

ECE 131A HW 3

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Problem 1

Let C_{ij} be the event that the i th person grabs the j th person's coat, then we have that in order for all of them to have grabbed the wrong coat we could have two possible sequences, $C_{13}C_{21}C_{32}$ or $C_{12}C_{23}C_{31}$, both of these cases can happen with probability $\frac{1}{6}$ therefore the probability that everyone grabs the

wrong coat is $\boxed{\frac{1}{3}}$

Problem 2

(a)

$$\boxed{\frac{1}{1000}}$$

(b)

Since 1003 is not in the range of pages then the probability

$$P(X = 1003) = \boxed{0}$$

(c)

$$\boxed{\frac{1}{1000}}$$

(d)

There are 123 possible pages therefore

$$P(X \leq 122) = \boxed{\frac{123}{1000}}$$

(e)

$$P(12 \leq X \leq 17) = \boxed{\frac{6}{1000}}$$

Problem 3

(a)

$$P(X = j) = \left(\frac{4}{5}\right)^{j-1} \frac{1}{5}$$

(b)

$$P(X = j) = \boxed{\frac{1}{5}, j = 1, 2, 3, 4, 5}$$

Problem 4

(a)

Let the bernoulli random variable X_i be the event that the i th plate is isolated, then we have that

$$P(X_i = 1) = \frac{10}{14} \frac{9}{13} \frac{5}{15} = 0.4945$$

Therefore

$$E[X] = 15E[X_i] = \boxed{7.417}$$

(b)

Let the bernoulli random variable Y_i be the event that the i th plate is semi happy, then we have

$$P(Y_i = 1) = 2 \frac{10}{13} \frac{5}{15} \frac{4}{14} = 0.439$$

Therefore

$$E[Y] = 15E[Y_i] = \boxed{6.593}$$

(c)

Let the bernoulli random variable Z_i be the event that the i th plate is joyous, We have

$$P(Z_i = 1) = 1 - P(X_i = 1) - P(Y_i = 1) = 0.0665$$

Therefore

$$E[Z] = 15E[Z_i] = \boxed{0.989}$$

Problem 5

(a)

$$\boxed{(1-p)^{n-1}p}$$

(b)

$$\boxed{(1-p)^{n-2}p^2}$$

(c)

The success are distributed in a way that is independent on where it occurs so the probability that the k success occurs at j_1, j_2, \dots, j_k is always the same no matter what j_1, j_2, \dots, j_k so long as $0 < j_1, j_2, \dots, j_k \leq n$

Problem 6

(a)

Let the number of accidents be denoted by a random variable N . N is distributed as a poisson random variable with $\lambda = 3$. Therefore

$$P(N = 5) = \boxed{\frac{3^5 e^{-3}}{5!}}$$

(b)

$$P(N < 3) = e^{-3} \left(1 + \frac{3}{2} + \frac{9}{3!} \right)$$

(c)

$$P(N \geq 2) = 1 - P(N < 2) = \boxed{1 - e^{-3} \left(1 + \frac{3}{5!} \right)}$$

Problem 7

(a)

Rate for 1 hr for fans of both teams is 14 per hour, so for 3 hours it would be $\boxed{42}$.

(b)

if the rat is 14 per hour, then the rate for 20 minutes would be $14/3$ per 20 minutes, since this will still be distributed as a Poisson we have that the probability of only 1 fan of either the Yankees or the Red Sox enter the store

is just $\boxed{e^{-\frac{14}{3}} \frac{14}{3}}$.

Problem 8

(a)

There are $\binom{N}{n}$ ways to pick out n balls from the N balls in the urn. But to choose k blue balls from the b balls we have $\binom{b}{k}$ ways to do so, and we have $\binom{r}{n-k}$ ways to choose the remaining $n-k$ red balls. Therefore the probability of choosing k blue balls is

$$\boxed{\frac{\binom{b}{k} \binom{r}{n-k}}{\binom{N}{n}}}$$

(b)

$$\begin{aligned} \lim_{N, b, r \rightarrow \infty} \frac{\binom{b}{k} \binom{r}{n-k}}{\binom{N}{n}} &= \lim_{N, b, r \rightarrow \infty} \binom{n}{k} \frac{b \cdot (b-1) \cdots (b-k+1) \cdot r \cdot (r-1) \cdots (r-(n-k)+1)}{N \cdot (N-1) \cdots (N-n+1)} \\ &= \lim_{N, b, r \rightarrow \infty} \binom{n}{k} \frac{b^k \cdot r^{n-k}}{N^n} \\ &= \binom{n}{k} p^k (1-p)^{n-k} \end{aligned}$$