ECE 113 HW 2

Lawrence Liu

October 14, 2022

Problem 1

(a)

$$x_{1_{even}}[n] = \frac{u(n-3) + u(-(n+3))}{2}$$
$$x_{1_{odd}}[n] = \frac{u(n-3) - u(-(n+3))}{2}$$

(b)

$$x_{2_{even}}[n] = \frac{\alpha^n u[n-1] + \alpha^{-n} u[-(n+1)]}{2}$$
$$x_{2_{odd}}[n] = \frac{\alpha^n u[n-1] - \alpha^{-n} u[-(n+1)]}{2}$$

(c)

$$x_{3_{even}}[n] = \frac{n\alpha^n u[n-1] - n\alpha^{-n} u[-(n+1)]}{2}$$

$$x_{2_{even}}[n] = \frac{\alpha^n u[n-1] + n\alpha^{-n}u[-(n+1)]}{2}$$

(d)

$$x_{4_{even}}[n] = x_4[n] = \alpha^{|n|}$$
$$x_{4_{odd}}[n] = 0$$

Problem 2

(a)

False, for instance, $x_n[n] = \cos(\pi n)$ is a power signal since it has a bounded power of $p(x) = \frac{1}{2}\sum_{n=0}^{1} |\cos(\pi n)|^2 = 1.$ while the energy is $\sum_{n=-\infty}^{\infty} |\cos(\pi n)|^2 = \infty$.

(b)

True, for an energy sequence we have that $\sum_{n=-\infty}^{\infty} |x_n[n]|^2 = e_x < \infty$, therefore for the power of the signal we have in general

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_n[n]|^2$$

therefore

$$\begin{aligned} P_x &= \lim_{N\to\infty} \frac{1}{2N+1} e_x \\ &= e_x \lim_{N\to\infty} \frac{1}{2N+1} \\ &= 0 \end{aligned}$$

(c)

True, first we start by proving that the energy of a signal $x[n-1]$ is the same as the energy of $x[n]$, e_x. We have that the energy of $x[n-1]$, e_{x-1} is

$$e_{x-1} = \sum_{n=-\infty}^{\infty} |x[n-1]|^2$$

Letting k=n-1 we have that

$$\begin{aligned}\sum_{n=-\infty}^{\infty} |x[n-1]|^2 &= \sum_{k=-\infty-1}^{\infty+1} |x[k]|^2 \\ &= \sum_{k=-\infty}^{\infty} |x[k]|^2 \\ &= e_x\end{aligned}$$

therefore we have that

$$\begin{aligned}e_x - e_{x-1} &= \sum_{n=-\infty}^{\infty} |x[n]|^2 - \sum_{k=-\infty-1}^{\infty+1} |x[k]|^2 \\ e_x - e_x &= |x[-\infty]|^2 + |x[\infty]|^2 \\ 0 &= |x[-\infty]|^2 + |x[\infty]|^2\end{aligned}$$

Since $|x[-\infty]|^2 \geq 0$ and $|x[\infty]|^2 \geq 0$ we have that $x[n] = 0$ as $n \to \infty$

(d)

True, let $x[n] = \sqrt{|n|}$, then we have that the power of the signal P_x is

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_n[n]|^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\sqrt{|n|}\right)^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |n| \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(1 + 2\sum_{n=1}^{N} n\right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(1 + 2\frac{N(N+1)}{2}\right) \\
&= \lim_{N\to\infty} \frac{N(N+1)+1}{2N+1} \\
&= \infty
\end{aligned}$$

Problem 3

For System 1 let $x[n] \leq B < \infty$ for all n, we have

$$y[n] = \log(|x[n-1]|)$$

since $|x[n-1]| \leq B$ for all n and since $\log(x)$ is an convex function

$$|y[n]| = |\log(|x[n-1]|)| \leq |\log(B)| < \infty$$

Therefore System I is BIBO stable. To test for time invariant, let the input $x[n-k]$ corresponds to the output y_1, then we have that

$$y_1[n] = \log(|x[(n-1)-k]|) = y[n-k]$$

therefore System I is time invariant.

For System II let $|x[n]| \leq B < \infty$ for all n, we have

$$y[n] = e^{x[2n]}$$

since $x[n] \leq B$ for all n and since e^x is an convex function, we have

$$|y[n]| = e^{x[2n]} \leq e^B < \infty$$

Therefore System II is BIBO stable. To test for time invariant, let the input $x[n-k]$ corresponds to the output y_2, then we have that

$$y_2[n] = e^{x[2n-k]}$$

this is not equal to

$$y[n-k] = e^{x[2(n-k)]}$$

therefore System II is time variant.

Thus we have that statement (a) is correct.

Problem 4

(a)

The system is not linear, let $x_1[n]$ corresponds to the output y_1, and $x_2[n]$ corresponds to the output y_2, then we have that, the output $y'[n]$ corresponding to the input $x_1[n] + x_2[n]$ is

$$y'[n] = \ln(|x_1[n-1] + x_2[n-1]| + 1)$$

which is not equal to $y_1[n] + y_2[n]$

The system is time invariant, let $x[n-k]$ corresponds to the output $y_3[n]$, then we have that

$$y_3[n] = \ln(|x[n-k-1]| + 1) = y[n-k]$$

The system is casual since it only depends on the input values at index n.

The system is BIBO stable, since given an input $x[n] \leq B < \infty$ for all n, we have that

$$|y[n]| = |\ln(|x[n-1]| + 1)| \leq |\ln(|B| + 1)| < \infty$$

The system is relaxed since given an input that goes to 0 as $n \to \infty$, the output also goes to $\ln(1) = 0$.

(b)

The system is linear, let $x_1[n]$ corresponds to the output y_1, and $x_2[n]$ corresponds to the output y_2, then we have that, the output $y'[n]$ corresponding to the input $x_1[n] + x_2[n]$ is

$$y'[n] = y'[n-1] + x[n]$$

$$y'[n] = y'[-1] + \sum_{n=0}^{n} x_1[n] + x_2[n]$$

$$y'[n] = \sum_{k=0}^{n} x_1[k] + \sum_{k=0}^{n} x_1[k]$$

which is equal to $y_1[n] + y_2[n]$

The system is time variant, let $x[n-k]$ corresponds to the output $y_3[n]$, then we have that

$$\begin{aligned} y_3[n] &= y_3[n-1] + x[n-k] \\ &= \sum_{m=0}^{n} x[m-k] \\ &= \sum_{m-=k}^{n-k} x[m] \end{aligned}$$

This is diffrent from

$$\begin{aligned} y[n-k] &= y[n-k-1] + x[n-k] \\ &= \sum_{m=0}^{n-k} x[m] \end{aligned}$$

Therefore the system is time variant.

The system is casual since it only depends on the input values at index n.

The system is not BIBO stable, since given an input $x[n] \leq B < \infty$ for all n, we have that

$$|y[n]| = |y[n-1] + x[n]| \leq \sum_{m=0}^{n} B = nB$$

This goes to ∞ as $n \to \infty$ so the system is not BIBO stable.

The system is not relaxed since it accumulates the input values over all indexs $0 \leq n$ so the output will not go to 0 as the input goes to 0 as $n \to \infty$.

(c)

The system is non linear, let $x_1[n]$ corresponds to the output y_1, and $x_2[n]$ corresponds to the output y_2, then we have that, the output $y'[n]$ corresponding to the input $x_1[n] + x_2[n]$ is

$$y'[n] = y'[n-1] + x[n]$$

$$y'[n] = y'[-1] + \sum_{n=0}^{n} x_1[n] + x_2[n]$$

$$y'[n] = 1 + \sum_{k=0}^{n} x_1[k] + \sum_{k=0}^{n} x_1[k]$$

which is not equal to $y_1[n] + y_2[n] = 2 + \sum_{k=0}^{n} x_1[k] + \sum_{k=0}^{n} x_1[k]$

The system is time variant, let $x[n-k]$ corresponds to the output $y_3[n]$, then we have that

$$\begin{aligned} y_3[n] &= y_3[n-1] + x[n-k] \\ &= 1 + \sum_{m=0}^{n} x[m-k] \\ &= 1 + \sum_{m-=k}^{n-k} x[m] \end{aligned}$$

This is diffrent from

$$\begin{aligned} y[n-k] &= y[n-k-1] + x[n-k] \\ &= 1 + \sum_{m=0}^{n-k} x[m] \end{aligned}$$

Therefore the system is time variant.

The system is casual since it only depends on the input values at index n.

The system is not BIBO stable, since given an input $x[n] \leq B < \infty$ for all n, we have that

$$|y[n]| = |y[n-1] + x[n]| \leq 1 + \sum_{m=0}^{n} B = nB$$

This goes to ∞ as $n \to \infty$ so the system is not BIBO stable.

The system is not relaxed since it accumulates the input values over all indexs $0 \leq n$ so the output will not go to 0 as the input goes to 0 as $n \to \infty$.

(d)

The system is non linear, let $x_1[n]$ corresponds to the output y_1, and $x_2[n]$ corresponds to the output y_2, then we have that, the output $y'[n]$ corresponding to the input $x_1[n] + x_2[n]$ is

$$y'[n] = 2 + x_1[n] + x_2[n]$$

which is not equal to $y_1[n] + y_2[n] = 4 + x_1[n] + x_2[n]$

The system is time invariant, let $x[n-k]$ corresponds to the output $y_3[n]$, then we have that

$$y_3[n] = 2 + x[n-k]$$

Therefore the system is time invariant since $y[n-k] = 2 + x[n-k]$.

The system is casual since it only depends on the input values at index n.

The system is BIBO stable, since given an input $x[n] \leq B < \infty$ for all n, we have that

$$|y[n]| = |2 + x[n]| \leq 2 + B < \infty$$

The system is relaxed since given an input that goes to 0 as $n \to \infty$, the output also goes to 0 since it only depends on the input at index n.

Problem 5

(a)

A system is stable if and only if the sum of the input response is stable ie that

$$S = \sum_{k=-\infty}^{\infty} |h[n]|$$

is finite.

We have

$$
\begin{aligned}
S &= \sum_{k=-\infty}^{\infty} |h[n]| \\
&= \sum k = -\infty^{-1} \left| a^k \right| \\
&= \sum_{k=-\infty}^{-1} |a|^k \\
&= \sum_{k=0}^{\infty} |a|^{-k}
\end{aligned}
$$

This is a geometric series, so if $|a| < 1$ then the sum is finite, and if $|a| > 1$ then the sum is infinite. Therefore the condition for stability is that $\boxed{|a| < 1}$.

(b)

We have that

$$
\begin{aligned}
S &= \sum_{k=-\infty}^{\infty} |h[n]| \\
&= \sum_{k=0}^{99} \left| a^k \right| \\
&= \sum_{k=0}^{99} |a|^k \\
&= \begin{cases} \frac{1-|a|^k}{1-|a|} & |a| \neq 1 \\ k & |a| = 1 \end{cases}
\end{aligned}
$$

Therefore this system is stable for $\boxed{\text{any } a}$.

(c)

We have that

$$
\begin{aligned}
S &= \sum_{k=-\infty}^{\infty} |h[n]| \\
&= \sum_{k=0}^{\infty} |r^n \sin[n\omega_0]| \\
&= \sum_{i=0}^{\frac{2\pi}{\omega_0}-1} \sum_{k=0}^{\infty} |r|^{k+i} \left|\sin[i\omega_0]\right| \\
&= \sum_{i=0}^{\frac{2\pi}{\omega_0}-1} \left|r^i \sin[i\omega_0]\right| \sum_{k=0}^{\infty} |r|^k
\end{aligned}
$$

Therefore the system is stable if either the geometric series converges, so $\boxed{|r| < 1}$, or if $\sin[n\omega_0] = 0$ for all n, so if $\boxed{\omega_0 = 0}$

(d)

We have that

$$
\begin{aligned}
S &= \sum_{k=-\infty}^{\infty} |h[n]| \\
&= \left(2 \sum_{k=0}^{\infty} |r^n|\right) - 1 \\
&= \left(2 \sum_{k=0}^{\infty} |r|^k\right) - 1
\end{aligned}
$$

Therefore the system will be stable if $\boxed{a < 1}$

(e)

We have that

$$\begin{aligned}
S &= \sum_{k=-\infty}^{\infty} |h[n]| \\
&= \sum_{k=0}^{\infty} |K(-1)^n| \\
&= |K| \sum_{k=0}^{\infty} |(-1)^n| \\
&= |K| \sum_{k=0}^{\infty} 1
\end{aligned}$$

Therefore the system will be stable if $\boxed{K = 0}$