

ECE 141 Homework 4

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Problem 4.11

(a)

We have that when, $W(s) = 0$

$$Y(s) = \frac{D_c(s)}{s^2 + K + D_c(s)} R(s)$$

$$E(s) = \frac{s^2 + K}{s^2 + K + D_c(s)}$$

Therefore from steady state theorem we have that the steady state error for a ramp input is

$$\lim_{s \rightarrow 0} sE(s) \frac{1}{s^2} = \lim_{s \rightarrow 0} \frac{s^2 + K}{s^3 + Ks + D_c(s)s}$$

Therefore we have that in order to have constant steady state error, $D_c(s)$ must have a pole at 0.

(b)

We have that

$$Y(s) = \frac{1}{s^2 + K + D_c(s)} W(s)$$

Therefore

$$Y(s) = \lim_{s \rightarrow 0} s \frac{1}{s^2 + K + D_c(s)} \frac{1}{s^n}$$

If $D(s)$ has a pole at 0, we have

$$Y(s) = \lim_{s \rightarrow 0} s \frac{1}{s^2 + K + \frac{C}{s}} \frac{1}{s^n}$$

For some C therefore we have that $Y(s) = 0$ only when $n = 1$, ie the system can reject unit step disturbances with 0 steady state error

Problem 4.26

(a)

We have

$$F_{car} = m\dot{v} = 10U - 10v$$

$$msV(s) = 10U - 10V(s)$$

$$(1000s + 10)V(s) = 10U(s)$$

$$\frac{V(s)}{U(s)} = \frac{1}{100s + 1}$$

(b)

We have

$$\begin{aligned} E(s) &= V_p - \frac{\frac{k_p}{s+0.02}}{1 + \frac{k_p}{s+0.02}} V_p - \frac{0.05 \frac{1}{s+0.02}}{1 + \frac{k_p}{s+0.02}} W(s) \\ &= \frac{(s+0.02)V_p - 0.05W(s)}{s+0.02+k_p} \end{aligned}$$

Since we are only considering the error from the gradient we have

$$E(s) = \frac{-0.05W(s)}{s+0.02+k_p}$$

therefore we have

$$\begin{aligned} e(t) &= \lim_{s \rightarrow 0} sE(s) \frac{2}{s} \\ &= \lim_{s \rightarrow 0} \frac{-0.1}{s+0.02+k_p} = \frac{-0.1}{0.02+k_p} \end{aligned}$$

And therefore in order for $|e(t)| < 1$, we must have

$$\begin{aligned} \frac{0.1}{|0.02+k_p|} &< 1 \\ 0.1 &< |0.02+k_p| < 0.02+|k_p| \\ 0.08 &< |k_p| \end{aligned}$$

Since we must have the root to be less than $s = 0$ we must have

$$\boxed{k_p > 0.08}$$

(c)

Ensures steady state tracking error to be 0

(d)

We have

$$\begin{aligned} E(s) &= V_p - \frac{\frac{k_p}{s} \frac{1}{s+0.02}}{1 + \frac{k_p}{s^2+0.02s}} V_p - \frac{0.05sW(s)}{s^2 + 0.02s + k_p} \\ &= \frac{(s^2 + 0.02s)V_p - 0.05sW(s)}{s^2 + 0.02s + k_p} \end{aligned}$$

Therefore if we have $\eta = 1$, we must have $\omega_n = 0.01$, and thus $\boxed{k_p = 0.01^2}$

Problem 4.29

(a)

When the reference speed is zero we have

$$E = \frac{1}{Js + b + \frac{10K}{0.5s+1}}$$

Therefore the steady state error is

$$\begin{aligned} e(\infty) &= \lim_{s \rightarrow 0} s \frac{1}{Js + b + \frac{10K}{0.5s+1}} \frac{1}{s} \\ &= \frac{1}{b + 10K} \end{aligned}$$

Therefore we have

$$|e(\infty)| \leq 0.01 \frac{1}{|b + 10K|} \leq 0.01$$

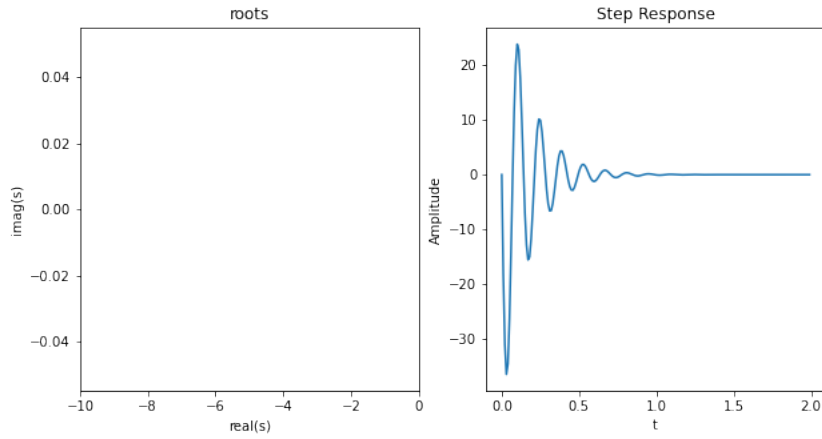
$$100 \leq |b + 10K| \leq b + 10|K|$$

$$\boxed{K \geq 9.9}$$

(b)

We have

$$\frac{\Omega_m}{\Omega_f} = \frac{(Js + b)(0.5s + 1)}{(0.5s + 1)(Js + b) + 10K}$$

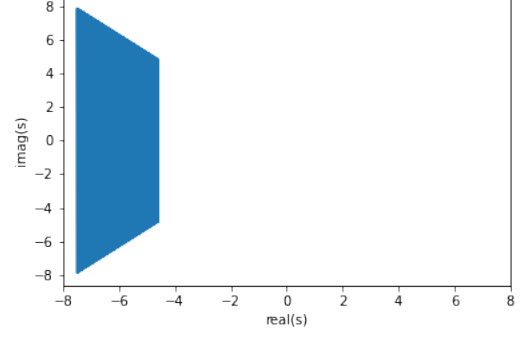


(c)

We have

$$M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}} \leq 5\%$$

$$t_s = \frac{4.6}{\zeta \omega_n} \leq 0.1$$



Therefore the region of the complex plane is

(d)

$$\frac{\Omega_m}{\Omega_f} = \frac{(Js + b)(0.5s + 1)}{(0.5s + 1)(Js + b) + 10(k_p + K_D s)}$$

Therefore we will have that the characterist polynomial is

$$s^2 + (12 + 200K_d)s + 20 + 200k_p = 0$$

Therefore we have

$$\zeta \omega_m = 6 + 100K_d \geq 46$$

$$\frac{\pi^2}{\ln^2(0.05)} \zeta^2 \geq 1 - \zeta^2$$

$$20 + 200k_p = \omega_m^2 \geq \left(\frac{\pi^2}{\ln^2(0.05)} + 1 \right) \zeta^2 \omega^2 \geq \left(\frac{\pi^2}{\ln^2(0.05)} + 1 \right) 46^2$$

$$\boxed{k_p \geq 22.115}$$

$$\boxed{k_D \geq 0.4}$$

(e)

$$E = \frac{1}{Js + b + \frac{10(k_D s + k_p)}{0.5s + 1}}$$

Therefore the steady state error is

$$\begin{aligned} e(\infty) &= \lim_{s \rightarrow 0} s \frac{1}{Js + b + \frac{10(k_D s + k_p)}{0.5s + 1}} \frac{1}{s} \\ &= \frac{1}{b + 10k_p} \\ &= 0.450\% \end{aligned}$$

To eliminate steady state error we could use PID controller