ECE C143A Homework 4

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Problem 1

(a)

$P(N=0) = 1 - 0.25 = \boxed{0.75}$

(b)

We want $P(N=0|R=0)$, we know that

$P(R=1|N=0) = P(E=0)P(R=1|E=0,N=0) + P(E=1)P(R=1|E=1,N=0) = 0.01$

$P(R=0|N=0) = 0.99$ and thus from bayes law we have

$$\begin{aligned} P(N=0|R=0) &= P(R=0|N=0)\frac{P(N=0)}{P(R=0)} \\ &= 0.9\frac{0.1 \cdot 0.75}{P(R=0)} \end{aligned}$$

To find $P(R=0)$ we must find $P(R=1)$,

$$\begin{aligned}P(R=1) = P(E=1)P(N=1)P(R=1|E=1,N=1) \\ + P(E=1)P(N=0)P(R=1|E=1,N=0) \\ + P(E=0)P(N=1)P(R=1|E=0,N=1) \\ + P(E=0)P(N=0)P(R=1|E=0,N=0)\end{aligned}$$

$P(R=1) = 0.9 \cdot 0.25 \cdot 1 + 0.1 \cdot 0.75 \cdot 0.1 + 0.1 \cdot 0.25 \cdot 0.1 = 0.235$

Therefore $P(R=0) = 1 - P(R=1) = 0.765$, thus

$$P(N=0|R=0) = 0.99\frac{0.75}{P(R=0)} = \boxed{0.97}$$

(c)

We have

$$\begin{aligned}P(E=0,N=0|R=0) &= \frac{P(E=0,N=0,R=0)}{P(R=0)} \\ &= \frac{P(R=0|E=0,N=0)P(E=0)P(N=0)}{P(R=0)} \\ &= \frac{(1-P(R=1|E=0,N=0))P(E=0)P(N=0)}{P(R=0)} \\ &= \frac{0.9 \cdot 0.1 \cdot 0.75}{0.765} \\ &= \boxed{0.088}\end{aligned}$$

This intuitively makes sense because we need two conidtions to occur, equipment broken and neuron spike not occuring.

(d)

Let us consider the case $E=1$, $N=0$ given $R=1$ we have

$$P(E=1,N=0|R=1) = \frac{P(R=1|E=1,N=0)P(E=1)P(N=0)}{P(R=1)}$$

Since $P(R=1|E=1,N=0)=0$, we thus have $P(E=1,N=0|R=1)=0$. However

$$\begin{aligned}P(E=1|R=1) = P(R=1|E=1)\frac{P(E=1)}{P(R=1)}\\ &= (P(R=1|E=1,N=0)P(N=0)+\\ &P(R=1|E=1,N=1)P(N=1))\frac{P(E=1)}{P(R=1)}\end{aligned}$$

This therefore $P(E=1|R=1)>1$, likewise

$$\begin{aligned}P(N=0|R=1) = P(R=1|N=0)\frac{P(N=0)}{P(R=1)}\\ &= (P(R=1|E=1,N=0)P(E=1)+\\ &P(R=1|E=0,N=0)P(E=0))\frac{P(N=0)}{P(R=1)}\end{aligned}$$

This therefore $P(E=1|R=1)>1$, therfore, $P(E=1|R=1)P(N=0|R=1)>0$ and is not equal to $P(E=1,N=0|R=1)$ therefore they are conditionally dependent. Ie they are not independent given R

Problem 2

(a)

$$\boxed{P(a,b,c,d) = P(c)P(a|c)P(d)P(b|a,d)}$$

(b)

$$\begin{aligned}
P(C,D) &= \sum_A \sum_B P(C,D,A,B) \\
&= \sum_A \sum_B P(C)P(D)P(A|C)P(B|A,D) \\
&= P(C)P(D) \sum_A \sum_B P(A|C)P(B|A,D) \\
&= P(C)P(D) \sum_A P(A|C) \sum_B P(B|A) \\
&= P(C)P(D)
\end{aligned}$$

Therefore C and D are independent.

(c)

$$\begin{aligned}
P(C,D|A,B) &= \frac{P(C,D,A,B)}{P(A,B)} \\
&= \frac{P(C)P(D)P(A|C)P(B|A,D)}{P(A,B)} \\
&= \frac{P(D)P(C|A)P(A)P(B|A,D)}{P(A,B)} \\
&= \frac{P(D)P(C|A)P(D|B,A)}{P(D|A)} \\
&= \frac{P(D)}{P(D|A)} P(C|A)P(D|B,A) \\
&= \frac{P(C|A)P(D|B,A)}{P(D,A)}
\end{aligned}$$

Therefore C and D are not independent given A and B.

(d)

$$\begin{aligned}
P(a,d) &= \sum_B P(a,d,b) \\
&= \sum_a P(d)P(b)P(b|d,a) \\
&= P(d)P(b)\sum_B P(b|d,a) \\
&= P(d)P(b)
\end{aligned}$$

Therefore a and d are independent

(e)

$$\begin{aligned}
P(a,d|b) &= \frac{P(a,d,b)}{P(b)} \\
&= \frac{P(a)P(d)P(b|a,d)}{P(b)} \\
&= \frac{P(a)P(d)}{P(b)}\frac{P(b|d)P(a|b,d)}{P(a|d)} \\
&= boxed P(a|b,d)P(d|b)
\end{aligned}$$

Therefore a and d are not independent given b

(f)

$$\begin{aligned}
P(c,b) &= \sum_A P(c,b,a) \\
&= \sum_A P(c)P(a|c)P(b|a) \\
&= P(c)\sum_A P(a|c)P(b|a)
\end{aligned}$$

Since $\sum_A P(a|c)P(b|a) \neq P(b)$ in general, c and b are not independent.

(g)

$$\begin{aligned} P(c, b|a) &= \frac{P(c, b, a)}{a} \\ &= \frac{P(c)P(a|c)P(b|a)}{P(a)} \\ &= P(c|a)P(b|a) \end{aligned}$$

Therefore c and b are independent given a