

# ECE 231A HW 3

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## Problem 1

(a)

$$\begin{aligned}\lim_{n \rightarrow \infty} [p(X_1, \dots, X_n)]^{\frac{1}{n}} &= 2^{\lim_{n \rightarrow \infty} \frac{1}{n} \log_2[p(X_1, \dots, X_n)]} \\ &= 2^{\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \log_2[p(X_i)]} \\ &= 2^{E[\ln[p(X_i)]]} \\ &= \boxed{2^{H(x)}}$$

(b)

$$\begin{aligned}
 E \left[ \left( \prod_{i=1}^n f(X_i) \right)^{\frac{1}{n}} \right] &= \exp \left[ \ln \left( E \left[ \left( \prod_{i=1}^n f(X_i) \right) \right] \right) \right] \\
 &\leq \exp \left[ \frac{1}{n} E \left[ \ln \left( \prod_{i=1}^n f(X_i) \right) \right] \right] \\
 &= \exp \left[ \frac{1}{n} \sum_{i=1}^n E [\ln (f(X_i))] \right] \\
 &\leq \exp \left[ \frac{1}{n} \sum_{i=1}^n E[X_i] \right] \\
 &= \exp [E[X_i]]
 \end{aligned}$$

Therefore we have that

$$E \left[ \left( \prod_{i=1}^n f(X_i) \right)^{\frac{1}{n}} \right] \leq \exp [E[X_i]]$$

## Problem 2

(a)

$$H(X) = \boxed{1.895}$$

(b)