

Arithmetic Circuits

Binary Addition

Binary addition works just like decimal addition. But these rules are to be kept in mind:

+	0	1
0	0	1
1	1	10

$$\begin{array}{rcccc} & \textcircled{1} & \textcircled{1} & \textcircled{1} & \\ & 1 & 1 & 0 & 1 \\ + & & 1 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 & 0 \\ \hline \end{array}$$

How to carry: When we get 2 bits, the second bit is taken as carry to the next side.

In the following example, we have:

$$\begin{array}{rcccc} & \textcircled{1} & \textcircled{1} & \textcircled{1} & \\ & 1 & 1 & 0 & 1 \\ + & & 1 & 1 & 1 \\ \hline 1 & 0 & 1 & 0 & 0 \\ \hline \end{array}$$

- $1 + 1 = 10$. So 1 of the 10 is taken as carry.
- $0 + 1 = 1$. But the 1 carry makes it $0 + 1 + 1 = 10$. Again we have 1 as carry from the 10.
- $1 + 1 = 10$. But the carry makes it $1 + 1 + 1 = 11$. So we have to carry 1 from 11.
- Then we have 1. But the carry makes it $1 + 1 = 10$.

So the final number is 10100.

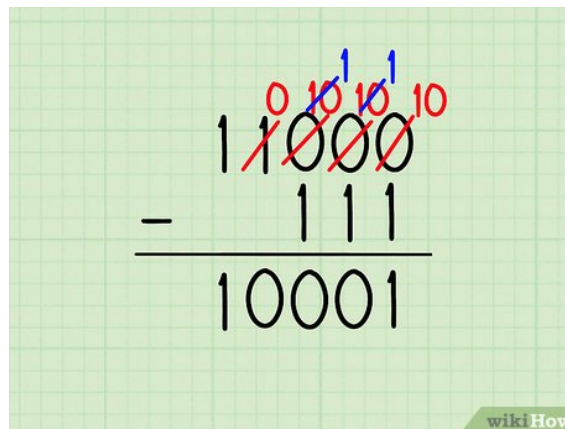
Binary Subtraction

Binary subtraction works very similar to decimal subtraction, except for these:

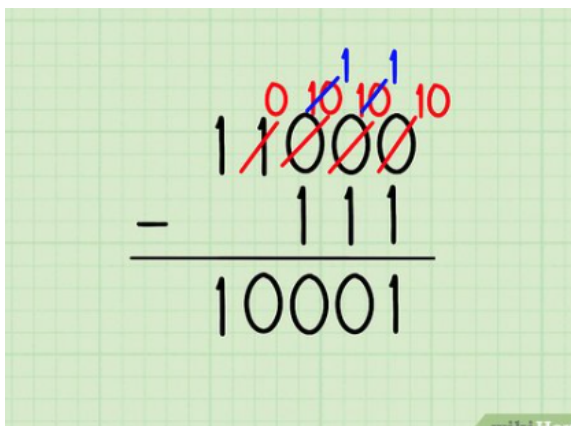
-	0	1
0	0	1
1	1 (Borrow: $10 - 1 = 1$)	0

How to borrow: When we have an expression like $0 - 1$, where the first number is less than the second, then we do borrowing. In this case:

1. We keep going left until we find a 1.
2. Make the 1 to 0, then the digit to the right becomes 10.
3. Now, borrow again. 10 becomes 1 and the digit to the right becomes 10.
4. Do this until you reach the same column you started at.



$$\begin{array}{r} 11000 \\ - 111 \\ \hline 10001 \end{array}$$



$$\begin{array}{r} 11000 \\ - 111 \\ \hline 10001 \end{array}$$

In the given example, we start with the first column on the right.

We have $0 - 1$. So we go left, and find $0 - 1$ again. We go left, and find $0 - 1$ again. Then, we find $1 - 0$. So we borrow from the 1 to make it 0, and the 0s on the right become 10. Now do the regular binary subtraction ($10 - 1 = 1$)

Binary Multiplication

Binary multiplication works in the same way as decimal multiplication does. In fact, it is just repeated binary addition! Here is the table to show how two single bits can be multiplied:

×	0	1
0	0	0
1	0	1

Multiplication for greater than or equal to 2 bits: If we have to multiply two numbers that hold 2 bits or greater, then we follow the same steps as in decimal multiplication for two numbers that have at least 2 digits.

$$\begin{array}{r}
 1\ 1\ 0\ 1\ 1 \\
 \times\ 1\ 0\ 1 \\
 \hline
 1 \\
 1\ 1\ 0\ 1\ 1 \\
 0\ 0\ 0\ 0\ 0\ 0\ x \\
 1\ 1\ 0\ 1\ 1\ x\ x \\
 \hline
 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1
 \end{array}$$

$$\begin{array}{r}
 1\ 1\ 0\ 1\ 1 \\
 \times\ 1\ 0\ 1 \\
 \hline
 1 \\
 1\ 1\ 0\ 1\ 1 \\
 0\ 0\ 0\ 0\ 0\ 0\ x \\
 1\ 1\ 0\ 1\ 1\ x\ x \\
 \hline
 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1
 \end{array}$$

In the given example, it can be observed that each bit on the first number is being multiplied by the bit on the second number, and the result is written with each row moved left once, and these new numbers are added, just like in decimal multiplication!

Binary Division

Binary division works in the same way as decimal division does. In fact, it is just repeated binary subtraction! Here is the table to show how two single bits can be divided:

÷	0	1
0	Not possible	Not possible
1	0	1

Long division for binary numbers: In binary, long division works similar to decimal long division, but it can get very tricky. Just like in decimal long division:

1. Multiply the divisor by the number that is the greatest number lesser than the dividend.
2. Do regular binary subtraction
3. Get the next bit to the new number, or if it does not exist, add a 0 bit and write a decimal point on the quotient.

Binary Division: Example



$$\begin{array}{r} 101 \\ 101 \overline{) 11010} \\ \underline{(-) 101} \downarrow \\ 11 \downarrow \\ \underline{(-) 00} \downarrow \\ 110 \downarrow \\ \underline{(-) 101} \\ 1 \end{array}$$