ECE 102 Homework 2

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Problem 1

(a)

After Laplace transform we have that

$$3s^2Y(S) + 19sY(s) + 20Y(S) = 2sX(s) - X(s)$$

$$H(s) = \boxed{\frac{2s-1}{3s^2+19s+20}}$$

(b)

$$X(s) = \frac{2}{2s+1}e^{-3s}$$

$$
\begin{aligned}
Y(s) &= H(s)X(s) \\
&= e^{-3s}\frac{(2s-1)}{(3s^2+19s+20)(s+\frac{1}{2})} \\
&= e^{-3s}\frac{(2s-1)}{(3s+4)(s+5)(s+\frac{1}{2})} \\
&= \frac{e^{-3s}}{3}\frac{(2s-1)}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})} \\
&= \frac{e^{-3s}}{3}\frac{(2s+1)-2}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})} \\
&= \frac{e^{-3s}}{3}\left(2\left(\frac{1}{(s+\frac{4}{3})(s+5)}\right) - \frac{2}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})}\right)
\end{aligned}
$$

We can rewrite

$$
\frac{1}{(s+\frac{4}{3})(s+5)} = \frac{3}{11}\left(\frac{1}{s+\frac{4}{3}} - \frac{1}{s+5}\right)
$$

and

$$
\frac{1}{(s+\frac{4}{3})(s+5)(s+\frac{1}{2})} = -\frac{18}{55}\frac{1}{s+\frac{4}{3}} + \frac{2}{33}\frac{1}{s+5} + \frac{4}{15}\frac{1}{s+\frac{1}{2}}
$$

Thus we have that

$$
y(t) = \boxed{\frac{2}{3}\left(\frac{-3}{55}e^{-\frac{4}{3}(t-3)} + \frac{-11}{33}e^{-5(t-3)} - \frac{4}{15}e^{-\frac{1}{2}(t-3)}\right)u(t-3)}
$$

Problem 2

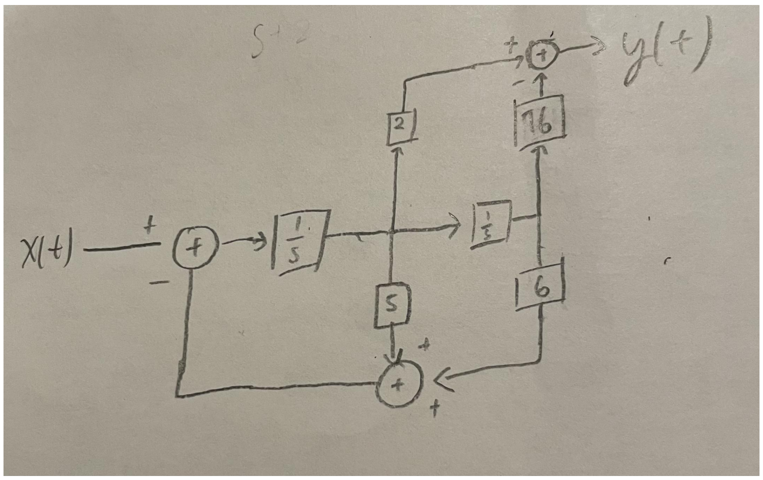
(a)

The system response function is

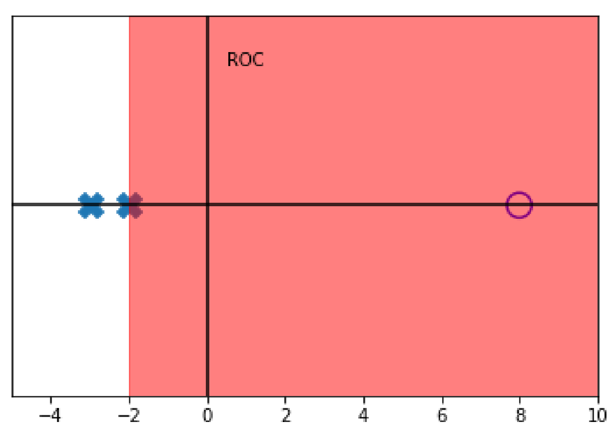
$$
h(s) = \frac{2s^2-14s-16}{s^2+6s^2+11s+6}
$$

$$h(s) = \frac{2}{s+2}\frac{1}{s+3}(s-8)$$

Thus the system looks like



(b)



This converges since the roc includes $s = 0$

(c)

$$X(s) = \frac{1}{s}\left(e^{2s} - e^{-2s}\right)$$

$$\begin{aligned} Y(s) &= X(s)H(s) \\ &= \left(e^{2s} - e^{-2s}\right)\frac{2(s-8)}{s(s+2)(s+3)} \\ &= -2\left(e^{2s} - e^{-2s}\right)\left(\frac{1}{(s+2)(s+3)} - 8\frac{1}{s(s+2)(s+3)}\right) \end{aligned}$$

Expanding

$$\frac{1}{(s+2)(s+3)} = \frac{1}{s+2} - \frac{1}{s+3}$$
$$\frac{1}{s(s+2)(s+3)} = \frac{1}{6s} - \frac{1}{2(s+2)} + \frac{1}{3(s+3)}$$

Thus we get that

$$\frac{1}{(s+2)(s+3)} - 8\frac{1}{s(s+2)(s+3)} = \frac{5}{s+2} - \frac{4}{3s} - \frac{11}{3(s+3)}$$

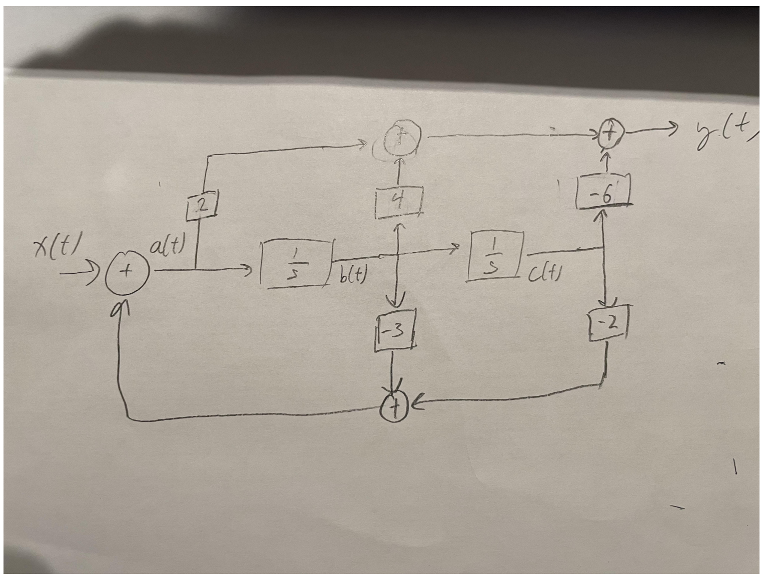
Reverse Laplace transforming we get that

$$y(t) = \boxed{2\left(-\frac{4}{3} + 5e^{-2(t-2)} - \frac{11}{3}e^{-3(t-2)}\right)u(t-2) - 2\left(-\frac{4}{3} + 5e^{-2(t+2)} - \frac{11}{3}e^{-3(t+2)}\right)u(t+2)}$$

Problem 3

(a)

Let $a(t)$, $b(t)$, and $c(t)$ be determined as depicted in the drawing below



Therefore in the s domain we have

$$A(s) = X(s) - 3B(s) - 2C(s)$$

$$B(s) = \frac{1}{s}A(s)$$

$$C(s) = \frac{1}{s}B(s) = \frac{1}{s^2}A(s)$$

Therefore we have that

$$A(s) = X(s) - \frac{3}{s}A(s) - \frac{2}{s^2}A(s)$$

$$A(s) = X(s)\frac{1}{1 + \frac{3}{s} + \frac{2}{s^2}}$$

$$A(s) = X(s)\frac{s^2}{s^2 + 3s + 2}$$

Thus we have that

$$\begin{aligned} Y(s) &= 2A(s) + 4B(s) - 6C(s) \\ &= X(s)\frac{s^2}{s^2+3s+2}\left(2 + 4\frac{1}{s} - 6\frac{1}{s^2}\right) \\ &= X(s)\frac{s^2}{s^2+3s+2}\frac{2s^2+4s-6}{s^2} \\ &= X(s)\frac{2s^2+4s-6}{s^2+3s+2} \end{aligned}$$

Thus we have that

$$H(s) = \boxed{\frac{2s^2+4s-6}{s^2+3s+2}}$$

(b)

We have that

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2s^2+4s-6}{s^2+3s+2}$$

$$Y(s)(s^2+3s+2) = X(s)(2s^2+4s-6)$$

Inverse laplace transforming we get that

$$\boxed{\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{d^2x(t)}{dt^2} + 4\frac{dx(t)}{dt} - 6x(t)}$$

(c)

$$
\begin{aligned}
H(s) &= \frac{2s^2+4s-6}{s^2+3s+2} \\
&= \frac{2s^2+4s-6}{(s+1)(s+2)} \\
&= \frac{2s}{s+1} - \frac{6}{(s+1)(s+2)} \\
&= \frac{2s+2-2}{s+1} - \frac{6}{s+1} + \frac{6}{s+2} \\
&= 2 - \frac{8}{s+1} + \frac{6}{s+2}
\end{aligned}
$$

Inverse laplace transforming we get that

$$h(t) = 2\delta(t) - 8e^{-t} + 6e^{-2t}$$

$$
\begin{aligned}
H(\frac{s}{3}) &\to \frac{1}{3}h(3t) \\
H(\frac{s}{3}-4) &\to \frac{1}{3}e^{4t}h(3t) \\
e^{-4s}H(\frac{s}{3}-4) &\to \frac{1}{3}e^{4(t-4)}h(3(t-4)) \\
e^{-4s}H(\frac{s}{3}-4) &\to \boxed{\frac{1}{3}e^{4(t-4)}\left(2\delta(3(t-4)) - 8e^{-3(t-4)} + 6e^{-6(t-4)}\right)}
\end{aligned}
$$

Problem 4

(a)

$$\Omega_k = \boxed{\frac{2k\pi}{T_0}}$$

(b)

The period of $\cos(2\pi t)$ is 1 and the period of $\cos(6\pi t + \frac{\pi}{4})$ is $\frac{1}{3}$, the period of $x(t)$ is the least common multiple of these two, so the period of $x(t)$ is 1.

Therefore we also have that

$$A_0 = 2$$
$$\Omega_0 = 0$$
$$\theta_0 = 0$$
$$A_1 = 1$$
$$\Omega_1 = 2\pi$$
$$\theta_1 = 0$$
$$A_3 = -3$$
$$\Omega_3 = 6\pi$$
$$\theta_3 = \frac{\pi}{4}$$

(c)

The period of $\cos(2\pi t)$ is 1 and the period of $\cos(20t + \frac{3}{4})$ is $\frac{\pi}{10}$. There is no least common multiple of these two. So the signal is not periodic. Since in order for a signal to have a fourier series, it must be periodic, a fourier series cannot be determined for $x_1(t)$

Problem 5

(a)

```
syms t
f=(t-5)^5* exp(-3*t)*heaviside(t);
F=laplace(f)
```

Thus we get

$$F(s) = \boxed{\frac{150}{(s+3)^4} - \frac{2500}{(s+3)^3} - \frac{600}{(s+3)^5} + \frac{120}{(s+3)^6} - \frac{3125(s+2)}{(s+3)^2}}$$

(b)

```
syms s
F=1/(s*((s+2)^2+(pi/3)^2));
f=vpa(ilaplace(F),4)
fplot(f,[0,2])
```

Thus we get

$$f(t) = 0.1962 - 0.1962e^{-2.0t}(\cos(1.047t) + 1.91\sin(1.047t))$$

The plot of which looks like