ECE 131A HW 3

Lawrence Liu

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Problem 1

Let C_{ij} be the event that the ith person grabs the jth person's coat, then we have that in order for all of them to have grabbed the wrong coat we could have two possible sequences, $C_{13}C_{21}C_{32}$ or $C_{12}C_{23}C_{31}$, both of these cases can happen with probability $\frac{1}{6}$ therefore the probability that everyone grabs the wrong coat is $\boxed{\frac{1}{3}}$

Problem 2

(a)

$$\boxed{\frac{1}{1000}}$$

(b)

Since 1003 is not in the range of pages then the probability

$$P(X = 1003) = \boxed{0}$$

(c)

$$\boxed{\frac{1}{1000}}$$

(d)

There are 123 possible pages therefore

$$P(X \leq 122) = \boxed{\frac{123}{1000}}$$

(e)

$$P(12 \leq X \leq 17) = \boxed{\frac{6}{1000}}$$

Problem 3

(a)

$$P(X = j) = \left(\frac{4}{5}\right)^{j-1} \frac{1}{5}$$

(b)

$$P(X=j)=\boxed{\frac{1}{5}, j=1,2,3,4,5}$$

Problem 4

(a)

Let the bernoulli random variable X_i be the event that the ith plate is isolated, then we have that

$$P(X_i=1)=\frac{10}{14}\frac{9}{13}\frac{5}{15}3=0.4945$$

Therefore

$$E[X]=15E[X_i]=\boxed{7.417}$$

(b)

Let the bernoulli random variable Y_i be the event that the ith plate is semi happy, then we have

$$P(Y_i=1)=2\frac{10}{13}\frac{5}{15}\frac{4}{14}3=0.439$$

Therefore

$$E[Y]=15E[Y_i]=\boxed{6.593}$$

(c)

Let the bernoulli random variable Z_i be the event that the ith plate is joyous, We have

$$P(Z_i=1)=1-P(X_i=1)-P(Y_i=1)=0.0665$$

Therefore

$$E[Z] = 15E[Z_i] = \boxed{0.989}$$

Problem 5

(a)

$$\frac{(1-p)^{n-1}p}{\binom{n}{1}(1-p)^{n-1}p} = \boxed{\frac{1}{n}}$$

(b)

$$\frac{(1-p)^{n-2}p^2}{\binom{n}{2}(1-p)^{n-2}p^2} = \boxed{\frac{1}{n(n-2)}}$$

(c)

The success are distrubted in a way that is independent on where it occurs so the probability that the k success occurs at $j_1, j_2, \ldots, j_k$ is always the same no matter what $j_1, j_2, \ldots, j_k$ so long as $0 < j_1, j_2, \ldots, j_k \leq n$, and given these k successes, the probability that a specific trail is successful is $\frac{1}{\binom{n}{k}}$,

Problem 6

(a)

Let the number of accidents be denoted by a random variable N. N is distributed as a possion random variable with $\lambda = 3$. Therefore

$$P(N = 5) = \boxed{\frac{3^5 e^{-3}}{5!}}$$

(b)

$$P(N < 3) = e^{-3}\left(1 + \frac{3}{2} + \frac{9}{3!}\right)$$

(c)

$$P(N \geq 2) = 1 - P(N < 2) = \boxed{1 - e^{-3}\left(1 + \frac{3}{5!}\right)}$$

Problem 7

(a)

Rate for 1 hr for fans of both teams is 14 per hour, so for 3 hours it would be $\boxed{42}$.

(b)

if the rat is 14 per hour, then the rate for 20 minutes would be 14/3 per 20 minutes, since this will still be distrubuted as a Poisson we have that the probability of only 1 fan of either the Yankees or the Red Sox enter the store is just $\boxed{e^{-\frac{14}{3}}\frac{14}{3}}$.

Problem 8

(a)

There are $\binom{N}{n}$ ways to pick out n balls from the N balls in the urn. But to choose k blue balls from the b balls we have $\binom{b}{k}$ ways to do so, and we have $\binom{r}{n-k}$ ways to choose the remaining $n-k$ red balls. Therefore the probability of choosing k blue balls is

$$\boxed{\frac{\binom{b}{k}\binom{r}{n-k}}{\binom{N}{n}}}$$

(b)

$$\begin{aligned}
\lim_{N,b,r\to\infty}\frac{\binom{b}{k}\binom{r}{n-k}}{\binom{N}{n}} &= \lim_{N,b,r\to\infty}\binom{n}{k}\frac{b\cdot(b-1)\cdots(b-k+1)\cdot r\cdot(r-1)\cdots(r-(n-k)+1)}{N\cdot(N-1)\cdots(N-n+1)} \\
&= \lim_{N,b,r\to\infty}\binom{n}{k}\frac{b^k\cdot r^{n-k}}{N^n} \\
&= \binom{n}{k}p^k(1-p)^{n-k}
\end{aligned}$$