ECE 102 Homework 2

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Problem 1

(a)

After Laplace transform we have that

$$3s^2Y(S) + 19sY(s) + 20Y(S) = 2sX(s) - X(s)$$

$$H(s) = \boxed{\frac{2s-1}{3s^2+19s+20}}$$

(b)

$$X(s) = \frac{2}{2s+1}e^{-3s}$$

$$Y(s) = H(s)X(s)$$

$$Y(s) = e^{-3s}\frac{(2s-1)}{(3s^2+19s+20)(s+\frac{1}{2})}$$

$$Y(s) = e^{-3s}\frac{(2s-1)}{(3s+4)(s+5)(s+\frac{1}{2})}$$

$$Y(s) = \frac{e^{-3s}}{3} \frac{(2s-1)}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})}$$

$$Y(s) = \frac{e^{-3s}}{3} \left(\frac{-\frac{5}{3}}{s+\frac{4}{3}} + \frac{-11}{s+5} + \frac{-2}{s+\frac{1}{2}} \right)$$

$$Y(t) = \boxed{-\frac{5}{9}e^{-\frac{4}{3}(t-3)}u(t-3) - \frac{11}{3}e^{-5(t-3)}u(t-3) - \frac{2}{3}e^{-\frac{t-3}{2}}u(t-3)}$$

Problem 2

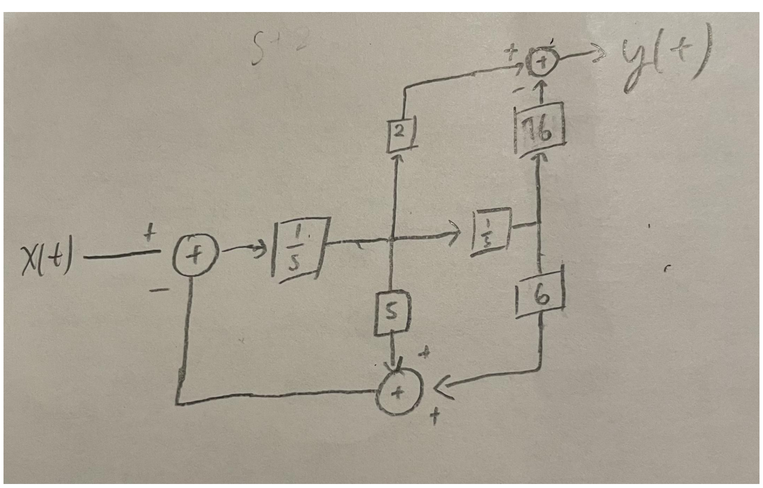
(a)

The system response function is

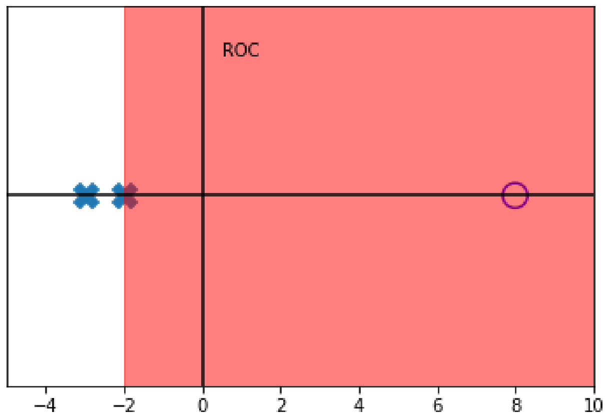
$$h(s) = \frac{2s^2 - 14s - 16}{s^2 + 6s^2 + 11s + 6}$$

$$h(s) = \frac{2}{s+2}\frac{1}{s+3}(s-8)$$

Thus the system looks like



(b)



This converges since the roc includes $s = 0$

(c)

$$X(s) = \frac{1}{s}\left(e^{2s} - e^{-2s}\right)$$

$$\begin{aligned} Y(s) &= X(s)H(s) \\ &= \left(e^{2s} - e^{-2s}\right)\frac{2(s-8)}{s(s+2)(s+3)} \\ &= -\left(e^{2s} - e^{-2s}\right)\left(\frac{16}{s} + \frac{20}{s+2} + \frac{22}{s+3}\right) \end{aligned}$$

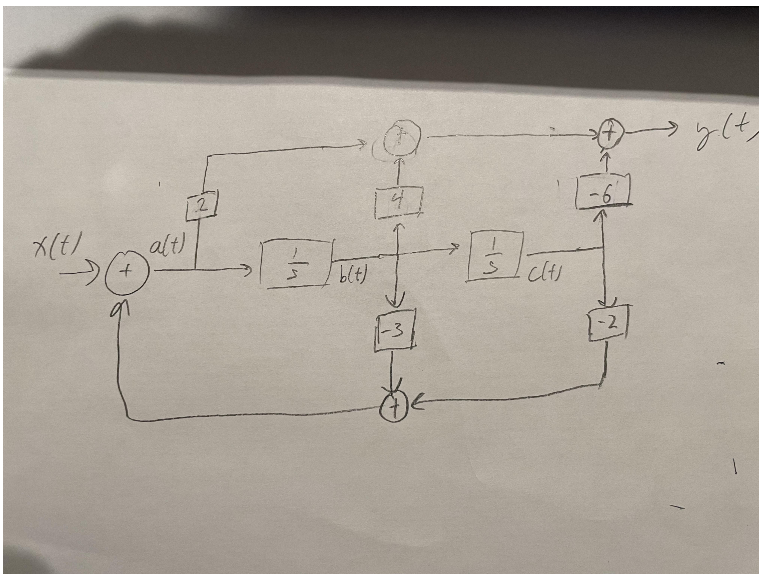
Reverse Laplace transforming we get that

$$y(t) = \boxed{\left(16 + 20e^{-2(t-2)} + 22e^{-3(t-2)}\right)u(t-2) - \left(16 + 20e^{-2(t+2)} + 22e^{-3(t+2)}\right)u(t+2)}$$

Problem 3

(a)

Let $a(t)$, $b(t)$, and $c(t)$ be determined as depicted in the drawing below



Therefore in the s domain we have

$$A(s) = X(s) - 3B(s) - 2C(s)$$

$$B(s) = \frac{1}{s}A(s)$$

$$C(s) = \frac{1}{s}B(s) = \frac{1}{s^2}A(s)$$

Therefore we have that

$$A(s) = X(s) - \frac{3}{s}A(s) - \frac{2}{s^2}A(s)$$

$$A(s) = X(s)\frac{1}{1+\frac{3}{s}+\frac{2}{s^2}}$$

$$A(s) = X(s)\frac{s^2}{s^2+3s+2}$$

Thus we have that

$$\begin{aligned} Y(s) &= 2A(s) + 4B(s) - 6C(s) \\ &= X(s)\frac{s^2}{s^2+3s+2}\left(2 + 4\frac{1}{s} - 6\frac{1}{s^2}\right) \\ &= X(s)\frac{s^2}{s^2+3s+2}\frac{2s^2+4s-6}{s^2} \\ &= X(s)\frac{2s^2+4s-6}{s^2+3s+2} \end{aligned}$$

Thus we have that

$$H(s) = \boxed{\frac{2s^2+4s-6}{s^2+3s+2}}$$

(b)

We have that

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s^2+4s-6}{s^2+3s+2}$$

$$Y(s)(s^2+3s+2) = X(s)(2s^2+4s-6)$$

Inverse laplace transforming we get that

$$\boxed{\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{d^2x(t)}{dt^2} + 4\frac{dx(t)}{dt} - 6x(t)}$$