ECE 141 Homework 4

Lawrence Liu

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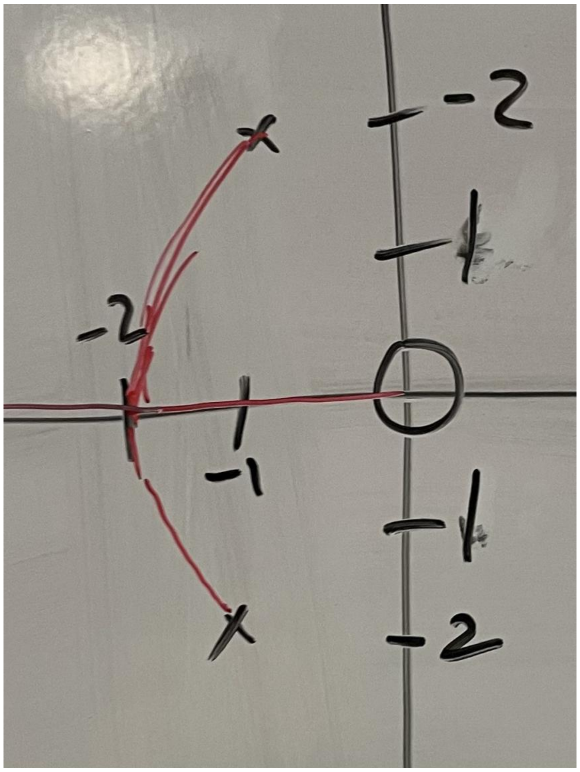
Problem 5.9

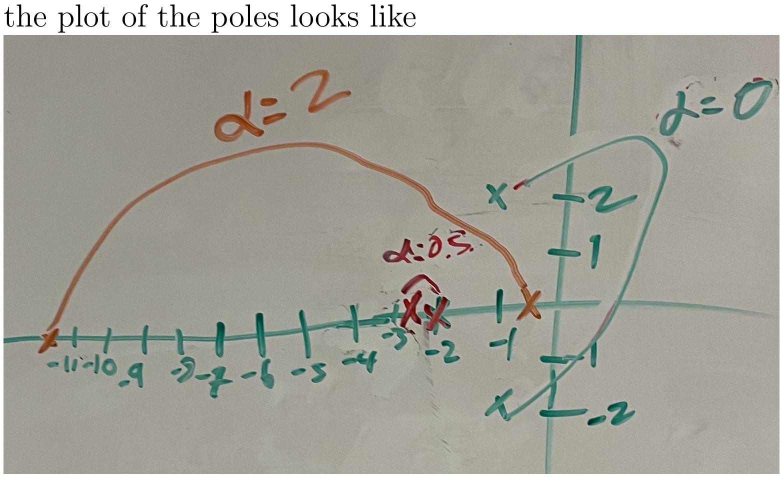
The transfer function is

$$\frac{Y}{R} = \frac{5}{(s(s+2)+5+5\alpha s)}$$

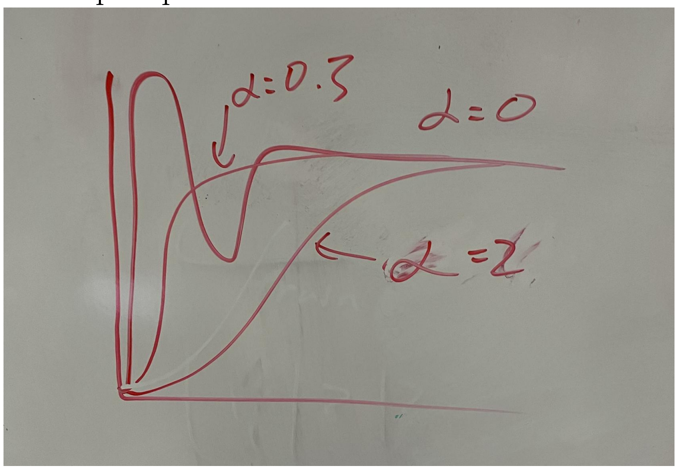
, therefore the characteristic equation is $s(s+2)+5+5\alpha s = 0$, therefore we have $b(s) = 5s$ and $a(s) = s^2+2s+1$ Therefore we have that $L(s) = \frac{5s}{s^2+2s+1}$, therefore 1 line will approach asymptotes centered at -2 and leaving at angles $180°$. Furthermore, the departure angle from the poles $-1 \pm 2j$ is $\mp 153.4°$ and the arival angle to the zero at 0 is $180°$, therefore the root locuses are at



When $\alpha = 0$, there are poles at $-1 \pm 2j$, when $\alpha = 0.5$, there are poles at -2 and -2.5, and when $\alpha = 2$, there are poles at -0.432 and -11.568, therefore the plot of the poles looks like



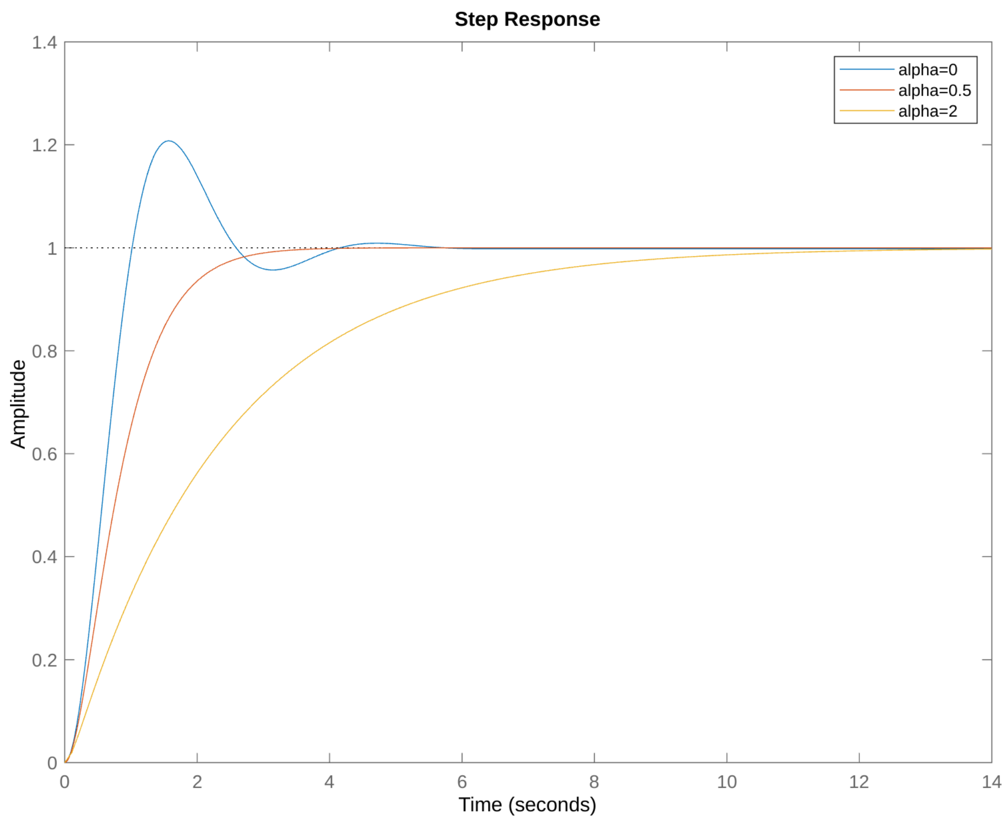
Therefore since $alpha = 0$ the step response will be underdamped since it has poles with imaginary components, and for $\alpha = 0.5$ the damping factor $\zeta = \frac{4.5}{2\sqrt{5}}$ and when $\alpha = 2$ the damping factor $\zeta = \frac{6}{\sqrt{5}}$, therefore the plot of the step responses would look like



With the following matlab code we can verify it

```
sys = tf([5],[1 2 5]);
step(sys)
stepinfo(sys)
hold on;

sys = tf([5],[1 2+2.5 5]);
step(sys)
stepinfo(sys)

sys = tf([5],[1 12 5]);
step(sys)
stepinfo(sys)

hold off;
legend('alpha=0','alpha=0.5','alpha=2')
```

Which produces the following plot.



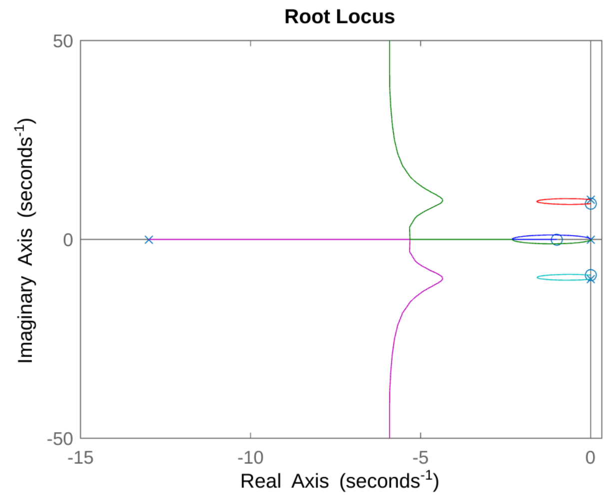
Problem 5.13

(a)

The transfer function is $\frac{K\frac{(s+1)(s^2+81)}{(s+13)s^2(s+100)}}{1+K\frac{(s+1)(s^2+81)}{(s+13)s^2(s+100)}}$, therefor $L(s) = \frac{(s+1)(s^2+81)}{(s+13)s^2(s+100)}$ and therefore with the following matlab code we can get that the Roots Locus looks like

```
sys = tf([1 1 81 81],[1 13 100 1300 0 0]);
```

```
rlocus(sys)
```



(b)

No there isn't, we can confirm this through the following matlab code

```
sys = tf([1 1 81 81],[1 13 100 1300 0 0]);
[r,k] = rlocus(sys);
phases=atan2(imag(r),real(r));
sum(sum(phases>0.5==5))
```

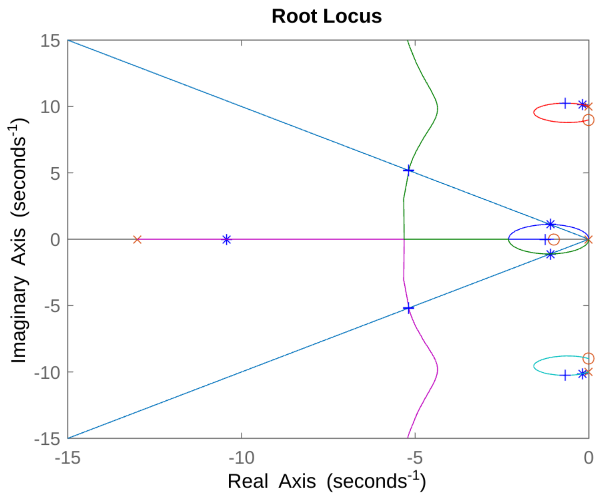
The outputs 0, so there is no values of K such that all roots have a damping factor $\zeta > 0.5$

(c)

From the following code we get that the possible values of K are $K = 32.6$ and $K = 88.3$

```
syms s

sys = tf([1 1 81 81],[1 13 100 1300 0 0]);
thresh=0.001;
zeta=0.707;
L=50;
plot([-zeta*L 0 -zeta*L],[-sqrt(1-zeta^2)*L 0 sqrt(1-zeta^2)*L])
hold on;
k = (20:0.1:40);
r = rlocus(sys,k);
rlocus(sys)
phases=atan2(-real(r),abs(imag(r)));
loc=sum((abs(phases-0.786)<thresh))==2;
Gain=k(loc)

plot(real(r(:,loc)),imag(r(:,loc)),'b*')

k = (80:0.1:120);
r = rlocus(sys,k);
rlocus(sys)
phases=atan2(-real(r),abs(imag(r)));
loc=sum((abs(phases-0.786)<thresh))==2;
Gain=k(loc)

plot(real(r(:,loc)),imag(r(:,loc)),'b+')
hold off;
xlim([-15 0])
```

Which also produces the following graph

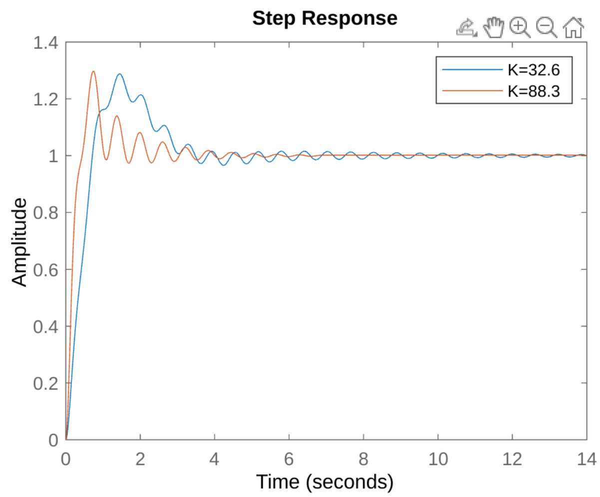


With the points denoted with a + sign being the poles from where $K = 88.3$, and the points * are poles from where $K = 32.6$

(d)

Using the code below, we can plot the step response

```
syms s

G = tf([1 1 81 81],[1 13 100 1300 0 0]);

K=32.6;
sys=K*G/(1+K*G);

step(sys)
hold on;
K=88.3;
sys=K*G/(1+K*G);

step(sys)
hold off;
legend('K=32.6',"K=88.3")
```


Step Response
K=32.6
K=88.3
Amplitude
Time (seconds)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
14