ECE 131A HW 1

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Problem 1

(a)

The sample space is just all the combinations of the two dice, with r denoting the bottom face of the red die and g denoting the bottom face of the green die. Therefore the sample space is just all the 16 orderd sets possible combinations of the possible results of r and g. In other words, the sample space is

$$\boxed{\begin{gathered}\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),\\(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}\end{gathered}}$$

(b)

(i)

$$E = \boxed{\{(1,1),(1,3),(3,1),(3,3)\}}$$
$$F = \boxed{\{(1,4),(2,3),(3,2),(4,1)\}}$$

(ii)

Since E requires all values to be odd, and thus their sum to be even and F requires the sum of the values to be odd (5), these sets are disjoint, thus:

$$E \cap F = \boxed{\emptyset}$$

(iii)

$$E \cup F = \boxed{\{(1,1),(1,3),(3,1),(3,3),(1,4),(2,3),(3,2),(4,1)\}}$$

(iv)

G^c is effectively just all value sof g and g such that $g + r \geq 7$, thus

$$G^c = \boxed{\{(3,4),(4,3),(4,4)\}}$$

Problem 2

Since $P(A) + P(B) = \frac{13}{12} > 1$ we have that the minimum for $P(A \cap B)$ is $1 - P(A) - P(B)$, which is $\frac{1}{12}$. Likewise, $P(A \cap B)$ is maximized when A and B totally overlap. In this case would be equal to the minimum of $P(A)$ or $P(B)$, so $P(A \cap B) = P(B) = \frac{1}{3}$.

Problem 3

The total possibilities for choosing M items from 100 items is $\binom{100}{M}$. Therefore we have that the probability p that m items are defective of the M chosen is

$$p = \begin{cases} 0 & \text{if } m > k \\ \frac{\binom{k}{m}\cdot\binom{(100-k)}{M-m}}{\binom{100}{M}} & \text{if } m \leq k \end{cases}$$

Problem 4

(a)

STATISTICS is a 10 letter word, therefore if every letter was unique, we would have 10! possible ways to arrange the words. However not every letter is unique, there are 2 occurences of I, and 3 occurences of S, and 3 occurences of T. Therefore the number of possible ways to arrange the letters is

$$\frac{10!}{2!3!3!} = \boxed{50400}$$

(b)

We can effectively treat the two "I"s together as one letter, so then we would have the total arrangements of the letters in STATISTICS with the two "I"s together is $\frac{9!}{3!3!}$. Therefore the probability of this occurence is

$$\frac{\frac{9!}{3!3!}}{50400} = \boxed{\frac{1}{5}}$$

Problem 5