Financial Modelling (BEAM046) Term 1 2022/3

Week 1 Tutorial Exercise

The first tutorial session asks you to solve three short exercises in Excel rather than finishing a practical case. These exercises will introduce several fundamental Excel functions and tools that we will use throughout this class.

Question 1: Calculating summary statistics

You are interested in calculating the summary statistics of two assets: stock XYZ and the market portfolio (MKT). Assume the risk-free rate is 0% throughout this question.

a. Suppose you somehow acquired the superpower to observe the true probability distribution of both XYZ and MKT. The worksheet titled "Population Info" provides information on the return distributions of the two assets. There are 100 possible states, and you know the true probability (column B) and the realized *annual* return (columns C and D) for each state. Calculate the population mean, variance, and standard deviation of the two assets. Additionally, calculate the covariance between the two assets and XYZ's beta.

   Population Mean (μ) = $\sum_{s=1}^{S} p_s r_s$
   Variance = $\sum_{s=1}^{S} p_s (r_s - \mu)^2$
   Covariance $\sigma_{i,j} = \sum_{s=1}^{S} p_s (r_{i,s} - \mu)(r_{j,s} - \mu)$

b. Although you know the true moments of XYZ and MKT, you are interested in knowing whether the sample moments are reasonable proxies for their population counterparts. Thus, you collected historical daily prices of XYZ and MKT from 2/1/2020 to 31/12/2021. The historical price data can be found in the "Daily Sample" worksheet. Calculate the ***annualised*** sample average, variance, and standard deviation of daily returns of these two assets. Also calculate XYZ's beta. (Hint: You can find the annualised average and variance from the lecture notes.)

c. Repeat part (b) using weekly returns, not daily returns. Use Wednesday as the base day to calculate weekly returns because fewer holidays fall on Wednesday.

d. Compare your estimation results from parts (b) and (c) to their population counterparts. Are the daily and weekly samples moments good proxies for the population moments? Why or why not? (Hint: Calculate the sample autocorrelation coefficients for both assets. The autocorrelation coefficient of asset *i* is the correlation between $r_{i,t}$ and $r_{i,t-1}$.)

Question 2: Saving for retirement

Today is your 25th birthday. Although your career has just started, you are already worried about retirement. To ensure that you will have enough money after retirement, you have decided to start a retirement saving fund. You will deposit £20,000 each year for the next ten years, with the first deposit due one year from today. After that, you will deposit £30,000 each year, with the last deposit to be made on your 65th birthday. (That is, you will make 40 deposits in total.)

a. Suppose the retirement fund offers an annual saving rate of 4%, and the annual inflation is 2%. What will be the amount of your ***real*** savings when you retire?
(Hint: Real rate: $\frac{1+nominal\ rate}{1+inflation} - 1$; Real value of CF_t at $t=0$: $\frac{Nominal\ CF_t}{(1+inflation)^t}$)

b. If you want the real value of your total savings to be £1.2 million when you retire, how high a rate must this account offer assuming you do not make any additional deposits?

c. You are worried that the inflation rate may increase and cannibalise your purchasing power. To understand the effect of inflation, perform a sensitivity analysis by allowing the inflation rate to change from 0% to 10% in increments of 1% and calculate the corresponding real value of your savings.

d. You are also interested in understanding the interaction between inflation and the saving rate offered by your account. Perform a two-way sensitivity analysis in which inflation and saving rates change simultaneously. The inflation rate ranges from 0% to 10% in increments of 1%, and the saving rate ranges from 2% to 10% in increments of 2%. Calculate the real value of your savings corresponding to different inflation and saving rates.

Question 3: Solving for unknowns

Consider a capital market in which the Capital Asset Pricing Model (CAPM) holds true. There are two assets in this market: A and B. Asset A has a beta of 1.5 and expected return of 11%. Asset B has a beta of 0.9 and expected return of 7.4%. Calculate the risk-free rate and the expected market return in this market.

(Note: This question can be solved by solving the following system of equations: $\begin{cases} r_f + 1.5 \times \left[E(r_M) - r_f\right] - 11\% = 0 \\ r_f + 0.9 \times \left[E(r_M) - r_f\right] - 7.4\% = 0 \end{cases}$. How do you do this in Excel?)