

ECE 102 Homework 2

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Problem 1

(a)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha y_1(t) + \beta y_2(t)$

$$\begin{aligned} S\{\alpha x_1(t) + \beta x_2(t)\} &= \int_{-\infty}^{2t} (\alpha x_1(\tau + 3) + \beta x_2(\tau + 3)) d\tau \\ &= \alpha \int_{-\infty}^{2t} x_1(\tau + 3) d\tau + \beta \int_{-\infty}^{2t} x_2(\tau + 3) d\tau \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Thus this system is linear.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$S\{x(t - \sigma)\} = \int_{-\infty}^{2t} x(\tau + 3 - \sigma) d\tau$$

let $\lambda = \tau - \sigma$, $d\lambda = d\tau$, and the limits of the integral become $-\infty$ to $2t - \sigma$, thus we get

$$S\{x(t - \sigma)\} = \int_{-\infty}^{2t - \sigma} x(\lambda + 3) d\lambda$$

Since $y(t - \sigma) = \int_{-\infty}^{2(t - \sigma)} x(\tau + 3) d\tau$ we get that the system is time variant.

This system is also non causal since $\int_{-\infty}^{2t} x(\tau + 3) d\tau$ depends on the value of $x()$ past time t

If $x(t) = u(t-2) - u(t-4)$ we get that

$$\begin{aligned}
 y(t) &= \int_{-\infty}^{2t} x(\tau+3) d\tau \\
 &= \int_{-\infty}^{2t} u(\tau+1) d\tau - \int_{-\infty}^{2t} u(\tau-1) d\tau \\
 &= \begin{cases} 0 & t \leq -0.5 \\ 2t+1 & -0.5 < t \leq 0.5 \\ 2 & t > 0.5 \end{cases}
 \end{aligned}$$

(b)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha y_1(t) + \beta y_2(t)$

$$\begin{aligned}
 S\{\alpha x_1(t) + \beta x_2(t)\} &= (\alpha x_1(t) + \beta x_2(t)) \sin(\pi t) \\
 &= \alpha x_1(t) \sin(\pi t) + \beta x_2(t) \sin(\pi t) \\
 &= \alpha y_1(t) + \beta y_2(t)
 \end{aligned}$$

Thus this system is linear.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t-\sigma)\} = y(t-\sigma)$

$$S\{x(t-\sigma)\} = x(t-\sigma) \sin(\pi t)$$

Since $y(t-\sigma) = x(t-\sigma) \sin(\pi(t-\sigma))$ we have that the system is time variant

This system is also causal since $x(t) \sin(\pi t)$ depends only on the value of $x()$ at time t .

If $x(t) = u(t-2) - u(t-4)$ we get that $y(t) = \boxed{u(t-2) \sin(\pi t) - u(t-4) \sin(\pi t)}$

(c)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha y_1(t) + \beta y_2(t)$

$$\begin{aligned}
 S\{\alpha x_1(t) + \beta x_2(t)\} &= \frac{d}{dt}(\alpha x_1(t) + \beta x_2(t)) \\
 &= \alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt} \\
 &= \alpha y_1(t) + \beta y_2(t)
 \end{aligned}$$

Thus this system is linear.

A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$\begin{aligned} S\{x(t - \sigma)\} &= \frac{dx(t - \sigma)}{dt} \\ &= \frac{dx(t - \sigma)}{d(t - \sigma)} \frac{d(t - \sigma)}{dt} \\ &= \frac{dx(t - \sigma)}{d(t - \sigma)} \end{aligned}$$

Since $y(t - \sigma) = \frac{dx(t - \sigma)}{d(t - \sigma)}$ therefore this system is time invariant

Furthermore, since $\frac{dx(t)}{dt} = \lim_{\Delta t \rightarrow 0} \frac{x(t + \Delta t) - x(t)}{\Delta t}$ the system is non causal.

If $x(t) = u(t - 2) - u(t - 4)$ we get that

$$\begin{aligned} y(t) &= \frac{dx(t)}{dt} \\ &= \boxed{\delta(t - 2) - \delta(t - 4)} \end{aligned}$$