

ECE 133A HW 1

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October 19, 2022

Exercise T8.8

For a specific t_i we have

$$\begin{aligned}f(t_i) &= y_i \\ \frac{c_1 + c_2 t_i + c_3 t_i^2}{1 + d_1 t_i + d_2 t_i^2} &= y_i \\ c_1 + c_2 t_i + c_3 t_i^2 &= y_i (1 + d_1 t_i + d_2 t_i^2) \\ c_1 + c_2 t_i + c_3 t_i^2 - y_i d_1 t_i - y_i d_2 t_i^2 &= y_i\end{aligned}$$

Therefore we can construct a matrix A and a vector b , such that $A\theta = b$, where $\theta = [c_1, c_2, c_3, d_1, d_2]^T$. We have that for 5 values of t_i and the corresponding 5 values of y_i ,

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & -y_1 t_1 & -y_1 t_1^2 \\ 1 & t_2 & t_2^2 & -y_2 t_2 & -y_2 t_2^2 \\ 1 & t_3 & t_3^2 & -y_3 t_3 & -y_3 t_3^2 \\ 1 & t_4 & t_4^2 & -y_4 t_4 & -y_4 t_4^2 \\ 1 & t_5 & t_5^2 & -y_5 t_5 & -y_5 t_5^2 \end{bmatrix}$$

and

$$B = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

Therefore we can then just solve for θ with $\theta = A^{-1}b$.

Thus for the values of t and y given, we can get the values of θ with the following code:

```
t=[1,2,3,4,5]
y=[-1,12,10,1,-3]
A=zeros(5,5)

for i=1:5
    A[i,:]=[1,t[i],t[i]^2,-y[i]*t[i],-y[i]*t[i]^2]
end
print("theta= ")
println(round.(A\y,digits=3))
```

We get that

$$\theta = \boxed{(-6.117, 6.99, -1.322, -0.709, 0.158)}$$

Exercise T8.10

Squaring these, we get that for $1 \leq i \leq 4$:

$$\begin{aligned}\rho_i^2 &= \|x - a_i\|^2 \\ &= (x_1 - a_{i1})^2 + (x_2 - a_{i2})^2 + (x_3 - a_{i3})^2\end{aligned}$$

Subtracting ρ_4^2 from the others, we get that for $1 \leq i \leq 3$

$$\begin{aligned}\rho_i^2 - \rho_4^2 &= (2a_{41} - 2a_{i1})x_1 + a_{i1}^2 - a_{41}^2 \\ &\quad + (2a_{42} - 2a_{i2})x_2 + a_{i2}^2 - a_{42}^2 \\ &\quad + (2a_{43} - 2a_{i3})x_3 + a_{i3}^2 - a_{43}^2\end{aligned}$$

Therefore for the values of ρ_i and a_i we get that:

$$x = (0.605, 0.405, -0.503)$$

Which can be found with the following code in Julia

```
using LinearAlgebra

A=transpose([[-10, 10, 10] [0, 10, 0] [-10, 10, 0] [-20, -10, -10]])
rho=[17.7518, 9.6417, 14.3198, 24.9654]

M=zeros(3,3)
b=zeros(3)
# println(size(A))
```

```

for i=1:3
    # println(b)
    b[i]=rho[i].^2-rho[4].^2
    for j=1:3
        M[i,j]=2*(A[4,j]-A[i,j])
        b[i]+=A[4,j].^2-A[i,j].^2
    end
end

x=M\b
print("x= ")
println(round.(x,digits=3))

for i=1:4
    println(norm(x-A[i,:]))
end

```

Exercise A3.8

(a)

We have that $f(s_k, t_k) = \sum_{i=1}^3 \sum_{j=1}^3 c_i j s_k^{i-1} t_k^{j-1} = y_k$, then we will have

$$A = \begin{bmatrix} 1 & t_1 & t_1^2 & s_1 & s_1 t_1 & s_1 t_1^2 & s_1^2 & s_1^2 t_1 & s_1^2 t_1^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & t_9 & t_9^2 & s_9 & s_9 t_9 & s_9 t_9^2 & s_9^2 & s_9^2 t_9 & s_9^2 t_9^2 \end{bmatrix}$$

$$x = [c_1 1, c_1 2, c_1 3, c_2 1, c_2 2, c_2 3, c_3 1, c_3 2, c_3 3]^T$$

$$b = \begin{bmatrix} y_1 \\ \vdots \\ y_9 \end{bmatrix}$$

(b)

Using the code below:

```

S=[0,0,0,1,1,1,2,2,2]
T=[0,1,2,0,1,2,0,1,2]
Y=[4,0,5,7,4,9,0,3,4]
M=zeros(9,9)

for k in 1:9
    for i in 1:3

```

```

        for j in 1:3
            M[k,3*(i-1)+j]=S[k]^ (i-1)*T[k]^ (j-1)
        end
    end
end

println(M\Y)

```

We get that

$$\begin{array}{rclclcl}
 c_{11} & = & 4.0 & c_{12} & = & -8.5 & c_{13} & = & 4.5 \\
 c_{21} & = & 8.0 & c_{22} & = & -3.25 & c_{23} & = & 1.75 \\
 c_{31} & = & -5.0 & c_{32} & = & 4.75 & c_{33} & = & -2.25
 \end{array}$$

Exercise A4.3

we have, that in order for $(A + uv^T)$ to be nonsingular, there must exist a matrix B such that $(A + uv^T)B = I$. Therefore need to prove that

$$\begin{aligned}
 (A + uv^T) \left(A^{-1} - \frac{1}{1 + v^T A^{-1} u} A^{-1} uv^T A^{-1} \right) &= I \\
 (A + uv^T) \left(A^{-1} - \frac{1}{1 + v^T A^{-1} u} A^{-1} uv^T A^{-1} \right) &= I - \frac{1}{1 + v^T A^{-1} u} uv^T A^{-1} \\
 &\quad + uv^T A^{-1} - \frac{1}{1 + v^T A^{-1} u} uv^T A^{-1} uv^T A^{-1} \\
 &= I - \frac{1}{1 + v^T A^{-1} u} uv^T A^{-1} \\
 &\quad + uv^T A^{-1} - \frac{(v^T A^{-1} u)}{1 + v^T A^{-1} u} uv^T A^{-1} \\
 &= I - \frac{1 + v^T A^{-1} u}{1 + v^T A^{-1} u} uv^T A^{-1} + uv^T A^{-1} \\
 &= I - uv^T A^{-1} + uv^T A^{-1} \\
 &= I
 \end{aligned}$$

Therefore we have that A is singular if $1 + v^T A^{-1} u \neq 0$, ie if $v^T A^{-1} u \neq -1$, and that $(A + uv^T)^{-1} = A^{-1} - \frac{1}{1 + v^T A^{-1} u} A^{-1} uv^T A^{-1}$.

Exercise A4.12

(a)

When $a = \pm 1$, because this would make the columns not linearly independent.

(b)

We will have that

$$A^{-1} = \begin{bmatrix} c_1 & 0 & \cdots & 0 & c_2 \\ 0 & c_1 & \cdots & 0 & c_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & c_2 & \cdots & c_1 & 0 \\ c_2 & 0 & \cdots & 0 & c_1 \end{bmatrix}$$

Therefore we must have that

$$c_1 + \alpha c_2 = 1$$

$$c_2 + \alpha c_1 = 0$$

thus we get that $c_2 = -\alpha c_1$ and thus

$$c_1 = \frac{1}{1 - \alpha^2}$$

$$c_2 = -\frac{\alpha}{1 - \alpha^2}$$

Exercise A4.9

(a)

We have

$$\begin{aligned}(YX)^T &= (AB)^T (B^\dagger A^\dagger)^T \\&= B^T A^T (A^\dagger)^T (B^\dagger)^T \\&= B^T A^T (A^T)^\dagger (B^\dagger)^T \\&= B^T (B^\dagger)^T \\&= B^T (B^T (BB^T)^{-1})^T \\&= B^T ((BB^T)^{-1} B) \\&= (B^T (BB^T)^{-1}) B \\&= B^\dagger B \\&= B^\dagger A^\dagger AB \\&= YX\end{aligned}$$

Therefore YX is symmetric.

(b)

We have that

$$\begin{aligned}(XY)^T &= Y^T X^T \\&= (B^\dagger A^\dagger)^T (AB)^T \\&= (A^\dagger)^T (B^\dagger)^T B^T A^T \\&= (A^\dagger)^T A^T \\&= (A(A^T A)^{-1}) A^T \\&= A((A^T A)^{-1} A^T) \\&= AA^\dagger \\&= ABB^\dagger A^\dagger \\&= XY\end{aligned}$$

Therefore XY is symmetric

(c)

We have that

$$\begin{aligned}YXY &= B^\dagger A^\dagger A B Y \\&= B^\dagger I B Y \\&= B^\dagger B Y \\&= I Y \\&= Y\end{aligned}$$

(d)

We have

$$\begin{aligned}XYX &= ABB^\dagger A^\dagger AB \\&= ABB^\dagger IB \\&= ABB^\dagger BY \\&= AB \\&= X\end{aligned}$$