

# ECE 113 HW 2

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## Problem 1

We have that

$$h[-n] = \{4, -6, 5 - 3\}, -1 \leq n \leq 2$$

Thus

$$h[-n - 1] = \{4, -6, 5 - 3\}, -2 \leq n \leq 1$$

$$\text{So } y[-1] = -12 - 15 - 12 = \boxed{-39}$$

## Problem 2

(a)

$$x_1[n] * x_1[n] = \boxed{\{1, -2, 3, -2, 1\}, -2 \leq n \leq 2}$$

(b)

$$x_2[n] * x_2[n] = \boxed{\{1, -2, 1, 2, -4, 2, 1, -2, 1\}, 0 \leq n \leq 8}$$

(c)

$$x_3[n] * x_3[n] = \boxed{\{1, -4, 4, 4, -10, 4, 4, -4, 1\}, -6 \leq n \leq 2}$$

### Problem 3

We have that the output  $y[n]$  is

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} 2^k u[k] 2^{-(n-k)} u[n-k] \end{aligned}$$

Therefore in the case that  $n < 0$  we have

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^n 2^{2k-n} \\ y[n] &= 2^{-n} \sum_{j=-n}^{\infty} 2^{-2j} \\ y[n] &= 2^n \sum_{i=0}^{\infty} 2^{-2i} \\ y[n] &= 2^n \frac{1}{1-2^{-2}} \\ y[n] &= 2^{n+2} \frac{1}{3} \end{aligned}$$

In the case that  $n \geq 0$  we have

$$\begin{aligned}
 y[n] &= \sum_{k=-\infty}^0 2^{2k-n} \\
 &= 2^{-n} \sum_{j=0}^{\infty} 2^{-2j} \\
 &= 2^{-n} \frac{1}{1 - 2^{-2}} \\
 &= 2^{-n+2} \frac{1}{3}
 \end{aligned}$$

Therefore we have

$$y[n] = \boxed{\begin{cases} 2^{n+2} \frac{1}{3} & n < 0 \\ 2^{-n+2} \frac{1}{3} & n \geq 0 \end{cases}}$$

## Problem 4

(a)

$$x[n] * y[n] = \boxed{\{3, 5, 6, 7, 7, 8, 9, 9, 7, 4, 2, 1\}, -3 \leq n \leq 8}$$

(b)

Then the response  $y_2[n]$  would be

$$\begin{aligned} y_2[n] &= h_1[n] * \left( \left( \frac{1}{3} \right)^n u[n] \right) \\ &= \frac{1}{2^{n+1}} x[n+2] u[n] * \left( \left( \frac{1}{3} \right)^n u[n] \right) - \frac{3}{2} \delta[n] u[n] * \left( \left( \frac{1}{3} \right)^n u[n] \right) \\ &\quad + \frac{1}{2^n} u[n-3] u[n] * \left( \left( \frac{1}{3} \right)^n u[n] \right) \end{aligned}$$

We have that  $x[n+2] * \left( \left( \frac{1}{3} \right)^n u[n] \right) = 0$  if  $n < 0$  and for  $n \geq 0$  we have:

$$\begin{aligned} \frac{1}{2^{(n+1)}} x[n+2] u[n] * \left( \left( \frac{1}{3} \right)^n u[n] \right) &= \sum_{k=-\infty}^{\infty} \frac{1}{2^{(k+1)}} x[k+2] u[k] \left( \left( \frac{1}{3} \right)^{n-k} u[n-k] \right) \\ &= \left( \frac{1}{3} \right)^n \sum_{k=0}^n 2^{-k-1} 3^k x[k+2] \end{aligned}$$

Likewise we have that  $\delta[n] * \left( \left( \frac{1}{3} \right)^n u[n] \right) = \left( \frac{1}{3} \right)^n u[n]$ , and for  $(2^{-n} u[n-3]) * (3^{-n} u[n])$  we get that for  $n < 3$  the sequence is 0 and otherwise we have

$$\begin{aligned} (2^{-n} u[n-3]) * (3^{-n} u[n]) &= \sum_{k=-\infty}^{\infty} 2^{-k} u[k-3] 3^{-k+n} u[n-k] \\ &= \sum_{k=3}^n 2^{-k} 3^{k-n} \\ &= 3^{-n} \sum_{k=3}^n \left( \frac{3}{2} \right)^k \\ &= 3^{-n} 2 \left( \left( \frac{3}{2} \right)^n - \left( \frac{3}{2} \right)^2 \right) \end{aligned}$$

Therefore we have that

$$y_2[n] = \begin{cases} 0 & n < 0 \\ -\frac{3}{2} \left( \frac{1}{3} \right)^n + 3^{-n} \sum_{k=0}^n 2^{-k-1} 3^k x[k+2] & 0 \leq n < 3 \\ -\frac{3}{2} \left( \frac{1}{3} \right)^n + 3^{-n} 2 \left( \left( \frac{3}{2} \right)^n - \left( \frac{3}{2} \right)^2 \right) + 3^{-n} \sum_{k=0}^4 2^{-k-1} 3^k x[k+2] & n \geq 3 \end{cases}$$

## Problem 5

(a)

The first index is the index  $n$  that satisfies the condition  $n - n_1 - n_3 = 0$ , thus we get that the first nonzero element occurs at  $\boxed{n_1 + n_3}$

(b)

The last index is the index  $n$  that satisfies the condition  $n - (n_2 - 1) - (n_4 - 1) = 0$ , thus we get that the last nonzero element occurs at  $\boxed{n_2 + n_4 - 2}$

(c)

Therefore the length of the output sequence is  $n_2 + n_4 - 2 - (n_1 + n_3) = \boxed{n_x + n_h - 2}$

(c)