CSE 215 - Homework 2

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Problem Set 1

52. Let m = 3. 3^2 - 4 = 5. 5 is prime and not composite, thus this statement is false.

53. Let n = 11. $(11)^2$ - 11 + 11 = 121. 121 is not prime as it is divisible by 11, thus this statement is false.

Problem Set 2

61. m + n + $2\sqrt{mn}$
    Let a^2 = m and b^2 = n.
    = a^2 + b^2 + $2\sqrt{a^2b^2}$ (Substitution in for m and n)
    = a^2 + b^2 + 2ab (Simplifying $\sqrt{a^2b^2}$)
    = a^2 + 2ab + b^2 (Commutative Property of Addition)
    = $(a+b)^2$ (Factoring a^2 + 2ab + b^2)
    Let (a + b) = x.
    = x^2 (Substitution in for (a + b))
    true because x^2 can always be written by a product of x and x.

Problem Set 3

30. x^2 + bx + c = (x - r)(x - s) (Given in question)
    = x^2 - rx - sx + rs (Expanding (x - r)(x - s))
    = c = rs (The two constants of the 2 polynomials must be equal).
    Supppose r is rational.

$= s = \frac{c}{r}$ (Divide both sides by r)
Since both c and r are both rational, then s must also be rational since it's the quotient of both c and r.

Problem Set 4

38. This person never proved that $\frac{a}{b} + \frac{c}{d}$ is equal to a fraction where both the numerator and denominator are both integers.

39. Never proved r + s is rational. Uses the assumption that r + s is rational in the equation.

Problem Set 5

30. If a | n^2, then n^2 is a factor of a (By definition)
n is a factor of n^2 (n $\times$ n = n^2)
n is a factor of a (By transitivity)
$\therefore$ a | n (By definition)

Problem Set 6

34. No it isn't possible because nickels are 0.05, quarters are 0.25, and dimes are 0.10 and quarters = 5 nickels and dimes = 2 nickels. 4.72 doesn't have a factor of 0.05 because 4.72 % 0.05 $\neq$ 0, thus it can't be written as a multiple of 0.05 and thus, can't be written as a combination of nickels or and of the other coins stated.

Problem Set 7

35. 40 is the least common multiple of both 8 and 10, therefore 4:00 + 40 = 4:40PM will be the first time the two will meet at the start line after beginning.

Problem Set 8

42c. 8 zeroes. In 20!, there contains 4 multiples of 5 (5, 10, 15, and 20) and each of these can produce a zero ($5 \times 2 = 10$ and $15 \times 4 = 60$). If we then multiply this by 2 due to the number being squared, we get 8 zeroes.

Problem Set 9

43. 102 men and 170 women. Since 2/3 of the men must be married, the number of men must be divisible by 3 since the only way to get rid of the denominator of 3 is to get a number which is a multiple of 3. Thus the least integer closest to 100 is 102. To find the number of women, we know that there are 102 married women and 3/5 of the total number of women are married. We have to divide 102 by 3/5 to get the total number of women in the town since it must fit the ratio of $\frac{102}{x} : \frac{3}{5}$.

Problem Set 10

30a. Case num % 3 == 0, then the number can be written as 3k (since it's divisible or a multiple of 3) and the next number would be 3k + 1.

3k(3k + 1) (First and Second Number Multiplied)
= $9k^2$ + 3k (Expanded Form)
= 3($3k^2$ + k) (Factor Out 3)
= 3l (If we substitute in l = $3k^2$ + k)

Case num % 3 == 1, then the number can be written as 3k + 1 (since the previous number is divisible or a multiple of 3) and the next number would be 3k + 2.

= (3k + 1)(3k + 2) (Multiplying the two numbers)
= $9k^2$ + 9k + 2 (Expanded Form)
= 3($3k^2$ + 3k) + 2 (Factor out 3)
= 3l + 2 (If we substitute in l = $3k^2$ + 3k)

Case num % 3 == 2, then the number can be written as 3k + 2 (since the previous previous number is divisible or a multiple of 3) and the next number would be 3k + 3.

= (3k + 2)(3k + 3) (Multiplying the two numbers)
= $9k^2$ + 15k + 6 (Expanded Form)
= 3($3k^2$ + 5k + 2) (Factor out 3)
= 3l (If we substitute in l = $3k^2$ + 5k + 2)

As we can see all the possible solutions can be written as either 3l or 3l + 2 and thus, this statement is true.

Problem Set 11

31a.

Case 1(Both even):	even + even = even even - even = even
Case 2(Both odd):	odd + odd = even odd - odd = even
Case 3(m even n odd):	even + odd = odd even - odd = odd
Case 4(m odd n even):	odd + even = odd odd - even = odd

As we can see above, in each of the cases, the outcomes are always either both even or both false, thus this statement is true.

Problem Set 12

40. = n(n - 1)(n + 1)(n + 2) (Factoring n^2 - 1)
= (n - 1) n (n + 1)(n + 2) (Commutative Property)
As we can see, these are all consective numbers where n - 1 is the first number followed by n, then n + 1, then n + 2. They are all also being multiplied Regardless of the value of n, one of these numbers will always be a multiple of 4 since it's 4 consective numbers, and they're being

multiplied with eachother which means the product will also always be a multiple of 4 and, thus divisible by 4.

Problem Set 13

24a. The reciprocal of any irrational number is irrational.
= If a number is rational, then its reciprocal is rational. (Contrapositive)

x = $\frac{a}{b}$ (By definition, if a number is rational there there must be a ratio of integers with a non-zero denominator).
= bx = a (Cross multiplying)
= $\frac{bx}{a}$ = 1 (Divide both sides by a)
= $\frac{b}{a} = \frac{1}{x}$ (Divide both sides by x)

Since the reciprocal of x ($\frac{1}{x}$) is equal to a ratio of 2 integers b and a, then by definition, its reciprocal is rational. This makes the statement that "If a number is rational, then its reciprocal is rational" true and therefore its contrapositive is also true.

24b. If the reciprocal of a number is irrational, the number is irrational.
= The reciprocal of a number is irrational and the number is rational. (Negation)

x = $\frac{a}{b}$ (By definition, if a number is rational there there must be a ratio of integers with a non-zero denominator).
= bx = a (Cross multiplying)
= $\frac{bx}{a}$ = 1 (Divide both sides by a)
= $\frac{b}{a} = \frac{1}{x}$ (Divide both sides by x)

Therefore, $\frac{1}{x}$ is rational by definition since it can be written as a ratio between non zero integers. There is a contradiction. The reciprocal of x cannot be both rational and irrational making this statement false, and thus the negation of this statement/the original statement must be true.

Problem Set 14

31b. For all integers n 1, if n is not prime, then there exists a prime number p such that p $\leq \sqrt{n}$ and n is divisible by p.
= For all integers n > 1, if a prime number p is > $\sqrt{n}$ or n is not divisible by p, then n is prime. (Contrapositive)

Since a number is never divisible by p when p is > $\sqrt{n}$, then this statement is true meaning its contrapositive or the original statement is true.

Problem Set 15

33.

	~~2~~	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Problem Set 16

In a number set of 1 trillion + 1 numbers, there will always be two numbers whose difference is a multiple of 1 trillion. Every number in this set can be written as (1 trillion) * n + (some remainder when the number is mod by 1 trillion). In order for the number to be divisible by 1 trillion, the constants at the end of this expression must be equal in order for the difference to be divisible by 1 trillion. Since there are 1 trillion + 1 numbers, there will always be a set where these two constants are equal since there are only 1 trillion different constants at the end (since any number mod 1 trillion can only give a number within the range [1, 1 trillion)) and the + 1 ensures that there will be a duplicate.