Redes Neurais com Recorrência Global

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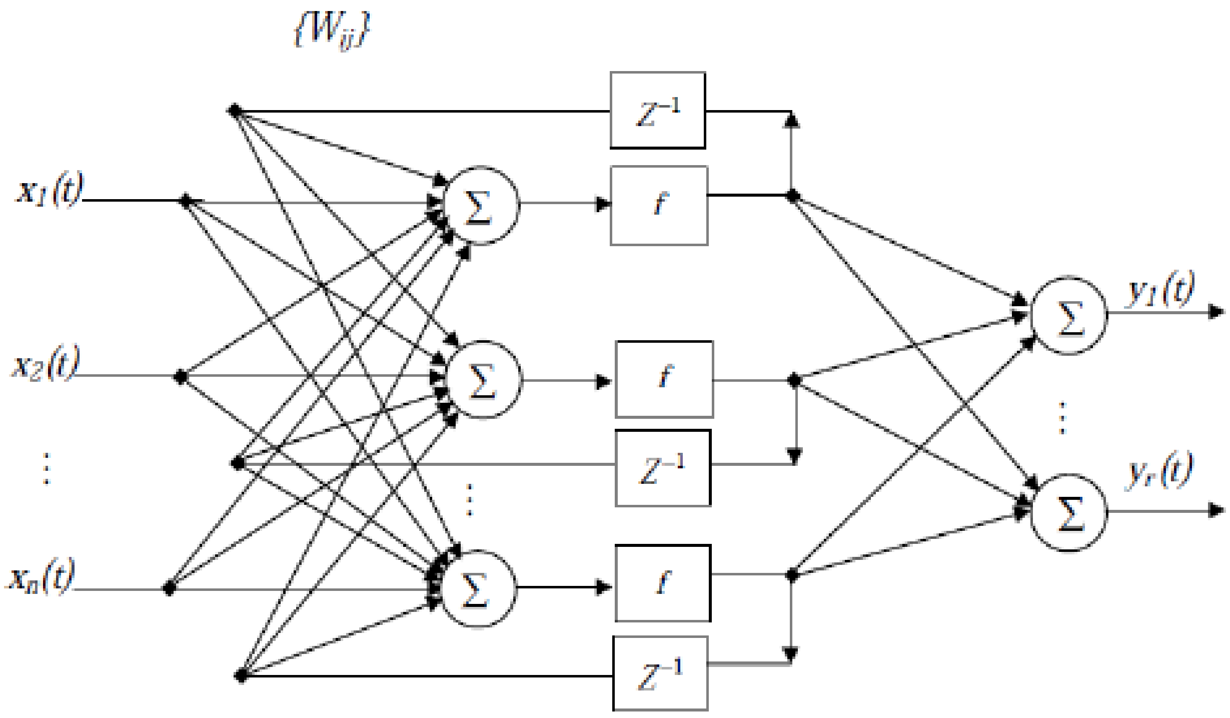
Sumário

- Formulação Matemática

Rede Neural com Recorrencia Global



Equacionamento

$$Zin_1(n) = b_{11}Z_1(n-1) + b_{12}Z_1(n-2) + b_{13}Z_2(n-1) + b_{14}Z_2(n-2)\ldots$$
$$+ a_{10}1 + a_{11}x_1(n) + a_{12}x_2(n)$$

$$Zin_2(n) = b_{21}Z_1(n-1) + b_{22}Z_1(n-2) + b_{23}Z_2(n-1) + b_{24}Z_2(n-2)\ldots$$
$$+ a_{20}1 + a_{21}x_1(n) + a_{22}x_2(n)$$

$$Zin_i(n) = \sum_{j=1}^{h}\sum_{k=1}^{L} b_{i(L*(j-1)+k)} Z_j(n-k) + \sum_{j=1}^{ne} a_{ij}x_j(n) + a_{i0}$$

$$\boxed{Zin_i(n) = \sum_{j=1}^{h}\sum_{k=1}^{L} b_{i(L*(j-1)+k)} Z_j(n-k) + \sum_{j=0}^{ne} a_{ij}x_j(n)}$$

EQUACIONAMENTO

$$Z_i(n) = f(Zin_i(n))$$

$$Yin_k(n) = c_{k1}Z_1(n) + c_{k2}Z_2(n) + c_{k0}1$$

$$Yin_k(n) = \sum_{i=1}^{h} c_{ki}Z_i(n) + c_{k0}1$$

$$Yin_k(n) = \sum_{k=0}^{h} c_{ki}Z_i(n)$$

$$Y_k(n) = f(Yin_k(n))$$

Propagação Direta

$$Zin_i(n) = \sum_{j=1}^{h} \sum_{k=1}^{L} b_{i(L*(j-1)+k)} Z_j(n-k) + \sum_{j=0}^{ne} a_{ij} x_j(n)$$

$$Z_i(n) = f(Zin_i(n))$$

$$Yin_k(n) = \sum_{i=0}^{h} c_{ki} Z_i(n)$$

$$Y_k(n) = g(Yin_k(n))$$

Cálculo do Erro

$$e_k(n) = Y_k(n) - Yd_k(n)$$

$$E(n) = \frac{1}{2}\sum_{k=1}^{ns} e_k(n)^2$$

$$E_T = \frac{1}{N}\sum_{n=1}^{N} E(n)$$

$$\begin{bmatrix} Zin_1(n) \\ Zin_2(n) \\ Zin_3(n) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix} \begin{bmatrix} Z_1(n-1) \\ Z_1(n-2) \\ Z_2(n-1) \\ Z_2(n-2) \\ Z_2(n-1) \\ Z_3(n-2) \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{10} \\ a_{21} & a_{22} & a_{23} & a_{20} \\ a_{31} & a_{32} & a_{33} & a_{30} \end{bmatrix} \begin{bmatrix} x_2(n) \\ x_2(n) \\ x_2(n) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Z_1(n) \\ Z_2(n) \\ Z_3(n) \end{bmatrix} = \begin{bmatrix} f(Zin_1(n)) \\ f(Zin_2(n)) \\ f(Zin_3(n)) \end{bmatrix} \Rightarrow \begin{bmatrix} Yin_1(n) \\ Yin_2(n) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{10} \\ b_{21} & b_{22} & b_{23} & b_{20} \end{bmatrix} \begin{bmatrix} Z_1(n) \\ Z_2(n) \\ Z_3(n) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} Y_1(n) \\ Y_2(n) \end{bmatrix} = \begin{bmatrix} g(Yin_1(n)) \\ g(Yin_2(n)) \end{bmatrix} \Rightarrow \begin{bmatrix} e_1(n) \\ e_2(n) \end{bmatrix} = \begin{bmatrix} Y_1(n) - Yd_1(n) \\ Y_2(n) - Yd_2(n) \end{bmatrix}$$

Calculando as derivadas

- Precisamos calcular

$$\frac{\partial E_T}{\partial a_{mp}}, \frac{\partial E_T}{\partial b_{\mathbf{mp}}}, \frac{\partial E_T}{\partial c_{mp}}$$

- Vamos calcular

$$E_T = \frac{1}{N}\sum_{n=1}^{N} E(n) \Rightarrow \frac{\partial E_T}{\partial b_{mp}} = \frac{1}{N}\sum_{n=1}^{N}\frac{\partial E(n)}{\partial c_{mp}}$$

$$E(n) = \frac{1}{2}\sum_{k=1}^{ns} e_k(n)^2 \Rightarrow \frac{\partial E(n)}{\partial c_{mp}} = \sum_{k=1}^{ns} e_k(n)\frac{\partial e_k(n)}{\partial c_{mp}}$$

$$e_k(n) = Y_k(n) - Yd_k(n) \Rightarrow \frac{\partial e_k(n)}{\partial c_{ml}} = \frac{\partial Y_k(n)}{\partial c_{ml}}$$

Equacionamento

$$Y_k(n) = g(Yin_k(n)) \quad \Rightarrow \quad \frac{\partial Y_k(n)}{\partial c_{mp}} = \frac{\partial g(Yin_k(n))}{\partial Yin_m(n)} \frac{\partial Yin_m(n)}{\partial c_{mp}}$$

$$Yin_k(n) = \sum_{i=0}^{h} c_{ki} Z_i(n) \quad \Rightarrow \quad \frac{\partial Yin_m(n)}{\partial c_{mp}} = Z_p(n)$$

$$\frac{\partial E_T}{\partial c_{mp}} = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{ns} e_k(n) \frac{\partial g(Yin_k(n))}{\partial Yin_m(n)} Z_p(n)$$

$$\frac{\partial E_T}{\partial c_{mp}} = \frac{1}{N} \sum_{n=1}^{N} \sum_{k=1}^{ns} e_k(n) \dot{g}_{km}(Yin_k(n)) Z_p(n)$$

EQUACIONAMENTO

- Se adotarmos função de ativação sigmoid ou tangente hiperbolica

$$\frac{\partial E_T}{\partial c_{mp}} = \frac{1}{N}\sum_{n=1}^{N} e_k(n)\,\dot{g}_{kk}(Yin_k(n))Z_p(n)$$

$$\begin{bmatrix} \frac{\partial E(n)}{\partial b_{11}} & \frac{\partial E(n)}{\partial c_{12}} & \cdots & \frac{\partial E(n)}{\partial c_{21}} & \frac{\partial E(n)}{\partial c_{22}} & \cdots \end{bmatrix} =$$

$$\begin{bmatrix} erro_1(n) & erro_2(n) \end{bmatrix} \begin{bmatrix} \overset{*}{g}_{11} & 0 \\ 0 & \overset{*}{g}_{11} \end{bmatrix} \begin{bmatrix} Z_1(n) & Z_2(n) & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_1(n) & Z_2(n) & 1 \end{bmatrix}$$

Equacionamento

- Vamos calcular

$$\frac{\partial E_T}{\partial a_{\mathbf{m}p}},$$

$$E_T = \frac{1}{N}\sum_{n=1}^{N} E(n) \quad \Rightarrow \quad \frac{\partial E_T}{\partial b_{mp}} = \frac{1}{N}\sum_{n=1}^{N}\frac{\partial E(n)}{\partial a_{mp}}$$

$$E(n) = \frac{1}{2}\sum_{k=1}^{ns} e_k(n)^2 \quad \Rightarrow \quad \frac{\partial E(n)}{\partial a_{mp}} = \sum_{k=1}^{ns} e_k(n)\frac{\partial e_k(n)}{\partial a_{mp}}$$

$$e_k(n) = Y_k(n) - Yd_k(n) \quad \Rightarrow \quad \frac{\partial e_k(n)}{\partial a_{ml}} = \frac{\partial Y_k(n)}{\partial a_{ml}}$$

CÁLCULO $\dfrac{\partial E_T}{\partial a_{\mathbf{m}p}}$,

$$Y_k(n) = g(Yin_k(n)) \Rightarrow \frac{\partial Y_k(n)}{\partial a_{mp}} = \sum_{l=1}^{ns} \frac{\partial g(Yin_k(n))}{\partial Yin_l(n)} \frac{\partial Yin_l(n)}{\partial a_{mp}}$$

Função ativação
Sigmoid ou Tangente

$$\Rightarrow \frac{\partial Y_k(n)}{\partial a_{mp}} = \dot{g}_{kk} \frac{\partial Yin_k(n)}{\partial a_{mp}}$$

$$Yin_k(n) = \sum_{i=0}^{h} c_{ki} Z_i(n) \Rightarrow \frac{\partial\, Yin_k(n)}{\partial a_{mp}} = \sum_{i=0}^{h} c_{ki} \frac{\partial Z_i(n)}{\partial a_{mp}}$$

$$\frac{\partial E(n)}{\partial a_{mp}} = \sum_{k=1}^{ns} e_k(n) \dot{g}_{kk} \sum_{l=1}^{h} c_{kl} \frac{\partial Z_l(n)}{\partial a_{mp}}$$

CÁLCULO $\dfrac{\partial E_T}{\partial a_{\mathbf{m}p}}$,

$$\frac{\partial E(n)}{\partial a_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{ml} \frac{\partial Z_l(n)}{\partial a_{mp}}$$

$$Z_m(n) = f(Zin_m(n)) \quad \Rightarrow \quad \frac{\partial Z_m(n)}{\partial a_{mp}} = f'(Zin_m(n)) \frac{\partial Zin_m(n)}{\partial a_{mp}}$$

$$Zin_m(n) = \sum_{j=1}^{h} \sum_{k=1}^{L} a_{m(L*(j-1)+k)} Z_j(n-k) + \sum_{j=0}^{ne} b_{mj} x_j(n)$$

$$\frac{\partial Zin_l(n)}{\partial a_{mp}} = \begin{cases} Z_m(n-(p-L(m-1))) + \displaystyle\sum_{k=1}^{L} a_{l(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial a_{mp}} & l = m \\ 0 + \displaystyle\sum_{k=1}^{L} a_{l(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial a_{mp}} & l \neq m \end{cases}$$

$$\frac{\partial Zin_l(n)}{\partial a_{mp}} = \begin{cases} Z_m(n-(p-L(m-1))) + \sum_{k=1}^{L} b_{l(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial a_{mp}} & l = m \\ 0 + \sum_{k=1}^{L} b_{l(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial a_{mp}} & l \neq m \end{cases}$$

$$\begin{bmatrix} Zin_1(n) \\ Zin_2(n) \\ Zin_3(n) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix} \begin{bmatrix} Z_1(n-1) \\ Z_1(n-2) \\ Z_2(n-1) \\ Z_2(n-2) \\ Z_2(n-1) \\ Z_3(n-2) \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{10} \\ a_{21} & a_{22} & a_{23} & a_{20} \\ a_{31} & a_{32} & a_{33} & a_{30} \end{bmatrix} \begin{bmatrix} x_2(n) \\ x_2(n) \\ x_2(n) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial Zin_1(n)}{\partial b_{11}} \\ \frac{\partial Zin_2(n)}{\partial b_{11}} \\ \frac{\partial Zin_3(n)}{\partial b_{11}} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix} \begin{bmatrix} \frac{\partial Z_1(n-1)}{\partial b_{11}} \\ \frac{\partial Z_1(n-2)}{\partial b_{11}} \\ \frac{\partial Z_2(n-1)}{\partial b_{11}} \\ \frac{\partial Z_2(n-2)}{\partial b_{11}} \\ \frac{\partial Z_3(n-1)}{\partial b_{11}} \\ \frac{\partial Z_3(n-2)}{\partial b_{11}} \end{bmatrix} + \begin{bmatrix} Z_1(n-1) \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} Zin_1(n) \\ Zin_2(n) \\ Zin_3(n) \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix} \begin{bmatrix} Z_1(n-1) \\ Z_1(n-2) \\ Z_2(n-1) \\ Z_2(n-2) \\ Z_2(n-1) \\ Z_3(n-2) \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{10} \\ a_{21} & a_{22} & a_{23} & a_{20} \\ a_{31} & a_{32} & a_{33} & a_{30} \end{bmatrix} \begin{bmatrix} x_2(n) \\ x_2(n) \\ x_2(n) \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{\partial Zin_1(n)}{\partial b_{12}} \\ \dfrac{\partial Zin_2(n)}{\partial b_{12}} \\ \dfrac{\partial Zin_3(n)}{\partial b_{12}} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix} \begin{bmatrix} \dfrac{\partial Z_1(n-1)}{\partial b_{12}} \\ \dfrac{\partial Z_1(n-2)}{\partial b_{12}} \\ \dfrac{\partial Z_2(n-1)}{\partial b_{12}} \\ \dfrac{\partial Z_2(n-2)}{\partial b_{12}} \\ \dfrac{\partial Z_3(n-1)}{\partial b_{12}} \\ \dfrac{\partial Z_3(n-2)}{\partial b_{12}} \end{bmatrix} + \begin{bmatrix} Z_1(n-2) \\ 0 \\ 0 \end{bmatrix}$$

$$
\begin{bmatrix} Zin_1(n) \\ Zin_2(n) \\ Zin_3(n) \end{bmatrix}
=
\begin{bmatrix}
b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\
b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\
b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36}
\end{bmatrix}
\begin{bmatrix} Z_1(n-1) \\ Z_1(n-2) \\ Z_2(n-1) \\ Z_2(n-2) \\ Z_2(n-1) \\ Z_3(n-2) \end{bmatrix}
+
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{10} \\
a_{21} & a_{22} & a_{23} & a_{20} \\
a_{31} & a_{32} & a_{33} & a_{30}
\end{bmatrix}
\begin{bmatrix} x_2(n) \\ x_2(n) \\ x_2(n) \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix}
\frac{\partial Zin_1(n)}{\partial b_{11}} & \frac{\partial Zin_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_1(n)}{\partial b_{36}} \\
\frac{\partial Zin_2(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \\
\frac{\partial Zin_3(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}}
\end{bmatrix}
=
\begin{bmatrix}
b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\
b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\
b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36}
\end{bmatrix}
\begin{bmatrix}
\frac{\partial Z_1(n-1)}{\partial b_{11}} & \frac{\partial Z_1(n-1)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n-1)}{\partial b_{36}} \\
\frac{\partial Z_1(n-2)}{\partial b_{11}} & \frac{\partial Z_1(n-2)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n-2)}{\partial b_{36}} \\
\frac{\partial Z_2(n-1)}{\partial b_{11}} & \frac{\partial Z_2(n-1)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n-1)}{\partial b_{36}} \\
\frac{\partial Z_2(n-2)}{\partial b_{11}} & \frac{\partial Z_2(n-2)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n-2)}{\partial b_{36}} \\
\frac{\partial Z_3(n-1)}{\partial b_{11}} & \frac{\partial Z_3(n-1)}{\partial b_{12}} & \cdots & \frac{\partial Z_3(n-1)}{\partial b_{36}} \\
\frac{\partial Z_3(n-2)}{\partial b_{11}} & \frac{\partial Z_3(n-2)}{\partial b_{12}} & \cdots & \frac{\partial Z_3(n-2)}{\partial b_{36}}
\end{bmatrix}
$$

$$
+
\begin{bmatrix}
Z_1(n-1) & Z_1(n-2) & \ldots & 0 \\
0 & 0 & \ldots & 0 \\
0 & 0 & \ldots & Z_3(n-2)
\end{bmatrix}
$$

$$\frac{\partial Z_m(n)}{\partial a_{mp}} = f'(Zin_m(n))\frac{\partial Zin_m(n)}{\partial a_{mp}}$$

$$\begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{11}} & \frac{\partial Z_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n)}{\partial b_{36}} \\ \frac{\partial Z_2(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \\ \frac{\partial Z_3(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \end{bmatrix} = \begin{bmatrix} \overset{*}{f}(Zin_1(n))\frac{\partial Zin_1(n)}{\partial b_{11}} & \overset{*}{f}(Zin_1(n))\frac{\partial Zin_1(n)}{\partial b_{12}} & \cdots & \overset{*}{f}(Zin_1(n))\frac{\partial Zin_1(n)}{\partial b_{36}} \\ \overset{*}{f}(Zin_2(n))\frac{\partial Zin_2(n)}{\partial b_{11}} & \overset{*}{f}(Zin_2(n))\frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \overset{*}{f}(Zin_2(n))\frac{\partial Zin_2(n)}{\partial b_{36}} \\ \overset{*}{f}(Zin_3(n))\frac{\partial Zin_3(n)}{\partial b_{11}} & \overset{*}{f}(Zin_3(n))\frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \overset{*}{f}(Zin_3(n))\frac{\partial Zin_2(n)}{\partial b_{36}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{11}} & \frac{\partial Z_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n)}{\partial b_{36}} \\ \frac{\partial Z_2(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \\ \frac{\partial Z_3(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \end{bmatrix} = \begin{bmatrix} \overset{*}{f}(Zin_1(n)) & 0 & 0 \\ 0 & \overset{*}{f}(Zin_2(n)) & 0 \\ 0 & 0 & \overset{*}{f}(Zin_3(n)) \end{bmatrix} \begin{bmatrix} \frac{\partial Zin_1(n)}{\partial b_{11}} & \frac{\partial Zin_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_1(n)}{\partial b_{36}} \\ \frac{\partial Zin_2(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \\ \frac{\partial Zin_3(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \end{bmatrix}$$

$$\frac{\partial E(n)}{\partial b_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{kl} \frac{\partial Z_l(n)}{\partial b_{mp}}$$

$$\frac{\partial E(n)}{\partial b_{11}} = \begin{bmatrix} e_1(n)\overset{*}{g}_{11} & e_1(n)\overset{*}{g}_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} \dfrac{\partial Z_1(n)}{\partial b_{11}} \\ \dfrac{\partial Z_2(n)}{\partial b_{11}} \\ \dfrac{\partial Z_3(n)}{\partial b_{11}} \end{bmatrix}$$

$$\frac{\partial E(n)}{\partial b_{11}} = \begin{bmatrix} e_1(n) & e_1(n) \end{bmatrix} \begin{bmatrix} \overset{*}{g}_{11} & 0 \\ 0 & \overset{*}{g}_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} \dfrac{\partial Z_1(n)}{\partial b_{11}} \\ \dfrac{\partial Z_2(n)}{\partial b_{11}} \\ \dfrac{\partial Z_3(n)}{\partial b_{11}} \end{bmatrix}$$

$$\frac{\partial E(n)}{\partial b_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{kl} \frac{\partial Z_l(n)}{\partial b_{mp}}$$

$$\frac{\partial E(n)}{\partial b_{12}} = \begin{bmatrix} e_1(n)\overset{*}{g}_{11} & e_1(n)\overset{*}{g}_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{12}} \\ \frac{\partial Z_2(n)}{\partial b_{12}} \\ \frac{\partial Z_3(n)}{\partial b_{12}} \end{bmatrix}$$

$$\frac{\partial E(n)}{\partial b_{12}} = \begin{bmatrix} e_1(n) & e_1(n) \end{bmatrix} \begin{bmatrix} \overset{*}{g}_{11} & 0 \\ 0 & \overset{*}{g}_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{12}} \\ \frac{\partial Z_2(n)}{\partial b_{12}} \\ \frac{\partial Z_3(n)}{\partial b_{12}} \end{bmatrix}$$

$$\frac{\partial E(n)}{\partial b_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{kl} \frac{\partial Z_l(n)}{\partial b_{mp}}$$

$$\begin{bmatrix} \frac{\partial E(n)}{\partial b_{11}} & \frac{\partial E(n)}{\partial b_{12}} & \cdots & \frac{\partial E(n)}{\partial b_{36}} \end{bmatrix} = \begin{bmatrix} e_1(n) & e_1(n) \end{bmatrix} \begin{bmatrix} \overset{*}{g}_{11} & 0 \\ 0 & \overset{*}{g}_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{11}} & \frac{\partial Z_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n)}{\partial b_{36}} \\ \frac{\partial Z_2(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \\ \frac{\partial Z_3(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{11}} & \frac{\partial Z_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n)}{\partial b_{36}} \\ \frac{\partial Z_2(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \\ \frac{\partial Z_3(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n)}{\partial b_{36}} \end{bmatrix} = \begin{bmatrix} \overset{*}{f}(Zin_1(n)) & 0 & 0 \\ 0 & \overset{*}{f}(Zin_2(n)) & 0 \\ 0 & 0 & \overset{*}{f}(Zin_3(n)) \end{bmatrix} \begin{bmatrix} \frac{\partial Zin_1(n)}{\partial b_{11}} & \frac{\partial Zin_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_1(n)}{\partial b_{36}} \\ \frac{\partial Zin_2(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \\ \frac{\partial Zin_3(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \end{bmatrix}$$

$$\begin{bmatrix} \frac{\partial Zin_1(n)}{\partial b_{11}} & \frac{\partial Zin_1(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_1(n)}{\partial b_{36}} \\ \frac{\partial Zin_2(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \\ \frac{\partial Zin_3(n)}{\partial b_{11}} & \frac{\partial Zin_2(n)}{\partial b_{12}} & \cdots & \frac{\partial Zin_2(n)}{\partial b_{36}} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & b_{15} & b_{16} \\ b_{21} & b_{22} & b_{23} & b_{24} & b_{25} & b_{26} \\ b_{31} & b_{32} & b_{33} & b_{34} & b_{35} & b_{36} \end{bmatrix} \begin{bmatrix} \frac{\partial Z_1(n-1)}{\partial b_{11}} & \frac{\partial Z_1(n-1)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n-1)}{\partial b_{36}} \\ \frac{\partial Z_1(n-2)}{\partial b_{11}} & \frac{\partial Z_1(n-2)}{\partial b_{12}} & \cdots & \frac{\partial Z_1(n-2)}{\partial b_{36}} \\ \frac{\partial Z_2(n-1)}{\partial b_{11}} & \frac{\partial Z_2(n-1)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n-1)}{\partial b_{36}} \\ \frac{\partial Z_2(n-2)}{\partial b_{11}} & \frac{\partial Z_2(n-2)}{\partial b_{12}} & \cdots & \frac{\partial Z_2(n-2)}{\partial b_{36}} \\ \frac{\partial Z_3(n-1)}{\partial b_{11}} & \frac{\partial Z_3(n-1)}{\partial b_{12}} & \cdots & \frac{\partial Z_3(n-1)}{\partial b_{36}} \\ \frac{\partial Z_3(n-2)}{\partial b_{11}} & \frac{\partial Z_3(n-2)}{\partial b_{12}} & \cdots & \frac{\partial Z_3(n-2)}{\partial b_{36}} \end{bmatrix}$$

$$+ \begin{bmatrix} Z_1(n-1) & Z_1(n-2) & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & Z_3(n-2) \end{bmatrix}$$

Cálculo $\frac{\partial E(n)}{\partial a_{mp}}$

$$\frac{\partial E(n)}{\partial a_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{ml} \frac{\partial Z_l(n)}{\partial a_{mp}}$$

$$Z_l(n) = f(Zin_l(n))$$

$$\frac{\partial Z_l(n)}{\partial b_{mp}} = f'(Zin_l(n)) \frac{\partial Zin_l(n)}{\partial b_{mp}}$$

$$Zin_l(n) = \sum_{j=1}^{h} \sum_{k=1}^{L} b_{l(L*(j-1)+k)} Z_j(n-k) + \sum_{j=0}^{ne} a_{lj} x_j(n)$$

$$\frac{\partial Zin_l(n)}{\partial b_{mp}} = \begin{cases} \sum_{j=1}^{h} \sum_{k=1}^{L} b_{l(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial b_{mp}} + x_p(n) & k = m \\ \sum_{j=1}^{h} \sum_{k=1}^{L} b_{l(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial b_{mp}} + 0 & k \neq m \end{cases}$$

$$\frac{\partial E(n)}{\partial b_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{ml} \frac{\partial Z_l(n)}{\partial b_{mp}}$$

$$\frac{\partial Z_m(n)}{\partial b_{mp}} = f'(Zin_m(n)) \frac{\partial Zin_m(n)}{\partial b_{mp}}$$

$$\frac{\partial Zin_m(n)}{\partial b_{mp}} = \sum_{k=1}^{L} a_{m(L*(j-1)+k)} \frac{\partial Z_j(n-k)}{\partial b_{mp}} + x_p(n)$$

$$\frac{\partial E(n)}{\partial b_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{ml} f'(Zin_l(n)) \sum_{k=1}^{L} a_{l(L*(j-1)+k)} \frac{\partial Z_l(n-k)}{\partial b_{mp}} + x_p(n)$$

$$\frac{\partial E(n)}{\partial b_{mp}} = \sum_{k=1}^{ns} e_k(n)\dot{g}_{kk} \sum_{l=1}^{h} c_{ml}\, f'(Zin_l(n)) \sum_{k=1}^{L} a_{l(L*(j-1)+k)} \frac{\partial Z_l(n-k)}{\partial b_{mp}} + x_p(n)$$

$$\begin{bmatrix} \frac{\partial Z_1(n)}{\partial b_{11}} & \frac{\partial Z_2(n)}{\partial b_{11}} & \frac{\partial Z_3(n)}{\partial b_{11}} & \frac{\partial Z_1(n)}{\partial b_{12}} & \cdots \end{bmatrix} =$$

$$\begin{bmatrix} f'(Zin_1(n)) & & \\ & f'(Zin_2(n)) & \\ & & f'(Zin_3(n)) \end{bmatrix} \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ & & \\ & & \end{bmatrix} \begin{bmatrix} \frac{\partial Z_1(n-1)}{\partial b_{11}} \\ \frac{\partial Z_2(n-1)}{\partial b_{11}} \\ \frac{\partial Z_3(n-1)}{\partial b_{11}} \end{bmatrix} + \begin{bmatrix} x_1(n) & x_2(n) & 1 & & \\ & & & & \\ & & & & \end{bmatrix} \right)$$

Propagação Direta (em Matlab)

- X = [X,ones(N,1)];
- Zold = zeros(h*L,1);
- for n = 1:N,
  - Zin(: ,n) = A*Zold+B*X(n,:)’;
  - Z(:,n) = f(Zin(: ,n));
  - Yin(:,n) = C*[Z(:,n);1];
  - Y(:,n) = g(Yin(:, n));
  - erro(n,:) = Y(n, :)’ - Yd(n**, :);**
  - Zold (2:end)= Zold(1:end-1);
  - Zold (1:L:end)= Z(:,n);
- **end**
- EQM=1/N*sum(sum(erro'*erro));