ECE 133A HW 6

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Exercise A11.8

(c)

We have that

$$R_1 1 = 1$$

And thus

$$R_{1,2:3} = [0, 1]$$

Thus now we need to compute the cholensky factorization of

$$\begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$$

Thus we have that

$$R_{22} = 1$$

And thus we have that

$$R_{23} = 0$$

And thus we have that

$$R_{33} = \sqrt{a}$$

Thus we have that A is positive definite if and only if $a \geq 0$, and if it exists we have

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{a} \end{bmatrix}$$

(e)

We have that

$$R_1 1 = \sqrt{a}$$

thus we must have that $a \geq 0$, and thus we also have that

$$R_{1,2:3} = [\frac{1}{\sqrt{a}}, 0]$$

Therefore we have that we want to find the cholesky factorization of of

$$\begin{bmatrix} -a & 1 \\ 1 & a \end{bmatrix} - \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -a - \frac{1}{a} & 1 \\ 1 & a \end{bmatrix}$$

This cannot be factorized since we already have that $a \geq 0$ and thus we have that $-a - \frac{1}{a} \leq 0$. Thus we have that A is not positive definite.

(h)

We have that

$$R_1 1 = 1$$

And thus

$$R_{1,2:3} = \frac{1}{1} A_{1,2:3} = [1, 1]$$

Thus we have that we want to find the cholesky factorization of

$$\begin{bmatrix} a & a \\ a & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a - 1 & a - 1 \\ a - 1 & 1 \end{bmatrix}$$

Thus we have that
$$R_{22} = \sqrt{a-1}$$
And thus we have that
$$R_{23} = \frac{a-1}{\sqrt{a-1}} = \sqrt{a-1}$$
Thus in order for these to exist we must have that $a \geq 1$, and thus we have that
$$R_{33} = \sqrt{1-a-1} = \sqrt{-a}$$
This cannot exist, since we have already shown that $a \geq 1$, and thus we have that A is not positive definite and that the cholesky factorization does not exist.

A11.14

(a)

We have that the cholensky factorization of B
$$B = R_B^T R_B$$
is of the form of
$$R_B = \begin{bmatrix} R & v \\ 0 & v_{n+1} \end{bmatrix}$$
Now we need to solve for v and v_{n+1}, we have that
$$\begin{aligned} R_B^t R_B &= \begin{bmatrix} R^T & 0 \\ v^T & v_{n+1} \end{bmatrix} \begin{bmatrix} R & v \\ 0 & v_{n+1} \end{bmatrix} \\ &= \begin{bmatrix} R^T R & R^T v \\ v^T R & v^T v + v_{n+1}^2 \end{bmatrix} \end{aligned}$$
Thus we have that
$$v = R^{-T} u$$

and

$$v_{n+1}^2 = u^T A^{-1} u - 1$$

and thus we have that

$$\boxed{R_B = \begin{bmatrix} R & R^{-T}u \\ 0 & \sqrt{u^T A^{-1} u - 1} \end{bmatrix}}$$

(b)

To solve for v, since R is a upper triangular matrix, we can just use forward substitution to solve for v, which will cost us n^2 flops, then to solve for v_{n+1} we take the dot product of v with itself, which will cost us $2n - 1$ flops, and then subtract one and take the square root, which will cost us 2 flops, thus we have that the total cost is $\boxed{n^2 + 2n + 1}$ flops.