Physics 115C HW 5

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May 22, 2023

Problem 1

(a)

We have that

$$\Delta = \frac{\Omega}{2}$$

Therefore we must have that

$$\frac{\lambda^2}{\Delta^2} = 4\frac{\lambda^2}{\Omega^2} = \frac{\gamma^2}{\hbar^2\Omega^2}$$

Therefore we have that

$$\lambda = \frac{\gamma}{2\hbar}$$

Since

$$\Delta^2 = \lambda^2 + \omega^2$$

We have that

$$\omega^2 = \frac{\gamma^2}{4\hbar^2} - \frac{\Omega^2}{4}$$

Since $\Omega^2 = \delta\omega^2 + \frac{\gamma^2}{\hbar^2}$ we have:

$$\omega^2 = -\frac{\delta\omega^2}{4}$$

Therefore we have that

$$\omega = i\frac{\delta\omega}{2}$$

(b)

Therefore we have that

$$H_0 = \begin{pmatrix} 2\hbar i\frac{\delta\omega}{2} & 0 \\ 0 & 0 \end{pmatrix}$$

And

$$H_1 = \begin{pmatrix} 0 & \frac{\gamma}{2} \\ \frac{\gamma}{2} & 0 \end{pmatrix}$$

Therefore we have that

$$H_0 + H_1 = \begin{pmatrix} 2\hbar i\frac{\delta\omega}{2} & \frac{\gamma}{2} \\ \frac{\gamma}{2} & 0 \end{pmatrix}$$

We know that the eigenvalues in terms of λ, ω and Δ are $\hbar(\omega \pm \Delta)$, therefore we have that the energy eigenvalues are

$$E_\pm = \hbar \left(i\frac{\delta\omega}{2} \pm \frac{\Omega}{2} \right)$$

And we have that the eigenvectors $v_+ = d \begin{pmatrix} \omega + \Delta \\ \lambda \end{pmatrix}$ and $v_- = d \begin{pmatrix} -\lambda \\ \omega + \Delta \end{pmatrix}$ where d is a normalization constant. Therefore we have that

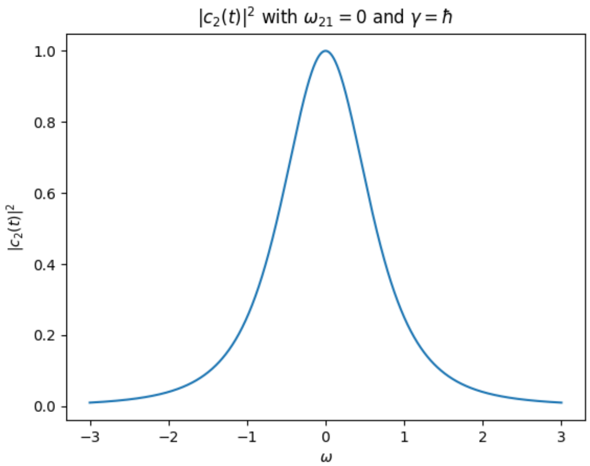
$$v_+ = d \begin{pmatrix} i\frac{\delta\omega}{2} + \frac{\Omega}{2} \\ \frac{\gamma}{2\hbar} \end{pmatrix}$$

And

$$v_- = d \begin{pmatrix} -\frac{\gamma}{2\hbar} \\ i\frac{\delta\omega}{2} + \frac{\Omega}{2} \end{pmatrix}$$

We have that $d^{-2} = \Omega$.

(c)



This is a lorentzian curve.

Problem 2

We have that the probability of a transition is given by

$$P_{if} = \left| \frac{i}{\hbar} \int_0^t e|E_0| e^{-\gamma t} \langle f| z |i\rangle \, dt \right|^2$$

We have that

$$\langle 211| z |100\rangle = 0$$

$$\langle 21-1| z |210\rangle = 0$$

From 6.3b. Also from applying what we have from 6.3e and changing L_z to L_y and y to z we get:

$$\langle 200| z |100\rangle = 0$$

And thus we just need to solve for

$$\begin{aligned}
\langle 210|\, z\, |100\rangle &= \int_0^\infty \int_0^{2\pi} \int_0^\pi r^3 \sin(\theta)\cos(\theta)\psi_{210}^*(r,\theta,\phi)\psi_{210}^*(r,\theta,\phi)d\theta d\phi dr \\
&= 2\pi \int_0^\infty Y_{210}Y_{100}r^3 \int_0^\pi \sin(\theta)\cos^2(\theta)dr \\
&= \frac{4\pi}{3}\int_0^\infty Y_{210}Y_{100}r^3 dr \\
&= \frac{4\pi}{3}\frac{1}{4\sqrt{2\pi}}\frac{1}{\sqrt{\pi}a_0^4}\int_0^\infty r^4 e^{-3r/2a_0}dr \\
&= \frac{2^{\frac{15}{2}}a_0}{3^5}
\end{aligned}$$

Thereforer we have that the probability of transistion is given by

$$P_{if} = \left| \frac{i}{\hbar}\frac{2^{\frac{15}{2}}a_0}{3^5}\int_0^t e|E_0|e^{-\gamma t}e^{\frac{it}{\hbar}E_f - E_i}dt \right|^2$$

Thus the limit as $t \to \infty$ is:

$$P_{if} = \left| \frac{i}{\hbar}\frac{2^{\frac{15}{2}}a_0}{3^5}\frac{e|E_0|}{-\gamma + \frac{i(E_f - E_i)}{\hbar}} \right|^2$$

$$P_{if} = \frac{2^{15}a_0^2}{3^{10}\hbar^2}\frac{e^2|E_0|^2}{\gamma^2 + \frac{(E_f - E_i)^2}{\hbar^2}}$$

We have that $E_2 - E_1 = \frac{3e^2}{8a_0}$ thus we get:

$$P_{if} = \boxed{\frac{2^{15}a_0^2}{3^{10}\hbar^2}\frac{e^2|E_0|^2}{\gamma^2 + \frac{9e^4}{64a_0^2\hbar^2}}}$$

Problem 3

(a)

We have that

$$\begin{aligned}
[L_z, x] &= [xp_y - yp_x, x] \\
&= [xp_y, x] - [yp_x, x] \\
&= x[p_y, x] + [x, p_y]x - [y, x]p_x - y[p_x, x] \\
&= -y[p_x, x] \\
&= yi\hbar
\end{aligned}$$

Likewise

$$\begin{aligned}
[L_z, y] &= [xp_y - yp_x, y] \\
&= [xp_y, y] - [yp_x, y] \\
&= x[p_y, y] + [x, p_y]y - [y, x]p_x - y[p_x, y] \\
&= x[p_y, y] - [y, x]p_x \\
&= -xi\hbar
\end{aligned}$$

and

$$\begin{aligned}
[L_z, z] &= [xp_y - yp_x, z] \\
&= [xp_y, z] - [yp_x, z] \\
&= x[p_y, z] + [x, p_y]z - [y, x]p_x - y[p_x, z] \\
&= x[p_y, z] - [y, x]p_x \\
&= 0
\end{aligned}$$

(b)

We have that

$$\begin{aligned}
\left\langle n'l'm' \right| [L_z, z] \left| nlm \right\rangle &= \left\langle n'l'm' \right| 0 \left| nlm \right\rangle \\
\left\langle n'l'm' \right| L_z z - z L_z \left| nlm \right\rangle &= 0 \\
\left\langle n'l'm' \right| (m' - m) z \left| nlm \right\rangle &= 0 \\
(m' - m) \left\langle n'l'm' \right| z \left| nlm \right\rangle &= 0
\end{aligned}$$

Thus we can see that unless $m' = m$ the matrix element must be zero, thus if light is polarized in the z direction, it cannot change the m quantum number. Thus we cannot go from $|100\rangle \to |211\rangle$ with light polarized in the z direction.

(c)

We have that

$$\begin{aligned}
\left\langle n'l'm'\right| [L_z, x] \left|nlm\right\rangle &= \left\langle n'l'm'\right| i\hbar y \left|nlm\right\rangle \\
\left\langle n'l'm'\right| L_z x - x L_z \left|nlm\right\rangle &= i\hbar \left\langle n'l'm'\right| y \left|nlm\right\rangle \\
\hbar \left\langle n'l'm'\right| (m' - m) x \left|nlm\right\rangle &= i\hbar \left\langle n'l'm'\right| y \left|nlm\right\rangle \\
(m' - m) \left\langle n'l'm'\right| x \left|nlm\right\rangle &= i \left\langle n'l'm'\right| y \left|nlm\right\rangle
\end{aligned}$$

(d)

We have that:

$$\begin{aligned}
[L_z, x \pm iy] &= [L_z, x] \pm i[L_z, y] \\
&= i\hbar y \pm \hbar x
\end{aligned}$$

$$\begin{aligned}
\left\langle n'l'm'\right| [L_z, x \pm iy] \left|nlm\right\rangle &= \left\langle n'l'm'\right| i\hbar y \pm \hbar x \left|nlm\right\rangle \\
(m' - m) \left\langle n'l'm'\right| x \pm iy \left|nlm\right\rangle &= \pm \left\langle n'l'm'\right| x \pm iy \left|nlm\right\rangle \\
(m' - m \mp 1) \left\langle n'l'm'\right| x \pm iy \left|nlm\right\rangle &= 0
\end{aligned}$$

Thus we can see that the only possible nonzero matrix elements are when $m' = m \pm 1$. This reminds us of the raising and lower operators.

(e)

We have that

$$\begin{aligned}
\left\langle n'l'm'\right| [L_z, y] \left|nlm\right\rangle &= -\left\langle n'l'm'\right| i\hbar x \left|nlm\right\rangle \\
(m'-m)\left\langle n'l'm'\right| y \left|nlm\right\rangle &= -i\left\langle n'l'm'\right| x \left|nlm\right\rangle \\
(m'-m)^2\left\langle n'l'm'\right| x \left|nlm\right\rangle &= \left\langle n'l'm'\right| x \left|nlm\right\rangle \\
\left((m'-m)^2-1\right)\left\langle n'l'm'\right| x \left|nlm\right\rangle = 0
\end{aligned}$$

Thus we can see that $\langle n'l'm'| x |nlm\rangle = \langle n'l'm'| y |nlm\rangle = 0$ unless $(m'-m)^2 = 1$. This means that for both the $bran'l'm'x\left|nlm\right\rangle$ and $\langle n'l'm'| y |nlm\rangle$ matricies will have 0 values beyond the diagonal above a and below the main diagonal.