

# ECE 113 HW 2

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October 13, 2022

## Problem 1

(a)

$$x_{1_{even}}[n] = \frac{u(n-3) + u(-(n+3))}{2}$$
$$x_{1_{odd}}[n] = \frac{u(n-3) - u(-(n+3))}{2}$$

(b)

$$x_{2_{even}}[n] = \frac{\alpha^n u[n-1] + \alpha^{-n} u[-(n+1)]}{2}$$
$$x_{2_{odd}}[n] = \frac{\alpha^n u[n-1] - \alpha^{-n} u[-(n+1)]}{2}$$

(c)

$$x_{3_{even}}[n] = \frac{n\alpha^n u[n-1] - n\alpha^{-n} u[-(n+1)]}{2}$$

$$x_{2_{even}}[n] = \frac{\alpha^n u[n-1] + n\alpha^{-n} u[-(n+1)]}{2}$$

(d)

$$\begin{aligned} x_{4_{even}}[n] &= x_4[n] = \alpha^{|n|} \\ x_{4_{odd}}[n] &= 0 \end{aligned}$$

## Problem 2

(a)

False, for instance,  $x_n[n] = \cos(\pi n)$  is a power signal since it has a bounded power of  $p(x) = \frac{1}{2} \sum_{n=0}^1 |\cos(\pi n)|^2 = 1$ . while the energy is  $\sum_{n=-\infty}^{\infty} |\cos(\pi n)|^2 = \infty$ .

(b)

True, for an energy sequence we have that  $\sum_{n=-\infty}^{\infty} |x_n[n]|^2 = e_x < \infty$ , therefore for the power of the signal we have in general

$$P_x = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x_n[n]|^2$$

therefore

$$\begin{aligned} P_x &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} e_x \\ &= e_x \lim_{N \rightarrow \infty} \frac{1}{2N+1} \\ &= 0 \end{aligned}$$

(c)

True, first we start by proving that the energy of a signal  $x[n-1]$  is the same as the energy of  $x[n]$ ,  $e_x$ . We have that the energy of  $x[n-1]$ ,  $e_{x-1}$  is

$$e_{x-1} = \sum_{n=-\infty}^{\infty} |x[n-1]|^2$$

Letting  $k=n-1$  we have that

$$\begin{aligned} \sum_{n=-\infty}^{\infty} |x[n-1]|^2 &= \sum_{k=-\infty-1}^{\infty+1} |x[k]|^2 \\ &= \sum_{k=-\infty}^{\infty} |x[k]|^2 \\ &= e_x \end{aligned}$$

therefore we have that

$$\begin{aligned} e_x - e_{x-1} &= \sum_{n=-\infty}^{\infty} |x[n]|^2 - \sum_{k=-\infty-1}^{\infty+1} |x[k]|^2 \\ e_x - e_x &= |x[-\infty]|^2 + |x[\infty]|^2 \\ 0 &= |x[-\infty]|^2 + |x[\infty]|^2 \end{aligned}$$

Since  $|x[-\infty]|^2 \geq 0$  and  $|x[\infty]|^2 \geq 0$  we have that  $x[n] = 0$  as  $n \rightarrow \infty$

(d)

True, let  $x[n] = \sqrt{|n|}$ , then we have that the power of the signal  $P_x$  is

$$\begin{aligned} P_x &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |x_n[n]|^2 \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N \left( \sqrt{|n|} \right)^2 \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{n=-N}^N |n| \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left( 1 + 2 \sum_{n=1}^N n \right) \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \left( 1 + 2 \frac{N(N+1)}{2} \right) \\ &= \lim_{N \rightarrow \infty} \frac{N(N+1) + 1}{2N+1} \\ &= \infty \end{aligned}$$

### Problem 3

For System 1 let  $x[n] \leq B < \infty$  for all  $n$ , we have

$$y[n] = \log(|x[n-1]|)$$

since  $|x[n-1]| \leq B$  for all  $n$  and since  $\log(x)$  is an convex function

$$|y[n]| = |\log(|x[n-1]|)| \leq |\log(B)| < \infty$$

Therefore System I is BIBO stable. To test for time invariant, let the input  $x[n-k]$  corresponds to the output  $y_1$ , then we have that

$$y_1[n] = \log(|x[(n-1)-k]|) = y[n-k]$$

therefore System I is time invariant.

For System II let  $|x[n]| \leq B < \infty$  for all  $n$ , we have

$$y[n] = e^{x[2n]}$$

since  $x[n] \leq B$  for all  $n$  and since  $e^x$  is an convex function, we have

$$|y[n]| = e^{x[2n]} \leq e^B < \infty$$

Therefore System II is BIBO stable. To test for time invariant, let the input  $x[n - k]$  corresponds to the output  $y_2$ , then we have that

$$y_2[n] = e^{x[2n-k]}$$

this is not equal to

$$y[n - k] = e^{x[2(n-k)]}$$

therefore System II is time variant.

Thus we have that statement (a) is correct.

## Problem 4

(a)

The system is not linear, let  $x_1[n]$  corresponds to the output  $y_1$ , and  $x_2[n]$  corresponds to the output  $y_2$ , then we have that, the output  $y'[n]$  corresponding to the input  $x_1[n] + x_2[n]$  is

$$y'[n] = \ln(|x_1[n - 1] + x_2[n - 1]| + 1)$$

which is not equal to  $y_1[n] + y_2[n]$

The system is time invariant, let  $x[n - k]$  corresponds to the output  $y_3[n]$ , then we have that

$$y_3[n] = \ln(|x[n - k - 1]| + 1) = y[n - k]$$

The system is casual since it only depends on the input values at index  $n$ .

The system is BIBO stable, since given an input  $x[n] \leq B < \infty$  for all  $n$ , we have that

$$|y[n]| = |\ln(|x[n-1]| + 1)| \leq |\ln(|B| + 1)| < \infty$$

The system is relaxed since given an input that goes to 0 as  $n \rightarrow \infty$ , the output also goes to  $\ln(1) = 0$ .

**(b)**

The system is linear, let  $x_1[n]$  corresponds to the output  $y_1$ , and  $x_2[n]$  corresponds to the output  $y_2$ , then we have that, the output  $y'[n]$  corresponding to the input  $x_1[n] + x_2[n]$  is

$$y'[n] = y'[n-1] + x[n]$$

$$y'[n] = y'[-1] + \sum_{n=0}^n x_1[n] + x_2[n]$$

$$y'[n] = \sum_{k=0}^n x_1[k] + \sum_{k=0}^n x_2[k]$$

which is equal to  $y_1[n] + y_2[n]$

The system is time variant, let  $x[n-k]$  corresponds to the output  $y_3[n]$ , then we have that

$$\begin{aligned} y_3[n] &= y_3[n-1] + x[n-k] \\ &= \sum_{m=0}^n x[m-k] \\ &= \sum_{m=-k}^{n-k} x[m] \end{aligned}$$

This is different from

$$\begin{aligned} y[n-k] &= y[n-k-1] + x[n-k] \\ &= \sum_{m=0}^{n-k} x[m] \end{aligned}$$

Therefore the system is time variant.

The system is casual since it only depends on the input values at index  $n$ .

The system is not BIBO stable, since given an input  $x[n] \leq B < \infty$  for all  $n$ , we have that

$$|y[n]| = |y[n-1] + x[n]| \leq \sum_{m=0}^n B = nB$$

This goes to  $\infty$  as  $n \rightarrow \infty$  so the system is not BIBO stable.

The system is not relaxed since it accumulates the input values over all indexes  $0 \leq n$  so the output will not go to 0 as the input goes to 0 as  $n \rightarrow \infty$ .

(c)

The system is non linear, let  $x_1[n]$  corresponds to the output  $y_1$ , and  $x_2[n]$  corresponds to the output  $y_2$ , then we have that, the output  $y'[n]$  corresponding to the input  $x_1[n] + x_2[n]$  is

$$\begin{aligned} y'[n] &= y'[n-1] + x[n] \\ y'[n] &= y'[-1] + \sum_{n=0}^n x_1[n] + x_2[n] \\ y'[n] &= 1 + \sum_{k=0}^n x_1[k] + \sum_{k=0}^n x_2[k] \end{aligned}$$

which is not equal to  $y_1[n] + y_2[n] = 2 + \sum_{k=0}^n x_1[k] + \sum_{k=0}^n x_1[k]$

The system is time variant, let  $x[n - k]$  corresponds to the output  $y_3[n]$ , then we have that

$$\begin{aligned} y_3[n] &= y_3[n - 1] + x[n - k] \\ &= 1 + \sum_{m=0}^n x[m - k] \\ &= 1 + \sum_{m=-k}^{n-k} x[m] \end{aligned}$$

This is different from

$$\begin{aligned} y[n - k] &= y[n - k - 1] + x[n - k] \\ &= 1 + \sum_{m=0}^{n-k} x[m] \end{aligned}$$

Therefore the system is time variant.

The system is casual since it only depends on the input values at index  $n$ .

The system is not BIBO stable, since given an input  $x[n] \leq B < \infty$  for all  $n$ , we have that

$$|y[n]| = |y[n - 1] + x[n]| \leq 1 + \sum_{m=0}^n B = nB$$

This goes to  $\infty$  as  $n \rightarrow \infty$  so the system is not BIBO stable.

The system is not relaxed since it accumulates the input values over all indexes  $0 \leq n$  so the output will not go to 0 as the input goes to 0 as  $n \rightarrow \infty$ .

**(d)**

The system is non linear, let  $x_1[n]$  corresponds to the output  $y_1$ , and  $x_2[n]$  corresponds to the output  $y_2$ , then we have that, the output  $y'[n]$  correspond-



ing to the input  $x_1[n] + x_2[n]$  is

$$y'[n] = 2 + x_1[n] + x_2[n]$$

which is not equal to  $y_1[n] + y_2[n] = 4 + x_1[n] + x_2[n]$

The system is time invariant, let  $x[n - k]$  corresponds to the output  $y_3[n]$ , then we have that

$$y_3[n] = 2 + x[n - k]$$

Therefore the system is time invariant since  $y[n - k] = 2 + x[n - k]$ .

The system is casual since it only depends on the input values at index  $n$ .

The system is BIBO stable, since given an input  $x[n] \leq B < \infty$  for all  $n$ , we have that

$$|y[n]| = |2 + x[n]| \leq 2 + B < \infty$$

The system is relaxed since given an input that goes to 0 as  $n \rightarrow \infty$ , the output also goes to 0 since it only depends on the input at index  $n$ .

## Problem 5