ECE 141 Homework 3

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Problem 3.53

(a)

Characterist equation

$$1 + KG(s) = 0$$
$$s(s^3 + 2s + 3s + 4) + 4(s + 2) = 0$$
$$s^4 + 2s^3 + 3s^2 + 8s + 8 = 0$$

Therefore we have

s^4:	1	3	8
s^3:	2	8	
s^2:	-2	8	
s^1:	24		
s^0:	8		

Unstable because change of two sign changes, equal two roots, so Unstable

(b)

$$1 + KG(s) = 0$$
$$s^2(s+1) + 2(s+4) = 0$$
$$s^3 + s^2 + 2s + 8 = 0$$

Therefore we have

s^3:	1	2
s^2:	1	8
s^1:	-6	
s^0:	8	

Unstable because change of two sign changes, equal two roots, so Unstable

Problem 3.54

(a)

s^4:	1	32	100
s^3:	8	80	
s^2:	22	100	
s^1:	43.6		
s^0:	100		

No sign change, so no roots

(b)

s^4:	1	7	8
s^3:	2	-2	
s^2:	8	8	
s^1:	-4		
s^0:	8		

Two sign changes, so two roots

Problem 3.57

We have that the transfer function is

$$\frac{KK_0\frac{s+z}{(s+p)(s^2-a^2)}}{1+KK_0\frac{s+z}{(s+p)(s^2-a^2)}} = \frac{KK_0(s+z)}{(s+p)(s^2-a^2)+KK_0(s+z)}$$

Therefore we have that the characterist polynomial is:

$$s^2 + ps^2 + (KK_0 - a^2)s + KK_0z - pa^2 = 0$$

Therefore we have the following routhe array

s^4:	1	$KK_0 - a^2$
s^3:	p	$KK_0z - pa^2$
s^1:	$KK_0\frac{p-z}{p}$	
s^0:	$KK_0z - pa^2$	

Therefore we have that

$$p > 0$$
$$KK_0\frac{p-z}{p} > 0$$
$$KK_0z - pa^2 > 0$$

therefore we have

$$KK_0p > KK_0z$$

$$KK_0 > pa^2 > 0$$

therefore we have

$$p > z$$

$$KK_0 > \frac{p}{z}a^2 > \frac{z}{z}a^2 > a^2$$

Therefore our conditions for stablity are

$$p > 0$$

$$p > z$$

$$KK_0 > a^2$$