Concepts

Topics covered are:

1. Differentiation
2. Chain Rule
3. Differentiability
4. Increasing and Decreasing Functions
5. Increment Theorem

Differentiation

The rate of change of a function is said to be the derivative of a function.

$f'(x) = \frac{dy}{dx} = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h},\ where\ h = \Delta x$

Here are different ways to write a derivative, considering $y = f(x)$:

$\frac{dy}{dx} = f'(x) = y'$

For differentiating it n times:

$\frac{d^n y}{(dx)^n} = f''''...\ n\ times(x) = y''''...\ n\ times$

Chain rule

If $y = f(u),\ u = g(x)$ or $y = f(g(x))$,

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

Differentiability

A function $f(x)$ is differentiable at a point if $RHD = LHD$ where

$RHD = \lim\limits_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and

$LHD = \lim\limits_{h \to 0} \frac{f(x-h)-f(x)}{-h}$

Graphically, A function which is continuous and has no sharp points throughout its domain is said to be differentiable.

Here is a list of some functions and their continuity and differentiability throughout its domain:

Function	Continuous?	Differentiable?
Polynomial ($\sum_{i=0}^{n} a^{i} x^{n-i}$, $n \in N$)	Yes	Yes
Modulus ($\lvert x \rvert$)	Yes	No (at 0)
Constant (a)	Yes	Yes
Signum ($\frac{\lvert x \rvert}{x}$, $x \neq 0$; 0, $x = 0$)	No (at 0)	No (at 0)
Greatest Integer / Floor ($\lfloor x \rfloor$)	No (at integers)	No (at integers)
Smallest Integer / Ceil ($\lceil x \rceil$)	No (at integers)	No (at integers)
Fractional ($\{x\} = x - \lfloor x \rfloor$)	No (at integers)	No (at integers)
Reciprocal ($\frac{1}{f(x)}$, $f(x) \neq 0$)	No (depends on $f(x)$)	No (depends on $f(x)$)

Increasing and Decreasing Functions

A function $f(x)$ is said to to be increasing throughout (a, b) if $f'(x) \geq 0,\ x \in (a, b)$
A function $f(x)$ is said to to be decreasing throughout (a, b) if $f'(x) \leq 0,\ x \in (a, b)$

A function $f(x)$ is said to to be strictly increasing throughout (a, b) if $f'(x) > 0,\ x \in (a, b)$
A function $f(x)$ is said to to be strictly decreasing throughout (a, b) if $f'(x) < 0,\ x \in (a, b)$

A function $f(x)$ is said to to be constant throughout (a, b) if $f'(x) = 0,\ x \in (a, b)$

Increment Theorem

If a function $f(x)$ is differentiable at a point x, then for a small increment Δx, the corresponding increment in the function, $\Delta y = f(x + \Delta x) - f(x)$, can be expressed as:

$$\Delta y = f'(x) \cdot \Delta x + \epsilon \cdot \Delta x$$

where $\epsilon \to 0$ as $\Delta x \to 0$.
Here, ϵ is a small number that cancels out the error achieved by approximating the change in the function. It becomes negligible as the change in x is reduced, because the value of y gets more accurate.

Uses

Differentiation: To find the rate of change in something at a given instant, and to obtain the tangent of a curve at any given input value.
Chain Rule: Finding derivatives of nested functions.
Differentiability: Determine whether a function can be differentiated at a given input value without having to look at the curve.
Increasing and Decreasing Functions: Determine whether a function is rising or declined over a range without looking at the curve.
Increment Theorem: Highly useful for error approximation and rigorous ways to prove derivatives.