ECE 102 Homework 2

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Problem 1

(a)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$\begin{aligned} S\{\alpha x_1(t) + \beta x_2(t)\} &= \int_{-\infty}^{2t} (\alpha x_1(\tau+3) + \beta x_2(\tau+3)) d\tau \\ &= \alpha \int_{-\infty}^{2t} x_1(\tau+3) d\tau + \beta \int_{-\infty}^{2t} x_2(\tau+3) d\tau \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Thus this system is $\boxed{\text{linear}}$.
A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t-\sigma)\} = y(t-\sigma)$

$$S\{x(t-\sigma)\} = \int_{-\infty}^{2t} x(\tau + 3 - \sigma) d\tau$$

let $\lambda = \tau - \sigma$, $d\lambda = d\tau$, and the limits of the integral become $-\infty$ to $2t - \sigma$, thus we get

$$S\{x(t-\sigma)\} = \int_{-\infty}^{2t-\sigma} x(\lambda + 3) d\lambda$$

Since $y(t-\sigma) = \int_{-\infty}^{2(t-\sigma)} x(\tau+3) d\tau$ we get that the system is $\boxed{\text{time variant}}$.
This system is also $\boxed{\text{non causual}}$ since $\int_{-\infty}^{2t} x(\tau+3) d\tau$ depends on the value of $x()$ past time t

If $x(t) = u(t-2) - u(t-4)$ we get that

$$\begin{aligned} y(t) &= \int_{-\infty}^{2t} x(\tau+3)d\tau \\ &= \int_{-\infty}^{2t} u(\tau+1)d\tau - \int_{-\infty}^{2t} u(\tau-1)d\tau \\ &= \boxed{\begin{cases} 0 & t \leq -0.5 \\ 2t+1 & -0.5 < t \leq 0.5 \\ 2 & t > 0.5 \end{cases}} \end{aligned}$$

(b)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$\begin{aligned} S\{\alpha x_1(t) + \beta x_2(t)\} &= (\alpha x_1(t) + \beta x_2(t)) \sin(\pi t) \\ &= \alpha x_1(t) \sin(\pi t) + \beta x_2(t) \sin(\pi t) \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Thus this system is $\boxed{\text{linear}}$.
A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t-\sigma)\} = y(t-\sigma)$

$$S\{x(t-\sigma)\} = x(t-\sigma)\sin(\pi t)$$

Since $y(t-\sigma) = x(t-\sigma)\sin(\pi(t-\sigma))$ we have that the system is $\boxed{\text{time variant}}$
This system is also $\boxed{\text{causual}}$ since $x(t)\sin(\pi t)$ depends only on the value of $x()$ at time t.
If $x(t) = u(t-2) - u(t-4)$ we get that $y(t) = \boxed{u(t-2)\sin(\pi t) - u(t-4)\sin(\pi t)}$

(c)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$\begin{aligned} S\{\alpha x_1(t) + \beta x_2(t)\} &= \frac{d}{dt}(\alpha x_1(t) + \beta x_2(t)) \\ &= \alpha \frac{dx_1(t)}{dt} + \beta \frac{dx_2(t)}{dt} \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Thus this system is $\boxed{\text{linear}}$.
A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t-\sigma)\} = y(t-\sigma)$

$$\begin{aligned} S\{x(t-\sigma)\} &= \frac{dx(t-\sigma)}{dt} \\ &= \frac{dx(t-\sigma)}{d(t-\sigma)} \frac{d(t-\sigma)}{dt} \\ &= \frac{dx(t-\sigma)}{d(t-\sigma)} \end{aligned}$$

Since $y(t-\sigma) = \frac{dx(t-\sigma)}{d(t-\sigma)}$ therefore this system is $\boxed{\text{time invariant}}$
Furthermore, since $\frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$ the system is $\boxed{\text{non causual}}$.
If $x(t) = u(t-2) - u(t-4)$ we get that

$$\begin{aligned} y(t) &= \frac{dx(t)}{dt} \\ &= \boxed{\delta(t-2) - \delta(t-4)} \end{aligned}$$

(d)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$\begin{aligned} S\{\alpha x_1(t) + \beta x_2(t)\} &= \alpha x_1(2-t) + \beta x_2(2-t) + \alpha x_1(2+t) + \beta x_2(2+t) \\ &= \alpha y_1(t) + \beta y_2(t) \end{aligned}$$

Thus this system is $\boxed{\text{linear}}$.
A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t-\sigma)\} = y(t-\sigma)$

$$S\{x(t-\sigma)\} = x(2-t-\sigma) + x(2+t-\sigma)$$

Since $y(t-\sigma) = x(2-(t-\sigma)) + x(2+(t-\sigma))$ therefore this system is $\boxed{\text{time variant}}$
This system is also $\boxed{\text{non causual}}$ since $y(t)$ depends on values of $x()$ past time t, specifically $x(t+2)$. If $x(t) = u(t-2) - u(t-4)$ we get that

$$\begin{aligned} y(t) &= x(2-t) + x(2+t) \\ &= u((2-t)-2) + u((t+2)-2) - u((2-t)-4) - u((2+t)-4) \\ &= u(-t) + u(t) - u(-2-t) - u(-2+t) \\ &= \boxed{0} \end{aligned}$$

Problem 2

(a)

$$
\begin{aligned}
y(t) &= x(t) - \int_{t-1}^{t+1} e^{|t-\tau|} x(\tau) d\tau \\
&= \int_{t-1}^{t+1} x(\tau)(\delta(\tau - t) - e^{|t-\tau|}) d\tau \\
&= \boxed{\int_{-\infty}^{\infty} x(\tau) \left(\delta(\tau - t) - e^{|t-\tau|} \left(u(\tau - 1) - u(\tau + 1)\right)\right) d\tau}
\end{aligned}
$$

(b)

A system is linear if given that $S\{x_1(t)\} = y_1(t)$ and $S\{x_2(t)\} = y_2(t)$, we get that $S\{\alpha x_1(t) + \beta x_2(t)\} = \alpha x_1(t) + \beta x_2(t)$

$$
\begin{aligned}
S\{\alpha x_1(t) + \beta x_2(t)\} &= \int_{-\infty}^{\infty} (\alpha x_1(\tau) + \beta x_2(\tau)) \left(\delta(\tau - t) - e^{|t-\tau|} \left(u(\tau - 1) - u(\tau + 1)\right)\right) d\tau \\
&= \alpha y_1(t) + \beta y_2(t)
\end{aligned}
$$

Thus this system is $\boxed{\text{linear}}$.
A system is time invariant if given $S\{x(t)\} = y(t)$, $S\{x(t - \sigma)\} = y(t - \sigma)$

$$
S\{x(t - \sigma)\} = x(t - \sigma) - \int_{t-1}^{t+1} e^{|t-\tau|} x(\tau - \sigma) d\tau
$$

$$
y(t - \sigma) = x(t - \sigma) - \int_{t-1-\sigma}^{t+1-\sigma} e^{|t-\tau-\sigma|} x(\tau) d\tau
$$

Let $\lambda = \tau - \sigma$, thus $d\lambda = d\sigma$ and the limits of the integral become $t - 1$ and $t + 1$, then we get

$$
y(t - \sigma) = x(t - \sigma) - \int_{t-1}^{t+1} e^{|t-\sigma|} x(\lambda - \sigma) d\tau
$$

Thus the system is $\boxed{\text{time invariant}}$
Furthermore, since $\int_{t-1}^{t+1} e^{|t-\tau|} x(\tau) d\tau$ depends on values of $x()$ past time t the system is $\boxed{\text{non causual}}$.

(c)

$$\begin{aligned} y(t) &= x(t) - \int_{t-1}^{t+1} e^{|t-\tau|} x(\tau) d\tau \\ &= \boxed{e^{-t}u(t+2) - \int_{t-1}^{t+1} e^{|t-\tau|} e^{-\tau} u(\tau+2) d\tau} \end{aligned}$$