ECE 131A, Fall 2022 **Homework #1**

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Due Sunday, 9 October 2022, uploaded to Gradescope.
Covers material up to Lecture 4.
100 points total.

1. (10 points) **Sample space of random experiments**

   (a) (1 point) Two tetrahedrons (4 sided), each with equal sides numbered from 1 to 4, are identical except that one is red and the other green. If the two tetrahedrons are tossed and the number on the bottom face of each is observed, what is the sample space for this experiment?

   (b) (9 points) Consider the sample space found in the previous part. Let E consist of those outcomes for which both (tetrahedron) dice show an odd number. Let F be the event that the sum of the two numbers on these dice is 5. Let G be the event that the sum of the two numbers is less than 7.

      i. (4 points) List the elements of E and F
      ii. (2 points) Find $E \cap F$
      iii. (2 points) Find $E \cup F$
      iv. (1 point) Find G^c

2. (10 points) **Probability bounds**

   Let A and B be events with probabilities $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Show the following bound holds:

$$\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3}$$

3. (10 points) **Combinatorics**

   A lot of 100 items contain k defective items. M items are chosen at random and tested. What is the probability that m are found defective?

4. (20 points) **Arranging letters in a word**

   (a) (10 points) How many ways you can arrange the letters in the word STATISTICS?

   (b) (10 points) If all arrangements are equally likely, what is the probability that two "I"s are next to each other?

5. (20 points) **Conditional probability**

   A man possesses five coins, two of which are double-headed, one is double-tailed, and two are normal.

   (a) (10 points) He shuts his eyes, picks a coin at random, and tosses it. What is the probability that the lower face of the coin is a head?

(b) (10 points) He opens his eyes and sees that the coin is showing heads; what is the probability that the lower face is a head?

6. (10 points) **Information error probability**

To communicate one bit of information reliably, cellular phones transmit the same binary symbol five times. Thus the information "zero" is transmitted as 00000 and "one" is 11111. The receiver detects the correct information if three or more binary symbols are received correctly. What is the information error probability $P[E]$, if the binary symbol error probability is $q = 0.1$?

7. (20 points) **Independence**

Jane has three children, each of which is equally likely to be a boy or a girl independently of the others. Define the events:

- $A = \{$ all the children are of the same sex $\}$
- $B = \{$ there is at most one boy $\}$
- $C = \{$ the family includes a boy and a girl $\}$

(a) (10 points) Show that A is independent of B, and that B is independent of C.

(b) (5 points) Is A independent of C?

(c) (5 points) Do the above results hold if boys and girls are not equally likely?