ECE 231A HW 3

Lawrence Liu

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Problem 1

(a)

$$
\begin{aligned}
\lim_{n\to\infty}[p(X_1,\cdots,X_n)]^{\frac{1}{n}} &= 2^{\lim_{n\to\infty}\frac{1}{n}\log_2[p(X_1,\cdots,X_n)]} \\
&= 2^{\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\log_2[p(X_i)]} \\
&= 2^{E[\ln[p(X_i)]]} \\
&= \boxed{2^{-H(x)}}
\end{aligned}
$$

(b)

$$
\begin{aligned}
E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] &= \left(\left(E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right]\right)^{n}\right)^{\frac{1}{n}} \\
&\leq \left(E\left[\prod_{i=1}^{n} f(X_i)\right]\right)^{\frac{1}{n}} \\
&= (E^n[f(X_1)])^{\frac{1}{n}} \\
&= E[f(X_1)]
\end{aligned}
$$

Therefore we have that

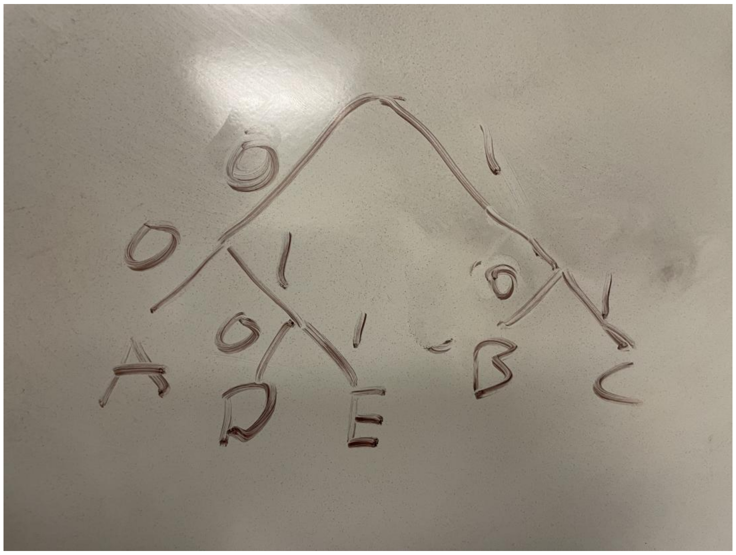
$$
\boxed{E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] \leq E[X_i]}
$$

Problem 2

(a)

$$
H(X) = \boxed{2.246 \text{ Shannons}}
$$

(b)



So we have that the average length is $\boxed{2.3}$ bits.

(c)

codeword A is 001
codeword B is 0110
codeword C is 1001
codeword D is 1100
codeword E is 11110
Therefore the SFE codeword average length is $\boxed{3.8} bits$.

(d)

We have that the CDF of BAC is

$$P(X_1 = A) + P(X_1 = B, X_2 = A)(P(X_3 = A) + P(X_3 = B) + P(X_3 = C)) = \boxed{0.342}$$

Therefore for the SFE code, we have that

$$\bar{F} = P(X_1 = A) + P(X_1 = B, X_2 = A)(P(X_3 = A) + P(X_3 = B) + P(X_3 = C)) - \frac{1}{2}P(X_1 = B, X_2 =$$

and

$$l = -\lceil \log_2(P(X_1 = B, X_2 = A, X_3 = c)) \rceil + 1 = 8$$

Thus we have that the SFE encoding is $\boxed{01010110}$

Problem 3

(a)

$$F(X^1) = \begin{cases} 0.2 & \text{if } X^1 = A \\ 0.5 & \text{if } X^1 = B \\ 1 & \text{if } X^1 = C \end{cases}$$

The interval for the first symbol is $[0, 0.2)$.

(b)

$$F(X^1X^2) = \begin{cases} 0.04 & \text{if } X^1X^2 = AA \\ 0.1 & \text{if } X^1X^2 = AB \\ 0.2 & \text{if } X^1X^2 = AC \\ 0.26 & \text{if } X^1X^2 = BA \\ 0.35 & \text{if } X^1X^2 = BB \\ 0.5 & \text{if } X^1X^2 = BC \\ 0.6 & \text{if } X^1X^2 = CA \\ 0.75 & \text{if } X^1X^2 = CB \\ 1 & \text{if } X^1X^2 = CC \end{cases}$$

Therefore the interval corresponding to AC is $[0.1, 0.2)$

(c)

For the interval $[0, 0.2)$, we have that the midpoint is 0.1 and the length is $-\lceil \log_2(0.2) \rceil + 1 = 4$ bits. Therefore the code word is $\boxed{0001}$
For the interval $[0.1, 0.2)$, we have that the midpoint is 0.15 and the length is $-\lceil \log_2(0.01) \rceil + 1 = 8$ bits, so the code word is $\boxed{00100}$

(d)

Therefore the cdf for

$$F(X^1X^2X^3X^4 = ACCB) = 0.1 + 0.1 * 0.6 = 0.175$$

And thus we have that interval for the ACCB is $[0.15, 0.16)$ so the midpoint is

$$\bar{F} = \frac{0.16 + 0.175}{2} = 0.1675$$

And since $p = 0.2 \cdot 0.5^2 \cdot 0.3 = 0.015$, we have that

$$l = -\lceil \log_2(0.015) \rceil + 1 = 8$$

Thus we have that the SFE code for ACCB is $\boxed{00101010}$

(e)

The binary encoding for 0.16 is 0.001010001111010111 and the binary encoding for 0.1675 is 0.001010101110000101 and the binary encoding for 0.175 is 0.00101100110011001101. Therefore 5 bits can be know if we do not know how ACCB continues.

Problem 4

$$AB, BB, C, ABBBCA, AA, B, B$$