

ECE 231A HW 1

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Problem 3

(a)

The entropy of $H(X)$ is

$$H(X) = - \sum_{x \in X} p(x) \log(p(x))$$

and we have that

$$\begin{aligned} H(X|Y) &= \sum_{x \in X, y \in Y} p(y) H(X|Y = y) \\ &= \left(\sum_{x \in S} p(x) \right) H(X|Y = 1) + \left(\sum_{x \notin S} p(x) \right) H(X|Y = 0) \end{aligned}$$

We have

$$\begin{aligned} H(X|Y = 1) &= - \sum_{x \in S} p(x|Y = 1) \log(p(x|Y = 1)) \\ &= - \sum_{x \in S} \frac{p(x)}{\sum_{x \in S} p(x)} \log \left(\frac{p(x)}{\sum_{x \in S} p(x)} \right) \end{aligned}$$

likewise we have

$$\begin{aligned} H(X|Y=0) &= - \sum_{x \notin S} p(x|Y=0) \log(p(x|Y=0)) \\ &= - \sum_{x \notin S} \frac{p(x)}{\sum_{x \notin S} p(x)} \log \left(\frac{p(x)}{\sum_{x \notin S} p(x)} \right) \end{aligned}$$

Therefore we have

$$\begin{aligned} H(X|Y) &= - \sum_{x \in S} p(x) \left(\log(p(x)) - \log \left(\sum_{x \in S} p(x) \right) \right) - \sum_{x \notin S} p(x) \left(\log(p(x)) - \log \left(\sum_{x \notin S} p(x) \right) \right) \\ &= H(X) + \sum_{x \in S} p(x) \log \left(\sum_{x \in S} p(x) \right) + \sum_{x \notin S} p(x) \log \left(\sum_{x \notin S} p(x) \right) \end{aligned}$$

Therefore we have

$$H(X) - H(X|Y) = \boxed{- \sum_{x \in S} p(x) \log \left(\sum_{x \in S} p(x) \right) - \sum_{x \notin S} p(x) \log \left(\sum_{x \notin S} p(x) \right)}$$

Problem 6

(a)

$$\begin{aligned} H(x) &= - \sum_{x \in X} p(x) \log(p(x)) \\ &= p(A) \log(p(A)) + p(B) \log(p(B)) + p(C) \log(p(C)) \\ &\quad + p(D) \log(p(D)) + p(E) \log(p(E)) + p(F) \log(p(F)) \\ &= \frac{1}{2} \log(4) + \frac{1}{4} \log(8) + \frac{3}{16} \log\left(\frac{16}{3}\right) + \frac{1}{16} \log(16) \\ &= \boxed{2.2452 \text{ shannons}} \end{aligned}$$