

ECE C143A Homework 3

Lawrence Liu

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Problem 1

(a)

$$\begin{aligned} P(M(s) = m) &= \sum_{n=m}^{\infty} \binom{n}{m} (1-p)^m p^{n-m} \frac{(\lambda s)^n}{n!} e^{-\lambda s} \\ &= \sum_{n=m}^{\infty} \frac{n!}{m!(n-m)!} (1-p)^m p^{n-m} \frac{(\lambda s)^n}{n!} e^{-\lambda s} \\ &= e^{-\lambda s} \frac{(1-p)^m}{m!} \sum_{n=m}^{\infty} \frac{p^{n-m}}{(n-m)!} (\lambda s)^n \\ &= e^{-\lambda s} \frac{(1-p)^m}{m!} (\lambda s)^m \sum_{n=m}^{\infty} \frac{p^{n-m}}{(n-m)!} (\lambda s)^{n-m} \\ &= e^{-\lambda s} \frac{(1-p)^m}{m!} (\lambda s)^m \sum_{i=0}^{\infty} \frac{p^i}{(i)!} (\lambda s)^i \\ &= e^{-\lambda s} \frac{(\lambda(1-p)s)^m}{m!} e^{p\lambda s} \\ &= \frac{(\lambda(1-p)s)^m}{m!} e^{-\lambda(1-p)s} \end{aligned}$$

Therefore this the distribution of M is Poisson($(1-p)\lambda s$).