ECE 231A HW 3

Lawrence Liu

October 24, 2022

Problem 1

(a)

$$\lim_{n \to \infty} [p(X_1, \cdots, X_n)]^{\frac{1}{n}} = 2^{\lim_{n \to \infty} \frac{1}{n} \log_2[p(X_1, \cdots, X_n)]}$$

$$= 2^{\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \log_2[p(X_i)]}$$

$$= 2^{E[\ln[p(X_i)]]}$$

$$= \boxed{2^{H(x)}}$$

(b)

$$E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] = \exp\left[\ln\left(E\left[\left(\prod_{i=1}^{n} f(X_i)\right)\right]\right)\right]$$

$$\leq \exp\left[\frac{1}{n}E\left[\ln\left(\prod_{i=1}^{n} f(X_i)\right)\right]\right]$$

$$= \exp\left[\frac{1}{n}\sum_{i=1}^{n} E\left[\ln\left(f(X_i)\right)\right]\right]$$

$$\leq \exp\left[\frac{1}{n}\sum_{i=1}^{n} E[X_i]\right]$$

$$= \exp\left[E[X_i]\right]$$

Therefore we have that

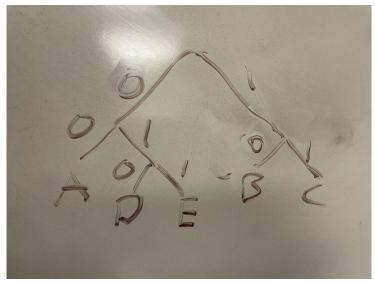$$\boxed{E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] \leq \exp\left[E[X_i]\right]}$$

Problem 2

(a)

$$H(X) = \boxed{1.895}$$

(b)

So we have that the average length is 2.3 bits.

(c)