ECE 133A HW 6

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November 27, 2022

Exercise A11.8

(c)

We have that

$$R_1 1 = 1$$

And thus

$$R_{1,2:3} = [0, 1]$$

Thus now we need to compute the cholensky factorization of

$$\begin{bmatrix} 1 & 1 \\ 1 & a \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix}$$

Thus we have that

$$R_{22} = 1$$

And thus we have that

$$R_{23} = 0$$

And thus we have that

$$R_{33} = \sqrt{a}$$

Thus we have that A is positive definite if and only if $a \geq 0$, and if it exists we have

$$R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{a} \end{bmatrix}$$

(e)

We have that

$$R_1 1 = \sqrt{a}$$

thus we must have that $a \geq 0$, and thus we also have that

$$R_{1,2:3} = [\frac{1}{\sqrt{a}}, 0]$$

Therefore we have that we want to find the cholesky factorization of of

$$\begin{bmatrix} -a & 1 \\ 1 & a \end{bmatrix} - \begin{bmatrix} \frac{1}{a} & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -a - \frac{1}{a} & 1 \\ 1 & a \end{bmatrix}$$

This cannot be factorized since we already have that $a \geq 0$ and thus we have that $-a - \frac{1}{a} \leq 0$. Thus we have that A is not positive definite.

(h)

We have that

$$R_1 1 = 1$$

And thus

$$R_{1,2:3} = \frac{1}{1} A_{1,2:3} = [1, 1]$$

Thus we have that we want to find the cholesky factorization of

$$\begin{bmatrix} a & a \\ a & 2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a - 1 & a - 1 \\ a - 1 & 1 \end{bmatrix}$$

Thus we have that

$$R_{22} = \sqrt{a-1}$$

And thus we have that

$$R_{23} = \frac{a-1}{\sqrt{a-1}} = \sqrt{a-1}$$

Thus in order for these to exist we must have that $a \geq 1$, and thus we have that

$$R_{33} = \sqrt{1-a-1} = \sqrt{-a}$$

This cannot exist, since we have already shown that $a \geq 1$, and thus we have that A is not positive definite and that the cholesky factorization does not exist.

A11.14