

ECE M16 Homework 1

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Problem 1

Since there are 26 letters in the English Alphabet, we would need $\lceil \log_2(26) \rceil = 5$ bits to represent this signal. Therefore we could create a way of encoding the English Alphabet as 5 bits with gray encoding. ie we would have the following table

a	00000	n	01011
b	00001	o	01001
c	00011	p	01000
d	00010	q	11000
e	00110	r	11001
f	00111	s	11011
g	00101	t	11010
h	00100	u	11110
i	01100	v	11111
j	01101	w	11101
k	01111	x	11100
l	01110	y	10100
m	01010	z	10101

Problem 2

(a)

The equation for the circuit is

$$f(a, b, c) = ((a \wedge \bar{b}) \vee \bar{c}) \wedge \overline{((c \vee \bar{a}) \wedge b)}$$

Expanding it we get

$$\begin{aligned} f(a, b, c) &= ((a \wedge \bar{b}) \vee \bar{c}) \wedge \overline{((c \vee \bar{a}) \wedge b)} \\ &= ((a \wedge \bar{b}) \vee \bar{c}) \wedge ((\bar{c} \wedge a) \vee \bar{b}) \\ &= ((a \wedge \bar{b}) \wedge ((\bar{c} \wedge a) \vee \bar{b})) \vee (\bar{c} \wedge ((\bar{c} \wedge a) \vee \bar{b})) \\ &= \boxed{(a \wedge \bar{b} \wedge \bar{c}) \vee (a \wedge \bar{b}) \vee (\bar{c} \wedge a) \vee (\bar{c} \wedge \bar{b})} \end{aligned}$$

(b)

$$\begin{aligned} f(a, b, c) &= (a \wedge \bar{b} \wedge \bar{c}) \vee (a \wedge \bar{b}) \vee (\bar{c} \wedge a) \vee (\bar{c} \wedge \bar{b}) \\ &= \boxed{(a \wedge \bar{b}) \vee (\bar{c} \wedge a) \vee (\bar{c} \wedge \bar{b})} \end{aligned}$$

Problem 3

(a)

we have $a.\bar{a} = 0$, therefore we have

$$\begin{aligned}a + 0 &= a \\a + (a.\bar{a}) &= a \\(a + a).(a + \bar{a}) &= a \\(a + a).1 &= a \\a + a &= a\end{aligned}$$

Likewise we have

$$\begin{aligned}a.1 &= a \\a.(a + \bar{a}) &= a \\a.a + a.\bar{a} &= a \\a.a + 0 &= a \\a.a &= a\end{aligned}$$

(b)

From the Boolean Algebra postulates we have:

$$1.\bar{1} = 0$$

Therefore we must have that $\bar{1} = 0$

(c)

Let us prove that \bar{a} is unique through contradiction, ie: for a given a , there are two variables, \bar{a}_1 and \bar{a}_2 , that satisfy the complement law:

$$\bar{a}_1.a = \bar{a}_2.a = 0$$

$$\bar{a}_1 + a = \bar{a}_2 + a = 1$$

We also have

$$\bar{a}_1 \cdot a \cdot \bar{a}_2 = (\bar{a}_1 \cdot a) \cdot \bar{a}_2 = \bar{a}_2$$

And

$$\bar{a}_1 \cdot a \cdot \bar{a}_2 = \bar{a}_1 \cdot (a \cdot \bar{a}_2) = \bar{a}_1$$

Therefore $\bar{a}_1 = \bar{a}_2$, and thus \bar{a} must be unique for a given a .

Problem 4

(a)

p	q	r	f
1	1	1	1
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

(c)

A Karnaugh map for three variables p, q, and r. The map is a 2x4 grid. The columns are labeled with qr values: 00, 01, 11, 10. The rows are labeled with p values: 0, 1. The cells contain the following values: (0,00)=0, (0,01)=1, (0,11)=1, (0,10)=0; (1,00)=0, (1,01)=0, (1,11)=1, (1,10)=1. There are three prime implicants circled: a vertical circle around the two 1s in the 01 column, a vertical circle around the two 1s in the 11 column, and a horizontal circle around the two 1s in the 10 column.

p \ q ^r	00	01	11	10
0	0	1	1	0
1	0	0	1	1

Therefore the prime implicants are

$$\bar{p}.r$$

$$q.r$$

$$p.q$$

(d)

A Karnaugh map for three variables p, q, and r, identical to the one in part (c). The prime implicants are circled: a vertical circle around the two 1s in the 01 column, a vertical circle around the two 1s in the 11 column, and a horizontal circle around the two 1s in the 10 column. The circles are drawn with more emphasis than in part (c), indicating they are essential prime implicants.

p \ q ^r	00	01	11	10
0	0	1	1	0
1	0	0	1	1

The essential prime implicants are

$$\bar{p}.r$$

and

$$p.q$$

Therefore the boolean expression for the function is

$$f(p, q, r) = (\bar{p} \wedge r) \vee (p \wedge q) = (\bar{p}.r) + (p.q)$$

Problem 5

x	y	z	$\overline{x+y+z}$	$\overline{x.y.z}$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	1

x	y	z	$\overline{x.y.z}$	$\overline{x+y+z}$
1	1	1	0	0
1	1	0	1	1
1	0	1	1	1
1	0	0	1	1
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	1

Problem 6

$$\begin{aligned}
 (y.\bar{z} + \bar{x}.w).(x.\bar{y} + z.\bar{w}) &= y.\bar{z}.(x.\bar{y} + z.\bar{w}) + \bar{x}.w.(x.\bar{y} + z.\bar{w}) \\
 &= \bar{z}.x.y.\bar{y} + y.\bar{w}.z.\bar{z} + w.\bar{y}.\bar{x}.x + z.\bar{x}.w.\bar{w} = \boxed{0}
 \end{aligned}$$

since $x.\bar{x} = y.\bar{y} = w.\bar{w} = z.\bar{z} = 0$

$$(x.y) + (x.(w.z + w.\bar{z})) = \boxed{(x.y) + (x.w)}$$