ECE 102 Homework 2

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January 19, 2022

Problem 1

(a)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$S\{\alpha x_1(t) + \beta x_2(t)\} = \int_{-\infty}^{2t} (\alpha x_1(\tau + 3) + \beta x_2(\tau + 3))d\tau$$

$$= \alpha \int_{-\infty}^{2t} x_1(\tau + 3)d\tau + \beta \int_{-\infty}^{2t} x_2(\tau + 3)d\tau$$

$$= \alpha y_1(t) + \beta y_2(t)$$

Thus this system is $\boxed{\text{linear}}$.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$S\{x(t - \sigma)\} = \int_{-\infty}^{2t} x(\tau + 3 - \sigma)d\tau$$

let $\lambda = \tau - \sigma$, $d\lambda = d\tau$, and the limits of the integral become $-\infty$ to $2t - \sigma$, thus we get

$$S\{x(t - \sigma)\} = \int_{-\infty}^{2t-\sigma} x(\lambda + 3)d\lambda$$

1

Since $y(t - \sigma) = \int_{-\infty}^{2(t-\sigma)} x(\tau + 3)d\tau$ we get that the system is $\boxed{\text{time variant}}$.

This system is also $\boxed{\text{non causual}}$ since $\int_{-\infty}^{2t} x(\tau + 3)d\tau$ depends on the value of $x()$ past time t

If $x(t) = u(t - 2) - u(t - 4)$ we get that

$$y(t) = \int_{-\infty}^{2t} x(\tau + 3)d\tau$$
$$= \int_{-\infty}^{2t} u(\tau + 1)d\tau - \int_{-\infty}^{2t} u(\tau - 1)d\tau$$
$$= \boxed{\begin{cases} 0 & t \leq -0.5 \\ 2t + 1 & -0.5 < t \leq 0.5 \\ 2 & t > 0.5 \end{cases}}$$

(b)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$S\{\alpha x_1(t) + \beta x_2(t)\} = (\alpha x_1(t) + \beta x_2(t)) \sin(\pi t)$$
$$= \alpha x_1(t) \sin(\pi t) + \beta x_2(t) \sin(\pi t)$$
$$= \alpha y_1(t) + \beta y_2(t)$$

Thus this system is $\boxed{\text{linear}}$.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$S\{x(t - \sigma)\} = x(t - \sigma) \sin(\pi t)$$

Since $y(t - \sigma) = x(t - \sigma) \sin(\pi(t - \sigma))$ we have that the system is $\boxed{\text{time variant}}$
This system is also $\boxed{\text{causual}}$ since $x(t) \sin(\pi t)$ depends only on the value of $x()$ at time t.

If $x(t) = u(t - 2) - u(t - 4)$ we get that $y(t) = \boxed{u(t - 2) \sin(\pi t) - u(t - 4) \sin(\pi t)}$

(c)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$
\begin{aligned}
S\{\alpha x_1(t) + \beta x_2(t)\} &= \frac{d}{dt}(\alpha x_1(t) + \beta x_2(t)) \\
&= \alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt} \\
&= \alpha y_1(t) + \beta y_2(t)
\end{aligned}
$$

Thus this system is $\boxed{\text{linear}}$.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$
\begin{aligned}
S\{x(t - \sigma)\} &= \frac{dx(t - \sigma)}{dt} \\
&= \frac{dx(t - \sigma)}{d(t - \sigma)} \frac{d(t - \sigma)}{dt} \\
&= \frac{dx(t - \sigma)}{d(t - \sigma)}
\end{aligned}
$$

Since $y(t - \sigma) = \frac{dx(t - \sigma)}{d(t - \sigma)}$ therefore this system is $\boxed{\text{time invariant}}$

Furthermore, since $\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$ the system is $\boxed{\text{non causual}}$.

If $x(t) = u(t - 2) - u(t - 4)$ we get that

$$
\begin{aligned}
y(t) &= \frac{dx(t)}{dt} \\
&= \boxed{\delta(t - 2) - \delta(t - 4)}
\end{aligned}
$$

3

(d)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(2 - t) + \beta x_2(2 - t) + \alpha x_1(2 + t) + \beta x_2(2 + t)$$
$$= \alpha y_1(t) + \beta y_2(t)$$

Thus this system is $\boxed{\text{linear}}$.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$S\{x(t - \sigma)\} = x(2 - t - \sigma) + x(2 + t - \sigma)$$

Since $y(t - \sigma) = x(2 - (t - \sigma)) + x(2 + (t - \sigma))$ therefore this system is $\boxed{\text{time variant}}$

This system is also $\boxed{\text{non causal}}$ since $y(t)$ depends on values of $x()$ past time t, specifically $x(t + 2)$. If $x(t) = u(t - 2) - u(t - 4)$ we get that

$$y(t) = x(2 - t) + x(2 + t)$$
$$= u((2 - t) - 2) + u((t + 2) - 2) - u((2 - t) - 4) - u((2 + t) - 4)$$
$$= u(-t) + u(t) - u(-2 - t) - u(-2 + t)$$
$$= \boxed{0}$$

Problem 2

(a)

$$y(t) = x(t) - \int_{t-1}^{t+1} e^{|t-\tau|} x(\tau) d\tau$$

$$= \int_{t-1}^{t+1} x(\tau)(\delta(\tau - t) - e^{|t-\tau|}) d\tau$$

$$\boxed{= \int_{-\infty}^{\infty} x(\tau) \left(\delta(\tau - t) - e^{|t-\tau|} \left(u(\tau - 1) - u(\tau + 1) \right) \right) d\tau}$$

(b)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$S\{\alpha x_1(t) + \beta x_2(t)\} = \int_{-\infty}^{\infty} (\alpha x_1(\tau) + \beta x_2(\tau)) \left(\delta(\tau - t) - e^{|t-\tau|} \left(u(\tau - 1) - u(\tau + 1) \right) \right) d\tau$$

$$= \alpha y_1(t) + \beta y_2(t)$$

Thus this system is $\boxed{\text{linear}}$.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$S\{x(t - \sigma)\} = x(t - \sigma) - \int_{t-1}^{t+1} e^{|t-\tau|} x(\tau - \sigma) d\tau$$

$$y(t - \sigma) = x(t - \sigma) - \int_{t-1-\sigma}^{t+1-\sigma} e^{|t-\tau-\sigma|} x(\tau) d\tau$$

Let $\lambda = \tau - \sigma$, thus $d\lambda = d\sigma$ and the limits of the integral become $t - 1$ and $t + 1$, then we get

$$y(t - \sigma) = x(t - \sigma) - \int_{t-1}^{t+1} e^{|t-\sigma|} x(\lambda - \sigma) d\tau$$

5

Thus the system is time invariant

Furthermore, since $\int_{t-1}^{t+1} e^{|t-\tau|} x(\tau) d\tau$ depends on values of $x()$ past time t the system is non causual.

(c)

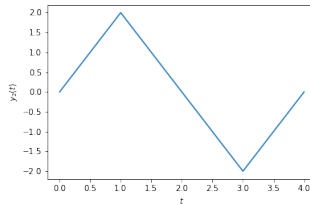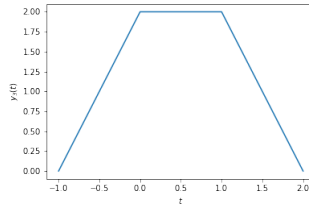$$y(t) = x(t) - \int_{t-1}^{t+1} e^{|t-\tau|} x(\tau) d\tau$$

$$= \boxed{e^{-t} u(t+2) - \int_{t-1}^{t+1} e^{|t-\tau|} e^{-\tau} u(\tau+2) d\tau}$$

Problem 3

(a)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$. Likewise a system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$.

We can express $x_2(t)$ as $x_2(t) = x_1(t) - x_1(t-2)$. Therefore $y_2(t) = y_1(t) - y_1(t-2)$, this plot of this is drawn below

(b)

We can express $x_3(t)$ as $x_3(t) = x_1(t+1) + x_1(t)$. Therefore $y_3(t) = y_1(t+1) + y_1(t)$, the plot of this is drawn below
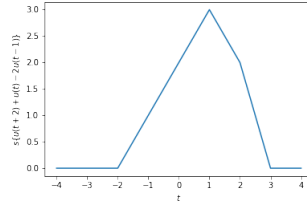

(c)

$$h(t) = \int_{t-2}^{t} \delta(\tau)d\tau$$
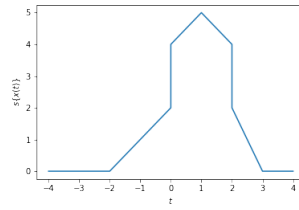$$= \boxed{u(t) - u(t-2)}$$

Because the system is LTI to find the output of the system when the input is $x(t) = u(t+2) + u(t) - 2u(t-1) + \delta(t-1)$ we just need to find the system response to a unit impulse response (which we have found) and the unit step response which is $s\{u(t)\} = \int_{t-2}^{t} = \begin{cases} 0 & t < 0 \\ t & 0 \leq t < 2 \\ 2 & t > 2 \end{cases}$. Thus $s\{u(t+2) + u(t) - 2u(t-1)\}$ looks like

And thus the system output where $x(t) = u(t+2) + u(t) - 2u(t-1) + \delta(t-1)$ is

Problem 4

(a)

$$h(t, \tau) = \int_{-\infty}^{\infty} e^{-t}(t - \sigma)^2 u(t + \sigma)\delta(\sigma - \tau - 2)d\sigma$$