Topic

Convolution and Correlation

Lecture by: Fatemeh Pishdadian

Convolution

- **convolution** is a mathematical operator which takes two functions *x* and *h* and produces a third function that represents the amount of overlap between *h* and a reversed and translated version of *x*.
- In signal processing, one of the functions (*h*) is taken to be a fixed filter *impulse response*, and the other (x) the input signal.

$$\underset{\substack{\uparrow \\ \text{Convolution operator}}}{(h * x)(t)} \equiv \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Discrete Convolution

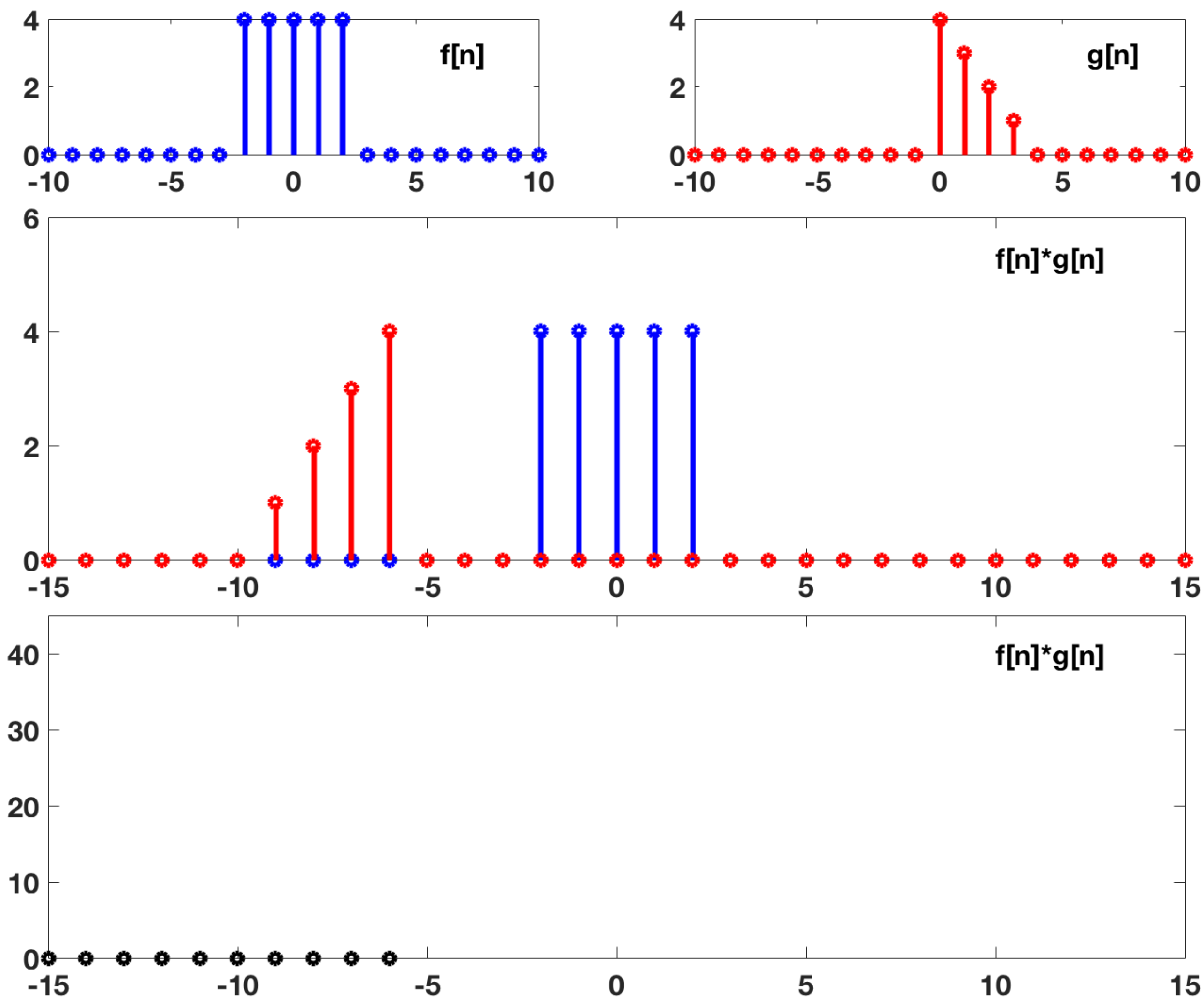
- **convolution** is a mathematical operator which takes two functions *x* and *h* and produces a third function that represents the amount of overlap between *h* and a reversed and translated version of *x*.
- In signal processing, one of the functions (h) is taken to be a fixed filter *impulse response*, and the other (x) the input signal.

$$(h \underset{\substack{\uparrow \\ \text{Convolution} \\ \text{operator}}}{*} x)[n] \equiv \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

Convolution In Python Code

```
import numpy as np

def convolution(A,B):

      lengthA=np.size(A)
      lengthB=np.size(B)

      C = np.zeros(lengthA + lengthB -1)

      for m in np.arange(lengthA):
           for n in np.arange(lengthB):
                C[m+n] = C[m+n] + A[m]*B[n]

      return C
```

Let’s look at Convolution



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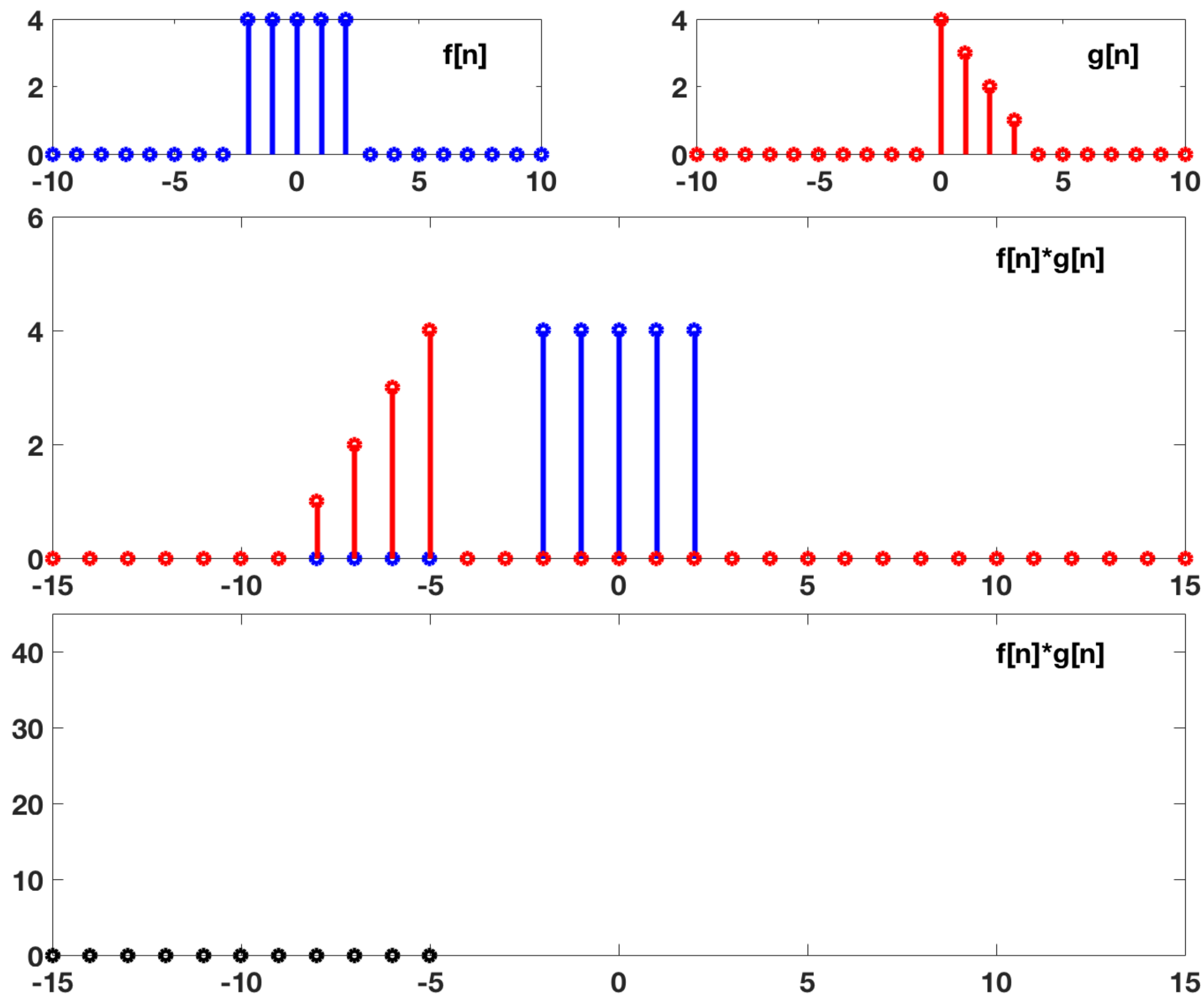

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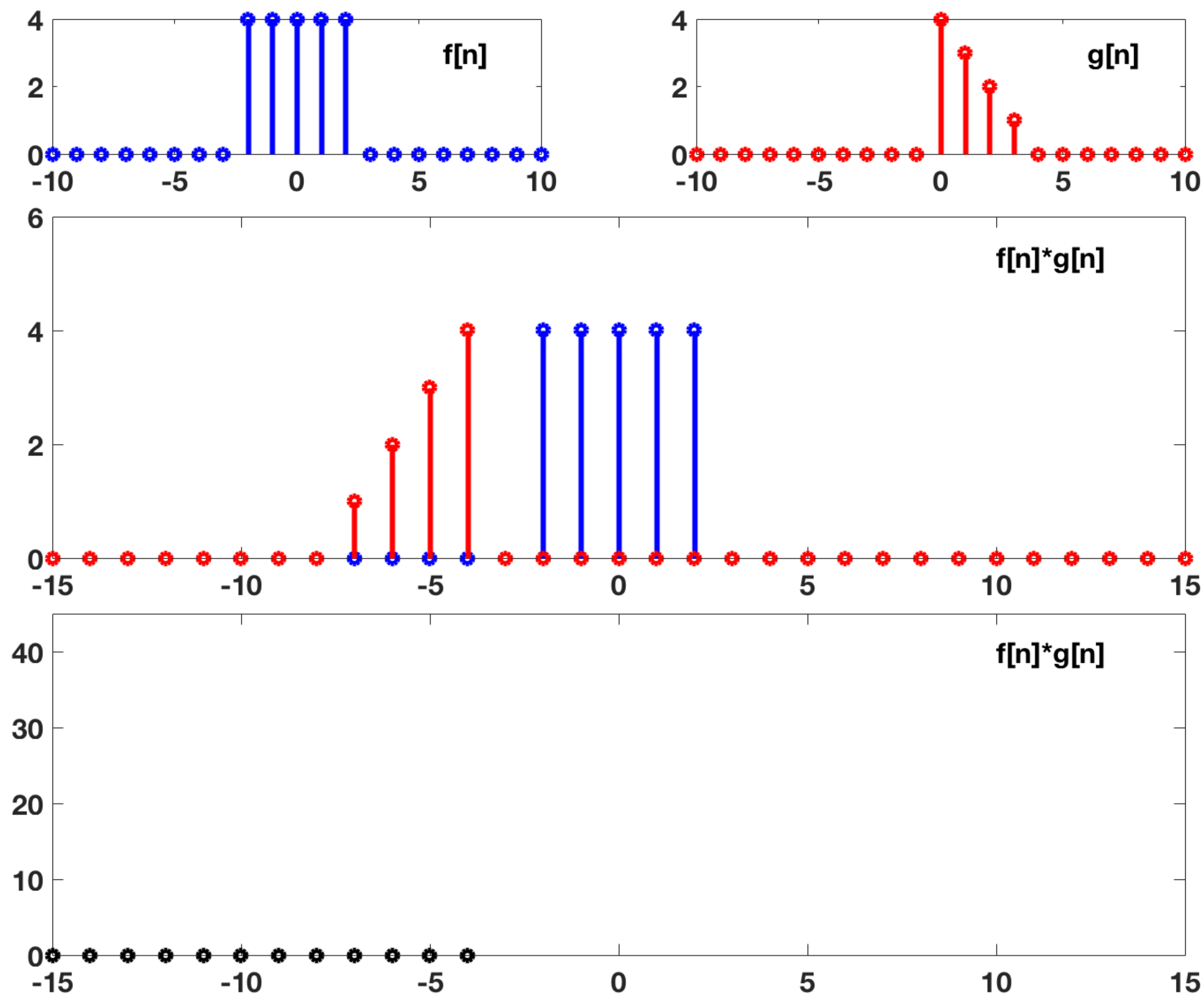

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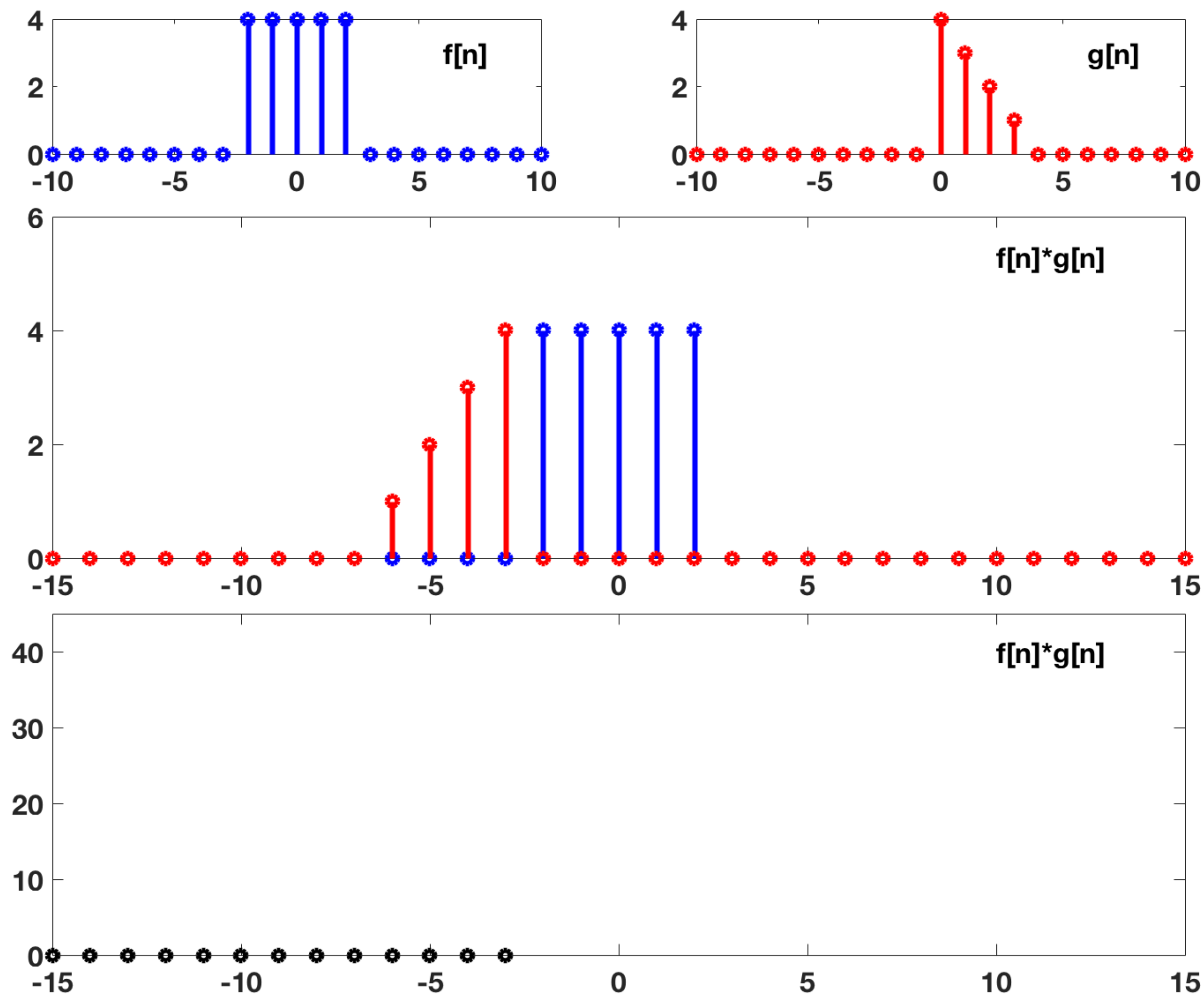

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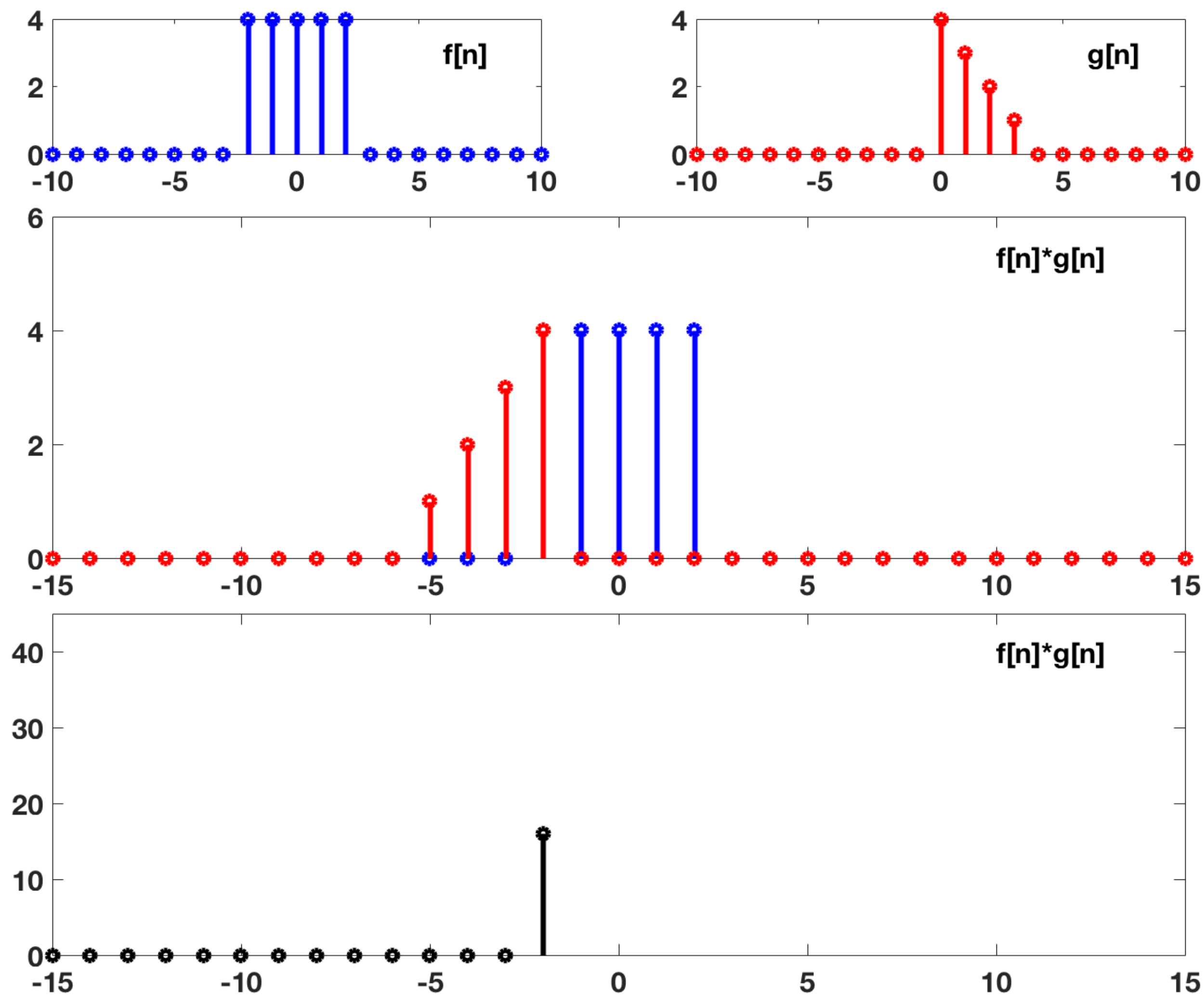

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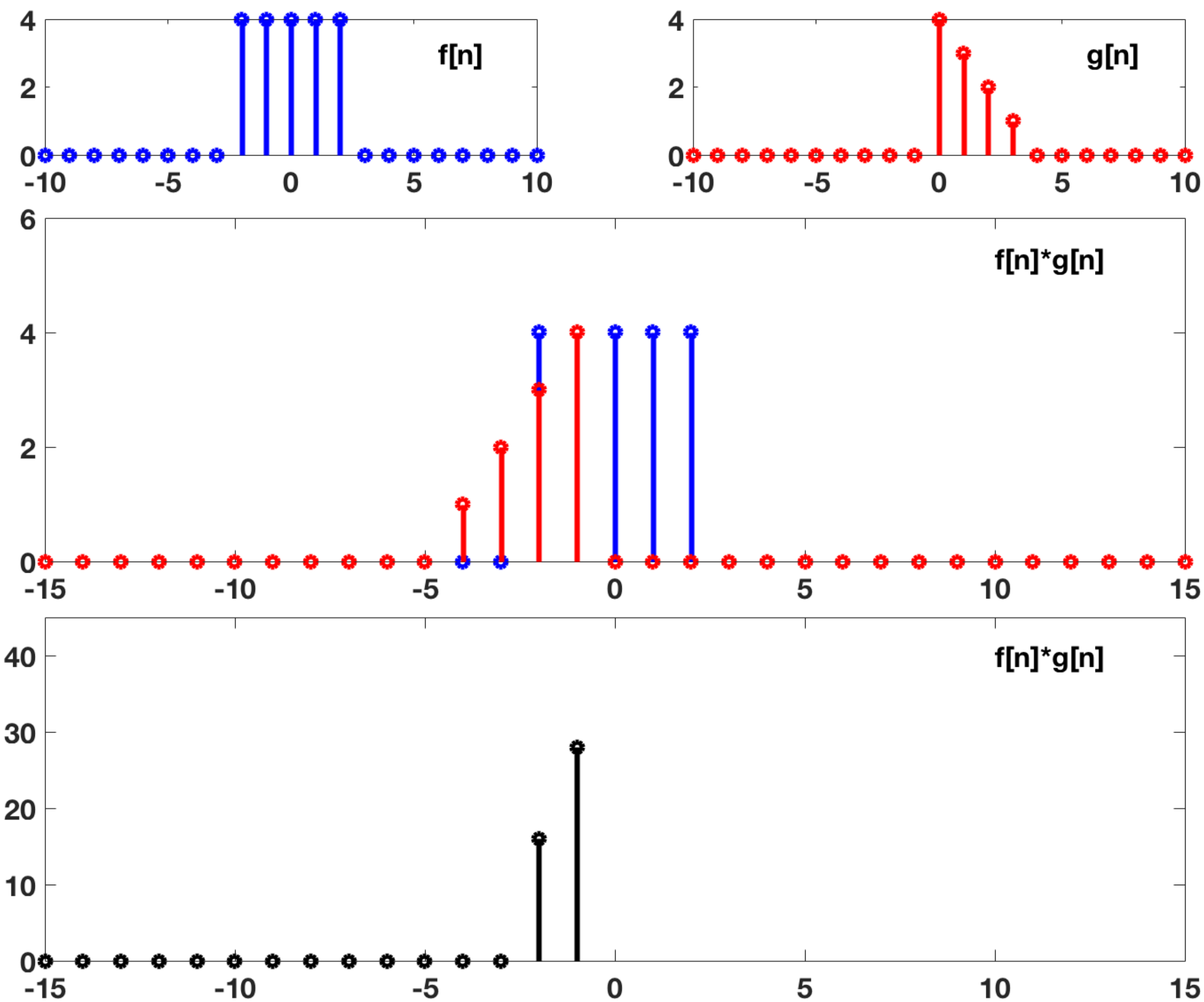

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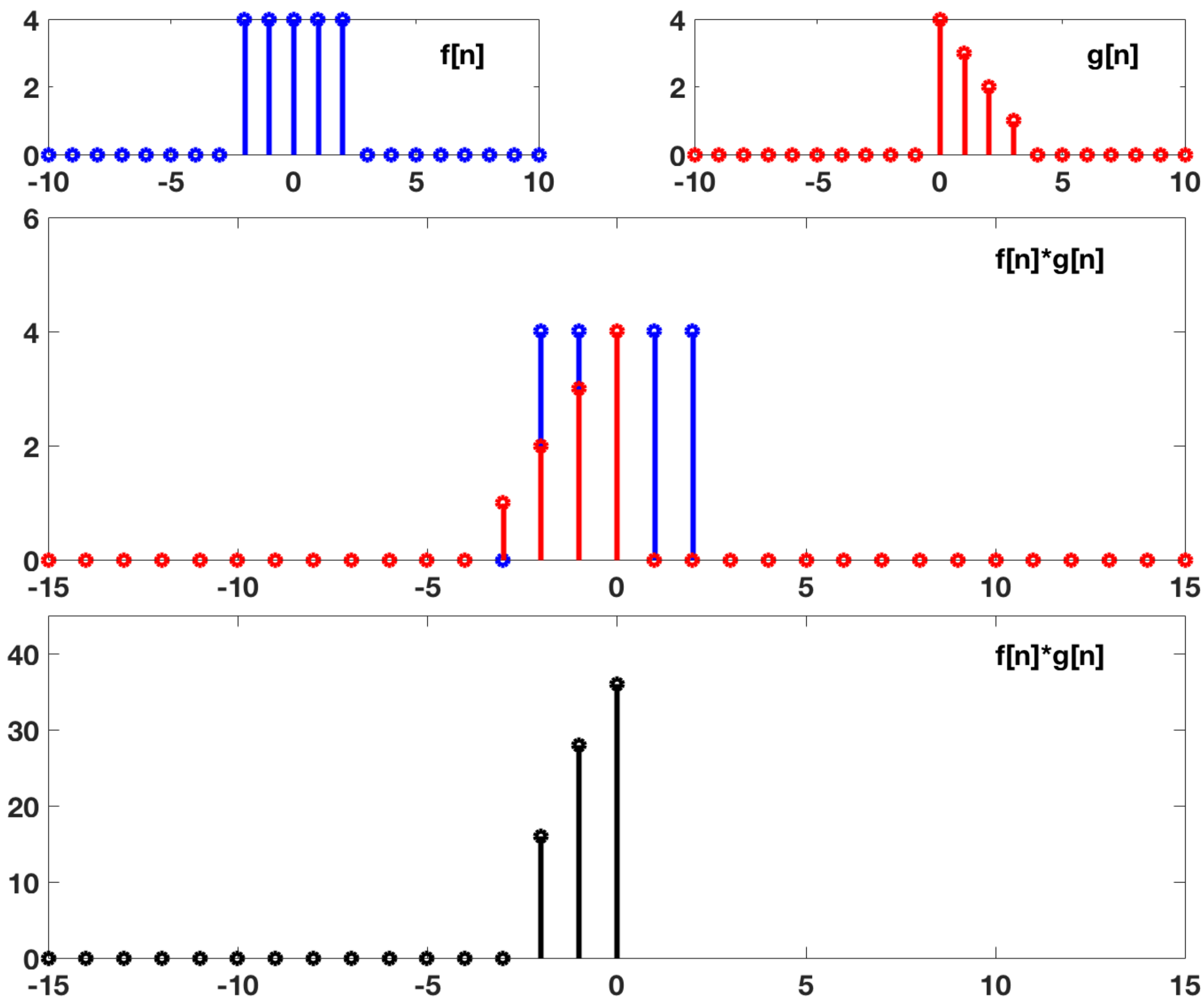

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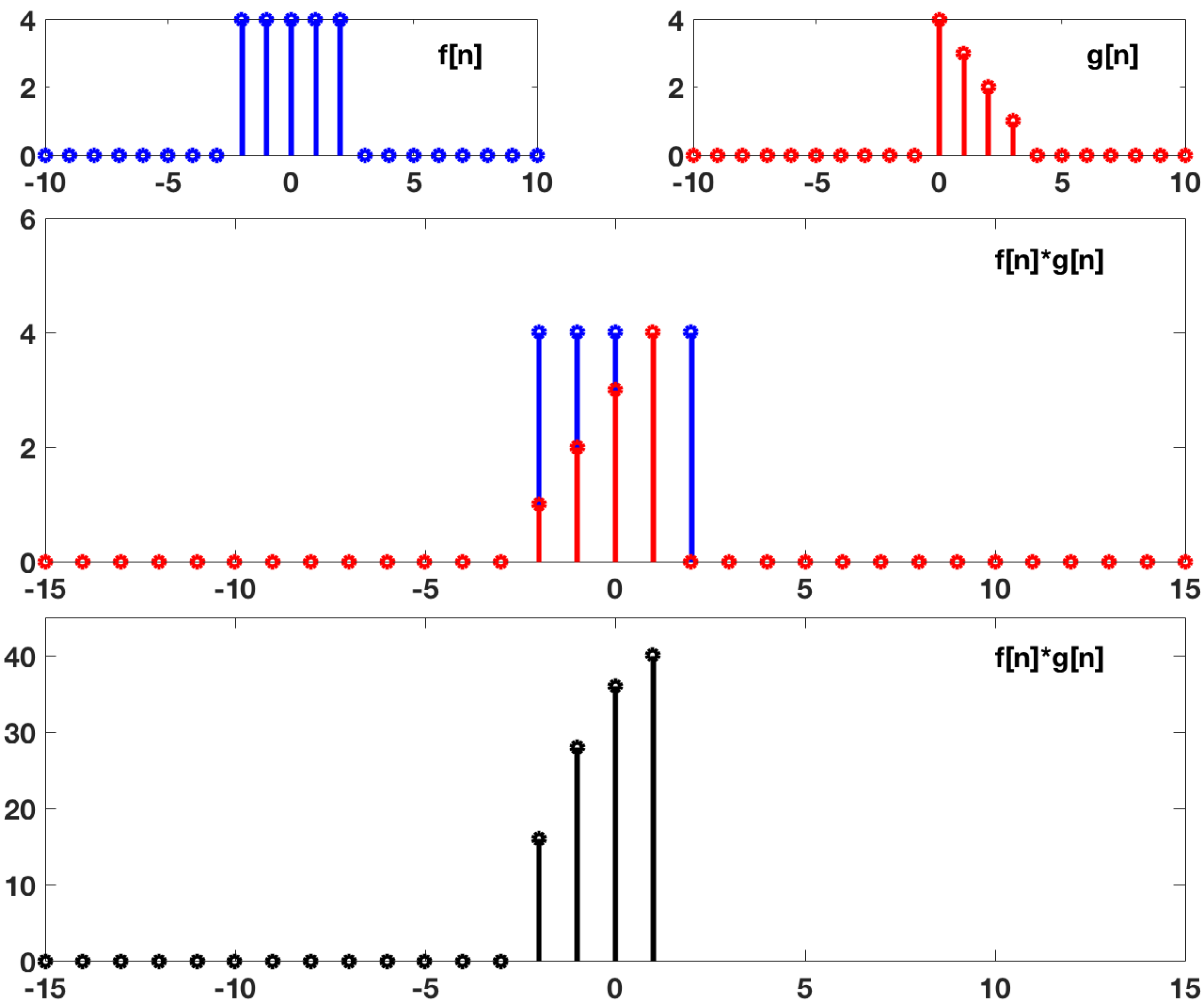

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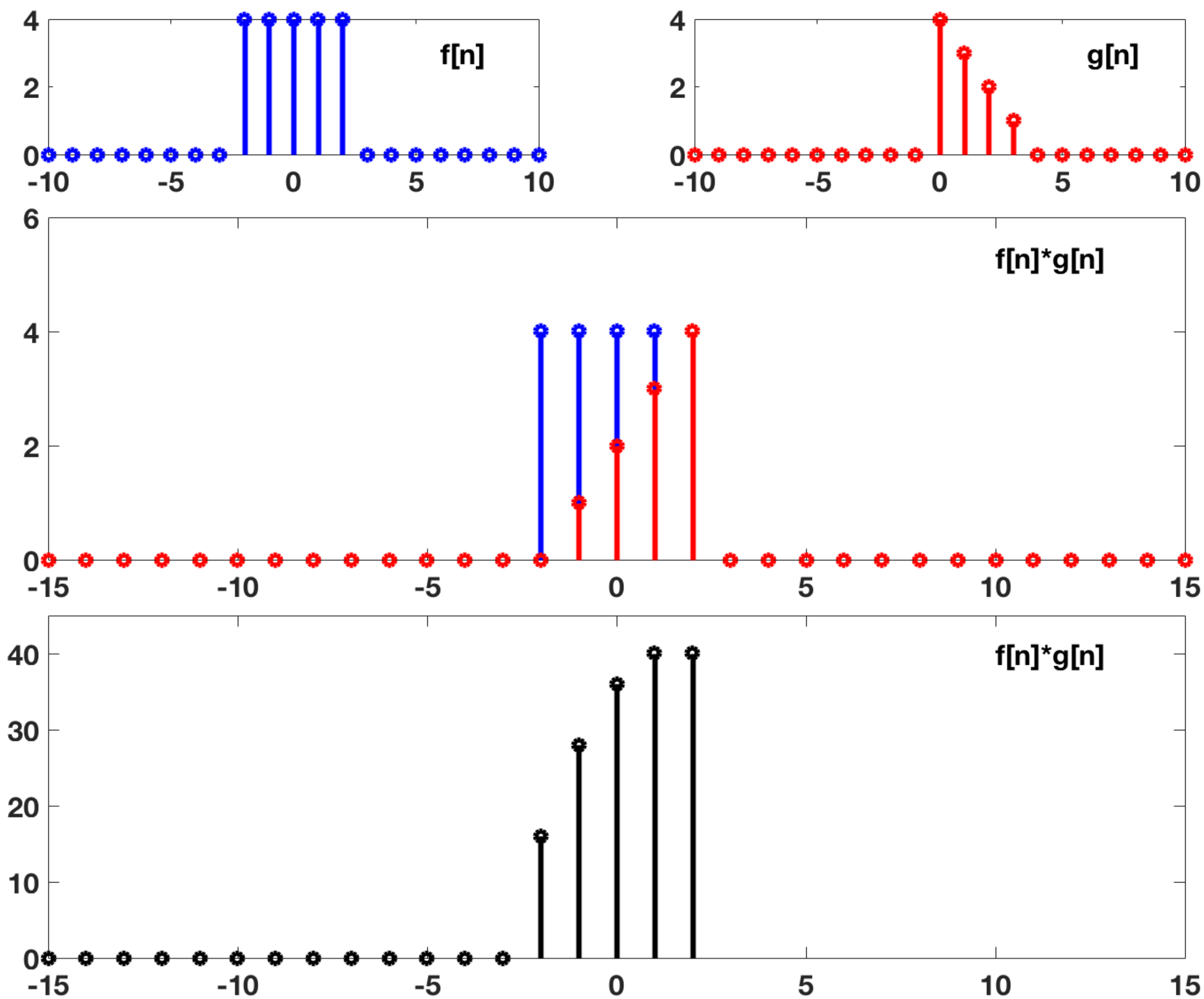

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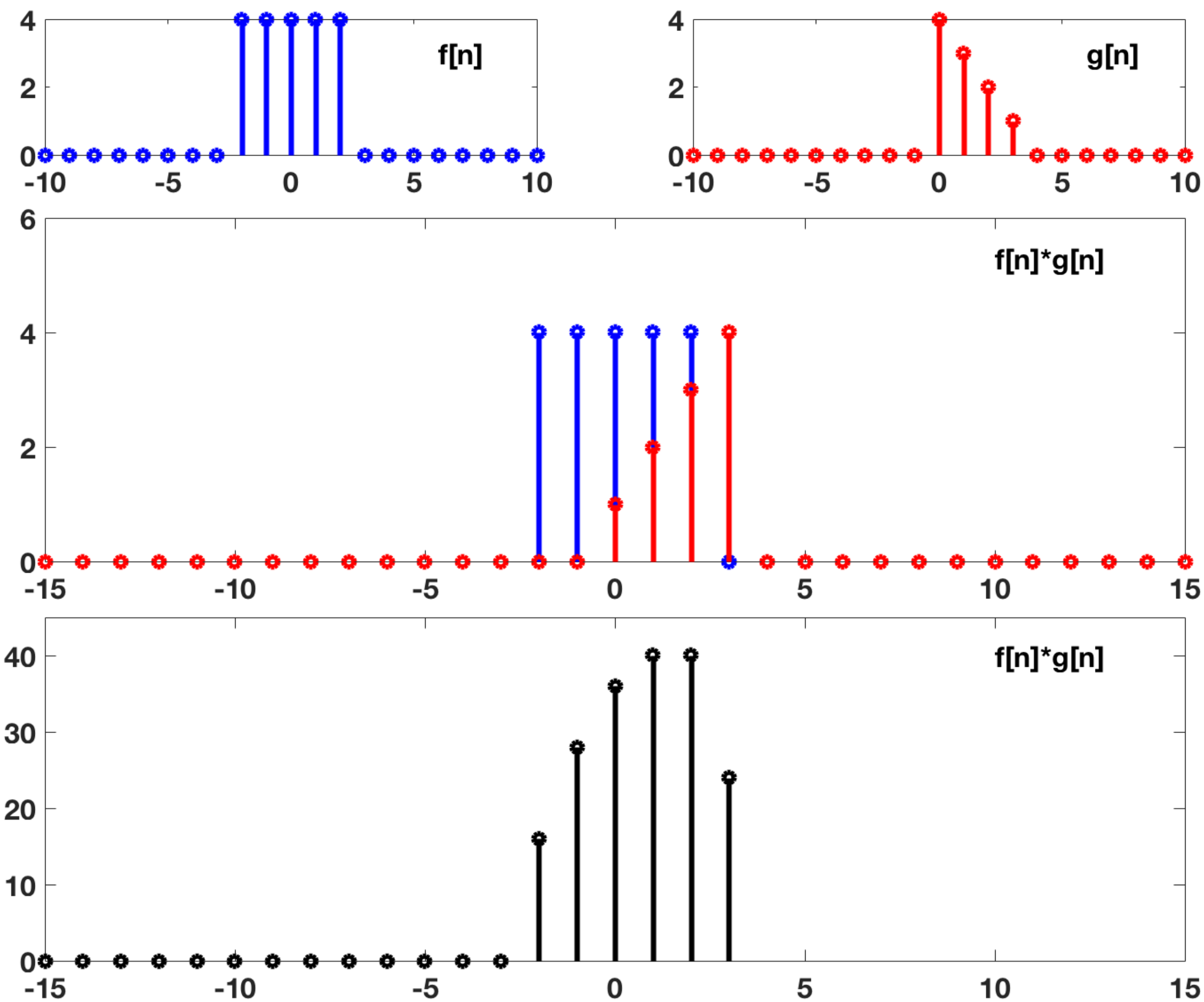

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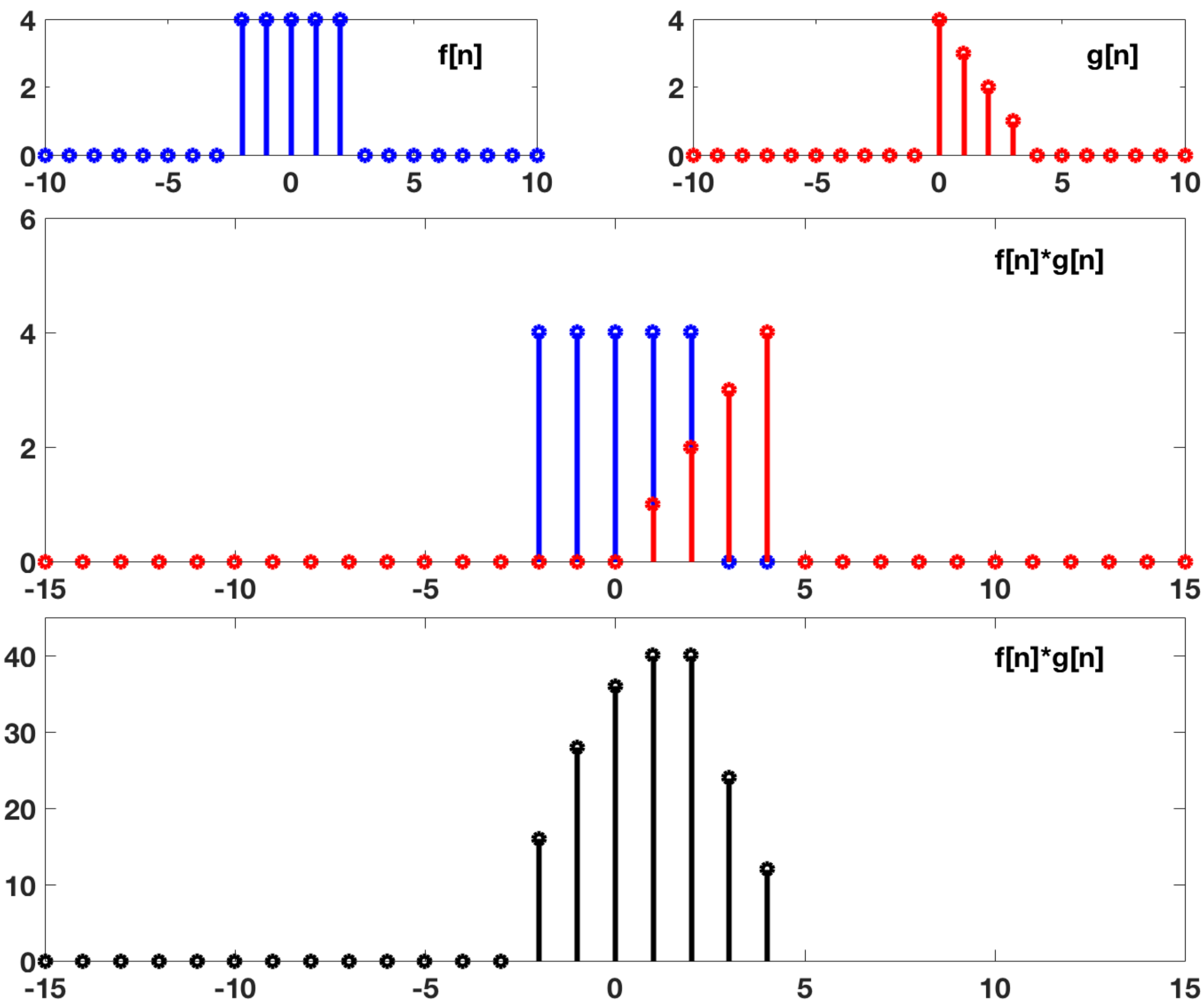

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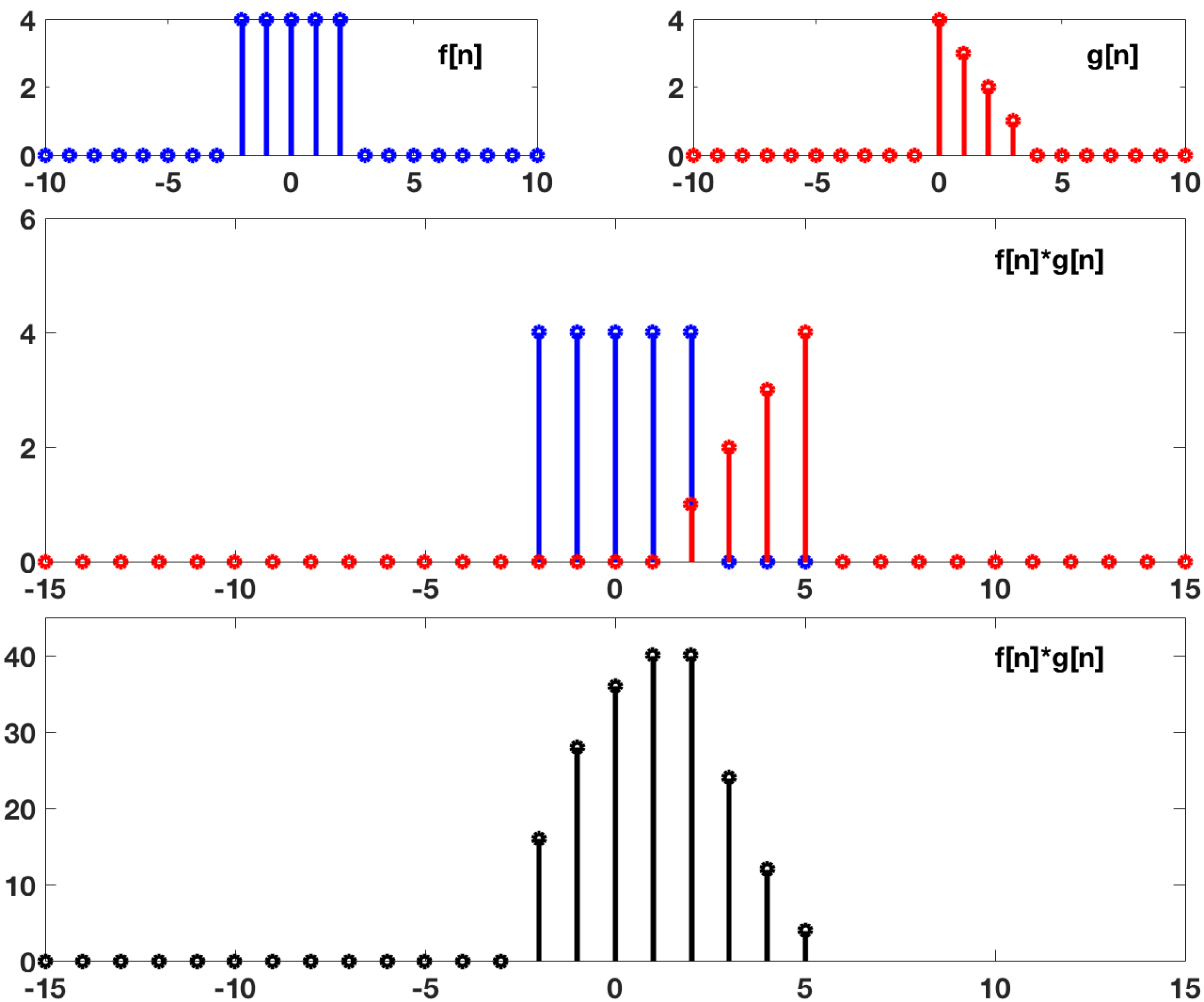

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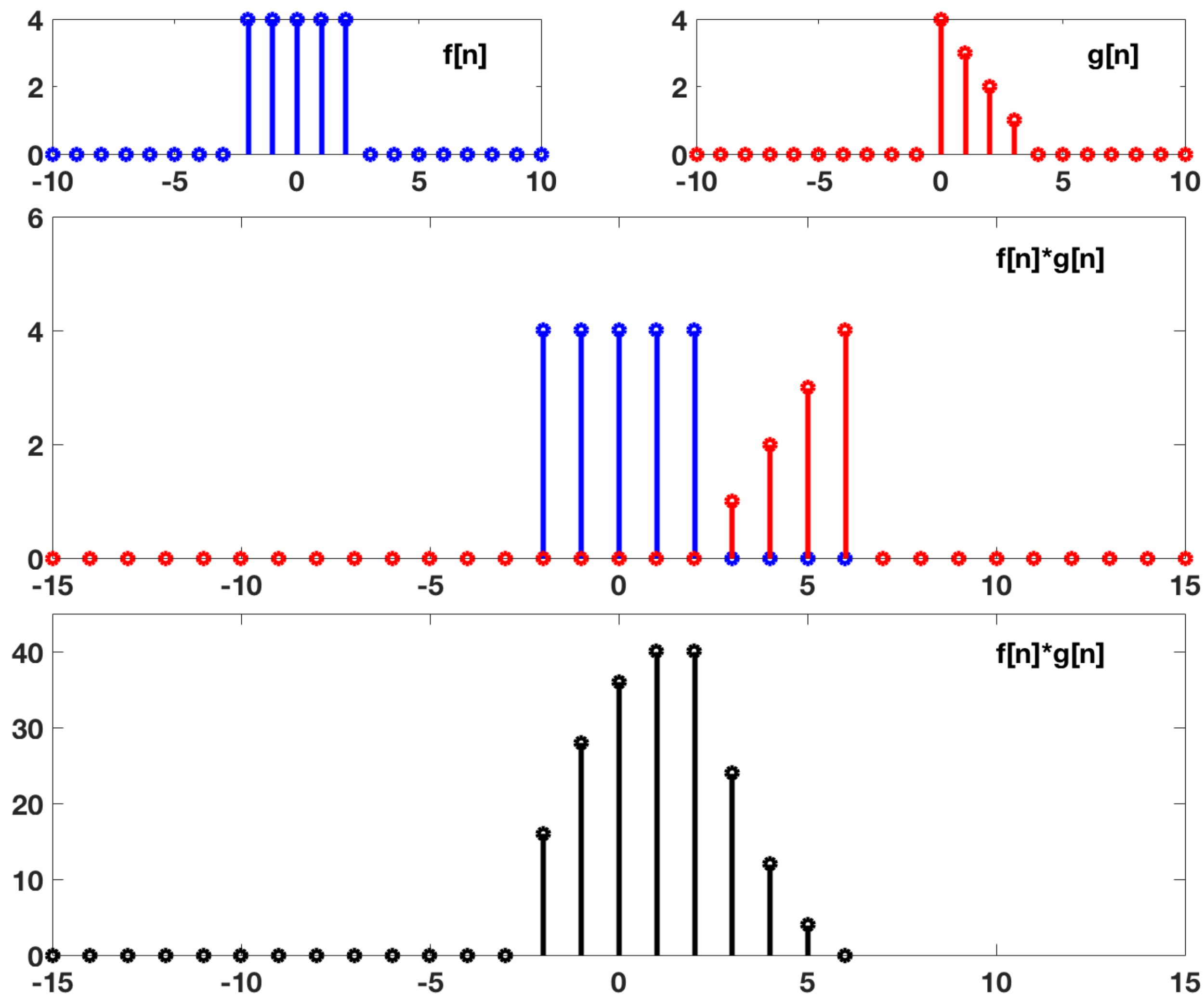

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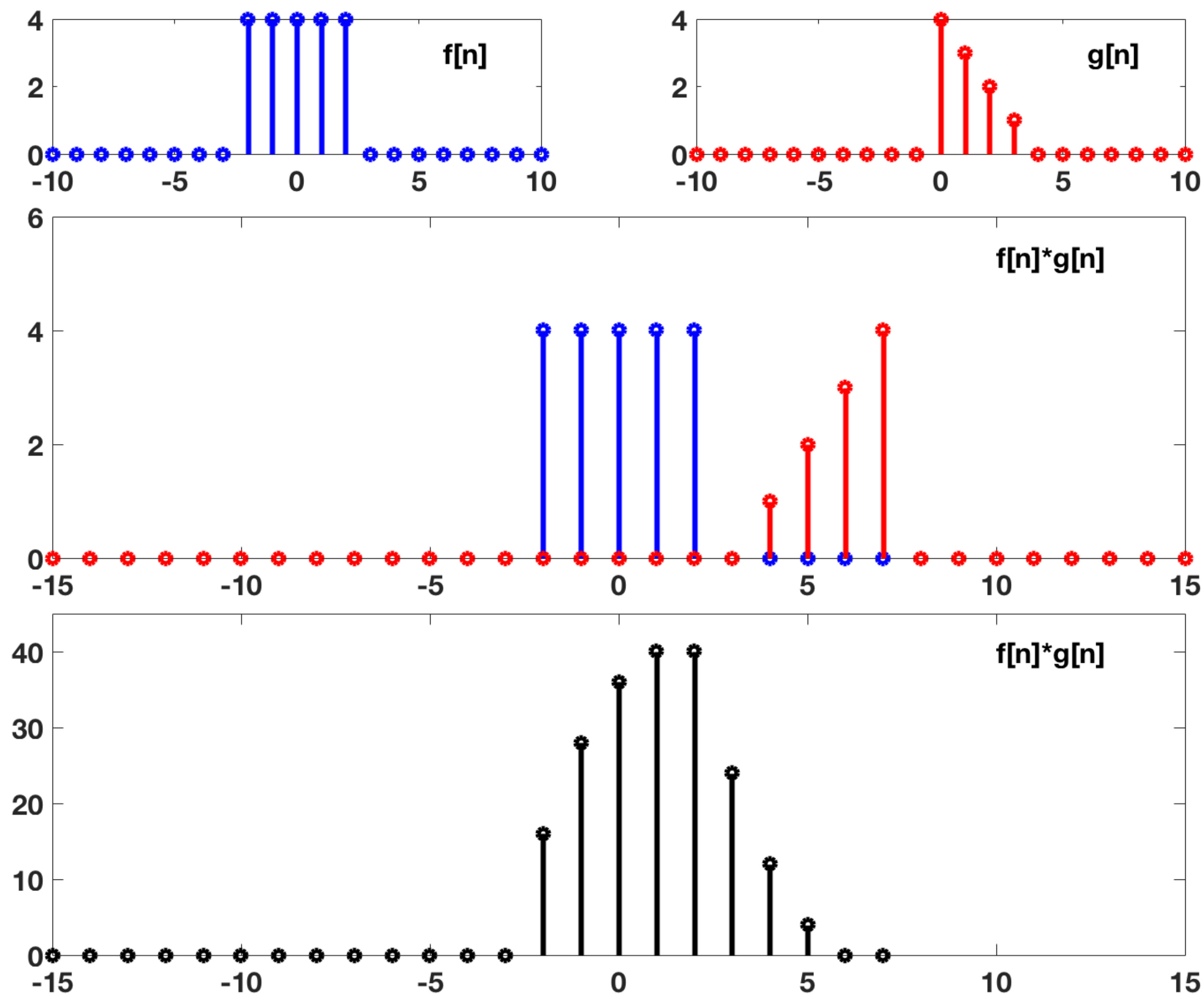

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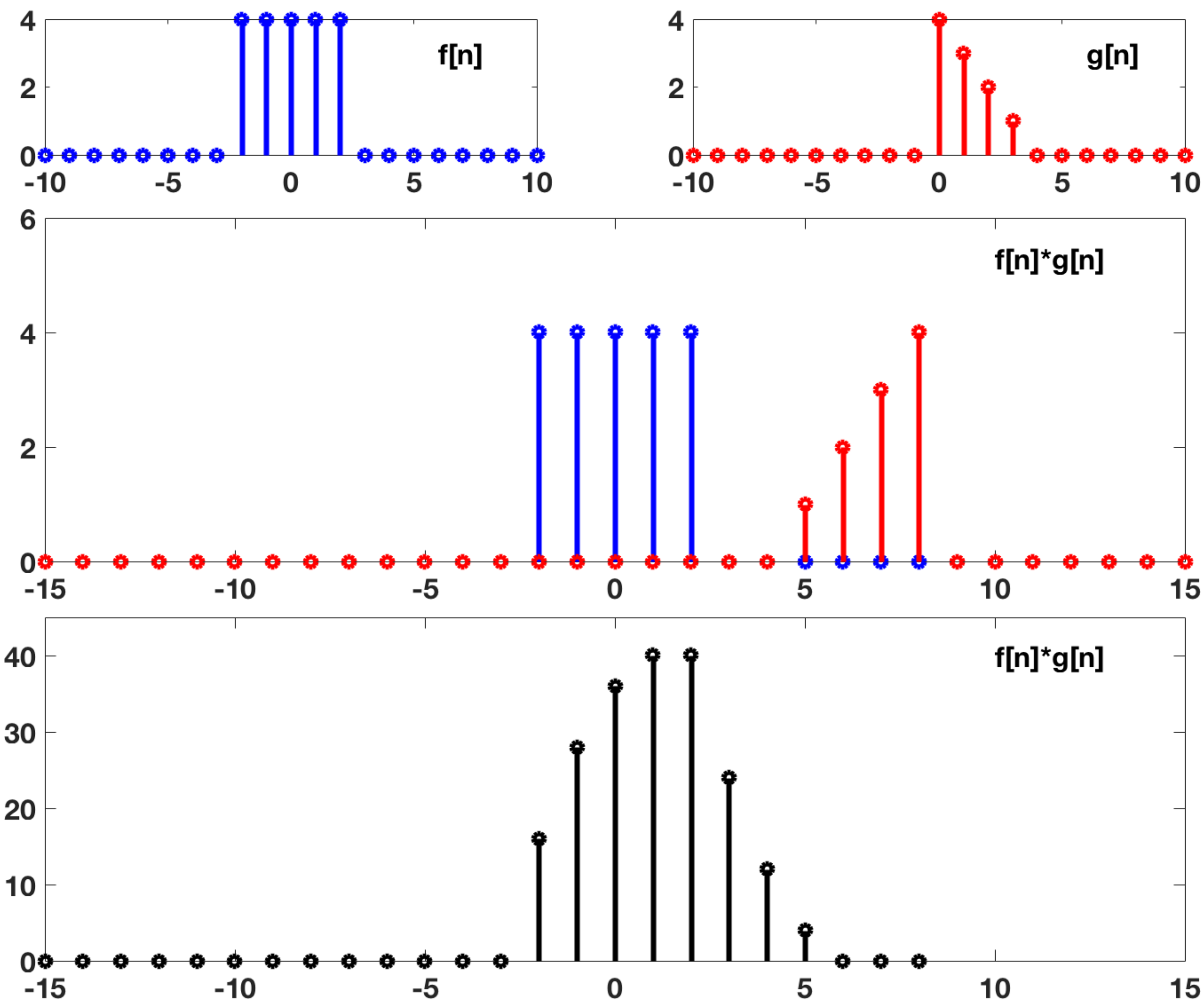

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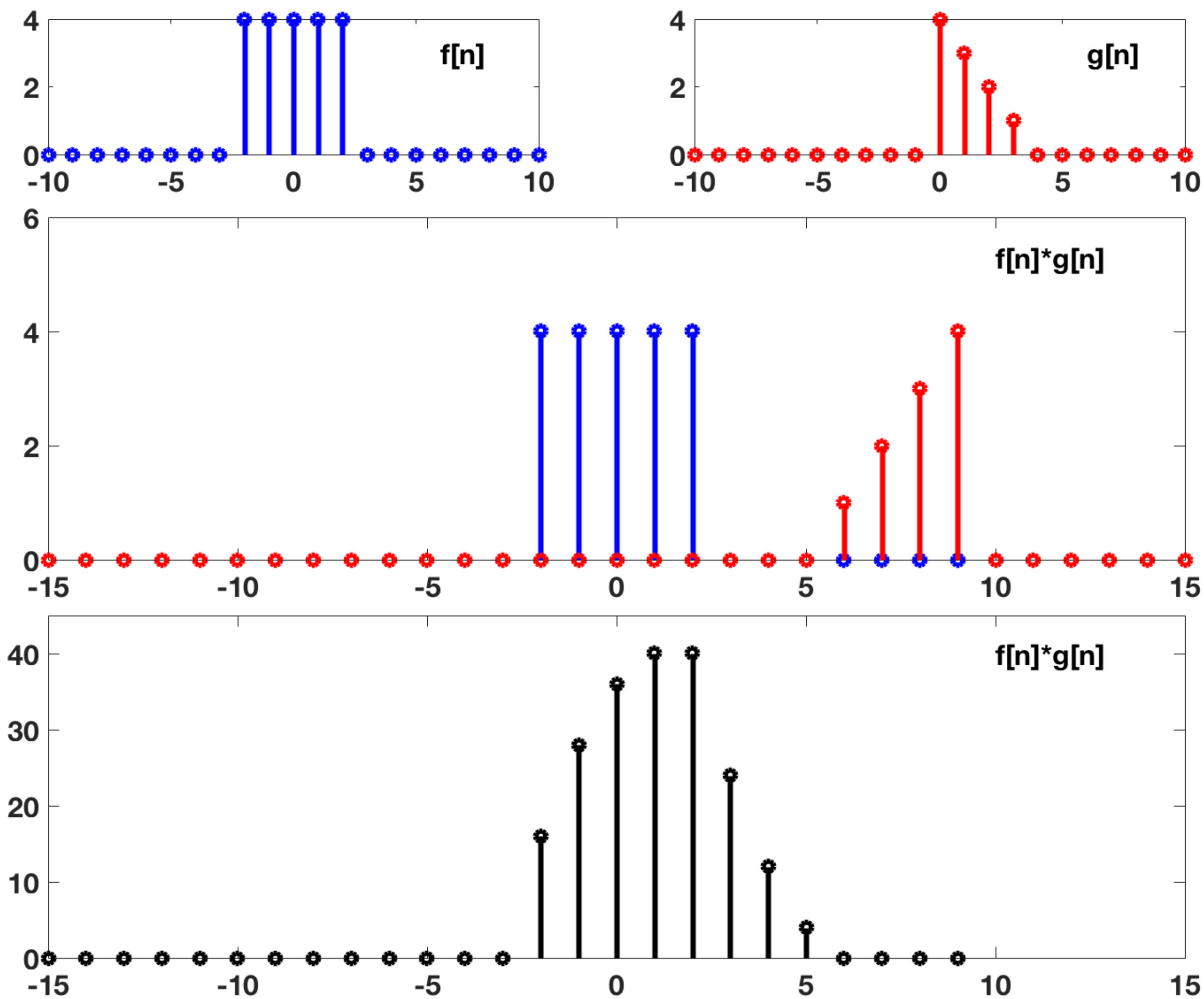

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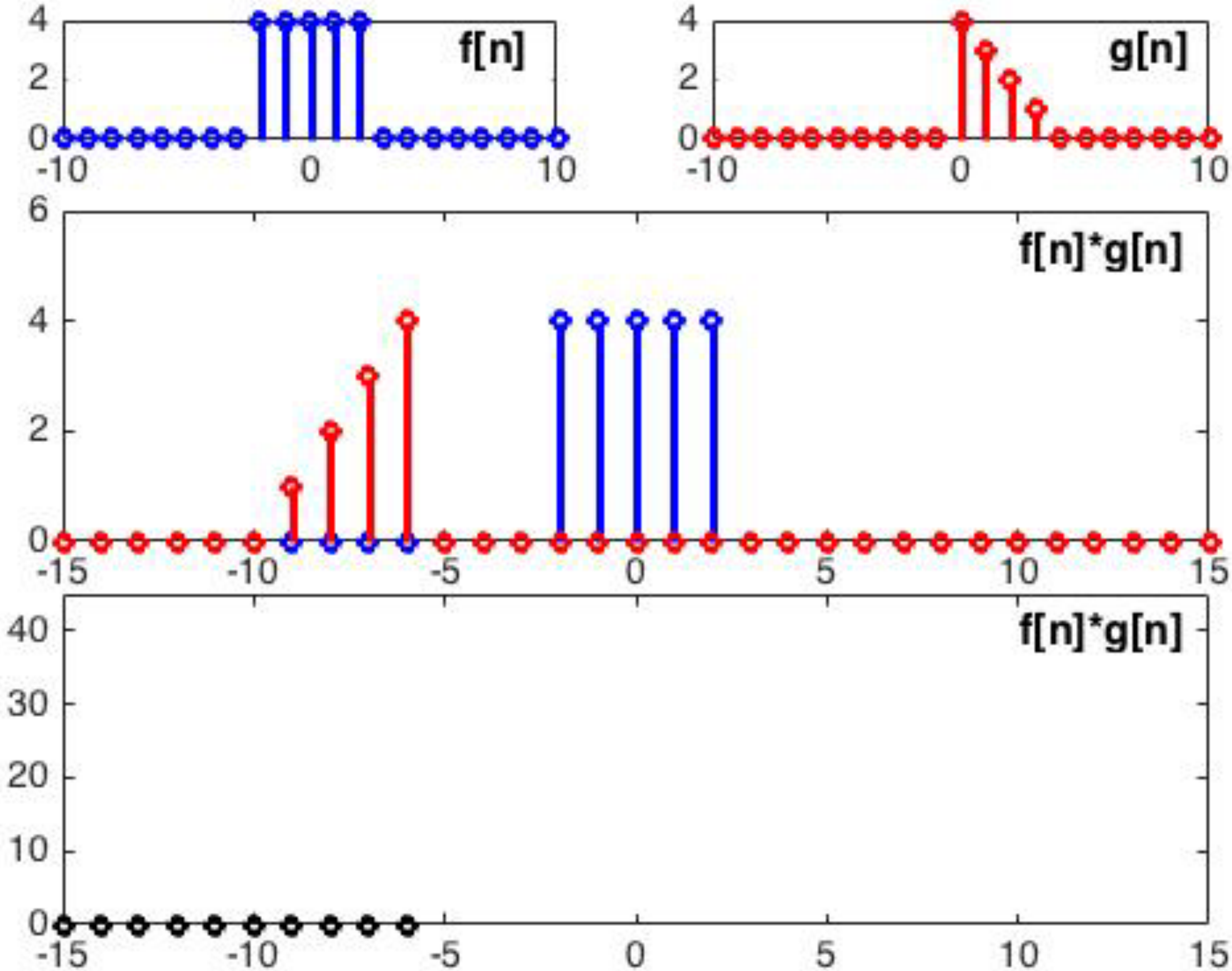

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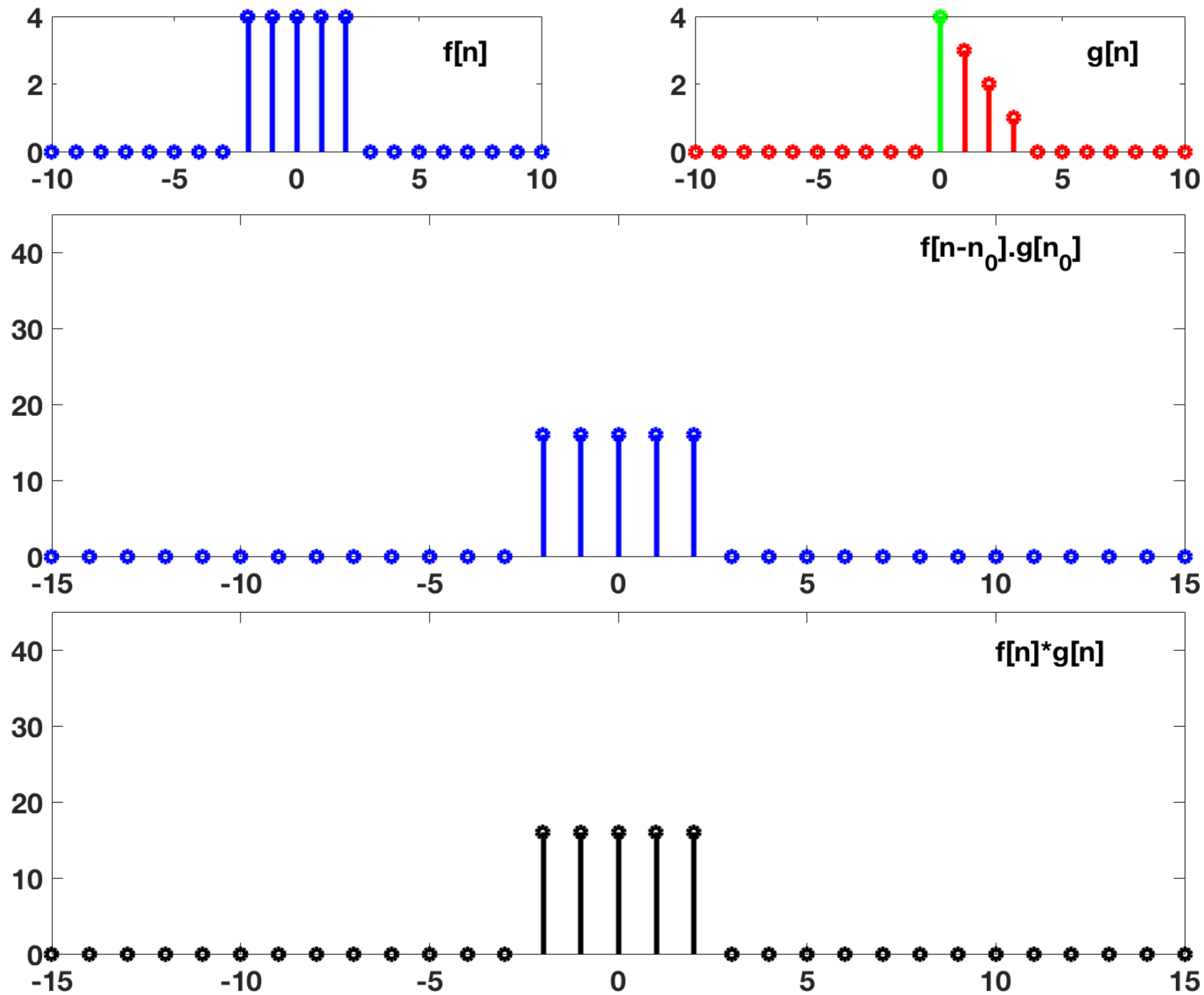

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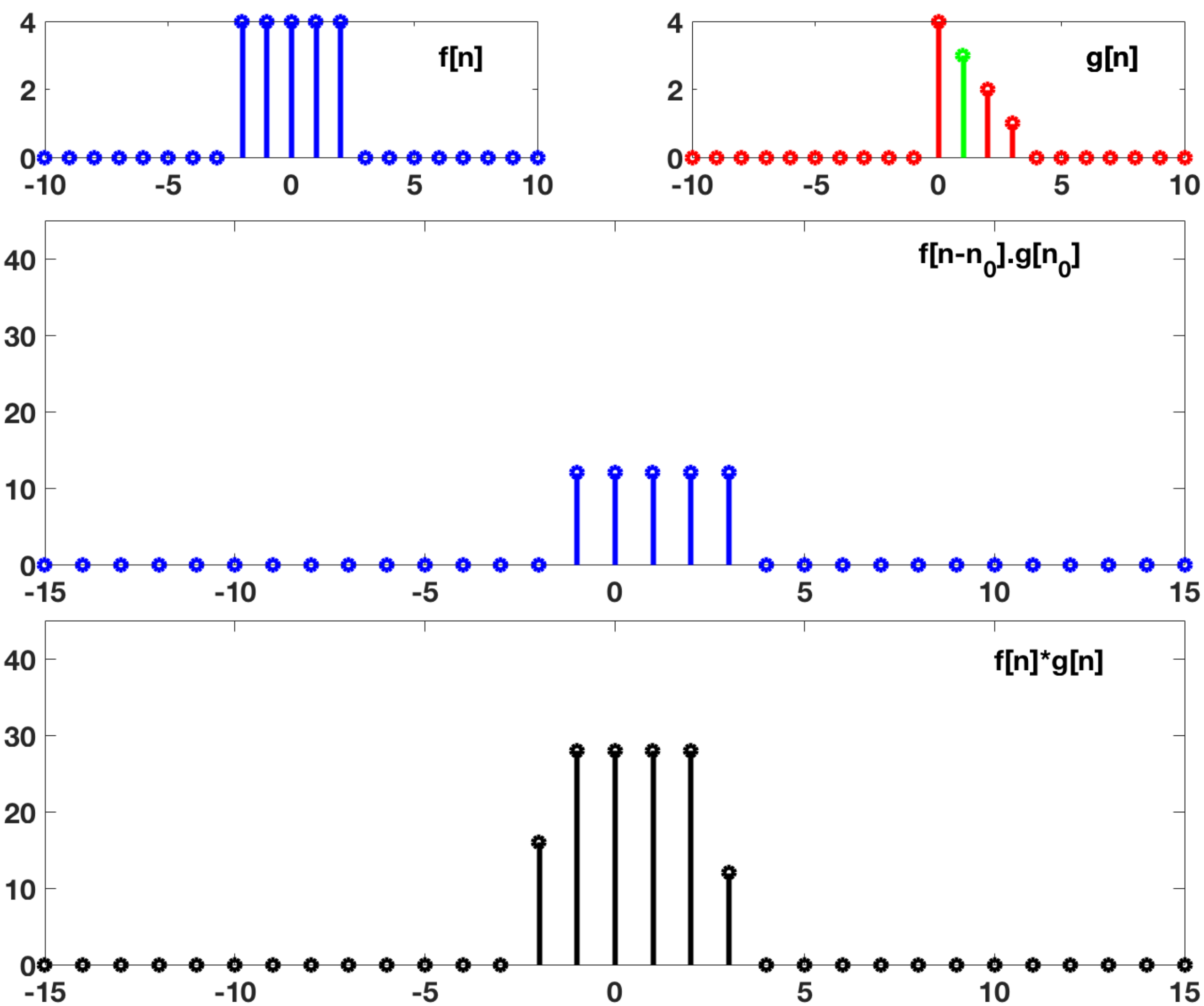

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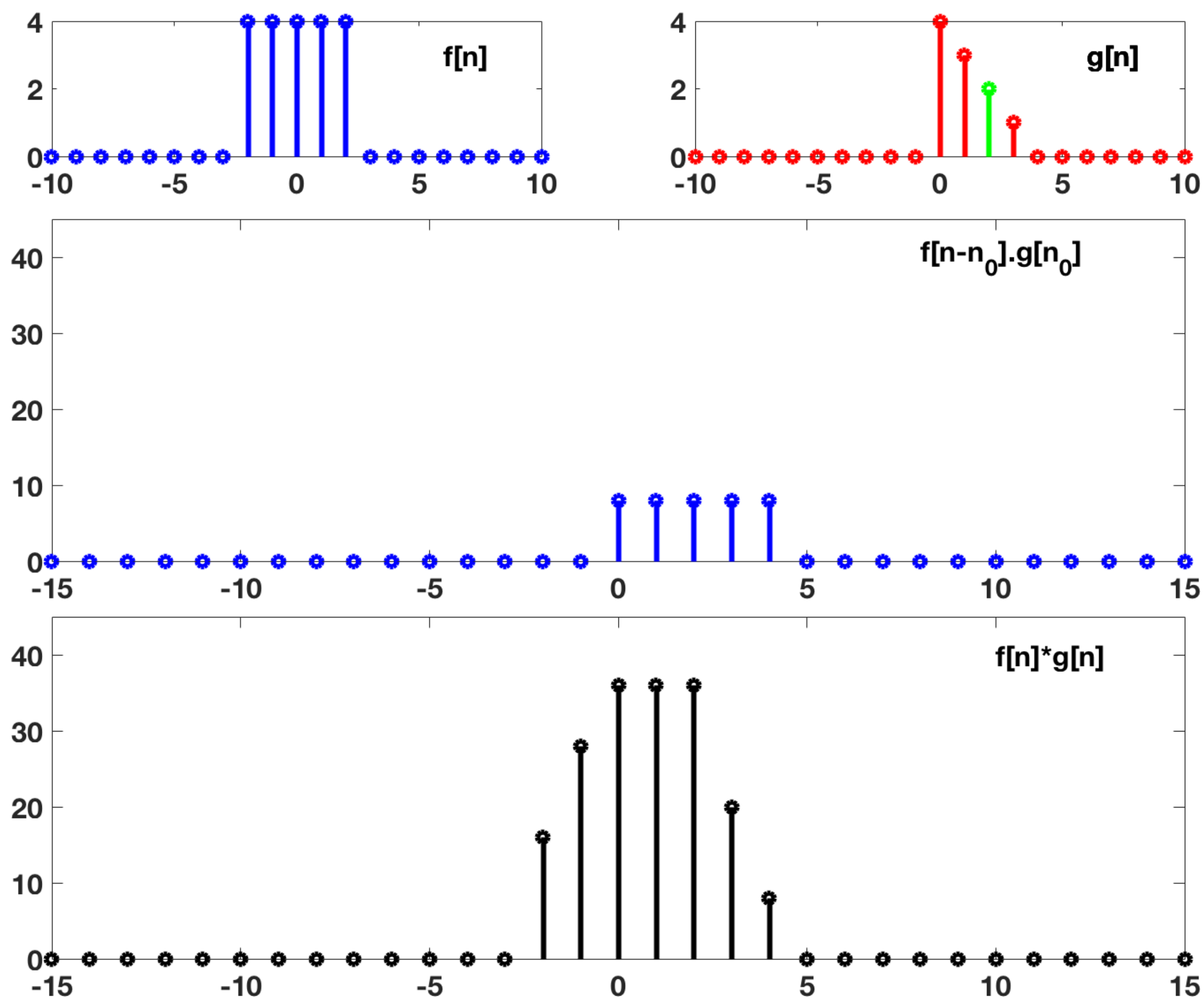

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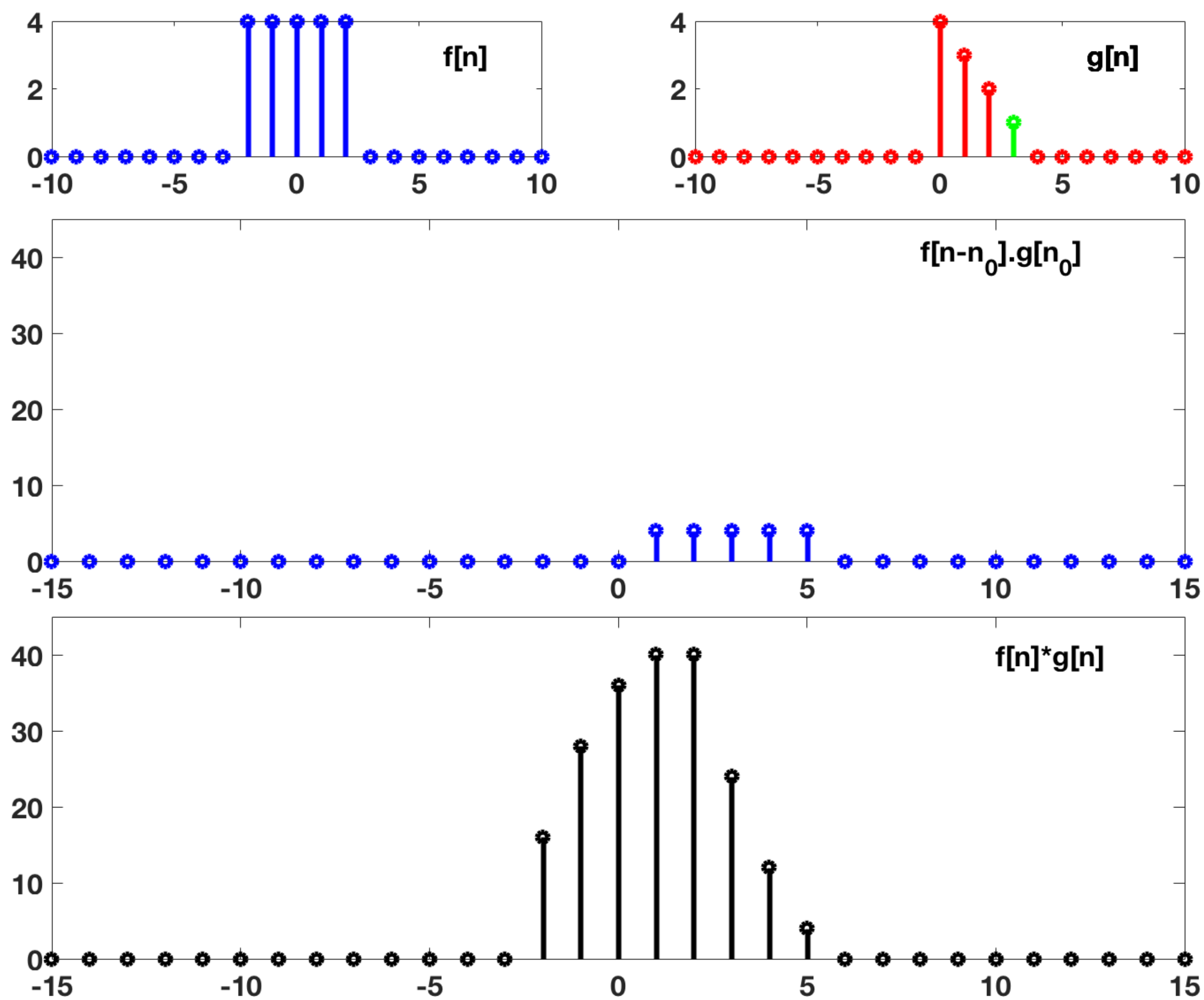

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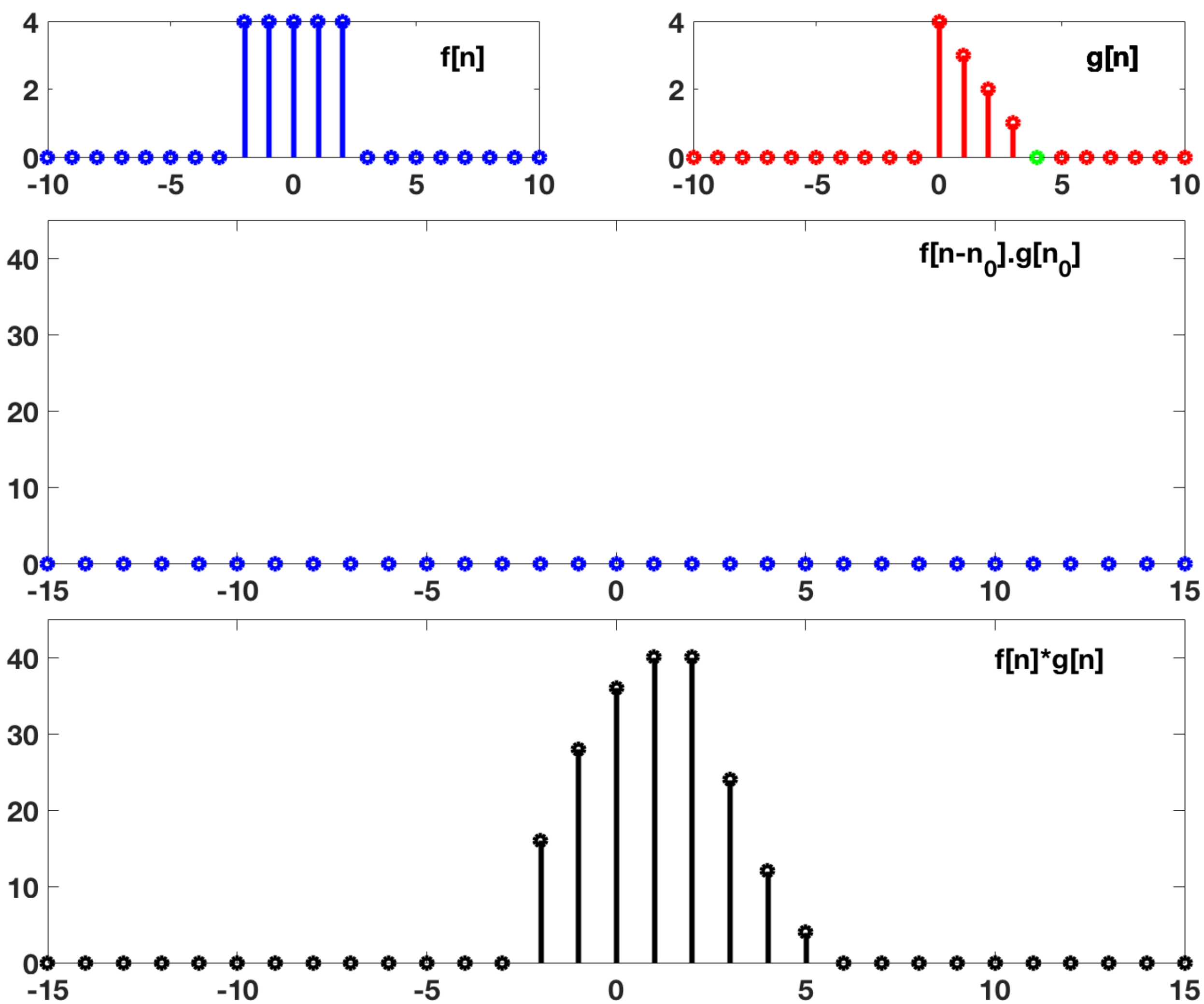

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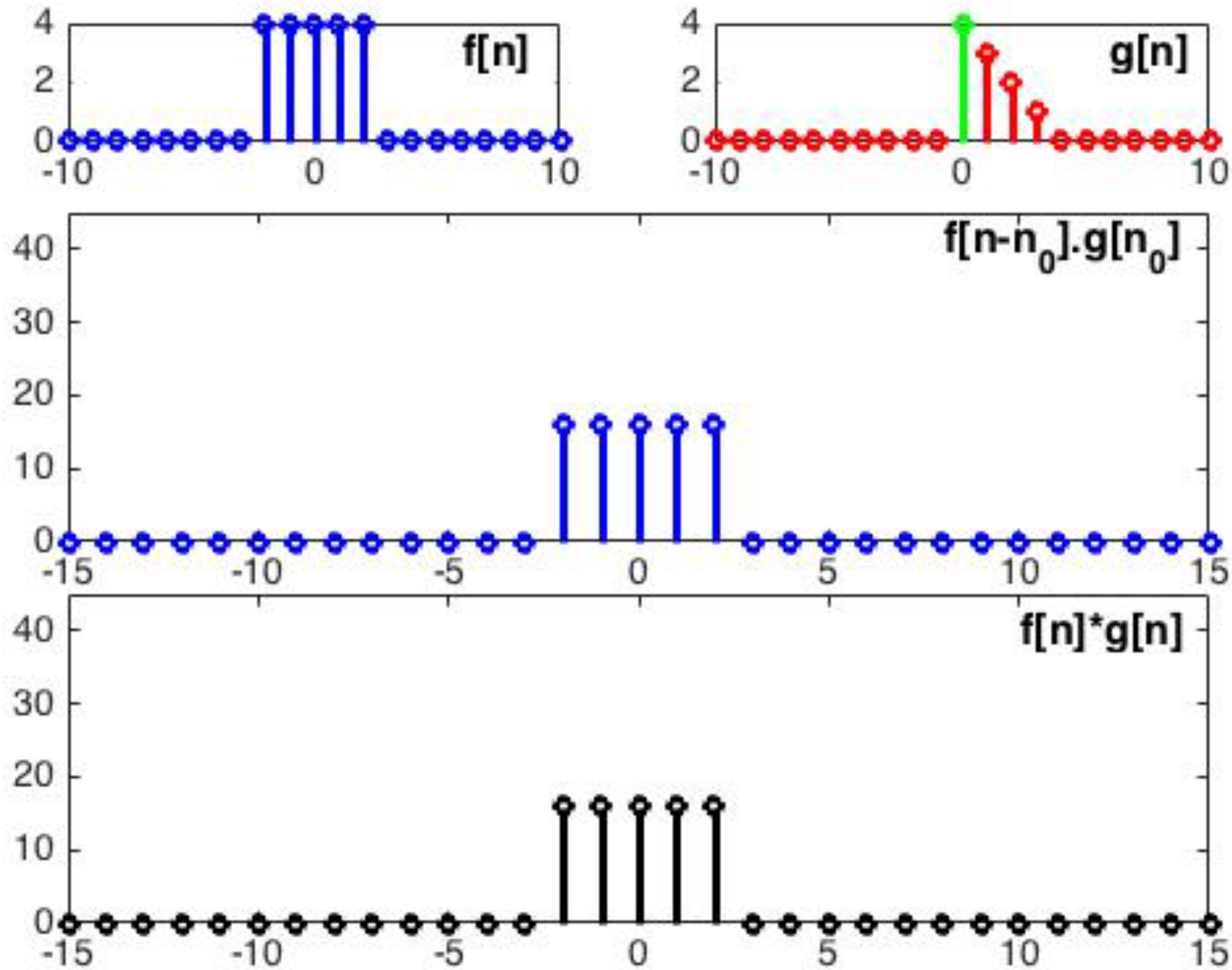

Cross-correlation

- **Cross-correlation** is a measure of similarity of two functions at time-lag ***t*** applied to one of them. It is a LOT like convolution…

Means “complex conjugate of h

$$(h \;\text{}\; \mathrm{x})(t) \equiv \int_{-\infty}^{\infty} h^{*}(\tau)\,x(t+\tau)\,d\tau$$

Cross-correlation operator
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VERY Similar

- Convolution

$$(h * x)(t) \equiv \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

- Cross-correlation

$$(h \star \mathrm{x})(t) \equiv \int_{-\infty}^{\infty} h^{*}(\tau)\, x(t+\tau)\, d\tau$$

Cross-correlation in Python Code

We can easily implement cross correlation with convolution as follows:

```
def crosscorrelation(A,B):
     return convolution(np.conj(A),B[::-1])
```

Better yet, use the built in Python functions…

```
np.convolve(A,B,"full")   # for convolution
np.correlate(A,B,"full")  # for cross correlation
```

Auto-correlation

- **Auto-correlation** is a measure of similarity of a function to itself at time-lag ***t***. It is a special case of cross-correlation (cross-correlation of a function with itself).

Means “complex conjugate of f”

$$(x \star \mathrm{x})(t) \equiv \int_{-\infty}^{\infty} x^{*}(\tau)\, x(t+\tau)\, d\tau$$

Cross correlation
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Relating them all

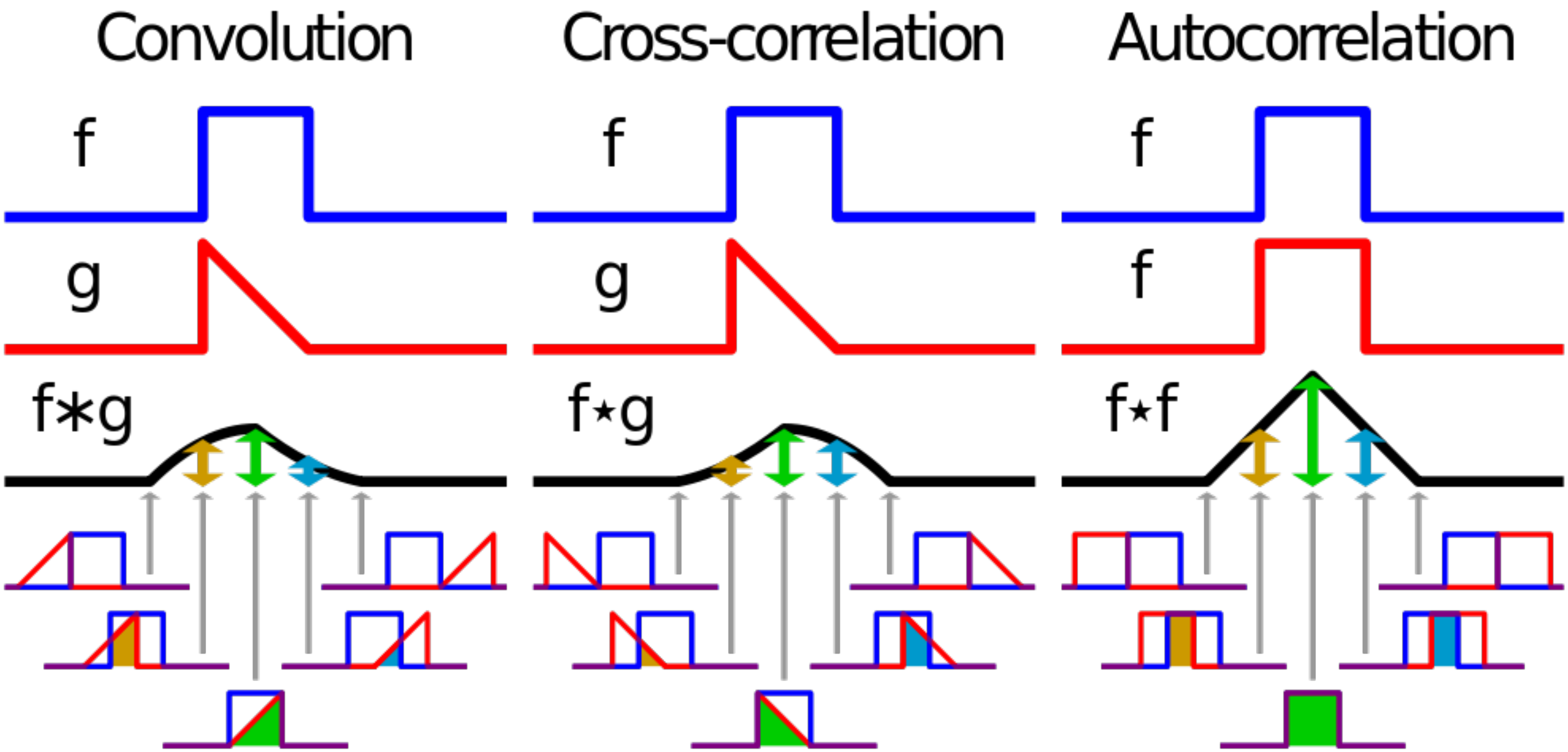

Image from http://commons.wikimedia.org/wiki/File:Comparison_convolution_correlation.svg

Convolution and Fourier transform

- An important property of the Fourier transform: converts convolution in the time domain into multiplication in the frequency domain.

Convolution… $$y(t) = h(t) * x(t)$$

In the time domain: $$y(t) = \int h(\tau)x(t-\tau)\,d\tau$$

In frequency domain: $$Y(\omega) = H(\omega)X(\omega)$$