

ECE 131A HW 1

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Problem 1

(a)

$$\begin{aligned} P[B_0] &= P[A_0]P[B_0|A_0] + P[A_1]P[B_0|A_1] \\ &= \boxed{\frac{1}{2} \cdot (1 - \epsilon_1 + \epsilon_2)} \end{aligned}$$

(b)

We have

$$P[B_1] = 1 - P[B_0] = \frac{1}{2} \cdot (\epsilon_1 + 1 - \epsilon_2)$$

Therefore from Bayes law we have

$$\begin{aligned} P[A_1|B_1] &= \frac{P[B_1|A_1]P[A_1]}{P[B_1]} \\ &= \frac{1 - \epsilon_2}{\epsilon_1 + 1 - \epsilon_2} \end{aligned}$$

$$\begin{aligned}
 P[A_0|B_1] &= \frac{P[B_1|A_0]P[A_0]}{P[B_1]} \\
 &= \frac{\epsilon_1}{\epsilon_1 + 1 - \epsilon_2}
 \end{aligned}$$

Therefore we have for $\epsilon_1 = 0.25$ and $\epsilon_2 = 0.5$: we will have

$$\begin{aligned}
 P[A_1|B_1] &= \frac{1 - 0.5}{0.25 + 1 - 0.5} \\
 &= \frac{2}{3} \\
 P[A_0|B_1] &= \frac{0.25}{0.25 + 1 - 0.5} \\
 &= \frac{1}{3}
 \end{aligned}$$

Therefore A_1 will be more likely.

Problem 2

(a)

(i)

$$\boxed{\binom{20}{15}}$$

(ii)

$$\boxed{\binom{15}{4}}$$

0.1 (b)

If we care about the order in which we place the balls in the buckets, the probability that we place all 5 balls in different buckets is $\frac{1}{5^5}$. There are $5!$ ways to order the balls to place into the buckets, so since we do not care about the order in which we place the balls in the buckets, the probability that we place all 5 balls in different buckets is $\frac{1}{5^5} \cdot 5! = 0.0384$.

(c)

This is a multinomial permutation so we have

$$\frac{9!}{4!2!3!} = \boxed{1260}$$

Problem 3

Problem 4

Problem 5

Problem 6

(a)

Let the random event denoting whether we received the correct answer be C and the random event denoting whether we asked a Tourist be T . Then we have

$$P(C) = P(C|T)P(T) + P(C|T^c)P(T^c)$$

$$P(C) = \frac{2}{3} \frac{3}{4} = \boxed{\frac{1}{2}}$$

(b)

Let the random event A_1 denote our first response, and A_2 denote our second response we have that

$$P(C|A_1 = A_2) = P(T|A_1 = A_2)P(C|A_1 = A_2, T) + P(T^c|A_1 = A_2)P(C|A_1 = A_2, T^c)$$

Since

$$\begin{aligned} P(T|A_1 = A_2) &= \frac{P(T)P(A_1 = A_2|T)}{P(A_1 = A_2)} \\ &= \frac{P(T)P(A_1 = A_2|T)}{P(A_1 = A_2|T)P(T) + P(A_1 = A_2|T^c)P(T^c)} \\ &= \frac{\left(\frac{3^2}{4^2} + \frac{1}{4^2}\right) \cdot \frac{2}{3}}{\left(\frac{3^2}{4^2} + \frac{1}{4^2}\right) \cdot \frac{2}{3} + \frac{1}{3}} \\ &= \frac{20}{36} \end{aligned}$$

And

$$P(C|A_1 = A_2, T) = \frac{\left(\frac{3}{4}\right)^2}{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2} = \frac{9}{10}$$

and

$$P(C|A_1 = A_2, T^c) = 0$$

Therefore we have

$$\begin{aligned} P(C|A_1 = A_2) &= \frac{20}{36} \cdot \frac{9}{10} + 0 \\ &= \boxed{\frac{1}{2}} \end{aligned}$$

(c)

Let the random event A_3 denote the third response we get

$$P(C|A_1 = A_2 = A_3) = P(T|A_1 = A_2 = A_3)P(C|A_1 = A_2 = A_3, T) + P(T^c|A_1 = A_2 = A_3)P(C|A_1 = A_2 = A_3, T^c)$$

Since

$$\begin{aligned} P(T|A_1 = A_2 = A_3) &= \frac{P(T)P(A_1 = A_2 = A_3|T)}{P(A_1 = A_2 = A_3)} \\ &= \frac{P(T)P(A_1 = A_2 = A_3|T)}{P(A_1 = A_2 = A_3|T)P(T) + P(A_1 = A_2 = A_3|T^c)P(T^c)} \\ &= \frac{\left(\frac{3^3}{4^3} + \frac{1}{4^3}\right) \cdot \frac{2}{3}}{\left(\frac{3^3}{4^3} + \frac{1}{4^3}\right) \cdot \frac{2}{3} + \frac{1}{3}} \\ &= \frac{56}{120} \end{aligned}$$

And

$$P(C|A_1 = A_2 = A_3, T) = \frac{\left(\frac{3}{4}\right)^3}{\left(\frac{3}{4}\right)^3 + \left(\frac{1}{4}\right)^3} = \frac{27}{28}$$

and

$$P(C|A_1 = A_2, T^c) = 0$$

Therefore we have

$$\begin{aligned} P(C|A_1 = A_2) &= \frac{56}{120} \cdot \frac{27}{28} + 0 \\ &= \boxed{\frac{9}{20}} \end{aligned}$$

(d)

Let the random event A_4 denote the fourth response we get

$$P(C|A_1 = A_2 = A_3 = A_4) = P(T|A_1 = A_2 = A_3 = A_4)P(C|A_1 = A_2 = A_3 = A_4, T) + P(T^c|A_1 = A_2 = A_3 = A_4)P(C|A_1 = A_2 = A_3 = A_4, T^c)$$

Since

$$\begin{aligned}
P(T|A_1 = A_2 = A_3 = A_4) &= \frac{P(T)P(A_1 = A_2 = A_3 = A_4|T)}{P(A_1 = A_2 = A_3 = A_4)} \\
&= \frac{P(T)P(A_1 = A_2 = A_3 = A_4|T)}{P(A_1 = A_2 = A_3 = A_4|T)P(T) + P(A_1 = A_2 = A_3 = A_4|T^c)P(T^c)} \\
&= \frac{\left(\frac{3^4}{4^4} + \frac{1}{4^4}\right) \cdot \frac{2}{3}}{\left(\frac{3^4}{4^4} + \frac{1}{4^4}\right) \cdot \frac{2}{3} + \frac{1}{3}} \\
&= \frac{82}{210}
\end{aligned}$$

And

$$P(C|A_1 = A_2 = A_3 = A + 4, T) = \frac{\left(\frac{3}{4}\right)^4}{\left(\frac{3}{4}\right)^4 + \left(\frac{1}{4}\right)^4} = \frac{81}{82}$$

and

$$P(C|A_1 = A_2 = A_3 = A_4, T^c) = 0$$

Therefore we have

$$\begin{aligned}
P(C|A_1 = A_2 = A_3 = A_4) &= \frac{82}{210} \cdot \frac{81}{82} + 0 \\
&= \boxed{\frac{27}{70}}
\end{aligned}$$