ECE 131A, Fall 2022 **Homework #3**
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Due Monday, 24 October 2022, uploaded to Gradescope.
Covers material up to Lecture 8.
100 points total.

1. (10 points) **Picking coats at a party**

   Consider 3 students who wear similar-looking black coats to a party. When they leave the party, they are in a hurry, and they each randomly grab a coat as they depart, with all possibilities equally likely. Use inclusion-exclusion to calculate the probability that none of them get their correct coat. [Hint: First find the probability that at least one of them gets their correct coat.]

2. (10 points) **Choosing a page at random**

   A student buys a brand new calculus textbook that has 1000 pages, each numbered with 3 digits, from 000 to 999. She randomly opens the book to a page and starts to read! Assume that any of the 1000 pages are equally likely to be chosen. Let X be the page number of the chosen page. Thus, X is an integer-valued random variable between 0 and 999.

   (a) (2 points) Find $P(X = 122)$.
   (b) (2 points) Find $P(X = 1003)$.
   (c) (2 points) When x is an integer between 0 and 999, find $P(X = x)$.
   (d) (2 points) Find $P(X \leq 122)$.
   (e) (2 points) Find $P(12 \leq X \leq 17)$.

3. (10 points) **Gloves**

   A matching pair of blue gloves, a matching pair of red gloves, and one lone white right-handed glove are in a drawer. The gloves are pulled out of the drawer, one at a time.

   (a) (5 points) Suppose that a person is looking for the white glove. He repeatedly does the following: He pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he replaces the glove in the drawer and starts to check again, i.e., he reaches blindly into the drawer of 5 gloves. He continues to do this over and over until he finds the white glove, and then he stops. Let X be the number of draws that are necessary to find the white glove for the first time. For each positive integer j, find $P(X = j)$.
   (b) (5 points) Now suppose that he searches for the white glove but, if he pulls a different colored glove from the drawer, he does not replace it. So he pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he permanently discards the glove and starts to check again, i.e., he reaches blindly into the drawer with the gloves that remain. He continues to do this over and over again until he finds the white glove, and then he stops. Let X be the number of draws that are necessary to find the white glove for the first time. For each integer j, with $1 \leq j \leq 5$, find $P(X = j)$.

4. (15 points) **Card with emotions**

   Consider a deck of 15 cards containing 5 blue cards, 5 red cards, and 5 green cards. Shuffle the cards and deal all 15 of the cards out, around a circular table, with one card per seat.

   (a) (5 points) A card is called "isolated" if its color does not agree with either of the nearby cards (i.e., if it has a different color than the card to its right and a different color than the card to its left). Let X denote the number of isolated cards. Find $E(X)$.

   (b) (5 points) A card is called "semi-happy" if its color agrees with exactly one (but not both) of the nearby cards (i.e., if its color agrees with the color of the card on its left or on its right, but not both). Let Y denote the number of semi-happy cards. Find $E(Y)$.

   (c) (5 points) A card is called "joyous" if its color agrees with both of the nearby cards (i.e., if its color agrees with the color of the card on its left and on its right). Let Z denote the number of joyous cards. Find $E(Z)$.

5. (15 points) **Randomness of successes in bernoulli trials**

   Let X be a binomial random variable that results from the performance of n Bernoulli trials with probability of success p.

   (a) (5 points) Suppose that $X = 1$. Find the probability that the single event occurred in the k^{th} Bernoulli trial.

   (b) (5 points) Suppose that $X = 2$. Find the probability that the two events occurred in the j^{th} and k^{th} Bernoulli trials where $j < k$

   (c) (5 points) In light of your answers to part (a) and part (b), in what sense are the successes distributed "completely at random" over the n Bernoulli trials?

6. (15 points) **Accidents**

   On average a certain intersection results in 3 traffic accidents per month. What is the probability that for any given month at this intersection

   (a) (5 points) exactly 5 accidents will occur?

   (b) (5 points) less than 3 accidents will occur?

   (c) (5 points) at least 2 accidents will occur?

7. (10 points) **Baseball Fans**

   The number of Yankees fans shopping at a sports store, per hour, is Poisson with mean 8 per hour. The number of Red Sox fans shopping at the same store is Poisson with mean 6 per hour. Assume that the numbers of fans of the two types are independent. In particular, there is no person who is simultaneously a fan of both teams.

   (a) (5 points) In a three hour period, how many Yankees and Red Sox fans do we expect altogether?

   (b) (5 points) Find the probability that exactly 1 person enters the store during the next 20 minutes who likes the Yankees or Red Sox.

8. (15 points) **Hypergeometric distribution**

An urn contains N balls, b of which are blue and r of which are red. A random sample of n balls is withdrawn without replacement from the urn. Let B be the random variable denoting the number of blue balls in the sample.

(a) (5 points) Find $P(B = k)$.

(b) (10 points) Show that if N, b and r approach ∞ in such a way that $\frac{b}{N} \to p$ and $\frac{r}{N} \to 1 - p$, then

$$P(B = k) \to \binom{n}{k} p^k (1 - p)^{n-k}$$