**Bertrand-Nash equilibrium prices and monopoly prices**

The Bertrand-Nash equilibrium condition is

,

where

and

This condition means that the firm can’t improve the reward by changing the price at the Nash-equilibrium.

Expanding the condition, we have

Consider the special case of homogenous agents (firms): and consider the homogenous solution , we have

Or,

This equation can be solved (numerically by an iterative procedure) to obtain the Bertrand-Nash price.

###We calculate 15 costs:

Given this equation, we can write the cost as a function of price:

It is a one-to-one mapping. Hence, with the price range in AER paper, we can have 15 different costs. The range can be found on p. 3274 in AER paper, we need to find the exact price in AER code. Here, we need to add a Betrand price with cost=1 in case the price range in AER does not include that price(cost=1).

To get the monopoly price each firm choose the price to maximize the total reward

For each k, we have

This leads to

Again, we consider the special case of homogenous agents (firms): , and look for solution , for all k. The above equation reduces to

This leads to

For the case of homogeneous firms, we can also get the monopoly price by setting all the prices = , and maximize the total reward with respect to :

The monopoly price should satisfy (by taking the derivative with respect to ,

This leads to

=0

The same equation for is obtained.

This equation also needs to be solved numerically.

###We cacluate the price for monoply with cost =1.

Given this equation, we can calcuate the optimal price for one firm, by setting n=1. We have pm

In particular, when the cost=1, pm=

This is different from obtaining the monopoly price, , by considering only one firm (obtained by setting n=1).

It is easy to see that

is the Bertrand-Nash price.