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Purchase Order Financing: Credit, Commitment, and Supply Chain Consequences

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Abstract. We study a supply chain where a retailer buys from a supplier who faces financial constraints. Informational problems about the supplier's demand prospects and production capabilities restrict her access to capital. By committing to a minimum purchase quantity, the retailer can mitigate these informational problems and expand the supplier's feasible production set. We assume a newsvendor model of operations and analyze the strategic interaction of the two parties as a sequential game. Key parameters in our model are the supplier's ex ante credit limit, her informational transparency—which conditions the amount of additional capital released by the commitment—and the demand characteristics of the final market. We show that in equilibrium the supplier can benefit from a lower ex ante credit limit or lower informational transparency. The retailer always benefits from an increase in these parameters. We also indicate limits to the commitment approach: under certain conditions, the retailer may prefer to relax the supplier's financial constraint by adjusting the wholesale price, or a combination of wholesale price and commitment. Our study provides a novel perspective on capital market frictions in supply chains.

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Keywords: purchase order financing • capital constraints • informational transparency • imperfect financial markets

1. Introduction

Supply chain contracts with purchase commitments have an established place in the study of operations management. Bassok and Anupindi (1997) were among the first to analyze the case that a customer guarantees to purchase at least a specified amount from a supplier over a given planning horizon. Scholars have subsequently shown that supply contracts and commitments can help firms create value, by enabling better planning of operations and minimizing risks of excess or shortage (e.g., Li and Kouvelis 1999, Durango-Cohen and Yano 2006). Nevertheless, these studies invariably assume—at least implicitly—that the firms can access a perfect capital market and are not subject to any financial constraints. Capital markets are generally imperfect, however, and supply contracts can also serve to mitigate the impact of these imperfections, yielding financial and operational benefits that have not yet been studied. Especially in the case that a supplier is a small or medium-sized enterprise (SME), a committed purchase order from a corporate customer may constitute valuable information about the supplier's demand prospects, thereby extending her access to capital. This *purchase order financing* expands the

feasible production set of the supplier, creating value for both firms. A purchase order commitment implies more risk for the customer, however, who must balance this risk against the value created. In this paper we develop and study a model for purchase order financing. We quantify the relevant capital market frictions, determine the resulting optimal commitment, and show how the operational decisions and profits are conditioned by the financial context.

Besides contributing a new dimension to the literature on supply contracts and commitments, our work addresses a topic that is of considerable practical importance. Purchase order financing is a form of pre-shipment finance, since it enables capital to be released before shipment of goods to the customer. Preshipment finance contrasts with postshipment finance, which denotes arrangements that allow firms to expedite access to the value of their receivables (Wuttke et al. 2013). Postshipment solutions have enjoyed significant attention and growth in the wake of the financial crisis of 2008, when credit to many SMEs was restricted (Demica 2013). As far as trade with emerging markets is concerned, however, some go so far as to claim that “pre-shipment financing is even more crucial than

post-shipment financing” (Demica 2011, p. 7). The use of purchase orders to enable financing is also recognized in trade literature as a means for SMEs to manage high inventories during a busy season or to meet large orders (Fullen 2006, Sinclair-Robinson 2010).

Capital market frictions—particularly informational problems—are highly likely to affect SMEs or trade with emerging markets. In such settings, demand prospects and management quality may highly be firm specific and not verifiable by providers of capital. In the absence of a long-lasting relationship between a bank and an SME, collecting and incorporating such “soft” demand information into the financing decisions becomes difficult (Degryse and Van Cayseele 2000). Lending decisions may be purely or primarily based on “hard” information, such as the value of the firm’s existing fixed assets, inventories, and accounts receivable (Berger and Udell 2002). The firm then effectively faces a *credit limit*, a hard financial constraint motivated by informational problems. Similar hard constraints have also been considered by Boyabatlı and Toktay (2011), Caldentey and Chen (2011), and Boyabatlı et al. (2016) in the operations management literature. See Caldentey and Haugh (2009) for a comprehensive discussion of their existence. Many SMEs have relatively few assets to use as a basis of collateral, however, so a hard constraint in the form of a credit limit may prevent them from operating at the first-best level.

Although it may seem that a customer could simply lend funds to a supplier facing a credit limit, such lending rarely occurs in practice. As noted in the finance literature, cash is a nondifferentiated commodity that can easily be diverted to nonproductive uses (see, e.g., Burkart and Ellingsen 2004). Monitoring usage of funds is typically either impossible or very expensive for nonfinancial firms. In contrast, financial intermediaries such as commercial banks are specialists in these matters. A committed purchase order from a reputable customer, for example, a large corporation, mitigates informational problems about a supplier’s demand prospects while leaving monitoring and technicalities of lending to a financial intermediary.

The commitment expands the supplier’s access to capital, although not necessarily to the point that the financial constraint is fully relaxed. Even when the capital market recognizes a new asset of a firm, claims can only be issued against a fraction of its full value (Turnbull 1979, Stiglitz and Weiss 1981, Boyle and Guthrie 2003). Similar to the ex ante credit limit that is imposed by collateral asset value, the *informational transparency* of the supplier conditions the amount of additional capital that is released by the customer’s commitment. In particular, although the commitment mitigates informational problems about

demand prospects, informational asymmetries regarding the supplier’s technology and its ability to deliver on the commitment may persist. Because of a lack of a financial track record and information on the operating capabilities of the supplier, finance providers may not be fully assured that she will meet the commitment. As the supplier matures and develops a track record, these informational problems are alleviated. She becomes more informationally transparent and can more easily access financing. The concept of informational transparency is well grounded in studies of financing for smaller firms and start-ups, where informational problems are particularly relevant. In that literature the inverse term *informational opacity* is commonly used (cf. Berger and Udell 1998, 2002), but the two concepts stand in one-to-one correspondence.

Building on this scenario of capital market imperfections, we study purchase order financing by means of a stylized supply chain model that fits with literature on “selling to the newsvendor” (Lariviere and Porteus 2001). Ours is rather a case of “buying from the newsvendor,” however, since we take the customer, a corporate retailer, to be the leader in the sequential game, he must decide what purchase order commitment to offer the supplier, who will respond with a production decision. In making a commitment, the retailer reduces the risk of shortage by enabling the supplier to produce more. Nevertheless, a greater commitment brings a greater risk of excess. The retailer’s optimal commitment decision is thus conditioned by the two key financial parameters: the supplier’s ex ante credit limit and her informational transparency. In this setting, our work contributes the following insights to the operations management literature.

1. The equilibrium profit levels that result from purchase order financing can exhibit properties that are not a priori evident. For example, it does not always benefit the supplier to have a high level of informational transparency: the modality of her profit as a function of informational transparency is conditioned by the relative gross margins of the firms. In some cases, the supplier’s profit will increase if her ex ante credit limit decreases. The supplier can then benefit from capital market frictions, because of her strategic interaction with the retailer.

2. Capital market frictions condition the impact of demand uncertainty on the supplier’s equilibrium profit. While an increase in demand uncertainty is always detrimental for the retailer, it may benefit the supplier if her level of informational transparency is low. As her informational transparency increases, increased demand uncertainty also becomes detrimental for the supplier.

3. A minimum purchase commitment can be the optimal recourse for the retailer even when he has the option of dictating wholesale price in the transaction

with the supplier. When informational transparency is sufficiently high, he can use commitment, price, or even both together to control the supplier's production decision. Otherwise, he exclusively uses commitment or wholesale price.

4. The supplier's credit limit and informational transparency are always substitutes for the retailer but may be substitutes or complements for the supplier. The retailer's marginal benefit from any increase in the supplier's credit limit decreases in the supplier's level of informational transparency. For suppliers with low credit limit and low informational transparency, these characteristics tend to be substitutes, irrespective of whether her profit is increasing or decreasing in either one. For suppliers with higher credit limit or informational transparency, the characteristics are complements: each mitigates the marginal effect of an increase in the other. To our knowledge, this is the first example of a supply chain model where the financial characteristics of a firm are shown to be complements or substitutes with respect to profits.

We present the research in the following seven sections. Section 2 places our work in the context of relevant literature. Section 3 presents the mathematical model and derives the equilibrium decisions, subject to the assumption that wholesale price is exogenous. Section 4 analyzes the response of the exogenous price equilibrium to changes in the financial parameters. Section 5 deconstructs the financial and risk-sharing effects underlying purchase order financing. Section 6 extends the analysis to the case where wholesale price is endogenously determined by the retailer. Section 7 considers the effect of demand variance and the value of purchase order financing from the perspective of supply chain efficiency. Section 8 summarizes the main insights.

2. Literature

An interaction between finance and operations can only arise when capital markets are in some respect imperfect and thus fail to satisfy the requirements of the Modigliani–Miller theorem (Modigliani and Miller 1958). Market imperfections such as information asymmetries, bankruptcy costs, taxes, and so forth entail that the source of financing may interact with other management decisions within the firm, and ultimately also firms' ability to create value (Mayers and Smith 1982, Smith and Stulz 1985, Froot et al. 1993).

Recognition of these interactions is not novel in finance literature (Ravid 1988), and researchers in operations management have started to show increasing concern to address them and bring financial realism to operational modeling. Several early contributions address the effect of capital constraints on manufacturing and/or inventory decisions (e.g., Archibald et al. 2002, Buzacott and Zhang 2004, Xu and Birge 2006,

Dada and Hu 2008). Others consider the coordination of operational decisions with financial decisions such as loan size (Babich and Sobel 2004). Work on these key matters continues (Alan and Gaur 2018), and additional perspectives on the interface of finance and operations have been explored. For example, the impact of potential bankruptcies in the supply chain on operational decisions (Babich 2010, Yang et al. 2015), or the effect of capital constraints on the choice between different production technologies (Boyabatlı and Toktay 2011, Chod and Zhou 2014). Our study addresses another new perspective: the potential of purchase commitments for mitigating capital market frictions. When purchase commitments are used, strategic interaction between firms can sometimes let one of them benefit from market frictions. Moreover, different types of frictions can serve as complements or substitutes for firms' profits.

The financially constrained supplier in our model presents a contrast to the orientation of other recent studies, where the financial constraints either impact the buyer (Caldentey and Haugh 2009, Kouvelis and Zhao 2011, Caldentey and Chen 2011) or both parties (Lai et al. 2009, Kouvelis and Zhao 2012). Caldentey and Chen (2011) show that it can be optimal for a wealthy supplier to take some demand risk by letting a buyer delay payment, in order to increase operating levels. Lai et al. (2009) show that when a supplier cannot fully finance her optimal production level, she will sell part of her inventory in advance to the retailer, so the latter assumes some of the demand risk. We show that purchase commitments involve a similar risk-sharing effect, but we also distinguish this from a concomitant financing effect: while our wealthy retailer's total benefit from a purchase commitment is positive, his benefit from the risk-shifting effect alone may be negative. Also, as Kouvelis and Zhao (2011) find that a supplier can lower the wholesale price in order to induce a financially constrained customer to purchase more, we show when our retailer can—or cannot—use wholesale price as an effective alternative to a minimum purchase commitment.

Minimum purchase commitments are well known in operations management. Bassok and Anupindi (1997) define the setting that has inspired much subsequent work: a buyer receives a price discount from his supplier if he commits to purchase a minimum amount. The optimal inventory policy for the buyer balances his reduction in procurement cost from commitment with his increased risk of excess. The problem has been extended to include multiple products (Anupindi and Bassok 1998), nonstationary demand (Chen and Krass 2001), or multiple commitments across a rolling horizon (Lian and Deshmukh 2009). Consistent with their focus on operations though, these studies implicitly assume that capital markets are perfect. We relax

this assumption and show how a minimum purchase commitment can also serve to mitigate capital market frictions. Even in absence of price discounts, a buyer can have an incentive to make a commitment: it can enhance his supplier's access to capital and allow her to produce more. The equilibrium commitment balances the buyer's reduced risk of shortage with his increased risk of excess.

3. Model and Equilibrium Solutions with Exogenous Wholesale Price

We consider a two-stage supply chain, where a retailer sells a product that he sources from one specific supplier. The retailer faces stochastic demand of X units; the corresponding probability distribution $F(x)$ is known to both firms. The supplier makes a single production run that concludes just prior to the revelation of demand. The supplier's unit production cost $c > 0$ and the price $p > c$ at which the retailer sells to the final market are both exogenous. The supplier realizes gross margin m_s per unit on sales to the retailer, that is, the wholesale price is $c(1 + m_s)$ per unit. The gross margin of the retailer is m_r per unit. Unsold inventory has no salvage value for either firm. Appendix A in the e-companion summarizes primary notation for the model.

In this section and the next we also take the specification of margins to be exogenous. This allows us to derive foundational results and insights. Exogenous prices are a reasonable assumption when the firms are price takers and the price is determined by market competition (Dong and Rudi 2004), when the firms have similar size, or when the wholesale price negotiations are settled in advance, so price and quantity decisions are decoupled (Erkoc and Wu 2005). In Section 6 we consider the possibility of an endogenous wholesale price.

The firms are risk neutral and seek to maximize their respective profit ("profit" here and henceforth is always "expected profit"), but the supplier has no significant liquid funds that she can invest in production. Moreover, prior to any purchase order commitment, the value of the prospective sale to the retailer is not recognized by the capital market. This may result from lack of reliable information about the business opportunity, regulatory restrictions, legal environment, etc. These initial conditions entail that the supplier can only finance production by assuming debt that can be fully secured by her current net asset value (Degryse and Van Cayseele 2000, Berger and Udell 2002). The supplier can raise funds to a maximum amount $\kappa \geq 0$ through this channel. As the debt is fully secured within this limit, it is risk free. At the end of the production run, the supplier must repay principal plus interest of $i\%$ on any debt taken. Besides the risk-free rate r_f , the rate i may reflect transaction costs

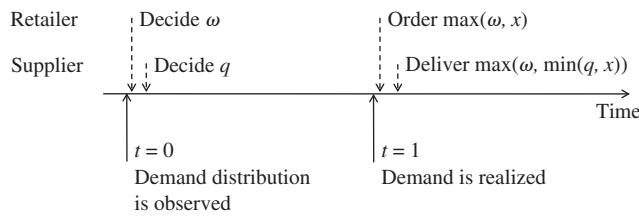
the capital market charges when issuing the loan. We must have $i < m_s$ in order for borrowing to be economically feasible for the supplier, but setting $i > 0$ entails only a constant shift in our results. We therefore set $i = 0$ without loss of generality. Similar models of risk-free borrowing with transaction costs are common in the finance literature (cf. Gamba and Triantis 2008). Allowing for risky loans to the supplier does not have a material impact on our results, provided the loans are fairly priced, but greatly limits analytical tractability.

For $j \in \{r, s\}$, let q_j^n denote the newsvendor optimum of each firm: $q_j^n \equiv F^{-1}(\alpha_j)$, where $\alpha_j = m_j/(1 + m_j)$ is the relevant critical fractile. The supplier would ideally borrow cq_s^n and produce q_s^n units. Purchase order financing becomes relevant when the supplier's net asset value and resulting credit limit are insufficient to allow this outcome. The credit limit then also constrains the maximum quantity the retailer will be able to purchase. To ensure that purchase order financing is relevant we require $0 \leq \kappa < cq_s^n$ when m_s is exogenous. In this case, the retailer may be able to improve his profit by committing to purchase $\omega > 0$ units, prior to the start of production.

The commitment relaxes the financial constraint on the supplier: it extends her credit limit, but only to a fraction of its total value. We denote this fraction by the financial parameter $\gamma \in [0, 1]$, that is, a commitment ω enables additional borrowing of $c\gamma\omega(1 + m_s)$. Like κ , γ results from capital market frictions. We assume that the retailer is risk free and bound to purchase the quantity committed, the supplier is willing and able to meet the commitment of the retailer, and the latter is aware of this through his specific knowledge of the supply chain. Nevertheless, the supplier will only be able to borrow against the full value of future revenue in the special case that providers of financing have no uncertainty about her willingness or ability to comply with the commitment contract (cf. Boyle and Guthrie 2003). The supplier generally will not have full *informational transparency* to the capital market. The lower the level of transparency, the less a commitment will extend her borrowing capacity.

At one extreme, when $\gamma = 1$, the supplier is fully informationally transparent and the capital market knows that the commitment of the retailer can and will be met. At the other extreme, when $\gamma = 0$, the supplier is fully informationally opaque and the capital market has no evidence that supplier will comply with the contract. Although a more general perspective may ultimately be of interest, we limit our attention here to commitment contracts that the supplier will always be willing to accept. Formally, this means that the supplier's informational transparency γ meets or exceeds a

Figure 1. Timing of Events



Notes. At $t = 0$, retailer and supplier observe $F(x)$. Retailer commits to purchase $\omega \geq 0$ units; supplier decides production quantity $q(\omega)$, subject to her financial constraint. At $t = 1$, demand x is observed. If $x \leq \omega$, retailer buys the committed quantity. Otherwise, he orders x and supplier delivers $\min(q, x)$.

lower threshold $\underline{\gamma} \equiv 1/(1 + m_s)$. When $\gamma = \underline{\gamma}$, a commitment from the retailer enables just enough financing to cover the variable costs associated with meeting the commitment.

Figure 1 shows the sequence of events. Upon acceptance of a commitment ω , the supplier decides a production quantity $q(\omega)$. The commitment enables her to produce at least ω units and requires the retailer to purchase at least ω units, so the supplier will choose $q \geq \omega$. (We suppress the dependence of q on ω unless explicit reference to it is needed.) Once demand x is revealed, the retailer places a final order, equal to $\max(\omega, x)$. The supplier accordingly delivers the greater of ω or $\min(q, x)$. The retailer then sells the product to the final market.

A commitment ω brings two—potentially opposing—effects for the retailer. First, by giving the supplier access to additional financing, it may lead her to increase her production level. Second, it shifts some of the risk of excess from the supplier to the retailer. A low ω entails little risk of excess for the retailer, but may provide only slight support to the supplier, leaving a large risk of shortage. A higher ω entails higher risk of excess, but may significantly enhance the supplier's production, reducing the risk of shortage. The exact extent to which ω expands the financial resources of the supplier is conditioned by γ , the supplier's informational transparency. The retailer must also recognize that the supplier will only produce *more* than ω if it is her own interest to do so.

Next we analyze the decisions of each firm. This is a sequential game where the retailer moves first, so we begin by determining the supplier's optimal response to an arbitrary commitment ω .

3.1. The Supplier's Problem

A commitment $\omega > 0$ enables the supplier to produce at least ω units and requires the retailer to purchase at least ω units. The supplier may choose to produce *more* than ω if sufficient financing is available. Given

$\omega \geq 0$, the supplier solves the following constrained optimization problem:

$$\begin{aligned} \text{maximize } \pi_s(q) &= E_X[c(1 + m_s)\max(\omega, \min(q, X))] \\ &\quad - cq \end{aligned} \quad (1)$$

$$\text{subject to } cq \leq \kappa + c\gamma\omega(1 + m_s). \quad (2)$$

The objective function (1) quantifies the supplier's profit. If the supplier chooses to produce more than ω , she will sell additional units to the extent that demand exceeds the commitment quantity. The inequality (2) represents the financial constraint: the production cost cannot exceed the financing available from debt and the retailer's commitment. Proposition 1 describes the unique optimal solution to the supplier's problem. (Proofs for this section are provided in Appendix B in the e-companion.)

Proposition 1 (Supplier's Optimal Production Decision). *Given the retailer's commitment ω , the supplier's optimal production quantity decision $q^*(\omega)$ is as follows:*

$$q^*(\omega) = \begin{cases} q_s^\kappa + \gamma\omega(1 + m_s) & \text{if } 0 \leq \omega < \omega^n, \\ q_s^n & \text{if } \omega^n \leq \omega < q_s^n, \\ \omega & \text{if } q_s^n \leq \omega, \end{cases}$$

where $q_s^\kappa \equiv \kappa/c$, $q_s^\Delta \equiv q_s^n - q_s^\kappa$, and $\omega^n \equiv q_s^\Delta/\gamma(1 + m_s)$.

The three cases identified for $q^*(\omega)$ in Proposition 1 represent solution types that will reappear through the remainder of our analysis. Consequently, we introduce the following descriptive terms for them: (i) $0 \leq \omega < \omega^n$ yields a *constrained* solution; (ii) $\omega^n \leq \omega < q_s^n$ yields an *unconstrained* solution; (iii) $q_s^n \leq \omega$ yields a *fulfilling* solution. The intuition behind each type is readily apparent. If the financial constraint (2) is not binding, the supplier faces a newsvendor problem: she produces q_s^n . Without any commitment the maximum production she can achieve is $q_s^\kappa \equiv \kappa/c$, which we assume to be less than q_s^n . The production shortfall resulting from the credit limit κ is thus $q_s^\Delta \equiv q_s^n - q_s^\kappa$. The smallest commitment that allows her to attain the unconstrained solution is $\omega^n \equiv q_s^\Delta/\gamma(1 + m_s)$. If $\omega < \omega^n$ we have the constrained solution: the supplier uses all available financing, in order to produce as close to q_s^n as possible. With this solution type, her production linearly increases with the commitment. Once $\omega \geq \omega^n$ is satisfied, we see the unconstrained solution: the supplier is able to produce q_s^n and will not produce more until $\omega \geq q_s^n$. When this latter condition is satisfied, we have the fulfilling solution: the supplier produces more than her unconstrained optimal newsvendor quantity, purely in order to satisfy the requirements of the commitment contract.

3.2. The Retailer's Problem

At time $t = 0$ the retailer chooses a commitment $\omega \geq 0$ in order to maximize his profit, anticipating $q^*(\omega)$, the

best response of the supplier. At $t = 1$, if the retailer observes final market demand in excess of ω , this demand can be met to the extent that the supplier has produced additional units. The retailer sells the lesser of realized demand and the quantity produced by the supplier. If ω exceeds realized demand, however, the retailer must fulfill his contractual obligation and buy ω units. Formally, he solves the following unconstrained optimization problem:

$$\text{maximize } \pi_r(\omega) = c(1 + m_s)E_X[(1 + m_r)\min(q^*(\omega), X) - \max(\omega, \min(q^*(\omega), X))]. \quad (3)$$

The retailer's revenue depends on demand and the supplier's production decision. His cost of procurement depends additionally on his commitment. The objective function (3) is not concave in general and may have multiple locally optimal solutions. Nevertheless, the objective function is locally concave on the regions that correspond to the three supplier solution types identified in Proposition 1. Consequently, we identify next the retailer's optimal commitment for each solution type. Proposition 2 then gives the optimal policy.

(i) Constrained solution: $0 \leq \omega < \omega^n$. In this case, the retailer's commitment does not enable the supplier reach her optimum q_s^n . The financial constraint (2) binds. The supplier produces $q^* = q_s^k + \gamma(1 + m_s)\omega$. The retailer's unique optimum $\tilde{\omega}$ here satisfies the following first-order condition:

$$m_r\gamma(1 + m_s)\bar{F}(q^*) - F(\tilde{\omega}) = 0. \quad (4)$$

The retailer faces a modified newsvendor problem, with cost of excess normalized to 1 per unit. In particular, for commitment ω , probability of excess is $F(\omega)$, cost of shortage is $m_r\gamma(1 + m_s)$ per unit, and probability of shortage is $\bar{F}(q^*)$. Condition (4) entails immediately that $\tilde{\omega} > 0$ must hold.

(ii) Unconstrained solution: $\omega^n \leq \omega < q_s^n$. Here the commitment enables the supplier to realize q_s^n , her optimal newsvendor production. In this range she will always produce $q^* = q_s^n$. Increasing the commitment would only transfer more risk of excess to the retailer: shortage risk would not change. His profit is thus decreasing with ω and it is optimal to set $\omega = \omega^n$.

(iii) Fulfilling solution: $q_s^n \leq \omega$. If the commitment meets or exceeds q_s^n , then $q^* = \omega$. The supplier meets the commitment exactly. The retailer assumes all risk of excess and effectively operates a make-to-stock business, while the supplier operates in make-to-order fashion. Only two types of fulfilling solutions are plausible. To see this, note first that the retailer's own newsvendor optimum is $q_r^n \equiv F^{-1}(m_r/(1 + m_r))$. If the retailer's gross margin is less than the supplier's, $m_r < m_s$, then $q_r^n < q_s^n$ and increasing the commitment beyond q_s^n decreases profit. If $m_r \geq m_s$, then $q_r^n \geq q_s^n$ and the retailer will prefer to set $\omega = q_r^n$. The optimum is therefore $\omega = \max(q_r^n, q_s^n)$.

Joint examination of the three local solutions yields the globally optimal policy for the retailer (Proposition 2). If $m_r < m_s$ the optimum can lie in the constrained region or at ω^n . If $m_r \geq m_s$ a point in the interior of the fulfilling region is also a possible location for the optimum: the endpoint q_s^n can never be globally optimal, as the smaller commitment ω^n induces the supplier to the same production level. The retailer will choose the fulfilling solution if it offers a higher profit than any smaller commitment. When the fulfilling solution offers lower profit, the assumption $m_r \geq m_s$ guarantees that the retailer's profit is increasing at ω^n . Along with local concavity of profit, this entails that the retailer will not commit less than ω^n . When $m_r \geq m_s$ the optimal commitment is thus exclusively ω^n or q_r^n . In the latter case the retailer takes all risk of excess in the supply chain; in the former he limits risk but only enables the supplier to produce q_s^n .

Proposition 2 (Retailer's Optimal Commitment). *In equilibrium, the retailer always makes a commitment. The equilibrium commitment quantity ω^* is*

$$\omega^* = \begin{cases} \min(\tilde{\omega}, \omega^n) & \text{if } m_r < m_s, \\ \arg \max_{\omega \in \{\omega^n, q_r^n\}} \pi_r(\omega) & \text{otherwise.} \end{cases}$$

Together, the retailer's optimal commitment and the supplier's optimal production decision entail three possible types of subgame perfect equilibrium in purchase order financing:

(i) a constrained equilibrium,

$$(\omega^*, q^*) = (\tilde{\omega}, q_s^k + \gamma(1 + m_s)\tilde{\omega});$$

(ii) an unconstrained equilibrium, $(\omega^*, q^*) = (\omega^n, q_s^n)$;

(iii) a fulfilling equilibrium, $(\omega^*, q^*) = (q_r^n, q_r^n)$.

These equilibria are mutually exclusive. Which one occurs depends on all model parameters, but we are particularly interested to see how the financial parameters γ and κ affect decisions and profits. Section 4 investigates this while maintaining the assumption of exogenous wholesale price.

4. Analysis of the Equilibrium Solution with Exogenous Wholesale Price

An immediate consequence of the preceding results is that the commitment and production quantity are locally insensitive to changes in either financial parameter when a fulfilling equilibrium occurs. In contrast, the effect of any change in γ or κ is not trivial when an unconstrained or a constrained equilibrium occurs. In these cases, the retailer's commitment, the production level of the supplier, and the profit for each will depend on the financial parameters.

Of course, a change in a financial parameter may be sufficient to entail a change of equilibrium type, and

thus also a different response to further changes. Consequently, in order to have a global understanding of the effect of changes in the financial parameters, we begin this section by determining how the parameter space is partitioned according to the ultimate type of equilibrium.

4.1. Partition of the Parameter Space with Exogenous Wholesale Price

Lemma 1 describes the partition of the parameter space. The set U denotes all combinations of the financial parameters that give rise to an unconstrained equilibrium. Conditions on the firms' gross margins specify what type of equilibrium occurs when a point in U is not reached.

Lemma 1 (Partition of the Parameter Space by Equilibrium Type). *The type of equilibrium in purchase order financing can be determined by means of the functions*

$$v(\gamma, \kappa) = m_r \gamma - F(\omega^n), \quad w(\gamma, \kappa) = \int_0^{\omega^n} F(x) dx + N(m_r),$$

where $\omega^n = q_s^\Delta / \gamma(1 + m_s)$ and $N(m_r)$ is a value that is independent of γ and κ . Specifically, if we denote the entire feasible financial parameter space as $Z \equiv \{(\gamma, \kappa): \gamma \geq 1/(1 + m_s), \kappa \geq 0\}$ and the set $U \subset Z$ as

$$U \equiv \begin{cases} \{(\gamma, \kappa): v(\gamma, \kappa) \geq 0\} \cap Z & \text{if } m_r < m_s, \\ \{(\gamma, \kappa): w(\gamma, \kappa) \leq 0\} \cap Z & \text{otherwise,} \end{cases}$$

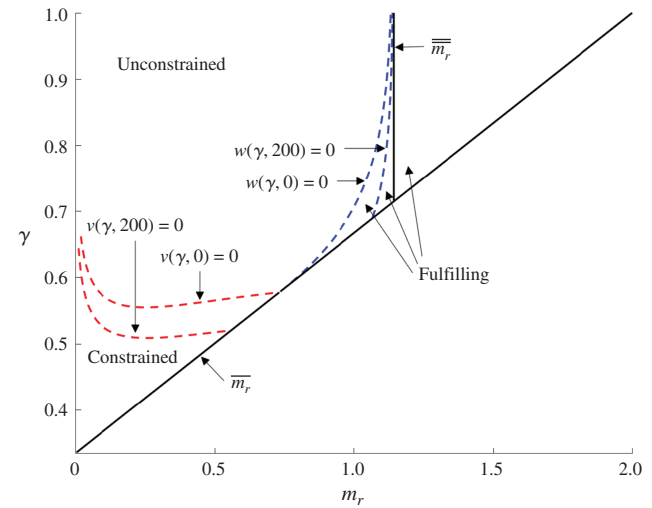
then

$$(\omega^*, q^*) = \begin{cases} (\tilde{\omega}, q_s^\kappa + \gamma(1 + m_s)\tilde{\omega}) & \text{if } (\gamma, \kappa) \notin U \text{ and } m_r < m_s, \\ (\omega^n, q_s^n) & \text{if } (\gamma, \kappa) \in U, \\ (q_r^n, q_r^n) & \text{if } (\gamma, \kappa) \notin U \text{ and } m_r > m_s. \end{cases}$$

Figure 2 illustrates the regions defined by Lemma 1 for our base case: $X \sim \mathcal{N}(2,000, 400)$, $c = 1$, and $p = 3$. The diagonal line shows the transparency requirement $\gamma \geq \underline{\gamma} \equiv 1/(1 + m_s)$. All solutions lie above this line. Since the firm's gross margins are related through $p \equiv c(1 + m_s)(1 + m_r)$, we can define the maximum level of m_r at each level of γ as $\bar{m}_r \equiv p\gamma/c - 1$. The dashed curves to the left show the set $v(\gamma, \kappa) = 0$ for $\kappa = 0$ and $\kappa = 200$. The dashed curves to the right show the set $w(\gamma, \kappa) = 0$ for the same levels of κ . As κ increases, the pairs of curves diverge, expanding the region between them.

Unconstrained equilibria occur either when the financial constraint is not very tight—that is, the financial parameters are not in the lower portion of their domain—or when retailer and supplier have similar gross margins. In either case, the retailer has a strong interest to enable just enough capital that the supplier can realize her unconstrained optimal newsvendor production.

Figure 2. Partition of the Parameter Space for Base Case



Notes. Base case has $X \sim \mathcal{N}(2,000, 400)$, $c = 1$, $p = 3$. For each γ , the diagonal line $\bar{m}_r \equiv p\gamma/c - 1$ shows the maximum value of m_r . All feasible equilibria lie above this line. Pairs of dashed curves indicate cross sections of the parameter space for $\kappa = 0$ and $\kappa = 200$. Unconstrained equilibria lie above these curves. The vertical solid line shows threshold \bar{m}_r . Only fulfilling equilibria are possible beyond this threshold.

If the financial constraint tightens—that is, simultaneously low values of κ and γ —and the retailer's gross margin is less than the supplier's, we move to the region of constrained equilibria. Here the retailer would need to make a relatively large commitment to motivate the supplier to produce her newsvendor optimum. The supplier's relatively low informational transparency means that a commitment enables little extra production, while the retailer's low margin means there is little potential loss from missed sales: low cost of shortage. The retailer therefore makes a commitment that enables some additional production, but the supplier's financing constraint still binds.

In contrast, if the financial parameters tighten and the retailer's gross margin is greater than the gross margin of the supplier, we move to the region of fulfilling equilibria in Figure 2. Here the retailer's margin is relatively high and he wants the supplier to produce large quantities, even if she has relatively low informational transparency. He commits $\omega^* = q_r^n$, his own newsvendor optimum, and thereby takes significantly larger risk of excess than in the unconstrained equilibrium.

The analysis of the parameter space reveals a further noteworthy point: there is a margin threshold \bar{m}_r beyond which only fulfilling equilibria are possible. It is the unique solution to $N(m_r) = 0$, exemplified by the vertical line in Figure 2. When the supplier's credit limit is fully relaxed, a retailer with gross margin \bar{m}_r is indifferent between the risk of committing to his own newsvendor quantity and the risk of procuring from

the supplier's own (smaller) optimal production. When $m_r > \bar{m}_r$, a retailer commits to his own newsvendor quantity even in absence of financial constraints.

We consider next how equilibrium decisions and profits respond to changes in financial conditions.

4.2. Comparative Statics for the Equilibrium Solution with Exogenous Wholesale Price

We consider first the possibility that financial conditions entail an unconstrained equilibrium. The profit functions (1) and (3) and the retailer's optimality criterion (4) then give the following results.

Theorem 1 (Comparative Statics for Unconstrained Equilibria). *Suppose (γ, κ) lies strictly inside the set U specified in Lemma 1, so an unconstrained equilibrium occurs. Each parameter then has a qualitatively similar effect on the equilibrium solution. Specifically,*

- (i) retailer's commitment ω^* is linearly decreasing in κ (convex decreasing in γ);
- (ii) supplier's production decision q^* is constant in κ (constant in γ);
- (iii) retailer's profit π_r is concave increasing in κ (concave increasing in γ);
- (iv) supplier's profit π_s is convex decreasing in κ (convex decreasing in γ); and
- (v) γ and κ are substitutes for the retailer's profit but complements for the supplier's profit.

Theorem 1 tells (i) that a reduction in either financial parameter leads to an increase of the retailer's equilibrium commitment. It is optimal (ii) for both firms that the supplier attain her newsvendor level of production, so the retailer must commit more when the supplier's access to capital is more restricted. Increased commitment implies increased risk of excess for the retailer, and thus (iii) a reduction in either financial parameter decreases his profit. The supplier meanwhile enjoys a greater commitment to enable the same level of production, so (iv) her profit increases. She benefits from capital market frictions, because of her strategic interaction with the retailer.

Part (v) of Theorem 1 adds a further dimension to the effects of the financial parameters. They are substitutes for the retailer's profit, so a reduction in one parameter reinforces the effect of a subsequent reduction in the other. For instance, if the supplier's credit limit decreases, the retailer will experience a sharper decrease in profit from a reduction of the supplier's informational transparency. The situation for the supplier is the reverse. The interaction effects are further discussed in Appendix D in the e-companion.

Theorem 1 is no longer valid if the financial parameters reduce to the point that an unconstrained equilibrium is no longer optimal. A switch from an unconstrained equilibrium to a fulfilling equilibrium can occur if and only if the prerequisite $m_r > m_s$ is satisfied. Otherwise, the switch will be to a constrained

equilibrium. Once a fulfilling equilibrium is realized, the firms' decisions and profits are insensitive to perturbation of the financial parameters. If a constrained equilibrium is realized, the supplier produces less than she would in a standard newsvendor setting. The profit functions (1) and (3) and the retailer's optimality criterion (4) then give the following results.

Theorem 2 (Comparative Statics for Constrained Equilibria). *Suppose $m_r < m_s$ and (γ, κ) does not lie in the set U specified in Lemma 1, so a constrained equilibrium occurs. Then a change in a financial parameter κ or γ will sometimes have a nonmonotone effect on the equilibrium solution and profits. Specifically,*

- (i) retailer's commitment ω^* is decreasing in κ (if the demand distribution has constant or increasing failure rate, C/IFR, then ω^* is strictly unimodal with respect to γ);
- (ii) supplier's production decision q^* is increasing in κ (increasing in γ);
- (iii) retailer's profit π_r is concave increasing in κ (increasing in γ);
- (iv) if the demand distribution is of C/IFR type, then the supplier's profit π_s is strictly unimodal with respect to κ (strictly unimodal with respect to γ); and
- (v) γ and κ are substitutes for the retailer, but complements or substitutes for the supplier.

Theorem 2 tells that the effects of the financial parameters on the supplier's production decision (ii) and the retailer's profit (iii) are qualitatively the same in the constrained case as they were in the unconstrained case. The commitment decision of the retailer (i) is however different. If the supplier's credit limit decreases in a constrained equilibrium, the retailer will still respond by increasing the commitment; but his decision is more complex in the case that the supplier's informational transparency deteriorates. The commitment proves to be a strictly unimodal function of γ , and all three qualitatively distinct outcomes are possible: the commitment can be strictly increasing, strictly decreasing, or have a mode on the interior of the constrained region. Which one of these possibilities occurs depends on the gross margins and the credit limit of the supplier. The proof of Theorem 2 identifies some special cases, using the generalized failure rate function for the demand distribution (Lariviere and Porteus 2001).

Although the retailer may increase his commitment as financial conditions tighten, such increases do not entail increased production by the supplier. The total capital available to the supplier always reduces with a decrease in either financial parameter, and since she fully uses available capital in a constrained equilibrium, her production level decreases (ii).

On account of the lower production level, it is plausible that the supplier's profit would decrease from a reduction in a financial parameter. Nevertheless, we

find that this outcome is only assured when the reduction also leads to a smaller commitment from the retailer. The latter may increase, however, so lower production does not necessarily entail lower profit for the supplier. The positive effect from the greater commitment may dominate the negative effect from reduced production. Consequently, as with the commitment itself, the supplier's profit proves to be a strictly unimodal function of each financial parameter (iv). As in the unconstrained case, we see that the supplier may benefit from capital market frictions. In light of the preceding discussion of the evolution of the commitment, it is evident that the gross margins of the firms will have an important role in determining whether such benefit is available.

The similarity in first-order profit effects for the retailer is carried through to the substitution effects (v): the financial parameters are again substitutes with respect to the retailer's profit, as in the case of an unconstrained equilibrium. In contrast to the unconstrained case, the financial parameters may here be substitutes or complements for the supplier's profit. The appearance of a substitution effect may be anticipated: the retailer's commitment, which was decreasing in each financial parameter in the unconstrained case, may here be increasing in one or both.

Appendix C in the e-companion summarizes all results from the comparative statics.

5. Financing Effect and Risk-Sharing Effect

In this section we quantify the underlying effects of purchase order financing—the financing effect and the risk-sharing effect—in order to be able to see how these effects change as the financial constraints are gradually relaxed. Although we do not model such relaxation, a heuristic motivation for this analysis is the possibility that the supply chain may accumulate wealth and/or transparency through repeated transactions.

Recall that a commitment ω has two simultaneous effects. It makes additional capital $c\gamma\omega(1+m_s)$ available to the supplier, which relaxes her financial constraint, but it also shifts risk from supplier to retailer: ω constitutes a new lower bound of sales (procurement) for the supplier (retailer). Accordingly, we begin by defining two modifications of the supplier's original problem (1)–(2). Each modification retains the essence of only one of the effects: modification (F) captures only the effect of providing additional capital, while modification (R) captures only the effect of shifting risk:

$$(F) \quad \begin{aligned} &\text{maximize } \pi_s(q) = E_X[c(1+m_s)\min(q, X)] - cq, & (5) \\ &\text{subject to } cq \leq \kappa + c\gamma\omega(1+m_s), & (6) \end{aligned}$$

$$(R) \quad \begin{aligned} &\text{maximize } \pi_s(q) = E_X[c(1+m_s) \\ &\quad \cdot \max(\omega, \min(q, X))] - cq, & (7) \\ &\text{subject to } cq \leq \kappa. & (8) \end{aligned}$$

Table 1. For $q_s^\Delta > 0$, a Summary of Financing Effects, Risk-Sharing Effects, and the Impact (\uparrow or \downarrow) of Increasing κ

EFFECT SEPARABILITY	$\Delta^F \pi_s$	$\Delta^R \pi_s$	$\Delta^F \pi_r$	$\Delta^R \pi_r$
Inseparable ($\kappa < \hat{\kappa}$)	Positive, \downarrow	Positive, \uparrow	Positive, \downarrow	Negative, \downarrow
Separable ($\kappa \geq \hat{\kappa}$)	Positive, \downarrow	Positive, \downarrow	Positive, \downarrow	Negative, \uparrow

In (F), the objective function (5) omits the risk-sharing effect that featured in (1), but the financing constraint (6) is still relaxed to the same extent as it was with the commitment. In (R), the objective function (7) is the same as (1), but the commitment does not relax the financing constraint (8).

Let $q^F(\omega)$, $q^R(\omega)$, $\pi_s^F(\omega)$, and $\pi_s^R(\omega)$ be the supplier's optimal production levels and profits from the respective modified problems. (Although there is no commitment in (F), the argument ω is needed to specify the relaxation of the financing constraint.) Given ω , the retailer's profit π_r^F from (F) is obtained by amending the production level and removing the commitment from the cost element of (3). The retailer's profit π_r^R from (R) requires only amendment of the production level in (3). The modified expressions are shown explicitly by (EC.41) and (EC.42) in the proofs for this section (Appendix E in the e-companion).

For $j \in \{s, r\}$, let π_j be the equilibrium profits from the retailer's optimal commitment $\omega = \omega^*$ (as derived in Section 3). Let π_j^0 be the optima that result when neither financing nor commitment is provided to the supplier, that is, when we simply set $\omega = 0$ throughout the original model. We propose the differences

$$\Delta^F \pi_j \equiv \pi_j^F(\omega^*) - \pi_j^0, \quad \Delta^R \pi_j \equiv \pi_j^R(\omega^*) - \pi_j^0, \quad (9)$$

to quantify, respectively, the financing and risk-sharing benefits of purchase order financing.

The effects defined in (9) account together for the entire effect of purchase order financing, except in cases where the supplier cannot take full advantage of the commitment unless additional financing is also provided. We distinguish these cases by defining $\hat{\kappa}$ to be the level of κ that gives $q_s^\kappa = \omega^*$. When $\kappa < \hat{\kappa}$ and no additional financing is provided, the supplier can only partially meet the commitment. This is possible with all equilibrium types. In contrast, when $\kappa \geq \hat{\kappa}$ we find

$$\pi_j = \pi_j^0 + \Delta^R \pi_j + \Delta^F \pi_j. \quad (10)$$

If (10) holds then we say that the effect of purchase order financing is *separable* with respect to the financing and risk-sharing effect. When $\kappa < \hat{\kappa}$, (10) does not hold and the effect of purchase order financing is *inseparable*. The supplier's inability to take full advantage of the commitment without financing then entails $\pi_s < \pi_s^0 + \Delta^R \pi_s + \Delta^F \pi_s$.

Table 1 summarizes the nature of the effects and their evolution with κ when $q_s^\Delta > 0$, that is, the supplier cannot independently realize her newsvendor optimum. The rows of Table 1 distinguish separable cases from inseparable cases. The initial entries (positive or negative) in the body of the table show the sign of each effect, and the associated arrow indicates whether the effect is increasing or decreasing in κ . Note that the sign of each effect is consistent for each firm and independent of the separability condition. Both effects are positive for the supplier: the financing effect allows her to move closer to her newsvendor optimum, while the risk-sharing effect reduces her risk of excess. The financing effect is also positive for the retailer, since product availability increases, but the risk-sharing effect is negative: product availability is unchanged while his risk of excess increases.

The financing effect for each firm is decreasing in κ . Theorems 1 and 2 show that the retailer's equilibrium commitment ω^* is decreasing in κ for constrained or unconstrained equilibria: as the supplier's credit limit increases, less additional financing is needed. With a fulfilling equilibrium, the commitment and thus the amount of additional financing are constant in κ , but the supplier's independent production ability increases, diminishing the financing effect for both firms.

The impact of κ on risk-sharing effects depends on the separability condition, $\kappa \geq \hat{\kappa}$. When the effects are inseparable (first row), the risk-sharing effect is increasing (decreasing) in κ for the supplier (retailer). The root of inseparability is the supplier's inability to take full advantage of the retailer's commitment, but as κ increases, she can increase production and shift more risk to the retailer. Once κ has increased enough that the effects become separable (second row), the impact of further increases takes the opposite sense. The equilibrium type is then necessarily constrained or unconstrained, so the retailer's commitment is less than the supplier's own newsvendor optimum. It is optimal for her to produce more than the committed amount, and this shifts risk from the retailer to her. The extent of this shift increases in κ .

Table 1 does not apply when κ increases sufficiently that $q_s^\Delta \leq 0$. The supplier can then independently finance her newsvendor optimum. If the retailer's margin is greater than the threshold \bar{m}_r , a fulfilling equilibrium will still be optimal (see discussion after Lemma 1). Otherwise, since $\omega^n \leq 0$ it will be optimal for him to make no commitment and simply procure from the supplier's independent production. This outcome reappears as an important possibility when we address the endogenization of margins in Section 6. We denote it as an *absolutely unconstrained equilibrium*.

Since the retailer optimally makes no commitment in an absolutely unconstrained equilibrium, the risk-sharing effect and financing effect are both zero. In the

case of a fulfilling equilibrium with $\omega^n \leq 0$, the financing effects are also zero, albeit for a different reason: the supplier can independently realize her newsvendor production level and will not produce more without a corresponding commitment. The risk-sharing effect for the supplier is positive and increasing in κ , as in the inseparable case of Table 1. The retailer's risk-sharing effect is initially negative when κ is just enough to give $\omega^n = 0$, but in contrast to the inseparable case of Table 1, his risk-sharing effect here is *increasing* in κ . The commitment induces the supplier to produce beyond her newsvendor optimum, and the benefit of this extra production increases faster than additional cost of commitment. The retailer's risk-sharing effect ultimately becomes positive and is maximized at $q_s^\kappa = q_r^n$. At this point, the supplier is independently able to finance production of the retailer's newsvendor optimum, and the retailer's profit from commitment is equal to his profit from purchase order financing, $\pi_r^R = \pi_r$.

Since increasing κ will always ultimately entail $q_s^\Delta \leq 0$, the previous analysis and discussion shows that the financing and risk-sharing effects will ultimately disappear if the supplier's independent credit limit increases from repeat business and the retailer's gross margin is below the threshold \bar{m}_r . If this margin condition is not satisfied, the risk-sharing effect will persist as κ increases.

6. Equilibrium Solutions with Endogenous Wholesale Price

The partition of the parameter space in Lemma 1 applies for an arbitrary specification of gross margins. To endogenize wholesale price, we have to extend our analysis in two respects. First, specify an endogenization criterion that leads to a well-defined choice of margins. Second, relax the assumption that the supplier cannot finance her newsvendor optimum. When the margins are endogenous, we cannot a priori exclude the possibility that the supplier's equilibrium margin is low enough for her to be able to finance and produce q_s^n independently.

For the first step, either retailer or supplier may plausibly choose the margins, in order to maximize his or her profit, or there may be bargaining. We assume that the retailer chooses the margins. This scenario is reasonable for supply chains with large and powerful retailers. We represent the retailer's choice of wholesale price as a choice of m_r , his own margin. Since the retail price is fixed, this is equivalent to determining m_s , the supplier's margin. The retailer determines m_r prior to contracting with the supplier, in order to optimize his ultimate profit. He then communicates m_r and his corresponding optimal commitment $\omega^*(m_r)$ to the supplier, who responds with her optimal production decision $q^*(m_r)$. Formally, the retailer's partial equilibrium profit is $\pi_r(\omega^*(m_r), q^*(m_r))$.

To solve this problem, we define two subgames where the retailer's profit is explicitly a function of m_r . These two subgames together represent all possible margin choices for the retailer. In the first subgame, the retailer chooses m_r subject to the requirement that the supplier is incapable of financing her newsvendor optimum q_s^n . Formally, the requirement is $q_s^\Delta > 0$. In this subgame, the constraint on the supplier entails that it is always optimal for the retailer to make a commitment, as we saw when the gross margins were exogenous. Moreover, the retailer's optimal choice of m_r in this subgame always entails one of the three solution types we found in the original analysis (constrained, unconstrained, of fulfilling). In the second subgame, the retailer chooses m_r subject to the requirement that the supplier can always finance her newsvendor optimum: $q_s^\Delta \leq 0$. The retailer's optimal choice of m_r in this subgame may be such that his optimal commitment is zero. We denote this choice as $m_r = m_r^0$. The supplier then responds by producing q_s^n , her newsvendor optimum. The outcome $(m_r^*, \omega^*, q^*) = (m_r^0, 0, q_s^n)$ corresponds to the *absolutely unconstrained* solution already seen in Section 5, since the supplier needs no financial assistance in order to realize her optimal newsvendor production level. The proofs for this section (Appendix F in the e-companion) include full definitions of the subgames.

When the retailer can choose his margin m_r as well as his commitment ω , he can use either or both to influence the supplier's production decision. Proposition 3 describes the equilibrium. It shows that the retailer has essentially three possible choices of margin. First, he may set it as high as possible: $m_r = \bar{m}_r$. At this level, he controls the supplier's production decision through commitment only. Second, he may set $m_r = m_r^\# < \bar{m}_r$. In this case, in addition to making a commitment, the retailer sacrifices some of his margin in exchange for a higher production level. He uses margin and commitment *together* to increase the production level of the supplier. Finally, the retailer may set $m_r = m_r^0$ and $\omega = 0$. He then uses only his margin decision to influence the supplier.

Proposition 3 (Retailer's Optimal Wholesale Price). *Let the thresholds $\gamma^u(\kappa)$ and $\gamma^f(\kappa)$ be defined as in Corollaries EC.1 and EC.2 (in the e-companion). Additionally, let*

- (i) $m_r^\#$ be the unique solution to (EC.84);
- (ii) $\bar{m}_r \equiv p[1 - F(q_s^k)]/c - 1$ be the level of m_r where $q_s^\Delta = 0$;
- (iii) m_r^0 be $\max\{\bar{m}_r, \widehat{m}_r\}$, where \bar{m}_r is the unique solution to (EC.95);
- (iv) m_r^w be the value of m_r that sets $w(\gamma, \kappa) = 0$ for $\gamma^f(\kappa) \leq \gamma \leq 1$.

The retailer's optimal choice m_r^ can then be specified by three cases, as follows.*

- (a) If $\gamma^f(\kappa) \leq \gamma \leq 1$, then

$$m_r^* = \begin{cases} \arg \max_{m_r \in \{m_r^\#, \bar{m}_r, m_r^0\}} \pi_r(m_r) & \text{if } m_r^\# < \min\{m_r^w, \widehat{m}_r\}, \\ \arg \max_{m_r \in \{\bar{m}_r, m_r^0\}} \pi_r(m_r) & \text{otherwise.} \end{cases}$$

- (b) If $\gamma^u(\kappa) \leq \gamma < \gamma^f(\kappa)$, then

$$m_r^* = \begin{cases} \arg \max_{m_r \in \{m_r^\#, m_r^0\}} \pi_r(m_r) & \text{if } m_r^\# < \min\{\bar{m}_r, \widehat{m}_r\}, \\ \arg \max_{m_r \in \{\bar{m}_r, m_r^0\}} \pi_r(m_r) & \text{if } \bar{m}_r < \widehat{m}_r \text{ and } m_r^\# \geq \bar{m}_r, \\ m_r^0 & \text{otherwise.} \end{cases}$$

- (c) If $c/p \leq \gamma < \gamma^u(\kappa)$, then $m_r^* = \arg \max_{m_r \in \{\bar{m}_r, m_r^0\}} \pi_r(m_r)$.

The option $m_r^\#$ in the first two cases comes from the first subgame. The option m_r^0 comes from the second subgame. The option \bar{m}_r results exclusively from one or the other subgame, depending on the condition $\bar{m}_r > \widehat{m}_r$. Since the first subgame is derived from the exogenous price analysis, any of the three original equilibrium types may result from it. The second subgame yields either an absolutely unconstrained equilibrium or a fulfilling equilibrium. Overall, the retailer's optimal margin decision entails the following four possible types of subgame perfect equilibrium. The proof of Proposition 3 explains the specific association of $m_r^\#$ and \bar{m}_r to possible equilibria.

- (i) A constrained equilibrium,

$$(m_r^*, \omega^*, q^*) = (\bar{m}_r, \tilde{\omega}, q_s^k + \gamma(1 + m_s)\tilde{\omega});$$

- (ii) An unconstrained equilibrium,

$$(m_r^*, \omega^*, q^*) = (m_r^\#, \omega^n, q_s^n) \text{ or } (\bar{m}_r, \omega^n, q_s^n);$$

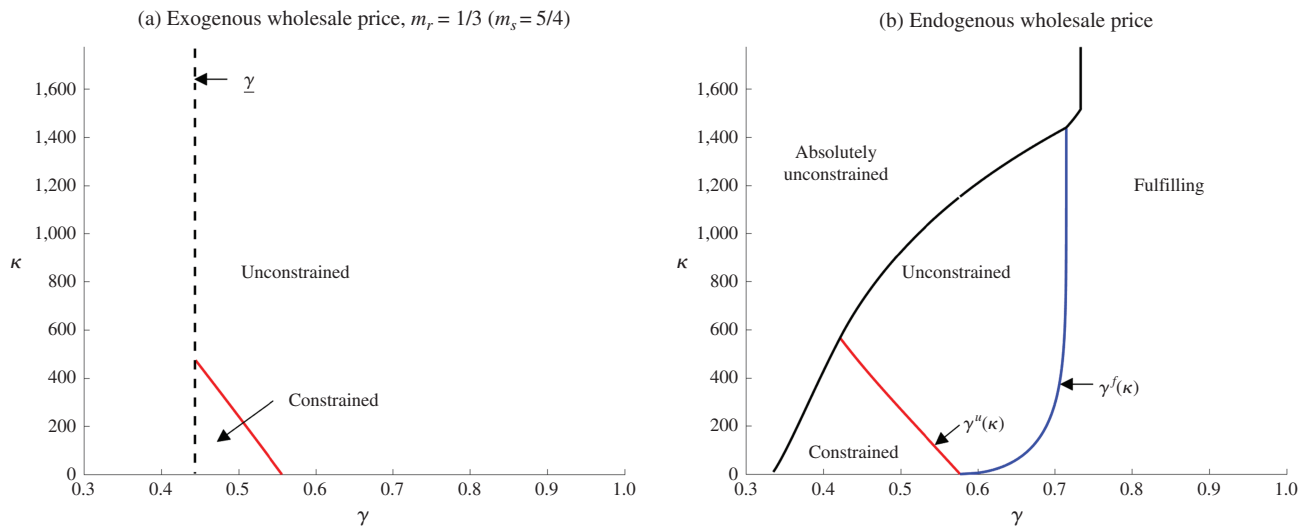
- (iii) A fulfilling equilibrium, $(m_r^*, \omega^*, q^*) = (\bar{m}_r, q_s^n, q_s^n)$;

- (iv) An absolutely unconstrained equilibrium,

$$(m_r^*, \omega^*, q^*) = (m_r^0, 0, q_s^n).$$

In a constrained or a fulfilling equilibrium, the retailer sets his margin at the upper limit: $m_r = \bar{m}_r$. The supplier's margin (and thus the wholesale price) is as low as possible: the additional funds from the commitment are just enough that she can meet the associated production cost. In the constrained case, the financial constraints are so tight that the retailer cannot increase his margin further without rendering the transaction infeasible for the supplier. In the fulfilling case, the supplier's operation is essentially a make-to-order business, so the lowest possible wholesale price is the evident choice for the retailer.

With an unconstrained equilibrium, the lowest possible wholesale price does not always occur. In addition to a commitment, the retailer may sacrifice some of his margin, in order to increase the supplier's production. Intuitively, this combined use of commitment and wholesale price occurs when retail price is relatively

Figure 3. Partition of the Financial Parameter Space: Exogenous Wholesale Price vs. Endogenous Wholesale Price

Notes. When wholesale price is exogenous (Figure 3(a)) two types of equilibrium are possible. The minimum transparency requirement is $\underline{\gamma} \equiv 1/(1 + m_s)$. When wholesale price is endogenous (Figure 3(b)) four types of equilibrium are possible. A transparency requirement is irrelevant to absolutely unconstrained equilibria; with the other types it is internalized by the optimal wholesale price.

high. The cost of shortage then justifies that the retailer lower his own margin in order to reduce the risk of shortage. This “interior” variant of unconstrained equilibrium first appears when we increase the retail price in our base case from $p = 3$ to around $p = 10$. (The precise threshold for p depends on the prevailing level of γ .)

For our base case, Figure 3 shows the effect of endogenizing wholesale price. In Figure 3(a) the price is exogenous and $m_r < m_s$. In accordance with Lemma 1 we see that only constrained or unconstrained equilibria are possible. (When $m_r > m_s$, the partition appears qualitatively similar, but fulfilling equilibria appear toward the lower left corner.) In Figure 3(b) the price is endogenous. In accordance with Proposition 3, four types of equilibrium are possible.

Mutatis mutandis, the intuition for the original three equilibrium types remains valid when price is endogenous. Unconstrained equilibria occur in the central region: either κ is relatively large, or the retailer’s decision yields similar margins for each firm. If κ is relatively small and γ decreases (increases), the margins diverge and we have constrained (fulfilling) equilibria. There is no fixed minimum transparency threshold in Figure 3(b), since absolutely unconstrained equilibria are possible. The retailer then makes no commitment, so transparency is irrelevant.

The main difference between Figures 3(a) and 3(b) is the region of absolutely unconstrained equilibria. When κ is sufficiently high, it can be optimal for the retailer to make no commitment: he can then set the supplier’s margin below what she would need to meet the cost of production implied by a commitment. The

supplier then independently produces a lower quantity than what the commitment strategy would have entailed, but the retailer has no risk of excess and realizes a significantly greater margin per unit. The threshold for the switch to an absolutely unconstrained equilibrium increases in γ , as the latter increases the value of a potential commitment.

For low levels of γ , either constrained or absolutely unconstrained equilibria are optimal in the endogenous price setting (see part (c) of Proposition 3). The retailer exclusively uses margin or commitment to influence the supplier’s production. When γ is sufficiently high that an unconstrained equilibrium is possible, the retailer may use margin and commitment together to influence the supplier (parts (a) and (b) of Proposition 3). Intuitively, lack of informational transparency constrains the retailer; as this constraint is relaxed, his optimal decision space expands.

The complexity of the equilibrium solution for endogenous margins precludes a full analytic treatment of the effect of varying γ or κ , but in the special case that \bar{m}_r is the equilibrium margin, it is clear that Theorems 1 and 2 are still valid for perturbations of κ .

7. Computational Studies: Demand Variance, Supply Chain Efficiency

In this penultimate section we investigate two aspects of our model that are not amenable to general analysis. Section 7.1 examines the effect of demand variance on the equilibrium solution. Section 7.2 compares total supply chain profit (retailer profit plus supplier profit) with and without purchase order financing, using total profit from an efficient (centralized), financially

unconstrained supply chain as a benchmark. The main insights from these experiments remain valid when wholesale price is endogenously determined, as in Section 6, but we let wholesale price be exogenous here, so that the focal effects are not obscured by changes in the underlying margins.

7.1. Demand Variance

Demand variance affects both operational decisions: the commitment of the retailer and production by the supplier. Intuitively, demand variance entails higher risk and cost for both firms, but the response of the retailer to this higher risk conditions the impact of variance on the supplier's profit. As our model is rooted in the newsvendor setting, the supplier's critical ratio α_s is essential for distinguishing the effect of demand variance. We consider two scenarios $\alpha_s > 1/2$ and $\alpha_s < 1/2$. We fix the supplier's credit limit at $\kappa = 0$ and increase the retail price in our base case to $p = 4$, as these allow the effect of demand variance to be more visibly pronounced.

Demand variance has two distinct and potentially opposing effects on the supplier's profit. First, higher variance tends to increase the risk of excess or shortage, thereby decreasing the supplier's profit. We call this the mismatch effect. It is well known from operations management theory in the absence of financial considerations. Second, the financial dimensions in our model entail that the retailer's commitment may increase with demand variance, reducing the supplier's risk of excess and tending to increase her profit. We refer to this as the commitment effect. The combination of these two effects determines the net impact of increased variance on the supplier's profit.

Figure 4 shows a case where $\alpha_s > 1/2$. The subfigures show the retailer's commitment (4(a)), the supplier's production (4(b)), retailer's profit (4(c)), and supplier's profit (4(d)) for $\sigma \in [0, 600]$ and different levels of γ , the latter being indicated directly on the lines. As with the financial parameters (see Section 4), changing variance may also change the type of equilibrium solution. In

Figure 4. Impact of Demand Uncertainty on Decisions and Profits for a Case with $\alpha_s = 7/12$

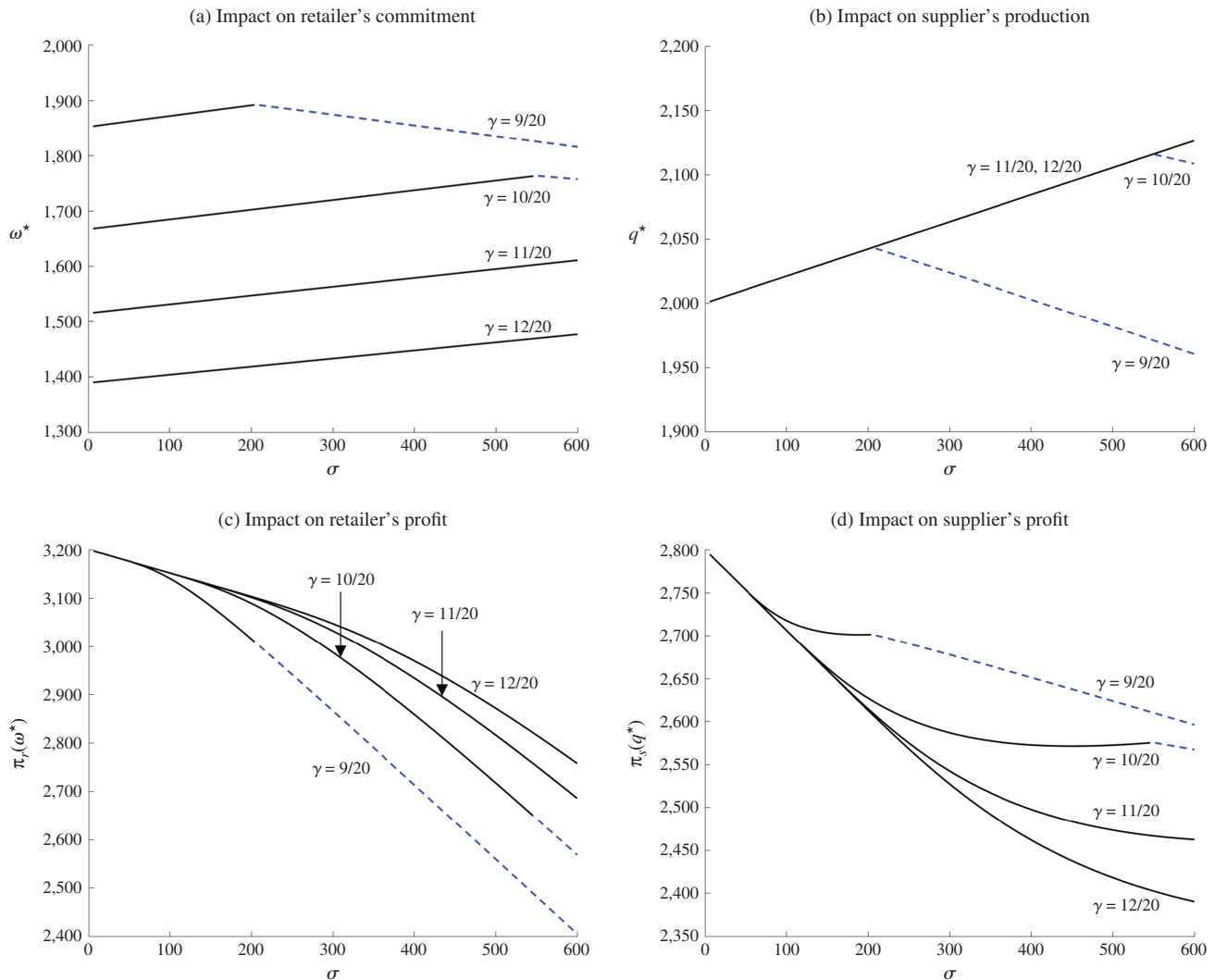


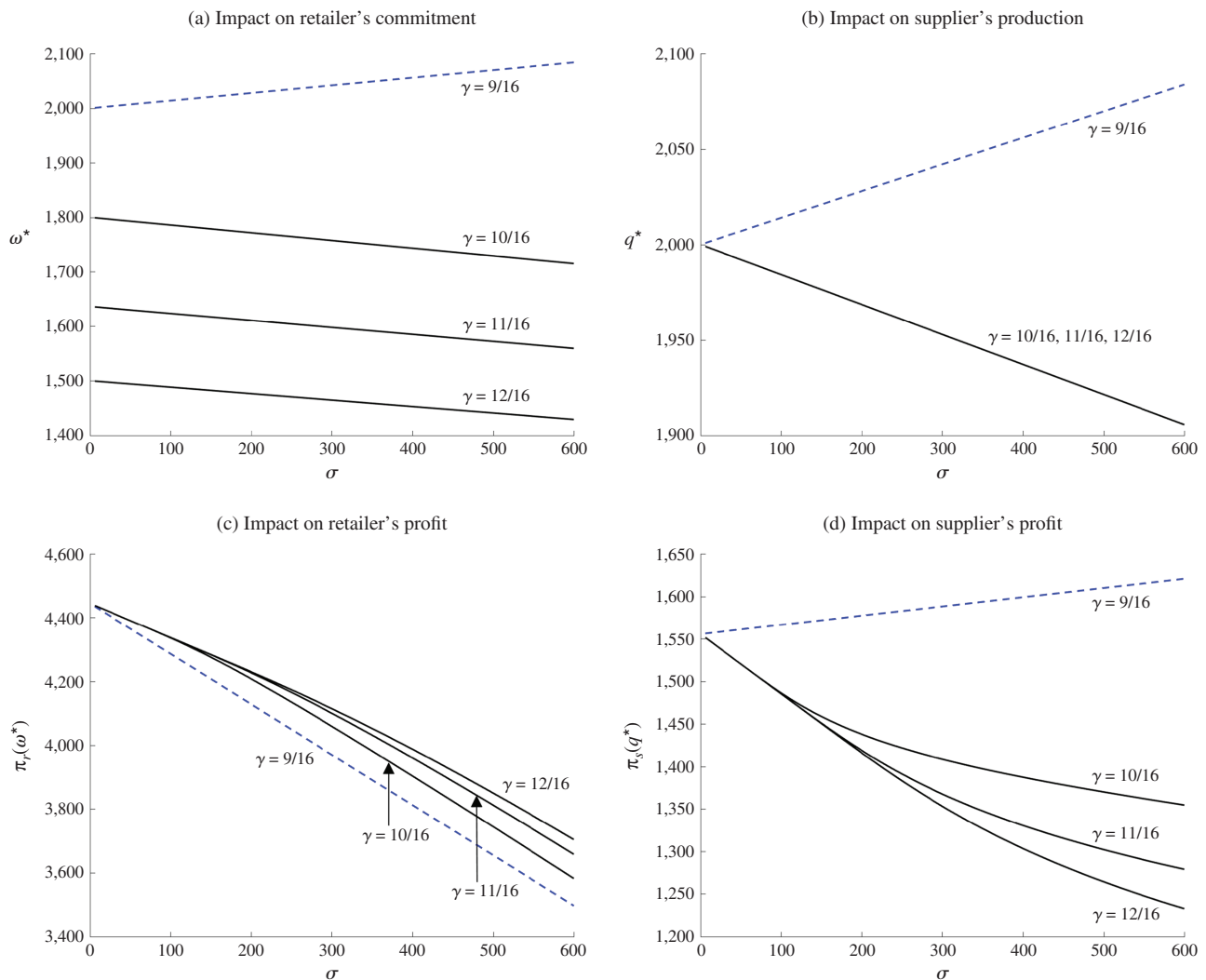
Figure 4, broken lines show constrained equilibria and solid lines show unconstrained equilibria.

The identification of equilibrium type proves to be useful for explaining the effect of demand variance. In the cases of unconstrained equilibrium, the supplier's equilibrium production quantity q_s^n increases with variance; the retailer's commitment ω^n therefore also increases with variance. These effects are evident in, respectively, Figures 4(b) and 4(a). The increase in commitment creates a positive counter effect to the negative mismatch effect of increased variance on the supplier profits. Whether the mismatch or the commitment effect dominates also depends on γ , which determines the sensitivity of the commitment to the change in demand variance. Since q_s^n is increasing in variance but independent of γ , the definition of ω^n (Proposition 1) implies that the commitment will increase more rapidly in variance at lower levels of γ . Consequently, in these cases the commitment effect may dominate the mismatch effect. This is what we see in Figure 4(d), for the

case $\gamma = 10/20$: the supplier can benefit from increased variance. For higher values of γ (e.g., $\gamma = 12/20$) the retailer can accommodate the increase in variance by a less sharp increase in commitment and the supplier's profit decreases. On the other hand, when variance is high enough relative to γ that a constrained equilibrium occurs (e.g., $\gamma = 9/20$ in Figure 4), then an increase in variance decreases the equilibrium quantity and the commitment, reducing the supplier's profit.

Figure 5 shows an analogous set of experiments, with $\alpha < 1/2$. Solid lines show unconstrained equilibria and broken lines show fulfilling equilibria. When an unconstrained equilibrium occurs, an increase in variance reduces the supplier's newsvendor level as well as the retailer's commitment. Mismatch and commitment both have a negative effect on the supplier's profit, which decreases in variance at all levels of γ . When a fulfilling equilibrium occurs, however, the retailer's commitment may increase with variance, increasing the supplier's production and profit.

Figure 5. Impact of Demand Uncertainty on Decisions and Profits for a Case with $\alpha_s = 7/16$



7.2. Supply Chain Efficiency

There are two distinct impediments to supply chain efficiency in our model: decentralization and capital market frictions. The former is well known and studied in literature on operations management, but there is limited work on the latter. We quantify the effect of each of these impediments for our base case, in order to see how well purchase order financing is able to mitigate the frictions.

Although our model features two types of market friction—the supplier’s credit limit κ and informational transparency γ —it is evident that the need for purchase order financing disappears as κ becomes arbitrarily large. When the supplier is independently able to finance her production, decentralization is the only source of supply chain inefficiency. Consequently, our experiments focus on total supply chain profit as a function of κ , and we note in passing how changes in γ affect the results. We increase demand variance to $\sigma = 600$ in the base case, in order to let the effect of decentralization be more visibly pronounced. Other parameters are $m_r = 1/2$ ($m_s = 1$) and $\gamma = 1/2$.

The benchmark scenario ① in Figure 6 is a centralized chain where the supplier is not financially constrained; total profit is thus independent of the level of κ indicated on the horizontal axis. For each level of κ , scenario ② is a decentralized supply chain without purchase order financing, and scenario ③ is the decentralized supply chain with purchase order financing. Based on these scenarios, we distinguish ④ the loss in profits due to decentralization from ⑤ the loss in profits due to frictions. This entails that we can ultimately

isolate ⑥ the additional profit enabled by purchase order financing.

The initial, increasing portion of ③ shows constrained equilibria. In this region, increases in κ relax the financial constraint on the supply chain and allow for higher total profit. For the same reason, although not shown in Figure 6, increasing γ at a low level of κ also increases total profit.

The level portion of ③ shows unconstrained equilibria. Once κ and γ are jointly sufficient to realize this outcome, further increases in either parameter do not affect total profit: they benefit the retailer at the expense of the supplier (see Theorem 1). Consequently, when a constrained equilibrium occurs, we see that increases in κ or γ increase the size of mitigation effect ⑥, relative to the loss due to frictions ⑤. Once κ and γ are jointly sufficient to induce an unconstrained equilibrium, purchase order financing fully mitigates capital market frictions: only the loss due to decentralization persists. As we should expect though, subsequent increases in κ further reduce the loss due to frictions and the mitigation effect. Both effects ultimately disappear, once κ is sufficient that the supplier can independently finance her optimal newsvendor production, that is, at the intersection of ② and ③.

8. Conclusion

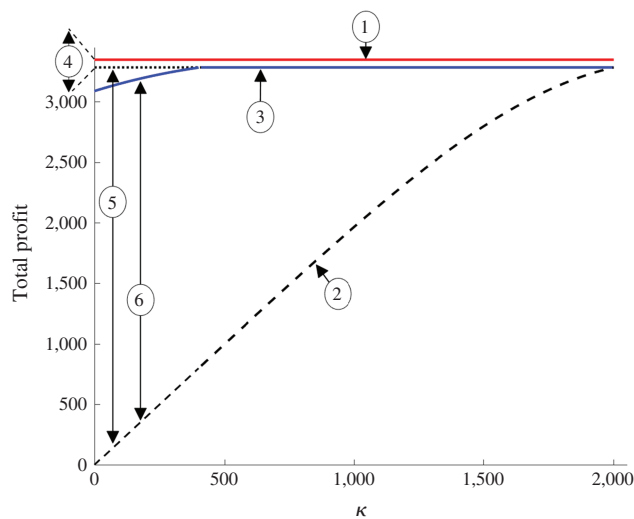
Our study illuminates the dynamics of purchase order financing. In a sequential game where a single transaction occurs between a retailer and his supplier, we show how the financial characteristics of the supplier influence the operational decisions and profits of both firms.

The retailer’s profit increases in the supplier’s ex ante credit limit and her informational transparency. In contrast, greater levels of either of these parameters may lead to a decrease in the supplier’s profit. A retailer needs to make a smaller commitment to induce a relatively creditworthy or informationally transparent supplier to the equilibrium level of production. Consequently, the supplier may have an interest to misrepresent her credit capacity, communicating it to be smaller than it truly is, or to present low informational transparency to the capital market. She then benefits from the retailer’s larger equilibrium commitment.

Nonetheless, the profitability of each firm has an important bearing on the equilibrium. If financial conditions are tight and the retailer’s gross margin is low, commitment will be small and the supplier can benefit from any relaxation in financing. If financial conditions are tight and the retailer’s gross margin is high, his optimal commitment may be large: he assumes all risk of excess.

We also observe that the financial parameters are substitutes for the retailer’s profit, but may be substitutes or complements for the supplier’s profit. Where the

Figure 6. Supply Chain Efficiency



Notes. The benchmark supply chain ① is centralized and financially unconstrained. Decentralized chains ② and ③ reflect, respectively, the absence or presence of purchase order financing. Differences ④ and ⑤ thus result, respectively, from decentralization and capital market frictions, and effect ⑥ is the mitigation of frictions by purchase order financing.

underlying effects of purchase order financing are concerned, we show that the risk-sharing effect is generally negative for the retailer, even though the total effect (including the financing effect) is positive.

When the firms' margins are available as a decision variable for the retailer, scenarios can still exist where commitment plays an important role. Informational transparency conditions the retailer's options. When the supplier has low informational transparency, the retailer exclusively uses margin or commitment to influence her production. At higher levels of transparency, these options remain, but the equilibrium can also entail using margin and commitment together.

Our computational studies reveal cases where the supplier benefits from increased demand variance. This contrasts with general operational intuition, and results from interactions triggered by the commitment in the presence of capital market frictions. Finally, we show that purchase order financing significantly mitigates the effect of capital market frictions, which enables the firms' combined profit to come close to the benchmark set by a coordinated, efficient supply chain.

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