ECE 141 Homework 4

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Problem 1

We have that

$$\beta = \arctan(\frac{l_r}{l_r + l_f}\tan(u))$$

therefore

$$\tan(\beta) = \frac{l_r}{l_r + l_f}\tan(u)$$

therefore since the range of tan is $-\infty$ to ∞ for any β we can find a u that satisfies the equation.

Problem 2

We have that

$$\frac{d}{dt}y = v\sin(\psi + \beta)$$

$$\frac{d}{dt}\psi = \frac{v}{l_R}\sin(\beta)$$

$$\beta = \arctan(\frac{l_r}{l_r + l_f}\tan(u))$$

Linearizing around $\psi = 0$ $\beta = 0$, we have

$$\frac{d}{dt} y = v(\psi + \beta)$$

$$\frac{d}{dt}\psi = \frac{v}{l_R}\beta$$

therefore taking the laplace transform we have

$$sY = v(\psi + \beta)$$

$$s\psi = \frac{v}{l_r}\beta$$

Therefore we get

$$sY = v(\frac{v}{l_r s} + 1)\beta$$

Therefore the transfer function is

$$\frac{Y(s)}{\beta} = \frac{v(v + l_r s)}{l_r s^2}$$

So now with a controller $D_c(s)$ and unity feedback we have that the transfer function is

$$\frac{Y}{R} = \frac{D_c(s)\frac{v(v+l_r s)}{l_r s^2}}{1 + D_c(s)\frac{v(v+l_r s)}{l_r s^2}}$$

Letting the controller be a PID controller, we have that the characteristic polynomial is

$$l_r s^3 + (k_p s + k_d s^2 + k_i)(v^2 + l_r v s) = 0$$

$$l_r(1 + k_d v)s^3 + (k_d v^2 + l_r v k_p)s^2 + (k_p v^2 + l_r v k_i)s + v^2 k_i = 0$$

$$s^3 + \frac{(k_d v^2 + l_r v k_p)}{l_r(1 + k_d v)}s^2 + \frac{(k_p v^2 + l_r v k_i)}{l_r(1 + k_d v)}s + \frac{v^2 k_i}{l_r(1 + k_d v)} = 0$$

picking $k_p = 1$, $k_i = 1$, $k_d = 1$ we get the following polynomial

$$s^3 + 9.5908s^2 + 9.5908s + 8.6509 = 0$$

Routh array of this polynomial can be computed using the following code in matlab

```
sympref('FloatingPointOutput',true)
syms  EPS
k_p=1;
k_d=1;
k_i=1;
v=15.6464;
l_r=1.7;
ra=routh([1  (k_d*v^2+l_r*v*k_p)/(l_r*(1+k_d*v)) (k_p*v^2+l_r*v*k_i)/(l_r*(1+k_d*v
simplify(ra)
```

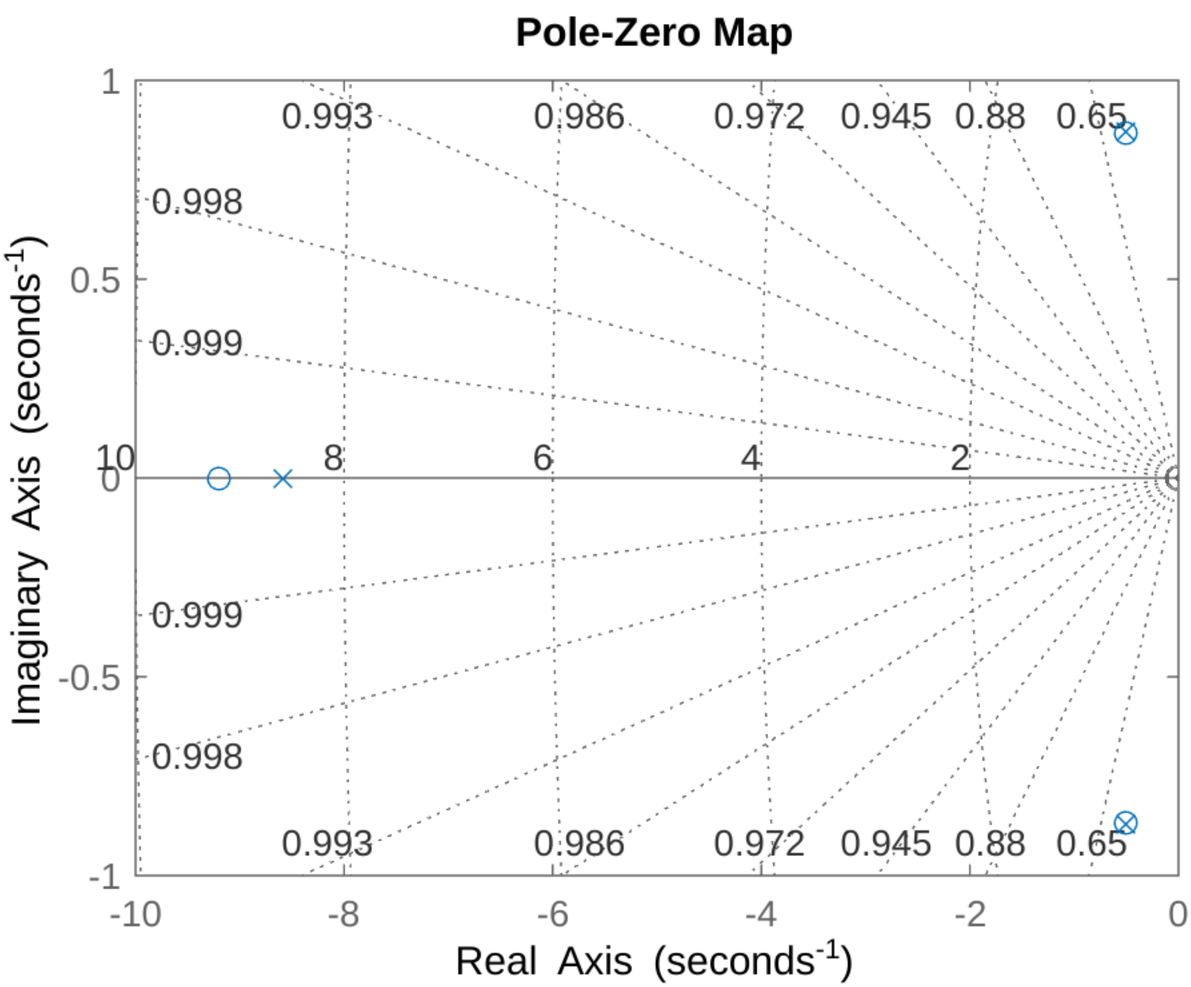
which outputs the following routh array

$$
\begin{array}{cc}
1 & 9.5908 \\
9.5908 & 8.6509 \\
8.6888 & 0 \\
8.6509 & 0
\end{array}
$$

Therefore the controller makes the system stable, we can confirm this by looking at the poles plotting out the pole zero plot with the following code

```
sympref('FloatingPointOutput',true)
syms  EPS
k_p=1;
k_d=1;
k_i=1;
v=15.6464;
l_r=1.7;
p=[(l_r*(1+k_d*v))  (k_d*v^2+l_r*v*k_p) (k_p*v^2+l_r*v*k_i) v^2*k_i]
sys = tf([(l_r*(k_d*v))  (k_d*v^2+l_r*v*k_p) (k_p*v^2+l_r*v*k_i) v^2*k_i],p);
h = pzplot(sys);
grid on
```

which outputs the following pole zero plot.


Pole-Zero Map
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.993
0.986
0.972
0.945
0.88
0.65
0.998
0.999
10
8
6
4
2
1
0.5
0
-0.5
-1
-10
-8
-6
-4
-2
0