ECE 131A Quiz 1

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Problem 1

(a)

For a specific color of paint there is only 1 way to paint the rocks, so for the 3 paints there is 3 ways to paint the rocks. Therefore the probability of all the rocks being the same color is

$$\frac{3}{3^6} = \frac{1}{3^5} = \boxed{0.004115226337}$$

(b)

For any two colors there are 2^6 ways to paint the rocks, but we need to subtract 2 for the 2 ways to paint the rocks with the same color. Therefore, since there is 3 paints there is $\binom{3}{2}$ ways to choose 2 paint colors, Therefore the probability is

$$\frac{\binom{3}{2}(2^6-2)}{3^6} = \boxed{0.2551440329}$$

(c)

For all three colors of paint, there are 3^6 ways to paint the rocks but we will need to subtract the $\binom{3}{2}(2^6 - 2)$ to paint the rocks exactly 2 colors and the 3 ways to paint the rocks one color, so we will have the probability is

$$\frac{3^6 - 3 - \binom{3}{2}(2^6 - 2) - 3}{3^6} = \boxed{0.7407407407}$$

Problem 2

(a)

The probability of getting the white glove for any draw is the same since he replaces the glove, therefore $P(A_j) = \boxed{\left(\frac{4}{5}\right)^{j-1} \frac{1}{5}}$ for any j

(b)

For the first draw the probability of getting the white glove is just 1/5 so $P(B_1) = \frac{1}{5}$

For the second draw we would have not have drawn the white glove in the first draw which can happen with probability $\frac{4}{5}$ and then drawn it in the second draw which can happen with probability $\frac{1}{4}$, therefore $P(B_2) = \frac{4}{5}\frac{1}{4} = \boxed{\frac{1}{5}}$

For the third draw we would have not have drawn the white glove in the first draw which can happen with probability $\frac{4}{5}$ and then drawn it in the second draw which can happen with probability $\frac{3}{4}$, and then drawn it in the third draw which can happen with probability $\frac{1}{3}$, therefore $P(B_3) = \frac{4}{5}\frac{3}{4}\frac{1}{3} = \boxed{\frac{1}{5}}$

For the fourth draw we would have not have drawn the white glove in the first draw which can happen with probability $\frac{4}{5}$ and then drawn it in the second draw which can happen with probability $\frac{3}{4}$, and then not have drawn it in the third draw which can happen with probability $\frac{2}{3}$, and then drawn it in the fourth draw which can happen with probability $\frac{1}{2}$, therefore $P(B_4) = \frac{4}{5}\frac{3}{4}\frac{2}{3}\frac{1}{2} = \boxed{\frac{1}{5}}$

For the fifth draw we would have not have drawn the white glove in the first draw which can happen with probability $\frac{4}{5}$ and then drawn it in the second draw which can happen with probability $\frac{3}{4}$, and then not drawn it in the third draw which can happen with probability $\frac{2}{3}$, and then not drawn it in the fourth draw which can happen with probability $\frac{1}{2}$, and then drawn it in the fifth draw which can happen with probability $\frac{1}{1}$, therefore $P(B_5) = \frac{4}{5}\frac{3}{4}\frac{2}{3}\frac{1}{2}\frac{1}{1} = \boxed{\frac{1}{5}}$

Problem 3

(a)

For each character there are 26 lower case letters, 26 upper case letters 10 numbers, and 7 special symbols, so for a specific entry code of length l there are

$$69^l - 62^l$$

. So therefore there are entry codes that are between 5 to 7 characters long

$$69^5 - 62^5 + 69^6 - 62^6 + 69^7 - 62^7 = \boxed{3,976,504,472,395}$$

(b)

Therefore it would take $3,976,504,472,395ms$ or $46,000$ days or $\boxed{126}$ years to try all the possible entry codes