ECE 141 Homework 2

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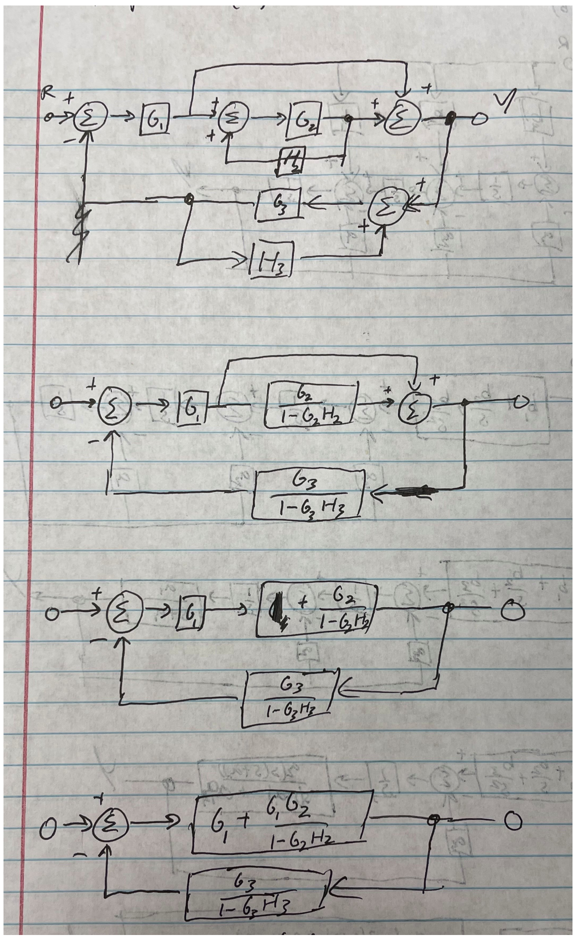
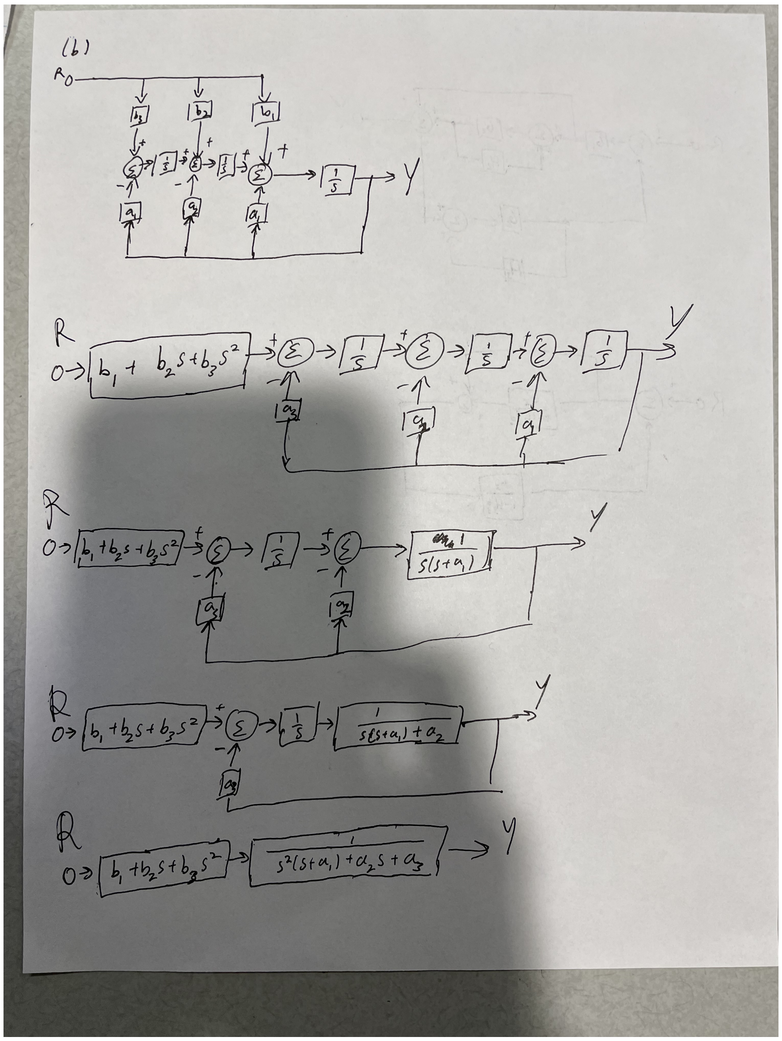
Problem 3.21

(a)



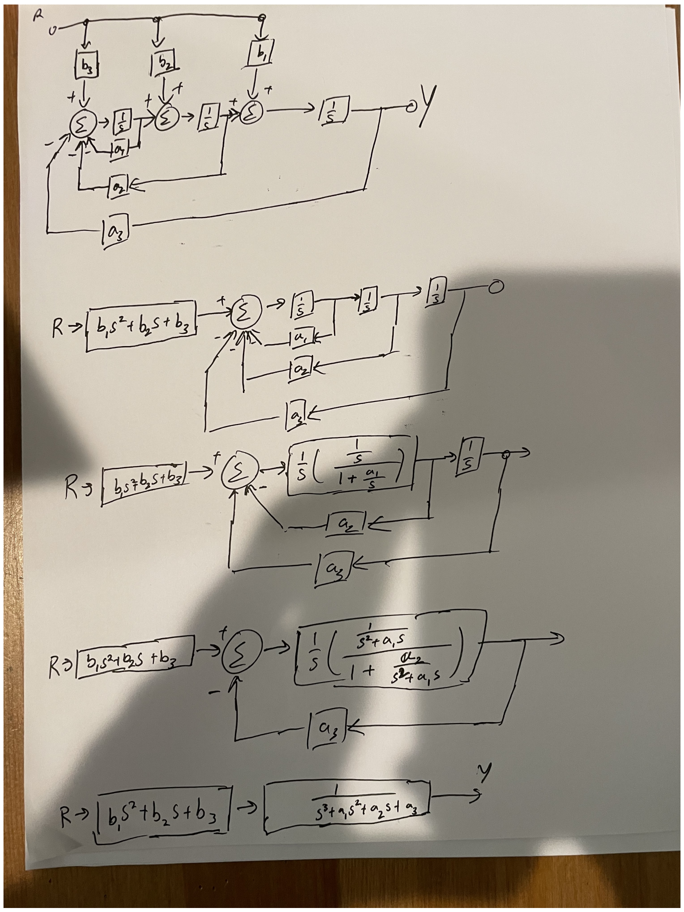
$$\frac{Y(s)}{R(s)} = \frac{G_1 + \frac{G_1 G_2}{1 - G_2 H_2}}{1 + \left(G_1 + \frac{G_1 G_2}{1 - G_2 H_2}\right) \frac{G_3}{1 - G_3 H_3}}$$

(b)



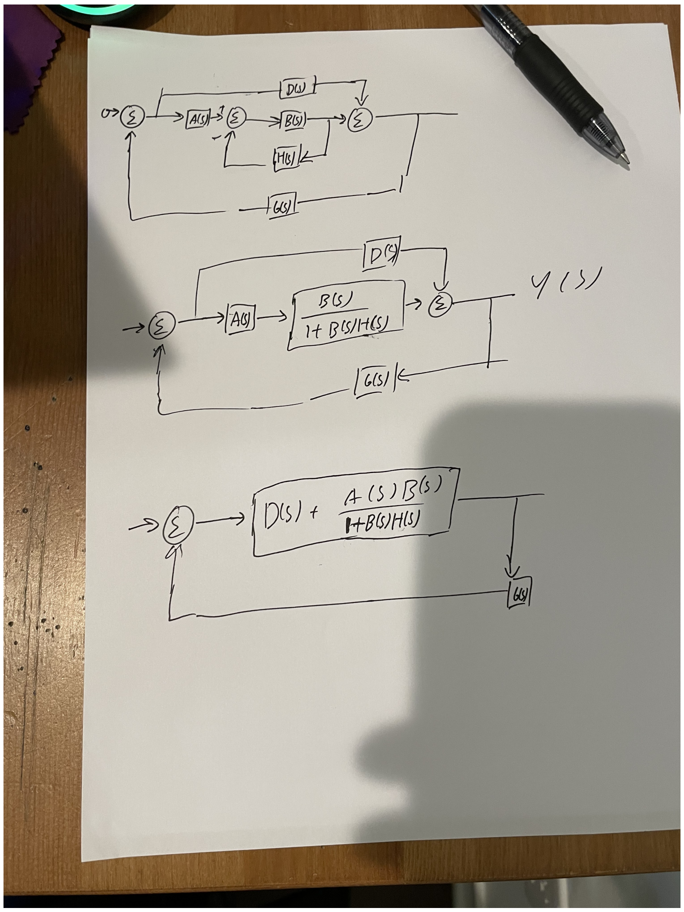
$$\frac{Y(s)}{R(s)} = \boxed{\frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}}$$

(c)



$$\frac{Y(s)}{R(s)} = \boxed{\frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}}$$

(d)



$$\frac{Y(s)}{R(s)} = \boxed{\frac{D(s) + D(s)B(s)H(s) + A(s)B(s)}{1 + B(s)H(s) + G(s)(D(s) + D(s)B(s)H(s) + A(s)B(s))}}$$

Problem 3.30

(a)

We have

$$16\% \geq M_p = e^{-\pi \frac{\zeta}{\sqrt{1-\zeta^2}}}$$

$$6.9 \geq t_s = \frac{4.6}{\zeta \omega_n}$$

$$1.8 \geq t_r = \frac{1.8}{\omega_n}$$

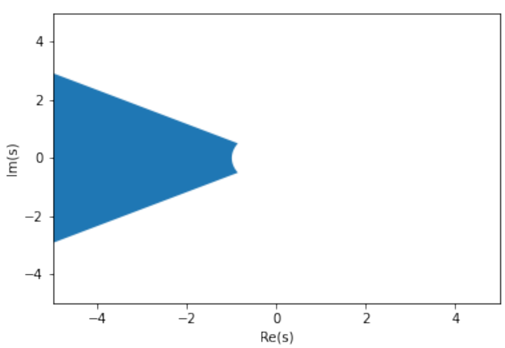
Therefore we get three conditions

$$\omega_n \geq 1$$

$$\sigma \leq -\frac{4.6}{6.9}$$

$$\sin^- 1(\zeta) \leq \tan^{-1} \frac{-\ln(0.16)}{\pi}$$

From these we get the following region



(b)

We therefore have

$$1.8 = t_r = \frac{1.8}{\omega_n}$$

Therefore $\omega_n = 1$, furthermore we have

$$6.9 = t_r = \frac{4.6}{\zeta\omega_n} = \frac{4.6}{\zeta}$$

Therefore $\zeta = \frac{4.6}{6.9} = \frac{2}{3}$ and thus we have that the overshoot

$$M_p = e^{-\pi\frac{2}{\sqrt{5}}}$$

$$M_p = 6\%$$

Problem 3.32

(a)

Therefore we have that the transfer function is

$$\begin{aligned}\frac{Y(s)}{R(s)} &= \frac{G(s)}{1+G(s)} \\ &= \frac{K}{s^2+2s+K}\end{aligned}$$

Therefore we have

$$K = \omega_n^2$$

$$\zeta\omega_n = 1$$

Furthermore from the peak time and overshoot criterion we have

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1$$

$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} = 0.05$$

Using the equation from t_p and applying $\omega_n = \frac{1}{\zeta}$ we have

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} = 1$$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{1}{\pi}$$

Plugging this into M_p we get

$$e^{-\pi\frac{1}{\pi}} = 0.367 \neq 0.05$$

Therefore we cannot meet both specifications just by selecting the right value of K

(b)

From the peak time and overshot criterion we have

$$\omega_n\sqrt{1-\zeta^2} = \pi$$

and

$$\omega_n\zeta = -\ln(0.05)$$

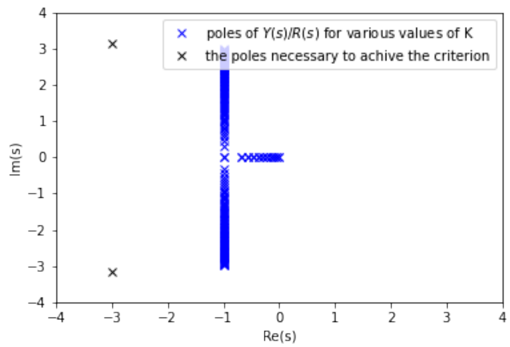
Therefore we have poles at

$$s = \ln(0.05) \pm \pi j$$

Furthermore, the poles for $\frac{Y(s)}{R(s)}$ are at

$$s = \frac{-2 \pm \sqrt{4-4K}}{2} = -1 \pm \sqrt{1-K}$$

Therefore we get the following sketch for the poles in the s plane



(c)

We relax the conditions by multiplying the peak time and overshoot with c therefore we have

$$t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = c$$

$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} = 0.05c$$

thererfore since $\omega_n = \frac{1}{\zeta}$

$$\frac{\zeta}{\sqrt{1-\zeta^2}} = \frac{c}{\pi}$$

therefore we have

$$-c = \ln(0.05) + \ln(c)$$

Solving this we have

$$c = 2.205$$

$$M_P = 11\%$$

$$t_p = 2.205s$$

And therefore we have

$$\frac{1}{\sqrt{\omega_n^2 - 1}} = \frac{c}{\pi} = 0.701$$

therefore we have

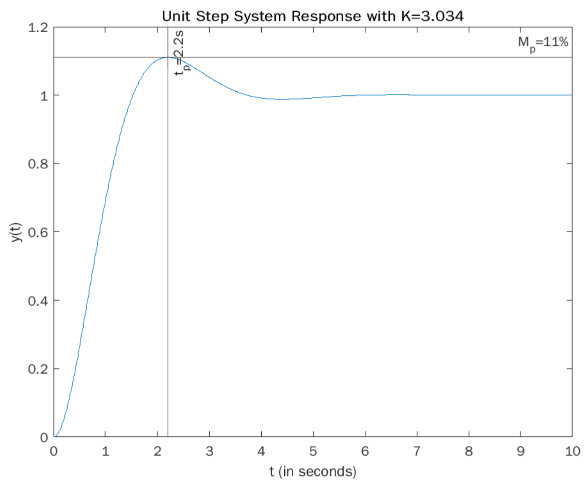
$$\frac{1}{0.701^2} = \omega_n^2 - 1$$

$$K = \omega_n^2 = \frac{1}{0.701^2} + 1 = 3.034$$

Using the following Matlab code, we can plot out the unit step response

```
syms s

K=3.034;
G=K/(s*(s+2));
H=G/(1+G);

SystemUnitResponse=H*1/s;

Y=ilaplace(SystemUnitResponse);

t=0:0.01:10;

plot(t,y)
[M_p,t_p]=max(y);
t_p=t(t_p);
M_p=double(M_p-1);
yline(M_p+1, "-", sprintf("M_p=%d%%",round(M_p,3)*100));
xline(t_p, "-",sprintf("t_p=%gs",round(t_p,1)));
xlabel('t (in seconds)')
ylabel('y(t)')
title('Unit Step System Response with K=3.034')
```

From which we get the following graph of the unite step response



As indicated on the graph, the peak time was $2.2s$ and the overshoot was 11% Which is inline with what we had expected

Problem 3.36

(a)

Applying the laplace transform we get

$$Js^2\Theta(s) + Bs\Theta(s) = T_c(s)$$

$$\frac{\Theta(s)}{T_c(s)} = \boxed{\frac{1}{Js^2 + Bs}}$$

(b)

Therefore we have

$$J\theta'' + B\theta' = K(\theta_r - \theta)$$

$$Js^2\Theta(s) + Bs\Theta(s) + K\Theta(s) = K\Theta_r(s)$$

$$\frac{\Theta_r(s)}{\Theta(s)} = \boxed{\frac{K}{Js^2 + Bs + K}}$$

(c)

For overshoot we have

$$M_p = e^{-\pi\frac{\zeta}{\sqrt{1-\zeta^2}}} < 0.1$$

$$-\pi\frac{\zeta}{\sqrt{1-\zeta^2}} < \ln(0.1)$$

$$-\pi\zeta < \ln(0.1)\sqrt{1-\zeta^2}$$

$$\pi^2\zeta^2 > \ln^2(0.1)(1-\zeta^2)$$

$$(\pi^2 + \ln^2(0.1))\zeta^2 > \ln^2(0.1)$$

$$\zeta > \sqrt{\frac{\ln^2(0.1)}{\pi^2 + \ln^2(0.1)}}$$

And since

$$\zeta\omega_n = \frac{B}{2J}$$

We have

$$\omega_n = \frac{B}{2\zeta} < \frac{B}{2J}\sqrt{\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}}$$

And since

$$\omega_n^2 = \frac{K}{J}$$

We have

$$\frac{K}{J} < \frac{B^2}{4J^2}\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}$$

$$K < \frac{B^2}{4J}\frac{\pi^2 + \ln^2(0.1)}{\ln^2(0.1)}$$

$$K < 476.92$$

(d)

In order for the rise time to be less than 80 seconds we must have

$$t_p = \frac{1.8}{\omega_n} < 80$$

$$\frac{1.8}{80} < \omega_n$$

and since we have $\omega_n^2 = \frac{K}{J}$ we have

$$\frac{1.8^2}{80^2} J < K$$

$$K > \boxed{303.75}$$

(e)

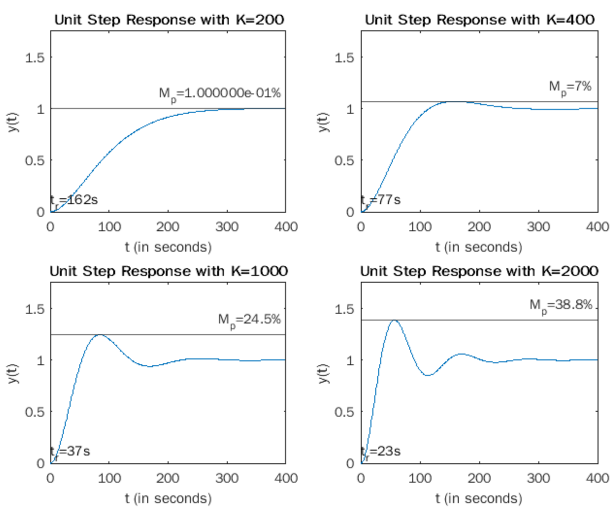
Using the following code, we plot the step responses for the various values of K

```
J=600000;
B=20000;
syms K
syms s
t=0:400;
H=K/(J*s^2+B*s+K);


figure;
axs1=subplot(2,2,1);
H200=subs(H,K,200)
y=subs(ilaplace(H200*1/s),t);
plot(t,y)
[M_p,t_p]=max(y);
t_r=t(min(find(y>0.9)))-t(max(find(y<0.1)));
M_p=double(M_p-1);
```

```
yline(M_p+1, "-", sprintf("M_p=%d%%",round(M_p,3)*100));
text( 0.5, 0.1, sprintf("t_r=%gs",round(t_r,1)));
xlabel('t (in seconds)')
ylabel('y(t)')
title("Unit Step Response with K=200")

axs2=subplot(2,2,2)
H400=subs(H,K,400);
y=subs(ilaplace(H400*1/s),t);
plot(t,y)
[M_p,t_p]=max(y);
t_r=t(min(find(y>0.9)))-t(max(find(y<0.1)));
M_p=double(M_p-1);
yline(M_p+1, "-", sprintf("M_p=%g%%",round(M_p,3)*100));
text( 0.5, 0.1, sprintf("t_r=%gs",round(t_r,1)));
xlabel('t (in seconds)')
ylabel('y(t)')
title("Unit Step Response with K=400")

axs3=subplot(2,2,3)
H1000=subs(H,K,1000);
y=subs(ilaplace(H1000*1/s),t);
plot(t,y)
[M_p,t_p]=max(y);
t_r=t(min(find(y>0.9)))-t(max(find(y<0.1)));
M_p=double(M_p-1);
yline(M_p+1, "-", sprintf("M_p=%g%%",round(M_p,3)*100));
text( 0.5, 0.1, sprintf("t_r=%gs",round(t_r,1)));
xlabel('t (in seconds)')
ylabel('y(t)')
title("Unit Step Response with K=1000")

axs4=subplot(2,2,4)
H2000=subs(H,K,2000);
y=subs(ilaplace(H2000*1/s),t);
plot(t,y)
[M_p,t_p]=max(y);
t_r=t(min(find(y>0.9)))-t(max(find(y<0.1)));
```

```
M_p=double(M_p-1);
yline(M_p+1, "-", sprintf("M_p=%g%%",round(M_p,3)*100));
text( 0.5, 0.1, sprintf("t_r=%gs",round(t_r,1)));
xlabel('t (in seconds)')
ylabel('y(t)')
title("Unit Step Response with K=2000")

linkaxes([axs1 axs2 axs3 axs4],'xy')
axs1.YLim =[0 1.75];
```

From which we get the following graph



This fits our calculations, since for $K = 400, 1000, 2000$ the rise time is less than $80s$ and for $K = 1000, 2000$ the overshoot is greater than 10%, and conversely for $K = 200, 400$ the overshoot is less than 10%.