ECE 113 HW 2

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Problem 1

We have that

$$h[-n] = \{4, -6, 5 - 3\}, -1 \leq n \leq 2$$

Thus

$$h[-n-1] = \{4, -6, 5 - 3\}, -2 \leq n \leq 1$$

So $y[-1] = -12 - 15 - 12 = \boxed{-39}$

Problem 2

(a)

$$x_1[n] * x_1[n] = \boxed{\{1, -2, 3, -2, 1\}, -2 \leq n \leq 2}$$

(b)

$$x_2[n] * x_2[n] = \boxed{\{1, -2, 1, 2, -4, 2, 1, -2, 1\}, 0 \leq n \leq 8}$$

(c)

$$x_3[n] * x_3[n] = \boxed{\{1, -4, 4, 4, -10, 4, 4, -4, 1\}, -6 \leq n \leq 2}$$

Problem 3

We have that the output $y[n]$ is

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} 2^k u[k] 2^{-(n-k)} u[n-k] \end{aligned}$$

Therefore in the case that $n < 0$ we have

$$y[n] = \sum_{k=-\infty}^{n} 2^{2k-n}$$

$$y[n] = 2^{-n} \sum_{j=-n}^{\infty} 2^{-2j}$$

$$y[n] = 2^{n} \sum_{i=0}^{\infty} 2^{-2i}$$

$$y[n] = 2^{n} \frac{1}{1-2^{-2}}$$

$$y[n] = 2^{n+2} \frac{1}{3}$$

In the case that $n \geq 0$ we have

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{0} 2^{2k-n} \\ &= 2^{-n} \sum_{j=0}^{\infty} 2^{-2j} \\ &= 2^{-n} \frac{1}{1-2^{-2}} \\ &= 2^{-n+2} \frac{1}{3} \end{aligned}$$

Therefore we have

$$y[n] = \boxed{\begin{cases} 2^{n+2}\frac{1}{3} & n < 0 \\ 2^{-n+2}\frac{1}{3} & n \geq 0 \end{cases}}$$