ECE 141 Homework 4

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May 29, 2022

Problem 1

We have that

$$\beta = \arctan(\frac{l_r}{l_r + l_f}\tan(u))$$

therefore

$$\tan(\beta) = \frac{l_r}{l_r + l_f}\tan(u)$$

therefore since the range of tan is $-\infty$ to ∞ for any β we can find a u that satisfies the equation.

Problem 2

We have that

$$\frac{d}{dt}y = v\sin(\psi + \beta)$$

$$\frac{d}{dt}\psi = \frac{v}{l_R}\sin(\beta)$$

$$\beta = \arctan(\frac{l_r}{l_r + l_f}\tan(u))$$

Linearizing around $\psi = 0$ $\beta = 0$, we have

$$\frac{d}{dt}y = v(\psi + \beta)$$

$$\frac{d}{dt}\psi = \frac{v}{l_R}\beta$$

therefore taking the laplace transform we have

$$sY = v(\psi + \beta)$$

$$s\psi = \frac{v}{l_r}\beta$$

Therefore we get

$$sY = v(\frac{v}{l_r s} + 1)\beta$$

Therefore the transfer function is

$$\frac{Y(s)}{\beta} = \frac{v(v + l_r s)}{l_r s^2}$$

So now with a controller $D_c(s)$ and unity feedback we have that the transfer function is

$$\frac{Y}{R} = \frac{D_c(s)\frac{v(v+l_r s)}{l_r s^2}}{1 + D_c(s)\frac{v(v+l_r s)}{l_r s^2}}$$

Letting the controller be a PID controller, we have that the characteristic polynomial is

$$l_r s^3 + (k_p s + k_d s^2 + k_i)(v^2 + l_r v s) = 0$$

$$l_r(1 + k_d v)s^3 + (k_d v^2 + l_r v k_p)s^2 + (k_p v^2 + l_r v k_i)s + v^2 k_i = 0$$

$$s^3 + \frac{(k_d v^2 + l_r v k_p)}{l_r(1 + k_d v)}s^2 + \frac{(k_p v^2 + l_r v k_i)}{l_r(1 + k_d v)}s + \frac{v^2 k_i}{l_r(1 + k_d v)} = 0$$

picking $k_p = 1$, $k_i = 1$, $k_d = 1$ we get the following polynomial

$$s^3 + 9.5908s^2 + 9.5908s + 8.6509 = 0$$

Routh array of this polynomial can be computed using the following code in matlab

```
sympref('FloatingPointOutput',true)
syms  EPS
k_p=1;
k_d=1;
k_i=1;
v=15.6464;
l_r=1.7;
ra=routh([1  (k_d*v^2+l_r*v*k_p)/(l_r*(1+k_d*v)) (k_p*v^2+l_r*v*k_i)/(l_r*(1+k_d*v
simplify(ra)
```

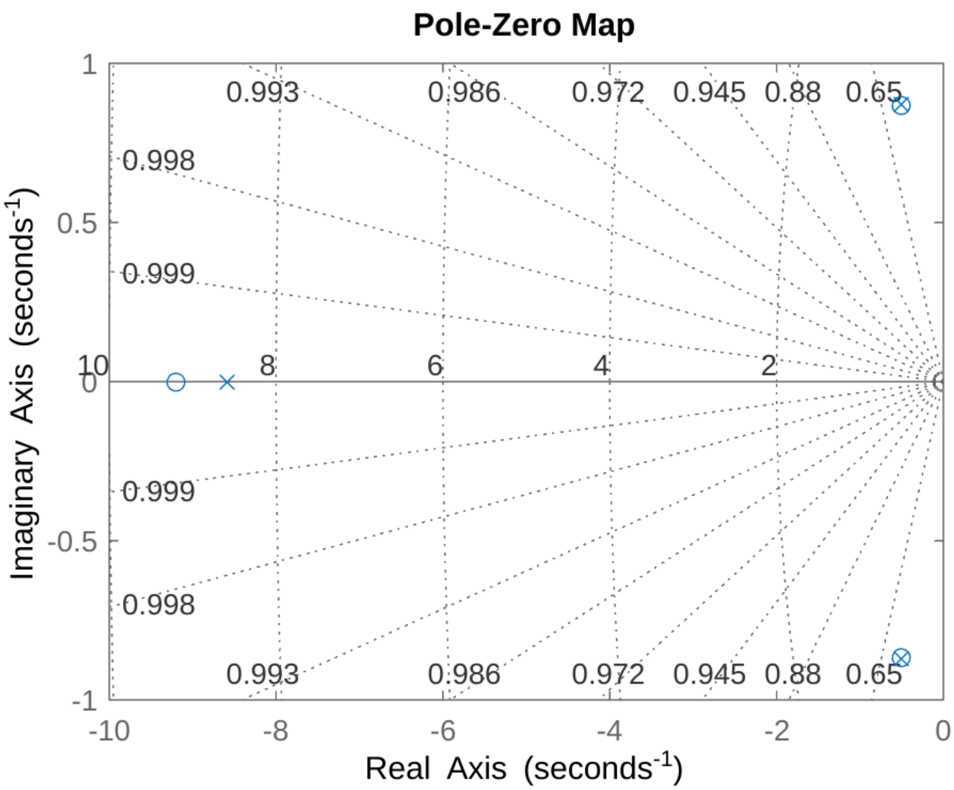
which outputs the following routh array

$$\begin{array}{cc} 1 & 9.5908 \\ 9.5908 & 8.6509 \\ 8.6888 & 0 \\ 8.6509 & 0 \end{array}$$

Therefore the controller makes the system stable, we can confirm this by looking at the poles plotting out the pole zero plot with the following code

```
sympref('FloatingPointOutput',true)
syms  EPS
k_p=1;
k_d=1;
k_i=1;
v=15.6464;
l_r=1.7;
p=[(l_r*(1+k_d*v))  (k_d*v^2+l_r*v*k_p) (k_p*v^2+l_r*v*k_i) v^2*k_i]
sys = tf([(l_r*(k_d*v))  (k_d*v^2+l_r*v*k_p) (k_p*v^2+l_r*v*k_i) v^2*k_i],p);
h = pzplot(sys);
grid on
```

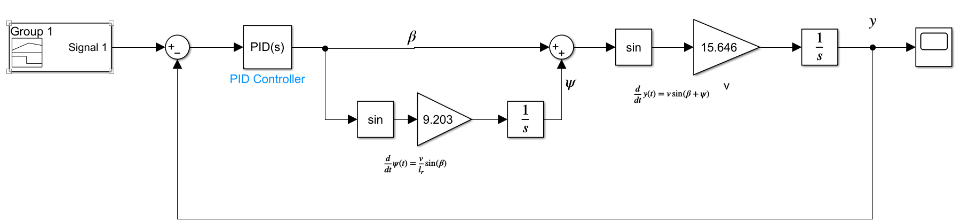
which outputs the following pole zero plot.



As we can see, the poles all have their real parts less than 0, which means the system is stable.

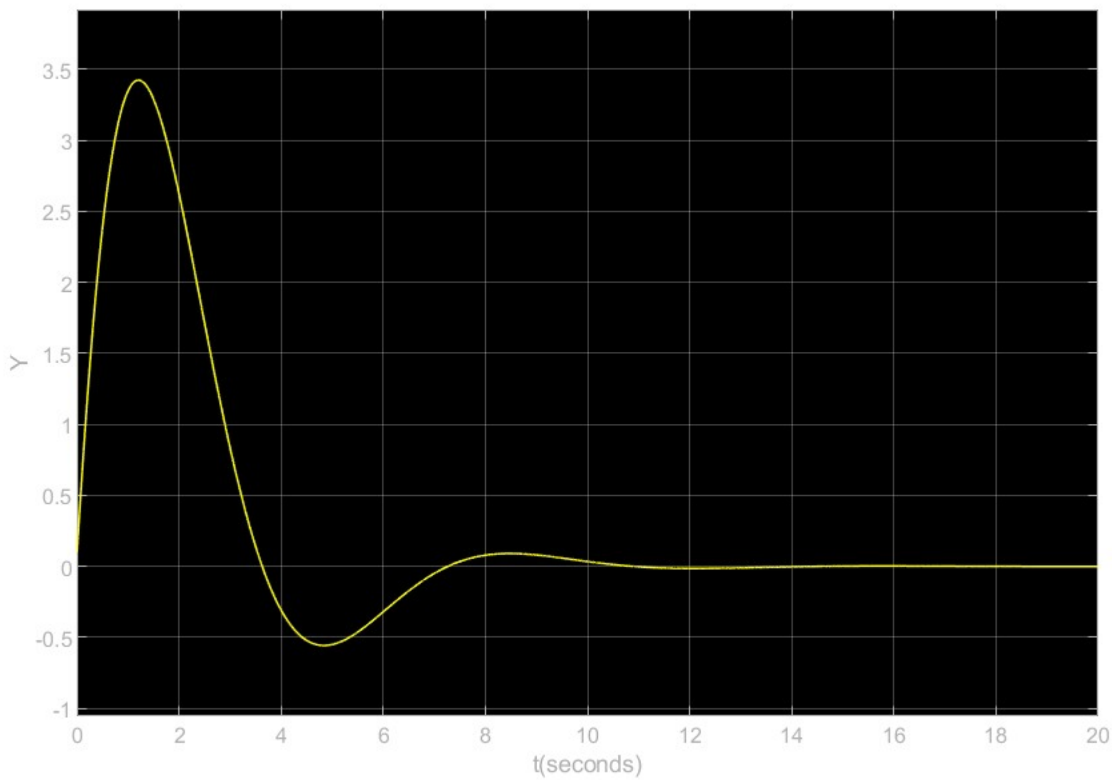
Problem 3

The blockdiagram for the nonlinear system looks like the following



We can modify the inital conditions by modifying the inital conditions for the itegrator blocks. We define good behaviour such that the absolute value of y is less than $\frac{3.0-1.8}{2} = 0.6$, ie the car cannot move out of its own line

With the initial condition $y(0^+) = 0.1$, and $\psi(0^+) = 0$ the controller display's unacceptabl behavior, see the plot below.



The car will initially move further away from $y = 0$, reaching a max of around $y = 3.5$ which is almost one lane away from the lane it needs to keep track off.