ECE 131A HW 5

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Problem 1

(a)

the standard score $z = 1.37996$ results in

$$P(Z \leq z) = 0.9162 = 1 - \frac{1}{2}(1 - 0.8324)$$

Thus we have that $c = \boxed{1.37996}$.

(b)

$c = 2.3 \cdot 1.37996 = \boxed{3.173908}$

Problem 2

(a)

The corresponding z value for a standard normal is

$$\frac{1-0.8875}{0.10625} = 1.05882352941$$

Thus we have that

$$P(Z \leq z) = 0.85515$$

(b)

We have that this is effectively

$$P\left(\frac{13-14.2}{1.7} \leq Z \leq \frac{15-14.2}{1.7}\right) = 0.68103 - 0.24013 = \boxed{0.4409}$$

(c)

We have that the probability that one book is heavy is

$$1 - P(Z \leq \frac{16-14.2}{1.7}) = 0.14484$$

Therefore the probability that there is 3 heavy books is a binomial distribution and thus the probability is

$$\binom{10}{3} 0.1448^3 (1-0.1448)^7 = \boxed{0.12189}$$

Problem 3

(a)

We have that the area of the quadrilateral is $6+2=8$ and that the area with $X \geq 3$ is 2 so $P(X \geq 3) = \frac{2}{8} = \boxed{\frac{1}{4}}$

(b)

we have that the area of the quadrilateral is $6+2=8$ and the area with $Y \geq 1$ is $4+\frac{1}{2} = 4.5$ so then we have that $P(Y \geq 1) = \frac{4.5}{8} = \boxed{\frac{9}{16}}$

(c)

The area of the quadrilateral is $6+2=8$ and the area with $X \leq 1$ and $Y \leq 1$ is 1 so then we have that the probability is $\frac{1}{8} = \boxed{\frac{1}{8}}$

Problem 4

we have

$$\begin{aligned} E[X] &= \int_0^\infty \int_0^\infty x \frac{1}{750} e^{-\left(\frac{x}{150}+\frac{y}{30} dydx\right)} \\ &= \int_0^\infty \frac{x}{150} e^{-\frac{x}{150}} \int_0^\infty \frac{1}{30} e^{-\frac{y}{30}} dydx \\ &= \boxed{150} \end{aligned}$$

Problem 5

(a)

let the triangle be denoted as R then we have

$$f(x,y) = \begin{cases} \frac{2}{9} & \text{if } (x,y) \in R \\ 0 & \text{otherwise} \end{cases}$$

Thus we have that

$$\begin{aligned} E[X] &= \int_0^3 x \int_0^{3-x} \frac{2}{9} dy dx \\ &= \int_0^3 \frac{2x}{9}(3-x) dx \\ &= \boxed{1} \end{aligned}$$

and also

$$\begin{aligned} E[Y] &= \int_0^3 \int_0^{3-x} \frac{2y}{9} dy dx \\ &= \int_0^3 \frac{(3-x)^2}{9} dx \\ &= \boxed{1} \end{aligned}$$

$$E[X+Y] = \boxed{2}$$

(b)

We have that

$$E[X] = 1$$

and

$$E[X] = \int_0^3 x^2 \int_0^{3-x} \frac{2}{9} dy dx$$
$$= \int_0^3 \frac{2x^2}{9}(3-x)dx$$
$$= \boxed{1.8}$$

Thus we have that

$$Var(X) = E[X^2] - E[X]^2 = \boxed{0.8}$$

Problem 6

we have that

$$P(X = l) = \begin{cases} \frac{3}{11}\frac{2}{10} & \text{if } l = 2 \\ \frac{16}{11}\frac{3}{10} & \text{if } l = 1 \\ \frac{8}{11}\frac{7}{10} & \text{if } l = 0 \\ 0 & \text{otherwise} \end{cases}$$

Then we have that

$$E[X] = 0.5454545454545454$$

and

$$E[X^2] = 0.6545454545454545$$

And thus we have that

$$Var(X) = \boxed{0.3570247933884298}$$