ECE 131A HW 1

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Problem 1

(a)

$$\begin{aligned}
P[B_0] &= P[A_0]P[B_0|A_0] + P[A_1]P[B_0|A_1] \\
&= \boxed{\frac{1}{2} \cdot (1 - \epsilon_1 + \epsilon_2)}
\end{aligned}$$

(b)

We have

$$P[B_1] = 1 - P[B_0] = \frac{1}{2} \cdot (\epsilon_1 + 1 - \epsilon_2)$$

Therefore from Bayes law we have

$$\begin{aligned}
P[A_1|B_1] &= \frac{P[B_1|A_1]P[A_1]}{P[B_1]} \\
&= \frac{1 - \epsilon_2}{\epsilon_1 + 1 - \epsilon_2}
\end{aligned}$$

$$
\begin{aligned}
P[A_0|B_1] &= \frac{P[B_1|A_0]P[A_0]}{P[B_1]} \\
&= \frac{\epsilon_1}{\epsilon_1 + 1 - \epsilon_2}
\end{aligned}
$$

Therefore we have for $\epsilon_1 = 0.25$ and $\epsilon_2 = 0.5$: we will have

$$
\begin{aligned}
P[A_1|B_1] &= \frac{1 - 0.5}{0.25 + 1 - 0.5} \\
&= \frac{2}{3} \\
P[A_0|B_1] &= \frac{0.25}{0.25 + 1 - 0.5} \\
&= \frac{1}{3}
\end{aligned}
$$

Therefore A_1 will be more likely.

Problem 2

(a)

(i)

$$\boxed{\binom{20}{15}}$$

(ii)

$$\boxed{\binom{15}{4}}$$

(b)

If we care about the order in which we place the balls in the buckets, the probability that we place all 5 balls in diffrent buckets is $\frac{1}{5^5}$. There are 5! ways to order the balls to place into the buckets, so since we do not care about the order in which we place the balls in the buckets, the probability that we place all 5 balls in diffrent buckets is $\frac{1}{5^5} \cdot 5! = 0.0384$.

(c)

This is a multnomial permutation so we have

$$\frac{9!}{4!2!3!} = \boxed{1260}$$

Problem 3

Problem 4

(a)

The probability that A is not hit is simply $\boxed{1 - p_A}$

(b)

The probability that both are hit is simply $\boxed{p_A p_B}$

(c)

Problem 5

(a)

There are $\binom{10}{3}$ to choose 3 socks from 10 if we do not care about the color of the socks we pick. If we do care about not selecting a pair of socks with the same colors we have $\frac{1}{3!} \cdot 10 \cdot 8 \cdot 7$ ways to choose 3 socks from 10. therefore the probability is $\frac{\frac{1}{3!} \cdot 10 \cdot 8 \cdot 7}{\binom{10}{3}} = \boxed{0.77777777777}$.

(b)

There are $\binom{2s}{r}$ ways to choose r socks from $2s$ if we do not care about the color of the socks we pick. If we do care about selecting the socks such that no two have the same color we have that we can do that in $\frac{2s}{r!}\frac{(2s-2)!}{(2s-r-1)!}$ ways. Therefore the probability is

$$\begin{aligned}\frac{\frac{2s}{r!}\frac{(2s-2)!}{(2s-r-1)!}}{\binom{2s}{r}} &= \frac{(2s)\frac{(2s-2)!}{(r!)(2s-r-1)!}}{\frac{2s!}{r!(2s-r)!}} \\ &= \boxed{\frac{2s-r}{2s-1}}\end{aligned}$$

Problem 6

(a)

Let the random event denoting whether we recived the correct answer be C and the random event denoting whether we asked a Tourist be T. Then we have

$$P(C) = P(C|T)P(T) + P(C|T^c)P(T^c)$$
$$P(C) = \frac{2}{3}\frac{3}{4} = \boxed{\frac{1}{2}}$$

(b)

Let the random event A_1 denote our first response, and A_2 denote our second response we have that

$P(C|A_1 = A_2) = P(T|A_1 = A_2)P(C|A_1 = A_2, T) + P(T^c|A_1 = A_2)P(C|A_1 = A_2, T^c)$

Since

$$\begin{aligned}
P(T|A_1 = A_2) &= \frac{P(T)P(A_1 = A_2|T)}{P(A_1 = A_2)} \\
&= \frac{P(T)P(A_1 = A_2|T)}{P(A_1 = A_2|T)P(T) + P(A_1 = A_2|T^c)P(T^c)} \\
&= \frac{\left(\frac{3^2}{4^2} + \frac{1}{4^2}\right) \cdot \frac{2}{3}}{\left(\frac{3^2}{4^2} + \frac{1}{4^2}\right) \cdot \frac{2}{3} + \frac{1}{3}} \\
&= \frac{20}{36}
\end{aligned}$$

And

$$P(C|A_1 = A_2, T) = \frac{\left(\frac{3}{4}\right)^2}{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \frac{9}{10}$$

and

$$P(C|A_1 = A_2, T^c) = 0$$

Therefore we have

$$\begin{aligned}
P(C|A_1 = A_2) &= \frac{20}{36} \cdot \frac{9}{10} + 0 \\
&= \boxed{\frac{1}{2}}
\end{aligned}$$

(c)

Let the random event A_3 denote the third response we get

$P(C|A_1 = A_2 = A_3) = P(T|A_1 = A_2 = A_3)P(C|A_1 = A_2 = A_3, T) + P(T^c|A_1 = A_2 = A_3)P(C|A_1$

Since

$$\begin{aligned}
P(T|A_1 = A_2 = A_3) &= \frac{P(T)P(A_1 = A_2 = A_3|T)}{P(A_1 = A_2 = A_3)} \\
&= \frac{P(T)P(A_1 = A_2 = A_3|T)}{P(A_1 = A_2 = A_3|T)P(T) + P(A_1 = A_2 = A_3|T^c)P(T^c)} \\
&= \frac{\left(\frac{3^3}{4^3} + \frac{1}{4^3}\right) \cdot \frac{2}{3}}{\left(\frac{3^3}{4^3} + \frac{1}{4^3}\right) \cdot \frac{2}{3} + \frac{1}{3}} \\
&= \frac{56}{120}
\end{aligned}$$

And

$$P(C|A_1 = A_2 = A_3, T) = \frac{\left(\frac{3}{4}\right)^3}{\left(\frac{3}{4}\right)^3 + \left(\frac{1}{4}\right)^3} = \frac{27}{28}$$

and

$$P(C|A_1 = A_2, T^c) = 0$$

Therefore we have

$$\begin{aligned}
P(C|A_1 = A_2) &= \frac{56}{120} \cdot \frac{27}{28} + 0 \\
&= \boxed{\frac{9}{20}}
\end{aligned}$$

(d)

Let the random event A_4 denote the fourth response we get

$P(C|A_1 = A_2 = A_3 = A_4) = P(T|A_1 = A_2 = A_3 = A_4)P(C|A_1 = A_2 = A_3 = A_4, T) + P(T^c|A_1 =$

Since

$$
\begin{aligned}
P(T|A_1 = A_2 = A_3 = A_4) &= \frac{P(T)P(A_1 = A_2 = A_3 = A_4|T)}{P(A_1 = A_2 = A_3 = A_4)} \\
&= \frac{P(T)P(A_1 = A_2 = A_3 = A_4|T)}{P(A_1 = A_2 = A_3 = A_4|T)P(T) + P(A_1 = A_2 = A_3 = A_4|T^c)P(T^c)} \\
&= \frac{\left(\frac{3^4}{4^4} + \frac{1}{4^4}\right) \cdot \frac{2}{3}}{\left(\frac{3^4}{4^4} + \frac{1}{4^4}\right) \cdot \frac{2}{3} + \frac{1}{3}} \\
&= \frac{82}{210}
\end{aligned}
$$

And

$$
P(C|A_1 = A_2 = A_3 = A + 4, T) = \frac{\left(\frac{3}{4}\right)^4}{\left(\frac{3}{4}\right)^4 + \left(\frac{1}{4}\right)^4} = \frac{81}{82}
$$

and

$$
P(C|A_1 = A_2 = A_3 = A_4, T^c) = 0
$$

Therefore we have

$$
\begin{aligned}
P(C|A_1 = A_2 = A_3 = A_4) &= \frac{82}{210} \cdot \frac{81}{82} + 0 \\
&= \boxed{\frac{27}{70}}
\end{aligned}
$$