ECE M16 Homework 1

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Problem 1

Since there are 26 letters in the English Alphabet, we would need $\lceil \log_2(26) \rceil = 5$ bits to represent this signal. Therefore we could create a way of encoding the English Alphabet as 5 bits with gray encoding. ie we would have the following table

a	00000	n	01011
b	00001	o	01001
c	00011	p	01000
d	00010	q	11000
e	00110	r	11001
f	00111	s	11011
g	00101	t	11010
h	00100	u	11110
i	01100	v	11111
j	01101	w	11101
k	01111	x	11100
l	01110	y	10100
m	01010	z	10101

Problem 2

(a)

The equation for the circuit is

$$f(a,b,c) = ((a \wedge \bar{b}) \vee \bar{c}) \wedge \overline{((c \vee \bar{a}) \wedge b)}$$

Expanding it we get

$$\begin{aligned}
f(a,b,c) &= ((a \wedge \bar{b}) \vee \bar{c}) \wedge (\overline{(c \vee \bar{a})} \vee \bar{b}) \\
&= ((a \wedge \bar{b}) \vee \bar{c}) \wedge ((\bar{c} \wedge a) \vee \bar{b}) \\
&= ((a \wedge \bar{b}) \wedge ((\bar{c} \wedge a) \vee \bar{b})) \vee (\bar{c} \wedge ((\bar{c} \wedge a) \vee \bar{b})) \\
&= \boxed{(a \wedge \bar{b} \wedge \bar{c}) \vee (a \wedge \bar{b}) \vee (\bar{c} \wedge a) \vee (\bar{c} \wedge \bar{b})}
\end{aligned}$$

(b)

$$\begin{aligned}
f(a,b,c) &= (a \wedge \bar{b} \wedge \bar{c}) \vee (a \wedge \bar{b}) \vee (\bar{c} \wedge a) \vee (\bar{c} \wedge \bar{b}) \\
&= \boxed{(a \wedge \bar{b}) \vee (\bar{c} \wedge a) \vee (\bar{c} \wedge \bar{b})}
\end{aligned}$$

Problem 3

(a)

we have $a.\bar{a} = 0$, therefore we have

$$\begin{aligned} a + 0 &= a \\ a + (a.\bar{a}) &= a \\ (a + a).(a + \bar{a}) &= a \\ (a + a).1 &= a \\ a + a &= a \end{aligned}$$

Likewise we have

$$\begin{aligned} a.1 &= a \\ a.(a + \bar{a}) &= a \\ a.a + a.\bar{a} &= a \\ a.a + 0 &= a \\ a.a &= a \end{aligned}$$

(b)

From the Boolean Algebra postulates we have:

$$1.\bar{1} = 0$$

Therefore we must have that $\bar{1} = 0$

(c)

Let us prove that $\bar{a}$ is unique through contradiction, ie: for a given a, there are two variables, $\bar{a_1}$ and $\bar{a_2}$, that satisfy the complement law:

$$\bar{a_1}.a = \bar{a_2}.a = 0$$

$$\bar{a_1} + a = \bar{a_2} + a = 1$$

We also have

$$\bar{a_1}.a.\bar{a_2} = (\bar{a_1}.a).\bar{a_2} = \bar{a_2}$$

And

$$\bar{a_1}.a.\bar{a_2} = \bar{a_1}.(a.\bar{a_2}) = \bar{a_1}$$

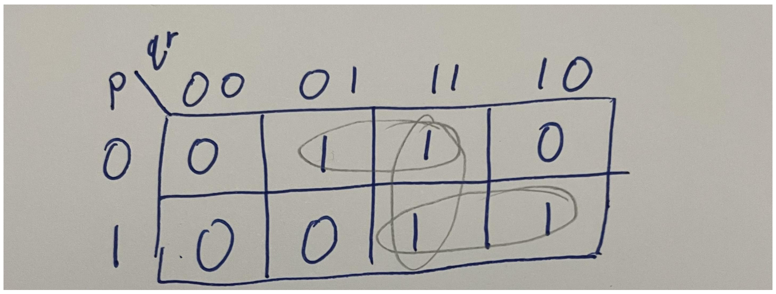
Therefore $\bar{a_1} = \bar{a_2}$, and thus $\bar{a}$ must be unique for a given a.

Problem 4

(a)

p	q	r	f
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

(c)



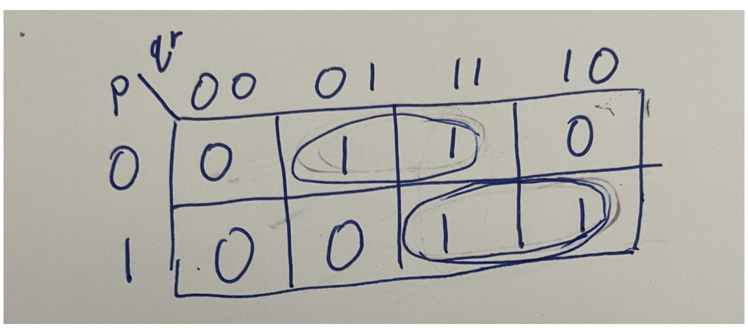
Therefore the prime implicants are

$$\boxed{\bar{p}.r}$$

$$\boxed{q.r}$$

$$\boxed{p.q}$$

(d)



The essential prime implicants are

$$\bar{p}.r$$

and

$$p.q$$

Therefore the boolean expression for the function is

$$f(p, q, r) = \boxed{(\bar{p} \wedge r) \vee (p \wedge q) = (\bar{p}.r) + (p.q)}$$

Problem 5

x	y	z	$\overline{x+y+z}$	$\overline{x}.\bar{y}.\bar{z}$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

x	y	z	$\overline{x.y.z}$	$\overline{x}+\bar{y}+\bar{z}$
1	1	1	0	0
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	1

Problem 6

$$\begin{aligned}(y.\bar{z}+\bar{x}.w).(x.\bar{y}+z.\bar{w}) &= y.\bar{z}.(x.\bar{y}+z.\bar{w})+\bar{x}.w.(x.\bar{y}+z.\bar{w}) \\ &= \bar{z}.x.y.\bar{y}+y.\bar{w}.z.\bar{z}+w.\bar{y}.\bar{x}.x+z.\bar{x}.w.\bar{w} \quad = \boxed{0}\end{aligned}$$

since $x.\bar{x}=y.\bar{y}=w.\bar{w}=z.\bar{z}=0$

$$(x.y)+(x.(w.z+w.\bar{z}))=\boxed{(x.y)+(x.w)}$$