Special Series

There are many special series, most of them named after famous mathematicians or related fields. This page covers three such series:

- Power series
- Taylor series
- Maclaurin series

Power Series

This is an infinite series of the form

$$f(x) = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n + \ldots$$

It is to be noted that:

- a is independent of x
- This series is used to approximate complex functions

The power series is about $x = p$ for a constant p if the series is $\sum_{n=0}^{\infty} a_n (x - p)^n$

Interval of Convergence

It is the values or range of x in $f(x)$ for which the series converges.

To solve questions based on this, use one of the convergence tests mentioned in another PDF to find the values of x. In most cases, it will be either ratio test or root test.

Taylor Series

For a function $f(x)$, if it is about $f(a)$ then according Taylor series:

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot f^n(a) \cdot (x - a)^n$$

Certain expansions such as e^x uses this series. Consider $f(x) = e^x$ about $f(0)$. Then

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Maclaurin Series

This series is a special case of the Taylor series when the function $f(x)$ is about $f(0)$. So basically

$$f(x) = \sum_{n=0}^{\infty} \frac{1}{n!} \cdot f^n(0) \cdot x^n$$