CSE 215 - Homework 5

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April 3, 2021

Problem Set 1

25a. $R_1 = \{x \in \mathbb{R} | 1 \leq x \leq 1 + \frac{1}{1}\} = [1, 2]$
$R_2 = \{x \in \mathbb{R} | 1 \leq x \leq 1 + \frac{1}{2}\} = [1, \frac{3}{2}]$
$R_3 = \{x \in \mathbb{R} | 1 \leq x \leq 1 + \frac{1}{3}\} = [1, \frac{4}{3}]$
$R_4 = \{x \in \mathbb{R} | 1 \leq x \leq 1 + \frac{1}{4}\} = [1, \frac{5}{4}]$
$\bigcup_{i=1}^{4} R_i = [1, 2]$

25b. $\bigcap_{i=1}^{4} R_i = [1, \frac{5}{4}]$

25c. No, $R_1, R_2, R_3...$ aren't mutually disjoint because they all contain at least 1. Not to mention, if they are mutually disjoint, 25b should be a null set, but it clearly doesn't result in one.

25d. $\bigcup_{i=1}^{n} R_i = [1, 2]$

25e. $\bigcap_{i=1}^{n} R_i = [1, \frac{n+1}{n}]$

25f. $\bigcup_{i=1}^{\infty} R_i = [1, 2]$

25g. $\bigcap_{i=1}^{n} R_i = \{1\}$

Problem Set 2

26a. $R_1 = \{x \in \mathbb{R} | 1 < x < 1 + \frac{1}{1}\} = (1, 2)$
$R_2 = \{x \in \mathbb{R} | 1 < x < 1 + \frac{1}{2}\} = (1, \frac{3}{2})$
$R_3 = \{x \in \mathbb{R} | 1 < x < 1 + \frac{1}{3}\} = (1, \frac{4}{3})$
$R_4 = \{x \in \mathbb{R} | 1 < x < 1 + \frac{1}{4}\} = (1, \frac{5}{4})$
$\bigcup_{i=1}^{4} R_i = (1, 2)$

26b. $\bigcap_{i=1}^{4} R_i = (1, \frac{5}{4})$

26c. They are not mutually disjoint. Take R_1 and R_2. R_1 goes from 1 to 2, which includes all the numbers from R_2 which is from 1 to $\frac{3}{2}$. Since there is overlap, these sets can not be disjoint, therefore the collection can not be mutually disjoint.

26d. $\bigcup_{i=1}^{n} R_i = (1, 2)$

26e. $\bigcap_{i=1}^{n} R_i = (1, \frac{n+1}{n})$

26f. $\bigcup_{i=1}^{\infty} R_i = (1, 2)$

26g. $\bigcap_{i=1}^{n} R_i = \{\phi\}$

Problem Set 3

33a. $\mathscr{P}(\phi) = \{\phi\}$

33b. $\mathscr{P}(\mathscr{P}(\phi)) = \{\phi, \{\phi\}\}$

33c. $\mathscr{P}(\mathscr{P}(\mathscr{P}(\phi))) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$

34a. $A_1 \times (A_2 \times A_3)$
$= A_1 \times (\{u, v\} \times \{m, n\})$
$= \{1, 2, 3\} \times \{(u, m), (u, n), (v, m), (v, n)\}$
= {(1, (u, m)), (1, (u, n)), (1, (v, m)), (1, (v, n)), (2, (u, m)), (2, (u,

n)), (2, (v, m)), (2, (v, n)), (3, (u, m)), (3, (u, n)), (3, (v, m)), (3, (v, n))}

34b. $(A_1 \times A_2) \times A_3$
$= (\{1, 2, 3\} \times \{u, v\}) \times A_3$
$= \{(1, u), (1, v), (2, u), (2, v), (3, u), (3, v)\} \times \{m, n\}$
= {((1, u), m), ((1, u), n), ((1, v), m), ((1, v), n), ((2, u), m), ((2, u), n), ((2, v), m), ((2, v), n), ((3, u), m), ((3, u), n), ((3, v), m), ((3, v), n)}

34c. $A_1 \times A_2 \times A_3$
$= \{1, 2, 3\} \times \{u, v\} \times \{m, n\}$
= {(1, u, m), (1, u, n), (1, v, m), (1, v, n), (2, u, m), (2, u, n), (2, v, m), (2, v, n), (3, u, m), (3, u, n), (3, v, m), (3, v, n)}

Problem Set 4

10. Proof that $(A - B) \cap (C - B) = (A \cap C) - B$.

Proof that $(A - B) \cap (C - B) \subseteq (A \cap C) - B$
Suppose $x \in (A - B) \cap (C - B)$
$x \in (A - B)$ and $x \in (C - B)$ ($\because$ Definition of Intersection)
$x \in A$ and $x \notin B$ and $x \in C$ and $x \notin B$
$x \in A$ and $x \in C$ and $x \notin B$ and $x \notin B$ ($\because$ Commutative Property)
$x \in A$ and $x \in C$ and $x \notin B$ ($\because$ Impodent Law)
$(x \in A$ and $x \in C)$ and $x \notin B$ ($\because$ Associative Property)
$x \in (A \cap C) - B$

Proof that $(A \cap C) - B \subseteq (A - B) \cap (C - B)$
Suppose $x \in (A \cap C)$ and $x \notin B$
$x \in A$ and $x \in C$ ($\because$ Definition of Intersection)
$x \in A$ and $x \notin B$, then $x \in (A - B)$
$x \in C$ and $x \notin B$, then $x \in (C - B)$
$x \in (A - B) \cap (C - B)$ ($\because$ Definition of Intersection)

Problem Set 5

19. Proof that $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

    Proof that $A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$
    Suppose $(x, y) \in A \times (B \cap C)$
    $x \in A$ and $y \in (B \cap C)$ ($\because$ Definition of Cartesian Product)
    $y \in B$ and $y \in C$ ($\because$ Definition of Intersection)
    $(x, y) \in (A \times B)$ ($\because$ $x \in A$ and $y \in B$)
    $(x, y) \in (A \times C)$ ($\because$ $x \in A$ and $y \in C$)
    $(A \times B) \cap (A \times C)$

    Proof that $(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$
    Suppose $(x, y) \in (A \times B) \cap (A \times C)$
    $(x, y) \in (A \times B)$ and $(x, y) \in (A \times C)$ ($\because$ Definition of Intersection)
    $x \in A$ and $y \in B$ and $x \in A$ and $y \in C$ ($\because$ Definition of Cartesian Product)
    $x \in A$ and $x \in A$ and $y \in B$ and $y \in C$ ($\because$ Commutative Property)
    $x \in A$ and $y \in B$ and $y \in C$ ($\because$ Impodent Law)
    $A \times (B \cap C)$

Problem Set 6

34. Proof that if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \phi$.

    Suppose not. Suppose $B \cap C \subseteq A$ and $(C - A) \cap (B - A) \neq \phi$

    Suppose $x \in (C - A) \cap (B - A)$
    $x \in (C - A)$ and $x \in (B - A)$ ($\because$ Definition of Intersection)
    $x \in C$ and $x \in B$, but $x \notin A$

    However, $B \cap C \subseteq A$ is assumed to be true and $x \notin A$ means that $x \notin (B \cap C)$, therefore we arrive at a contradiction.

    This means this statement is false and the original statement must be true.