

ECE 133A HW 1

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Exercise A1.7

For the optimal coefficients we have:

$$\begin{aligned} J &= \frac{1}{n} \|c_1 \mathbf{1} + c_2 a - b\|^2 \\ &= \frac{1}{n} \|(m_b - m_a c_2) \mathbf{1} + c_2 a - b\|^2 \\ &= \frac{1}{n} \sum_{k=1}^n ((m_b - m_a c_2) + c_2 a_k - b_k)^2 \\ &= \frac{1}{n} \sum_{k=1}^n (c_2(a_k - m_a) - (b_k - m_b))^2 \\ &= \frac{1}{n} \left(c_2^2 \sum_{k=1}^n (a_k - m_a)^2 - 2c_2 \sum_{k=1}^n (a_k - m_a)(b_k - m_b) + \sum_{k=1}^n (b_k - m_b)^2 \right) \\ &= c_2^2 s_a^2 - \frac{2c_2}{n} (a - m_a \mathbf{1})^T (b - m_b \mathbf{1}) + s_b^2 \\ &= \rho^2 s_b^2 - 2 \frac{\rho s_b}{s_a} \rho s_a s_b + s_b^2 \\ &= s_b^2 - \rho^2 s_b^2 \\ &= \boxed{(1 - \rho^2) s_b^2} \end{aligned}$$

Exercise A1.8

(a)

We want to maximize J with respect to c_1 , so we take the derivative of J with respect to c_1 and find the value of c_1 that makes the derivative equal to zero:

$$\begin{aligned}\frac{\partial J}{\partial c_1} &= \frac{2}{n} \sum_{k=1}^n \frac{c_1 + c_2 a_k - b_k}{1 + c^2} = 0 \\ \frac{1}{n} \sum_{k=1}^n (c_1 + c_2 a_k - b_k) &= 0 \\ c_1 + \frac{1}{n} \sum_{k=1}^n (c_2 a_k - b_k) &= 0 \\ c_1 + c_2 m_a - m_b &= 0 \\ c_1 &= m_b - c_2 m_a\end{aligned}$$

(b)

We have for the optimal c_1 ,

$$\begin{aligned}J &= \frac{\|c_2(a - m_a \mathbf{1}) - (b - m_b \mathbf{1})\|^2}{n(1 + c_2^2)} \\ &= \frac{1}{n(1 + c_2^2)} \left(\sum_{k=1}^n (c_2 a_k - m_a c_2 - b_k + m_b)^2 \right) \\ &= \frac{1}{(1 + c_2^2)} (c_2^2 s_a^2 + s_b^2 - 2\rho s_a s_b c_2)\end{aligned}$$

Taking the derivative with respect to c_2 and setting it equal to zero gives us that the optimal value of c_2 must satisfy:

$$\rho c_2^2 + \left(\frac{s_a}{s_b} - \frac{s_b}{s_a} \right) = 0$$

When $\rho \neq 0$ the optimal solution is one with the same sign as ρ , since that makes $-2\rho s_a s_b c_2$ negative.

(c)

TODO

Exercise A2.4

Expanding $(1 + A)^{n-1}$, we get

$$(1 + A)^{n-1} = \sum_{k=1}^{n-1} A^k$$

Since $A_{i,j}^k$ represents whether a path of length k exists from j to i , (with $A_{i,j}^k > 0$ representing that such a path exists) we can see that the sum of the A^k matrices for $1 \leq k \leq n-1$ represents all the paths that can possibly exist. Therefore A is irreducible if it is possible to travel from one node to any other node, ie that G_A is strongly connected.

Exercise A2.8

We can start by multiplying each