

# ECE 133A HW 6

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## Exercise A10.1

(a)

Let  $u = [u(0), u(1), \dots, u(N-1)]^T$ , then we have that  $x = u$  we want to minimize the energy or

$$\|u\|^2$$

Furthermore we can express

$$s_1(N) = 0.1u(N-2) + (0.95+1) \cdot 0.1u(N-3) + (0.95^2+0.95+1) \cdot 0.1u(N-4) + \dots + \left(\sum_{i=0}^{N-2} 0.95^i\right) \cdot 0.1u(0)$$

$$s_2(N) = 0.1u(N-1) + (0.95) \cdot 0.1u(N-2) + (0.95^2) \cdot 0.1u(N-3) + \dots + (0.95^{N-1}) \cdot 0.1u(0)$$

So therefore we have that

$$C = \begin{bmatrix} 0.1 \sum_{i=0}^{N-2} 0.95^i & \dots & 0.1(0.95+1) & 0.1 & 0 \\ 0.1(0.95^{N-1}) & \dots & 0.1(0.95^2) & 0.1(0.95) & 0.1 \end{bmatrix}$$

And

$$d = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

(b)

using PyPlot

```
function create_C(N)
    C = zeros(2,N)

    #make the first row of C
    for i=1:N-1
        for j=1:N-i
            C[1,i]+=0.1*(0.95^(j-1))
        end
    end
    # C[1,N-1]=0.1

    #make the second row of C
    for i=1:N
        C[2,i] = 0.1*(0.95^(N-i))
    end
    return C
end

# calculate u
N=30
C = create_C(N)
d=[10,0]
u = C\d
s1=zeros(N+1)
s2=zeros(N+1)
for i=1:N
    s1[i+1]=s1[i]+s2[i]
    s2[i+1]=0.95*s2[i]+0.1*u[i]
end

fig,axs=subplots(3,1,figsize=(10,12))
axs[1].plot(s1)
axs[1][:set_title]("s1")
```

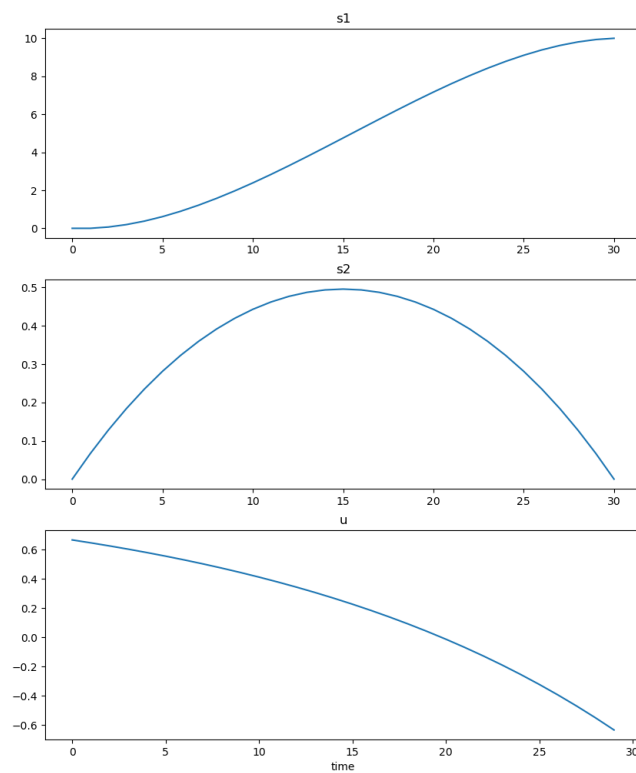
```

    axs[2][:plot](s2)
    axs[2][:set_title]("s2")
    axs[3][:plot](u)
    axs[3][:set_title]("u")
    axs[3][:set_xlabel]("time")

    savefig("problem2a.png")
    close()

```

We get the following plot



(c)

With the following code:

using PyPlot

```
function create_C(N)
    C = zeros(2,N)

    #make the first row of C
    for i=1:N-1
        for j=1:N-i
            C[1,i]+=0.1*(0.95^(j-1))
        end
    end
    # C[1,N-1]=0.1

    #make the second row of C
    for i=1:N
        C[2,i] = 0.1*(0.95^(N-i))
    end
    return C
end

N=2:29
println(N)
E=zeros(length(N))
for i=1:length(N)
    C = create_C(N[i])
    d=[10,0]
    u = C\d
    E[i]=sum(u.^2)
end
plot(N,E)
xlabel("N")
ylabel("E")
yscale("log")
```

```
savefig("problem2b.png")  
close()
```

We get the following plot

