ECE 131A HW 1

Lawrence Liu

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Problem 1

(a)

$$
\begin{aligned}
P[B_0] &= P[A_0]P[B_0|A_0] + P[A_1]P[B_0|A_1] \\
&= \boxed{\frac{1}{2} \cdot (1 - \epsilon_1 + \epsilon_2)}
\end{aligned}
$$

(b)

We have

$$
P[B_1] = 1 - P[B_0] = \frac{1}{2} \cdot (\epsilon_1 + 1 - \epsilon_2)
$$

Therefore from Bayes law we have

$$
\begin{aligned}
P[A_1|B_1] &= \frac{P[B_1|A_1]P[A_1]}{P[B_1]} \\
&= \frac{1 - \epsilon_2}{\epsilon_1 + 1 - \epsilon_2}
\end{aligned}
$$

$$
\begin{aligned}
P[A_0|B_1] &= \frac{P[B_1|A_0]P[A_0]}{P[B_1]} \\
&= \frac{\epsilon_1}{\epsilon_1 + 1 - \epsilon_2}
\end{aligned}
$$

Therefore we have for $\epsilon_1 = 0.25$ and $\epsilon_2 = 0.5$: we will have

$$
\begin{aligned}
P[A_1|B_1] &= \frac{1 - 0.5}{0.25 + 1 - 0.5} \\
&= \frac{2}{3} \\
P[A_0|B_1] &= \frac{0.25}{0.25 + 1 - 0.5} \\
&= \frac{1}{3}
\end{aligned}
$$

Therefore A_1 will be more likely.

Problem 2

(a)

(i)

$$\boxed{\binom{20}{15}}$$

(ii)

$$\boxed{\binom{15}{4}}$$

0.1 (b)

If we care about the order in which we place the balls in the buckets, the probability that we place all 5 balls in diffrent buckets is $\frac{1}{5^5}$. There are 5! ways to order the balls to place into the buckets, so since we do not care about the order in which we place the balls in the buckets, the probability that we place all 5 balls in diffrent buckets is $\frac{1}{5^5} \cdot 5! = 0.0384$.