ECE 131A, Fall 2022 **Homework #2**

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Due Monday, 17 October 2022, uploaded to Gradescope.
Covers material up to Lecture 6.
100 points total.

1. (10 points) **Non-symmetric Binary Channel**

   Consider a Non-symmetric Binary Channel with $P[B_0|A_0] = 1-\epsilon_1$, $P[B_1|A_0] = \epsilon_1$, $P[B_0|A_1] = \epsilon_2$ and $P[B_1|A_1] = 1 - \epsilon_2$. Assume all inputs are equally probable.

   (a) (2 points) Find the probability that the output is B_0.

   (b) (8 points) Find the probability that the input was A_0 given that the output is B_1. Find the probability that the input was A_1 given that the output is B_1. Which input is more probable if $\epsilon_1 = 0.25$ and $\epsilon_2 = 0.5$

2. (20 points) **Combinatorics**

   (a) Ordering a deluxe pizza means you have 4 choices from 15 available toppings. Assume that the order in which the toppings are selected does not matter.

      i (5 points) How many combinations are possible if toppings can be repeated?

      ii (5 points) How many combinations are possible if toppings cannot be repeated?

   (b) (5 points) Five balls are placed at random in five buckets. What is the probability that each bucket has a ball?

   (c) (5 points) How many distinct permutations are there of four red balls, two white balls, and three black balls?

3. (20 points) **Lottery**

   You choose r of the first n positive integers, and a lottery chooses a random subset L of the same size. What is the probability that:

   (a) (5 points) the numbers in L are drawn in increasing order?

   (b) (5 points) your choice of numbers is the same as L?

   (c) (10 points) there are exactly k of your numbers matching members of L?

4. (20 points) **Shooting game**

   A and B are involved in a duel. The rules of the duel are that they are to pick up their guns and shoot at each other simultaneously. If one or both are hit, then the duel is over. If both shots miss, then they repeat the process. Suppose that the results of the shots are independent and that each shot of A will hit B with probability p_A and each shot of B will hit A with probability p_B. What is:

   (a) (4 points) the probability that A is not hit?

(b) (4 points) the probability that both duelists are hit?

(c) (4 points) the probability that the duel ends after the n^{th} round of shots?

(d) (4 points) the conditional probability that the duel ends after the n^{th} round of shots given that A is not hit?

(e) (4 points) the conditional probability that the duel ends after the n^{th} round of shots given that both duelists are hit?

5. (10 points) **Sock matching**

(a) (5 points) Suppose a drawer has 5 pairs of socks (10 socks total), with each pair having its own unique color. If 3 socks are chosen, what is the probability that none of them match?

(b) (5 points) In general, if a drawer has s pairs of socks ($2s$ socks total), with each pair having its own unique color. If r socks are chosen (with $r \leq s$), what is the probability that none of them match?

6. (20 points) **Lost in the national park**

You are lost in the National Park of Bandrika. Tourists comprise two-thirds of the visitors to the park, and give a correct answer to requests for directions with probability $\frac{3}{4}$. (Answers to repeated questions are independent, even if the question and the person are the same.) If you ask a Bandrikan for directions, the answer is always false.

(a) (5 points) You ask a passer-by whether the exit from the Park is East or West. The answer is East. What is the probability this is correct?

(b) (5 points) You ask the same person again, and receive the same reply. Show the probability that it is correct is $\frac{1}{2}$.

(c) (5 points) You ask the same person again, and receive the same reply. What is the probability that it is correct?

(d) (5 points) You ask for the fourth time, and receive the answer East. Show that the probability it is correct is $\frac{27}{70}$.