ECE C143A Homework 4

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Problem 1

(a)

$P(N = 0) = 1 - 0.25 = \boxed{0.75}$

(b)

We want $P(N = 0|R = 0)$, we know that

$P(R = 1|N = 0) = P(E = 0)P(R = 1|E = 0, N = 0)+P(E = 1)P(R = 1|E = 1, N = 0) = 0.01$

$P(R = 0|N = 0) = 0.99$ and thus from bayes law we have

$$\begin{aligned} P(N = 0|R = 0) &= P(R = 0|N = 0)\frac{P(N = 0)}{P(R = 0)} \\ &= 0.9\frac{0.1 \cdot 0.75}{P(R = 0)} \end{aligned}$$

To find $P(R=0)$ we must find $P(R=1)$,

$$\begin{aligned}P(R=1) = P(E=1)P(N=1)P(R=1|E=1,N=1)\\ + P(E=1)P(N=0)P(R=1|E=1,N=0)\\ + P(E=0)P(N=1)P(R=1|E=0,N=1)\\ + P(E=0)P(N=0)P(R=1|E=0,N=0)\end{aligned}$$

$$P(R=1) = 0.9 \cdot 0.25 \cdot 1 + 0.1 \cdot 0.75 \cdot 0.1 + 0.1 \cdot 0.25 \cdot 0.1 = 0.235$$

Therefore $P(R=0) = 1 - P(R=1) = 0.765$, thus

$$P(N=0|R=0) = 0.99\frac{0.75}{P(R=0)} = \boxed{0.97}$$

(c)

We have

$$\begin{aligned}P(E=0,N=0|R=0) &= \frac{P(E=0,N=0,R=0)}{P(R=0)}\\ &= \frac{P(R=0|E=0,N=0)P(E=0)P(N=0)}{P(R=0)}\\ &= \frac{(1-P(R=1|E=0,N=0))P(E=0)P(N=0)}{P(R=0)}\\ &= \frac{0.9 \cdot 0.1 \cdot 0.75}{0.765}\\ &= \boxed{0.088}\end{aligned}$$

This intuitively makes sense because we need two conidtions to occur, equipment broken and neuron spike not occuring.

(d)

Let us consider the case $E=1$, $N=0$ given $R=1$ we have

$$P(E=1,N=0|R=1) = \frac{P(R=1|E=1,N=0)P(E=1)P(N=0)}{P(R=1)}$$

Since $P(R=1|E=1,N=0)=0$, we thus have $P(E=1,N=0|R=1)=0$. However

$$\begin{aligned}
P(E=1|R=1) = P(R=1|E=1)\frac{P(E=1)}{P(R=1)} \\
&= (P(R=1|E=1,N=0)P(N=0)+ \\
&P(R=1|E=1,N=1)P(N=1))\frac{P(E=1)}{P(R=1)}
\end{aligned}$$

This therefore $P(E=1|R=1)>1$, likewise

$$\begin{aligned}
P(N=0|R=1) = P(R=1|N=0)\frac{P(N=0)}{P(R=1)} \\
&= (P(R=1|E=1,N=0)P(E=1)+ \\
&P(R=1|E=0,N=0)P(E=0))\frac{P(N=0)}{P(R=1)}
\end{aligned}$$

This therefore $P(E=1|R=1)>1$, therfore, $P(E=1|R=1)P(N=0|R=1)>0$ and is not equal to $P(E=1,N=0|R=1)$ therefore they are conditionally dependent. Ie they are not independent given R

Problem 2

(a)

$$\boxed{P(a,b,c,d)=P(c)P(a|c)P(d)P(b|a,d)}$$

(b)

$$\begin{aligned}
P(C,D) &= \sum_A P(C,D,A) \\
&= \sum_A P(A)P(C|A)P(D|A) \\
&= \sum_A P(C)P(A|C)P(D|A) \\
&= P(C)\sum_A P(A|C)P(D|A)
\end{aligned}$$

In order for this to be independent we must have that $\sum_A P(A|C)P(D|A) = P(D)$ This will occur if and only if $P(D|A) = P(D)$ or $P(A|C) = P(A)$, this generally will not hold true because that would mean that either A is independent of C or A is independent of D, neither of which should hold true. Therefore C and D are not independent.

(c)