ECE 131A HW 1

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Problem 1

(a)

The sample space is just all the combinations of the two dice, with r denoting the bottom face of the red die and g denoting the bottom face of the green die. Therefore the sample space is just all the 16 orderd sets possible combinations of the possible results of r and g. In other words, the sample space is

$$\boxed{\begin{gathered}\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),\\(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}\end{gathered}}$$

(b)

(i)

$$E = \boxed{\{(1,1),(1,3),(3,1),(3,3)\}}$$
$$F = \boxed{\{(1,4),(2,3),(3,2),(4,1)\}}$$

(ii)

Since E requires all values to be odd, and thus their sum to be even and F requires the sum of the values to be odd (5), these sets are disjoint, thus:

$$E \cap F = \boxed{\emptyset}$$

(iii)

$$E \cup F = \boxed{\{(1,1),(1,3),(3,1),(3,3),(1,4),(2,3),(3,2),(4,1)\}}$$

(iv)

G^c is effectively just all value sof g and g such that $g + r \geq 7$, thus

$$G^c = \boxed{\{(3,4),(4,3),(4,4)\}}$$

Problem 2

Since $P(A) + P(B) = \frac{13}{12} > 1$ we have that the minimum for $P(A \cap B)$ is $1 - P(A) - P(B)$, which is $\frac{1}{12}$. Likewise, $P(A \cap B)$ is maximized when A and B totally overlap. In this case would be equal to the minimum of $P(A)$ or $P(B)$, so $P(A \cap B) = P(B) = \frac{1}{3}$.

Problem 3

The total possibilities for choosing M items from 100 items is $\binom{100}{M}$. Therefore we have that the probability p that m items are defective of the M chosen is

$$p = \begin{cases} 0 & \text{if } m > k \\ \frac{\binom{k}{m} \cdot \binom{(100-k)}{M-m}}{\binom{100}{M}} & \text{if } m \leq k \end{cases}$$

Problem 4

(a)

STATISTICS is a 10 letter word, therefore if every letter was unique, we would have 10! possible ways to arrange the words. However not every letter is unique, there are 2 occurences of I, and 3 occurences of S, and 3 occurences of T. Therefore the number of possible ways to arrange the letters is

$$\frac{10!}{2!3!3!} = \boxed{50400}$$

(b)

We can effectively treat the two "I"s together as one letter, so then we would have the total arrangements of the letters in STATISTICS with the two "I"s together is $\frac{9!}{3!3!}$. Therefore the probability of this occurence is

$$\frac{\frac{9!}{3!3!}}{50400} = \boxed{\frac{1}{5}}$$

Problem 5

(a)

Let us create a random variable C that represents which coin we flipped. Furthermore let the double headed coins be denoted as C_h and the double tailed coin denoted as C_t and C_n denote a normal coin. Likewise let F be a random variable that represents the lower face of the coin after it has been tossed, then we have:

$$\begin{aligned} P(F = \text{heads}) = P(C = C_h)P(F = \text{heads}|C = C_h)+ \\ &P(C = C_t)P(F = \text{heads}|C = C_t) + P(C = C_n)P(F = \text{heads}|C = C_n) \\ &= \frac{2}{5} + \frac{2}{5}\frac{1}{2} \\ &= \boxed{\frac{3}{5}} \end{aligned}$$

(b)

The only way for both the face showing him and the lower face are both heads, is only possible if both sides are if the coin is a double headed coin. Therefore we have that

$$\begin{aligned} P(C = C_n|F = \text{heads}) &= \frac{P(C = C_n)P(F = \text{heads}|C = C_n)}{P(F = \text{heads})} \\ &= \frac{\frac{2}{5}}{\frac{3}{5}} \\ &= \boxed{\frac{2}{3}} \end{aligned}$$

Problem 6

The probability of getting an error is effectively the probability of getting 3 or more binary errors. This is 1 minus the probability of getting 2 or less binary errors, in 5 transmissions. This is just the CDF of a binomial distribution with $n = 5$ and $p = 0.2$, evaluated at $x = 2$, which is 0.99144. Therefore we have that

$$P_{error} = 1 - 0.99144 = \boxed{0.00856}$$

Problem 7

(a)

Since order doesn't matter, we have the following possibilities for Jane's children BBB,GGG, BBG, GGB, with boy denoted by B, and girl denoted as G. Since BBG and GGB can have three possible permutations, we have that the probabilities for each occurrence is $\frac{1}{8}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$ for BBB, GGG, GBB, BGG respectively. We have that

$$P(B|A) = \frac{1}{2} = P(B)$$

Therefore B is independent of A. Likewise we have

$$P(C|B) = \frac{1}{2} = P(C)$$

(b)

No, because if C is true, then the family has both a boy and a girl, and thus A is not true.

(c)

No, let the probability of getting an boy be p, and thus the probability of getting a girl is $1-p$, then we have that the probabilities of getting BBB is p^3, GGG is $(1-p)^3$, GBB $3p(1-p)^2$, and BBG $3p^2(1-p)$. Thus we have that

$$P(B) = 3p(1-p)^2 + (1-p)^3$$

but

$$P(B|A) = \frac{(1-p)^3}{p^3 + (1-p)^3}$$

Which are different unless $p = \frac{1}{2}$, which is not the case.