

**ECE C143A/C243A, Spring 2022**

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**Homework #2**

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Due Monday, 25 April 2022, uploaded to Gradescope.

Covers material up to Poisson Processes I.

100 points total.

1. (15 pts) True / False. Determine if the following statements are true or false. If a statement is false, please correct the statement to receive full credit. Note that we use “spike” and “action potential” interchangeably. Each statement is worth 1 point.
  - (a) When an action potential is fired, there is first a larger current attributable to  $K^+$  channels opening before  $Na^+$  channels.
  - (b) During an action potential,  $Na^+$  currents and  $K^+$  currents both serve to depolarize the cell.
  - (c) The patch-clamp allows experimenters to measure the current flowing through a single ion channel.
  - (d) It is possible to record action potentials with electroencephalograms (EEG).
  - (e) Imagine a neuron perfectly modeled by a Poisson process with a homogeneous firing rate of 1 spike per second. Consider two scenarios. In scenario (a), the last spike occurred 100 ms ago. In scenario (b), the last spike occurred 1.2 s ago. It is more likely that in the next 100 ms, a spike will fire in scenario (b) than it will in scenario (a).
  - (f) If the Fano factor of a neuron is greater than 1, then its firing rate mean is greater than its firing rate variance.
  - (g) A Poisson process will *always* have a Fano factor of 1.
  - (h) An exponential interspike interval distribution models the refractory period well.
  - (i) Chronic, multi-site electrode arrays allow the measurement of action potentials from several neurons at millisecond resolution.
  - (j) There is stimulation electrode technology that can remain functional and effective in humans for years.
  - (k) Using single-electrode technology, it is possible to record spikes from different neurons at the same time.
  - (l) To accurately and reliably detect action potentials from a single neuron using a static threshold, a high-pass filter, which removes DC and low-frequency components of raw electrode voltage waveforms, is required.
  - (m) Convolution of spike trains with a Gaussian kernel to approximate a spike rate,  $r(t)$ , is a type of high-pass filtering.
  - (n) Tuning curves describe neural activity in the visual and motor systems well.
  - (o) During the relative refractory period, it is impossible for a spike to be generated.

2. (30 points) Tuning curves. One way to model the firing rate of motor cortical neurons is with tuning curves. The tuning curve models the average firing rate,  $f(\theta)$ , for a neuron when a reach is made in the direction  $\theta$ . The cosine tuning model asserts that:

$$f(\theta) = c_0 + c_1 \cos(\theta - \theta_0)$$

In this question, we will learn how to derive values for the parameters  $c_0$ ,  $c_1$ , and  $\theta_0$  given that we know the average firing rates for reaches in certain directions.

- (a) (1 point) Show that  $\theta_0$  is the *preferred direction* of the neuron. The preferred direction is the direction for which the neuron fires most.
- (b) (3 points) Your colleague writes a script to find the values of  $c_0$ ,  $c_1$ , and  $\theta_0$  given some neural data. He runs it and finds that  $c_0 = -11$ ,  $c_1 = 8$ , and  $\theta_0 = 125^\circ$ . Do you tell him “Great job! This is a completely reasonable model!” or do you tell him “You’ve made a mistake.” Why?
- (c) (3 points) You decide to take a stab at writing a script that will find the parameters of the tuning model. However, you’re going to go about it a different way than your colleagues. You’re first going to simplify the term  $\cos(\theta - \theta_0)$ . By using Euler’s formula,  $e^{j\theta} = \cos(\theta) + j \sin(\theta)$ , derive the formula:

$$\cos(\theta - \theta_0) = \cos(\theta) \cos(\theta_0) + \sin(\theta) \sin(\theta_0).$$

- (d) (3 points) To simplify our parameter estimation, we will re-write  $f(\theta)$  as

$$f(\theta) = k_0 + k_1 \sin(\theta) + k_2 \cos(\theta).$$

Find  $k_0$ ,  $k_1$ , and  $k_2$  in terms of  $c_0$ ,  $c_1$ , and  $\theta_0$ .

- (e) (5 points) We define  $y_\theta$  to be the measured average firing rates for a reach in direction  $\theta$ . For simplicity in this question, assume that we’ve only measured the firing rate when the monkey reaches to three unique directions:  $y_0$  for the reach at an angle of  $0^\circ$  (i.e., to the right),  $y_{120}$  for the reach to  $120^\circ$  (up and to the left) and  $y_{240}$  for the reach to  $240^\circ$  (down and to the left). Find  $k_0$ ,  $k_1$ , and  $k_2$  in terms of  $y_0$ ,  $y_{120}$  and  $y_{240}$ .
- (f) (5 points) Plot the tuning curve,  $f(\theta)$ , when  $y_0 = 25$ ,  $y_{120} = 70$ , and  $y_{240} = 10$ . Also include  $y_0$ ,  $y_{120}$ , and  $y_{240}$  on the plot. Finally, provide the values of  $c_0$ ,  $c_1$ , and  $\theta_0$ . You will use python to solve this problem, please download the jupyter notebook from BruinLearn. If you are new to python/jupyter notebook, please refer to instructions on BruinLearn. Please print the Jupyter notebook and submit it to Gradescope as well.
- (g) (10 points) Now consider that we sampled the workspace much more effectively. We now have the following data:

$$y_0 = 25, y_{60} = 40, y_{120} = 70, y_{180} = 30, y_{240} = 10 \text{ and } y_{300} = 15.$$

Report the values of  $c_0$ ,  $c_1$ , and  $\theta_0$  that minimize the mean-square error between the tuning curve and the observed data.

3. (5 points) Refractory periods

- (a) (2 points) In class, we introduced the exponential distribution to model inter-spike intervals (ISI). Does the exponential distribution incorporate the concept of refractory periods? Please explain.

- (b) (3 points) If a model neuron spikes at 50 spikes per second according to a homogeneous Poisson process, what percentage of spikes would violate a 1 ms refractory period?
4. (34 points) A neuron spikes according to a homogeneous Poisson process with rate  $\lambda$ .
- (a) (2 points) What is the mean ISI of this neuron?
- (b) (4 points) What is the probability that a given ISI is greater than the mean ISI?
- (c) (7 points) What is the expected ISI *given* that it is larger than the mean ISI?
- Hint:** You may find it useful to calculate

$$\Pr(T = t | T > 1/\lambda)$$

with Bayes' theorem, and subsequently calculate the (conditional) expectation:

$$\mathbb{E}[T | T > 1/\lambda] = \int_0^\infty t \Pr(T = t | T > 1/\lambda) dt$$

- (d) (7 points) What is the expected ISI *given* that it is smaller than the mean ISI?
- (e) (7 points) What is the expected number of spikes that will be fired before one sees an ISI greater than the mean ISI?
- (f) (7 points) What is the expected waiting time until (and including) an ISI greater than the mean ISI?
5. (16 points) You insert a pair of electrodes into the brain. Unbeknownst to you, electrode 1 sits next to a neuron with mean ISI of 20 ms, and electrode 2 sits next to a different neuron with mean ISI of 30 ms. Each neuron spikes independently according to a homogeneous Poisson process. A neuron is “detected” when it fires its first spike.
- (a) (4 points) What is the probability that no neurons are detected (on either electrode) during the first 60 ms?
- (b) (4 points) Given that no neurons are detected in the first  $s$  seconds, what is the probability that no neurons are detected in the first  $s + t$  seconds?
- (c) (8 points) A single spike comes in, and thus a neuron is detected. What is the probability that a neuron is detected on electrode 1 before electrode 2? (Hint: your answer should not be a function of time.)