ECE 231A HW 3

Lawrence Liu

October 28, 2022

Problem 1

(a)

$$\begin{aligned}
\lim_{n\to\infty}[p(X_1,\cdots,X_n)]^{\frac{1}{n}} &= 2^{\lim_{n\to\infty}\frac{1}{n}\log_2[p(X_1,\cdots,X_n)]} \\
&= 2^{\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}\log_2[p(X_i)]} \\
&= 2^{E[\ln[p(X_i)]]} \\
&= \boxed{2^{H(x)}}
\end{aligned}$$

(b)

$$\begin{aligned}
&\leq \exp\left[\frac{1}{n}E\left[\ln\left(\prod_{i=1}^{n}f(X_i)\right)\right]\right] \\
&= \exp\left[\frac{1}{n}\sum_{i=1}^{n}E\left[\ln\left(f(X_i)\right)\right]\right] \\
&\leq \exp\left[\frac{1}{n}\sum_{i=1}^{n}E[X_i]\right]
\end{aligned}$$

$= \exp\left[E[X_i]\right] ======= \left(\left(E\left[(\prod_{i=1}^{n} f(X_i))^{\frac{1}{n}}\right]\right)^n\right)^{\frac{1}{n}}$
$\leq \left(E\left[\prod_{i=1}^{n} f(X_i)\right]\right)^{\frac{1}{n}}$
$= (E^n[f(X_1)])^{\frac{1}{n}}$
$= E[f(X_1)] >>>>>>> 4d89209173117b00349cd54a99f87928dd152443$

Therefore we have that

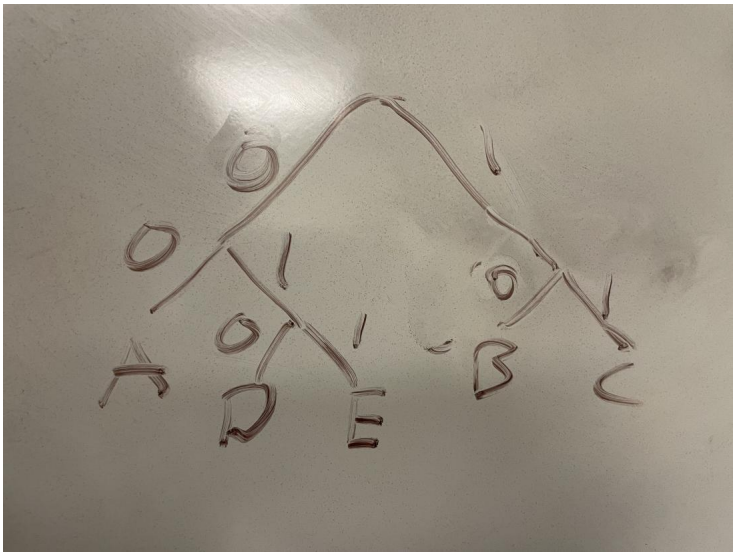
$$\boxed{E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] \leq E[X_i]}$$

Problem 2

(a)

$$H(X) = \boxed{1.895}$$

(b)



So we have that the average length is $\boxed{2.3}$ bits.

(c)

codeword A is 001
codeword B is 0011
codeword C is 0101
codeword D is 1101
codeword E is 10011
Therefore the SFE codeword average length is $\boxed{3.8}$*bits*.

(d)