ECE113, Fall 2022 — **Homework #6**

Digital Signal Processing — Prof. A. Kadambi

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Due Monday, 28 Nov 2022, by 11:59pm to Gradescope.
50 points total.

Submission instructions: Generate a .pdf file for the written problems and submit to it to the assignment called HW6 on Gradescope. There is no coding for this assignment.
Note: Problems appearing in blue are practice problems and are not graded.

1. (10 points) **Compression**
   For this problem, assume we are sampling a real-valued signal at 10,000 Hz for 5 seconds. Each sample is a float datatype that is 4 bytes. After sampling the signal, we apply the DFT to the discrete signal. Our goal is to send our friend the DFT representation of our signal using as few bytes as possible.

   (a) How many bytes are needed to naively represent the time-domain signal? And how many are needed for the DFT representation of the signal? (Remember, in the frequency domain, a discrete frequency is complex, so it would take 8 bytes to represent the real and imaginary part).

   (b) Since this is a real-valued signal, is there some kind of symmetry we can take advantage of in the frequency domain so that we do not have to send all the complex frequency components? How many bytes would we need to send our friend and are there any special instructions to reconstruct the original DFT?

   (c) After analyzing the signal, you found that most of the frequency content of the signal is between 2,000 and 4,000 Hz. Can you use this information to send even less data? How many bytes would we need to send our friend and are there any special instructions to reconstruct the original DFT?

2. (10 points) **Sampling Rate**
   For this problem, we are sampling a real-valued signal for 5 seconds, but we have not determined a sampling frequency. Each sample is a float datatype that is 4 bytes. Our goal is to figure out a sampling rate such that once we get the DFT representation of the signal we use as few bytes as possible to send it to our friend.

   (a) Assume that the max frequency we would ever see in the signal is 8,000 Hz. What sampling frequency should we use and how many bytes of data would we require to naively store the time-domain and frequency-domain representation of the signal.

   (b) It turns out after some analysis of the signals we are recording, that the minimum frequency we would ever see is 4,000 Hz. Can we lower the sampling frequency while still retaining all the information present in the original signal?

3. (10 points) **FFT and Polynomial Multiplication**
   In this question we will explore how we can apply the FFT algorithm to polynomial multiplication.

(a) Express in Big-O notation what the computational complexity of polynomial multiplication is.

(b) See if you can rewrite polynomial multiplication as convolution. Is this circular or linear convolution?
(Hint: rewrite the polynomial equation into a signal whose values are the coefficients of the polynomials. Also, zero-padding will help)

(c) Show the steps needed to accelerate polynomial multiplication with FFT algorithm. Express in Big-O notation what the computational complexity of polynomial multiplication would be using this method.

4. (10 points) **DFT Properties**
Let x[n] be a finite length sequence. Its DFT is defined as, $DFT(x[n]) = [0, 1-j, 1, 1+j]$. Using the DFT properties, solve for the DFTs of the following sequences:

   (a) $y[n] = x[n]cos(\frac{\pi}{2}n)$

   (b) $y[n] = [0, 0, 1, 0] \circ x[n]$

   (c) $y[n] = g[n]x[n]$, where $g[n] = [0, 0, 1, 0]$

   (d) $y[n] = x[n-1]$

5. (10 points) **Z-transform**
Find the Z-transform of the following sequences and indicate their region of convergence. Additionally, check if the DFT exists. For these problems, a is an arbitrary constant.

   (a) $x[n] = a^n u[n]$

   (b) $x[n] = au[n]$

Practice Problems:

1. You are given a N length signal (where N is even) such that:

$$x[n] = u[n] - u[n - N/2], \quad n = 0, 1, 2, \ldots N-1 \tag{1}$$

where $u[n]$ is a unit step function defined as:

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases} \tag{2}$$

Find the sequence $y[n]$ such that its DFT is $Y[k] = X[k]^2$.

2. **CYU: DFT**

   (a) You are given a 64 point DFT of the real sequence $x[n]$. Unfortunately, the value at k=40 was accidentally changed. Can we get the original value back? If so, what was the value originally?

   (b) You sample an analog signal at a sampling frequency of 20 kHz and acquire 800 samples. You then zero-pad the signal up to N=2,000 samples. The resulting zero-padded signal is $x[n]$ and its DFT is $X[k]$.

i. What is the maximum analog frequency recoverable from these samples?

ii. What is the frequency spacing between neighboring frequency bins? (What is the difference in the analog frequencies corresponding to the indices k and k+1)

iii. What analog frequency does the index at $k = 250$ in the DFT of $x[n]$ correspond to?