HW8

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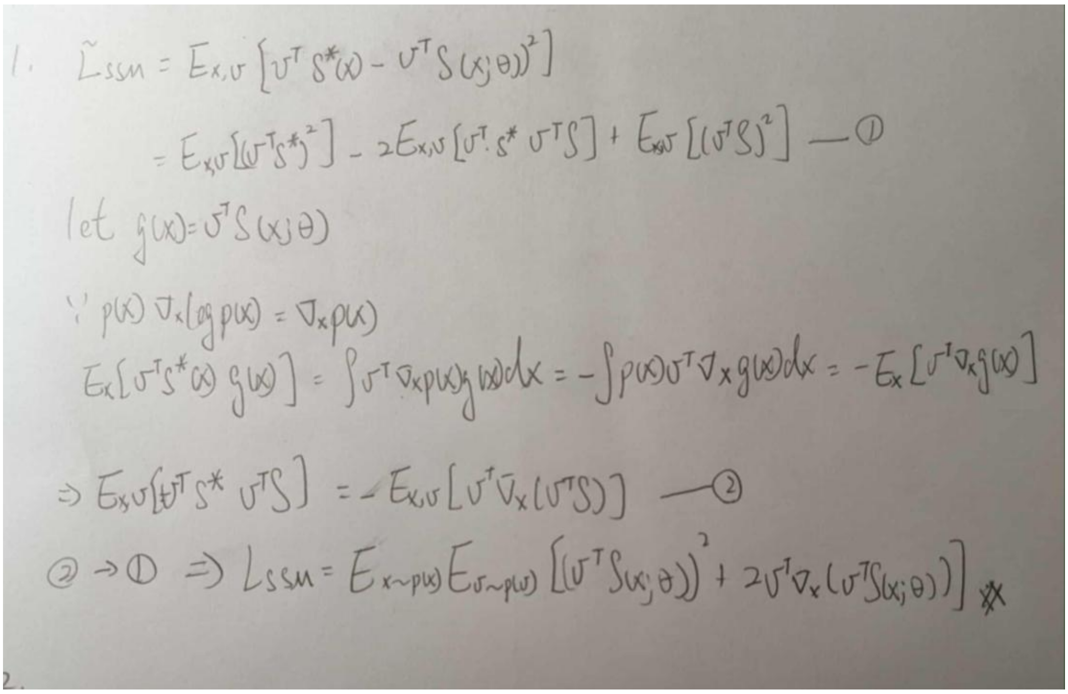
1.



1. $\tilde{L}_{ssm} = E_{x,v}\left[(v^T s^*(x) - v^T s(x;\theta))^2\right]$

$= E_{x,v}\left[(v^T s^*)^2\right] - 2E_{x,v}\left[v^T s^* \, v^T s\right] + E_{x,v}\left[(v^T s)^2\right]$ — ①

let $g(x) = v^T s(x;\theta)$

$\because p(x)\nabla_x \log p(x) = \nabla_x p(x)$

$E_x\left[v^T s^*(x)\, g(x)\right] = \int v^T \nabla_x p(x)\, g(x)\, dx = -\int p(x) v^T \nabla_x g(x)\, dx = -E_x\left[v^T \nabla_x g(x)\right]$

$\Rightarrow E_{x,v}\left[v^T s^* \, v^T s\right] = -E_{x,v}\left[v^T \nabla_x (v^T s)\right]$ — ②

② → ① $\Rightarrow L_{ssm} = E_{x\sim p(x)} E_{v\sim p(v)}\left[(v^T s(x;\theta))^2 + 2 v^T \nabla_x (v^T s(x;\theta))\right]$ ※

2.

SDE 描述含隨機擾動的連續時間動態：

$$dX_t = f(X_t, t)\, dt + g(X_t, t)\, dW_t$$

其中 f為漂移（drift）、g 為擴散（diffusion/噪聲強度）、W_t 為布朗運動。對應的密度 $p_t(x)$依 Fokker–Planck 方程 演化。

在擴散模型中：前向 SDE 逐步向資料加噪，使 p_0走向高斯 p_T；反向時間的 reverse SDE

$$dX_t = \left[f(X_t, t) - g(X_t, t)^2 \nabla_x \log p_t(X_t)\right] dt + g(X_t, t)\, d\bar{W}_t$$

利用 score 把分佈推回資料分佈，據此從噪聲逐步生成樣本。

3.

How does the Ito integral differ conceptually from the standard Riemann integral, and why can't we define it in the usual way?