

ECE 133A HW 1

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Exercise T2.4

ϕ is not linear, since point 2 is equal to the negative of point 3, but the output of the function at point 2 is not the negative of the output at point 3.

Exercise T2.8

(a)

$$\begin{aligned}\int p(x)dx &= \sum_{i=1}^n c_i \frac{1}{i} x^i \\ \int_{\alpha}^{\beta} &= \sum_{i=1}^n c_i \frac{1}{i} (\beta^i - \alpha^i)\end{aligned}$$

therefore we have

$$a = \boxed{\left((\beta - \alpha), \dots, \frac{1}{n}(\beta^n - \alpha^n) \right)}$$

(b)

we have

$$p'(\alpha) = \sum_{i=1}^n (i-1)c_i\alpha^{i-2}$$

thus we get

$$b = \boxed{(0, 1, \dots, (n-1)\alpha^{n-2})}$$

Exercise A1.2

let $u = (\sqrt{x_1}, \dots, \sqrt{x_n})$ and $v = \left(\sqrt{\frac{1}{x_1}}, \dots, \sqrt{\frac{1}{x_n}}\right)$, from Cauchy-Schwarz we have

$$\begin{aligned}\|u^T v\| &\leq \|u\| \|v\| \\ \|u^T v\|^2 &\leq \|u\|^2 \|v\|^2 \\ n^2 &\leq \left(\sum_{k=1}^n x_k\right) \left(\sum_{k=1}^n \frac{1}{x_k}\right) \\ \frac{1}{n} \sum_{k=1}^n x_k &\geq n \left(\sum_{k=1}^n \frac{1}{x_k}\right)^{-1} \\ \frac{1}{n} \sum_{k=1}^n x_k &\geq \left(\frac{1}{n} \sum_{k=1}^n \frac{1}{x_k}\right)^{-1}\end{aligned}$$

Exercise T3.25

(a)

$$E[p] = \boxed{\theta\mu + (1 - \theta)\mu^{\text{rf}}\mathbf{1}}$$

$$\begin{aligned} \text{Var}(p) &= \theta^2\sigma^2 \\ \sqrt{\text{Var}(p)} &= \boxed{|\theta|\sigma} \end{aligned}$$

(b)

Therefore we have that

$$\theta_{\text{optimal}} = \pm \frac{\sigma^{\text{tar}}}{\sigma}$$

Therefore if our risky asset has a positive rate of return, we will pick

$$\theta_{\text{optimal}} = \boxed{\frac{\sigma^{\text{tar}}}{\sigma}}$$

And if our risky asset has a negative rate of return we will choose

$$\theta_{\text{optimal}} = \boxed{-\frac{\sigma^{\text{tar}}}{\sigma}}$$

(c)

If we want our portfolio to have a lower risk level than the asset, ie $\sigma^{\text{tar}} < \sigma$, but if the expected return rate of the asset is still positive, then we will hedge, since $\theta_{\text{optimal}} = \frac{\sigma^{\text{tar}}}{\sigma} < 1$.

If we are comfortable with a greater level of risk than the asset, ie $\sigma^{\text{tar}} > \sigma$,

and if the expected return rate of the asset is still positive, we will Leverage since $\theta_{optimal} = \frac{\sigma^{tar}}{\sigma} > 1$.

And, if the expected return rate of the asset is negative, we will short the asset.