

ECE 141 Homework 1

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Problem 3.3

(a)

$$\begin{aligned}\cos(6t) &\rightarrow \frac{s}{s^2 + 36} \\ f(t) = 4\cos(6t) &\rightarrow \boxed{\frac{4s}{s^2 + 36}}\end{aligned}$$

(b)

$$\begin{aligned}\cos(3t) &\rightarrow \frac{s}{s^2 + 9} \\ \sin(3t) &\rightarrow \frac{3}{s^2 + 9} \\ e^{-t}\sin(3t) &\rightarrow \frac{3}{(s+1)^2 + 9} \\ f(t) = \sin(3t) + 2\cos(3t) + e^{-t}\sin(3t) &\rightarrow \frac{s+3}{s^2 + 9} + \frac{3}{(s+1)^2 + 9}\end{aligned}$$

Problem 3.7

(f)

We can rewrite $F(s)$ as

$$F(s) = \frac{a}{s+1} + \frac{bs+c}{s^2+16}$$

Therefore we have

$$f(t) = ae^{-t} + b\cos(4s) + \frac{c}{4}\sin(4s)$$

We have to solve for a, b, c such that

$$a(s^2+16) + (s+1)(bs+c) = 2(s+3)$$

therefore we have

$$a = -b$$

$$bs + cs = 2s$$

$$16a + c = 6$$

Solving these we get

$$f(t) = \boxed{(0.2353e^{-t} - 0.2353\cos(4s) + 0.5588\sin(4s)) u(t)}$$

(j)

$$\frac{1}{s^2} \rightarrow tu(t)$$

$$\frac{e^{-s}}{s^2} \rightarrow \boxed{(t-1)u(t-1)}$$

Problem 3.8

(b)

$$F(s) = \frac{1}{s-1}$$
$$f(t) = e^{-t}u(t)$$

Problem 3.9

(a)

$$\begin{aligned}y(t) &\rightarrow Y(s) \\y'(t) &\rightarrow sY(s) - 1 \\y''(t) &\rightarrow s^2Y(s) - s - 2 \\y''(t) - 2y'(t) + 4y(t) &\rightarrow s^2Y(s) - s - 2 - 2(sY(s) - 1) + 4Y(s) \\&= (s^2 - 2s + 4)Y(s) - s\end{aligned}$$

Therefore we have

$$Y(s) = \frac{s}{s^2 - 2s + 4} = \frac{s - 1 + 3}{(s - 1)^2 + 3}$$
$$y(t) = e^t(\cos(\sqrt{3}t) + \sqrt{3}\sin(\sqrt{3}t))u(t)$$

(b)

$$\begin{aligned}y''(t) + y'(t) &\rightarrow s^2Y(s) - s - 2 + sY(s) - 1 \\(s^2 + s)Y(s) - s - 3 &= \frac{1}{s^2 + 1}\end{aligned}$$

$$Y(s) = \frac{1}{(s^2 + 1)(s^2 + s)} + \frac{s + 3}{s^2 + s}$$

$$Y(s) = \frac{s + 1}{s^2 + s} + \frac{-1}{s^2 + 1} + \frac{3}{s} - \frac{2}{s + 1}$$

$$y(t) = \boxed{(4 - \sin(t) - 2e^{-t})u(t)}$$