ECE 131A HW 6

Lawrence Liu

December 2, 2022

Problem 1

We have that assuming the number of pens is large, then the total lifetime of the pens is distributed according to a normal with mean n and variance n therefore we have that we want to find a n such that

$$-2.32635 = \frac{15 - n}{\sqrt{n}}$$

solving this we get that $n = 27.11$ therefore the student would need $\boxed{28}$ pens in order for the probability of having a pen to run out during a semester to be greater than 0.99

Problem 2

The number of errors in 100 transmission can be estimated as a normal distribution with mean of 15 and variance of $100 \cdot 0.15 \cdot 0.85 = 12.75$, therefore the probability of having more than 20 errors has a corresponding Z value of 0.392 and therefore a corresponding probability of $\boxed{0.65247}$.

Problem 3

(a)

We have that
$$\Sigma^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$$
And
$$det(\Sigma) = 5$$
Therefore we have that
$$\Sigma^{-1}(\mu - X) = \frac{1}{5}\begin{bmatrix} 3(\mu_1 - x_1) + (\mu_2 - x_2) \\ (\mu_1 - x_1) + 2(\mu_2 - x_2) \end{bmatrix}$$
and therefore we have that
$$(\mu - X)^T\Sigma^{-1}(\mu - X) = \frac{3}{5}(\mu_1 - x_1)^2 + \frac{2}{5}(\mu_2 - x_2)(\mu_1 - x_1) + \frac{2}{5}(\mu_2 - x_2)^2$$
Therefore we have that
$$f(x_1, x_2) = (2\pi)^{-1}\frac{1}{\sqrt{det(5)}}e^{-\frac{3}{10}(\mu_1 - x_1)^2 - \frac{1}{5}(\mu_2 - x_2)(\mu_1 - x_1) - \frac{1}{5}(\mu_2 - x_2)^2}$$

(b)

We have that Y is distributed as an normal distribution with mean $A\mu + B = 4$ and variance $A\Sigma A^T = 3$ therefore we have that the pdf of Y is
$$f(y) = \frac{1}{\sqrt{2\pi 3}}e^{-\frac{(y-4)^2}{6}}$$

(c)

Therefore we have that given that Z is a standard normal distribution
$$P(Y \geq 2) = P(Z \geq \frac{2-4}{\sqrt{3}}) = 1 - 0.12425 = \boxed{0.87575}$$

Problem 4

(a)

We have that

$$Cov(Y_i, Y_i) = 2$$

and when $i \neq j$

$$Cov(Y_i, Y_j) = 1$$

Therefore we have that the covariance matrix is

$$\Sigma = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

(b)

We have that the pdf of Y is

$$f(y_1, y_2, y_3) = (2\pi)^{-\frac{3}{2}} \frac{1}{2} e^{\frac{1}{4}(y_1y_2+y_2y_3+y_3y_1)-\frac{3}{8}(y_1^2+y_2^2+y_3^2)}$$

(c)

We have that the distirbution for Y_1 and Y_2 is a given by a linear transformation of $[Y_1, Y_2, Y_3]$ with $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. Therefore we have that this mu is $A\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and the covariance matrix is $A\Sigma A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Therefore we have that

$$f(y_1, y_2) = \frac{1}{2\pi} \frac{1}{\sqrt{3}} e^{-\frac{1}{3}(y_1^2+y_2^2-y_1y_2)}$$

Likewise we have that the distribution for Y_1 and Y_3 is a given by a linear transformation of $[Y_1, Y_2, Y_3]$ with $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Therefore we have that this mu is $A\mu = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ and the covariance matrix is $A\Sigma A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$. Therefore we have that

$$f(y_1, y_3) = \frac{1}{2\pi} \frac{1}{\sqrt{3}} e^{-\frac{1}{3}(y_1^2 + y_3^2 - y_1 y_3)}$$

(d)

we can simply use a linear transformation $A = \Sigma^{-1} = \begin{bmatrix} \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{4} & \frac{3}{4} \end{bmatrix}$. and with this we will have the resulting covariance matrix is I.