

# ECE 231A Project 3 Part 2

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November 7, 2022

## Problem 1

We can rewrite  $P(l(x) < l'(x)) > P(l(x) > l'(x))$  as

$$\sum_x p(x) \text{sign}(l(x) - l'(x)) \leq 0$$

using the identity  $\text{sign}(a - b) \leq D^t - 1$ , we get

$$\begin{aligned} \sum_x p(x) \text{sign}(l(x) - l'(x)) &\leq \sum_x p(x) (D^{l(x)-l'(x)} + 1) \\ &= \sum_x D^{l'(x)} - 1 \end{aligned}$$

From the kraft inequality, we then get that  $\sum_x D^{l'(x)} \leq 1$ . and thus we get that  $\sum_x D^{l'(x)} - 1 \leq 0$ .

## Problem 2

(a)

let  $l'(x) = -\log(q(x))$ , then we have

$$\begin{aligned}
 p(l'(x) \leq l^*(x) - \gamma) &= p(-\log_D(q(x)) \leq -\log_D(p(x)) - \gamma) \\
 &= p(q(x) \geq p(x)D^\gamma) \\
 &= \sum_{x:p(x)D^\gamma \leq q(x)} p(x) \\
 &\leq \sum_{x:p(x)D^\gamma \leq q(x)} q(x)D^{-\gamma} \\
 &= \sum_x q(x)D^{-\gamma} \\
 &= D^{-\gamma}
 \end{aligned}$$

Thus we get

$$p(l'(x) \leq l^*(x) - \gamma) \leq |D|^{-\gamma}$$

(b)

We have

$$\begin{aligned}
 p(l(x) > l'(x) + 1) &= p\left(\lceil \log_D\left(\frac{1}{p(x)}\right) \rceil > l'(x) + 1\right) \\
 &\leq p(1 - \log_D(p(x)) > l'(x) + 1) \\
 &= p(-\log_D(p(x)) < l'(x))
 \end{aligned}$$

Since  $l(x)$  must be an integer we have that this is equal to

$$p(-\log_D(p(x)) < l'(x)) = p(l'(x) \leq -\log_D(p(x)) - 1)$$

from part (a) we know that

$$p(l'(x) \leq -\log_D(p(x)) - 1) \leq D^{-1} \leq \frac{1}{2}$$

since  $D \geq 2$ . thus we have that

$$p(l(x) > l'(x) + 1) \leq \frac{1}{2}$$

and thus

$$p(l(x) > l'(x) + 1) \leq p(l(x) \leq l'(x) + 1)$$