ECE C143A Homework 3

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April 29, 2022

Problem 1

(a)

$$
\begin{aligned}
P(M(s)=m) &= \sum_{n=m}^{\infty} \binom{n}{m}(1-p)^m p^{n-m}\frac{(\lambda s)^n}{n!}e^{-\lambda s}\\
&= \sum_{n=m}^{\infty} \frac{n!}{m!(n-m)!}(1-p)^m p^{n-m}\frac{(\lambda s)^n}{n!}e^{-\lambda s}\\
&= e^{-\lambda s}\frac{(1-p)^m}{m!}\sum_{n=m}^{\infty}\frac{p^{n-m}}{(n-m)!}(\lambda s)^n\\
&= e^{-\lambda s}\frac{(1-p)^m}{m!}(\lambda s)^m\sum_{n=m}^{\infty}\frac{p^{n-m}}{(n-m)!}(\lambda s)^{n-m}\\
&= e^{-\lambda s}\frac{(1-p)^m}{m!}(\lambda s)^m\sum_{i=0}^{\infty}\frac{p^{i}}{(i)!}(\lambda s)^{i}\\
&= e^{-\lambda s}\frac{(\lambda(1-p)s)^m}{m!}e^{p\lambda s}\\
&= \frac{(\lambda(1-p)s)^m}{m!}e^{-\lambda(1-p)s}
\end{aligned}
$$

Therefore this the distribution of M is Poisson($(1-p)\lambda s$).

(b)

The rate is

$$(1-p)\lambda$$

(c)

We have that the probability of d drops over a time period of τ is

$$\begin{aligned}
P(D(\tau)=d) &= \sum_{n=d}^{\infty}\binom{n}{d}p^d(1-p)^{n-d}\frac{(\lambda\tau)^n}{n!}e^{-\lambda\tau} \\
&= \frac{1}{d!}p^d(\lambda\tau)^d e^{-\lambda\tau}\sum_{n=d}^{\infty}(1-p)^{n-d}(\lambda\tau)^{n-d}e^{-\lambda\tau} \\
&= \frac{1}{d!}p^d(\lambda\tau)^d e^{-p\lambda\tau}
\end{aligned}$$

Therefore the distribution of the number of drops over a time period τ is a Poisson distribution with a rate of $\lambda p\tau$