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Due Saturday, 22 October 2022 by 11:59 PM, uploaded to Gradescope. Resources: Lectures 1-5, Homework 2, Discussions 3-4. 10 points total.

1. (3 points) Coloring rocks

Six rocks are sitting in a straight line. We paint them, using up to three colors (say, R's, W's, and B's). Suppose all of the $3^6 = 729$ outcomes are equally likely.

- (a) (1 point) Find the probability that exactly 1 color of paint is used.
- (b) (1 point) Find the probability that exactly 2 colors of paint are used.
- (c) (1 point) Find the probability that all 3 colors of paint are used.

2. (4 points) Searching for gloves

A matching pair of blue gloves, a matching pair of red gloves, and one lone white right-handed glove are in a drawer. The gloves are pulled out of the drawer, one at a time.

- (a) (2 points) Suppose that a person is looking for the white glove. He repeatedly does the following: He pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he replaces the glove in the drawer and starts to check again, i.e., he reaches blindly into the drawer of 5 gloves. He continues to do this over and over until he finds the white glove, and then he stops. Let A_j denote the event that he successfully discovers the white glove (for the first time) on his j^{th} attempt. Find $P(A_j)$.
- (b) (2 points) Now suppose that he searches for the white glove but, if he pulls a different colored glove from the drawer, he does not replace it. So he pulls out a glove, checks the color, and if it is white, he stops. If it is not white, then he permanently discards the glove and starts to check again, i.e., he reaches blindly into the drawer with the gloves that remain. He continues to do this over and over again until he finds the white glove, and then he stops. Let B_j denote the event that he successfully discovers the white glove (for the first time) on his j^{th} attempt. Find $P(B_1), P(B_2), P(B_3), P(B_4), \text{and} P(B_5)$.

3. (3 points) Decoding the entry code of Engineering IV

The entry code of Engineering IV building is created using the following specifications:

- Length of 5 to 7 characters
- Includes at least one special symbols {!,@,#,\$,%,&,*}
- No spaces
- May contain numbers {0-9}, lower and upper case letters(a-z,A-Z).
- Is case-sensitive.
- (a) (2 points) How many entry codes are there to Engineering IV?
- (b) (1 points) How long would it take to try (in years) all entry codes if an entry code could be entered in a millisecond?