ECE 102 Project

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1 Part I

Step 1

(a)

We have

$$(X(s) - kY(s))\,H_1(s)H_2(s) = Y(s)$$
$$X(s)H_1(s)H_2(s) = Y(s)\,(1 + kH_1(s)H_2(s))$$
$$H(s) = \frac{Y(s)}{X(s)} = \boxed{\frac{H_1(s)H_2(s)}{1 + kH_1(s)H_2(s)}}$$

(b)

We have

$$Z(s) = (s-1)X(s)$$
$$H_1(s) = \boxed{(s-1)}$$

And we have

$$y(t) = \int_{-\infty}^{t} z(\tau)e^{4(\tau-t)}d\tau$$

$$y(t) = z(t) * e^{-4t}u(t)$$

$$Y(s) = Z(s)\frac{1}{s+4}$$

$$H_2(s) = \boxed{\frac{1}{s+4}}$$

(c)

We have

$$H(s) = \frac{Y(s)}{X(s)} = \frac{H_1(s)H_2(s)}{1 + kH_1(s)H_2(s)}$$

$$H(s) = \frac{s-1}{s+4+ks-k}$$

$$H(s) = \boxed{\frac{s-1}{(k+1)s+4-k}}$$

(d)

The system has a pole at

$$s = \frac{k-4}{k+1}$$

Therefore in order for this system to be stable we must have that the ROC covers $s = 0$, ie that

$$\frac{k-4}{k+1} < 0$$

and thus

$$k - 4 < 0$$

and

$$k + 1 > 0$$

thus we get

$$\boxed{-1 \neq k < 4}$$

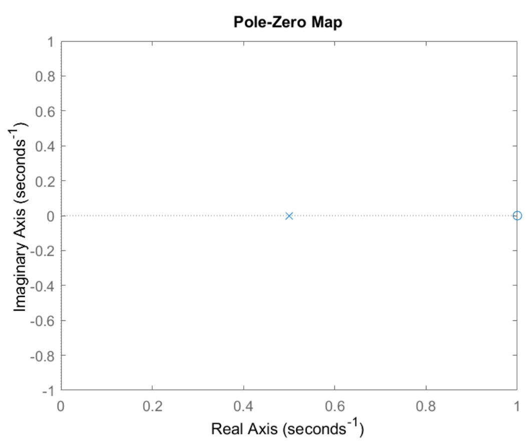
(e)

using this code, we get

```
k=-3;

sys = tf([1  -1],[(k+1)  (4+k)]);

h = pzplot(sys);
```

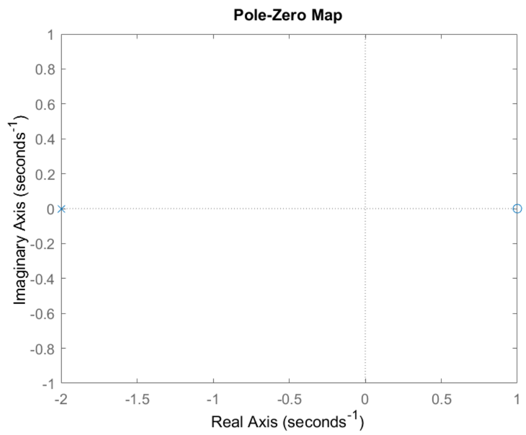


This plot, with $K = K_1 = -3$ is unstable since there is a pole greater than 0. But by changing K to be $K = K_2 = 2$ the systems is stable since there are no poles greater than 0, the pole zero plot is included below.

```
k=2;

sys = tf([1  -1],[(k+1)  (4+k)]);

h = pzplot(sys);
```

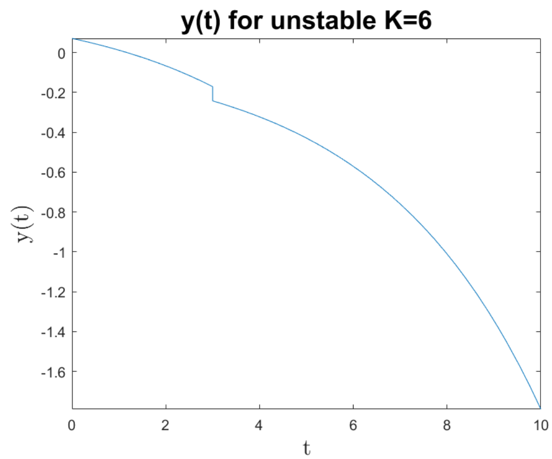


Step 2

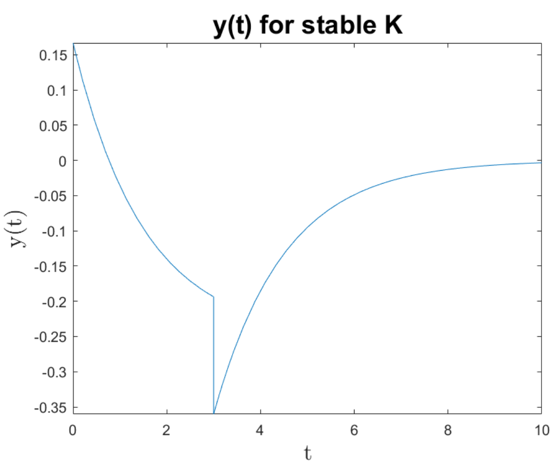
(a)

```
k=6;

syms t
x=(1/2).*rectangularPulse(-1.5, 1.5, t-1.5);
X_l = laplace(x);
syms s
sys = (s-1)/((k+1)*s+4-k);
y=ilaplace(sys.*X_l);
fplot(y,[0,10])
xlabel('t','Interpreter','latex','fontsize',16);
ylabel('y(t)','Interpreter','latex','fontsize',16);
title('y(t) for unstable K=6','fontsize',18);
```



And by changing the K we get the following response for stable system

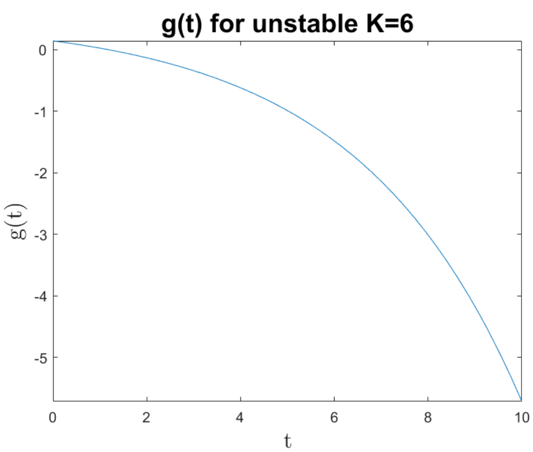


(b)

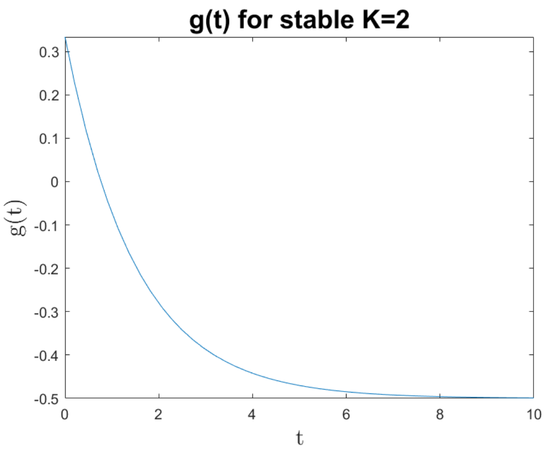
```
k=6;

syms t
x=heaviside(t);
```

```
X_l = laplace(x);
syms s
sys = (s-1)/((k+1)*s+4-k);
y=ilaplace(sys.*X_l);
fplot(y,[0,10])
xlabel('t','Interpreter','latex','fontsize',16);
ylabel('g(t)','Interpreter','latex','fontsize',16);
title('g(t) for unstable K=6','fontsize',18);
```



And by changing the K we get the following response for stable system



Part 2

Step 3

(a)

When $K = 2$ we have

$$H(s) = \frac{s-1}{3s+2}$$

$$H(j\omega) = \boxed{\frac{j\omega - 1}{3j\omega + 2}}$$

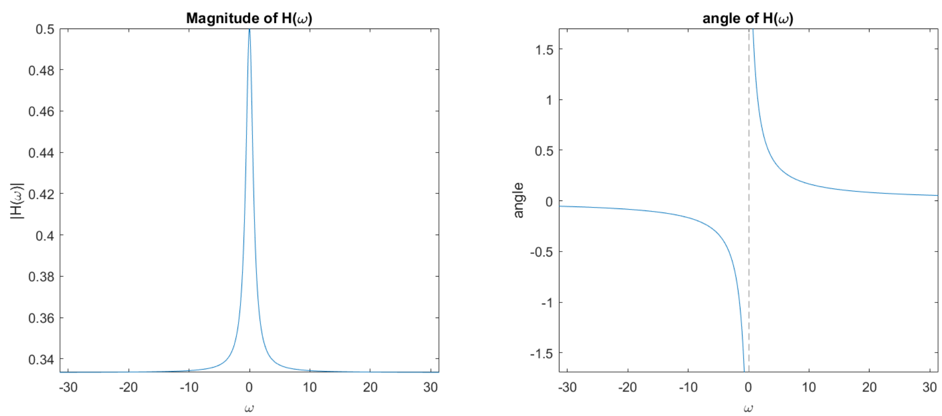
(b)

```
syms omega

H=(1j*omega-1)./(3j*omega+2);

figure;
subplot(1,2,1)
fplot(abs(H),[-10*pi,10*pi])
```

```
xlabel('\omega')
ylabel('|H(\omega)|')
title('Magnitude of H(\omega)')
subplot(1,2,2)
fplot(angle(H),[-10*pi,10*pi])
xlabel('\omega')
ylabel('angle')
title('angle of H(\omega)')
```



(c)

High frequency inputs are more heavily attenuated compared with low frequency inputs.

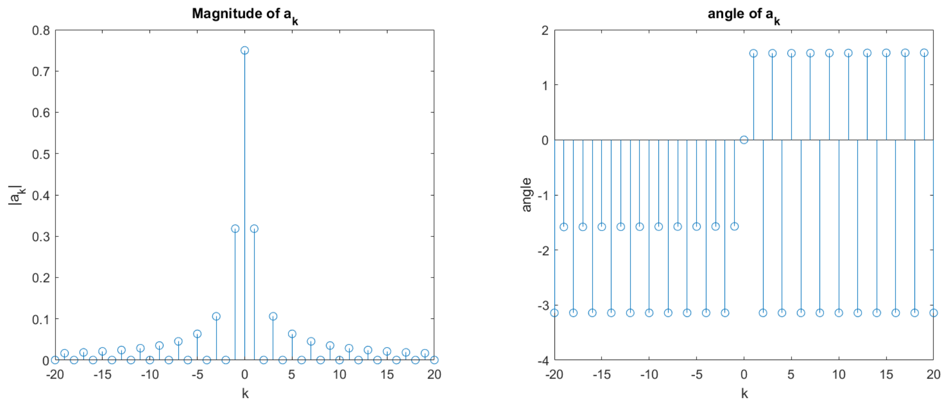
Step 4

(a)

$$
\begin{aligned}
a_k &= \frac{1}{4}\left(\int_0^2 \frac{t}{2} e^{-jk\frac{\pi}{2}t} dt + \int_2^4 e^{-jk\frac{\pi}{2}t} dt\right) \\
&= \frac{1}{4}\left(\frac{2\left((\mathrm{j}\pi k + 1)\cos(\pi k) - 1\right)}{\pi^2 k^2} + \frac{2\left(\mathrm{j}\cos(2\pi k) - \mathrm{j}\cos(\pi k)\right)}{\pi k}\right) \\
&= \boxed{\frac{1}{2}\left(\frac{\left((\mathrm{j}\pi k + 1)(-1)^k - 1\right)}{\pi^2 k^2} + \frac{\left(\mathrm{j} - \mathrm{j}(-1)^k\right)}{\pi k}\right)}
\end{aligned}
$$

(b)

```
k=-20:20;
ak= 1/4.*(2.*((1i.*pi.*k+1).*((-1).^k)-1)/ ...
     (pi.^2.*k.^2)+2.*(1i-1i.*(-1).^k)./(pi.*k));
ak(21)=0.75;
figure;
subplot(1,2,1)
stem(k,abs(ak))
xlabel('k')
ylabel('|a_k|')
title('Magnitude of a_k')
subplot(1,2,2)
stem(k,angle(ak))
xlabel('k')
ylabel('angle')
title('angle of a_k')
hold off
```



(c)

$$
\begin{aligned}
Y_k &= H(jk\frac{\pi}{2})a_k \\
&= \boxed{\frac{jk\frac{\pi}{2}-1}{2(3jk\frac{\pi}{2}+2)}\left(\frac{\left((\mathrm{j}\pi k+1)\left(-1\right)^k-1\right)}{\pi^2k^2}+\frac{\left(\mathrm{j}-\mathrm{j}(-1)^k\right)}{\pi k}\right)}
\end{aligned}
$$

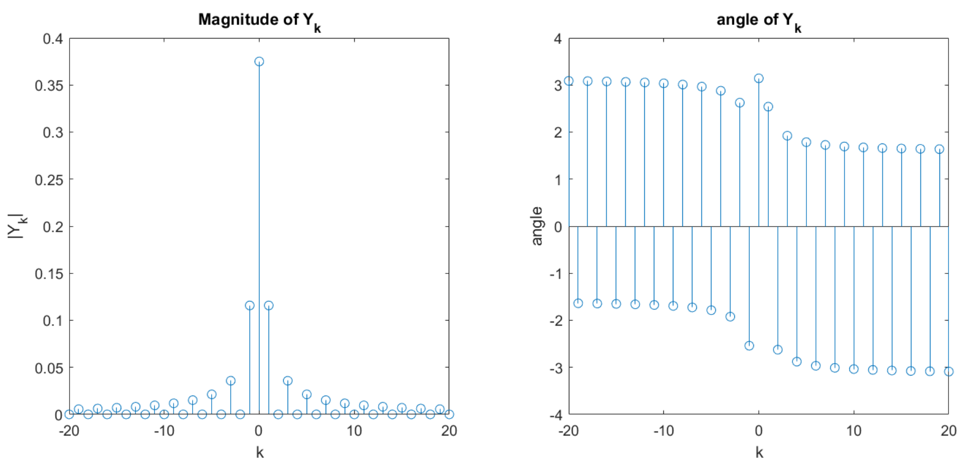
(d)

```
k=-20:20;
ak= 1/4.*(2.*((1i.*pi.*k+1).*(-1).^k-1)/  ...
     (pi.^2.*k.^2)+2.*(1i-1i.*(-1).^k)./(pi.*k));
ak(21)=0.75;


H_k=(1j.*k.*pi/2-1)./(3j.*k.*pi/2+2);

figure;
subplot(1,2,1)
stem(k,abs(H_k.*ak))
xlabel('k')
ylabel('|Y_k|')
title('Magnitude of Y_k')
```

```
subplot(1,2,2)
stem(k,angle(H_k.*ak))
xlabel('k')
ylabel('angle')
title('angle of Y_k')
hold off
```



(e)

```

t=0:0.1:12;
a =(rem(t,4)/2).*heaviside(rem(t,4))-(rem(t,4)/2).*heaviside(rem(t,4)-2
+heaviside(rem(t,4)-2) - heaviside(rem(t,4)-4);

x = zeros(1,121);
y=zeros(1,121);
for k=-20:20
    if k~=0
        ak= 1/4.*(2.*((1i.*pi.*k+1).*(-1).^k-1)/ ...
        (pi.^2.*k.^2)+2.*(1i-1i.*(-1).^k)./(pi.*k));

        hk=(1j.*k.*pi/2-1)./(3j.*k.*pi/2+2);
```

```
        x=x+ak.*exp(1i*k*pi/2*t);
        y=y+hk*ak.*exp(1i*k*pi/2*t);
    end

end

plot(t,real(y))
hold on
plot(t,a)

xlabel('t')
ylabel('y(t) and a(t)')
title('Plot of y(t) and a(t)')
legend('y(t)','a(t)')
hold off
```