ECE 131A Quiz 4

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November 18, 2022

Problem 1

(a)

We have that the corresponding standard score is $z = \frac{5-4}{0.75}$ And thus we have that the probability that a blade of grass is 5 inches or taller is

$$P(Z \geq \frac{4}{3}) = 1 - 0.90878 = \boxed{0.09122}$$

(b)

We have that the probability of picking up a blade of grass that is 5 inches or taller is 0.09122. so the number of blades of grass a child would need to pick up before reaching a blade of grass that is 5 inches or taller is a geometric random variable with parameter 0.09122. Thus the expected number of blades is $1/0.09122 = \boxed{10.9625}$

(c)

This is a binomial random variable, with $n = 10$ and $p = 0.09122$. Thus the expected number of blades of grass that are 5 inches or taller is $np = \boxed{0.9122}$

Problem 2

(a)

$$E[X] = \int_0^{10} x \frac{(10-x)^3}{2500} dx = \boxed{2}$$
$$E[X^2] = \int_0^{10} x^2 \frac{(10-x)^3}{2500} dx = \frac{20}{3}$$
$$Var(X) = \boxed{\frac{8}{3}}$$

(b)

Using the central limit theorem, we can approximate the distribution of the sum of 200 of these random variables as a normal distribution with mean $200 \cdot 2 = 400$ and variance $200 \cdot \frac{8}{3} = 533.33$. Thus the standard score of 420 is

$$z = \frac{420 - 400}{\sqrt{533.33}} = 0.866028110127$$

Thus

$$P(Z > z) = 1 - 0.80676 = \boxed{0.19324}$$

Problem 3

(a)

We have that

$$\begin{aligned}
F(X,Y) &= \int_0^x \int_0^y f(x,y)dydx \\
&= \int_0^X \int_0^Y (x+y)dydx \\
&= \int_0^X xY + \frac{Y^2}{2}dx \\
&= \boxed{\frac{X^2Y}{2} + \frac{Y^2X}{2}}
\end{aligned}$$

And thus we have that

$$F(1,1) = \boxed{1}$$

Which is what we intuitively expect.

(b)

We have that

$$f(x) = \int_0^1 f(x,y)dy = \int_0^1 (x+y)dy = \boxed{x + \frac{1}{2}}$$

and

$$f(y) = \int_0^1 f(x,y)dx = \int_0^1 (x+y)dx = \boxed{y + \frac{1}{2}}$$

(c)

No since

$$f(x)f(y) = (x + \frac{1}{2})(y + \frac{1}{2}) \neq x + y$$