ECE 131A HW 1

Lawrence Liu

October 3, 2022

Problem 1

(a)

The sample space is just all the combinations of the two dice, with r denoting the bottom face of the red die and g denoting the bottom face of the green die. Therefore the sample space is just all the 16 orderd sets possible combinations of the possible results of r and g. In other words, the sample space is

$$\boxed{\begin{aligned}\{&(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),\\&(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}\end{aligned}}$$

(b)

(i)

$$E = \boxed{\{(1,1),(1,3),(3,1),(3,3)\}}$$
$$F = \boxed{\{(1,4),(2,3),(3,2),(4,1)\}}$$

(ii)

Since E requires all values to be odd, and thus their sum to be even and F requires the sum of the values to be odd (5), these sets are disjoint, thus:

$$E \cap F = \boxed{\emptyset}$$

(iii)

$$E \cup F = \boxed{\{(1,1),(1,3),(3,1),(3,3),(1,4),(2,3),(3,2),(4,1)\}}$$

(iv)

G^c is effectively just all value sof g and g such that $g + r \geq 7$, thus

$$G^c = \boxed{\{(3,4),(4,3),(4,4)\}}$$

Problem 2

Since $P(A) + P(B) = \frac{13}{12} > 1$ we have that the minimum for $P(A \cap B)$ is $1 - P(A) - P(B)$, which is $\frac{1}{12}$. Likewise, $P(A \cap B)$ is maximized when A and B totally overlap. In this case would be equal to the minimum of $P(A)$ or $P(B)$, so $P(A \cap B) = P(B) = \frac{1}{3}$.

Problem 3

The total possibilities for choosing M items from 100 items is $\binom{100}{M}$. Therefore we have that the probability p that m items are defective of the M chosen is

$$p = \begin{cases} 0 & \text{if } m > k \\ \frac{\binom{k}{m}\cdot\binom{(100-k)}{M-m}}{\binom{100}{M}} & \text{if } m \leq k \end{cases}$$

Problem 4

(a)

STATISTICS is a 10 letter word, therefore if every letter was unique, we would have 10! possible ways to arrange the words. However not every letter is unique, there are 2 occurences of I, and 3 occurences of S, and 3 occurences of T. Therefore the number of possible ways to arrange the letters is

$$\frac{10!}{2!3!3!} = \boxed{50400}$$

(b)

We can effectively treat the two "I"s together as one letter, so then we would have the total arrangements of the letters in STATISTICS with the two "I"s together is $\frac{9!}{3!3!}$. Therefore the probability of this occurence is

$$\frac{\frac{9!}{3!3!}}{50400} = \boxed{\frac{1}{5}}$$

Problem 5

(a)

Let us create a random variable C that represents which coin we flipped. Furthermore let the double headed coins be denoted as C_h and the double tailed coin denoted as C_t and C_n denote a normal coin. Likewise let F be a random variable that represents the lower face of the coin after it has been tossed, then we have:

$$
\begin{aligned}
P(F = \text{heads}) = P(C = C_h)P(F = \text{heads}|C = C_h) + \\
&P(C = C_t)P(F = \text{heads}|C = C_t) + P(C = C_n)P(F = \text{heads}|C = C_n) \\
&= \frac{2}{5} + \frac{2}{5}\frac{1}{2} \\
&= \boxed{\frac{3}{5}}
\end{aligned}
$$

(b)

The only way for both the face showing him and the lower face are both heads, is only possible if both sides are if the coin is a double headed coin. Therefore we have that

$$
\begin{aligned}
P(C = C_n|F = \text{heads}) &= \frac{P(C = C_n)P(F = \text{heads}|C = C_n)}{P(F = \text{heads})} \\
&= \frac{\frac{2}{5}}{\frac{3}{5}} \\
&= \boxed{\frac{2}{3}}
\end{aligned}
$$

Problem 6

The probability of getting an error is effectively the probability of getting 3 or more binary errors. This is 1 minus the probability of getting 2 or less binary errors, in 5 transmissions. This is just the CDF of a binomial distribution with $n = 5$ and $p = 0.2$, evaluated at $x = 2$, which is 0.99144. Therefore we have that

$$P_{error} = 1 - 0.99144 = \boxed{0.00856}$$

Problem 7

(a)

Since order doesn't matter, we have the following possibilities for Jane's children BBB,GGG, BBG, GGB, with boy denoted by B, and girl denoted as G. We have that

$$P(B|A) = \frac{1}{2} = P(B)$$

Therefore B is independent of A. Likewise we have

$$P(C|B) = \frac{1}{2} = P(C)$$

(b)

No, because if C is true, then the family has both a boy and a girl, and thus A is not true.

(c)

No, let