

# ECE 131A HW 3

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## Problem 1

Let  $C_{ij}$  be the event that the  $i$ th person grabs the  $j$ th person's coat, then we have that in order for all of them to have grabbed the wrong coat we could have two possible sequences,  $C_{13}C_{21}C_{32}$  or  $C_{12}C_{23}C_{31}$ , both of these cases can happen with probability  $\frac{1}{6}$  therefore the probability that everyone grabs the

wrong coat is  $\boxed{\frac{1}{3}}$

## Problem 2

(a)

$$\boxed{\frac{1}{1000}}$$

(b)

Since 1003 is not in the range of pages then the probability

$$P(X = 1003) = \boxed{0}$$

(c)

$$\boxed{\frac{1}{1000}}$$

(d)

There are 123 possible pages therefore

$$P(X \leq 122) = \boxed{\frac{123}{1000}}$$

(e)

$$P(12 \leq X \leq 17) = \boxed{\frac{6}{1000}}$$

### Problem 3

(a)

$$P(X = j) = \left(\frac{4}{5}\right)^{j-1} \frac{1}{5}$$

(b)

$$P(X = j) = \boxed{\frac{1}{5}, j = 1, 2, 3, 4, 5}$$

## Problem 4

(a)

Let the bernoulli random variable  $X_i$  be the event that the  $i$ th plate is isolated, then we have that

$$P(X_i = 1) = \frac{10}{14} \frac{9}{13} \frac{5}{15} = 0.4945$$

Therefore

$$E[X] = 15E[X_i] = \boxed{7.417}$$

(b)

Let the bernoulli random variable  $Y_i$  be the event that the  $i$ th plate is semi happy, then we have

$$P(Y_i = 1) = 2 \frac{10}{13} \frac{5}{15} \frac{4}{14} = 0.439$$

Therefore

$$E[Y] = 15E[Y_i] = \boxed{6.593}$$

(c)

Let the bernoulli random variable  $Z_i$  be the event that the  $i$ th plate is joyous, We have

$$P(Z_i = 1) = 1 - P(X_i = 1) - P(Y_i = 1) = 0.0665$$

Therefore

$$E[Z] = 15E[Z_i] = \boxed{0.989}$$

## Problem 5

(a)

$$\boxed{(1-p)^{n-1}p}$$

(b)

$$\boxed{(1-p)^{n-2}p^2}$$

(c)

The success are distributed in a way that is independent on where it occurs so the probability that the  $k$  success occurs at  $j_1, j_2, \dots, j_k$  is always the same no matter what  $j_1, j_2, \dots, j_k$  so long as  $0 < j_1, j_2, \dots, j_k$

## Problem 6

(a)

Let the number of accidents be denoted by a random variable  $N$ .  $N$  is distributed as a poisson random variable with  $\lambda = 3$ . Therefore

$$P(N = 5) = \boxed{\frac{3^5 e^{-3}}{5!}}$$

(b)

$$P(N < 3) = e^{-3} \left( 1 + \frac{12}{5!} \right)$$

(c)

$$P(N \geq 2) = 1 - P(N < 2) = \boxed{1 - e^{-3} \left( 1 + \frac{3}{5!} \right)}$$

## Problem 7

(a)

Rate for 1 hr for fans of both teams is 14 per hour, so for 3 hours it would be  $\boxed{42}$ .

(b)

if the rat is 14 per hour, then the rate for 20 minutes would be  $14/3$  per 20 minutes, since this will still be distributed as a Poisson we have that the probability of only 1 fan of either the Yankees or the Red Sox enter the store

is just  $\boxed{e^{-\frac{14}{3}} \frac{14}{3}}$ .

## Problem 8

(a)

There are  $\binom{N}{n}$  ways to pick out  $n$  balls from the  $N$  balls in the urn. But to choose  $k$  blue balls from the  $b$  balls we have  $\binom{b}{k}$  ways to do so, and we have  $\binom{r}{n-k}$  ways to choose the remaining  $n-k$  red balls. Therefore the probability of choosing  $k$  blue balls is

$$\boxed{\frac{\binom{b}{k} \binom{r}{n-k}}{\binom{N}{n}}}$$

(b)

$$\begin{aligned} \lim_{N, b, r \rightarrow \infty} \frac{\binom{b}{k} \binom{r}{n-k}}{\binom{N}{n}} &= \lim_{N, b, r \rightarrow \infty} \binom{n}{k} \frac{b \cdot (b-1) \cdots (b-k+1) \cdot r \cdot (r-1) \cdots (r-k+1)}{N \cdot (N-1) \cdots (N-n+1)} \\ &= \lim_{N, b, r \rightarrow \infty} \binom{n}{k} \frac{b^k \cdot r^{n-k}}{N^n} \\ &= \binom{n}{k} p^k (1-p)^{n-k} \end{aligned}$$