ECE M16 Homework 1

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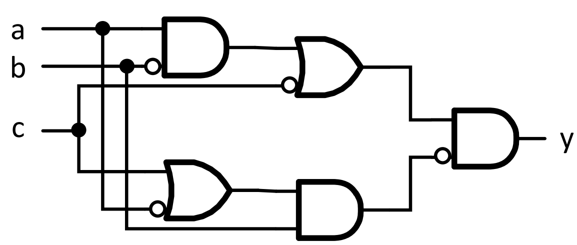
Sections covered: Digital representation of logic systems, binary numbers, Boolean algebra, K-maps, logic gates.

Total of 7 questions, 14 points each. Due: 11:59PM Friday of week 2.

For all Logisim question, you must use version logisim-evolution-2.14.8.4-cornell.jar which you can download from:
http://www.cs.cornell.edu/courses/cs3410/2019sp/logisim/logisim-evolution.jar.
Submit a copy of your Logisim schematic, your design process, and any results.

1. Conceive a way of encoding English alphabets into an n-bit binary signal so that, (i) representations for adjacent letters differ in only one bit, (ii) n is as small as possible.
2. For following logic circuit,
   a. Express the function computed in full disjunctive normal form as a Boolean expression in terms of a, b, and c.
   b. Simplify the expressions using Boolean algebra techniques.



3. Using Boolean algebra first postulates, prove:
   a. $a + a = a$ and $a.a = a$
   b. $\bar{1} = 0$
   c. $\bar{a}$ is unique for a given a (use contradiction).
4. Consider a combinational logic gate that implements the ternary operator ? : as follows: The system takes three 1-bit inputs p, q, and r; outputs a 1-bit value f; and, implements the following logic: "if p is logic-1, the system outputs on f the logic value of q, and otherwise it outputs the logic value of r.
   a. Write a truth table for the ? : operator.
   b. Write the entries of the Karnaugh map of the ternary operator.

c. Identify and write the Boolean expressions for the prime implicants of the operator.
d. Identify which of the prime implicants are essential, and using that find a minimal cover of the function in sum-of-products form, and write its Boolean expression.

5. Using perfect induction (a method of proving Boolean theorems by substituting all possible values of the variables), prove De Morgan's theorem with three variables. Specifically:

$$\overline{x + y + z} = \bar{x}.\bar{y}.\bar{z} \qquad \& \qquad \overline{x.y.z} = \bar{x} + \bar{y} + \bar{z}$$

6. Reduce the following Boolean expressions to the minimum number of literals:

$$(y.\bar{z} + \bar{x}.w).(x.\bar{y} + z.\bar{w})$$
$$(x.y) + (x.(w.z + w.\bar{z}))$$

7. In this problem you have to design a circuit that will tell whether a given month has 31 days in it. The month is specified by a 4-bit input $M[3:0]$ such that 0001 is January, 0010 is February, . . . 1100 is December. The circuit output y should be 1 only when the month specified at the inputs has 31 days in it.
   a. Write the truth table of the function.
   b. With the help of the Karnaugh map, find and write a simplified expression for the function where you minimize cost.
   c. Using the prescribed version of Logisim, draw a logic gate diagram using only NOT and NOR gates. Remember, you should seek to minimize the overall cost of the circuit where the cost of a NOR gate equals the number of inputs, while the cost of NOT is 1.