

# ECE 141 Homework 4

Lawrence Liu

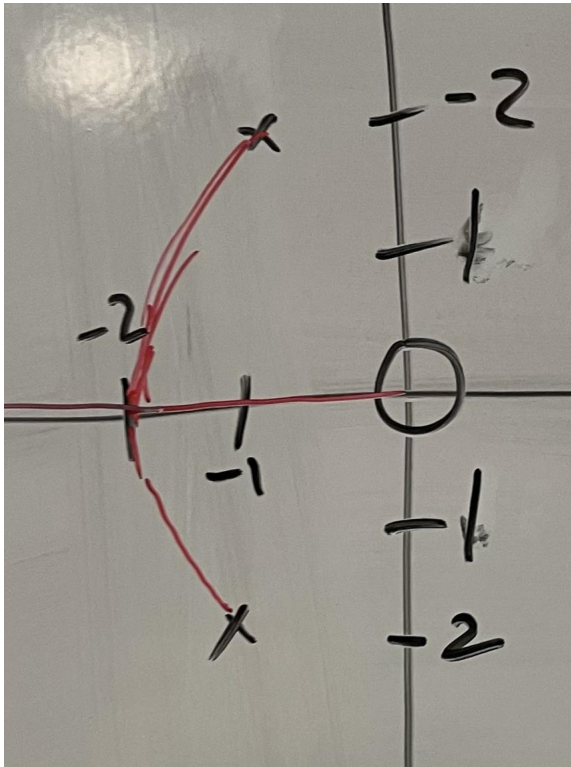
May 23, 2022

## Problem 5.9

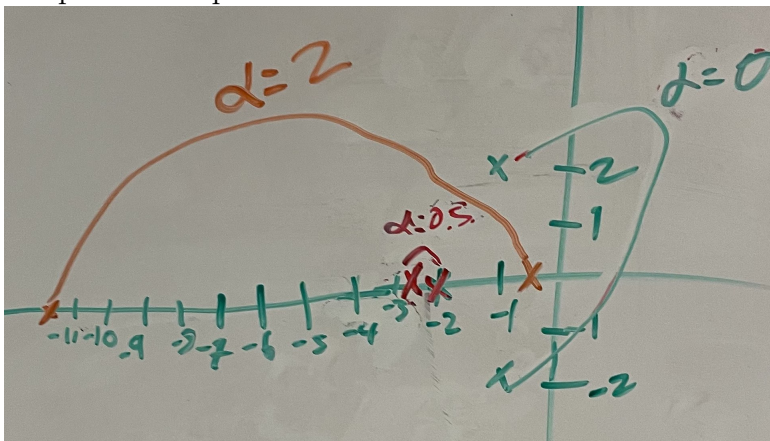
The transfer function is

$$\frac{Y}{R} = \frac{5}{(s(s+2) + 5 + 5\alpha s)}$$

, therefore the characteristic equation is  $s(s+2) + 5 + 5\alpha s = 0$ , therefore we have  $b(s) = 5s$  and  $a(s) = s^2 + 2s + 1$  Therefore we have that  $L(s) = \frac{5s}{s^2 + 2s + 1}$ , therefore 1 line will approach asymptotes centered at  $-2$  and leaving at angles  $180^\circ$ . Furthermore, the departure angle from the poles  $-1 \pm 2j$  is  $\mp 153.4^\circ$  and the arrival angle to the zero at 0 is  $180^\circ$ , therefore the root locuses are at

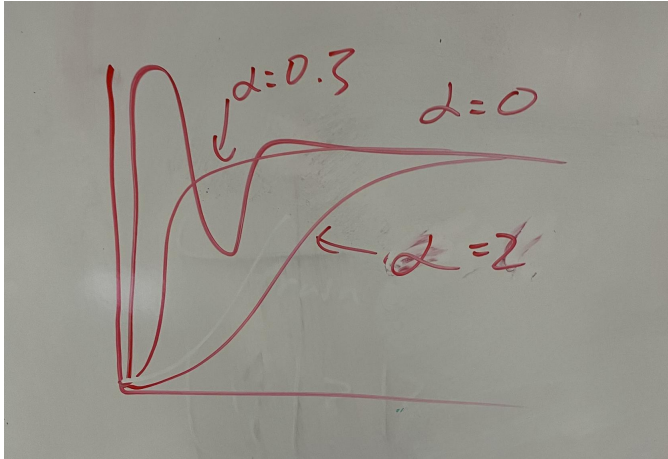


When  $\alpha = 0$ , there are poles at  $-1 \pm 2j$ , when  $\alpha = 0.5$ , there are poles at  $-2$  and  $-2.5$ , and when  $\alpha = 2$ , there are poles at  $-0.432$  and  $-11.568$ , therefore the plot of the poles looks like



Therefore since  $\alpha = 0$  the step response will be underdamped since it has poles with imaginary components, and for  $\alpha = 0.5$  the damping factor  $\zeta = \frac{4.5}{2\sqrt{5}}$  and when  $\alpha = 2$  the damping factor  $\zeta = \frac{6}{\sqrt{5}}$ , therefore the plot of

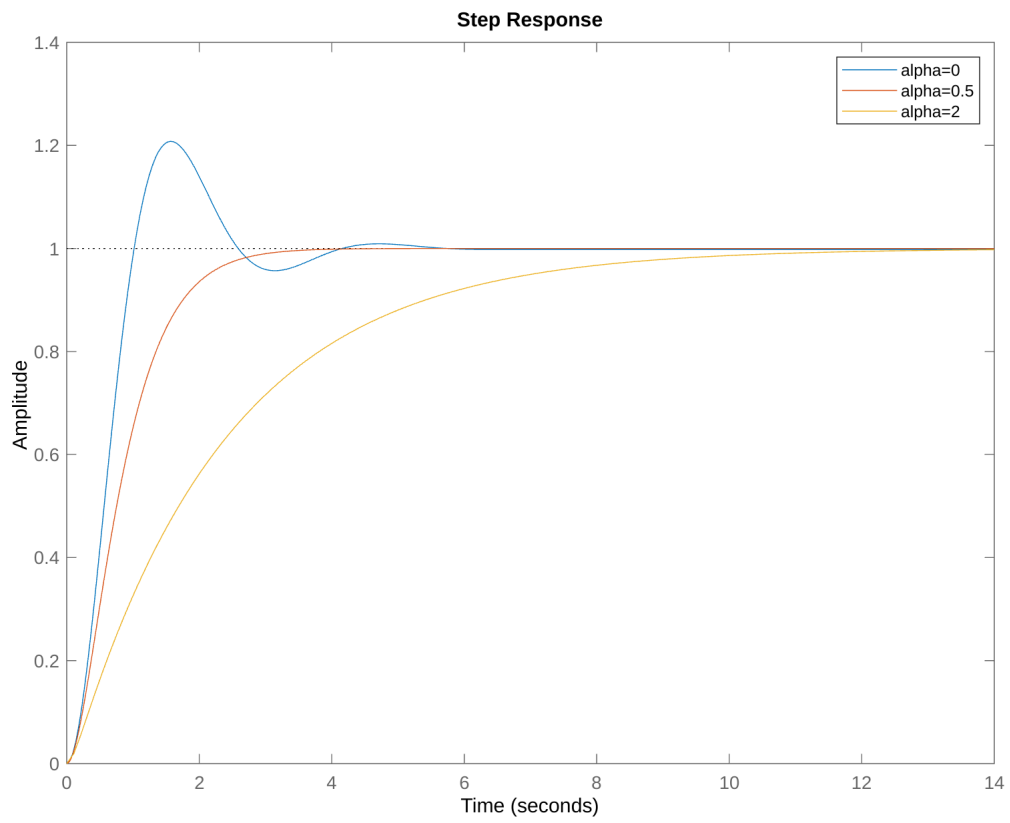
the step responses would look like



With the following matlab code we can verify it

```
sys = tf([5],[1 2 5]);  
step(sys)  
stepinfo(sys)  
hold on;  
  
sys = tf([5],[1 2+2.5 5]);  
step(sys)  
stepinfo(sys)  
  
sys = tf([5],[1 12 5]);  
step(sys)  
stepinfo(sys)  
  
hold off;  
legend('alpha=0','alpha=0.5','alpha=2')
```

Which produces the following plot.



## Problem 5.13

(a)