

ECE 231A HW 3

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Problem 1

(a)

$$\begin{aligned}\lim_{n \rightarrow \infty} [p(X_1, \dots, X_n)]^{\frac{1}{n}} &= 2^{\lim_{n \rightarrow \infty} \frac{1}{n} \log_2[p(X_1, \dots, X_n)]} \\ &= 2^{\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \log_2[p(X_i)]} \\ &= 2^{E[\ln[p(X_i)]]} \\ &= \boxed{2^{H(x)}}$$

(b)

$$\begin{aligned} E \left[\left(\prod_{i=1}^n f(X_i) \right)^{\frac{1}{n}} \right] &= \exp \left[\ln \left(E \left[\left(\prod_{i=1}^n f(X_i) \right) \right] \right) \right] \\ &\leq \exp \left[\frac{1}{n} E \left[\ln \left(\prod_{i=1}^n f(X_i) \right) \right] \right] \\ &= \exp \left[\frac{1}{n} \sum_{i=1}^n E [\ln (f(X_i))] \right] \\ &\leq \exp \left[\frac{1}{n} \sum_{i=1}^n E[X_i] \right] \\ &= \exp [E[X_i]] \end{aligned}$$

Therefore we have that

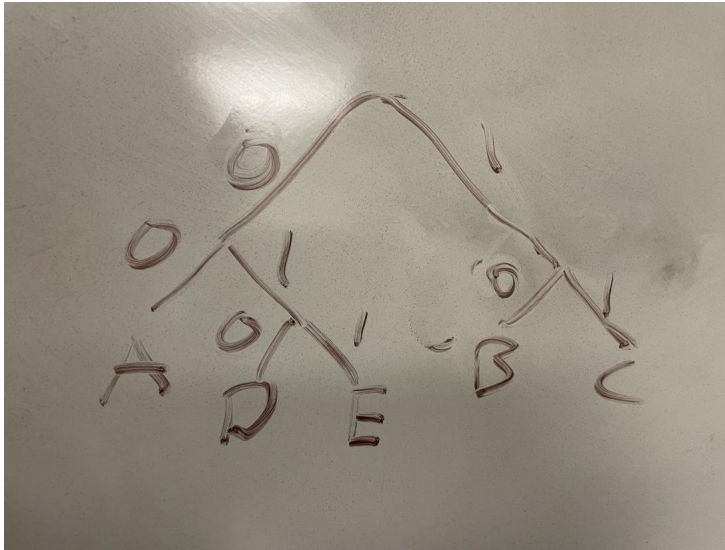
$$E \left[\left(\prod_{i=1}^n f(X_i) \right)^{\frac{1}{n}} \right] \leq \exp [E[X_i]]$$

Problem 2

(a)

$$H(X) = \boxed{1.895}$$

(b)



So we have that the average length is $\boxed{2.3}$ bits.

(c)

codeword A is 001

codeword B is 0011

codeword C is 0101

codeword D is 1101

codeword E is 10011

Therefore the SFE codeword average length is $\boxed{3.8}$ bits.

(d)