ECE 133A HW 6

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Exercise A11.8

(c)

We have that

$$R_11 = 1$$

And thus

$$R_{1,2:3} = [0, 1]$$

Thus now we need to compute the cholensky factorization of

$$\begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$$

Thus we have that

$$R_{22} = 1$$

And thus we have that

$$R_{23} = 0$$

And thus we have that

$$R_{33} = \sqrt{a}$$

Thus we have that A is positive definite if and only if $a \geq 0$, and if it exists we have

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{a} \end{bmatrix}$$

(e)

We have that

$$R_1 1 = \sqrt{a}$$

thus we must have that $a \geq 0$, and thus we also have that

$$R_{1,2:3} = [\frac{1}{\sqrt{a}}, 0]$$

Therefore we have that we want to find the cholesky factorization of of

$$\begin{bmatrix} -a & 1 \\ 1 & a \end{bmatrix} - \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -a - \frac{1}{a} & 1 \\ 1 & a \end{bmatrix}$$

This cannot be factorized since we already have that $a \geq 0$ and thus we have that $-a - \frac{1}{a} \leq 0$. Thus we have that A is not positive definite.

(h)

We have that

$$R_1 1 = 1$$

And thus

$$R_{1,2:3} = \frac{1}{1} A_{1,2:3} = [1, 1]$$

Thus we have that we want to find the cholesky factorization of

$$\begin{bmatrix} a & a \\ a & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a - 1 & a - 1 \\ a - 1 & 1 \end{bmatrix}$$

Thus we have that

$$R_{22} = \sqrt{a-1}$$

And thus we have that

$$R_{23} = \frac{a-1}{\sqrt{a-1}} = \sqrt{a-1}$$

Thus in order for these to exist we must have that $a \geq 1$, and thus we have that

$$R_{33} = \sqrt{1-a-1} = \sqrt{-a}$$

This cannot exist, since we have already shown that $a \geq 1$, and thus we have that A is not positive definite and that the cholesky factorization does not exist.

A11.14

(a)

We have that the cholensky factorization of B

$$B = R_B^T R_B$$

is of the form of

$$R_B = \begin{bmatrix} R & v \\ 0 & v_{n+1} \end{bmatrix}$$

Now we need to solve for v and v_{n+1}, we have that

$$\begin{aligned} R_B^t R_B &= \begin{bmatrix} R^T & 0 \\ v^T & v_{n+1} \end{bmatrix} \begin{bmatrix} R & v \\ 0 & v_{n+1} \end{bmatrix} \\ &= \begin{bmatrix} R^T R & R^T v \\ v^T R & v^T v + v_{n+1}^2 \end{bmatrix} \end{aligned}$$

Thus we have that

$$v = R^{-T} u$$

and

$$v_{n+1}^2 = u^T A^{-1} u - 1$$

and thus we have that

$$\boxed{R_B = \begin{bmatrix} R & R^{-T}u \\ 0 & \sqrt{u^T A^{-1} u - 1} \end{bmatrix}}$$

(b)

To solve for v, since R is a upper triangular matrix, we can just use forward substitution to solve for v, which will cost us n^2 flops, then to solve for v_{n+1} we take the dot product of v with itself, which will cost us $2n - 1$ flops, and then subtract one and take the square root, which will cost us 2 flops, thus we have that the total cost is $\boxed{n^2 + 2n + 1}$ flops.

Exercise A14.8

Exercise A12.2

(a)

We want to find the two values of t, t_1 and t_2, such that $n(t_1) = n(t_2) = 2$, we can write the newton's update as

$$\begin{bmatrix} t_1^{(k+1)} \\ t_2^{(k+1)} \end{bmatrix} = \begin{bmatrix} t_1^{(k)} \\ t_2^{(k)} \end{bmatrix}$$
$$- \begin{bmatrix} -20t_1 e^{-2t_1^{(k)}} - e^{-t_1^{(k)}} + 10e^{-t_1^{(k)}} & 0 \\ 0 & -20t_2 e^{-2t_2^{(k)}} - e^{-t_2^{(k)}} + 10e^{-t_2^{(k)}} \end{bmatrix}^{-1} \begin{bmatrix} 10t_1 e^{-2t_1^{(k)}} + e^{-t_1^{(k)}} - 2 \\ 10t_2 e^{-2t_2^{(k)}} + e^{-t_2^{(k)}} - 2 \end{bmatrix}$$

We can compute this newton's method in Julia with the following code:

```
    using LinearAlgebra

function n(T)
    n=zeros(length(T))
    for i in 1:length(T)
        t=T[i]
        n[i]=10*t*exp(-2*t)+exp(-t)-2
    end
    return n
end

function dn(T)
    dn=zeros(length(T))
    for i in 1:length(T)
        t=T[i]
        dn[i]=10*exp(-2*t)*(1-2*t)-exp(-t)
    end
    return dn
end

t_1=0
t_2=1

let T=[t_1,t_2]
    #newton's method
    thresh=0.00001
    while norm(n(T))>thresh
        T=T.-n(T)./dn(T)
    end
    println(T)
    println(n(T).+[2,2])
end
```

With which we get $t_1 = 0.15661093823882483$, and $t_2 = 0.8383819928969031$

(b)

for a value to reach the maximum, we would want its derivative to go to zero, so then we have that the newton's update to find $t_{\max}$

$$t_{\max}^{(k+1)} = t_{\max}^{(k)} - \frac{10e^{-2t_{\max}^{(k)}}(1 - 2t_{\max}^{(k)}) + e^{-t_{\max}^{(k)}}}{e^{-2t_{\max}^{(k)}}(e^{t_{\max}^{(k)}} + 40t_{\max}^{(k)} - 40)}$$

With the following code in addition to the code we wrote for part a we get that $t_{\max} = \boxed{0.4236255212307657}$

```
    function d2n(T)
    d2n=zeros(length(T))
    for i in 1:length(T)
        t=T[i]
        d2n[i]=exp(-2*t)*(exp(t)+40*t-40)
    end
    return d2n
end

let t_max=[0]
    thresh=0.00001
    while norm(dn(t_max))>thresh
        t_max=t_max.-dn(t_max)./d2n(t_max)
    end
    println("t_max=",t_max[1])
    println("n(t_max)=",n(t_max)[1]+2)
end
```