ECE 131A Quiz 1

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November 4, 2022

Problem 1

Problem 2

(a)

This would be a hypergeometric distribution. The parameters would be $K = 7$, $N = 1000$, $n = 10$. So the pmf for the number of green envolopes would be

$$p(k) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

(b)

The probability of getting 1 envelope is:

$$p(1) = \frac{\binom{7}{1}\binom{993}{9}}{\binom{1000}{10}} = 0.06629133689$$

(c)

This would be a binomial distribution. The parameters would be $N = 10$ and $p = \frac{7}{1000}$, so the pmf for the number of green envolopes k is:

$$p(k) = \binom{N}{k} p^k (1-p)^{N-k}$$

(d)

The probability of getting 1 envelope is:

$$10 \left(\frac{7}{1000}\right) \left(\frac{993}{1000}\right)^9 = \boxed{0.0657}$$

Problem 3

(a)

The phone ringing, email, arriving, and next computer beeping are possion processes, so the probability of no phone ringing for the next 10 seconds is

$$\frac{(\frac{10}{30})^0}{0!} e^{-\frac{10}{30}} = e^{-\frac{1}{3}}$$

Likewise the probability of no email arriving for the next 10 seconds is

$$\frac{(\frac{10}{20})^0}{0!} e^{-\frac{10}{20}} = e^{-\frac{1}{2}}$$

And the probability of no computer beeping for the next 10 seconds is

$$\frac{(\frac{10}{15})^0}{0!} e^{-\frac{10}{15}} = e^{-\frac{2}{3}}$$

Thus the total probability of all of these events is

$$e^{-\frac{1}{3}} e^{-\frac{1}{2}} e^{-\frac{2}{3}} = \boxed{e^{-\frac{3}{2}}}$$

(b)

The probability of the phone ringing, email arriving, and computer beeping are possion processes. So the probability of none of these events happening for t seconds is:

$$e^{-\frac{t}{30}}e^{-\frac{t}{20}}e^{-\frac{t}{15}} = \boxed{e^{-\frac{3t}{20}}}$$

(c)

Since these are possion processes, the sum of them would also be a possion process with rate of $30 + 20 + 15 + 65$ seconds, so the probability density function for the time between events is an exponential with $\lambda = 65$ so it would be:

$$\boxed{f(t) = 65e^{-65t}}$$