COMP -582: Design and Analysis of Algorithms

Flipped Class Offering

Fall 2017

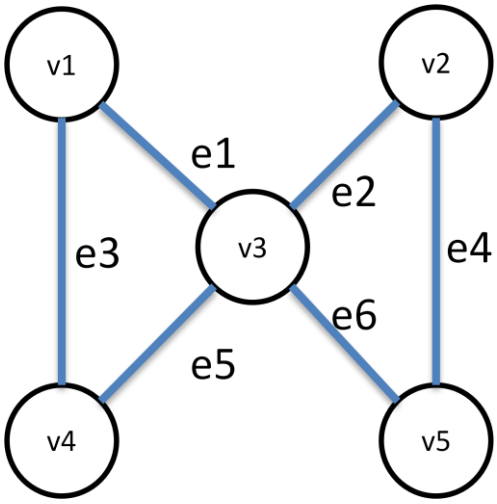
Instructor: Krishna V. Palem

Lead TA: Mohamed Abdelrahman (ma65)

Worksheet 4: Binary Search Trees, Balanced Binary Search Trees, Undirected Graphs

Date: Thursday – September 14, 2017
Due in Class on: Thursday - September 21, 2017

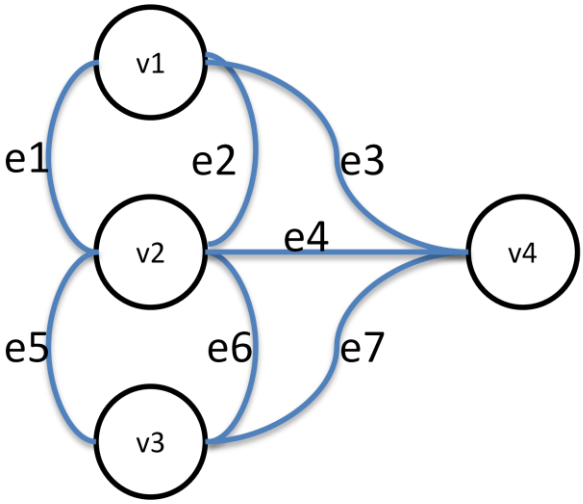
1. (20) **Definition: A walk** is an alternating sequence of vertices and edges, beginning and ending with a vertex, where each vertex is incident to **both** the edge that precedes it and the edge that follows it in the sequence, and where the vertices that precede and follow an edge are the end vertices of that edge. Consider the following graph:



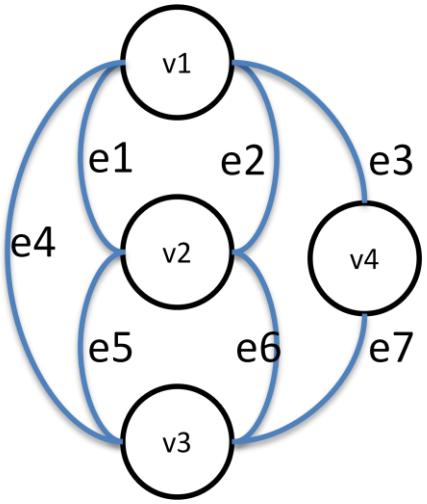
A walk could be v1, e1, v3, e6, v5, e4, v2, e2, v3, e5, v4.

An Eulerian path is a walk in a graph *which visits every edge exactly once and the start vertex is the same as the end vertex*. Note that a node can be revisited.

In our example, an Eulerian path would be v1, e1, v3, e6, v5, e4, v2, e2, v3, e5, v4, **e3, v1** (the edge and node in bold are added to make the path Eulerian).



Graph A



Graph B

Given the above two graphs A and B different only through edge e4.

a) **(10)** Does A contain an Eulerian path? If so write down such a path.

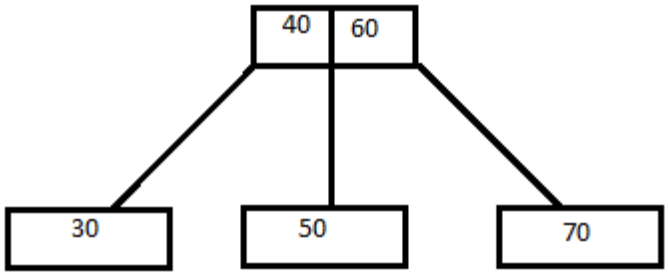
b) **(10)** Does B contain an Eulerian path? If so write down such a path.

2. (20) Graph Enumeration.

How many different **undirected graphs** are possible with V vertices? Give some reasoning
Note: Parallel-edges or Multi-edges are not allowed. It means a pair of vertices can be joined by at maximum 1 edge.

3. (20) Reverse a BST**:** Given a standard binary search tree (BST) design a linear-time algorithm to transform it into a **reverse binary search tree** (rBST). The resulting tree shape should be symmetric to the original one. In BST, each key is greater than the keys in its left subtree and smaller than the keys in its right subtree. In rBST, each key is smaller than the keys in its left subtree and greater than the keys in its right subtree.

4. (20) Consider the following 2-3 tree:



What would be the result for the following insert operations, assuming that the result of one operation affects the results of operations after it. Draw the resulting 2-3 tree after each insert operation.

a) **(10)** Insert (20).

b) **(10)** Insert (10).

5. (20) Given the set {6, 22, 9, 14, 13, 1, 8}, construct a left-leaning red-black tree that has:

a) **(10)** the maximum number of red edges.

b) **(10)** the minimum number of red edges.

Note: You have to give reasoning to your answers.