ECE C143A Homework 3

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May 17, 2022

Problem 1

(a)

$P(N=0) = 1 - 0.25 = \boxed{0.75}$

(b)

We want $P(N=0|R=0)$, we know that

$P(R=1|N=0) = P(E=0)P(R=1|E=0,N=0) + P(E=1)P(R=1|E=1,N=0) = 0.01$

$P(R=0|N=0) = 0.99$ and thus from bayes law we have

$$P(N=0|R=0) = P(R=0|N=0)\frac{P(N=0)}{P(R=0)}$$
$$= 0.9\frac{0.1 \cdot 0.25}{P(R=0)}$$

To find $P(R=0)$ we must find $P(R=1)$,

$$\begin{aligned}P(R=1) = P(E=1)P(N=1)P(R=1|E=1,N=1) \\ + P(E=1)P(N=0)P(R=1|E=1,N=0) \\ + P(E=0)P(N=1)P(R=1|E=0,N=1) \\ + P(E=0)P(N=0)P(R=1|E=0,N=0)\end{aligned}$$

$$P(R=1) = 0.9 \cdot 0.25 \cdot 1 + 0.1 \cdot 0.25 \cdot 0.1 + 0.1 \cdot 0.75 \cdot 0.1 = 0.235$$

Therefore $P(R=0) = 1 - P(R=1) = 0.765$, thus

$$P(N=0|R=0) = 0.99\frac{0.25}{P(R=0)} = \boxed{0.3235}$$

(c)

We have

$$\begin{aligned}P(E=0,N=0|R=0) &= \frac{P(E=0,N=0,R=0)}{P(R=0)} \\ &= \frac{P(R=0|E=0,N=0)P(E=0)P(N=0)}{P(R=0)} \\ &= \frac{(1-P(R=1|E=0,N=0))P(E=0)P(N=0)}{P(R=0)} \\ &= \frac{0.9 \cdot 0.1 \cdot 0.75}{0.765} \\ &= \boxed{0.088}\end{aligned}$$

This intuitively makes sense because we need two conidtions to occur, equipment broken and neuron spike not occuring.

(d)

Let us consider the case $E=1$, $N=0$ given $R=1$ we have

$$P(E=1,N=0|R=1) = \frac{P(R=1|E=1,N=0)P(E=1)P(N=0)}{P(R=1)}$$

Since $P(R=1|E=1,N=0)=0$, we thus have $P(E=1,N=0|R=1)=0$. However

$$\begin{aligned}P(E=1|R=1) &= P(R=1|E=1)\frac{P(E=1)}{P(R=1)}\\ &= (P(R=1|E=1,N=0)P(N=0)+\\ &P(R=1|E=1,N=1)P(N=1))\frac{P(E=1)}{P(R=1)}\end{aligned}$$

This therefore $P(E=1|R=1)>1$, likewise

$$\begin{aligned}P(N=0|R=1) &= P(R=1|N=0)\frac{P(N=0)}{P(R=1)}\\ &= (P(R=1|E=1,N=0)P(E=1)+\\ &P(R=1|E=0,N=0)P(E=0))\frac{P(N=0)}{P(R=1)}\end{aligned}$$

This therefore $P(E=1|R=1)>1$, therfore, $P(E=1|R=1)P(N=0|R=1)>0$ and is not equal to $P(E=1,N=0|R=1)$ therefore they are conditionally dependent. Ie they are not independent given R