

ECE C143A Homework 4

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Problem 1

(a)

$$P(N = 0) = 1 - 0.25 = \boxed{0.75}$$

(b)

We want $P(N = 0|R = 0)$, we know that

$$P(R = 1|N = 0) = P(E = 0)P(R = 1|E = 0, N = 0) + P(E = 1)P(R = 1|E = 1, N = 0) = 0.01$$

$P(R = 0|N = 0) = 0.99$ and thus from bayes law we have

$$\begin{aligned} P(N = 0|R = 0) &= P(R = 0|N = 0) \frac{P(N = 0)}{P(R = 0)} \\ &= 0.9 \frac{0.1 \cdot 0.75}{P(R = 0)} \end{aligned}$$

To find $P(R = 0)$ we must find $P(R = 1)$,

$$\begin{aligned} P(R = 1) &= P(E = 1)P(N = 1)P(R = 1|E = 1, N = 1) \\ &\quad + P(E = 1)P(N = 0)P(R = 1|E = 1, N = 0) \\ &\quad + P(E = 0)P(N = 1)P(R = 1|E = 0, N = 1) \\ &\quad + P(E = 0)P(N = 0)P(R = 1|E = 0, N = 0) \end{aligned}$$

$$P(R = 1) = 0.9 \cdot 0.25 \cdot 1 + 0.1 \cdot 0.75 \cdot 0.1 + 0.1 \cdot 0.25 \cdot 0.1 = 0.235$$

Therefore $P(R = 0) = 1 - P(R = 1) = 0.765$, thus

$$P(N = 0|R = 0) = 0.99 \frac{0.75}{P(R = 0)} = \boxed{0.97}$$

(c)

We have

$$\begin{aligned} P(E = 0, N = 0|R = 0) &= \frac{P(E = 0, N = 0, R = 0)}{P(R = 0)} \\ &= \frac{P(R = 0|E = 0, N = 0)P(E = 0)P(N = 0)}{P(R = 0)} \\ &= \frac{(1 - P(R = 1|E = 0, N = 0))P(E = 0)P(N = 0)}{P(R = 0)} \\ &= \frac{0.9 \cdot 0.1 \cdot 0.75}{0.765} \\ &= \boxed{0.088} \end{aligned}$$

This intuitively makes sense because we need two conditions to occur, equipment broken and neuron spike not occurring.

(d)

Let us consider the case $E = 1, N = 0$ given $R = 1$ we have

$$P(E = 1, N = 0|R = 1) = \frac{P(R = 1|E = 1, N = 0)P(E = 1)P(N = 0)}{P(R = 1)}$$

Since $P(R = 1|E = 1, N = 0) = 0$, we thus have $P(E = 1, N = 0|R = 1) = 0$. However

$$\begin{aligned} P(E = 1|R = 1) &= P(R = 1|E = 1) \frac{P(E = 1)}{P(R = 1)} \\ &= (P(R = 1|E = 1, N = 0)P(N = 0) + \\ &\quad P(R = 1|E = 1, N = 1)P(N = 1)) \frac{P(E = 1)}{P(R = 1)} \end{aligned}$$

This therefore $P(E = 1|R = 1) > 1$, likewise

$$\begin{aligned} P(N = 0|R = 1) &= P(R = 1|N = 0) \frac{P(N = 0)}{P(R = 1)} \\ &= (P(R = 1|E = 1, N = 0)P(E = 1) + \\ &\quad P(R = 1|E = 0, N = 0)P(E = 0)) \frac{P(N = 0)}{P(R = 1)} \end{aligned}$$

This therefore $P(E = 1|R = 1) > 1$, therefore, $P(E = 1|R = 1)P(N = 0|R = 1) > 0$ and is not equal to $P(E = 1, N = 0|R = 1)$ therefore they are conditionally dependent. Ie they are not independent given R

Problem 2

(a)

$$\boxed{P(a, b, c, d) = P(c)P(a|c)P(d)P(b|a, d)}$$

(b)

$$\begin{aligned} P(C, D) &= \sum_A \sum_B P(C, D, A, B) \\ &= \sum_A \sum_B P(C)P(D)P(A|C)P(B|A, D) \\ &= P(C)P(D) \sum_A \sum_B P(A|C)P(B|A, D) \\ &= P(C)P(D) \sum_A P(A|C) \sum_B P(B|A) \\ &= P(C)P(D) \end{aligned}$$

Therefore C and D are independent.

(c)

$$\begin{aligned} P(C, D|A, B) &= \frac{P(C, D, A, B)}{P(A, B)} \\ &= \frac{P(C)P(D)P(A|C)P(B|A, D)}{P(A, B)} \\ &= \frac{P(D)P(C|A)P(A)P(B|A, D)}{P(A, B)} \\ &= \frac{P(D)P(C|A)P(D|B, A)}{P(D|A)} \\ &= \frac{P(D)}{P(D|A)} P(C|A)P(D|B, A) \\ &= \frac{P(C|A)P(D|B, A)}{P(D, A)} \end{aligned}$$

Therefore C and D are not independent given A and B .

(d)

$$\begin{aligned}P(a, d) &= \sum_B P(a, d, b) \\&= \sum_a P(d)P(b)P(b|d, a) \\&= P(d)P(b) \sum_B P(b|d, a) \\&= P(d)P(b)\end{aligned}$$

Therefore a and d are independent

(e)

$$\begin{aligned}P(a, d|b) &= \frac{P(a, d, b)}{P(b)} \\&= \frac{P(a)P(d)P(b|a, d)}{P(b)} \\&= \end{aligned}$$