

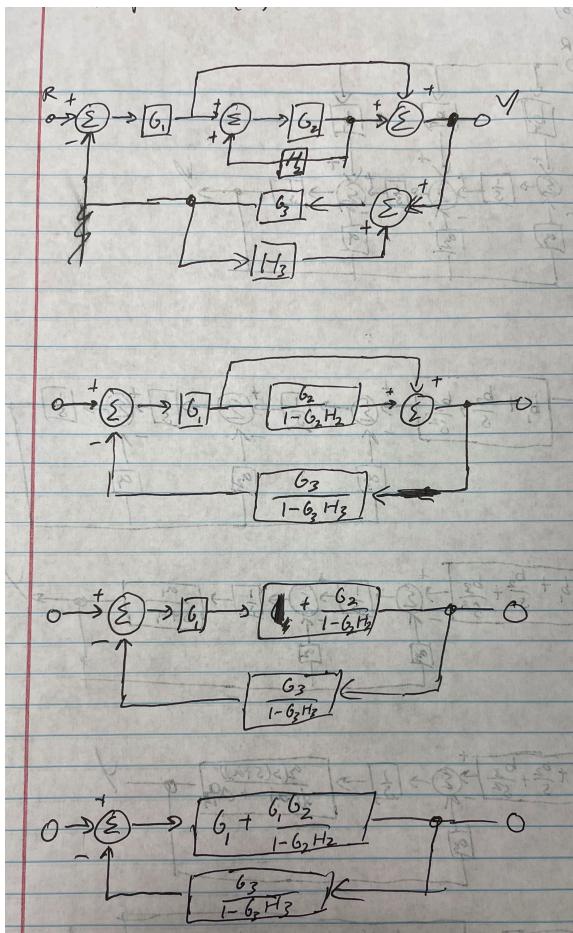
# ECE 141 Homework 2

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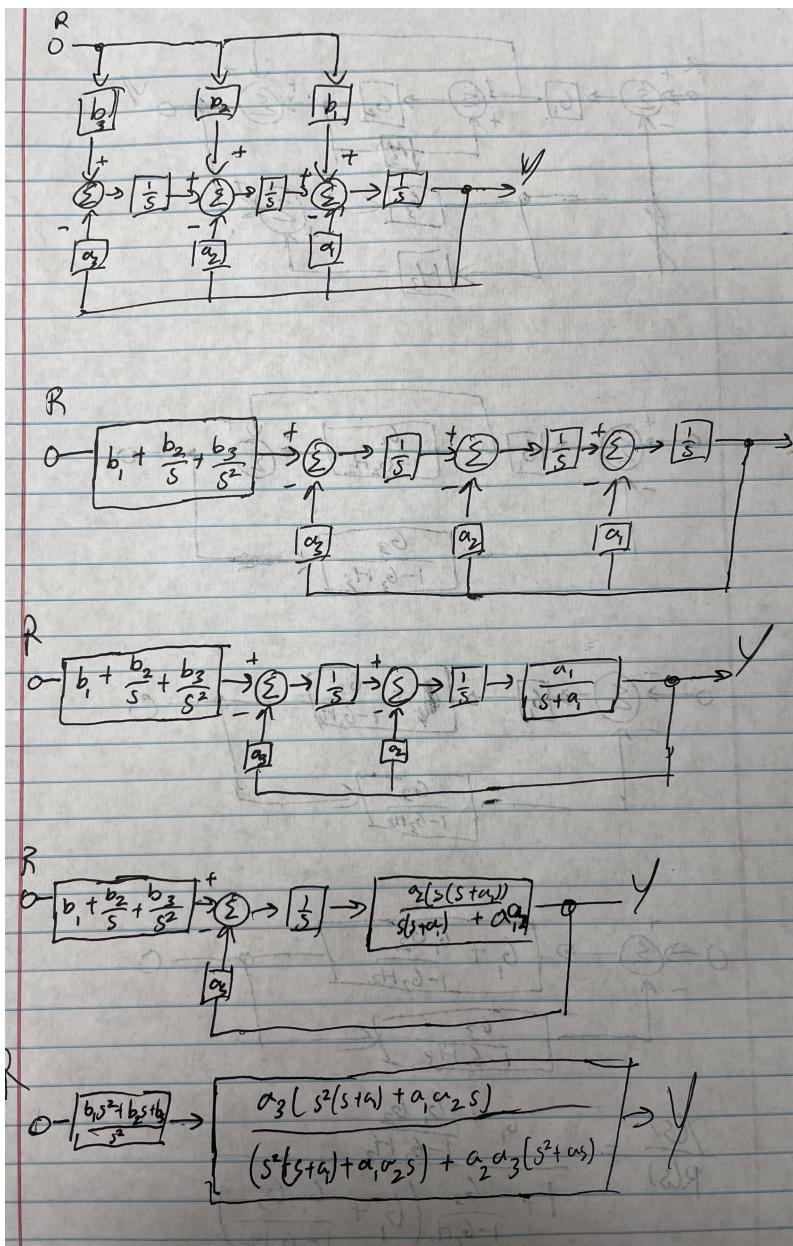
## Problem 3.21

(a)



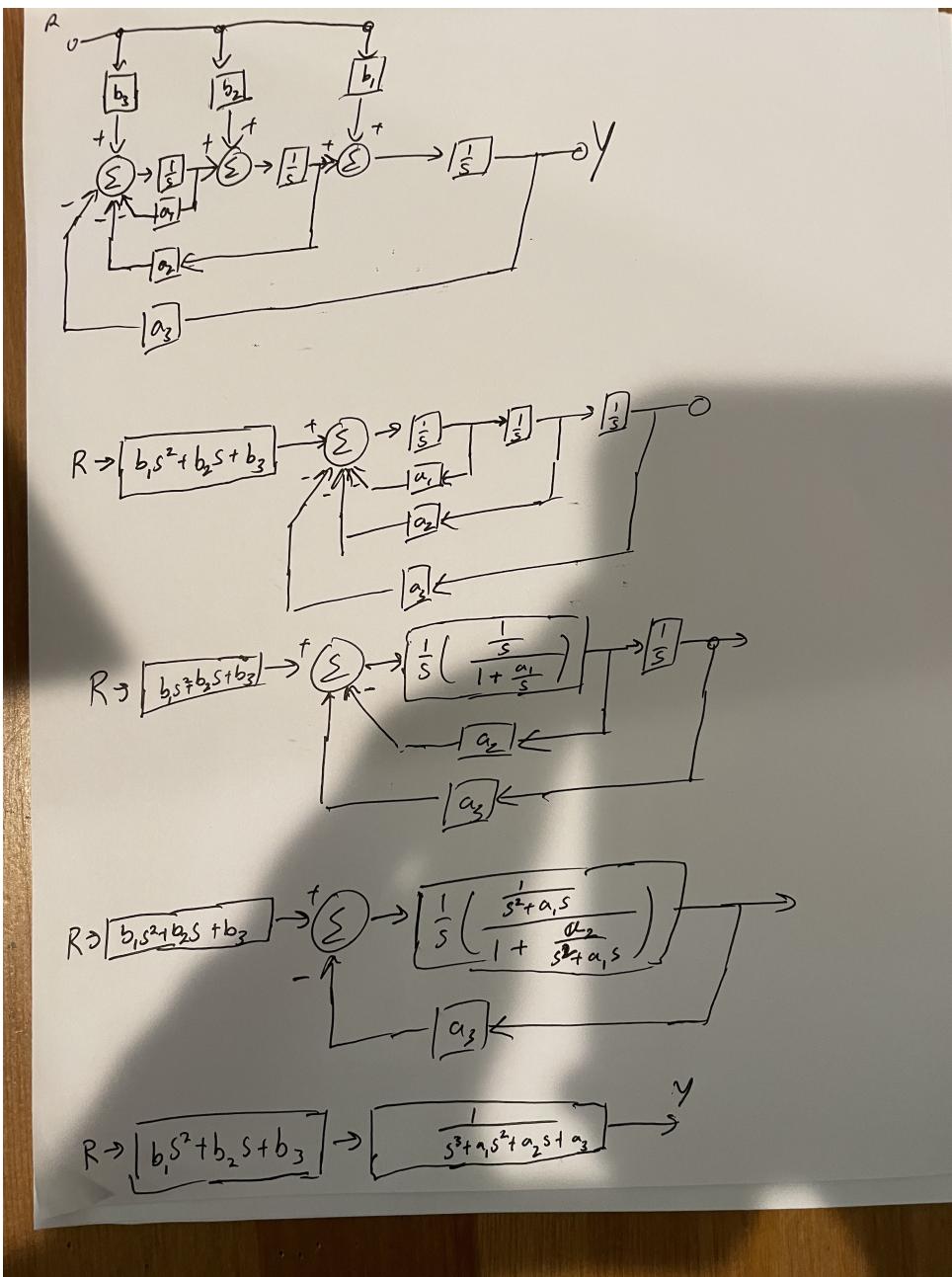
$$\frac{Y(s)}{R(s)} = \frac{G_1 + \frac{G_1 G_2}{1-G_2 H_2}}{1 + \left( G_1 + \frac{G_1 G_2}{1-G_2 H_2} \right) \frac{G_3}{1-G_3 H_3}}$$

(b)



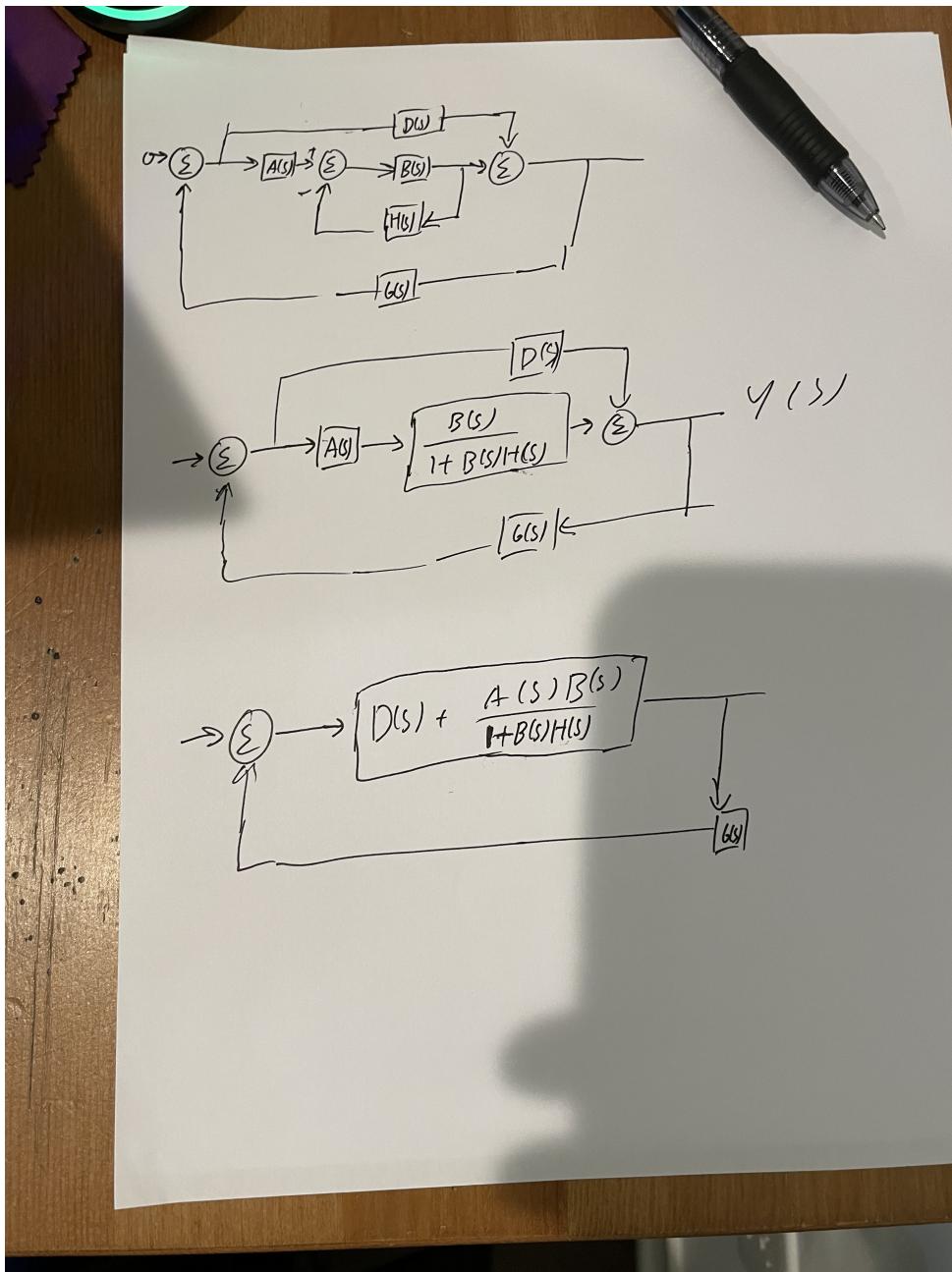
$$\frac{Y(s)}{R(s)}=\boxed{\frac{b_1s^2+b_2s+b_3}{s^2}+\frac{a_3(s^2(s+a_1)+a_1a_2s)}{s^2(s+a_1)+a_1a_2s+a_2a_3(s^2+a_1s)}}$$

(c)



$$\frac{Y(s)}{R(s)}=\boxed{\frac{b_1s^2+b_2s+b_3}{s^3+a_1s^2+a_2s+a_3}}$$

(d)



$$\frac{Y(s)}{R(s)} = \boxed{\frac{D(s) + D(s)B(s)H(s) + A(s)B(s)}{1 + B(s)H(s) + G(s)(D(s) + D(s)B(s)H(s) + A(s)B(s))}}$$

## Problem 3.30

(a)

We have

$$16\% \geq M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}}$$

$$6.9 \geq t_s = \frac{4.6}{\zeta \omega_n}$$

$$1.8 \geq t_r = \frac{1.8}{\omega_n}$$

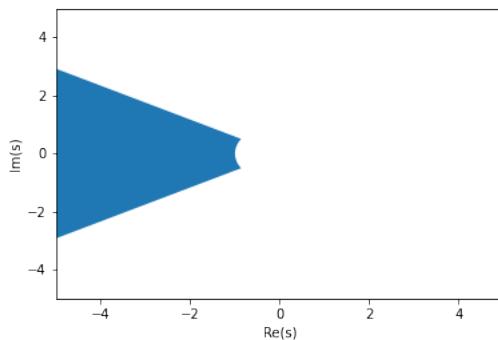
Therefore we get three conditions

$$\omega_n \geq 1$$

$$\sigma \leq -\frac{4.6}{6.9}$$

$$\sin^{-1}(\zeta) \leq \tan^{-1} \frac{-\ln(0.16)}{\pi}$$

From these we get the following region



(b)

We therefore have

$$1.8 = t_r = \frac{1.8}{\omega_n}$$

Therefore  $\omega_n = 1$ , furthermore we have

$$6.9 = t_r = \frac{4.6}{\zeta \omega_n} = \frac{4.6}{\zeta}$$

Therefore  $\zeta = \frac{4.6}{6.9} = \frac{2}{3}$  and thus we have that the overshoot

$$M_p = e^{-\pi \frac{2}{\sqrt{5}}}$$

$$M_p = 6\%$$

## Problem 3.32

(a)

Therefore we have that the transfer function is

$$\begin{aligned} \frac{Y(s)}{R(s)} &= \frac{G(s)}{1 + G(s)} \\ &= \frac{K}{s^2 + 2s + K} \end{aligned}$$

Therefore we have

$$K = \omega_n^2$$

$$\zeta \omega_n = 1$$

Furthermore from the peak time and overshoot criterion we have

$$t_p = \frac{1.8}{\omega_n} = 1$$

$$M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}} = 0.05$$

Using the equation for  $t_p$  we get  $\omega_n = 1.8$ , therefore we have  $\zeta = \frac{1}{1.8}$ , plugging this into  $M_p$  we get

$$e^{-\pi \frac{1}{\sqrt{1.8^2-1}}} = 0.014 \neq 0.05$$

Therefore we cannot meet both specifications just by selecting the right value of  $K$

**(b)**

From the peak time and overshoot criterion we have

$$\omega_n = 1.8$$

and

$$\sin^{-1}(\zeta) = \tan^{-1} \left( \frac{-\ln(0.05)}{\pi} \right)$$

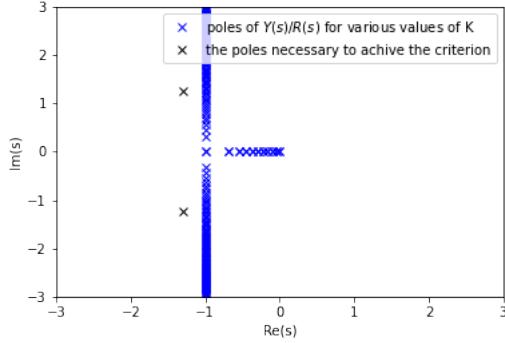
Therefore we have poles at

$$s = -1.303 \pm 1.242j$$

Furthermore, the poles for  $\frac{Y(s)}{R(s)}$  are at

$$s = \frac{-2 \pm \sqrt{4 - 4K}}{2} = -1 \pm \sqrt{1 - K}$$

Therefore we get the following sketch for the poles in the s plane



(c)

We relax the conditions by multiplying the peak time and overshoot with  $c$  therefore we have

$$t_p = \frac{1.8}{\omega_n} = c$$

$$M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}} = 0.05c$$

thererfore we have

$$\omega_n = \frac{1.8}{c}$$

And since  $\zeta\omega_n = 1$

$$\zeta = \frac{c}{1.8}$$

therefore we have

$$-\pi \frac{c}{\sqrt{1.8^2 - c^2}} = \ln(0.05) + \ln(c)$$

Solving this we have

$$c = 1.201$$

$$\omega_n = 1.5$$

$$\zeta = 0.667$$

$$K = 2.25$$

## Problem 3.36

(a)

Applying the laplace transform we get

$$Js^2\Theta(s) + Bs\Theta(s) = T_c(s)$$

$$\frac{\Theta(s)}{T_c(s)} = \boxed{\frac{1}{Js^2 + Bs}}$$

(b)

Therefore we have

$$\begin{aligned} J\theta'' + B\theta' &= K(\theta_r - \theta) \\ Js^2\Theta(s) + Bs\Theta(s) + K\Theta(s) &= K\Theta_r(s) \\ \frac{\Theta_r(s)}{\Theta(s)} &= \boxed{\frac{K}{Js^2 + Bs + K}} \end{aligned}$$

(c)

For overshoot we have

$$\begin{aligned} M_p &= e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}} < 0.1 \\ -\pi \frac{\zeta}{\sqrt{1-\zeta^2}} &< \ln(0.1) \\ -\pi\zeta &< \ln(0.1)\sqrt{1-\zeta^2} \\ \pi^2\zeta^2 &> \ln^2(0.1)(1-\zeta^2) \\ (\pi^2 + \ln^2(0.1))\zeta^2 &> \ln^2(0.1) \end{aligned}$$

$$\zeta > \sqrt{\frac{\ln^2(0.1)}{\pi^2 + \ln^2(0.1)}}$$

And since

$$\zeta \omega_n = \frac{B}{2J}$$

We have

$$\omega_n = \frac{B}{2\zeta} < \frac{B}{2J} \sqrt{\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}}$$

And since

$$\omega_n^2 = \frac{K}{J}$$

We have

$$\frac{K}{J} < \frac{B^2}{4J^2} \frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}$$

$$K < \frac{B^2}{4J} \frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}$$

$$K < [476.92]$$

**(d)**

In order for the rise time to be less than 80 seconds we must have

$$t_p = \frac{1.8}{\omega_n} < 80$$

$$\frac{1.8}{80} < \omega_n$$

and since we have  $\omega_n^2 = \frac{K}{J}$  we have

$$\frac{1.8^2}{80^2} J < K$$

$$K > [303.75]$$