ECE M16 Homework 1

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Problem 1

Since there are 26 letters in the English Alphabet, we would need $\lceil \log_2(26) \rceil = 5$ bits to represent this signal. Therefore we could create a way of encoding the English Alphabet as 5 bits with each letter being encoded as 1+ the encoding of the previous letter, for instance A=00000 and B=00001, etc.

Problem 2

(a)

The equation for the circuit is

$$f(a, b, c) = ((a \vee \bar{b}) \wedge \bar{c}) \vee \overline{((c \wedge \bar{a}) \vee b)}$$

Expanding it we get

$$\begin{aligned}
f(a,b,c) &= ((a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c})) \vee \overline{((c \wedge \bar{a}) \vee b)} \\
&= ((a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c})) \vee \overline{((c \vee b) \wedge (\bar{a} \vee b))} \\
&= ((a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c})) \vee (\overline{(c \vee b)} \vee \overline{(\bar{a} \vee b)}) \\
&= (a \wedge \bar{c}) \vee (\bar{b} \wedge \bar{c}) \vee (\bar{c} \wedge \bar{b}) \vee (a \wedge \bar{b}) \\
&= \boxed{(a \wedge \bar{c}) \vee (\bar{c} \wedge \bar{b}) \vee (a \wedge \bar{b})}
\end{aligned}$$

(b)

$$\boxed{(a \wedge \bar{c}) \vee (\bar{c} \wedge \bar{b}) \vee (a \wedge \bar{b})}$$

Problem 3

(a)

we have $a.\bar{a} = 0$, therefore we have

$$\begin{aligned}
a + 0 &= a \\
a + (a.\bar{a}) &= a \\
(a + a).(a + \bar{a}) &= a \\
(a + a).1 &= a \\
a + a &= a
\end{aligned}$$

Likewise we have

$$\begin{aligned}
a.1 &= a \\
a.(a + \bar{a}) &= a \\
a.a + a.\bar{a} &= a \\
a.a + 0 &= a \\
a.a &= a
\end{aligned}$$

(b)

From the Boolean Algebra postulates we have:

$$1.\bar{1} = 0$$

Therefore we must have that $\bar{1} = 0$

(c)

Let us consider the case where $\bar{a}$ was not unique, ie for $a_1 \neq a_2$, we have $\bar{a_1} = \bar{a_2} = \bar{a}$. Since $\bar{a}.(a_1 + a_2) = 0$ and $\bar{a} + (a_1.a_2) = 1$, we have that

$$\begin{aligned} a_1 + (\bar{a}.(a_1 + a_2)) &= a_1 \\ (a_1 + \bar{a}).(a_1 + a_1 + a_2) &= a_1 \\ (a_1 + a_2) &= a_1 \end{aligned}$$

And that

$$\begin{aligned} a_1.(\bar{a} + (a_1.a_2)) &= a_1 \\ (a_1.\bar{a}) + (a_1.a_1.a_2) &= a_1 \\ (a_1.a_2) &= a_1 \end{aligned}$$

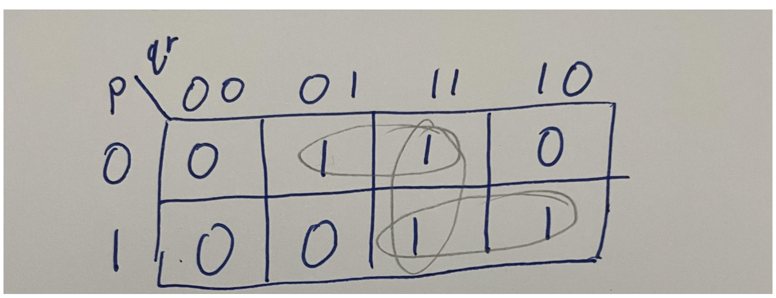
Therefore $a_1 = a_2$, and thus $\bar{a}$ must be unique

Problem 4

(a)

p	q	r	f
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

(c)



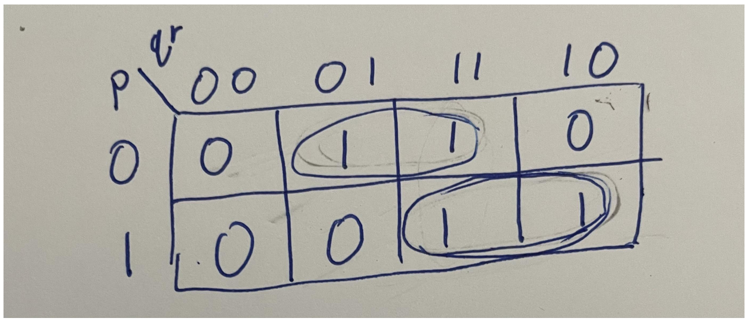
Therefore the prime implicants are

$$\boxed{\bar{p}.r}$$

$$\boxed{q.r}$$

$$\boxed{p.q}$$

(d)



The essential prime implicants are

$$\bar{p}.r$$

and

$$p.q$$

Therefore the boolean expression for the function is

$$f(p,q,r) = \boxed{(\bar{p} \wedge r) \vee (p \wedge q) = (\bar{p}.r) + (p.q)}$$

Problem 5

x	y	z	$\overline{x+y+z}$	$\bar{x}.\bar{y}.\bar{z}$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

x	y	z	$\overline{x.y.z}$	$\overline{x}+\bar{y}+\bar{z}$
1	1	1	0	0
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	1

Problem 6

$$\begin{aligned}(y.\bar{z}+\bar{x}.w).(x.\bar{y}+z.\bar{w}) &= y.\bar{z}.(x.\bar{y}+z.\bar{w})+\bar{x}.w.(x.\bar{y}+z.\bar{w}) \\ &= \bar{z}.x.y.\bar{y}+y.\bar{w}.z.\bar{z}+w.\bar{y}.\bar{x}.x+z.\bar{x}.w.\bar{w} \quad = \boxed{0}\end{aligned}$$

since $x.\bar{x}=y.\bar{y}=w.\bar{w}=z.\bar{z}=0$

$$(x.y)+(x.(w.z+w.\bar{z})) = \boxed{(x.y)+(x.w)}$$