ECE113, Fall 2022 **Homework #4**

Digital Signal Processing — Prof. A. Kadambi

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Due Friday, 11 Nov 2022, by 11:59pm to Gradescope.
50 points total.

Note: Unless specified, you are free to use any of the properties of DTFT that were taught in class. Also, all repeated derivations can be referenced with appropriate equation/result numbers.

1. (10 points) Let $x[n] = 3^n$.
   a.) Show that the DTFT does not exist.
   b.) Evaluate whether the DTFT exists for the following modifications to $x[n]$. If they exist, compute DTFT:
      i.) $x[n]\ u[n]$
      ii.) $x[-n]\ u[n]$
      iii.) $x[-|n|\ ]$
      iv.) $x[n]\ u[-n]$
2. (10 points) Let $x[n] = sin(\Omega_o n)$.
   (a) Derive the DTFT of $x[n]$.
   (b) Write the real and imaginary part of $X(\Omega)$ as well as its magnitude and phase.
3. (10 points) Determine the periodic convolution of the following sequences:
   (a) $\widetilde{x}[n] \otimes \widetilde{x}[n]$, where $\widetilde{x}[n] = [1, 1, 1, 1, 0]$ with a periodicity of $N = 5$.
   (b) $\widetilde{x}[n] \otimes \widetilde{y}[n]$, where $\widetilde{x}[n] = [1, -1, 1, -1, 1, -1]$ and $\widetilde{y}[n] = [1, 1, -1, -1, 1, 1]$ with a periodicity of $N = 6$.
4. (10 points) Derive the modulation property, $x[n]cos(\Omega_o n) \xrightarrow{\mathcal{F}} \frac{1}{2}(X(\Omega - \Omega_o) + X(\Omega + \Omega_o))$. Then determine the DTFT of the following signal, $x[n] = 0.2^n cos(\Omega_o n)u[n]$ using the DTFT Table Properties.
5. (10 points) Given the following difference equation:

$$y[n] + \frac{4}{9}y[n-1] + \frac{1}{27}y[n-2] = x[n] - \frac{8}{9}x[n-1] - \frac{1}{3}x[n-2], \quad (1)$$

   where $x[n]$ is the input and $y[n]$ is the output, determine the impulse response of the system, i.e. h[n].