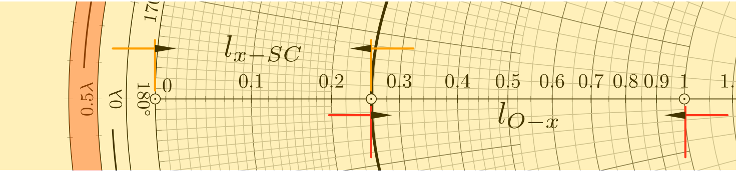


- Some Smith charts provide radially scaled parameters at the bottom of the sheet.
- By drawing a vertical line from the left of the circle to the bottom,

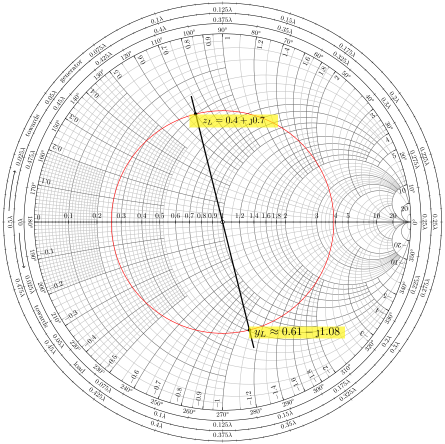


- Reading from the scales, we get, VSWR ≈ 3.9, $|\Gamma| \approx .59$, and the return loss $\approx$ 4.600 dB

- The ratio of the length of the line segments l_{O-x} and l_{O-SC} gives us $|\Gamma|$
- The point on the left of the Smith chart is the short-circuit point ($r = 0, x = 0$)

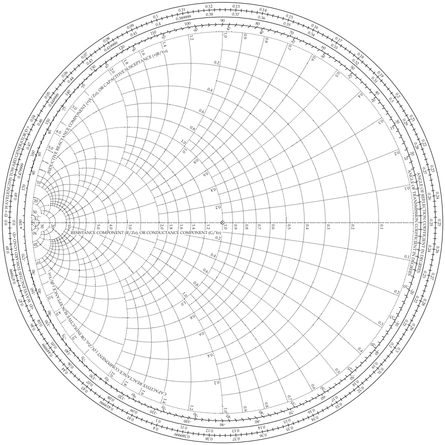


- Admittance is just the reciprocal of the impedance
- On the Smith chart, it represents the diametrically opposite point on the $|\Gamma|$-circle
- For $z_L = 0.4 + j.7$, $y_L = 1/z_L = 0.6 - j1.08$
- Alternatively, we can use an admittance Smith chart

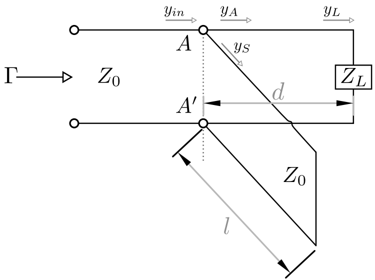


STUB MATCHING

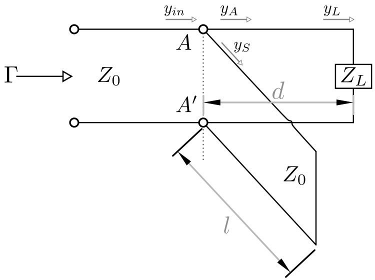
- Stub matching introduces additional impedances/admittances in the line (often in parallel)
- As parallel *admittances* are added up, it is convnenient to use a Smith chart showing admittance rather than impedance.
    - The result is a horizontally flipped admittance Smith chart.



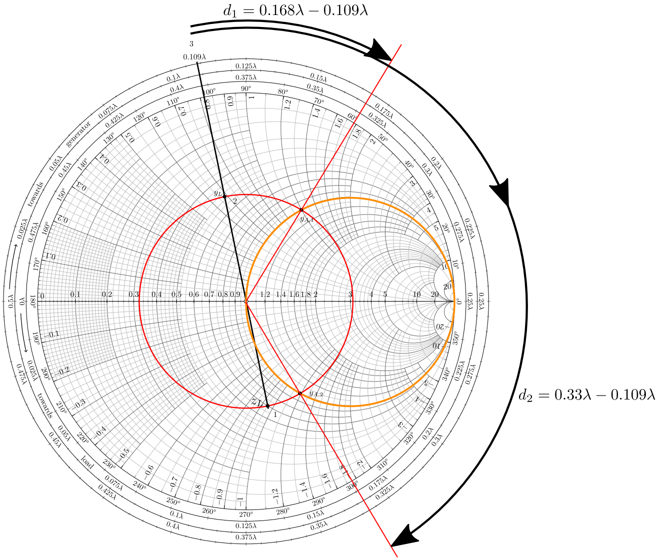
- A short circuit stub of length l is introduced in *parallel* at distance d from the load
- As seen in the figure, we need the input impedance of the parallel combination to be Z_0 at the point A-A'
- In other words, $y_s + y_A = y_{in} = 1$
- As we are using a short circuit stub, $y_s = -jb_A$
- The objective is to find the lengths d and l that generate a unity real part and zero imaginary part of admittance respectively.

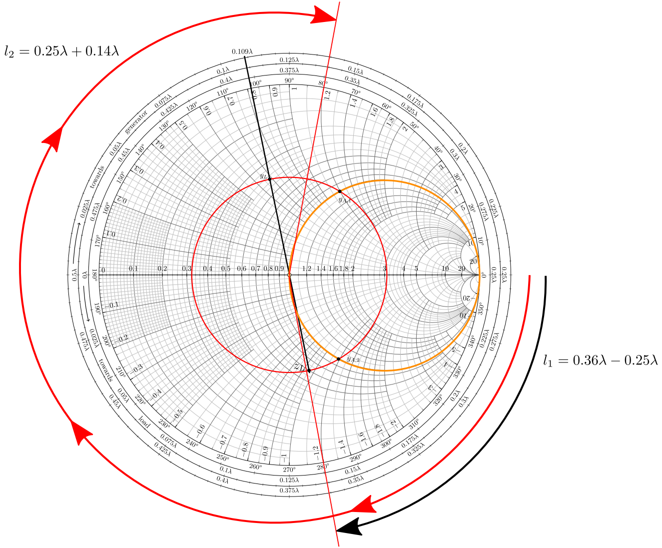


- For a $50\,\Omega$ transmission line connected to a load impedance $Z_L = (35 - j45.50)\,\Omega$
- Find the position and length of the short circuit stub that matches the load to the line.
- z_L becomes $0.70 - j0.95$

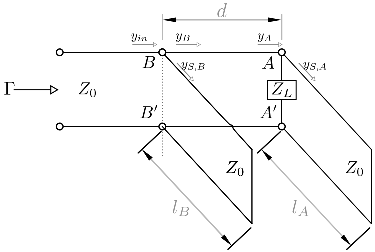


1. Draw z_L and draw the $|\Gamma|$-circle
2. Find y_L using a diametrically opposite line
3. Extend the line to the perimeter and note down the *wavelengths toward generator* value
4. Plot the $g = 1$ circle and note down the two points of intersection $y_{A,1} = 1 + jb_{A,1}$ and $y_{A,2} = 1 + jb_{A,2}$.
5. Find the distances d_1 and d_2 from the generator for the two points above
6. Find the lengths l_1 and l_2 to get the admittances $-jb_{A,1}$ and $-jb_{A,2}$

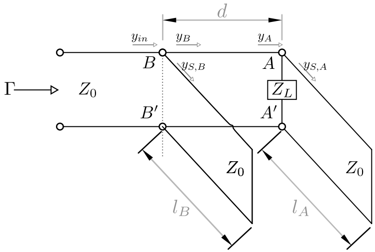

$d_1 = 0.168\lambda - 0.109\lambda$
$d_2 = 0.33\lambda - 0.109\lambda$
0.109λ
y_L
$y_{\lambda 1}$
$y_{\lambda 2}$
z_L
towards generator
towards load


$l_2 = 0.25\lambda + 0.14\lambda$
$l_1 = 0.36\lambda - 0.25\lambda$
0.109λ
y_L
$y_{\lambda,1}$
$y_{\lambda,2}$
z_L
towards generator
towards load

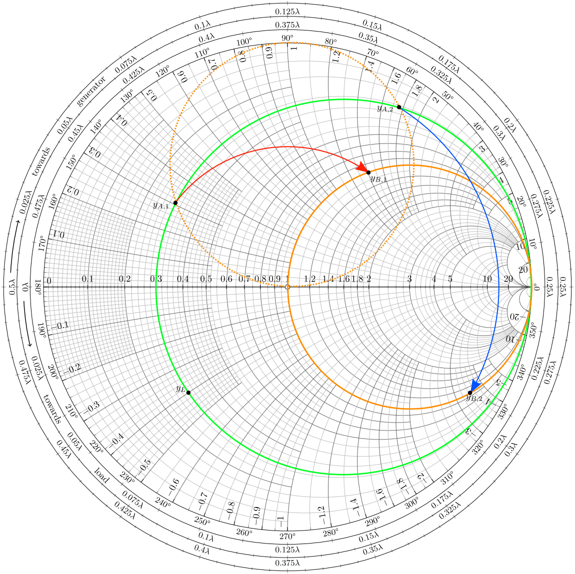
- Single stub matching requires a precise placement of the stub from the load
  - This distance is a function of frequency and therefore, changes if the source frequency is changed
  - Also, we can't engineer the length of the stub of any given value
- To avoid this and use matching at more than one frequency, we use double stub matching



- For a 50 Ω transmission line connected to a looad impedance $Z_L = (60 + j80)\,\Omega$
- A double-stub tuner spaced $d = \lambda/8$ distance from the load for matching.
- Find the lengths of the short circuit stubs that match the load to the line.
- z_L becomes $1.2 + j1.6$
- y_L becomes $0.3 - j0.40$



1. Plot the $g = 1$-circle
2. Rotate the above circle by $\lambda/8$ *towards the load*
3. Plot $y_L = 0.3 - j0.40$ and draw the $g = 0.3$-circle
4. Mark the points $y_{A,1} = 0.30 + j0.30$ and $y_{A,2} = 0.30 + j1.75$ that are points of intersection between the $g = 0.3$-circle and rotated $g = 1$-circle
5. The corresponding points on the $g = 1$-circle are $y_{B,1} = 1 + j1.40$ and $y_{B,2} = 1 - j3.50$.


$y_{A,1}$
$y_{A,2}$
$y_{B,1}$
$y_{B,2}$
y_L
towards generator
towards load

- At the load $(A - A')$, the admittance is $y_A = y_L + y_{S,A}$
- The admittances of the stub are
  $y_{A,1} - y_L = 0.3 + j0.30 - (0.30 - j0.40) = j0.70$ and
  $y_{A,2} - y_L = 0.30 + j1.75 - (0.30 - j0.40) = j2.15$
- Similarly, the admittances are simply the conjugate of of y_B
- They are $y_{B,1} = -j1.40$ and $y_{B,2} = -j3.50$
- The lengths of the stub can be found in the same manner as we did for the single stub.