

ECE C143A Homework 6

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Problem 1

(a)

False, the Na^+ channel opens first

(b)

False, only Na^+ serve to depolarize the cell.

(c)

True

(d)

False, EEG's cannot record action potentials.

(e)

False because λ does not vary with time and a poisson process is memoryless.

(f)

False, if the Fano factor is greater than one, the firing variance is greater than the firing mean

(g)

True

(h)

False, the Exponential distribution does not model the refractory period well.

(i)

False

(j)

True

(k)

True

(l)

True

(m)

False, a Gaussian kernel is a low pass.

(n)

False, such a tuning curve can describe the visual system well, but it cannot describe the motor system well.

(o)

False, Absolute not relative

Problem 2

(a)

$f(\theta)$ reaches a max at $\theta = \theta_0$ therefore this is the preferred direction.

(b)

No because the values of the tuning curve would all be negative

(c)

$$\begin{aligned}\cos(\theta - \theta_0) &= e^{j(\theta - \theta_0)} \\ &= (\cos(\theta) + j \sin(\theta))(\cos(\theta_0) - j \sin(\theta_0)) \\ &= \cos(\theta) \cos(\theta_0) + \sin(\theta) \sin(\theta_0)\end{aligned}$$

(d)

$$\begin{aligned}k_0 &= c_0 \\ k_1 &= c_1 \sin(\theta_0) \\ k_2 &= c_1 \cos(\theta_0)\end{aligned}$$

(e)

We have

$$\begin{aligned}y_0 &= 25 = k_0 + k_2 \\ y_{120} &= 70 = k_0 + \frac{k_1 \sqrt{3}}{2} - \frac{k_2}{2} \\ y_{240} &= 10 = k_0 - \frac{k_2}{2} - \frac{k_1 \sqrt{3}}{2}\end{aligned}$$

Therefore we have

$$\begin{aligned}y_{120} + y_{240} &= 2k_0 - k_2 \\ 2y_0 - y_{120} - y_{240} &= 2k_0 + 2k_2 - 2k_0 + k_2\end{aligned}$$

$$k_2 = \frac{2y_0 - y_{120} - y_{240}}{3}$$

$$k_0 = \frac{y_0 + y_{120} + y_{240}}{3}$$

$$k_1 = \frac{y_{120} - y_{240}}{\sqrt{3}}$$

(f) and (g)

Problem 2 Jupyter

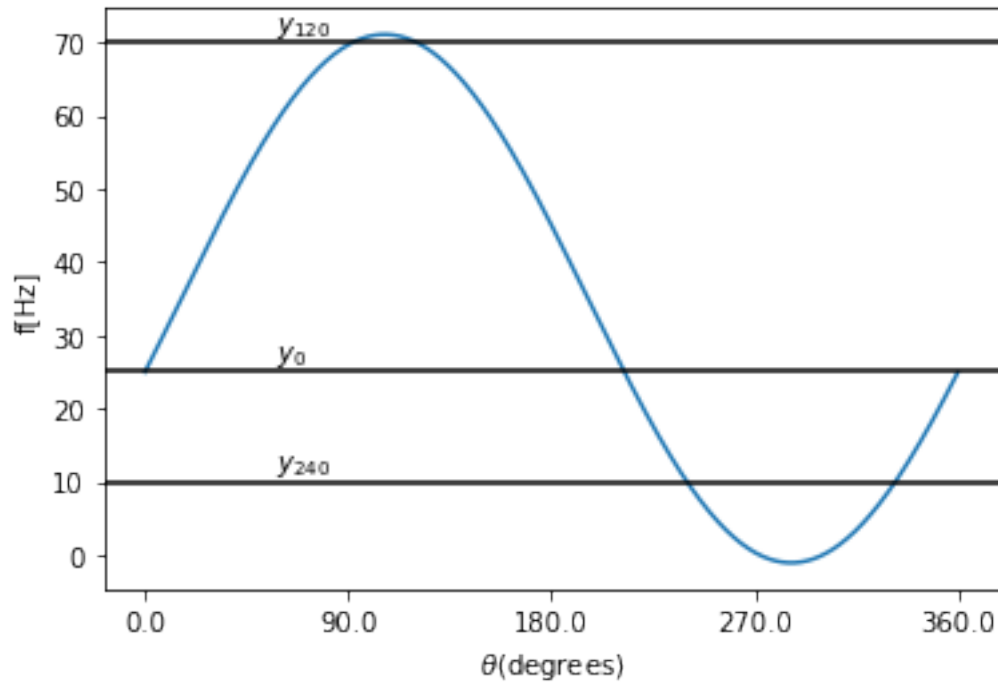
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```
[2]: import numpy as np
import matplotlib.pyplot as plt
```

1 Part (f)

```
[24]: y0=25
y120=70
y240=10
k0,k1,k2=np.dot(np.linalg.inv([[1,0,1],
                                [1,np.sqrt(3)/2,-1/2],
                                [1,-np.sqrt(3)/2,-1/2]]),[y0,y120,y240])
```

```
[25]: theta=np.arange(0,2*np.pi,0.01)
f=lambda theta: k0+k1*np.sin(theta)+k2*np.cos(theta)
plt.plot(theta,f(theta))
plt.axhline(y0,color="black")
plt.text(1,y0,"$y_{0}$",va="bottom")
plt.axhline(y120,color="black")
plt.text(1,y120,"$y_{120}$",va="bottom")
plt.axhline(y240,color="black")
plt.text(1,y240,"$y_{240}$",va="bottom")
plt.ylabel("f [Hz] ")
plt.xlabel(r"$\theta$(degrees)")
plt.xticks(np.linspace(0,2*np.pi,5),np.linspace(0,360,5))
plt.show()
```



```
[49]: c1=round(np.sqrt(k1**2+k2**2),3)
      c0=k0
      theta0=round(np.degrees(np.arctan2(k1,k2)),3)
      print(f"c0={c0}")
      print(f"c1={c1}")
      print(f"theta0={theta0} degrees")
```

```
c0=35.0
c1=36.056
theta0=106.102 degrees
```

2 Part (G)

```
[66]: theta=np.radians([0,60,120,180,240,300])
      y=[25,40,70,30,10,15]

      X=np.array([np.sin(theta),np.cos(theta)]).T
```

we can solve for the values of mean squared error by performing a linear regression over k_0 , k_1 , k_2

```
[67]: from sklearn.linear_model import LinearRegression

      reg = LinearRegression().fit(X, y)
      k0=reg.intercept_
```

```

k1,k2=reg.coef_
c1=round(np.sqrt(k1**2+k2**2),3)
c0=k0
theta0=round(np.degrees(np.arctan2(k1,k2)),3)
print(f"c0={c0}")
print(f"c1={c1}")
print(f"theta0={theta0} degrees")

```

```

c0=31.666666666666664
c1=25.221
theta0=103.373 degrees

```

```

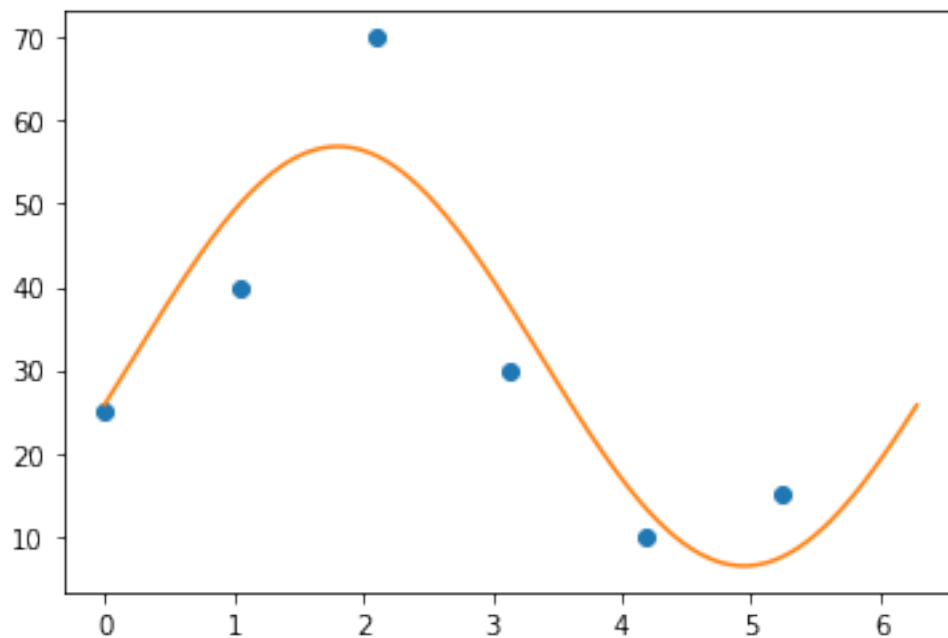
[68]: plt.plot(theta,y,"o")
      theta=np.arange(0,2*np.pi,0.01)
      f=lambda theta: k0+k1*np.sin(theta)+k2*np.cos(theta)
      plt.plot(theta,f(theta))

```

```

[68]: [<matplotlib.lines.Line2D at 0x7fdb0039a910>]

```



Problem 3

(a)

No, because the pdf has a maximum at $t = 0$ and it decreases exponentially. If it did then the pdf would be 0 for $t < t_1$ for some t_1

(b)

Therefore we have $\lambda = 50$ and the probability that a spike would violate the 1ms refractory period is

$$\int_0^{1 \cdot 10^{-3}} \lambda e^{-\lambda t} dt = 1 - e^{-0.05} = 0.048$$

So therefore we have the percentage spikes which would violate the 1ms refractory period is around 4.8%.

Problem 4

(a)

The ISI is an exponential distribution with parameter λ so therefore the mean is

$$\frac{1}{\lambda}$$

(b)

The probability that a given ISI is greater than the mean ISI is

$$\int_{\frac{1}{\lambda}}^{+\infty} \lambda e^{-\lambda t} dt = \boxed{e^{-1}}$$

(c)

from bayes theorem we have

$$\begin{aligned} Pr\left(T = t | T > \frac{1}{\lambda}\right) &= \frac{Pr(T > \frac{1}{\lambda} | T = t) f(T = t)}{Pr(T > \frac{1}{\lambda})} \\ E[T | T > \frac{1}{\lambda}] &= \int_0^{\infty} t Pr\left(T = t | T > \frac{1}{\lambda}\right) dt \\ &= \int_{\frac{1}{\lambda}}^{\infty} t e \lambda e^{-\lambda t} dt \\ &= \boxed{\frac{e}{\lambda}} \end{aligned}$$

(d)

we have

$$Pr\left(T = t | T < \frac{1}{\lambda}\right) = \frac{Pr(T < \frac{1}{\lambda} | T = t) f(T = t)}{Pr(T < \frac{1}{\lambda})}$$

therefore

$$\begin{aligned} E[T | < \frac{1}{\lambda}] &= \int_0^{\infty} t Pr\left(T = t | T < \frac{1}{\lambda}\right) dt \\ &= \int_0^{\frac{1}{\lambda}} t \frac{1}{1 - e^{-1}} \lambda e^{-\lambda t} dt \\ &= \boxed{\frac{1}{(1 - e^{-1})\lambda}} \end{aligned}$$

(e)

The probability for an ISI to be greater than the mean ISI is e^{-1} , therefore the number of spikes before one sees an ISI greater than the mean ISI is a geometric distribution with $p = e^{-1}$. The mean of this distribution is $\frac{1}{p} = e$ therefore the expected number of spikes that need to be fired before one sees an ISI less than the mean is $\boxed{e + 1}$, since one spike needs to be fired first to measure the ISI.

(f)

Let the expected waiting time be T_w we have

$$T_w = \sum_{n=1}^{\infty} p(n) \sum_{i=1}^n E(t_i)$$

Where t_i is the i th ISI time, and $p(n)$ is the probability that the n th waiting time will be the first greater than the mean. Thus $p(n)$ is a geometric distribution with $p = \frac{1}{e}$

$$E(T_w) = \sum_{n=1}^{\infty} p(n) \frac{n}{\lambda}$$

$$E(T_w) = \boxed{\frac{e}{\lambda}}$$

Problem 5

(a)

The probability that electrode 1's neuron does not spike in the first 60ms is

$$P_1 = \int_0^{\frac{3}{\lambda_1}} \lambda_1 e^{-\lambda_1 t} dt$$

$$P_1 = 1 - e^{-3}$$

And the probability that electrode 2's neuron does not spike in the first 60ms is

$$P_2 = \int_0^{\frac{2}{\lambda_2}} \lambda_2 e^{-\lambda_2 t} dt$$

$$P_2 = 1 - e^{-2}$$

Therefore the probability that neither of them are detected is

$$P_2 P_1 = (1 - e^{-2})(1 - e^{-3})$$

(b)

By the memoryless property, this is just

$$\int_0^t \lambda_1 e^{-\lambda_1 t} dt \int_0^t \lambda_2 e^{-\lambda_2 t} dt = (1 - e^{-\lambda_1 t})(1 - e^{-\lambda_2 t})$$

$$= \boxed{(1 - e^{-50t})(1 - e^{-33.333t})}$$

(c)

This is just the probability that neuron 1 fires before neuron 2, which is

$$\int_0^\infty \lambda_1 e^{-\lambda_1 t} \int_0^t \lambda_2 e^{-\lambda_2 \tau} d\tau dt = \int_0^\infty \lambda_1 e^{-\lambda_1 t} (1 - e^{-\lambda_2 t})$$

$$= 1 - \lambda_1 \int_0^\infty e^{-(\lambda_1 + \lambda_2)t}$$

$$= 1 - \lambda_1 \frac{1}{\lambda_1 + \lambda_2}$$

$$= \boxed{1 - \frac{50}{83.333}}$$