ECE 131A, Fall 2022 **Quiz #3**

Department of Electrical and Computer Engineering Prof. V. Roychowdhury
University of California, Los Angeles TA: T. Monsoor

Due Saturday, 5 November 2022 by 11:59 PM, uploaded to Gradescope.
Resources: Lectures 5-9, Homework 3, Discussions 4-6.
10 points total.

1. (3 points) **Happy bear pairs**

   Consider a collection of 6 bears. There is a pair of red bears consisting of one father bear and one mother bear. There is a similar green bear pair, and a similar blue bear pair. These 6 bears are all placed in a straight line, and all arrangements in such a line are equally likely. A bear pair is happy if it is sitting together. Let X denote the number of happy bear pairs. Find $E[X]$.

2. (4 points) **Selecting envelopes**

   (a) (1 point) Consider a stack of 1000 envelopes. Exactly 7 of them are green inside, and the other 993 are red inside. We choose 10 of the envelopes (without replacement). Let X denote the number of chosen envelopes that are green inside. What kind of random variable is X? What are the parameters?

   (b) (1 point) Find the probability that we select exactly 1 envelope which is green inside.

   (c) (1 point) Now reconsider 2(a) but replace and reshuffle the order of envelopes, in between the selections. Let Y denote the number of chosen envelopes that are green inside. What kind of random variable is Y ? What are the parameters?

   (d) (1 point) In this scheme with replacement and reshuffling, find the probability that we select exactly 1 envelope which is green inside.

3. (3 points) **Silence in a newsroom**

   In a busy newsroom, suppose that the times until the next phone ringing, next email arriving, or next computer beeping are independent exponential random variables, with respective means of 30 seconds, 20 seconds, and 15 seconds.

   (a) (1 point) Find the probability that the newsroom is silent for the next 10 seconds.

   (b) (1 point) Find the probability that the newsroom is silent for the next t seconds, where $t \geq 0$.

   (c) (1 point) If we define the random variable T as the amount of time the newsroom is silent for, then find the probability density function of T.