Impedence and Phasors Tutorial

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Impedence of Circuit compents

Impedence of a Capacitor

A capacitor's IV relation is

$$i = C\frac{dv}{dt}$$

Now let us consider an voltage that takes the form of $v = Ae^{j\omega t}$ where j is the imaginary unit, ie $j = \sqrt{-1}$, ω is the frequency, and A is a scaling factor, therefore we have

$$i = C\frac{dv}{dt} = j\omega CAe^{j\omega t} = j\omega Cv$$

Rearangeing this we get

$$i\frac{1}{j\omega C} = v$$

This reminds us of ohms law! As a matter of fact we can generalize ohms law to AC ciruits, with the generalized version of resistance being impedence. In this case our impedence is

$$Z_C = \frac{1}{j\omega C}$$

Impedence of a Inductor

Let us repeat our derivation for the impedence of a capcitor, this time with an Inductor. We have that the IV relation for a inductor is

$$v = L\frac{di}{dt}$$

now let the current take the form $i = Ae^{j\omega t}$ therefore we have

$$v = j\omega LAe^{j\omega t} = j\omega Li$$

Therefore we have

$$Z_L = j\omega L$$

Impedence of a Resistor

The impedence of a resistor is easy, since we have for any v,

$$v = iR$$

$$Z_R = R$$

Phasors

Let us consider an AC voltage source, with

$$V = A\cos(\omega t + \theta)$$

, adding on a imaginary compent $Aj\sin(\omega t + \theta)$ we have

$$A\cos(\omega t + \theta) + Ai\sin(\omega t + \theta) = Ae^{j\omega t + j\theta} = Ae^{j\theta}e^{j\omega t}$$

Let us define the value $Ae^{j\theta}$ to be the phasor, therefore for a AC voltage source $V = A\cos(\omega t + \theta)$, its corresponding phasor is $\underline{V} = Ae^{j\theta}$.

Now lets consider an example of a AC voltage source

$$V = 10\sin(\omega t)$$

How would we find the phasor for this source? well first of all from the trig identies, we know $\cos(\theta - \frac{\pi}{2}) = \sin(\theta)$, therefore we can rewrite V as

$$V = 10\cos(\omega t - \frac{\pi}{2})$$

Therefore adding on an imaginary compent $10j\cos(\omega t - \frac{\pi}{2})$ we get

$$10\cos(\omega t - \frac{\pi}{2}) + 10j\cos(\omega t - \frac{\pi}{2}) = 10e^{j\omega t - \frac{\pi}{2}j}$$

Therefore the corresponding phasor for this voltage source is $10e^{-j\frac{\pi}{2}} = -10j$
Now what would the corresponding phasor be for the voltage source in the HW?