

ECE C143A Homework 3

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Problem 1

(a)

$$P(N = 0) = 1 - 0.25 = \boxed{0.75}$$

(b)

We want $P(N = 0|R = 0)$, we know that

$$P(R = 1|N = 0) = P(E = 0)P(R = 1|E = 0, N = 0) + P(E = 1)P(R = 1|E = 1, N = 0) = 0.01$$

$P(R = 0|N = 0) = 0.99$ and thus from bayes law we have

$$\begin{aligned} P(N = 0|R = 0) &= P(R = 0|N = 0) \frac{P(N = 0)}{P(R = 0)} \\ &= 0.9 \frac{0.1 \cdot 0.25}{P(R = 0)} \end{aligned}$$

To find $P(R = 0)$ we must find $P(R = 1)$,

$$\begin{aligned} P(R = 1) &= P(E = 1)P(N = 1)P(R = 1|E = 1, N = 1) \\ &\quad + P(E = 1)P(N = 0)P(R = 1|E = 1, N = 0) \\ &\quad + P(E = 0)P(N = 1)P(R = 1|E = 0, N = 1) \\ &\quad + P(E = 0)P(N = 0)P(R = 1|E = 0, N = 0) \end{aligned}$$

$$P(R = 1) = 0.9 \cdot 0.25 \cdot 1 + 0.1 \cdot 0.25 \cdot 0.1 + 0.1 \cdot 0.75 \cdot 0.1 = 0.235$$

Therefore $P(R = 0) = 1 - P(R = 1) = 0.765$, thus

$$P(N = 0|R = 0) = 0.99 \frac{0.25}{P(R = 0)} = \boxed{0.3235}$$

(c)

We have

$$\begin{aligned} P(E = 0, N = 0|R = 0) &= \frac{P(E = 0, N = 0, R = 0)}{P(R = 0)} \\ &= \frac{P(R = 0|E = 0, N = 0)P(E = 0)P(N = 0)}{P(R = 0)} \\ &= \frac{(1 - P(R = 1|E = 0, N = 0))P(E = 0)P(N = 0)}{P(R = 0)} \\ &= \frac{0.9 \cdot 0.1 \cdot 0.75}{0.765} \\ &= \boxed{0.088} \end{aligned}$$

This intuitively makes sense because we need two conditions to occur, equipment broken and neuron spike not occurring.