ECE 141 Homework 3

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April 25, 2022

Problem 3.53

(a)

Characterist equation

$$1 + KG(s) = 0$$

$$s(s^3 + 2s + 3s + 4) + 4(s + 2) = 0$$

$$s^4 + 2s^3 + 3s^2 + 8s + 8 = 0$$

Therefore we have

$$
\begin{array}{llll}
s^4: & 1 & 3 & 8 \\
s^3: & 2 & 8 & \\
s^2: & \text{-2} & 8 & \\
s^1: & 24 & & \\
s^0: & 8 & &
\end{array}
$$

Unstable because change of two sign changes, equal two roots, so Unstable

(b)

$$1 + KG(s) = 0$$
$$s^2(s+1) + 2(s+4) = 0$$
$$s^3 + s^2 + 2s + 8 = 0$$

Therefore we have

$$
\begin{array}{llll}
s^3: & 1 & 2 \\
s^2: & 1 & 8 \\
s^1: & \text{-6} \\
s^0: & 8 \\
\end{array}
$$

Unstable because change of two sign changes, equal two roots, so Unstable

Problem 3.54

(a)

$$
\begin{array}{llll}
s^4: & 1 & 32 & 100 \\
s^3: & 8 & 80 \\
s^2: & 22 & 100 \\
s^1: & 43.6 \\
s^0: & 100 \\
\end{array}
$$

No sign change, so no roots

(b)

$$
\begin{array}{llll}
s^4: & 1 & 7 & 8 \\
s^3: & 2 & \text{-2} & \\
s^2: & 8 & 8 & \\
s^1: & \text{-4} & & \\
s^0: & 8 & &
\end{array}
$$

Two sign changes, so two roots

Problem 3.57

We have that the transfer function is

$$
\frac{KK_0 \frac{s+z}{(s+p)(s^2-a^2)}}{1 + KK_0 \frac{s+z}{(s+p)(s^2-a^2)}} = \frac{KK_0(s+z)}{(s+p)(s^2-a^2) + KK_0(s+z)}
$$

Therefore we have that the characterist polynomial is:

$$
s^2 + ps^2 + (KK_0 - a^2)s + KK_0 z - pa^2 = 0
$$

Therefore we have the following routhe array

$$
\begin{array}{lcc}
s^4: & 1 & KK_0 - a^2 \\
s^3: & p & KK_0 z - pa^2 \\
s^1: & KK_0 \frac{p-z}{p} & \\
s^0: & KK_0 z - pa^2 &
\end{array}
$$

Therefore we have that

$$
p > 0
$$

$$
KK_0 \frac{p-z}{p} > 0
$$

$$
KK_0 z - pa^2 > 0
$$

therefore we have

$$KK_0p > KK_0z$$

$$KK_0 > pa^2 > 0$$

therefore we have

$$p > z > 0$$

$$KK_0 > \frac{p}{z}a^2 > \frac{z}{z}a^2 > a^2$$

Therefore our conditions for stablity are

$$z > 0$$

$$p > z$$

$$KK_0 > a^2$$