ECE 131A Quiz 1

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Problem 1

(a)

Since $P(A^c \cap B^c) = \frac{2}{3}$, we have

$$P((A \cap B^c) \cup (A^c \cap B) \cup (A \cap B)) = 1 - P(A^c \cap B^c) = \boxed{\frac{1}{3}}$$

.

(b)

We have

$$P(C \cap D) = P(C, D)$$

$$P(C|D) = \frac{P(C, D)}{P(D)}$$

$$P(C^c|D) = 1 - \frac{P(C, D)}{P(D)}$$

$$P(C^c \cap D) = P(C^c, D) = P(D)P(C^c|D) = P(D)(1 - \frac{P(C,D)}{P(D)})$$

$$P(C^c \cap D) = P(D) - P(C,D) = 0.45 - 0.1 = \boxed{0.35}$$

(c)

We have

$$P(A \cup B) = 1 - P\left((A \cup B)^c\right) = 0.58$$

Furthermore we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.12$$

This is equal to $P(A)P(B) = 0.12$, therefore A and B are $\boxed{\text{independent}}$.

Problem 2

Let the bernoulli random variable T reprsent if a page from a textbook have a theorem on it. Likewise let M be a random variable that represents the type of textbook a student is reading given it is a math textbook, therefore we have

$$P(M = \text{probability}) = \frac{10}{100 - 72}$$

$$P(M = \text{algebra}) = \frac{13}{100 - 72}$$

$$P(M = \text{real analysis}) = \frac{5}{100 - 72}$$

And thus we have

$$P(M = \text{probability}|T = 1) = \frac{P(M = \text{probability})P(T = 1|M = \text{probability})}{P(T = 1)}$$

Since

$$\begin{aligned}
P(T=1) =& P(M=\text{probability})P(T=1|M=\text{probability}) \\
& + P(M=\text{algebra})P(T=1|M=\text{algebra}) \\
& + P(M=\text{real analysis})P(T=1|M=\text{real analysis}) \\
=& \frac{10}{100-72}\frac{20}{100} + \frac{13}{100-72}\frac{29}{100} + \frac{5}{100-72}\frac{37}{100} \\
=& 0.272142857143
\end{aligned}$$

Therefore we have

$$\begin{aligned}
P(M=\text{probability}|T=1) &= \frac{P(M=\text{probability})P(T=1|M=\text{probability})}{P(T=1)} \\
&= \frac{\frac{10}{100-72}\frac{20}{100}}{0.272142857143} \\
&= \boxed{0.262467191601}
\end{aligned}$$

Problem 3

The probability of getting a speeding ticket (denoted by a bernoulli random variable T_1) from L_1 is:

$$P(T_1=1) = 0.2 \cdot 0.4$$

Therefore the probability of not getting a speeding ticket is:

$$P(T_1=0) = 1 - P(T_1=1) = 0.92$$

Likewise the probability of getting a speeding ticket from L_2 is:

$$P(T_2=1) = 0.1 \cdot 0.3$$

Therefore the probability of not getting a speeding ticket is:

$$P(T_2=0) = 1 - P(T_2=1) = 0.97$$

The probability of getting a speeding ticket from L_3 is:

$$P(T_3=1) = 0.5 \cdot 0.2$$

Therefore the probability of not getting a speeding ticket is:

$$P(T_3 = 0) = 1 - P(T_3 = 1) = 0.9$$

The probability of getting a speeding ticket from L_4 is:

$$P(T_4 = 1) = 0.3 \cdot 0.2$$

Therefore the probability of not getting a speeding ticket is:

$$P(T_4 = 0) = 1 - P(T_4 = 1) = 0.94$$

Therefore the probability of not getting any speeding ticket is:

$$P(T_1 = 0)P(T_2 = 0)P(T_3 = 0)P(T_4 = 0) = 0.92 \cdot 0.97 \cdot 0.9 \cdot 0.94 = 0.7549704$$

Therefore the probability of getting at least one speeding ticket is $1 - 0.7549704 = \boxed{0.2450296}$.