# ECE 141 Homework 2

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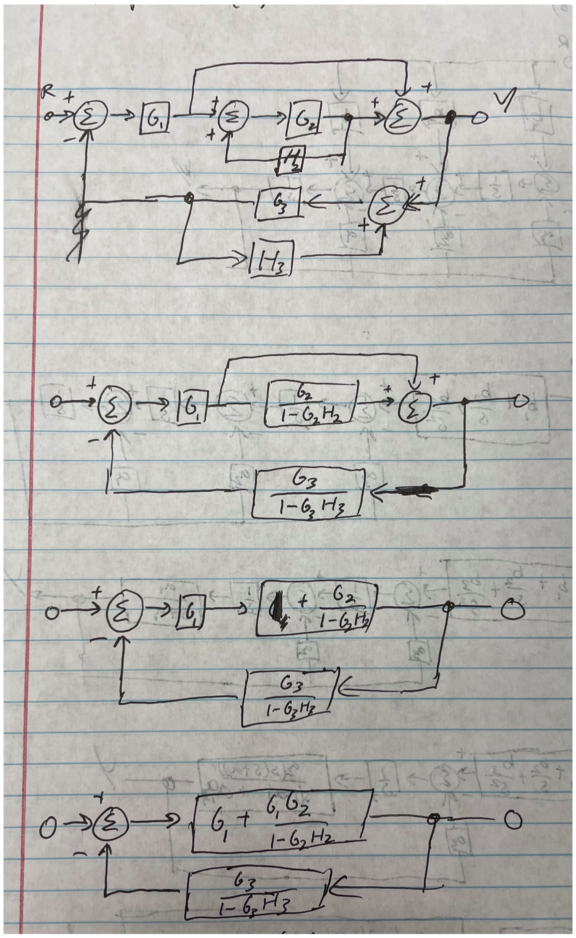
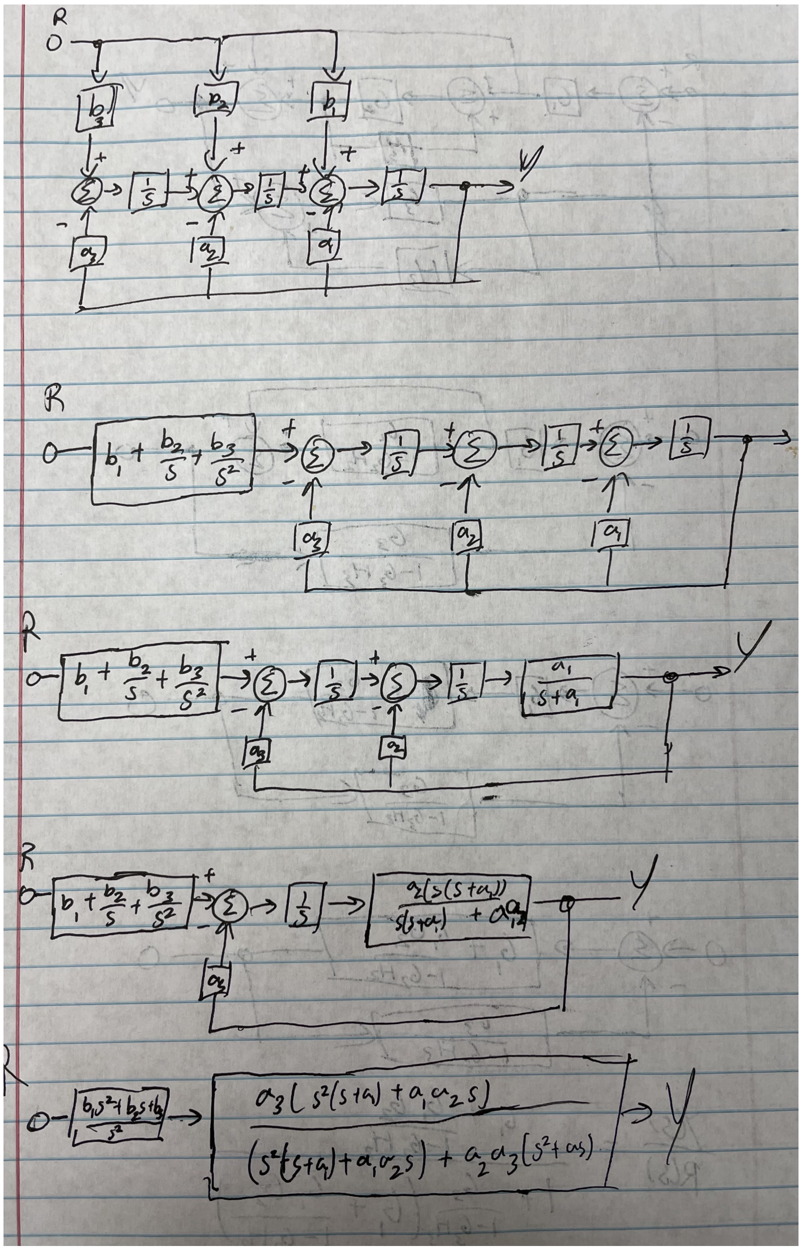
# Problem 3.21

## (a)



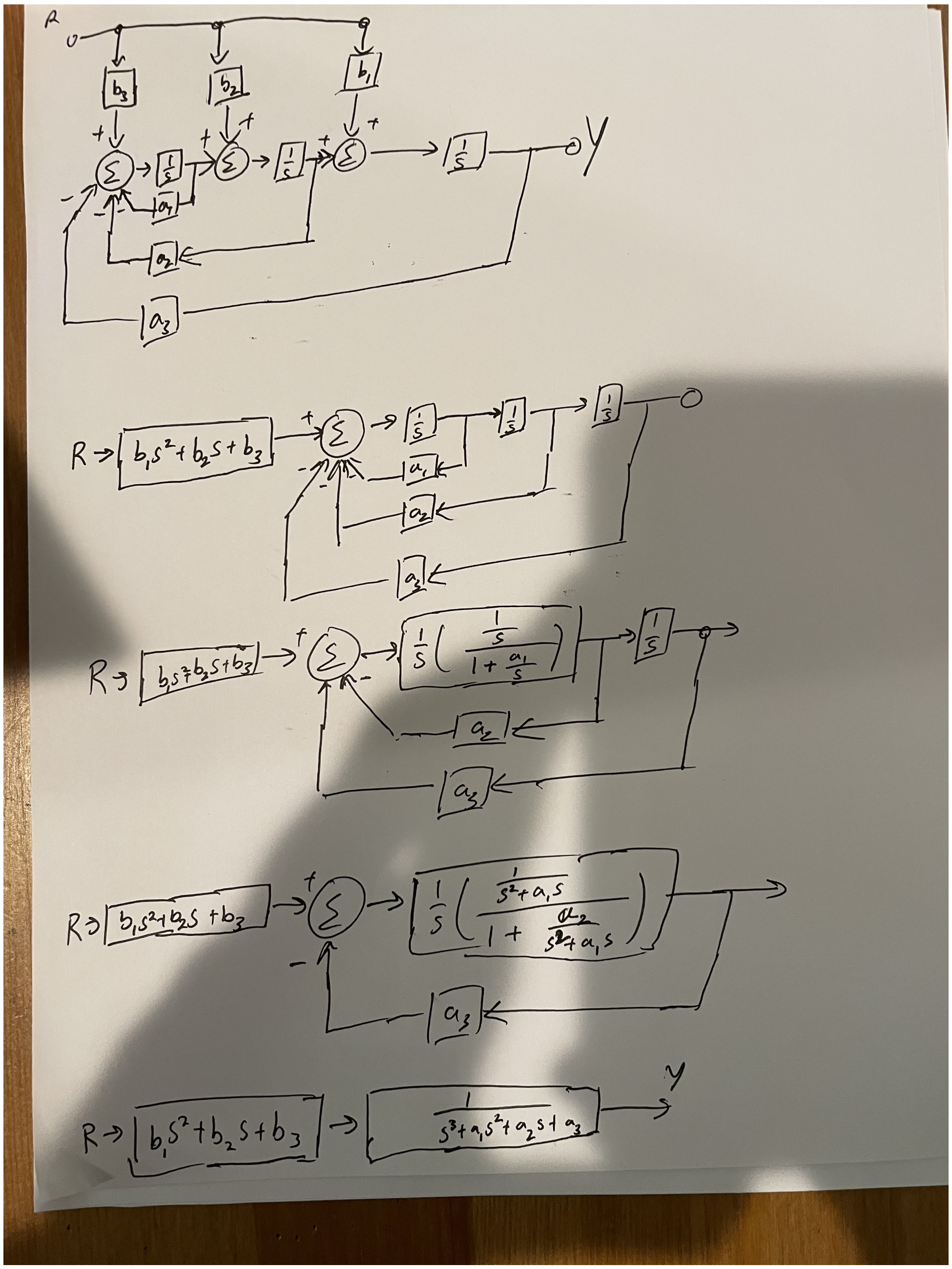
$$\frac{Y(s)}{R(s)} = \frac{G_1 + \frac{G_1 G_2}{1 - G_2 H_2}}{1 + \left(G_1 + \frac{G_1 G_2}{1 - G_2 H_2}\right) \frac{G_3}{1 - G_3 H_3}}$$

(b)



$$\frac{Y(s)}{R(s)} = \boxed{\frac{b_1 s^2 + b_2 s + b_3}{s^2} + \frac{a_3(s^2(s+a_1) + a_1 a_2 s)}{s^2(s+a_1) + a_1 a_2 s + a_2 a_3(s^2 + a_1 s)}}$$

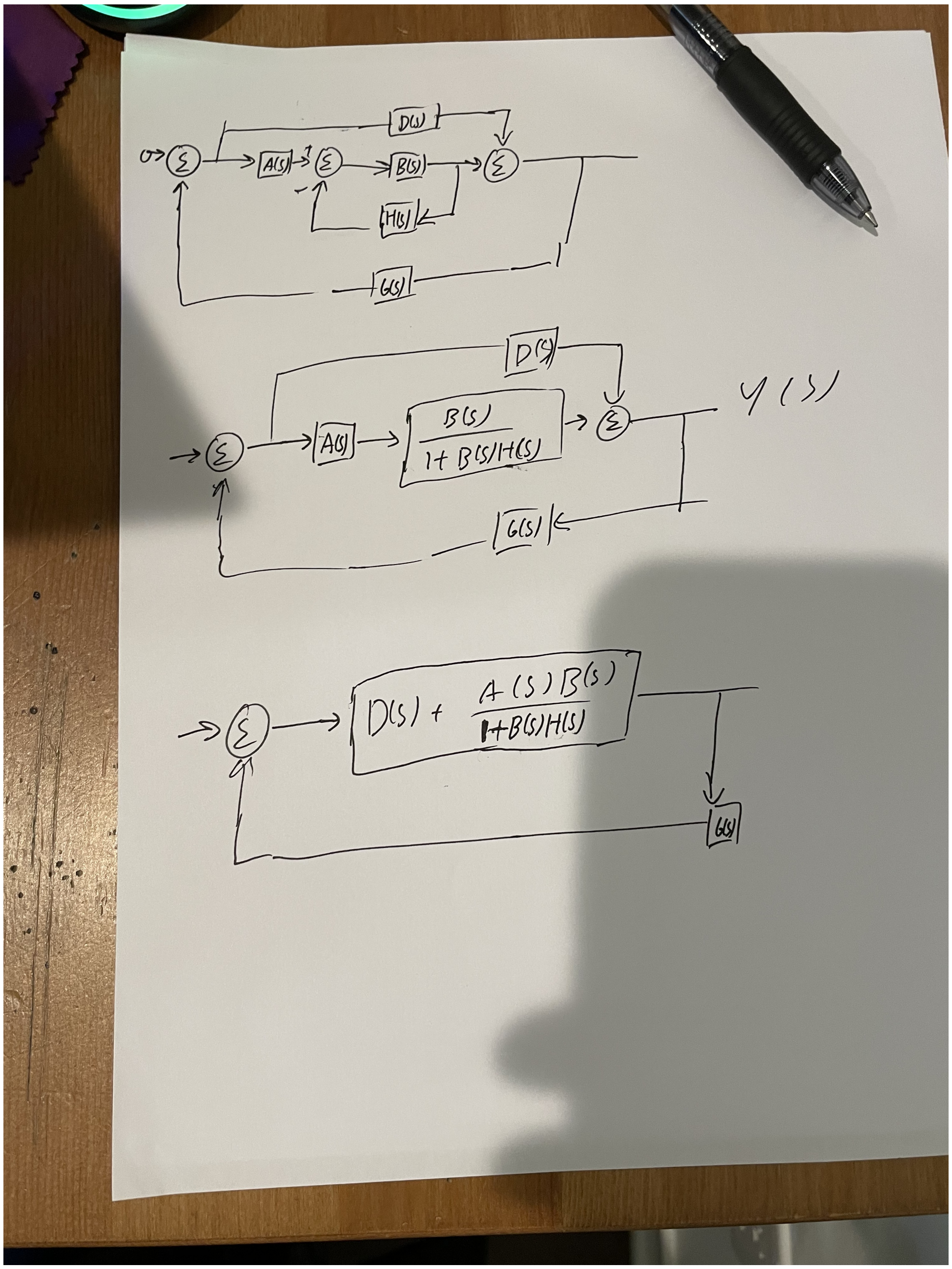
**(c)**



R
$b_3$
$b_2$
$b_1$
$\frac{1}{s}$
$a_1$
$a_2$
$a_3$
Y
$b_1 s^2 + b_2 s + b_3$
$\frac{1}{s}\left(\frac{\frac{1}{s}}{1+\frac{a_1}{s}}\right)$
$\frac{1}{s}\left(\frac{\frac{1}{s^2+a_1 s}}{1+\frac{a_2}{s^2+a_1 s}}\right)$
$\frac{1}{s^3+a_1 s^2+a_2 s+a_3}$

$$\frac{Y(s)}{R(s)} = \boxed{\frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}}$$

(d)



$$\frac{Y(s)}{R(s)} = \boxed{\frac{D(s) + D(s)B(s)H(s) + A(s)B(s)}{1 + B(s)H(s) + G(s)(D(s) + D(s)B(s)H(s) + A(s)B(s))}}$$

# Problem 3.30

## (a)

We have

$$16\% \geq M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}}$$

$$6.9 \geq t_s = \frac{4.6}{\zeta\omega_n}$$

$$1.8 \geq t_r = \frac{1.8}{\omega_n}$$

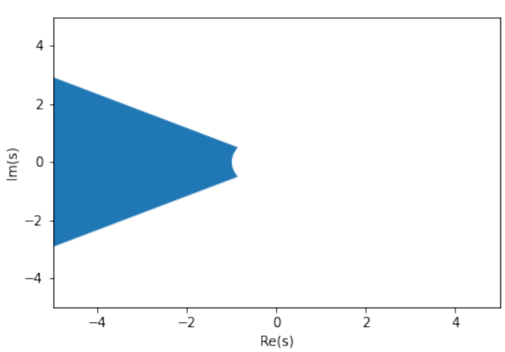
Therefore we get three conditions

$$\omega_n \geq 1$$

$$\sigma \leq -\frac{4.6}{6.9}$$

$$\sin^- 1(\zeta) \leq \tan^{-1} \frac{-\ln(0.16)}{\pi}$$

From these we get the following region



## (b)

We therefore have

$$1.8 = t_r = \frac{1.8}{\omega_n}$$

Therefore $\omega_n = 1$, furthermore we have

$$6.9 = t_r = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\zeta}$$

Therefore $\zeta = \frac{4.6}{6.9} = \frac{2}{3}$ and thus we have that the overshoot

$$M_p = e^{-\pi\frac{2}{\sqrt{5}}}$$

$$M_p = 6\%$$

# Problem 3.32

## (a)

Therefore we have that the transfer function is

$$\begin{aligned}\frac{Y(s)}{R(s)} &= \frac{G(s)}{1+G(s)} \\ &= \frac{K}{s^2+2s+K}\end{aligned}$$

Therefore we have

$$K = \omega_n^2$$

$$\zeta\omega_n = 1$$

Furthermore from the peak time and overshoot criterion we have

$$t_p = \frac{1.8}{\omega_n} = 1$$

$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} = 0.05$$

Using the equation for $t_p$ we get $\omega_n = 1.8$, therefore we have $\zeta = \frac{1}{1.8}$, plugging this into $M_p$ we get

$$e^{-\pi\frac{1}{\sqrt{1.8^2-1}}} = 0.014 \neq 0.05$$

Therefore we cannot meet both specifications just by selecting the right value of $K$

## (b)

From the peak time and overshoot criterion we have

$$\omega_n = 1.8$$

and

$$\sin^{-} 1(\zeta) = \tan^{-1}\left(\frac{-\ln(0.05)}{\pi}\right)$$

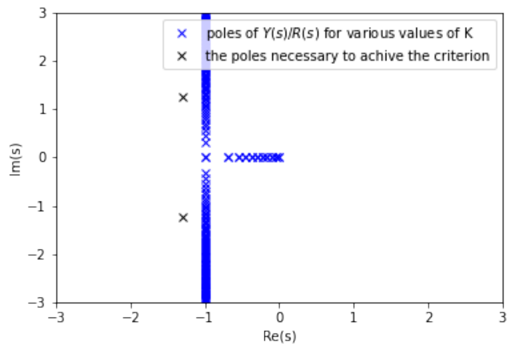
Therefore we have poles at

$$s = -1.303 \pm 1.242j$$

Furthermore, the poles for $\frac{Y(s)}{R(s)}$ are at

$$s = \frac{-2 \pm \sqrt{4-4K}}{2} = -1 \pm \sqrt{1-K}$$

Therefore we get the following sketch for the poles in the s plane




## (c)

We relax the conditions by multiplying the peak time and overshoot with $c$ therefore we have

$$t_p = \frac{1.8}{\omega_n} = c$$

$$M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}} = 0.05c$$

thererfore we have

$$\omega_n = \frac{1.8}{c}$$

And since $\zeta\omega_n = 1$

$$\zeta = \frac{c}{1.8}$$

therefore we have

$$-\pi \frac{c}{\sqrt{1.8^2 - c^2}} = \ln(0.05) + \ln(c)$$

Solving this we have

$$c = 1.201$$

$$\omega_n = 1.5$$

$$\zeta = 0.667$$

$$K = 2.25$$

# Problem 3.36

## (a)

Applying the laplace transform we get

$$Js^2\Theta(s) + Bs\Theta(s) = T_c(s)$$

$$\frac{\Theta(s)}{T_c(s)} = \boxed{\frac{1}{Js^2 + Bs}}$$

## (b)

Therefore we have

$$J\theta'' + B\theta' = K(\theta_r - \theta)$$

$$Js^2\Theta(s) + Bs\Theta(s) + K\Theta(s) = K\Theta_r(s)$$

$$\frac{\Theta_r(s)}{\Theta(s)} = \boxed{\frac{K}{Js^2 + Bs + K}}$$

## (c)

For overshoot we have

$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} < 0.1$$

$$-\pi\frac{\zeta}{\sqrt{1-\zeta^2}} < \ln(0.1)$$

$$-\pi\zeta < \ln(0.1)\sqrt{1-\zeta^2}$$

$$\pi^2\zeta^2 > \ln^2(0.1)(1-\zeta^2)$$

$$(\pi^2 + \ln^2(0.1))\zeta^2 > \ln^2(0.1)$$

$$\zeta > \sqrt{\frac{\ln^2(0.1)}{\pi^2 + \ln^2(0.1)}}$$

And since

$$\zeta\omega_n = \frac{B}{2J}$$

We have

$$\omega_n = \frac{B}{2\zeta} < \frac{B}{2J}\sqrt{\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}}$$

And since

$$\omega_n^2 = \frac{K}{J}$$

We have

$$\frac{K}{J} < \frac{B^2}{4J^2}\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}$$

$$K < \frac{B^2}{4J}\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}$$

$$K < 476.92$$

## (d)

In order for the rise time to be less than 80 seconds we must have

$$t_p = \frac{1.8}{\omega_n} < 80$$

$$\frac{1.8}{80} < \omega_n$$

and since we have $\omega_n^2 = \frac{K}{J}$ we have

$$\frac{1.8^2}{80^2}J < K$$

$$K > \boxed{303.75}$$