Signals

A discrete time signal can be described as a mathematical function $x[n]$ where n is the index of the sample. Or as an Array/List of the significant samples, with any sample not listed being 0. Or by plotting. We can modify a signal by multiplying it by something or adding something to it. We can also modify the signal by time shifting it: ie $X(t-3)$ will delay the signal by shifting it to the right, and $X(t+3)$ will advance the signal by shifting it to the left. Likewise we can multiply the signal by a constant $c > 1$ will effectively "downsample" the signal, and multiplying it by $c < 1$ will "upsample" the signal. Likewise we can time reverse the signal.

Delta and Unit Step Signals

We define the **delta signal** as $\boxed{\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}}$. From this we have the following sampling property: $\boxed{x[n] \cdot \delta[n-k] = x[k]\delta[n-k]}$.

From this delta signal we can define the **unit step** signal $\boxed{u[n] = \sum_{k=0}^{\infty} \delta[n-k] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}}$.

Periodicity

A signal is periodic if it can be written as $x[n] = x[n+N]$ for some integer N and all n. A signal's fundamental period is the smallest integer N such that $x[n] = x[n+N]$ for all n.

Even and Odd Signals

A signal is even if $x[n] = x[-n]$. A signal is odd if $x[n] = -x[-n]$. We can decompose any singal into its even and odd parts with the even part $x_e[n] = \frac{1}{2}(x[n] + x[-n])$ and the odd part $x_o[n] = \frac{1}{2}(x[n] - x[-n])$.

Energy and Power Signals

We define the energy of a signal as $\boxed{E = \sum_{n=-\infty}^{\infty} |x[n]|^2}$. We define the power of a signal as $\boxed{P = \lim_{m\to\infty} \frac{1}{2m+1} \sum_{n=-\infty}^{M} |x[n]|^2}$. If the signal is periodic, this can be simplified to $\boxed{P = \frac{1}{N}\sum_{n=0}^{N-1} |x[n]|^2}$. We call a signal a **energy signal** if its energy is finite, and a **power signal** if its power is finite. A power signal will have infinite energy.

System Properties

A system is **linear** if given the outputs $y_1[n]$, and $y_2[n]$ of two inputs $x_1[n]$ and $x_2[n]$ respectively, then the output of the input $x_3 = \alpha x_1[n] + \beta x_2[n]$ the output is $y_3[n] = \alpha y_1[n] + \beta y_2[n]$. A system is **time invariant** if given the output $y[n]$ of an input $x[n]$, then the output of the input $x[n-k]$ is $y[n-k]$. A system is **causal** if the output of an input $x[n]$ is only dependent on the input $x[n]$ and not $x[n-k]$ for $k < 0$. A system is **stable** if the output of an input $x[n] < \alpha < \infty$ for all n, the output $y[n] < \beta < \infty$ for all n. A system is **relaxed** if $y[n] \to 0$ for $n \to \infty$ and 0 when the input is 0. In general for a system that is lti, given its impulse response $h[n]$ (the system response to $\delta[n]$), the output of an input $x[n]$ is given by $y[n] = x[n] * h[n]$ and the system will be stable if and only if $\sum_{k=-\infty}^{\infty} |h[k]| < \infty$.

Convolution

We define the convolution of two signals $x[n]$ and $h[n]$ as $\boxed{y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]}$. A shortcut on how to do convolution is shown below:

We have that if given two signals $x_1[n]$ and $x_2[n]$ that start at n_1 and n_2 respectively and have lengths of L_1 and L_2 respectively, then the convolution of the two signals will have nonzeros values for $n_1 + n_2 \leq n \leq n_1 + n_2 + L_1 + L_2$, and thus the convolution will have $L_1 + L_2 + 1$ nonzero values. $u[n] * u[n] = r[n+1]$ where $r[n] = nu[n]$ is the unit ramp signal.

➲ The "n" dependency of $y[n]$ deserves some care: for each value of "n" the convolution sum must be computed ***separately*** over all values of a dummy variable "m". So, for each "n"

1. Rename the independent variable as m. You now have $x[m]$ and $h[m]$. Flip $h[m]$ over the origin. This is $h[-m]$
2. Shift $h[-m]$ as far left as possible to a point "n", where the two signals barely touch. This is $h[n-m]$
3. Multiply the two signals and sum over all values of m. This is the convolution sum for the specific "n" picked above.
4. Shift / move $h[-m]$ to the right by one sample, and obtain a new $h[n-m]$. Multiply and sum over all m.
5. Repeat 2~4 until $h[n-m]$ no longer overlaps with $x[m]$, i.e., shifted out of the $x[m]$ zone.

Discrete Time Fourier Transform

If a signal is periodic with period N, then we have that we can represnt it as a discrete fourier series, $\boxed{x[n] = \frac{1}{N}\sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn}}$.

Where c_k is derived from $\boxed{c_k = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}}$.

Aditional Notes

conjugate symmetric: $x^*[n] = x[-n]$ **conjugate anti-symmetric:** $x^*[n] = -x[-n]$. $\boxed{\sum_{k=1}^{n} ar^{k-1} = \begin{cases} \frac{a(1-r^n)}{1-r} & r \neq 1 \\ an & r = 1 \end{cases}}$.

The sum of an arithmetic sequence consisting of n values is $\boxed{\frac{n(a_1 + a_n)}{2}}$