EE 141 – Project
Spring 2022

Due on June 3rd by 5pm

In this project you will design a controller for lane-keeping and velocity regulation of an autonomous car.

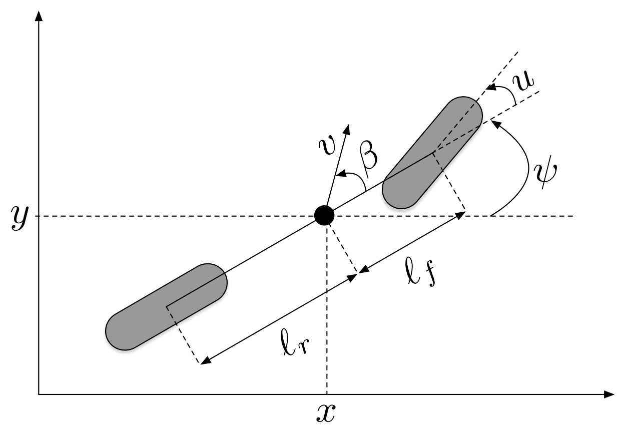

Figure 1: Not to scale graphical representation of the bicycle model for a car.

We will use the kinematic bicycle model depicted in Figure 1. The position of the car is denoted by $(x, y) \in \mathbb{R}^2$ and its orientation with respect to an inertial frame is given by $\psi \in [0, 2\pi[$. The front wheels' angle is denoted by u while β denotes the angle between the car's velocity v and car's longitudinal axis. Acceleration is denoted by a and the distance between the front and rear axles and the center of mass is given by ℓ_f and ℓ_r, respectively. The (kinematic) equations of motion are given by:

$$\dot{x} = v\cos(\psi + \beta) \tag{1}$$
$$\dot{y} = v\sin(\psi + \beta) \tag{2}$$
$$\dot{v} = a \tag{3}$$
$$\dot{\psi} = \frac{v}{\ell_r}\sin(\beta) \tag{4}$$
$$\beta = \tan^{-1}\left(\frac{\ell_r}{\ell_r + \ell_f}\tan(u)\right). \tag{5}$$

We have two control inputs, the steering angle u and acceleration a. We will use the following values for the length parameters:

$$\ell_f = 1.1m, \quad \ell_r = 1.7m.$$

1. Show that for any desired β in $[0, 2\pi[$ there exists a u in $[0, 2\pi[$ so that (5) holds. We can thus treat β as the input since for any β computed by a controller we can compute the steering angle u via the relation (5) and apply this command to the motor steering the wheels. This will greatly simplify the equations you have to work with.

2. We first consider the lane keeping problem, i.e., the design of a controller that keeps the car in the center of its lane. For this purpose we assume the car's velocity to be constant at 35 mph, that the lane center corresponds to $y = 0$, and that we have a sensor measuring y (in reality, the position of the car on the lane would be computed by using vision to detect the location of the lane markers). Linearize the equations of motion and design a controller that stabilizes the car at $y = 0$ using the linearized model (use the transfer function from β to y). You don't need to work with equation (1) since x will not be at equilibrium. Provide some plots showing the controller works as intended.

3. Simulate the controller from the previous question with the nonlinear car model. Determine the range of initial conditions for which the controller has adequate performance. Define what you consider adequate performance for a lane that is 3 meters wide and an autonomous car that is 1.8 meters wide. Provide some plots to justify your answer.

4. We now consider the velocity regulation problem. For this purpose we assume that $y = 0$. Linearize the equations of motion to obtain the transfer function from a to v and design a controller so that the closed-loop system tracks step inputs with zero steady-state error. You don't need to work with equation (1) since x will not be at equilibrium. Provide some plots showing the controller works as intended.

5. Simulate the nonlinear car model in closed-loop with *both* controllers. Note the assumptions made when designing the controllers are no longer satisfied: velocity is no longer constant and y no longer equals zero. Find the range of initial conditions for which the designed controllers have adequate performance when the commanded velocity is 35 mph. What happens when the controller regulating velocity is much slower than the controller regulating the position in the lane? Provide some plots to justify your answer.

6. How does the answer to the previous question changes as the commanded velocity increases? Provide some plots to justify your answer.