ECE 133A HW 1

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Exercise T2.4

ϕ is not linear, since point 2 is equal to the negative of point 3, but the output of the function at point 2 is not the negative of the output at point 3.

Exercise T2.8

(a)

$$\int p(x)dx = \sum_{i=1}^{n} c_i \frac{1}{i} x^i$$
$$\int_{\alpha}^{\beta} = \sum_{i=1}^{n} c_i \frac{1}{i}(\beta^i - \alpha^i)$$

therefore we have

$$a = \boxed{\left((\beta - \alpha)), ..., \frac{1}{n}(\beta^n - \alpha^n)\right)}$$

(b)

we have

$$p'(\alpha) = \sum_{i=1}^{n} (i-1) c_i \alpha^{i-2}$$

thus we get

$$b = \boxed{\left(0, 1, ..., (n-1)\alpha^{n-2}\right)}$$

Exercise A1.2

let $u = (\sqrt{x_1}, ..., \sqrt{x_n})$ and $v = \left(\sqrt{\frac{1}{x_1}}, ..., \sqrt{\frac{1}{x_n}}\right)$, from Cauchy-Schwarz we have

$$\begin{aligned}
\|u^T v\| &\leq \|u\|\|v\| \\
\|u^T v\|^2 &\leq \|u\|^2\|v\|^2 \\
n^2 &\leq \left(\sum_{k=1}^{n} x_n\right)\left(\sum_{k=1}^{n} \frac{1}{x_k}\right) \\
\frac{1}{n}\sum_{k=1}^{n} x_k &\geq n\left(\sum_{k=1}^{n} \frac{1}{x_k}\right)^{-1} \\
\frac{1}{n}\sum_{k=1}^{n} x_k &\geq \left(\frac{1}{n}\sum_{k=1}^{n} \frac{1}{x_k}\right)^{-1}
\end{aligned}$$

Exercise T3.25

(a)

$$E[p] = \boxed{\theta\mu + (1-\theta)\mu^{\mathrm{rf}}\mathbf{1}}$$

$$Var(p) = \theta^2\sigma^2$$
$$\sqrt{Var(p)} = \boxed{|\theta|\sigma}$$

(b)

Therefore we have that

$$\theta_{optimal} = \pm\frac{\sigma^{\mathrm{tar}}}{\sigma}$$

Therefore if our risky asset has a possitive rate of return, we will pick

$$\theta_{optimal} = \boxed{\frac{\sigma^{\mathrm{tar}}}{\sigma}}$$

And if our risky asset has a negative rate of return we will choose

$$\theta_{optimal} = \boxed{-\frac{\sigma^{\mathrm{tar}}}{\sigma}}$$

(c)

If we want our portfolio to have a lower risk level than the asset, ie $\sigma^{\mathrm{tar}} < \sigma$, but if the expected return rate of the asset is still positive, then we will hedge, since $\theta_{optimal} = \frac{\sigma^{\mathrm{tar}}}{\sigma} < 1$.

If we are comfortable with a greater level of risk than the asset, ie $\sigma^{\mathrm{tar}} > \sigma$, and if the expected return rate of the asset is still positive, we will Leverage since $\theta_{optimal} = \frac{\sigma^{\text{tar}}}{\sigma} > 1$.

And, if the expected return rate of the asset is negative, we will short the asset.