

# ECE 133A HW 1

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## Exercise A1.7

For the optimal coefficients we have:

$$\begin{aligned} J &= \frac{1}{n} \|c_1 \mathbf{1} + c_2 a - b\|^2 \\ &= \frac{1}{n} \|(m_b - m_a c_2) \mathbf{1} + c_2 a - b\|^2 \\ &= \frac{1}{n} \sum_{k=1}^n ((m_b - m_a c_2) + c_2 a_k - b_k)^2 \\ &= \frac{1}{n} \sum_{k=1}^n (c_2(a_k - m_a) - (b_k - m_b))^2 \\ &= \frac{1}{n} \left( c_2^2 \sum_{k=1}^n (a_k - m_a)^2 - 2c_2 \sum_{k=1}^n (a_k - m_a)(b_k - m_b) + \sum_{k=1}^n (b_k - m_b)^2 \right) \\ &= c_2^2 s_a^2 - \frac{2c_2}{n} (a - m_a \mathbf{1})^T (b - m_b \mathbf{1}) + s_b^2 \\ &= \rho^2 s_b^2 - 2 \frac{\rho s_b}{s_a} \rho s_a s_b + s_b^2 \\ &= s_b^2 - \rho^2 s_b^2 \\ &= \boxed{(1 - \rho^2) s_b^2} \end{aligned}$$