ECE C143A Homework 3

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Problem 1

(a)

$$P(M(s) = m) = \sum_{n=m}^{\infty} \binom{n}{m} (1-p)^m p^{n-m} \frac{(\lambda s)^n}{n!} e^{-\lambda s}$$

$$= \sum_{n=m}^{\infty} \frac{n!}{m!(n-m)!} (1-p)^m p^{n-m} \frac{(\lambda s)^n}{n!} e^{-\lambda s}$$

$$= e^{-\lambda s} \frac{(1-p)^m}{m!} \sum_{n=m}^{\infty} \frac{p^{n-m}}{(n-m)!} (\lambda s)^n$$

$$= e^{-\lambda s} \frac{(1-p)^m}{m!} (\lambda s)^m \sum_{n=m}^{\infty} \frac{p^{n-m}}{(n-m)!} (\lambda s)^{n-m}$$

$$= e^{-\lambda s} \frac{(1-p)^m}{m!} (\lambda s)^m \sum_{i=0}^{\infty} \frac{p^i}{(i)!} (\lambda s)^i$$

$$= e^{-\lambda s} \frac{(\lambda(1-p)s)^m}{m!} e^{p\lambda s}$$

$$= \frac{(\lambda(1-p)s)^m}{m!} e^{-\lambda(1-p)s}$$

Therefore this the distribution of M is Poisson$((1-p)\lambda s)$.

(b)

The rate is

$$(1 - p)\lambda$$

(c)

We have that the probability of d drops over a time period of τ is

$$
\begin{aligned}
P(D(\tau) = d) &= \sum_{n=d}^{\infty} \binom{n}{d} p^d (1-p)^{n-d} \frac{(\lambda\tau)^n}{n!} e^{-\lambda\tau} \\
&= \frac{1}{d!} p^d (\lambda\tau)^d \sum_{n=d}^{\infty} (1-p)^{n-d} (\lambda\tau)^{n-d} e^{-\lambda\tau} \\
&= \frac{1}{d!} p^d (\lambda\tau)^d e^{-p\lambda\tau}
\end{aligned}
$$

Therefore the distribution of the number of drops over a time period τ is a Poisson distribution with a rate of $\lambda p \tau$