ECE 113 HW 2

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Problem 1

(a)

The discrete fourier transform doesn't exist since we have

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \\ &= \sum_{n=0}^{2\frac{1}{\omega}-1} 3^n e^{-j\omega n} \sum_{k=-\infty}^{\infty} 3^{\frac{2k}{\omega}} \end{aligned}$$

Since this second sum does not converge since $3 > 1$, this means that the DTFT doesn't exist.

(b)

(i)

This DTFT doesn't exist since once again we can write the fourier transform as

$$X(\omega) = \sum_{n=0}^{2\frac{1}{\omega}-1} 3^n e^{-j\omega n} \sum_{k=0}^{\infty} 3^{\frac{2k}{\omega}}$$

And once again this second sum does not converge since $3 > 1$.

(ii)

This does exist, since we can write

$$X[-n]u[n] = 3^{-n}u[n]$$

Whose fourier transform is

$$3^{-n}u[n] \to \frac{1}{1 - \frac{e^{-j\omega}}{3}}$$

(iii)

$$x[|n|] = x[-n]u[n] + x[n]u[-n] - \delta[n]$$

from the time reversal property we have that

$$x[-n]u[n] + x[n]u[-n] - \delta[n] \to \frac{1}{1 - \frac{e^{-j\omega}}{3}} + \frac{1}{1 - \frac{e^{j\omega}}{3}} - 1$$

(iv)

From the time reversal property we have that

$$x[n]u[-n] \rightarrow \frac{1}{1-\frac{e^{j\omega}}{3}}$$

Problem 2

(a)

We have that

$$\sin(\Omega_0 n) = \frac{1}{2j}\left(e^{j\Omega_0 n} - e^{-j\Omega_0 n}\right)$$

And so we have that

$$X(\Omega) = \frac{\pi}{j}\left(\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)\right)$$

(b)

We have that

$$|X(\Omega)| = \pi\left(\delta(\Omega-\Omega_0) + \delta(\Omega+\Omega_0)\right)$$

and

$$\angle X(\Omega) = \frac{\pi}{2}\left(\delta(\Omega-\Omega_0) - \delta(\Omega+\Omega_0)\right)$$

Problem 3

(a)

$$\tilde{x}[n] \bigotimes \tilde{x}[n] = [3,3,3,4,3]$$

(b)

$$\tilde{x}[n] \bigotimes \tilde{y}[n] = [0, 0, 0, 0, 0, 0]$$

Problem 4

We have that

$$\begin{aligned} x[n]\cos(\Omega_0 n) &= \frac{x[n]}{2}\left(e^{j\Omega_0 n} + e^{-j\Omega_0 n}\right) \\ &\to \frac{1}{2}\left(X(\Omega - \Omega_0) + X(\Omega + \Omega_0)\right) \end{aligned}$$

Problem 5

We have that taking the fourier transform of the diffrence equation

$$Y(\omega)(1 + \frac{4}{9}e^{-j\omega} + \frac{1}{27}e^{-2j\omega}) = X(\omega)(1 - \frac{8}{9}e^{-j\omega} + \frac{1}{3}e^{-2j\omega})$$

Since $x[n]$ is the delta function we have that $X(\omega) = 1$ and thus we have

$$\begin{aligned} Y(\omega) &= \frac{3(9 - 8e^{-j\omega} - 3e^{-2j\omega})}{(3 + e^{-j\omega})(9 + e^{-j\omega})} \\ &= 3\left(\frac{1}{3 + e^{-j\omega}} - \frac{3e^{-j\omega}}{9 + e^{-j\omega}}\right) \\ &\to \boxed{(-3)^n u[n] - (-9)^{n-1} u[n-1]} \end{aligned}$$