ECE 231A HW 2

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Problem 1

Problem 2

Problem 3

(a)

Let the increase in Alice's score after the ith round be represented by the random variable Z_A^i we have that

$$Z_A^i = \begin{cases} 4 & \text{w.p. } 1/15 \\ 7 & \text{w.p. } 5/15 \\ 0 & \text{w.p. } 9/15 \end{cases}$$

And the increase in Bob's score after the ith round be represented by the random variable Z_B^i we have that

$$Z_B^i = \begin{cases} 3 & \text{w.p. } 1/15 \\ 5 & \text{w.p. } 2/15 \\ 6 & \text{w.p. } 6/15 \\ 0 & \text{w.p. } 6/15 \end{cases}$$

Then we have that

$$S_A^n = \sum_{i=1}^{n} Z_A^i$$

and

$$S_B^n = \sum_{i=1}^{n} Z_B^i$$

As $n \to \infty$ we have that

$$\lim_{n\to\infty} S_A^n = \lim_{n\to\infty} \sum_{i=1}^{n} Z_A^i = E[Z_A^i] = \boxed{2.6}$$

and

$$\lim_{n\to\infty} S_B^n = \lim_{n\to\infty} \sum_{i=1}^{n} Z_B^i = E[Z_B^i] = \boxed{3.225}$$

(b)

We will need to make $\alpha > 7$ since we need to increase the expected value, therefore the probabilites for Z_A would not change, however instead of being 7 with the probability of 5/15 we would have α with the probability of 5/15. Likewise Z_B would not change. Therefore we would have that our new

$$E[Z_A^i] = \frac{4}{15} + \frac{5}{15}\alpha$$

To make this greater than 3.225 we would have that $\alpha > 8.875$, so the least value of alpha to make Alice's score greater than Bob's after n rounds is $\boxed{\alpha = 9}$ if α has to be an integer.