ECE 113 HW 2

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Problem 1

(a)

The discrete fourier transform doesn't exist since we have

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \\ &= \sum_{n=0}^{2\frac{1}{\omega}-1} 3^n e^{-j\omega n} \sum_{k=-\infty}^{\infty} 3^{\frac{2k}{\omega}} \end{aligned}$$

Since this second sum does not converge since $3 > 1$, this means that the DTFT doesn't exist.

(b)

(i)

This DTFT doesn't exist since once again we can write the fourier transform as

$$X(\omega) = \sum_{n=0}^{2\frac{1}{\omega}-1} 3^n e^{-j\omega n} \sum_{k=0}^{\infty} 3^{\frac{2k}{\omega}}$$

And once again this second sum does not converge since $3 > 1$.

(ii)

This does exist, since we can write

$$X[-n]u[n] = 3^{-n}u[n]$$

Whose fourier transform is

$$3^{-n}u[n] \to \frac{1}{1 - \frac{e^{-j\omega}}{3}}$$

(iii)

$$x[|n|] = x[-n]u[n] + x[n]u[-n]$$

from the time reversal property we have that

$$x[-n]u[n] + x[n]u[-n] \to \frac{1}{1 - \frac{e^{-j\omega}}{3}} + \frac{1}{1 - \frac{e^{j\omega}}{3}}$$

(iv)

From the time reversal property we have that

$$x[n]u[-n] \rightarrow \frac{1}{1-\frac{e^{j\omega}}{3}}$$

Problem 2

(a)

We have that

$$\sin(\Omega_0 n) = \frac{1}{2j}\left(e^{j\Omega_0 n} - e^{-j\Omega_0 n}\right)$$

And so we have that

$$X(\Omega) = \frac{\pi}{j}\left(\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)\right)$$