

# ECE C143A Homework 4

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## Problem 1

(a)

$$P(N = 0) = 1 - 0.25 = \boxed{0.75}$$

(b)

We want  $P(N = 0|R = 0)$ , we know that

$$P(R = 1|N = 0) = P(E = 0)P(R = 1|E = 0, N = 0) + P(E = 1)P(R = 1|E = 1, N = 0) = 0.01$$

$P(R = 0|N = 0) = 0.99$  and thus from bayes law we have

$$\begin{aligned} P(N = 0|R = 0) &= P(R = 0|N = 0) \frac{P(N = 0)}{P(R = 0)} \\ &= 0.9 \frac{0.1 \cdot 0.75}{P(R = 0)} \end{aligned}$$

To find  $P(R = 0)$  we must find  $P(R = 1)$ ,

$$\begin{aligned} P(R = 1) &= P(E = 1)P(N = 1)P(R = 1|E = 1, N = 1) \\ &\quad + P(E = 1)P(N = 0)P(R = 1|E = 1, N = 0) \\ &\quad + P(E = 0)P(N = 1)P(R = 1|E = 0, N = 1) \\ &\quad + P(E = 0)P(N = 0)P(R = 1|E = 0, N = 0) \end{aligned}$$

$$P(R = 1) = 0.9 \cdot 0.25 \cdot 1 + 0.1 \cdot 0.75 \cdot 0.1 + 0.1 \cdot 0.25 \cdot 0.1 = 0.235$$

Therefore  $P(R = 0) = 1 - P(R = 1) = 0.765$ , thus

$$P(N = 0|R = 0) = 0.99 \frac{0.75}{P(R = 0)} = \boxed{0.97}$$

(c)

We have

$$\begin{aligned} P(E = 0, N = 0|R = 0) &= \frac{P(E = 0, N = 0, R = 0)}{P(R = 0)} \\ &= \frac{P(R = 0|E = 0, N = 0)P(E = 0)P(N = 0)}{P(R = 0)} \\ &= \frac{(1 - P(R = 1|E = 0, N = 0))P(E = 0)P(N = 0)}{P(R = 0)} \\ &= \frac{0.9 \cdot 0.1 \cdot 0.75}{0.765} \\ &= \boxed{0.088} \end{aligned}$$

This intuitively makes sense because we need two conditions to occur, equipment broken and neuron spike not occurring.

(d)

Let us consider the case  $E = 1, N = 0$  given  $R = 1$  we have

$$P(E = 1, N = 0|R = 1) = \frac{P(R = 1|E = 1, N = 0)P(E = 1)P(N = 0)}{P(R = 1)}$$

Since  $P(R = 1|E = 1, N = 0) = 0$ , we thus have  $P(E = 1, N = 0|R = 1) = 0$ . However

$$\begin{aligned} P(E = 1|R = 1) &= P(R = 1|E = 1) \frac{P(E = 1)}{P(R = 1)} \\ &= (P(R = 1|E = 1, N = 0)P(N = 0) + \\ &\quad P(R = 1|E = 1, N = 1)P(N = 1)) \frac{P(E = 1)}{P(R = 1)} \end{aligned}$$

This therefore  $P(E = 1|R = 1) > 1$ , likewise

$$\begin{aligned} P(N = 0|R = 1) &= P(R = 1|N = 0) \frac{P(N = 0)}{P(R = 1)} \\ &= (P(R = 1|E = 1, N = 0)P(E = 1) + \\ &\quad P(R = 1|E = 0, N = 0)P(E = 0)) \frac{P(N = 0)}{P(R = 1)} \end{aligned}$$

This therefore  $P(E = 1|R = 1) > 1$ , therefore,  $P(E = 1|R = 1)P(N = 0|R = 1) > 0$  and is not equal to  $P(E = 1, N = 0|R = 1)$  therefore they are conditionally dependent. Ie they are not independent given R

## Problem 2

(a)

$$\boxed{P(a, b, c, d) = P(c)P(a|c)P(d)P(b|a, d)}$$

(b)

$$\begin{aligned} P(C, D) &= \sum_A \sum_B P(C, D, A, B) \\ &= \sum_A \sum_B P(C)P(D)P(A|C)P(B|A, D) \\ &= P(C)P(D) \sum_A \sum_B P(A|C)P(B|A, D) \\ &= P(C)P(D) \sum_A P(A|C) \sum_B P(B|A) \\ &= P(C)P(D) \end{aligned}$$

Therefore  $C$  and  $D$  are independent.

(c)

$$\begin{aligned} P(C, D|A, B) &= \frac{P(C, D, A, B)}{P(A, B)} \\ &= \frac{P(C)P(D)P(A|C)P(B|A, D)}{P(A, B)} \\ &= \frac{P(D)P(C|A)P(A)P(B|A, D)}{P(A, B)} \\ &= \frac{P(D)P(C|A)P(D|B, A)}{P(D|A)} \\ &= \frac{P(D)}{P(D|A)} P(C|A)P(D|B, A) \\ &= \frac{P(C|A)P(D|B, A)}{P(D, A)} \end{aligned}$$

Therefore  $C$  and  $D$  are not independent given  $A$  and  $B$ .

(d)

$$\begin{aligned}P(a, d) &= \sum_B P(a, d, b) \\&= \sum_a P(d)P(b)P(b|d, a) \\&= P(d)P(b) \sum_B P(b|d, a) \\&= P(d)P(b)\end{aligned}$$

Therefore  $a$  and  $d$  are independent

(e)

$$\begin{aligned}P(a, d|b) &= \frac{P(a, d, b)}{P(b)} \\&= \frac{P(a)P(d)P(b|a, d)}{P(b)} \\&= \frac{P(a)P(d)}{P(b)} \frac{P(b|d)P(a|b, d)}{P(a|d)} \\&= boxed{P(a|b, d)P(d|b)}\end{aligned}$$

Therefore  $a$  and  $d$  are not independent given  $b$

(f)

$$\begin{aligned}P(c, b) &= \sum_A P(c, b, a) \\&= \sum_A P(c)P(a|c)P(b|a) \\&= P(c) \sum_A P(a|c)P(b|a)\end{aligned}$$

Since  $\sum_A P(a|c)P(b|a) \neq P(b)$  in general,  $c$  and  $b$  are not independent.

(g)

$$\begin{aligned} P(c, b|a) &= \frac{P(c, b, a)}{a} \\ &= \frac{P(c)P(a|c)P(b|a)}{P(a)} \\ &= P(c|a)P(b|a) \end{aligned}$$

Therefore  $c$  and  $b$  are independent given  $a$ .