ECE 231A HW 3

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Problem 1

(a)

$$\lim_{n \to \infty} [p(X_1, \cdots, X_n)]^{\frac{1}{n}} = 2^{\lim_{n \to \infty} \frac{1}{n} \log_2 [p(X_1, \cdots, X_n)]}$$

$$= 2^{\lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \log_2 [p(X_i)]}$$

$$= 2^{E[\ln[p(X_i)]]}$$

$$= \boxed{2^{H(x)}}$$

(b)

$$E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] = \left(\left(E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right]\right)^{n}\right)^{\frac{1}{n}}$$

$$\leq \left(E\left[\prod_{i=1}^{n} f(X_i)\right]\right)^{\frac{1}{n}}$$

$$= (E^n[f(X_1)])^{\frac{1}{n}}$$

$$= E[f(X_1)]$$
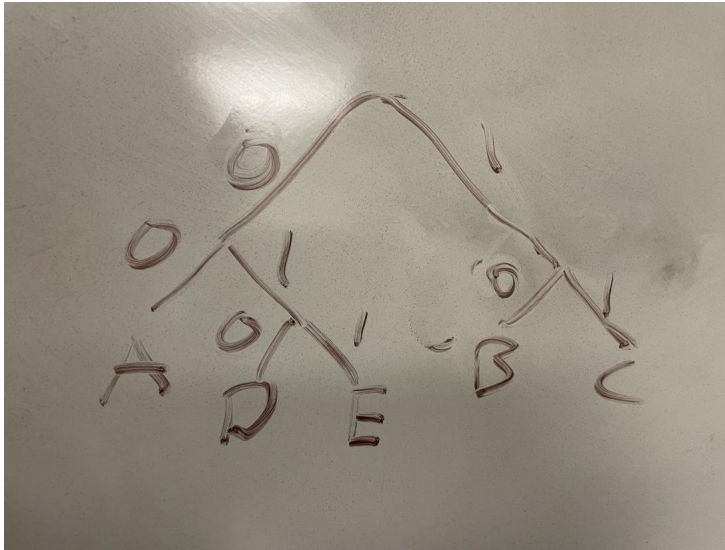
Therefore we have that

$$\boxed{E\left[\left(\prod_{i=1}^{n} f(X_i)\right)^{\frac{1}{n}}\right] \leq E[X_i]}$$

Problem 2

(a)

$$H(X) = \boxed{1.895}$$

(b)

So we have that the average length is $\boxed{2.3}$ bits.

(c)

codeword A is 001
codeword B is 0011
codeword C is 0101
codeword D is 1101
codeword E is 10011
Therefore the SFE codeword average length is $\boxed{3.8}$ *bits*.

(d)