ECE 231A HW 3

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Problem 1

(a)

$$\begin{aligned}
\lim_{n\to\infty}[p(X_1,\cdots,X_n)]^{\frac{1}{n}} &= 2^{\lim_{n\to\infty}\frac{1}{n}\log_2[p(X_1,\cdots,X_n)]} \\
&= 2^{\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\log_2[p(X_i)]} \\
&= 2^{E[\ln[p(X_i)]]} \\
&= \boxed{2^{-H(x)}}
\end{aligned}$$

(b)

$$\begin{aligned} E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] &= \left(\left(E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right]\right)^{n}\right)^{\frac{1}{n}} \\ &\leq \left(E\left[\prod_{i=1}^{n} f(X_i)\right]\right)^{\frac{1}{n}} \\ &= \left(E^n[f(X_1)]\right)^{\frac{1}{n}} \\ &= E[f(X_1)] \end{aligned}$$

Therefore we have that

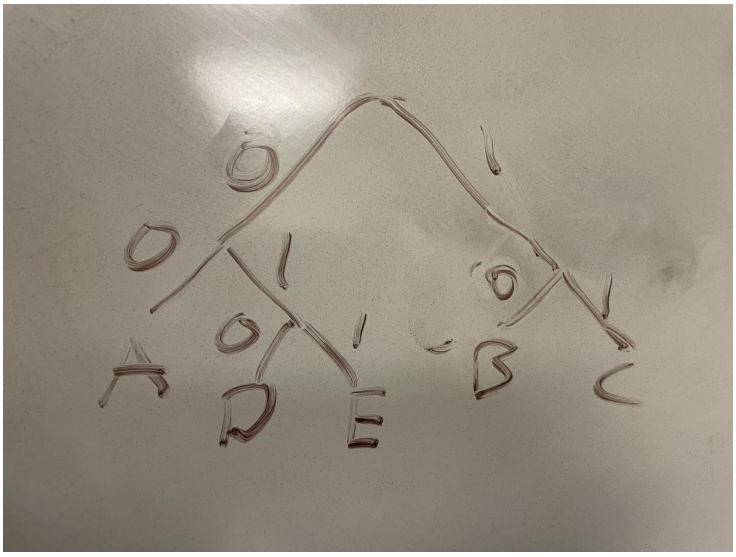
$$\boxed{E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] \leq E[X_i]}$$

Problem 2

(a)

$$H(X) = \boxed{2.246 \text{ Shannons}}$$

(b)



So we have that the average length is $\boxed{2.3}$ bits.

(c)

codeword A is 001
codeword B is 0110
codeword C is 1001
codeword D is 1100
codeword E is 11110
Therefore the SFE codeword average length is $\boxed{3.8}$*bits*.

(d)

We have that the CDF of BAC is

$P(X_1 = A)+P(X_1 = B, X_2 = A)(P(X_3 = A)+P(X_3 = B)+P(X_3 = C)) = \boxed{0.342}$

Therefore for the SFE code, we have that

$$\bar{F} = P(X_1 = A) + P(X_1 = B, X_2 = A)(P(X_3 = A) + P(X_3 = B) + P(X_3 = C)) - \frac{1}{2}P(X_1 = B, X_2 =$$

and

$$l = -\lceil \log_2(P(X_1 = B, X_2 = A, X_3 = c)) \rceil + 1 = 8$$

Thus we have that the SFE encoding is $\boxed{01010110}$

Problem 3

(a)

$$F(X^1) = \begin{cases} 0.2 & \text{if } X^1 = A \\ 0.5 & \text{if } X^1 = B \\ 1 & \text{if } X^1 = C \end{cases}$$

The interval for the first symbol is $[0, 0.2)$.

(b)

$$F(X^1X^2) = \begin{cases} 0.04 & \text{if } X^1X^2 = AA \\ 0.1 & \text{if } X^1X^2 = AB \\ 0.2 & \text{if } X^1X^2 = AC \\ 0.26 & \text{if } X^1X^2 = BA \\ 0.35 & \text{if } X^1X^2 = BB \\ 0.5 & \text{if } X^1X^2 = BC \\ 0.6 & \text{if } X^1X^2 = CA \\ 0.75 & \text{if } X^1X^2 = CB \\ 1 & \text{if } X^1X^2 = CC \end{cases}$$

Therefore the interval corresponding to AC is $[0.1, 0.2)$

(c)

Therefore the cdf for

$$F(X^1X^2X^3X^4) = 0.1 + 0.1 * 0.6 = 0.16$$

Therefore we have that

$$\bar{F} = 0.16 - 0.1 * 0.6/2 = 0.14$$