# Homogeneous Functions and Euler's Theorem

## Homogeneous Functions

A function $f(x, y)$ is homogeneous of degree $n$ if it can be expressed as:

$$f(x, y) = x^n \, \phi\left(\frac{y}{x}\right)$$

where $\phi$ is another rational function.

For example, consider $f(x, y) = x^3 + y^3$. This can be written as $x^3\left(1 + \left(\frac{y}{x}\right)^3\right)$ and hence, it is a homogeneous function of degree $3$.

## Euler's Theorem

If a function $u(x, y)$ is homogeneous and $n$ is the degree, then

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = n \cdot u(x, y)$$