ECE M16 Homework 1

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Problem 1

Since there are 26 letters in the English Alphabet, we would need $\lceil \log_2(26) \rceil = 5$ bits to represent this signal. Therefore we could create a way of encoding the English Alphabet as 5 bits with gray encoding. ie we would have the following table

a	00000	n	01011
b	00001	o	01001
c	00011	p	01000
d	00010	q	11000
e	00110	r	11001
f	00111	s	11011
g	00101	t	11010
h	00100	u	11110
i	01100	v	11111
j	01101	w	11101
k	01111	x	11100
l	01110	y	10100
m	01010	z	10101

Problem 2

(a)

The equation for the circuit is

$$f(a,b,c) = ((a \lor \bar{b}) \land \bar{c}) \lor \overline{((c \land \bar{a}) \lor b)}$$

Expanding it we get

$$\begin{aligned}
f(a,b,c) &= ((a \land \bar{c}) \lor (\bar{b} \land \bar{c})) \lor \overline{((c \land \bar{a}) \lor b)} \\
&= ((a \land \bar{c}) \lor (\bar{b} \land \bar{c})) \lor \overline{((c \lor b) \land (\bar{a} \lor b))} \\
&= ((a \land \bar{c}) \lor (\bar{b} \land \bar{c})) \lor (\overline{(c \lor b)} \lor \overline{(\bar{a} \lor b)}) \\
&= (a \land \bar{c}) \lor (\bar{b} \land \bar{c}) \lor (\bar{c} \land \bar{b}) \lor (a \land \bar{b}) \\
&= \boxed{(a \land \bar{c}) \lor (\bar{c} \land \bar{b}) \lor (a \land \bar{b})}
\end{aligned}$$

(b)

$$\boxed{(a \land \bar{c}) \lor (\bar{c} \land \bar{b}) \lor (a \land \bar{b})}$$

Problem 3

(a)

we have $a.\bar{a} = 0$, therefore we have

$$\begin{aligned}
a + 0 &= a \\
a + (a.\bar{a}) &= a \\
(a + a).(a + \bar{a}) &= a \\
(a + a).1 &= a \\
a + a &= a
\end{aligned}$$

Likewise we have

$$
\begin{aligned}
a.1 &= a \\
a.(a + \bar{a}) &= a \\
a.a + a.\bar{a} &= a \\
a.a + 0 &= a \\
a.a &= a
\end{aligned}
$$

(b)

From the Boolean Algebra postulates we have:

$$1.\bar{1} = 0$$

Therefore we must have that $\bar{1} = 0$

(c)

Let us consider the case where $\bar{a}$ was not unique, ie for $a_1 \neq a_2$, we have $\bar{a_1} = \bar{a_2} = \bar{a}$. Since $\bar{a}.(a_1 + a_2) = 0$ and $\bar{a} + (a_1.a_2) = 1$, we have that

$$
\begin{aligned}
a_1 + (\bar{a}.(a_1 + a_2)) &= a_1 \\
(a_1 + \bar{a}).(a_1 + a_1 + a_2) &= a_1 \\
(a_1 + a_2) &= a_1
\end{aligned}
$$

And that

$$
\begin{aligned}
a_1.(\bar{a} + (a_1.a_2)) &= a_1 \\
(a_1.\bar{a}) + (a_1.a_1.a_2) &= a_1 \\
(a_1.a_2) &= a_1
\end{aligned}
$$

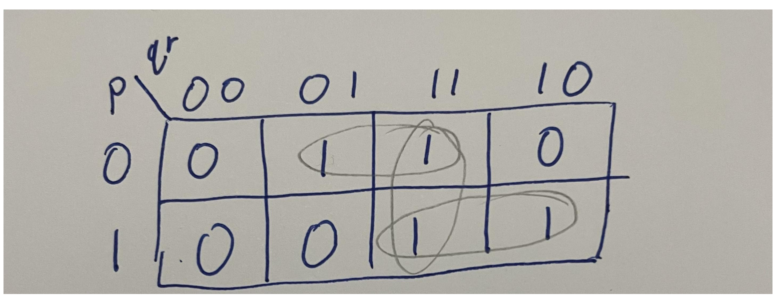
Therefore $a_1 = a_2$, and thus $\bar{a}$ must be unique

Problem 4

(a)

p	q	r	f
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

(c)



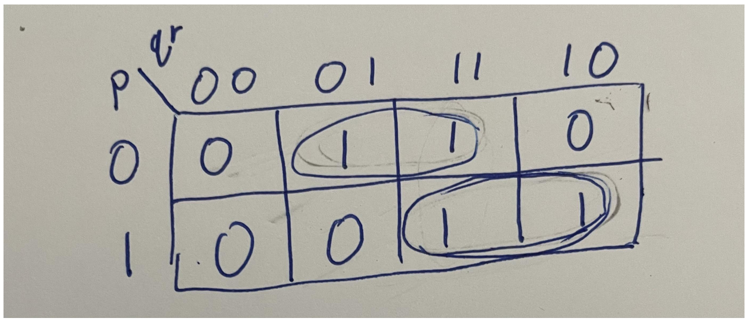
Therefore the prime implicants are

$$\boxed{\bar{p}.r}$$

$$\boxed{q.r}$$

$$\boxed{p.q}$$

(d)



The essential prime implicants are

$$\bar{p}.r$$

and

$$p.q$$

Therefore the boolean expression for the function is

$$f(p,q,r) = \boxed{(\bar{p} \wedge r) \vee (p \wedge q) = (\bar{p}.r) + (p.q)}$$

Problem 5

x	y	z	$\overline{x+y+z}$	$\bar{x}.\bar{y}.\bar{z}$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

x	y	z	$\overline{x.y.z}$	$\overline{x} + \bar{y} + \bar{z}$
1	1	1	0	0
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	1

Problem 6

$$\begin{aligned}(y.\bar{z} + \bar{x}.w).(x.\bar{y} + z.\bar{w}) &= y.\bar{z}.(x.\bar{y} + z.\bar{w}) + \bar{x}.w.(x.\bar{y} + z.\bar{w}) \\ &= \bar{z}.x.y.\bar{y} + y.\bar{w}.z.\bar{z} + w.\bar{y}.\bar{x}.x + z.\bar{x}.w.\bar{w} \quad = \boxed{0}\end{aligned}$$

since $x.\bar{x} = y.\bar{y} = w.\bar{w} = z.\bar{z} = 0$

$$(x.y) + (x.(w.z + w.\bar{z})) = \boxed{(x.y) + (x.w)}$$