ECE 113 HW 2

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Problem 1

We have that
$$h[-n] = \{4, -6, 5 - 3\}, -1 \leq n \leq 2$$
Thus
$$h[-n-1] = \{4, -6, 5 - 3\}, -2 \leq n \leq 1$$
So $y[-1] = -12 - 15 - 12 = \boxed{-39}$

Problem 2

(a)

$$x_1[n] * x_1[n] = \boxed{\{1, -2, 3, -2, 1\}, -2 \leq n \leq 2}$$

(b)

$$x_2[n] * x_2[n] = \boxed{\{1, -2, 1, 2, -4, 2, 1, -2, 1\}, 0 \leq n \leq 8}$$

(c)

$$x_3[n] * x_3[n] = \boxed{\{1, -4, 4, 4, -10, 4, 4, -4, 1\}, -6 \leq n \leq 2}$$

Problem 3

We have that the output $y[n]$ is

$$\begin{aligned} y[n] &= h[n] * x[n] \\ &= \sum_{k=-\infty}^{\infty} h[k]x[n-k] \\ &= \sum_{k=-\infty}^{\infty} 2^k u[k] 2^{-(n-k)} u[n-k] \end{aligned}$$

Therefore in the case that $n < 0$ we have

$$y[n] = \sum_{k=-\infty}^{n} 2^{2k-n}$$

$$y[n] = 2^{-n} \sum_{j=-n}^{\infty} 2^{-2j}$$

$$y[n] = 2^n \sum_{i=0}^{\infty} 2^{-2i}$$

$$y[n] = 2^n \frac{1}{1-2^{-2}}$$

$$y[n] = 2^{n+2} \frac{1}{3}$$

In the case that $n \geq 0$ we have

$$\begin{aligned} y[n] &= \sum_{k=-\infty}^{0} 2^{2k-n} \\ &= 2^{-n} \sum_{j=0}^{\infty} 2^{-2j} \\ &= 2^{-n} \frac{1}{1-2^{-2}} \\ &= 2^{-n+2} \frac{1}{3} \end{aligned}$$

Therefore we have

$$y[n] = \boxed{\begin{cases} 2^{n+2}\frac{1}{3} & n < 0 \\ 2^{-n+2}\frac{1}{3} & n \geq 0 \end{cases}}$$

Problem 4

(a)

$$x[n] * y[n] = \boxed{\{3, 5, 6, 4, 5, 7, 8, 6, 4, 2, 1\}, -3 \leq n \leq 7}$$

(b)

Then the response $y_2[n]$ would be

$$\begin{aligned}
y_2[n] =& h_1[n] * \left(\left(\frac{1}{3}\right)^n u[n]\right) \\
=& \frac{1}{2^{n+1}} x[n+2]u[n] * \left(\left(\frac{1}{3}\right)^n u[n]\right) - \frac{3}{2}\delta[n]u[n] * \left(\left(\frac{1}{3}\right)^n u[n]\right) \\
&+ \frac{1}{2^n} u[n-3]u[n] * \left(\left(\frac{1}{3}\right)^n u[n]\right)
\end{aligned}$$

We have that $x[n+2] * \left(\left(\frac{1}{3}\right)^n u[n]\right) = 0$ if $n < 0$ and for $n \geq 0$ we have:

$$\begin{aligned}
\frac{1}{2(n+1)} x[n+2]u[n] * \left(\left(\frac{1}{3}\right)^n u[n]\right) &= \sum_{k=-\infty}^{\infty} \frac{1}{2^{(k+1)}} x[k+2]u[k] \left(\left(\frac{1}{3}\right)^{n-k} u[n-k]\right) \\
&= \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} 2^{-k-1} 3^k x[k+2]
\end{aligned}$$

Likewise we have that $\delta[n] * \left(\left(\frac{1}{3}\right)^n u[n]\right) = \left(\frac{1}{3}\right)^n u[n]$, and for $(2^{-n}u[n-3]) * (3^{-n}u[n])$ we get that for $n < 3$ the sequence is 0 and otherwise we have

$$\begin{aligned}
\left(2^{-n}u[n-3]\right) * \left(3^{-n}u[n]\right) &= \sum_{k=-\infty}^{\infty} 2^{-k}u[k-3]3^{-k+n}u[n-k] \\
&= \sum_{k=3}^{n} 2^{-k}3^{k-n} \\
&= 3^{-n} \sum_{k=3}^{n} \left(\frac{3}{2}\right)^k \\
&= -2\left(3^{-n}\right)\left(\left(\frac{3}{2}\right)^n - \left(\frac{3}{2}\right)^2\right)
\end{aligned}$$

Therefore we have that

$$y_2[n] = \boxed{\begin{cases} 0 & n < 0 \\ -\frac{3}{2}\left(\frac{1}{3}\right)^n + 3^{-n}\sum_{k=0}^{n} 2^{-k-1}3^k x[k+2] & 0 \leq n < 3 \\ -\frac{3}{2}\left(\frac{1}{3}\right)^n - 2\left(3^{-n}\right)\left(\left(\frac{3}{2}\right)^n - \left(\frac{3}{2}\right)^2\right) + 3^{-n}\sum_{k=0}^{4} 2^{-k-1}3^k x[k+2] & n \geq 3 \end{cases}}$$

Problem 5

(a)

The first index is the index n that satisfies the condition $n - n_1 - n_3 = 0$, thus we get that the first nonzero element occurs at $\boxed{n_1 + n_3}$

(b)

The last index is the index n that satisfies the condition $n-(n_2-1)-(n_4-1) = 0$, thus we get that the last nonzero element occurs at $\boxed{n_2 + n_4 - 2}$

(c)

Therefore the length of the output sequence is $n_2 + n_4 - 2 - (n_1 + n_3) + 1 = \boxed{n_x + n_h - 1}$