ECE 102 Homework 2

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Problem 1

(a)

$$\begin{aligned}
h(t) &= \int_{-\infty}^{t-1} e^t \cos(2\tau + 2 - 2t)\delta(\tau)e^{2-\tau} d\tau \\
&= \int_{-\infty}^{t-1} e^t \cos(2 - 2t)\delta(\tau)e^{2} d\tau \\
&= e^{t+2} \cos(2 - 2t)u(t-1)
\end{aligned}$$

Since $h(t, \tau) = h(t - \tau) = 0$ for $t < \tau$, this system is causal. Furthermore, since e^t is not bounded (it goes to ∞ as $t \to \infty$), this system is not BIBO.

(b)

$$
\begin{aligned}
h(t) &= e^{-t}\int_{-\infty}^{t} e^{\tau}[\cos(t)\cos(\tau) - \sin(t)\sin(\tau)]\delta(\tau)d\tau \\
&= e^{-t}\int_{-\infty}^{t} e^{\tau}\cos(t+\tau)\delta(\tau)d\tau \\
&= e^{-t}\int_{-\infty}^{t} \cos(t)\delta(\tau)d\tau \\
&= e^{-t}\cos(t)u(t)
\end{aligned}
$$

Since $h(t,\tau) = h(t-\tau) = 0$ for $t < \tau$, this system is causal. To find if the system is BIBO, let us input a bounded signal $|x(t)| < M < +\infty$ and determine if the output is bounded as well

$$
|y(t)| = \left| e^{-t}\int_{-\infty}^{t} e^{\tau}[\cos(t)\cos(\tau) - \sin(t)\sin(\tau)]x(\tau)d\tau \right|
$$

Since e^t and e^-t are positive for any t we get

$$
\begin{aligned}
|y(t)| &= e^{-t}\int_{-\infty}^{t} e^{\tau}|\cos(t+\tau)x(\tau)|d\tau \\
&= e^{-t}\int_{-\infty}^{t} e^{\tau}|\cos(t+\tau)||x(\tau)|d\tau \\
&\leq e^{-t}\int_{-\infty}^{t} e^{\tau}|\cos(t+\tau)|M d\tau \\
&\leq Me^{-t}\int_{-\infty}^{t} e^{\tau}d\tau \\
&= Me^{-t}e^{t} \\
&= M < +\infty
\end{aligned}
$$

Thus this system is BIBO.

(c)

$$
\begin{aligned}
h(t) &= \int_{-\infty}^{t-1} e^{-(t-\tau)}\delta(\tau-2)d\tau \\
&= \int_{-\infty}^{t-1} e^{-(t-2)}\delta(\tau-2)d\tau \\
&= e^{2-t}u(t-3)
\end{aligned}
$$

Since $h(t,\tau) = h(t-\tau) = 0$ for $t < \tau$, this system is causal.