

COMP 540 HW 6
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1: Hidden Markov Models

1. Identifying the set of observables O , the set of hidden states S and the parameters $\lambda = [\pi, a, b]$, the initial state distribution, the transition matrix and the emission matrix. Draw the corresponding graphical model.

Solution. The set of observables is $O = \{low, medium, high\}$. The set of hidden states is $S = \{healthy, unhealthy\}$

The parameters λ , here we use H and N to represent hidden states, and L, M and H to represent observables low, medium and high.

The transition matrix a :	State	H	N
	H	0.8	0.2
	N	0.2	0.8

The emission matrix b :	O/S	L	M	H
	H	0.5	0.3	0.2
	N	0.3	0.3	0.4

The initial state distribution $\pi = [0.5, 0.5]$

2. Calculate the probability that the patient is healthy at $t = 2$ given that the test readings at $t = 1$ and $t = 2$ are both low.

Solution. What we want to find is $p(x_2|L, L)$, so we can use filtering to calculate that. When $t = 1$

$$\alpha_1(H) = b_H(e_1) \sum_{i=0}^n \alpha_0(i) a_{iH} = 0.5 \times 0.5 = 0.25$$

$$\alpha_1(N) = 0.3 \times 0.5 = 0.15$$

also we can calculate when $t = 2$

$$\alpha_2(H) = 0.5 \times 0.23 = 0.115$$

$$\alpha_2(N) = 0.3 \times 0.17 = 0.051$$

so the probability that the patient is healthy at $t = 2$ given that the test readings at $t = 1$ and $t = 2$ is that

$$p(x_2 = H|L, L) = \frac{0.115}{0.115 + 0.051} = 0.693$$

3. What is the most likely state sequence for $t = 0, 1, 2$ given the evidence from the previous subpart?

Solution. H-H-H