exercise3 (Score: 12.0 / 12.0)

1. Task (Score: 12.0 / 12.0)

Lab 5

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 演算法的實作可以參考lab-5 (https://yuanyuyuan.github.io/itcm/lab-5.html), 有任何問題歡迎找助教詢問。
4. **Deadline: 12/11(Wed.)**

In [1]:

```
name = "林以翎"
student_id = "B06201024"
```

(Top)

Exercise 3

Analyse the convergence properties of the Jacobi and Gauss-Seidel methods for the solution of a linear system whose matrix is

$$\left[\begin{matrix}

```
\alpha &&0 &&1\\
0 &&\alpha &&0\\
1 &&0 &&\alpha
\end{matrix}\right],
\quad \quad
\alpha \in \mathbb{R}.$$
```

By lecture slide, we have the convergence property holds for $\rho(G) = \max\{|\lambda| \mid \lambda$ is eigenvalue of $G\} < 1$.

Given

$$A = \begin{bmatrix} \alpha & 0 & 1 \\ 0 & \alpha & 0 \\ 1 & 0 & \alpha \end{bmatrix}, \quad \alpha \neq 0 D = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix}, \quad L = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad U = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Jacobi method:

$$G = -D^{-1}(L+U) = I - D^{-1}A = I - \begin{bmatrix} 1 & 0 & -\frac{1}{\alpha} \\ 0 & 1 & 0 \\ -\frac{1}{\alpha} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\frac{1}{\alpha} \\ 0 & 0 & 0 \\ -\frac{1}{\alpha} & 0 & 0 \end{bmatrix}.$$

$\begin{vmatrix} -x & 0 & -\frac{1}{\alpha} \\ 0 & -x & 0 \\ -\frac{1}{\alpha} & 0 & -x \end{vmatrix} = x\left(-x + \frac{1}{\alpha^2}\right)$, the eigenvalues are 0 and $\frac{1}{\alpha^2}$.

Then $\rho(G) < 1 \implies \frac{1}{\alpha^2} < 1 \implies |\alpha| > 1$.

Gauss-Seidel method:

$$G = -(L+D)^{-1}U = \begin{bmatrix} 0 & 0 & -\frac{1}{\alpha} \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\alpha^2} \end{bmatrix}, \text{ the eigenvalues are 0 and } \frac{1}{\alpha^2}.$$

Then $\rho(G) < 1 \implies \frac{1}{\alpha^2} < 1 \implies |\alpha| > 1$.