ECE M16 Homework 1

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Problem 1

Since there are 26 letters in the English Alphabet, we would need $\lceil \log_2(26) \rceil = 5$ bits to represent this signal. Therefore we could create a way of encoding the English Alphabet as 5 bits with each letter being encoded as 1+ the encoding of the previous letter, for instance A=00000 and B=00001, etc.

Problem 2

(a)

The equation for the circuit is

$$f(a,b,c) = ((a \vee \bar{b}) \wedge \bar{c}) \vee \overline{((c \wedge \bar{a}) \vee b)}$$

Expanding it we get

$$\begin{split} f(a,b,c) &= ((a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c})) \vee \overline{((c \wedge \bar{a}) \vee b)} \\ &= ((a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c})) \vee \overline{((c \vee b) \wedge (\bar{a} \vee b))} \\ &= ((a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c})) \vee (\overline{(c \vee b)} \vee \overline{(\bar{a} \vee b)}) \\ &= (a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c}) \vee (\bar{c} \wedge \bar{b}) \vee (a \wedge \bar{b}) \\ &= \overline{(a \wedge \bar{c}) \vee (\bar{c} \wedge \bar{b}) \vee (a \wedge \bar{b})} \end{split}$$

(b)

$$\boxed{(a \wedge \bar{c}) \vee (\bar{c} \wedge \bar{b}) \vee (a \wedge \bar{b})}$$

Problem 3

(a)

we have $a.\bar{a} = 0$, therefore we have

$$a + 0 = a$$

$$a + (a.\bar{a}) = a$$

$$(a + a).(a + \bar{a}) = a$$

$$(a + a).1 = a$$

$$a + a = a$$

Likewise we have

$$a.1 = a$$

$$a.(a + \bar{a}) = a$$

$$a.a + a.\bar{a} = a$$

$$a.a + 0 = a$$

$$a.a = a$$

(b)

From the Boolean Algebra postulates we have:

$$1.\bar{1} = 0$$

Therefore we must have that $\bar{1} = 0$

(c)

Let us consider the case where \bar{a} was not unique, ie for $a_1 \neq a_2$, we have $\bar{a}_1 = \bar{a}_2 = \bar{a}$. Since $\bar{a}.(a_1 + a_2) = 0$ and $\bar{a} + (a_1.a_2) = 1$, we have that

$$a_1 + (\bar{a}.(a_1 + a_2)) = a_1$$
$$(a_1 + \bar{a}).(a_1 + a_1 + a_2) = a_1$$
$$(a_1 + a_2) = a_1$$

And that

$$a_1.(\bar{a} + (a_1.a_2)) = a_1$$

 $(a_1.\bar{a}) + (a_1.a_1.a_2) = a_1$
 $(a_1.a_2) = a_1$

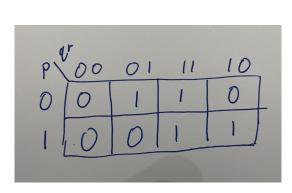
Therefore $a_1=a_2,$ and thus \bar{a} must be unique

Problem 4

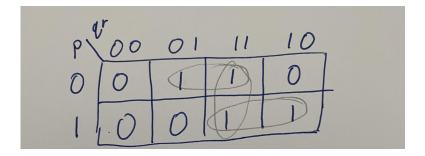
(a)

p	q	r	f
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

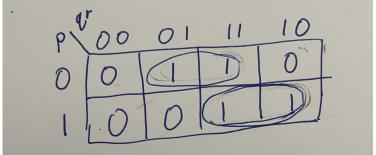
(b)



(c)



(d)



Therefore the boolean

expression for the function is

$$f(p,q,r) = \boxed{(\bar{p} \wedge r) \vee (p \wedge q)}$$

Problem 5

x	у	z	$\overline{x+y+z}$	$\overline{x}.\overline{y}.\overline{z}$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

x	у	z	$\overline{x.y.z}$	$\overline{x} + \overline{y} + \overline{z}$
1	1	1	0	0
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	1

Problem 6

since $x.\bar{x} = y.\bar{y} = w.\bar{w} = z.\bar{z} = 0$

$$(x.y) + (x.(w.z + w.\bar{z})) = (x.y) + (x.w)$$