# ECE 113 HW 2

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## Problem 1

### (a)

$$x_{1_{even}}[n] = \frac{u(n-3) + u(-(n+3))}{2}$$
$$x_{1_{odd}}[n] = \frac{u(n-3) - u(-(n+3))}{2}$$

### (b)

$$x_{2_{even}}[n] = \frac{\alpha^n u[n-1] + \alpha^{-n} u[-(n+1)]}{2}$$
$$x_{2_{odd}}[n] = \frac{\alpha^n u[n-1] - \alpha^{-n} u[-(n+1)]}{2}$$

### (c)

$$x_{3_{even}}[n] = \frac{n\alpha^n u[n-1] - n\alpha^{-n} u[-(n+1)]}{2}$$

$$x_{2_{even}}[n] = \frac{\alpha^n u[n-1] + n\alpha^{-n} u[-(n+1)]}{2}$$

### (d)

$$x_{4_{even}}[n] = x_4[n] = \alpha^{|n|}$$
$$x_{4_{odd}}[n] = 0$$

## Problem 2

### (a)

False, for instance, $x_n[n] = \cos(\pi n)$ is a power signal since it has a bounded power of $p(x) = \frac{1}{2}\sum_{n=0}^{1} |\cos(\pi n)|^2 = 1.$ while the energy is $\sum_{n=-\infty}^{\infty} |\cos(\pi n)|^2 = \infty$.

### (b)

True, for an energy sequence we have that $\sum_{n=-\infty}^{\infty} |x_n[n]|^2 = e_x < \infty$, therefore for the power of the signal we have in general

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_n[n]|^2$$

therefore

$$\begin{aligned} P_x &= \lim_{N\to\infty} \frac{1}{2N+1} e_x \\ &= e_x \lim_{N\to\infty} \frac{1}{2N+1} \\ &= 0 \end{aligned}$$

### (c)

True, first we start by proving that the energy of a signal $x[n-1]$ is the same as the energy of $x[n]$, $e_x$. We have that the energy of $x[n-1]$, $e_{x-1}$ is

$$e_{x-1} = \sum_{n=-\infty}^{\infty} |x[n-1]|^2$$

Letting k=n-1 we have that

$$\begin{aligned} \sum_{n=-\infty}^{\infty} |x[n-1]|^2 &= \sum_{k=-\infty-1}^{\infty+1} |x[k]|^2 \\ &= \sum_{k=-\infty}^{\infty} |x[k]|^2 \\ &= e_x \end{aligned}$$

therefore we have that

$$\begin{aligned} e_x - e_{x-1} &= \sum_{n=-\infty}^{\infty} |x[n]|^2 - \sum_{k=-\infty-1}^{\infty+1} |x[k]|^2 \\ e_x - e_x &= |x[-\infty]|^2 + |x[\infty]|^2 \\ 0 &= |x[-\infty]|^2 + |x[\infty]|^2 \end{aligned}$$

Since $|x[-\infty]|^2 \geq 0$ and $|x[\infty]|^2 \geq 0$ we have that $x[n] = 0$ as $n \to \infty$

### (d)

True, let $x[n] = \sqrt{|n|}$, then we have that the power of the signal $P_x$ is

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x_n[n]|^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\sqrt{|n|}\right)^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |n| \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(1 + 2\sum_{n=1}^{N} n\right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(1 + 2\frac{N(N+1)}{2}\right) \\
&= \lim_{N\to\infty} \frac{N(N+1)+1}{2N+1} \\
&= \infty
\end{aligned}$$

## Problem 3

For System 1 let $x[n] \leq B < \infty$ for all $n$, we have

$$y[n] = \log(|x[n-1]|)$$

since $x[n-1] \leq B$ for all $n$ and since $\log(x)$ is an convex function

$$y[n] = \log(|x[n-1]|) \leq \log(B) < \infty$$

Therefore System I is BIBO stable. To test for time invariant, let the input $x[n-k]$ corresponds to the output $y_1$, then we have that

$$y_1[n] = \log(|x[(n-1)-k]|) = y[n-k]$$

therefore System I is time invariant.

For System II let $x[n] \leq B < \infty$ for all $n$, we have

$$y[n] = e^{x[2n]}$$

since $x[n] \leq B$ for all $n$ and since $e^x$ is an convex function, we have

$$y[n] = e^{x[2n]} \leq e^B < \infty$$

Therefore System II is BIBO stable. To test for time invariant, let the input $x[n-k]$ corresponds to the output $y_2$, then we have that

$$y_2[n] = e^{x[2n-k]}$$

this is not equal to

$$y[n-k] = e^{x[2(n-k)]}$$

therefore System II is time variant.

Thus we have that statement (a) is correct.