ECE 141 Homework 4

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Problem 1

We have that

$$\beta = \arctan(\frac{l_r}{l_r + l_f}\tan(u))$$

therefore

$$\tan(\beta) = \frac{l_r}{l_r + l_f}\tan(u)$$

therefore since the range of tan is $-\infty$ to ∞ for any β we can find a u that satisfies the equation.

Problem 2

We have that

$$\frac{d}{dt}y = v\sin(\psi + \beta)$$

$$\frac{d}{dt}\psi = \frac{v}{l_R}\sin(\beta)$$

$$\beta = \arctan(\frac{l_r}{l_r + l_f}\tan(u))$$

Linearizing around $\psi = 0$ $\beta = 0$, we have

$$\frac{d}{dt}y = v(\psi + \beta)$$

$$\frac{d}{dt}\psi = \frac{v}{l_R}\beta$$

therefore taking the laplace transform we have

$$sY = v(\psi + \beta)$$

$$s\psi = \frac{v}{l_r}\beta$$

Therefore we get

$$sY = v(\frac{v}{l_r s} + 1)\beta$$

Therefore the transfer function is

$$\frac{Y(s)}{\beta} = \frac{v(v + l_r s)}{l_r s^2}$$

So now with a controller $D_c(s)$ and unity feedback we have that the transfer function is

$$\frac{Y}{R} = \frac{D_c(s)\frac{v(v+l_r s)}{l_r s^2}}{1 + D_c(s)\frac{v(v+l_r s)}{l_r s^2}}$$

Letting the controller be a PID controller, we have that the characteristic polynomial is

$$l_r s^3 + (k_p s + k_d s^2 + k_i)(v^2 + l_r v s) = 0$$

$$l_r(1 + k_d v)s^3 + (k_d v^2 + l_r v k_p)s^2 + (k_p v^2 + l_r v k_i)s + v^2 k_i = 0$$

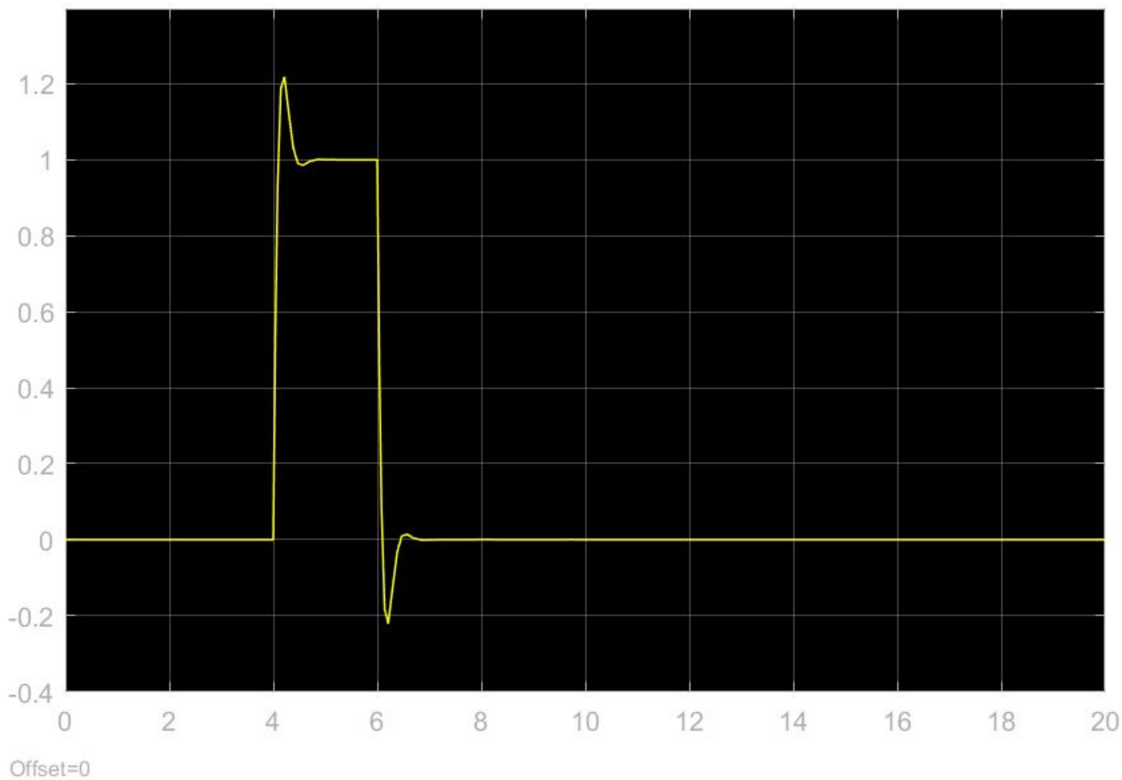
$$s^3 + \frac{(k_d v^2 + l_r v k_p)}{l_r(1 + k_d v)}s^2 + \frac{(k_p v^2 + l_r v k_i)}{l_r(1 + k_d v)}s + \frac{v^2 k_i}{l_r(1 + k_d v)} = 0$$

Therefore the error is

$$E(s) = R(s) - Y(s) = R(s) - \frac{D_c(s)\frac{v(v+l_r s)}{l_r s^2}}{1 + D_c(s)\frac{v(v+l_r s)}{l_r s^2}}R(s)$$

To model a disturbance of length d starting at time t_1 and ending at time t_2, we let $r(t) = du(t - t_1) - du(t - t_2)$. and let the controller be a simple k_p controller.

simulating in simulink with $k_p = 1$ and $d = 1$ and $t_1 = 4$ and $t_2 = 6$, we get the following plot for $y(t)$.



As we can see, the controller stabilizes the car at $y = 0$.