Lecture 6: Spiking statistics

Announcements.

1. HW #1 is due tonight, uploaded to Gradescope by 11:59pm.

2. HW #2 was uploaded and is due Friday, April 22, 2022. It contains a Jupyter Notebook.

- Question 1.c. is true.

- 2g. to solve $\min_{W} \| Y - XW \|_2^2$

  the least squares solution is: $W = (X^T X)^{-1} X^T Y$

- 5. $\mathbb{E}(X|Y) = \int_{-\infty}^{\infty} x \cdot p(x|y)\, dx$

3. Please look @ Bruin Learn Announcements (first one) for info on how to sign up for Piazza and Gradescope.

Firing Rates and Spike Statistics

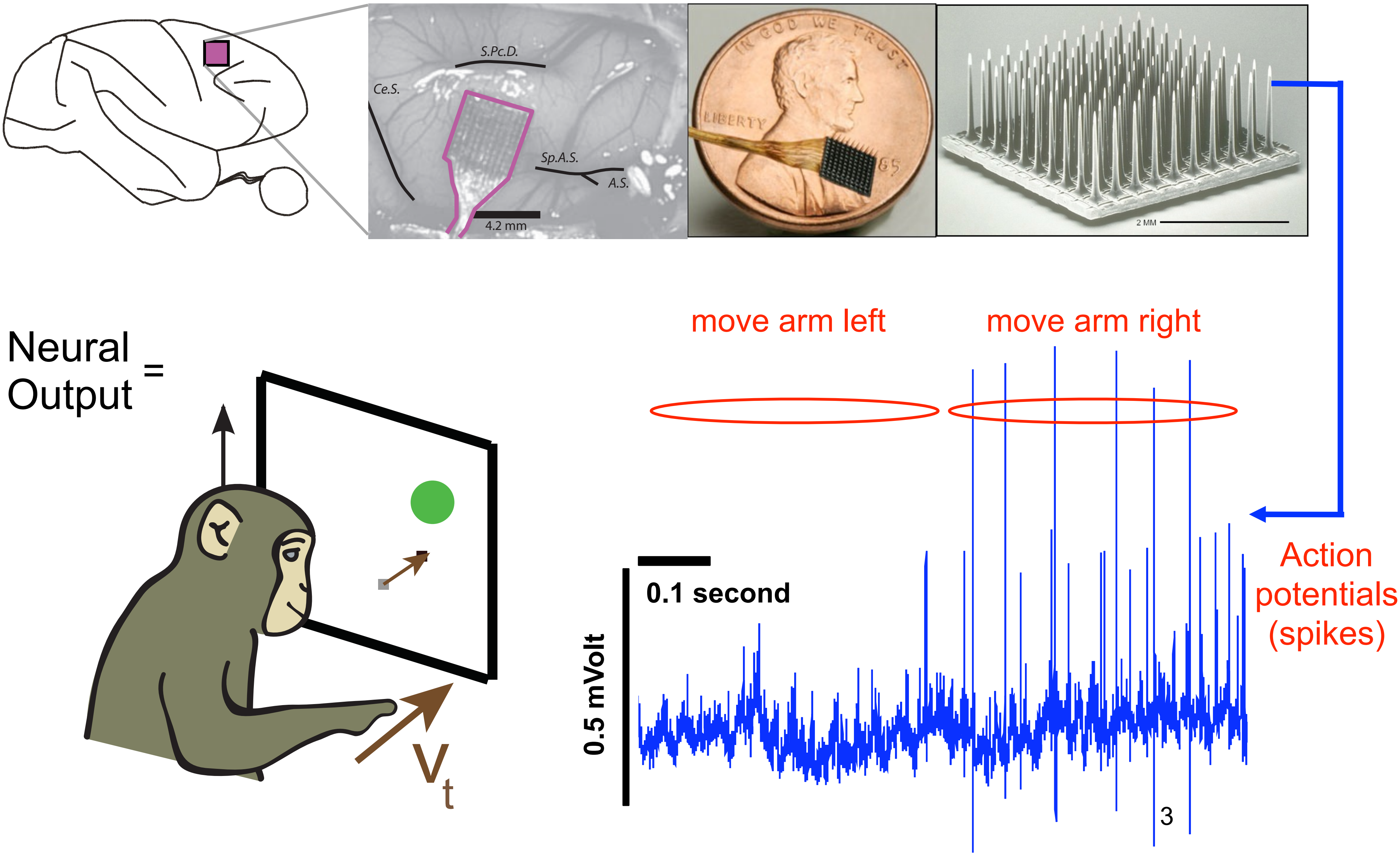
- Reading assignment from Dayan & Abbott:
  - Chapter 1 – Neural Encoding I: Firing Rates and Spike Statistics
  - p. 12-28 of PRML.

- We now know quite a bit about the electrical properties of neurons, including a first-principles understanding of:
  - Ion channels
  - Membrane potential
  - Action potential generation
  - Action potential propagation

- As discussed in class, we could continue learning about various fundamental neuroscience topics including neurotransmitters, synapses, development, genetics, etc…

- Though this would (hopefully!) be interesting and fun, it would constitute a course in neuroscience – not a course in “NeuroEngineering”.

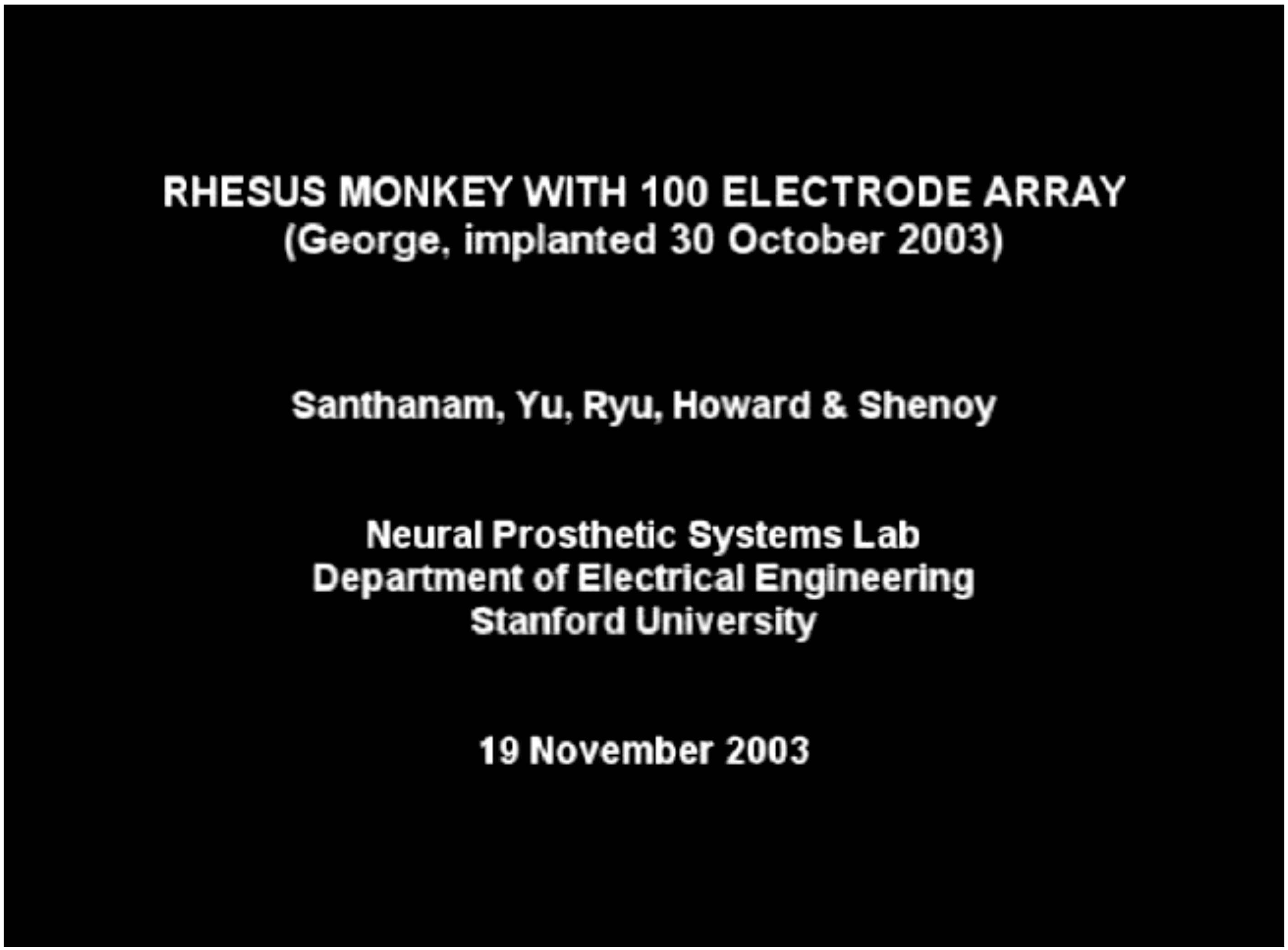
Neural Recordings Encode Arm Movements



Virtual lab tour

• We will be looking at various representations of spikes (full AP waveform, rasters, histograms, etc.) but it is useful have a “look and feel” in mind.

• Thus, a virtual lab tour including listening to action potentials stream in:



Neural Encoding and Decoding

- Neurons represent and transit information by firing sequences of spikes.
- Spikes are fired in various temporal patterns.

- The study of neural coding involves measuring and characterizing how stimulus attributes (light, sound intensity, or motor actions) are represented by spikes.

- **Neural encoding** – the map from stimulus to neural response.
  - Can measure how neurons respond to a wide variety of stimuli.
  - Then construct models; attempt to predict responses to other stimuli.
  - We will discuss encoding at a high level in this lecture.

- **Neural decoding** – the map from response to stimulus.
  - Attempt to reconstruct a stimulus, or certain aspects of that stimulus, from the spike sequence it evokes.
  - We will discuss decoding extensively in the rest of this course.

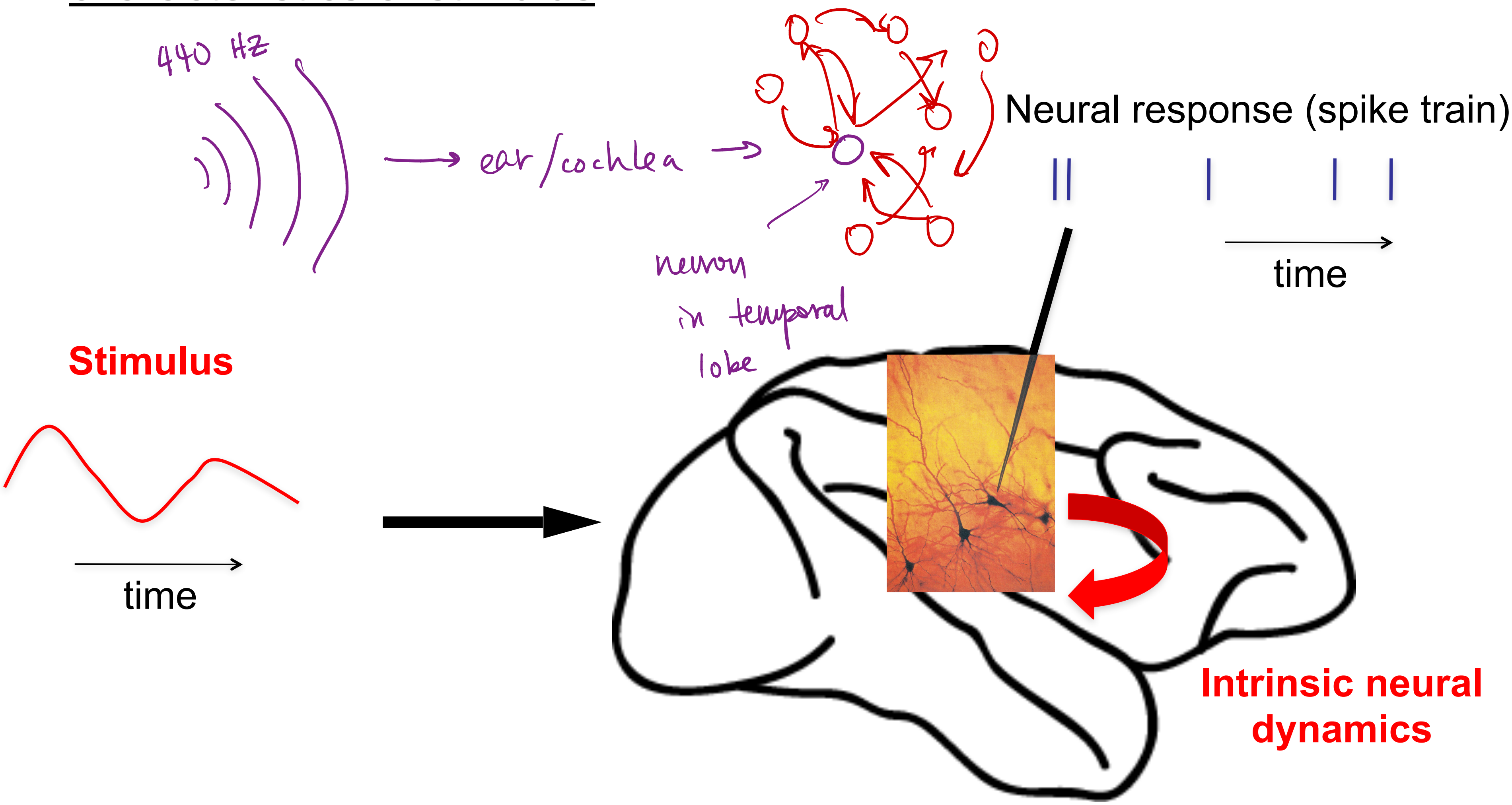
- neural data

From Stimulus to Response

Characterizing the stimulus-to-response relationship is difficult because neural responses are “complex” and variable. In particular,

time

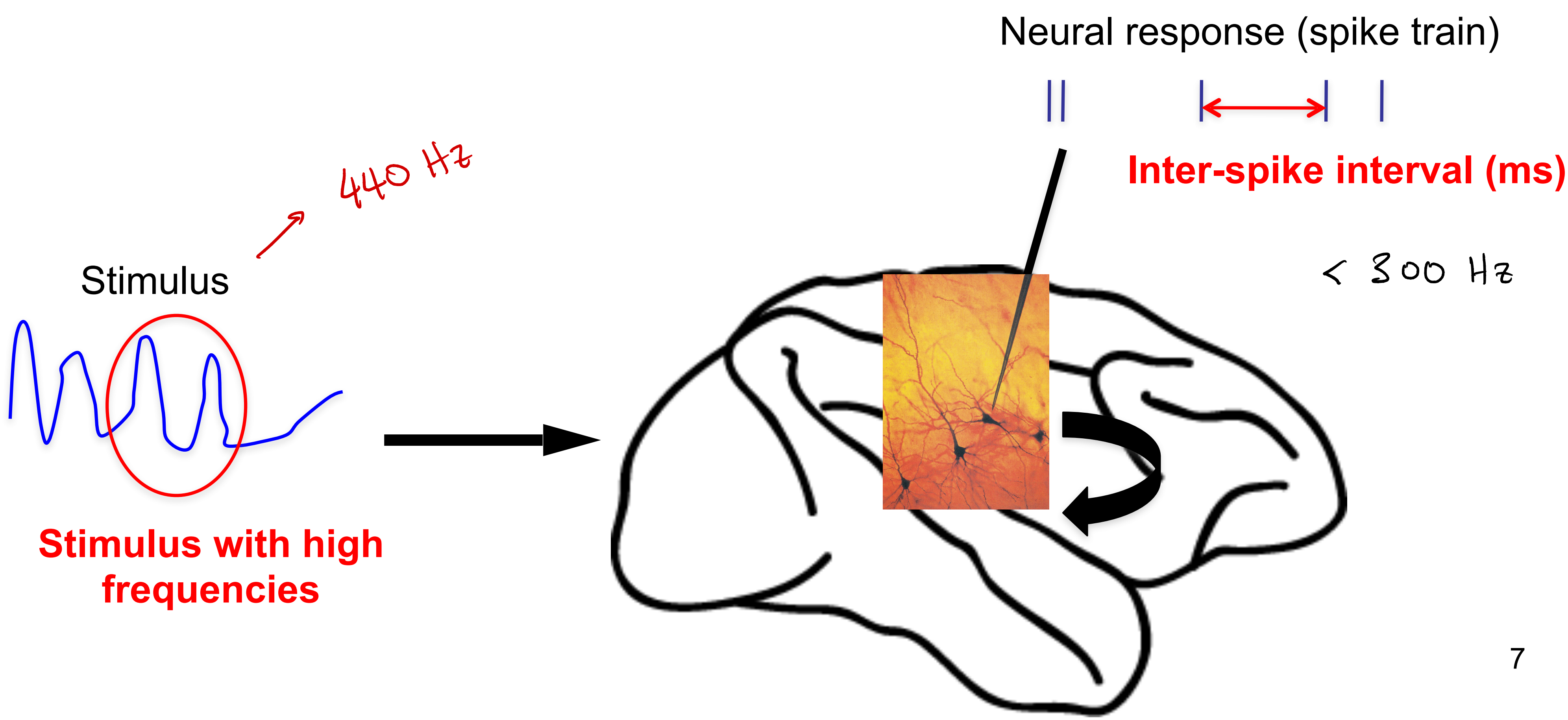
1) Spike sequences reflect both <u>intrinsic neural dynamics</u> and <u>temporal characteristics of stimulus</u>.



From Stimulus to Response

Characterizing the stimulus → response relationship is difficult because neural responses are "complex" and variable. In particular,

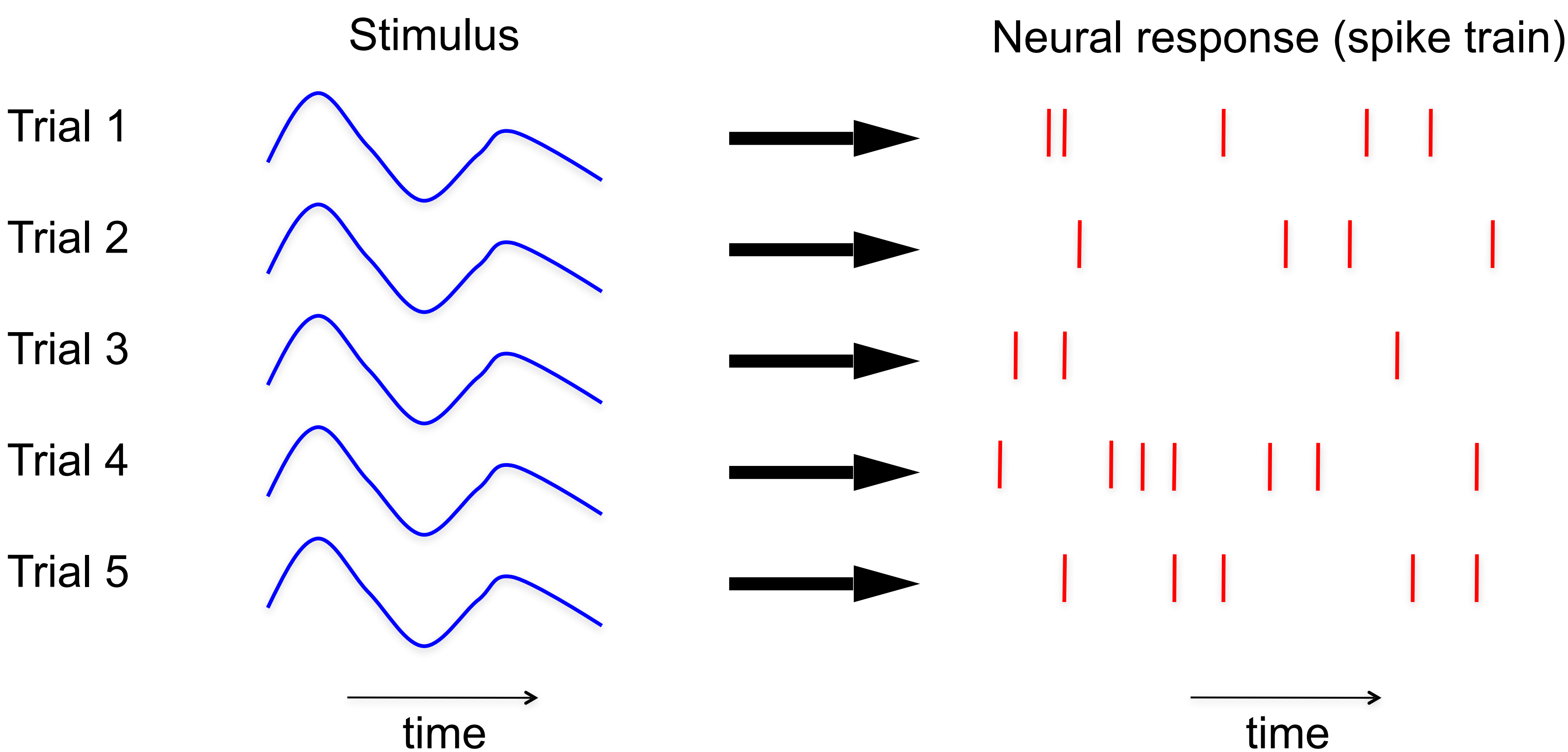
2) Identifying features of response that encode changes in stimulus is difficult, especially if stimulus changes on times scale of inter-spike interval.



From Stimulus to Response

Characterizing the stimulus → response relationship is difficult because neural responses are "complex" and variable. In particular,

3) Neural responses vary from trial-to-trial even when the same stimulus is presented repeatedly.

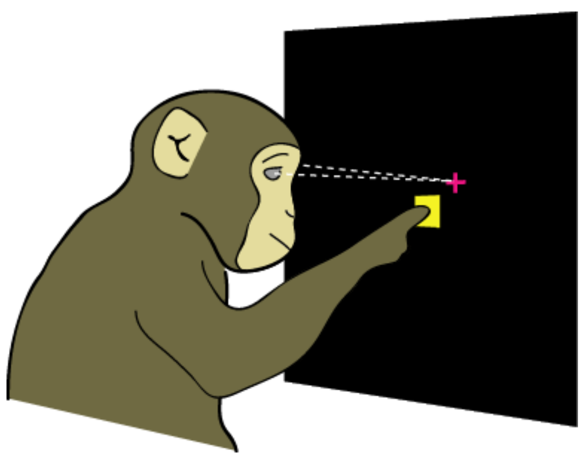


Why are neural responses variable?

- Randomness associated with biophysical processes involved in spike generation and transmission (e.g., neurotransmitter release at presynaptic terminal, opening / closing of ion channels)

- Variable levels of arousal and attention

- Effects of other cognitive processes:



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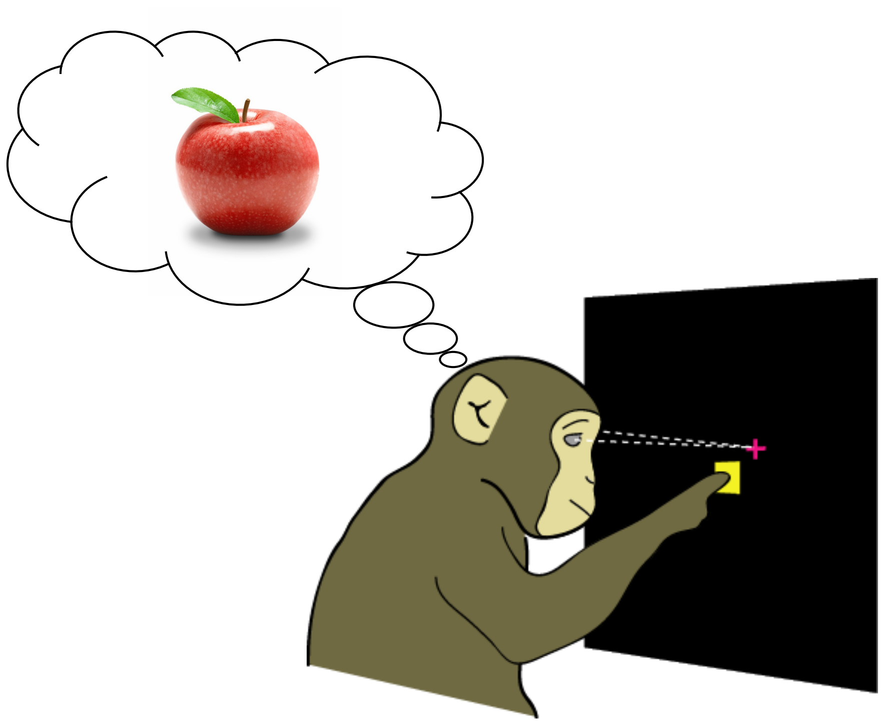

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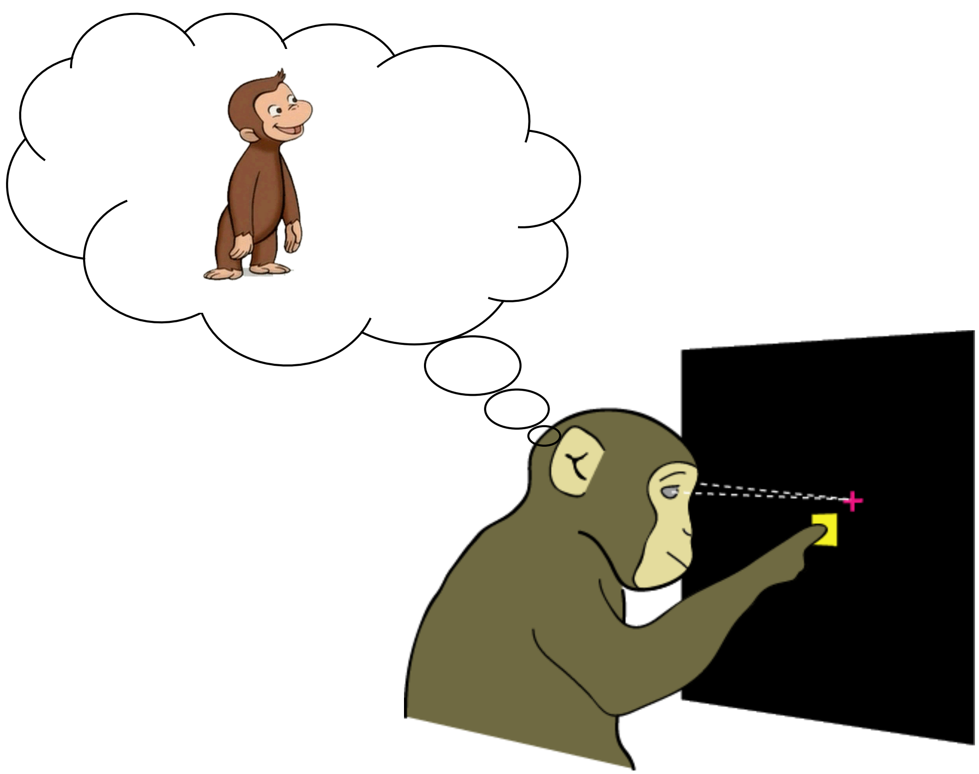

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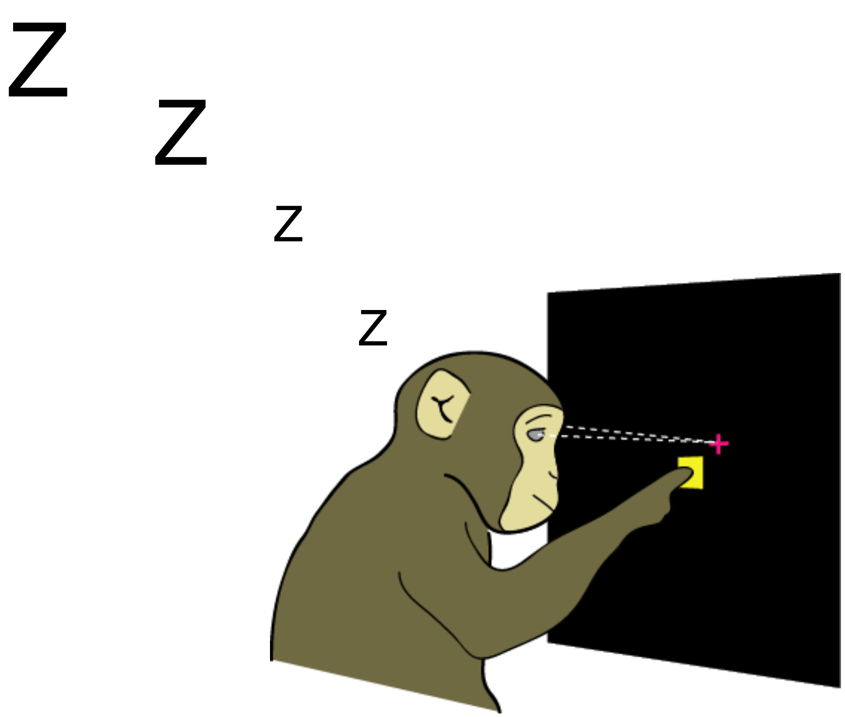


From Stimulus to Response

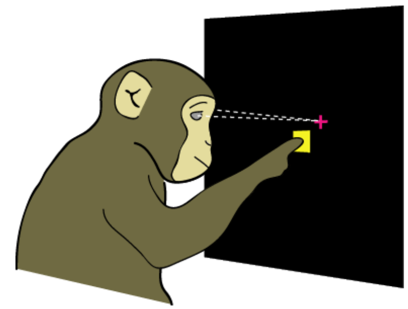
Characterizing the stimulus → response relationship is difficult because neural responses are "complex" and variable.

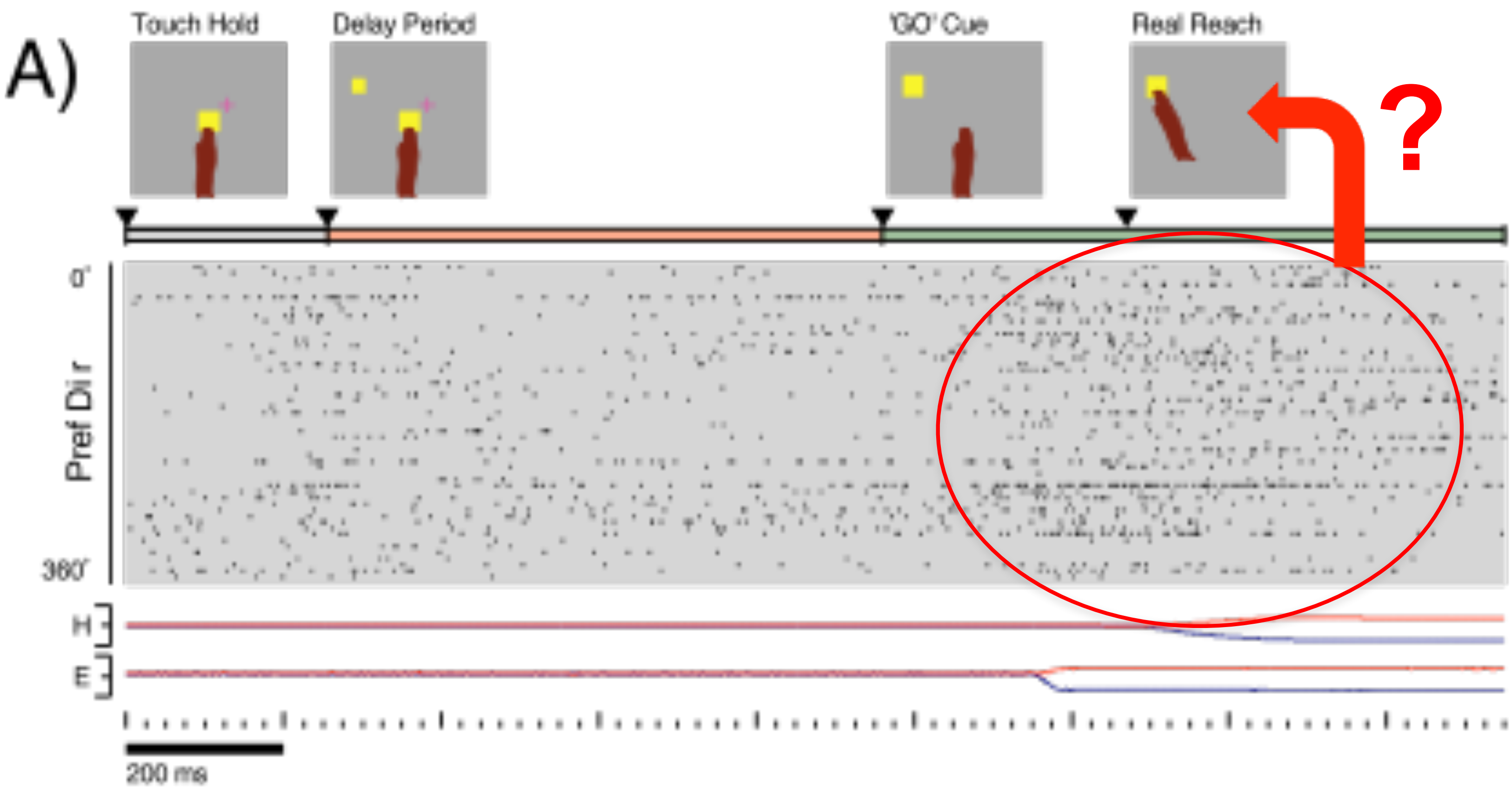
Thus, we cannot predict the exact timing of every spike.

Our goal is to find a model for the **probability** that different spike sequences are evoked by a specific stimulus.

Example to Clarify the Challenge of Encoding/Decoding

- An example of what real data looks like:
  - How does a population of neurons (in motor cortex) encode, with spiking, where the arm will move next?
  - How is the actual arm movement encoded?





Spike Trains

- APs encode and convey information through their rate of firing.

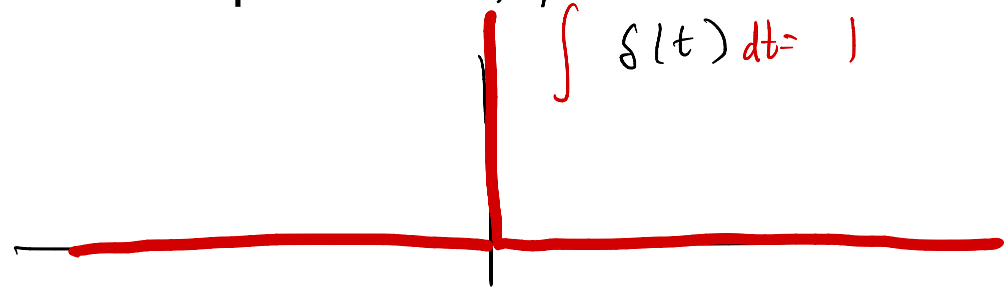
- AP duration, amplitude and shape are highly stereotyped (don't encode).

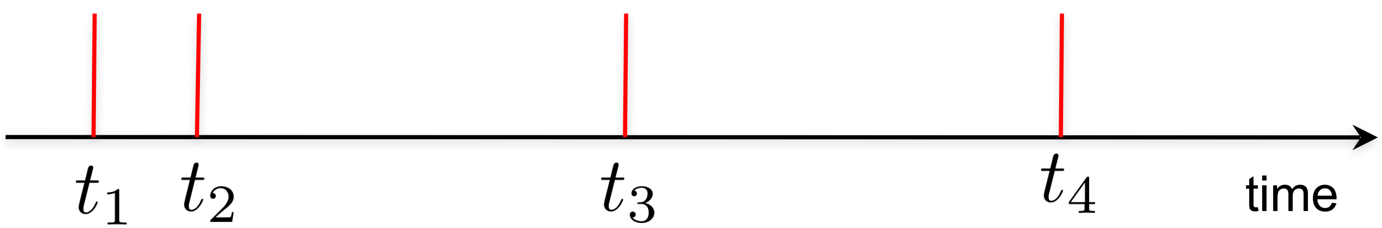
- Neglecting the brief duration of the actual AP (~1 ms), we can characterize an AP sequence with a list of spike times, t_i.

spike train

$$\rho(t) = \sum_{i=1}^{n} \delta(t - t_i)$$

$\int \delta(t)\, dt = 1$





Firing Rates

- Recall that the sequence of APs generated by a given stimulus varies from trial to trial.

- Thus neural responses are typically treated statistically / probabilistically.

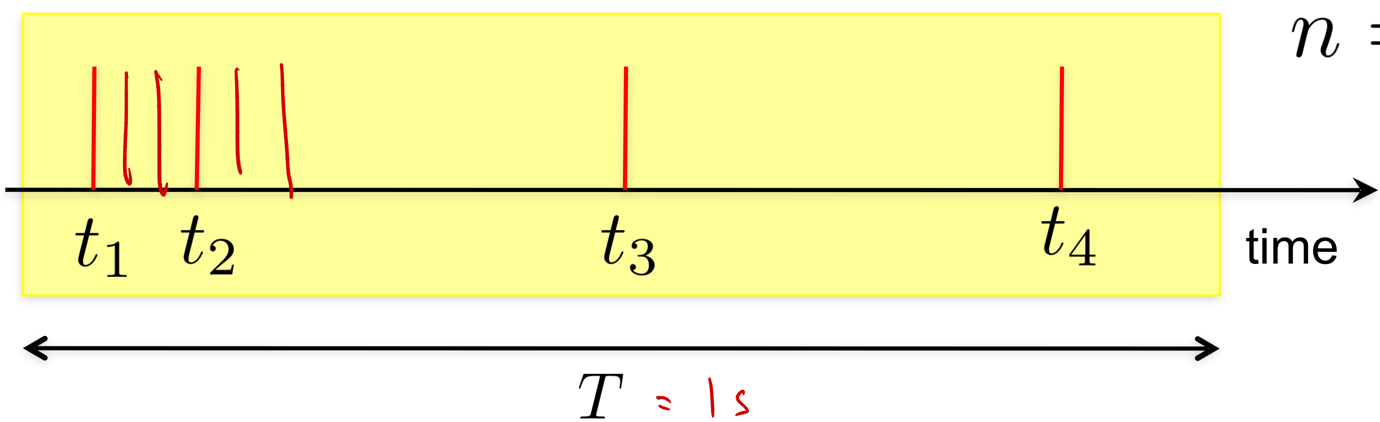
- Neural responses can be characterized by **firing rates**, rather than by specific spike sequences.

Firing Rates

In its simplest form, the firing rate is obtained by counting the number of spikes in a time window.

Thus, firing rate has units of *spikes per second*, or *Hz*.

$$n = 4$$



Firing rate

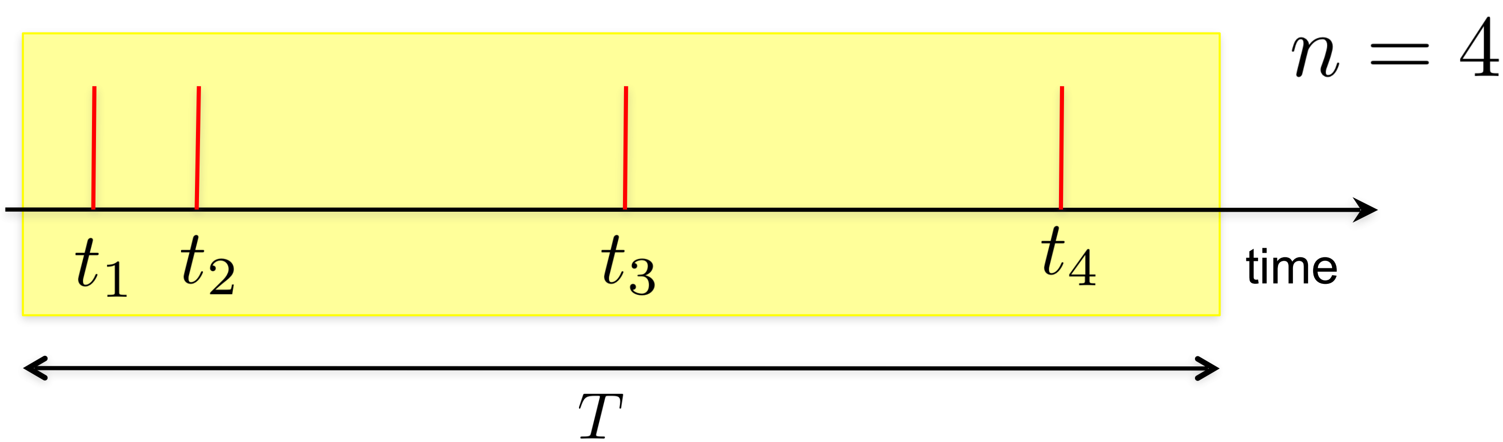
Spike count in window

$$\lambda = \frac{n}{T} = \frac{4\,sp}{1s} = 4\,sp/s \text{ or } 4\,Hz \quad \text{where} \quad n = \int_0^T \rho(\tau)d\tau$$

Firing Rates

With this definition of firing rate, what are we missing?

Time-varying properties of the neural response.



Firing rate

Spike count in window

$$\lambda = \frac{n}{T}$$

where $n = \int_0^T \rho(\tau) d\tau$

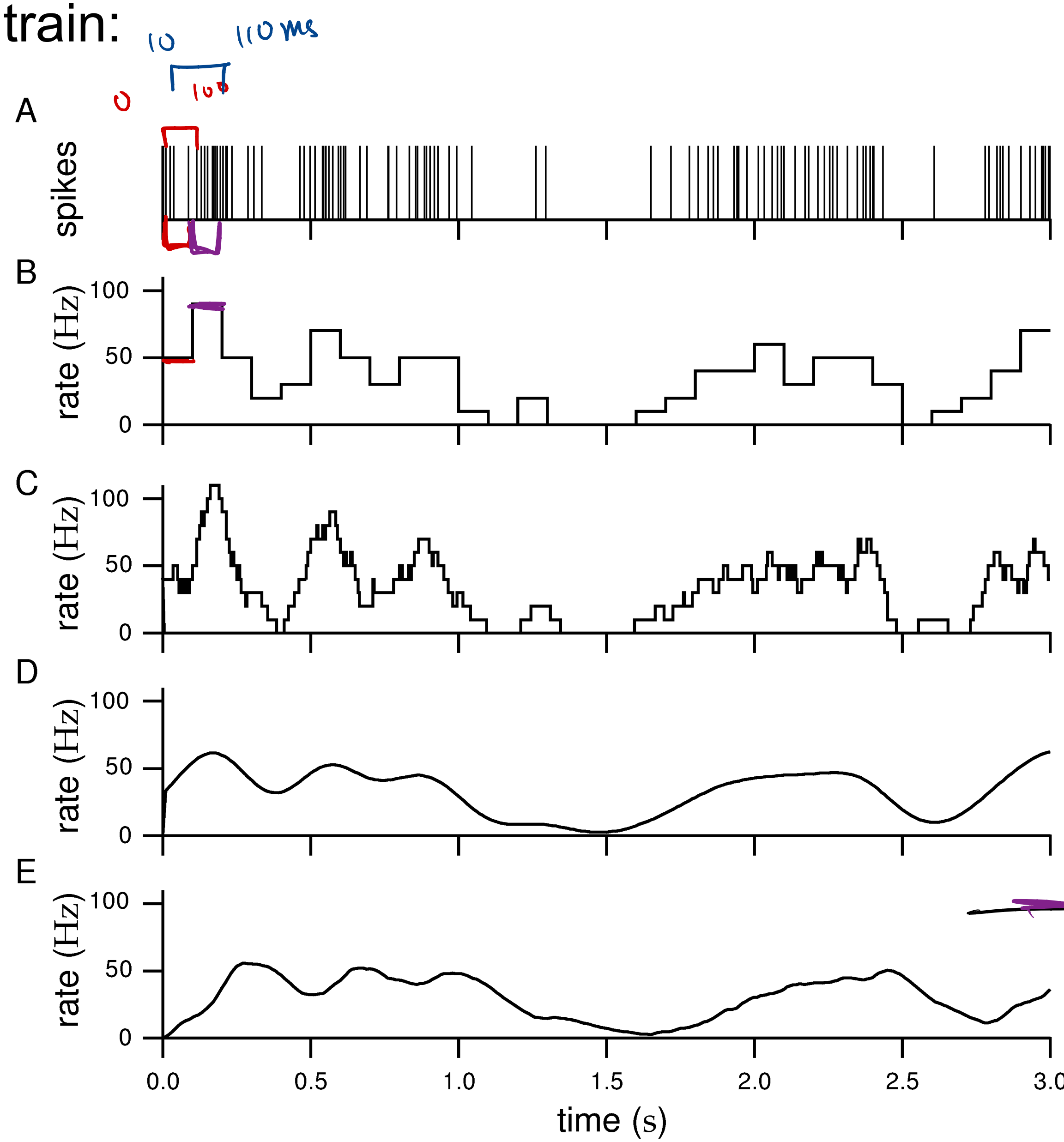
Motivation for Estimating Time-Varying Firing Rates

A neuron’s firing rate typically varies over time, so we’re likely to lose information by collapsing across the entire trial.

It’s like averaging all the frames of a movie into a single frame. That averaged frame is not likely to tell you much about what happened during the movie.

Estimating Time-Varying Firing Rates

There are many ways to approximate a time-varying firing rate from a spike train:



Raw spike train

Counts in 100 ms windows (non-overlapping)

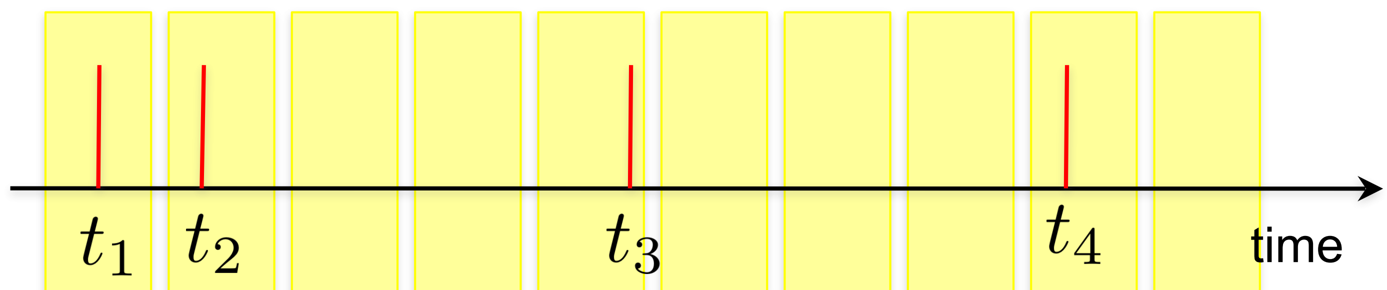
Counts in 100 ms windows (sliding)

Convolution with Gaussian

Convolution with one-sided exponential

Challenges of Estimating a Time-Varying Firing Rate from a Single Spike Train

1) If we want high temporal resolution, bins must be made small. But counts are then primarily zero or one.



2) Firing rate estimate is sensitive to randomness ("noise") in spike generation. We would like to discard this random component.

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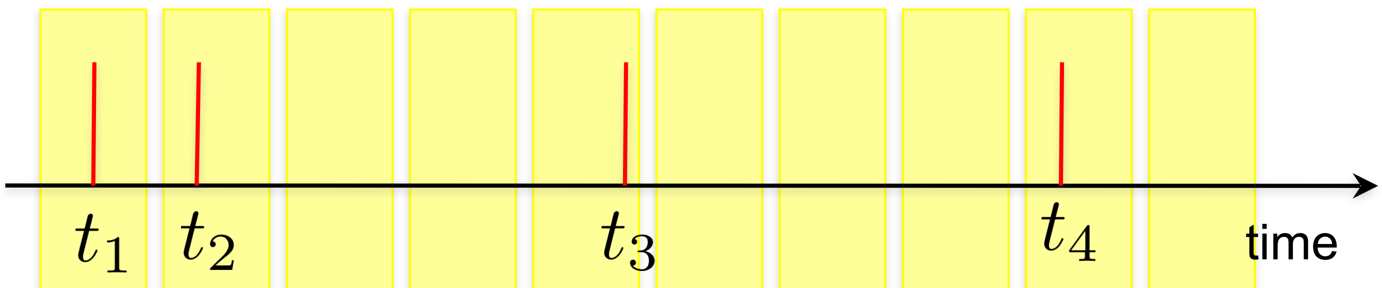
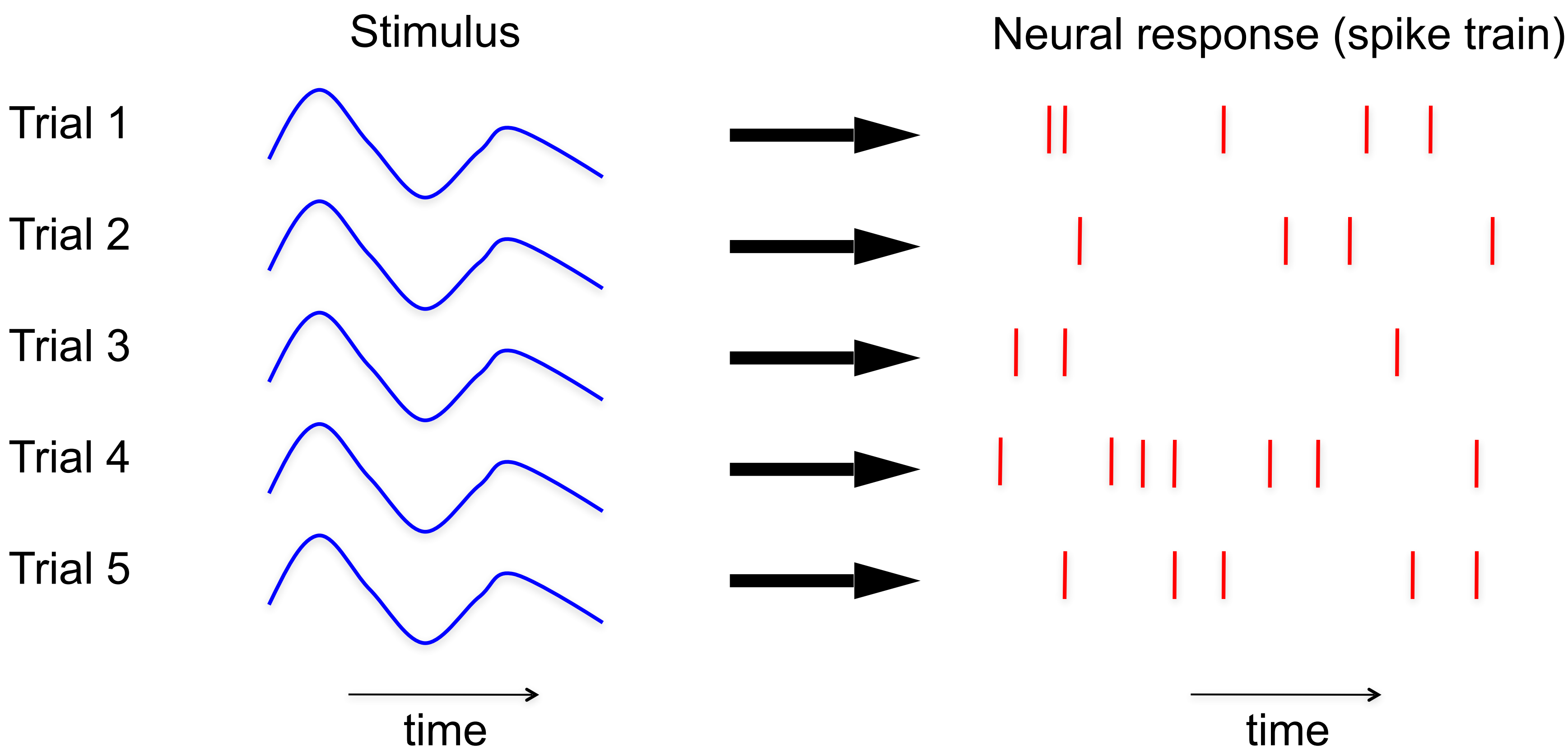


2) Firing rate estimate is sensitive to randomness ("noise") in spike generation. We would like to discard this random component.

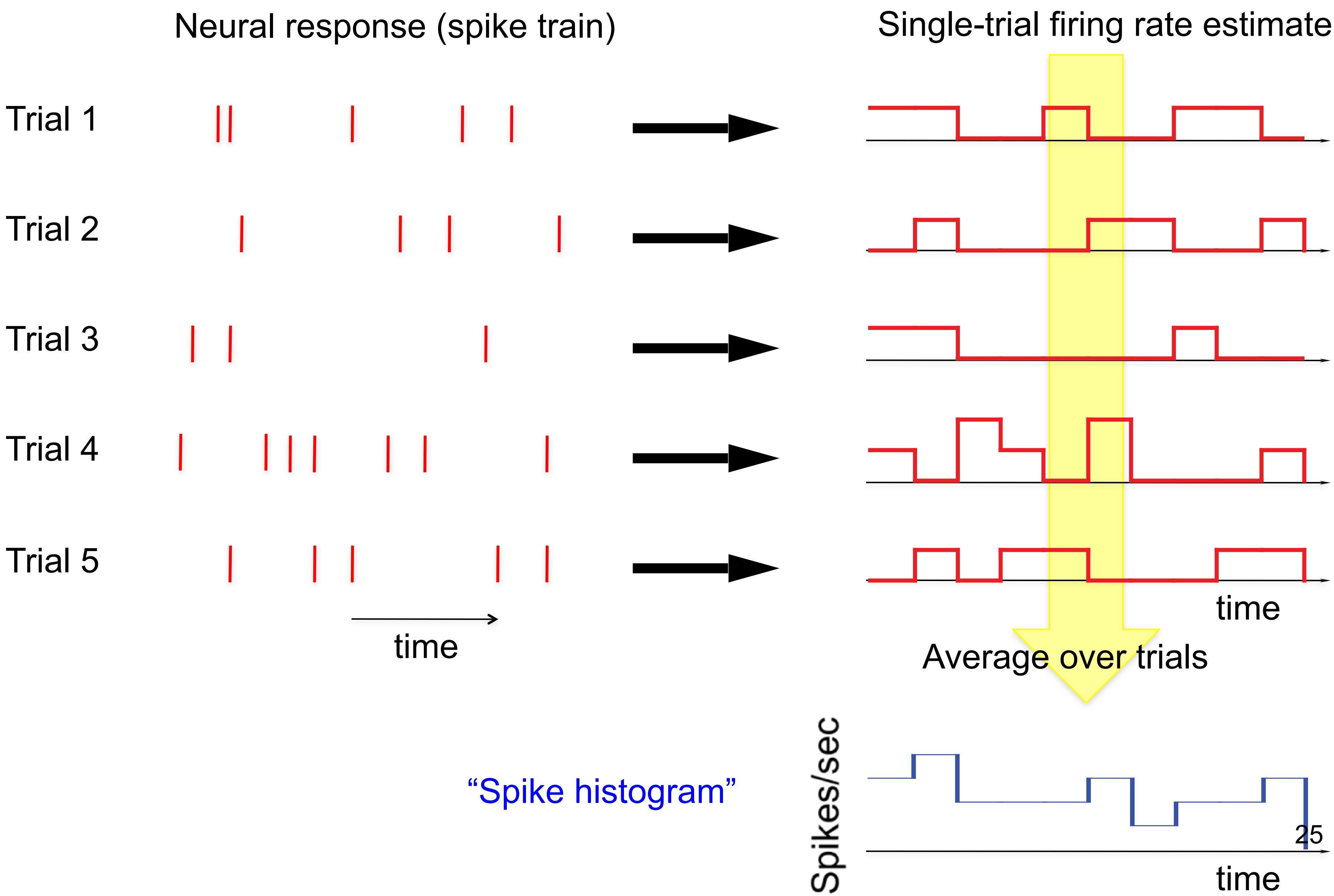
How can we get both high temporal resolution and beat down the noise when estimating firing rates?

One common way: Average across many trials.

Trial-Averaged Firing Rates

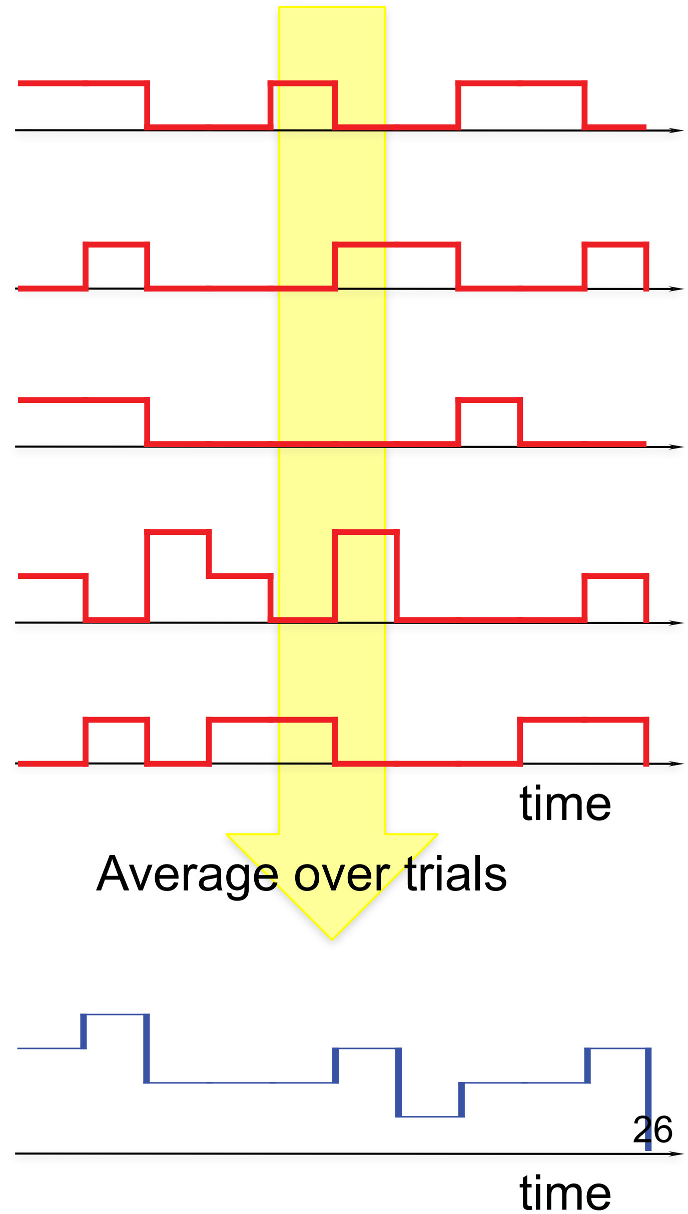


Trial-Averaged Firing Rates



Trial-Averaged Firing Rates

Single-trial firing rate estimate

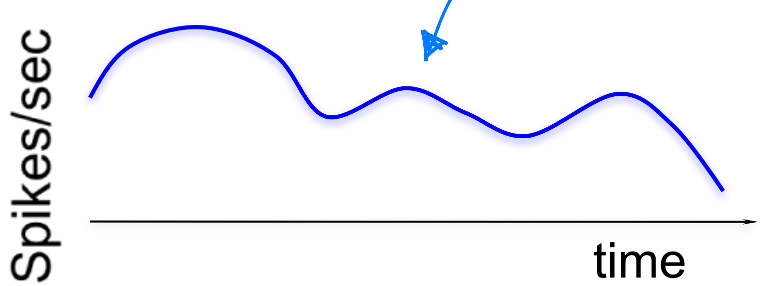


To make a spike histogram look nice,

- use small spike count windows
- average over a large number of trials

In the limit, this will produce a smoothly-varying firing rate.

Peri-stimulus Time Histogram (PSTH)



MORE trials

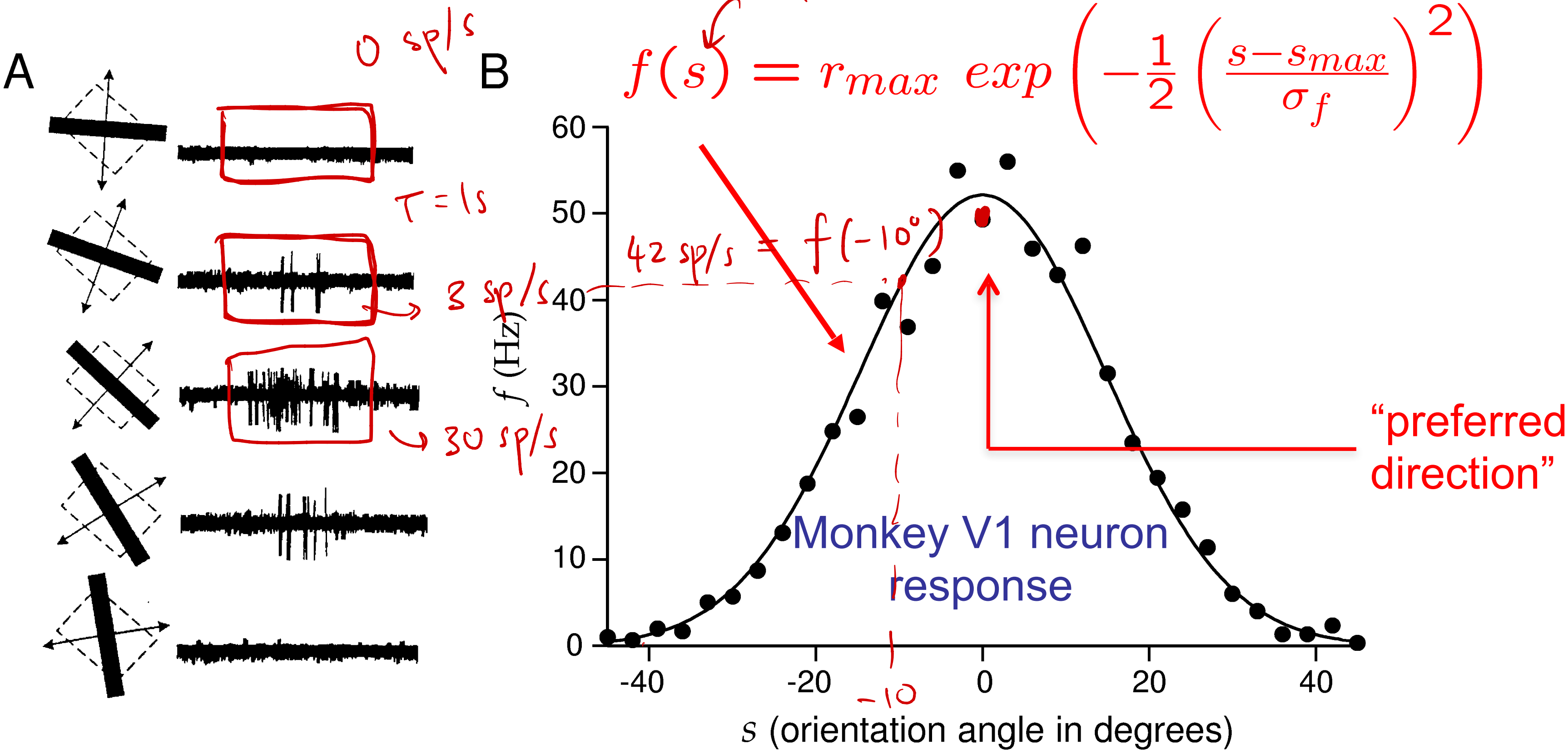
How are neural responses related to sensory stimulus or motor action?

To keep things simple for now, we will take spike counts across a large window, thus ignoring the time-varying structure that may be present in the spike trains.

Tuning Curves

- Neural responses typically depend on many different stimulus properties.
- Here we consider the dependence on just one stimulus attribute.
- Simple approach:
  - Count the number of spikes fired during the presentation of a stimulus.
  - Repeat stimulus presentation many times to better estimate the mean count.
  - Vary the stimulus attribute of interest, *s*.
  - Plot result and (optionally) fit parameterized function to data.

$$f(s) = r_{max}\ exp\left(-\frac{1}{2}\left(\frac{s-s_{max}}{\sigma_f}\right)^2\right)$$

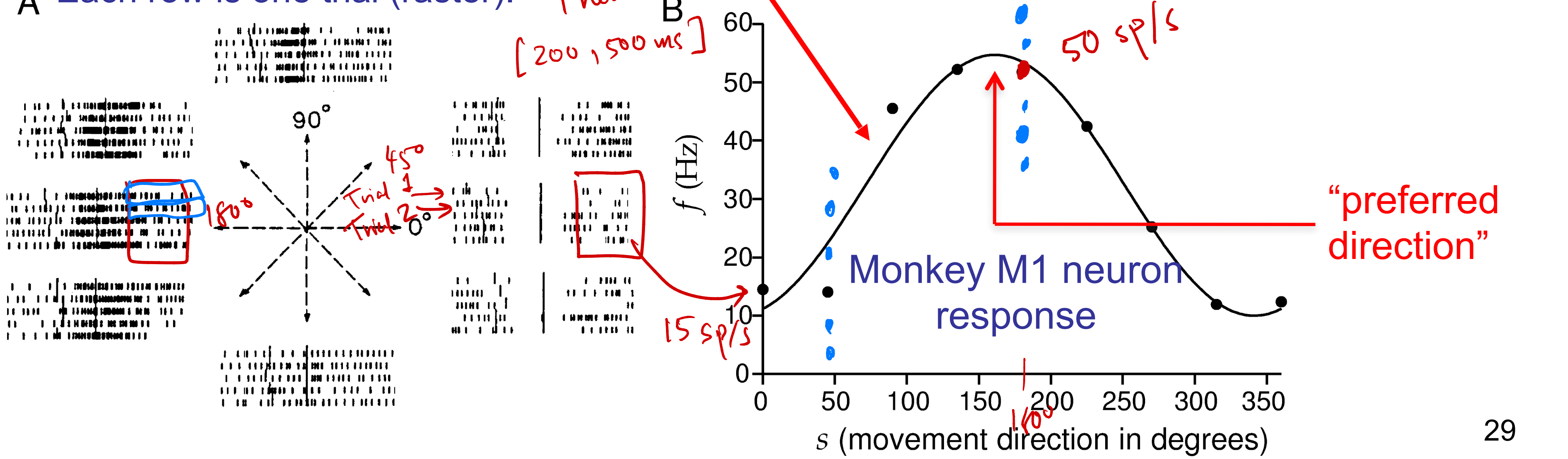


Tuning Curves

- Tuning curves also characterize responses from neurons in motor areas.
- In this example, a monkey is trained to reach in different directions, *s*.
- Count number of spikes firing during arm movement.
- Repeat movement many times to better estimate the mean count.

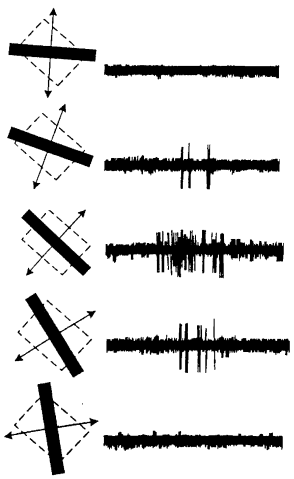
$$f(s) = r_0 + (r_{max} - r_0)\cos(s - s_{max})$$

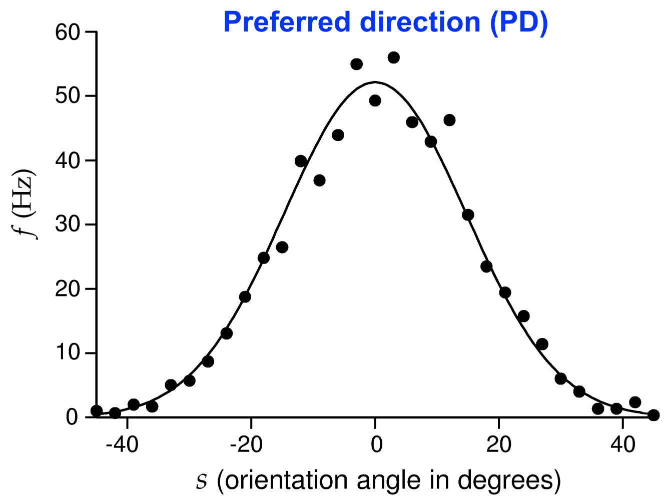
5 repeated trials in each direction.
Each row is one trial (raster).



Food for thought: Should we think of visual and motor systems in the exact same way (i.e., tuning curves)?

Visual system: Hubel & Wiesel and colleagues



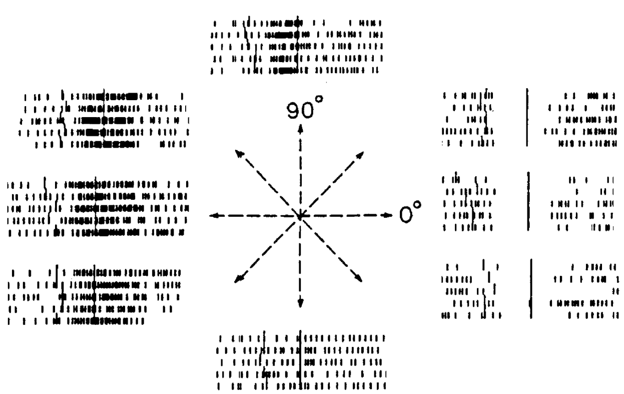


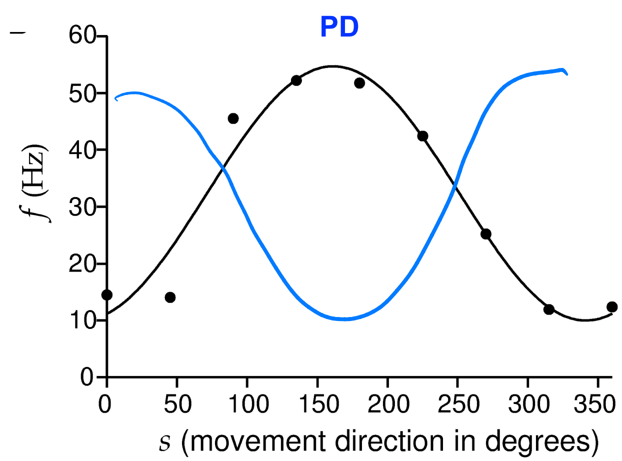
Role of visual system presumably to internally represent the outside world

Neurons extract key visual features, summarized by largely static tuning curves

Thus PDs are largely invariant to contrast and speed of moving edges (i.e., neurons encode movement direction)

Motor system: Georgopoulos & Schwartz and colleagues





Role of **motor system** presumably to **generate time-varying signals** to drive muscles, **not to represent movement**

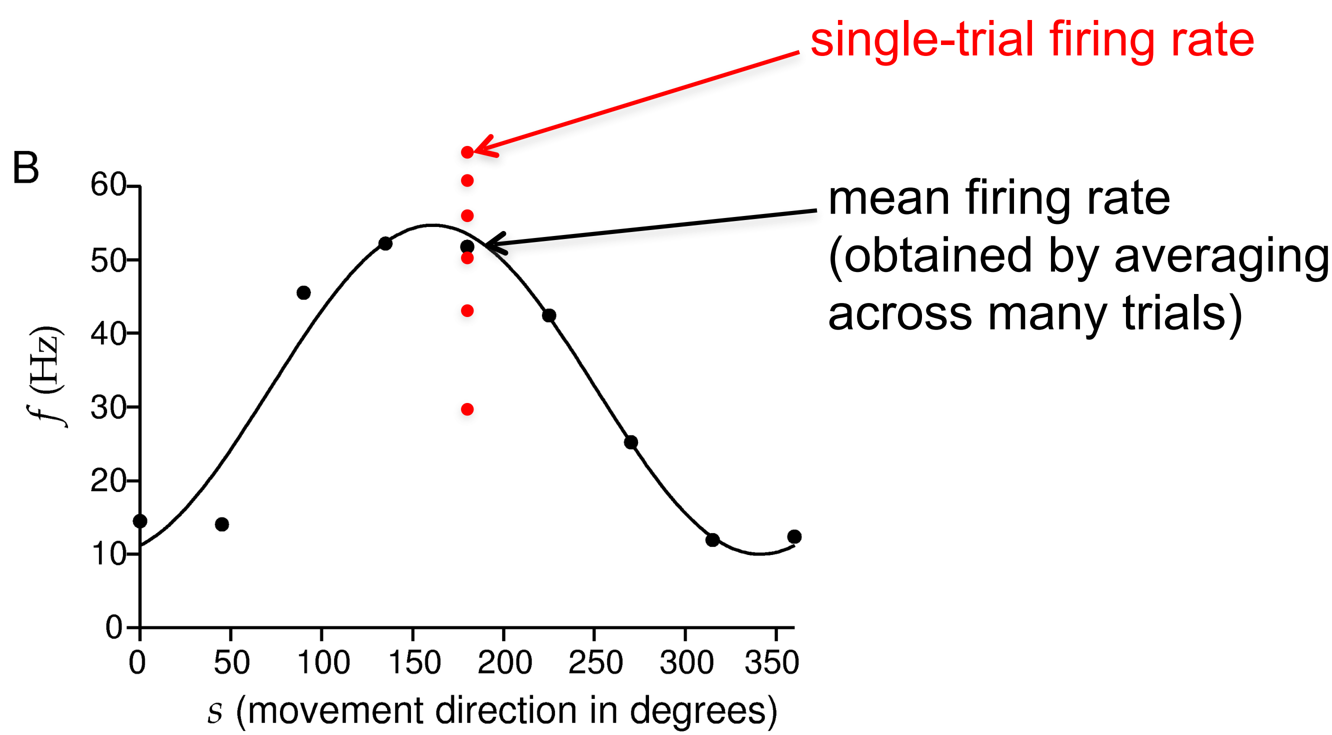
Neurons **generate (cause)** time-varying signals, and most are **not well captured by static tuning curves with simple functions**

PDs are **not invariant** in time, to movement speed, or to movement extent (i.e., tuning curves are time varying, complex & heterogeneous)

Shenoy, Churchland, Sahani (2013) *Ann. Rev. Neurosci.*
Churchland & Shenoy (2007b) *J Neurophysiol.*
Also: Fetz, Todorov, Cisek, Scott, Kalaska, Graziano, …

Noise

- Tuning curves allow us to predict the mean firing rate, given a stimulus.
- They do not describe how firing rate varies from trial to trial.



Noise

- Single-trial responses are probabilistic, not deterministic.

- Noise models describe the probability distribution, representing the firing rate on any given trial, about the mean $f(s)$.

- The standard deviation for the noise distribution can be:

  - Independent of the mean $f(s)$ → additive noise.

  - Dependent on the mean $f(s)$ → e.g., Poisson noise

- We will soon discuss a stochastic spike-generator model (Poisson) that will allow us to examine noise in finer detail.

What Makes a Neuron Fire?

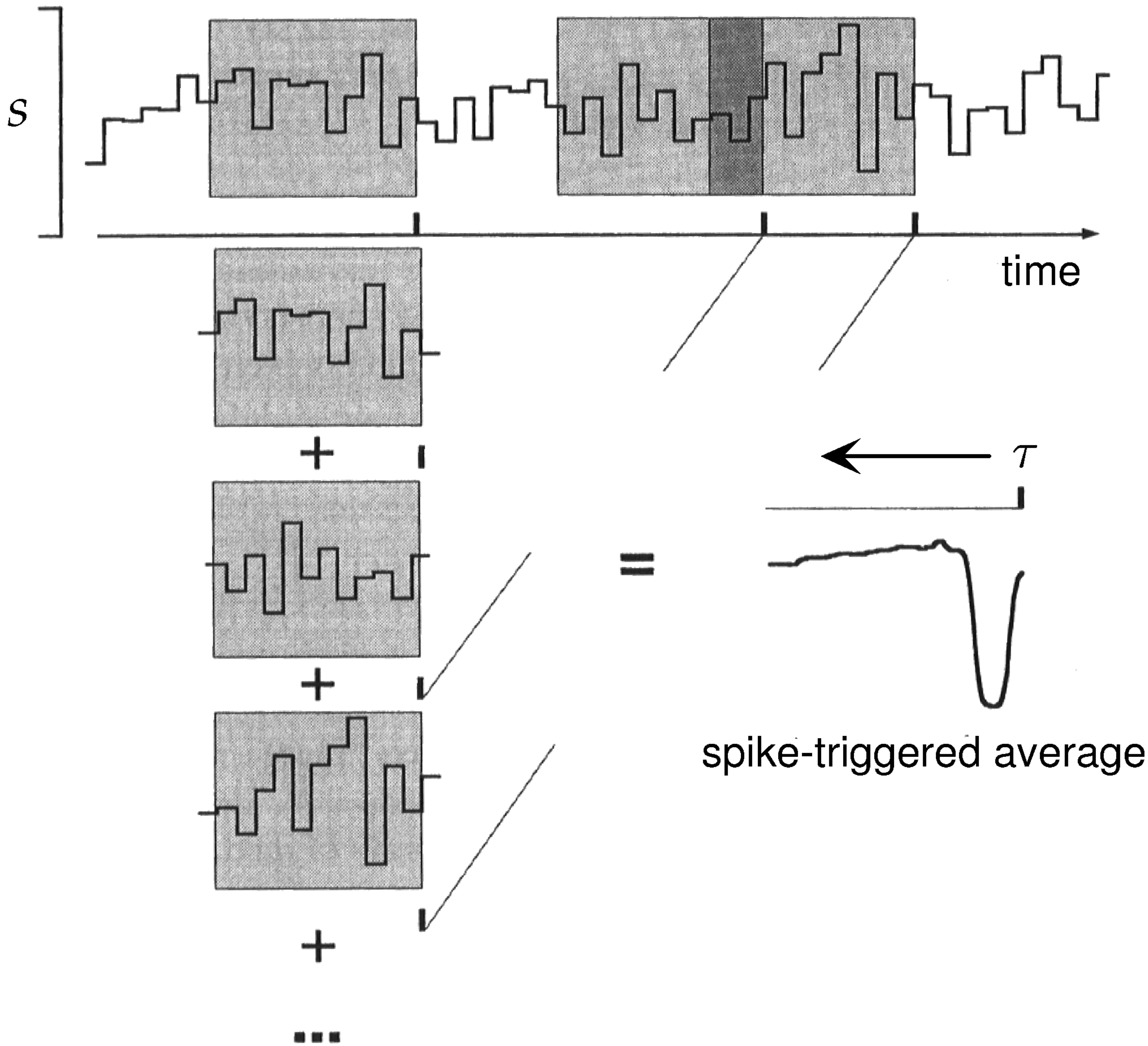
- Response tuning curves characterize the average response of a neuron to a given stimulus.

- What about averaging the stimuli that produce a given response? Yes, can do this too.

- Can trigger on action potential and ask, “What, on average, did the stimulus do before an action potential was fired?”

- Called “spike-triggered average”.

Spike-Triggered Average



Spike-Train Statistics

• A complete description of the stochastic relationship between a stimulus and a response would require us to know the probabilities corresponding to *every* sequence of spikes that can be evoked by the stimulus.

• However, the number of possible spike sequences is typically so large that determining or even roughly estimating all of their probabilities of occurrence is impossible.

• Instead, we must rely on some statistical model that allows us to estimate the probability of an arbitrary spike sequence.

Spike-Train Statistics

spikes

- *Point process* – stochastic process that generates a sequence of events, such as APs.
- *Renewal process* – point process where the probability of an event depends only on the immediately preceding event (intervals between successive events are independent).
- *Poisson process* – point process where no dependence at all on preceding events (events are statistically independent).
- The Poisson process is a useful and widely used approximation of stochastic neuronal firing.

SEE “POISSON PROCESSES” HANDOUT

Probability Refresher

Random Variables : A, B, ... (capital)

Events : a, b, ... (lower case)

Prob. that A takes on value (event) a and B takes on b,

$$\Pr\{A=a,\ B=b\} \triangleq p_{A,B}(a,b)$$

$$\triangleq p(a,b)$$

A is a R.V. = # of times a neuron spikes in T seconds

B is a R.V. = freq. of an auditory stimulus $\{200, 300, 400\}$ Hz

$$p(2,200) = \Pr\{A=2,\ B=200\text{ Hz}\}$$

Law of Total Prob.

$\mathcal{B} = \{200, 300, 400\}$

$$p(a) = \sum_{b \in \mathcal{B}} p(a, b)$$

$$\begin{aligned} p(2) = \Pr\{A=2\} &= \Pr\{A=2, B=200\} \\ &+ \Pr\{A=2, B=300\} \\ &+ \Pr\{A=2, B=400\} \end{aligned}$$

"Marginalization"

Intuition: if I want to know the $\Pr\{A=2\}$, this prob. is irrespective of what B is, so I'll sum across all possible values of B.

$$p_{A,B}(2, 200) \leq p_A(2)$$

Chain rule for probability:

"Prob that $A = a$"

$$p(a, b) = p(a) \cdot p(b \mid a)$$

"Prob $B = b$ given that $A = a$"

"Prob that $A = a$ and $B = b$"

$$= p(b) \cdot p(a \mid b)$$

$$p(a)\, p(b \mid a) = p(b)\, p(a \mid b)$$

Bayes rule: $$p(b \mid a) = \frac{p(b)\, p(a \mid b)}{p(a)}$$

$$p(a,b,c) = p(a) \cdot p(b \mid a) \cdot p(c \mid a,b)$$

$$= p(c) \cdot p(a \mid c) \cdot p(b \mid a,c)$$

$$= p(a,b) \cdot p(c \mid a,b)$$

<u>Intuition</u>: every R.V. gets to go in front of the conditioning bar <u>ONCE</u>. After that, its value is fixed and the remaining probs. assume the R.V. took on that value.

143_0413_1

pw: neuron

$$p(a,b)\ p(c \mid a,b)\ \ p(d,e)$$

$$p(a,b,c,d,e) = p(a,b,c)\ \ p(d,e \mid a,b,c)$$

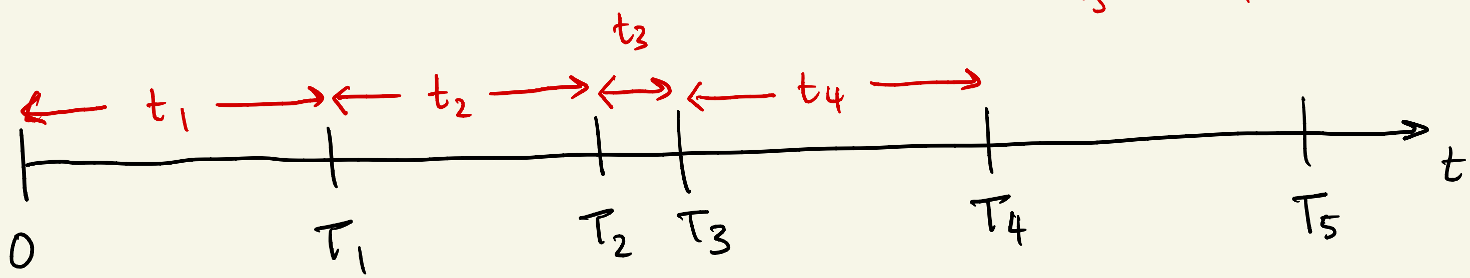
Poisson Processes

Model events that occur through time in a probabilistic framework

↓

spikes

$$T_1 = t_1$$
$$T_2 = t_1 + t_2$$
$$T_3 = t_1 + t_2 + t_3$$



"Interspike interval" (ISI)

$t_i \sim \exp(\lambda)$

t_i iid — "independent and identically distributed"

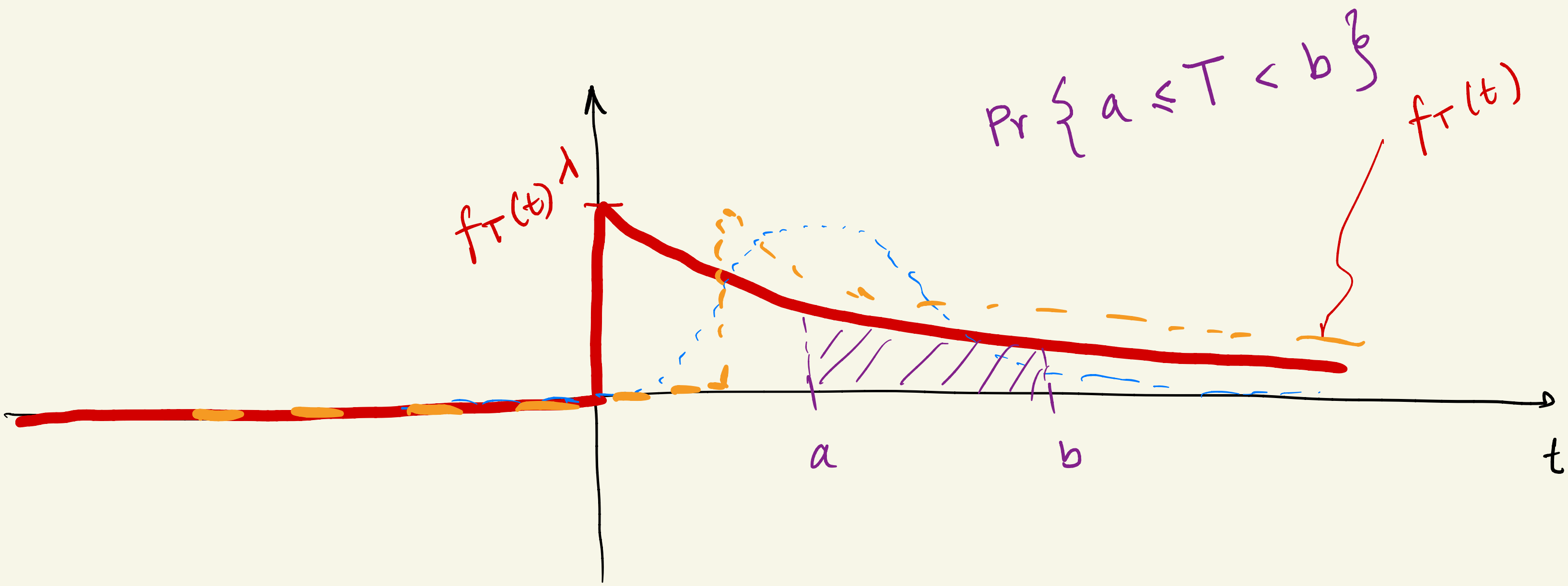
$$t_1 \perp\!\!\!\perp t_2 \perp\!\!\!\perp t_3$$

Exp. distribution

$[=]$ spikes/s

A R.V., T, is exp. distributed with rate $\lambda \geq 0$ if its probability density function (pdf) is:

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t}, & \text{if } t \geq 0 \\ 0, & \text{else} \end{cases}$$



T can also be described by its cdf

$$F_T(t) = \Pr\{T \leq t\}$$

$$= \int_0^t f_T(\tau)\, d\tau$$

$$= \begin{cases} 1 - e^{-\lambda t}, & \text{if } t \geq 0 \\ 0, & \text{else} \end{cases}$$

$$\Pr\{T > t\} = 1 - \Pr\{T \leq t\}$$

$$= \begin{cases} e^{-\lambda t}, & t \geq 0 \\ 1, & \text{else} \end{cases}$$

$$f_T(t) = \begin{cases} \lambda e^{-\lambda t}, & t \geq 0 \\ 0, & \text{else} \end{cases}$$

Mean, variance:

$$\mathbb{E}[T] = \int_{-\infty}^{\infty} t \, f_T(t) \, dt$$