ECE 131A HW 5

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Problem 1

(a)

the standard score $z = 1.37996$ results in

$$P(Z \leq z) = 0.9162 = 1 - \frac{1}{2}(1 - 0.8324)$$

Thus we have that $c = \boxed{1.37996}$.

(b)

$c = \sqrt{2.3} \cdot 1.37996 = \boxed{2.092}$

Problem 2

(a)

The corresponding z value for a standard normal is

$$\frac{1-0.8875}{0.10625} = 1.05882352941$$

Thus we have that

$$P(Z \leq z) = 0.85515$$

(b)

We have that this is effectively

$$P\left(\frac{13-14.2}{1.7} \leq Z \leq \frac{15-14.2}{1.7}\right) = 0.68103 - 0.24013 = \boxed{0.4409}$$

(c)

We have that the probability that one book is heavy is

$$1 - P(Z \leq \frac{16-14.2}{1.7}) = 0.14484$$

Therefore the probability that there is 3 heavy books is a binomial distribution and thus the probability is

$$\binom{10}{3} 0.1448^3 (1-0.1448)^7 = \boxed{0.12189}$$

Problem 3

(a)

We have that the area of the quadrilateral is $6+2=8$ and that the area with $X \geq 3$ is 2 so $P(X \geq 3) = \frac{2}{8} = \boxed{\frac{1}{4}}$

(b)

we have that the area of the quadrilateral is $6+2=8$ and the area with $Y \geq 1$ is $4+\frac{1}{2}=4.5$ so then we have that $P(Y \geq 1) = \frac{4.5}{8} = \boxed{\frac{9}{16}}$

(c)

The area of the quadrilateral is $6+2=8$ and the area with $X \leq 1$ and $Y \leq 1$ is 1 so then we have that the probability is $\frac{1}{8} = \boxed{\frac{1}{8}}$

Problem 4

we have

$$\begin{aligned} E[X] &= \int_0^\infty \int_0^\infty x \frac{1}{750} e^{-\left(\frac{x}{150}+\frac{y}{30}dydx\right)} \\ &= \int_0^\infty \frac{x}{25} e^{-\frac{x}{150}} \int_0^\infty \frac{1}{30} e^{-\frac{y}{30}} dydx \\ &= \boxed{900} \end{aligned}$$

Problem 5

(a)

let the triangle be denoted as R then we have

$$f(x,y) = \begin{cases} \frac{2}{9} & \text{if } (x,y) \in R \\ 0 & \text{otherwise} \end{cases}$$

Thus we have that

$$\begin{aligned} E[X] &= \int_0^3 x \int_0^{3-x} \frac{2}{9} dy dx \\ &= \int_0^3 \frac{2x}{9}(3-x) dx \\ &= \boxed{1} \end{aligned}$$

and also

$$\begin{aligned} E[Y] &= \int_0^3 \int_0^{3-x} \frac{2y}{9} dy dx \\ &= \int_0^3 \frac{(3-x)^2}{9} dx \\ &= \boxed{1} \end{aligned}$$

$$E[X+Y] = \boxed{2}$$

(b)

We have that

$$E[X] = 1$$

and

$$\begin{aligned}
E[X] &= \int_0^3 x^2 \int_0^{3-x} \frac{2}{9} dy dx \\
&= \int_0^3 \frac{2x^2}{9}(3-x) dx \\
&= \boxed{1.8}
\end{aligned}$$

Thus we have that

$$Var(X) = E[X^2] - E[X]^2 = \boxed{0.8}$$

Problem 6

we have that

$$P(X = l) = \begin{cases} \frac{3}{11}\frac{2}{10} & \text{if } l = 2 \\ \frac{16}{11}\frac{3}{10} & \text{if } l = 1 \\ \frac{8}{11}\frac{7}{10} & \text{if } l = 0 \\ 0 & \text{otherwise} \end{cases}$$

Then we have that

$$E[X] = 0.5454545454545454$$

and

$$E[X^2] = 0.6545454545454545$$

And thus we have that

$$Var(X) = \boxed{0.3570247933884298}$$

Problem 7

(a)

We have that there are $6!$ ways to arrange th 6 bears, and there are $10 \cdot 4!$ ways to do so and have the pair of bears be adjacent then we have that

$P(X_i = 1) = \frac{1}{3}$ thus we have that

$$Cov(X_i, X_i) = Var(X_i) = \frac{1}{3} \cdot \frac{2}{3} = \boxed{\frac{2}{9}}$$

(b)

We have that there is $2 \cdot 2 \cdot 2 \cdot (2 \cdot 3 + 3 \cdot 2) = 48$ ways to arrange the 6 bears So therefore $P(X_i X_j = 1) = 0.1333333$ and thus we have that

$$Cov(X_i, X_j) = \boxed{0.02221888888}$$

(c)

$$Var(X) = 3Var(X_i) + 6Cov(X_i, X_j) = boxed0.79997999994$$

Problem 8

We have that the distributions of the number of birds we see over UCLA after 40 hrs of observation is a Possion distribution with

$$\lambda = 40 \cdot 2 = 80$$

So then we have that

$$P(N \geq 75) = \boxed{0.72684}$$

Problem 9

(a)

we have that

$$E[\sum_{i=1}^{500} X_i] = \sum_{i=1}^{500} E[X_i] = \boxed{500}$$

$$Var(\sum_{i=1}^{500} X_i) = sum_{i=1}^{500} Var[X_i] = \boxed{300}$$

(b)

From the center limit theorem we have that

$$\sum_{i=1}^{500} X_i \to N(500, 300)$$

So therefore

$$P(780 < \sum_{i=1}^{500} X_i < 820) \approx P(\frac{28}{30} < Z < \frac{32}{30}) = \boxed{0.03226}$$

(c)

we have that Y can be approximated as a a normal distribution with

$$\mu = 500$$

and

$$\sigma^2 = 1500$$

so then we have that

$$P(Y < \sum_{i=1}^{500} X_i) = P(N(500, 1500) - N(500, 300) < 0) = \boxed{\frac{1}{2}}$$

Problem 10

(a)

We have that

$$\begin{aligned} g(t) &= M_X(t) \\ &= E(e^{tX}) \\ &= \int_0^\infty e^{tX} f(x) dx \\ &= \int_0^\infty e^{tx} \lambda e^{-\lambda x} dx \\ &= \frac{\lambda}{\lambda - t} & = \boxed{\frac{3}{3-t}} \end{aligned}$$

(b)

We have that

$$g'(t) = M_X'(t) = \boxed{\frac{3}{(3-t)^2}}$$

(c)

$$M_X'(0) = \frac{3}{(3-0)^2} = \boxed{\frac{1}{3}}$$