ECE 102 Homework 2

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Problem 1

(a)

After Laplace transform we have that

$$3s^2 Y(S) + 19sY(s) + 20Y(S) = 2sX(s) - X(s)$$

$$H(s) = \boxed{\frac{2s - 1}{3s^2 + 19s + 20}}$$

(b)

$$X(s) = \frac{2}{2s + 1} e^{-3s}$$

$$Y(s) = H(s)X(s)$$

$$= e^{-3s} \frac{(2s-1)}{(3s^2 + 19s + 20)(s + \frac{1}{2})}$$

$$= e^{-3s} \frac{(2s-1)}{(3s+4)(s+5)(s+\frac{1}{2})}$$

$$= \frac{e^{-3s}}{3} \frac{(2s-1)}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})}$$

$$= \frac{e^{-3s}}{3} \frac{(2s+1)-2}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})}$$

$$= \frac{e^{-3s}}{3} \left(2 \left(\frac{1}{(s+\frac{4}{3})(s+5)} \right) - \frac{2}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})} \right)$$

We can rewrite

$$\frac{1}{(s+\frac{4}{3})(s+5)} = \frac{3}{11} \left(\frac{1}{s+\frac{4}{3}} - \frac{1}{s+5} \right)$$

and

$$\frac{1}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})} = -\frac{18}{55} \frac{1}{s+\frac{4}{3}} + \frac{2}{33} \frac{1}{s+5} + \frac{4}{15} \frac{1}{s+\frac{1}{2}}$$

Thus we have that

$$\boxed{y(t) = \frac{2}{3} \left(\frac{-3}{55} e^{-\frac{4}{3}(t-3)} + \frac{-11}{33} e^{-5(t-3)} - \frac{4}{15} e^{-\frac{1}{2}(t-3)} \right) u(t-3)}$$
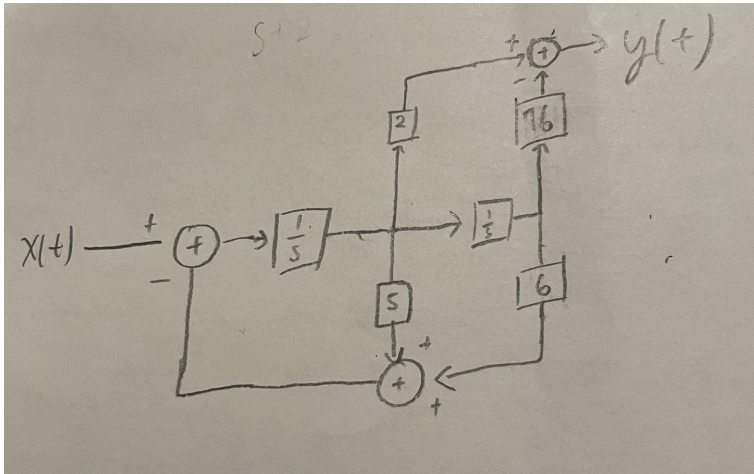
Problem 2

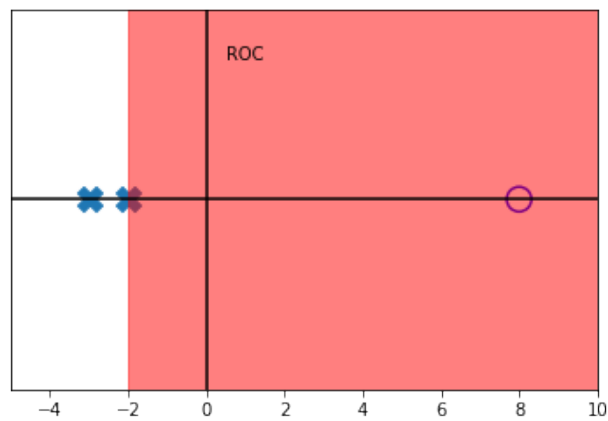## (a)

The system response function is

$$h(s) = \frac{2s^2 - 14s - 16}{s^2 + 6s^2 + 11s + 6}$$

$$h(s) = \frac{2}{s+2}\frac{1}{s+3}(s-8)$$

Thus the system looks like

(b)

This converges since the roc includes $s = 0$

(c)

$$X(s) = \frac{1}{s}\left(e^{2s} - e^{-2s}\right)$$

$$
\begin{aligned}
Y(s) &= X(s)H(s) \\
&= \left(e^{2s} - e^{-2s}\right)\frac{2(s-8)}{s(s+2)(s+3)} \\
&= -2\left(e^{2s} - e^{-2s}\right)\left(\frac{1}{(s+2)(s+3)} - 8\frac{1}{s(s+2)(s+3)}\right)
\end{aligned}
$$

Expanding

$$\frac{1}{(s+2)(s+3)} = \frac{1}{s+2} - \frac{1}{s+3}$$

$$\frac{1}{s(s+2)(s+3)} = \frac{1}{6s} - \frac{1}{2(s+2)} + \frac{1}{3(s+3)}$$

Thus we get that

$$\frac{1}{(s+2)(s+3)} - 8\frac{1}{s(s+2)(s+3)} = \frac{5}{s+2} - \frac{4}{3s} - \frac{11}{3(s+3)}$$

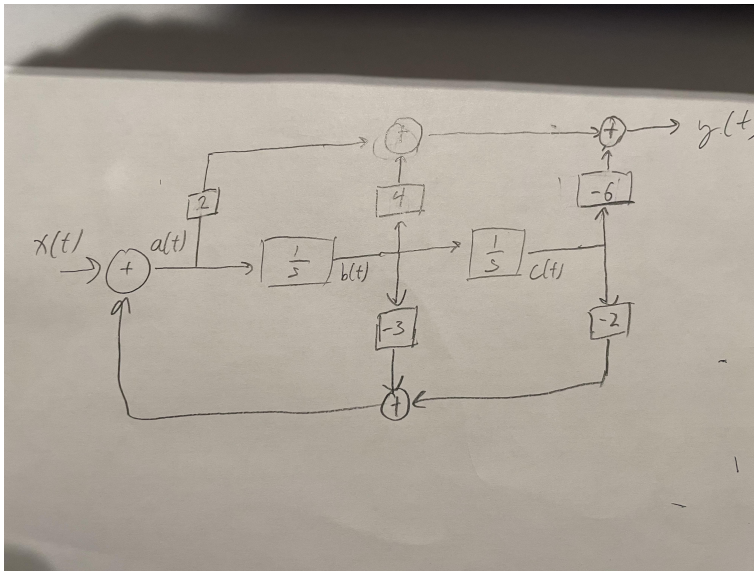Reverse Laplace transforming we get that

$$y(t) = \boxed{2\left(-\frac{4}{3} + 5e^{-2(t-2)} - \frac{11}{3}e^{-3(t-2)}\right)u(t-2) - 2\left(-\frac{4}{3} + 5e^{-2(t+2)} - \frac{11}{3}e^{-3(t+2)}\right)u(t+2)}$$

Problem 3

(a)

Let $a(t)$, $b(t)$, and $c(t)$ be determined as depicted in the drawing below

Therefore in the s domain we have

$$A(s) = X(s) - 3B(s) - 2C(s)$$

$$B(s) = \frac{1}{s}A(s)$$

$$C(s) = \frac{1}{s}B(s) = \frac{1}{s^2}A(s)$$

Therefore we have that

$$A(s) = X(s) - \frac{3}{s}A(s) - \frac{2}{s^2}A(s)$$

$$A(s) = X(s)\frac{1}{1 + \frac{3}{s} + \frac{2}{s^2}}$$

$$A(s) = X(s)\frac{s^2}{s^2 + 3s + 2}$$

Thus we have that

$$Y(s) = 2A(s) + 4B(s) - 6C(s)$$

$$= X(s)\frac{s^2}{s^2 + 3s + 2}\left(2 + 4\frac{1}{s} - 6\frac{1}{s^2}\right)$$

$$= X(s)\frac{s^2}{s^2 + 3s + 2}\frac{2s^2 + 4s - 6}{s^2}$$

$$= X(s)\frac{2s^2 + 4s - 6}{s^2 + 3s + 2}$$

Thus we have that

$$\boxed{H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}}$$

(b)

We have that

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}$$

$$Y(s)(s^2 + 3s + 2) = X(s)(2s^2 + 4s - 6)$$

Inverse laplace transforming we get that

$$\boxed{\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{d^2x(t)}{dt^2} + 4\frac{dx(t)}{dt} - 6x(t)}$$

(c)

$$H(s) = \frac{2s^2 + 4s - 6}{s^2 + 3s + 2}$$

$$= \frac{2s^2 + 4s - 6}{(s+1)(s+2)}$$

$$= \frac{2s}{s+1} - \frac{6}{(s+1)(s+2)}$$

$$= \frac{2s + 2 - 2}{s+1} - \frac{6}{s+1} + \frac{6}{s+2}$$

$$= 2 - \frac{8}{s+1} + \frac{6}{s+2}$$

Inverse laplace transforming we get that

$$h(t) = 2\delta(t) - 8e^{-t} + 6e^{-2t}$$

$$H(\frac{s}{3}) \rightarrow \frac{1}{3}h(3t)$$

$$H(\frac{s}{3} - 4) \rightarrow \frac{1}{3}e^{4t}h(3t)$$

$$e^{-4s}H(\frac{s}{3} - 4) \rightarrow \frac{1}{3}e^{4(t-4)}h(3(t-4))$$

$$e^{-4s}H(\frac{s}{3} - 4) \rightarrow \boxed{\frac{1}{3}e^{4(t-4)}\left(2\delta(3(t-4)) - 8e^{-3(t-4)} + 6e^{-6(t-4)}\right)}$$

Problem 4

(a)

$$\Omega_k = \boxed{\frac{2k\pi}{T_0}}$$

(b)