ECE 141 Homework 4

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Problem 4.11

(a)

We have that when, $W(s) = 0$

$$Y(s) = \frac{D_c(s)}{s^2 + K + D_c(s)} R(s)$$

$$E(s) = \frac{s^2 + K}{s^2 + K + D_c(s)}$$

Therefore from steady state theorem we have that the steady state error for a ramp input is

$$\lim_{s \to 0} sE(s)\frac{1}{s^2} = \lim_{s \to 0} \frac{s^2 + K}{s^3 + Ks + D_c(s)s}$$

Therefore we have that in order to have constant steady state error, $D_c(s)$ must have a pole at 0.

(b)

We have that

$$Y(s) = \frac{1}{s^2 + K + D_c(s)} W(s)$$

Therefore

$$Y(s) = \lim_{s \to 0} s \frac{1}{s^2 + K + D_c(s)} \frac{1}{s^n}$$

If $D(s)$ has a pole at 0, we have

$$Y(s) = \lim_{s \to 0} s \frac{1}{s^2 + K + \frac{C}{s}} \frac{1}{s^n}$$

For some C therefore we have that $Y(s) = 0$ only when $n = 1$, ie the system can reject unit step disturbances with 0 steady state error

Problem 4.26

(a)

We have

$$F_{car} = m\dot{v} = 10U - 10v$$
$$msV(s) = 10U - 10V(s)$$
$$(1000s + 10)V(s) = 10U(s)$$
$$\frac{V(s)}{U(s)} = \frac{1}{100s + 1}$$

(b)

We have

$$
\begin{aligned}
E(s) &= V_p - \frac{\frac{k_p}{s+0.02}}{1+\frac{k_p}{s+0.02}}V_p - \frac{0.05\frac{1}{s+0.02}}{1+\frac{k_p}{s+0.02}}W(s) \\
&= \frac{(s+0.02)V_p - 0.05W(s)}{s+0.02+k_p}
\end{aligned}
$$

Since we are only considering the error from the gradient we have

$$E(s) = \frac{-0.05W(s)}{s+0.02+k_p}$$

therefore we have

$$
\begin{aligned}
e(t) &= \lim_{s\to 0} sE(s)\frac{2}{s} \\
&= \lim_{s\to 0} \frac{-0.1}{s+0.02+k_p} && = \frac{-0.1}{0.02+k_p}
\end{aligned}
$$

And therefore in order for $|e(t)| < 1$, we must have

$$
\begin{gathered}
\frac{0.1}{|0.02+k_p|} < 1 \\
0.1 < |0.02+k_p| < 0.02 + |k_p| \\
0.08 < |k_p|
\end{gathered}
$$

Since we must have the root to be less than $s = 0$ we must have

$$\boxed{k_p > 0.08}$$

(c)

Ensures steady state tracking error to be 0

(d)

We have

$$E(s) = V_p - \frac{\frac{k_p}{s}\frac{1}{s+0.02}}{1+\frac{k_p}{s^2+0.02s}}V_p - \frac{0.05sW(S)}{s^2+0.02s+k_p}$$
$$= \frac{(s^2+0.02s)V_p - 0.05sW(s)}{s^2+0.02s+k_p}$$

Therefore if we have $\eta = 1$, we must have $\omega_n = 0.01$, and thus $\boxed{k_p = 0.01^2}$

Problem 4.29

(a)

When the refernce speed is zero we have

$$E = \frac{1}{Js+b+\frac{10K}{0.5s+1}}$$

Therefore the steady state error is

$$e(\infty) = \lim_{s\to 0} s\frac{1}{Js+b+\frac{10K}{0.5s+1}}\frac{1}{s}$$
$$= \frac{1}{b+10K}$$

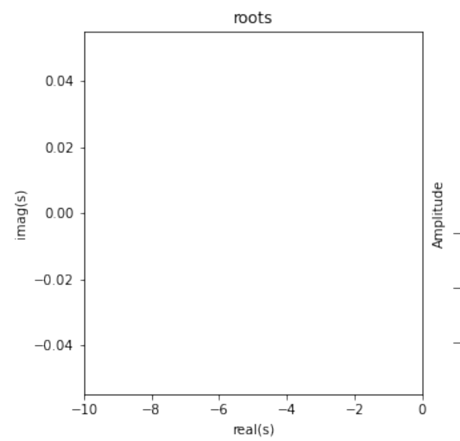
Therefore we have

$$|e(\infty)| \leq 0.01 \frac{1}{|b+10K|} \qquad\qquad \leq 0.01$$
$$100 \leq |b+10K| \leq b+10|K|$$
$$\boxed{K \geq 9.9}$$

(b)

We have

$$\frac{\Omega_m}{\Omega_f} = \frac{(Js+b)(0.5s+1)}{(0.5s+1)(Js+b)+10K}$$



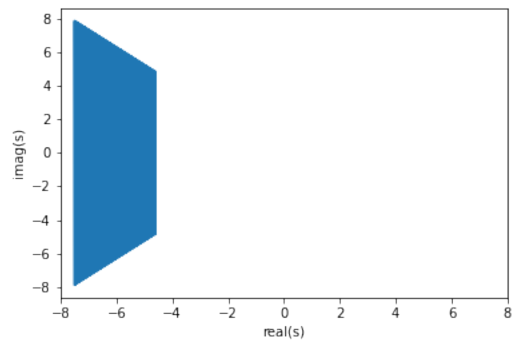
let $K = 10$, then we have the following poles and time response

(c)

We have

$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} \leq 5\%$$

$$t_s = \frac{4.6}{\zeta\omega_n} \leq 0.1$$

Therfore the region of the complex plane is 

(d)

$$\frac{\Omega_m}{\Omega_f} = \frac{(Js+b)(0.5s+1)}{(0.5s+1)(Js+b)+10(k_p+K_D s)}$$

Therefore we will have that the characterist polynomial is

$$s^2 + (12+200K_d)s + 20 + 200k_p = 0$$

Therefore we have

$$\zeta\omega_m = 6 + 100K_d \geq 46$$

$$\frac{\pi^2}{\ln^2(0.05)}\zeta^2 \geq 1-\zeta^2$$

$$20 + 200k_p = \omega_m^2 \geq \left(\frac{\pi^2}{\ln^2(0.05)} + 1\right)\zeta^2\omega^2 \geq \left(\frac{\pi^2}{\ln^2(0.05)} + 1\right)46^2$$

$$\boxed{k_p \geq 22.115}$$

$$\boxed{k_D \geq 0.4}$$

(e)

$$E = \frac{1}{Js + b + \frac{10(k_D s + k_p)}{0.5s+1}}$$

Therefore the steady state error is

$$
\begin{aligned}
e(\infty) &= \lim_{s\to 0} s \frac{1}{Js + b + \frac{10(k_D s + k_p)}{0.5s+1}} \frac{1}{s} \\
&= \frac{1}{b + 10k_p} \\
&= 0.450\%
\end{aligned}
$$

To eliminate steady state error we could use PID controller