Derivatives

Partial Differentiation

It is just like regular derivative, but it works well on functions of several variables. When differentiating a function with respect to a variable, we treat every constant and other variables in the function as constants, and differentiate with respect to that variable only.

Total Derivative

Consider a function $f(x, y)$ where x and y are written in parametric forms, $\phi(t)$ and $\psi(t)$, then the total derivative of f is

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$$

Implicit Differentiation

Let $f(x, y) = c$ be an implicit function, then upon differentiating, we get

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} = 0$$

Hence, we get:

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$