

ECE 131A HW 1

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Problem 1

(a)

The sample space is just all the combinations of the two dice, with r denoting the bottom face of the red die and g denoting the bottom face of the green die. Therefore the sample space is just all the 16 ordered sets possible combinations of the possible results of r and g . In other words, the sample space is

$$\{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), \\ (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$$

(b)

(i)

$$E = \{(1, 1), (1, 3), (3, 1), (3, 3)\}$$

$$F = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$$

(ii)

Since E requires all values to be odd, and thus their sum to be even and F requires the sum of the values to be odd (5), these sets are disjoint, thus:

$$E \cap F = \boxed{\emptyset}$$

(iii)

$$E \cup F = \boxed{\{(1, 1), (1, 3), (3, 1), (3, 3), (1, 4), (2, 3), (3, 2), (4, 1)\}}$$

(iv)

G^c is effectively just all value sof g and g such that $g + r \geq 7$, thus

$$G^c = \boxed{\{(3, 4), (4, 3), (4, 4)\}}$$

Problem 2

Since $P(A) + P(B) = \frac{13}{12} > 1$ we have that the minimum for $P(A \cap B)$ is $1 - P(A) - P(B)$, which is $\frac{1}{12}$. Likewise, $P(A \cap B)$ is maximized when A and B totally overlap. In this case would be equal to the minimum of $P(A)$ or $P(B)$, so $P(A \cap B) = P(B) = \frac{1}{3}$.

Problem 3

The total possibilities for choosing M items from 100 items is $\binom{100}{M}$. Therefore we have that the probability p that m items are defective of the M chosen is

$$p = \begin{cases} 0 & \text{if } m > k \\ \frac{\binom{k}{m} \binom{100-k}{M-m}}{\binom{100}{M}} & \text{if } m \leq k \end{cases}$$