

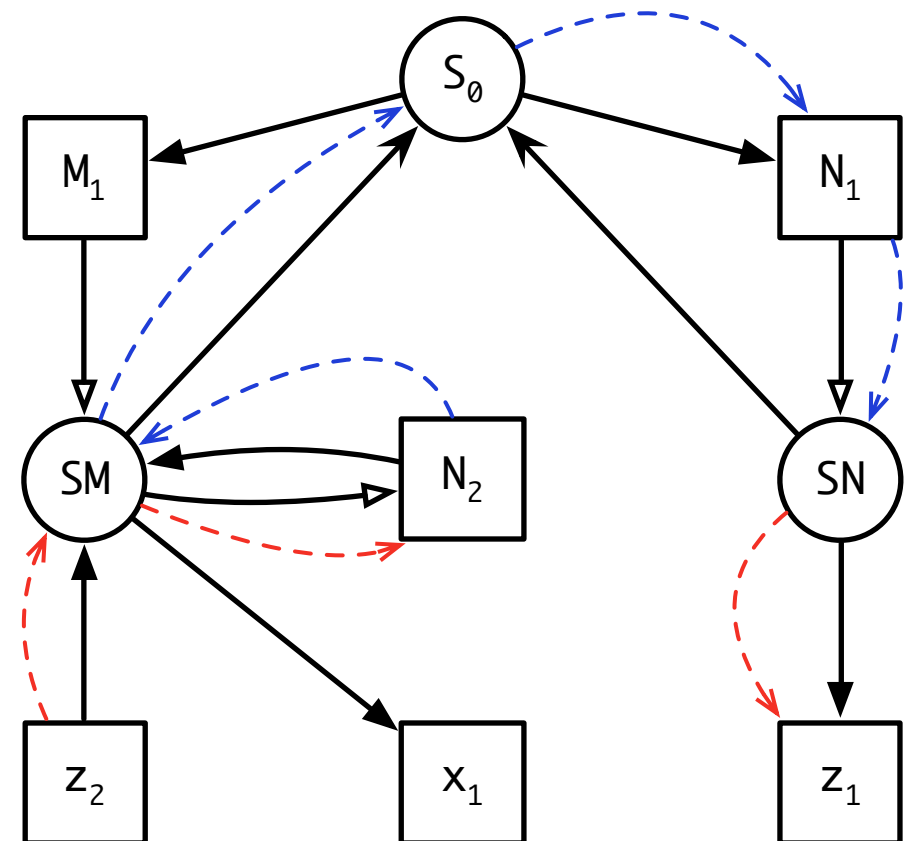
A Theory of Name Resolution

Pierre Neron¹

Andrew Tolmach²

Eelco Visser¹

Guido Wachsmuth¹



Name Resolution?

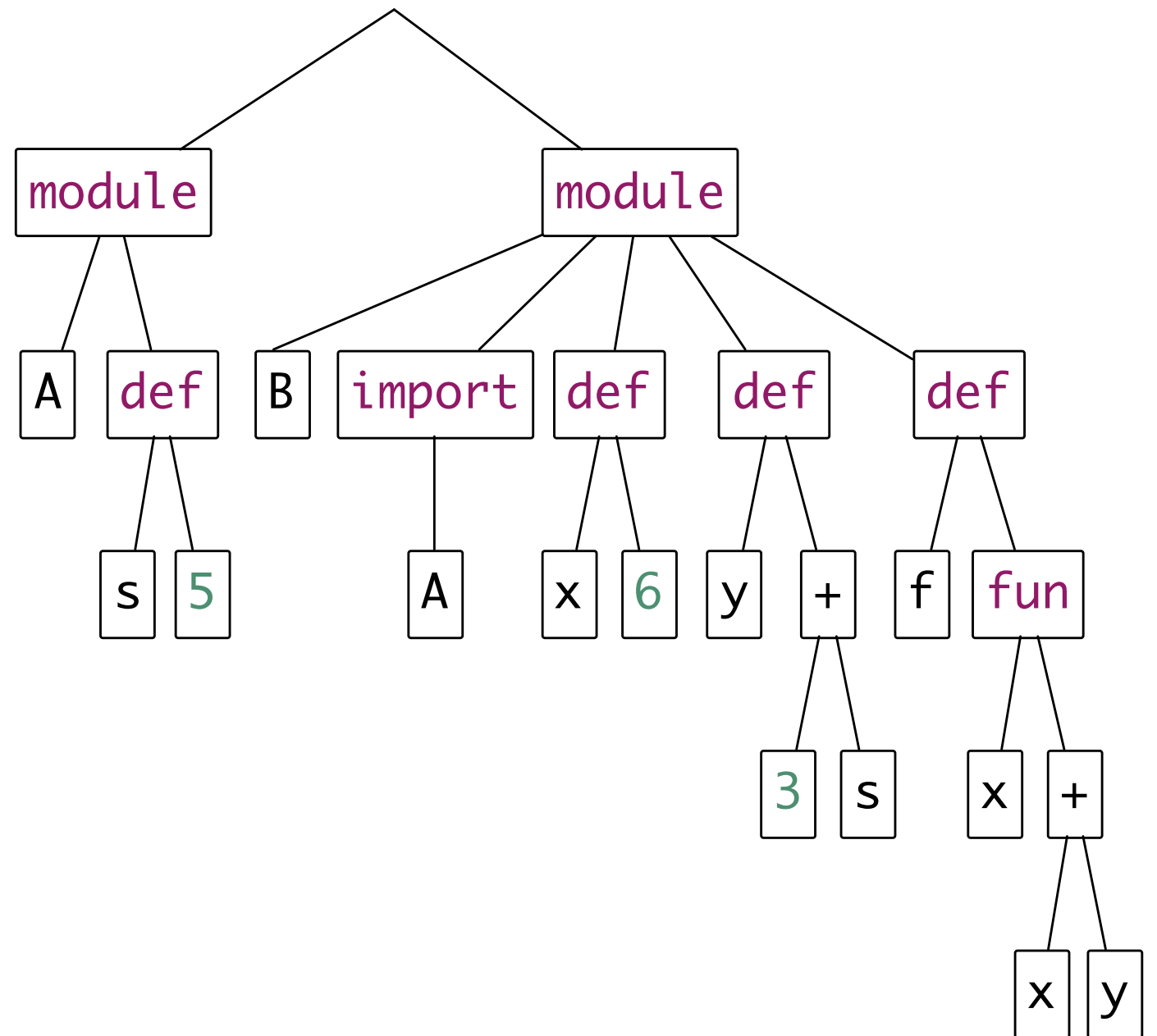
```
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```

```
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        fun x { x + y }  
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```

Name Resolution?

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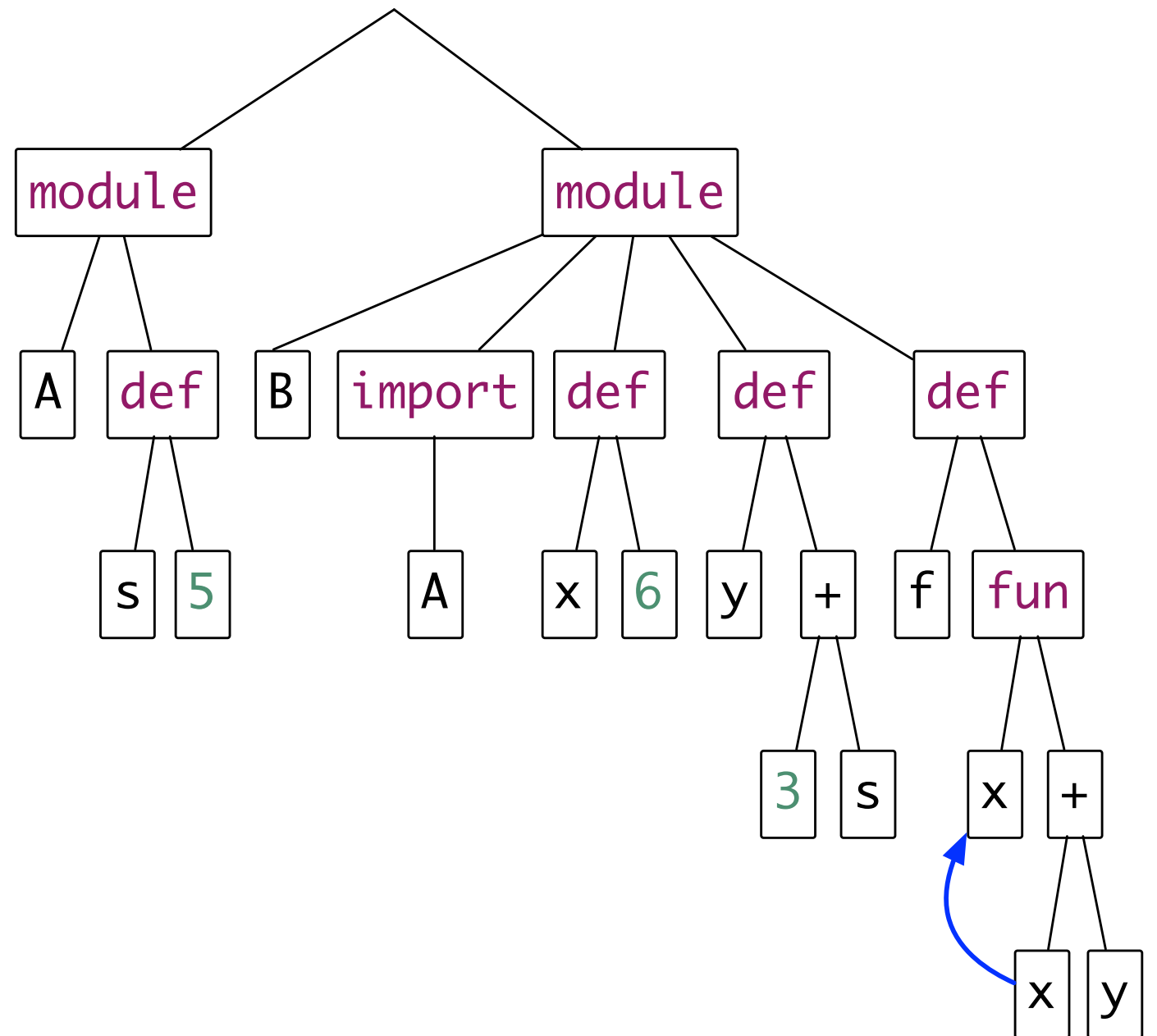
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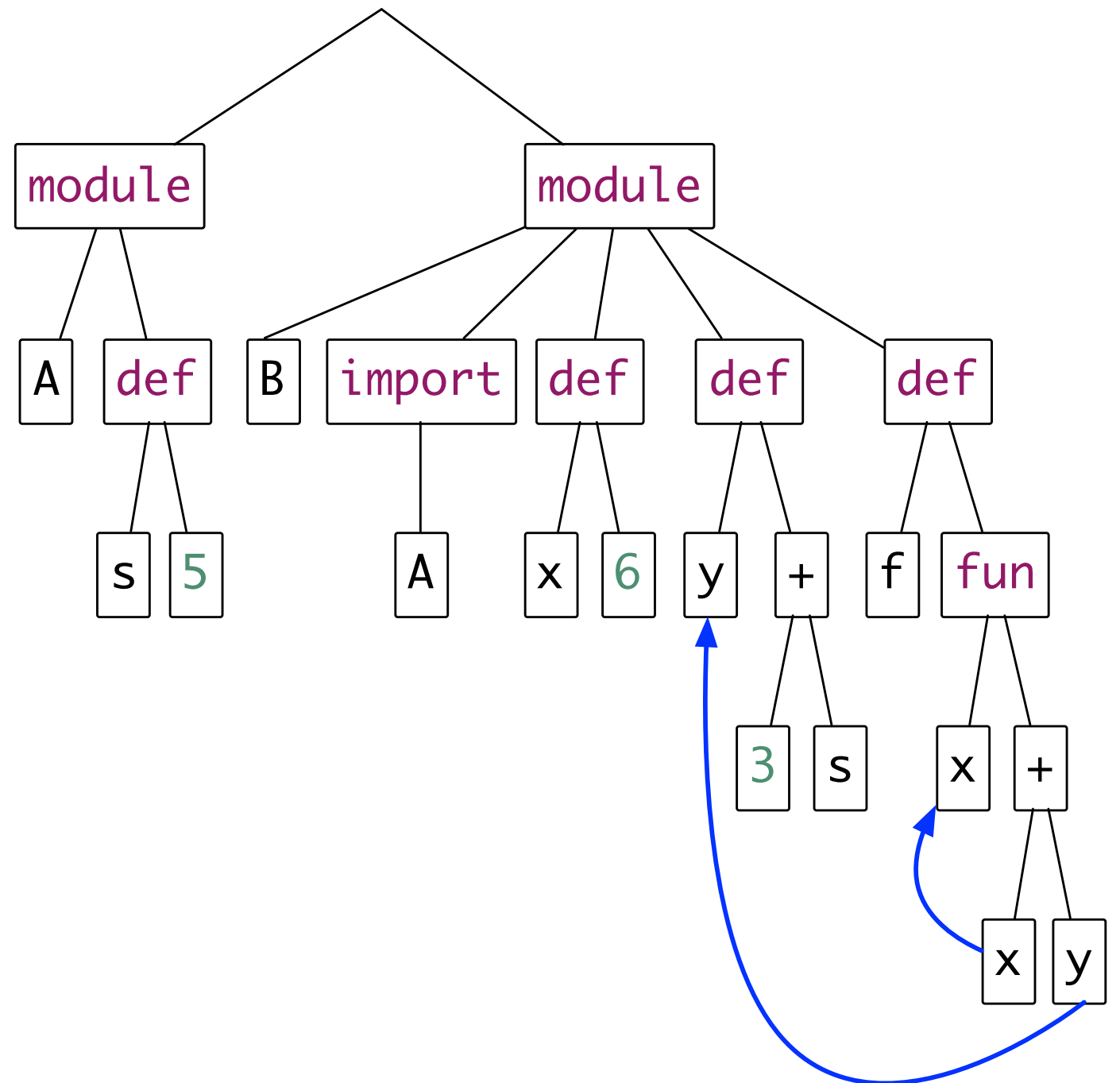
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Name Resolution?

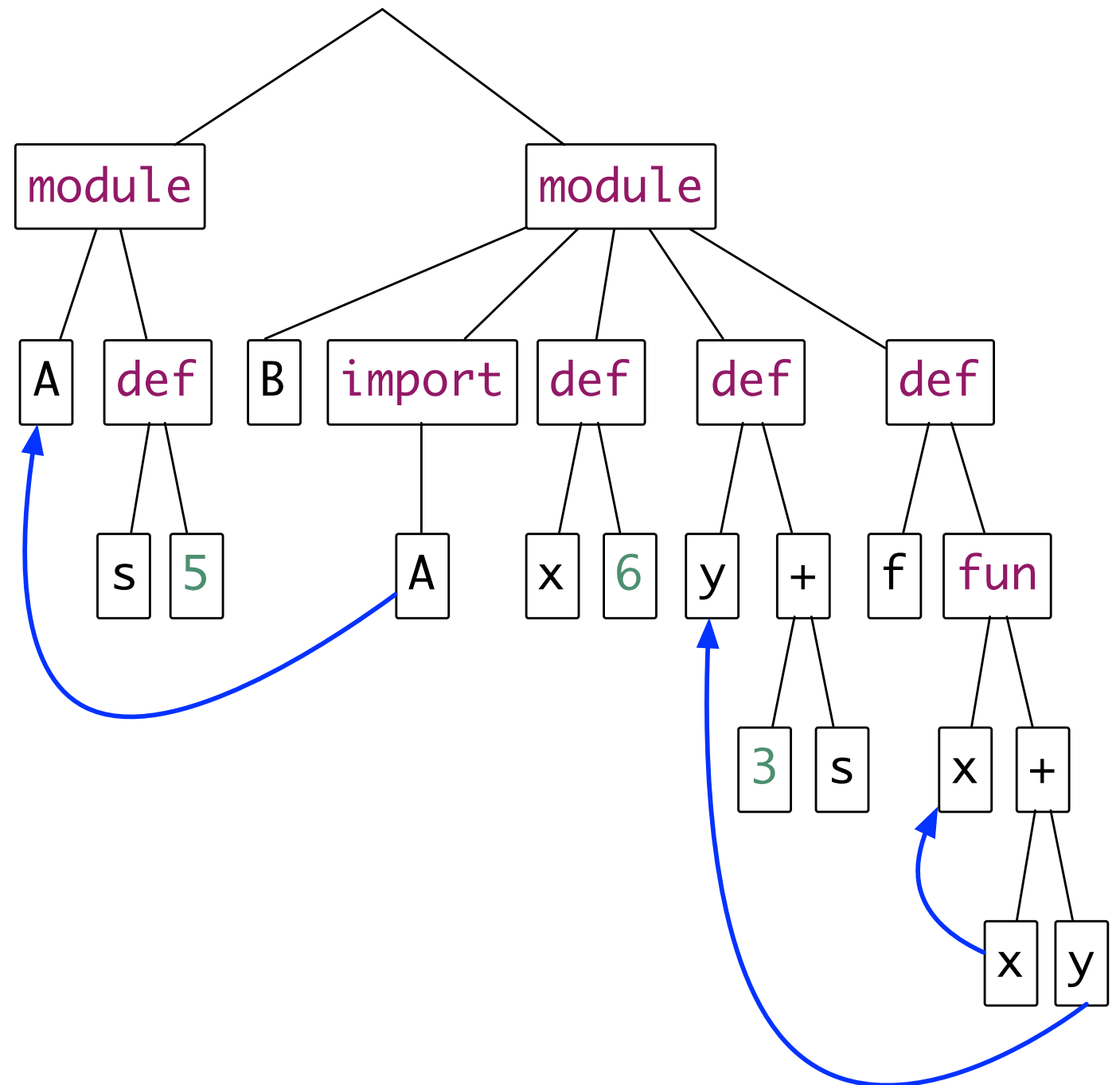
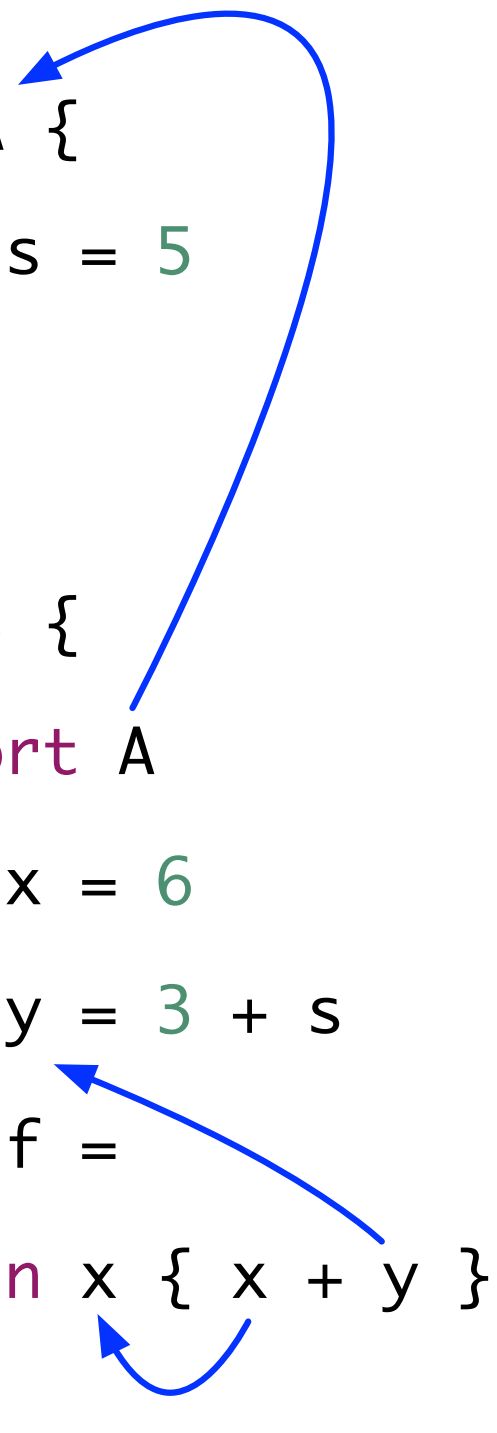
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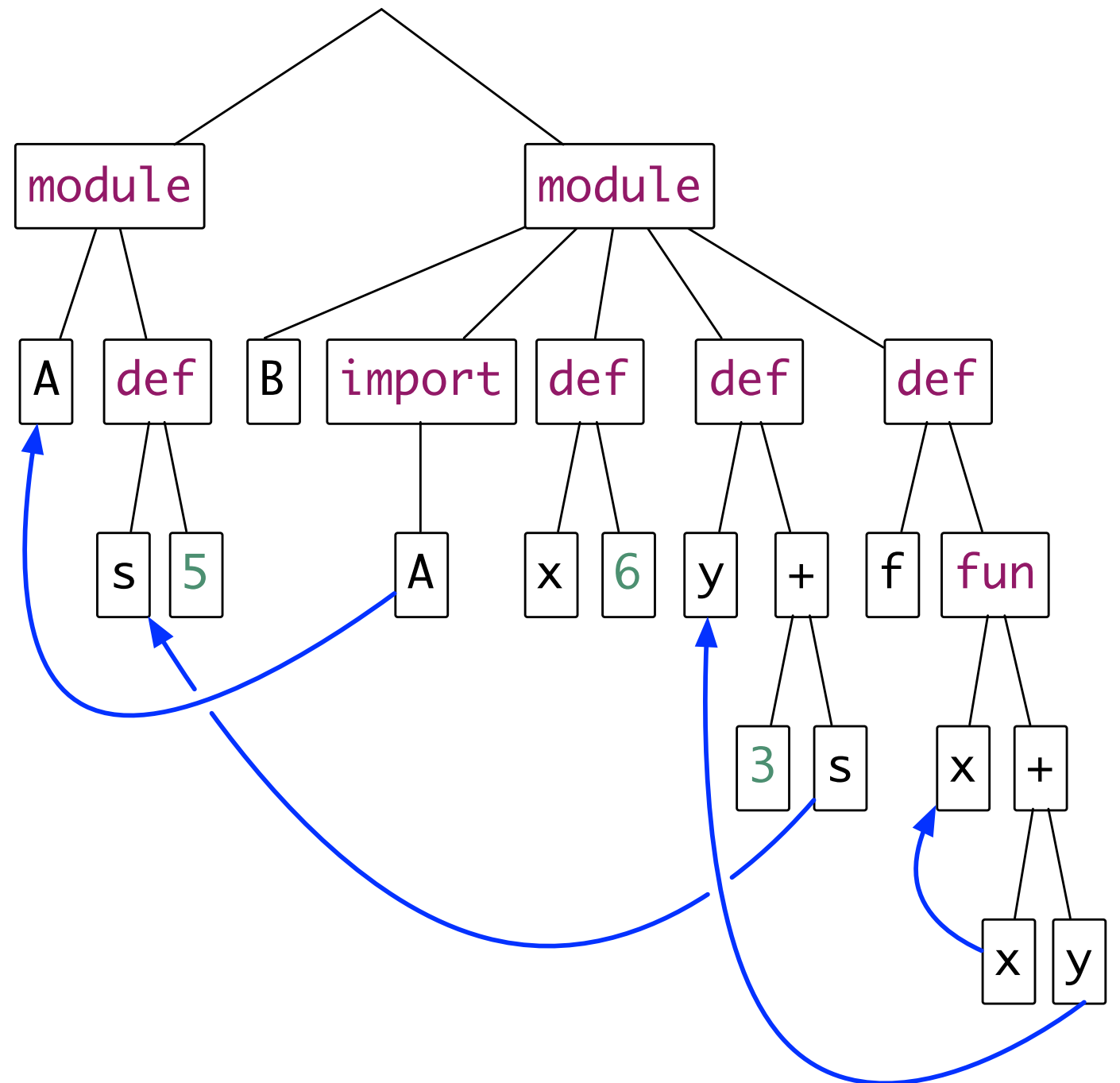
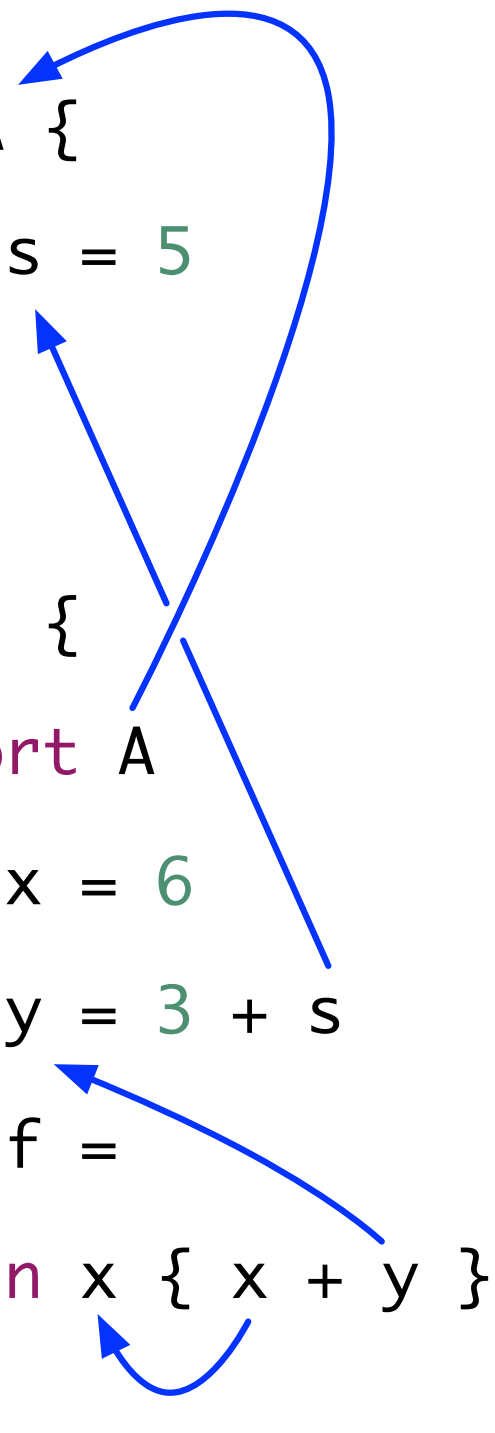
Name Resolution?

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Name Resolution Concerns

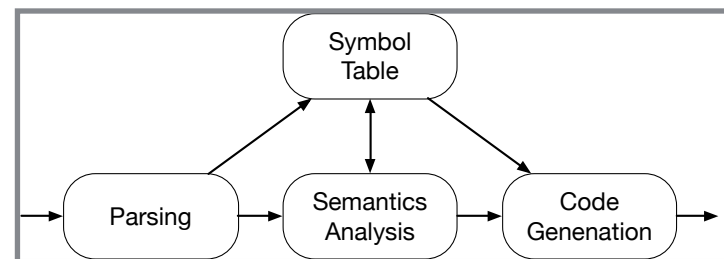
Multiple uses for each programming language ...

Name Resolution Concerns

Multiple uses for each programming language ...

$$\frac{x : \tau_1, \Gamma \vdash e : \tau_2}{\Gamma \vdash \lambda x. e : \tau_1 \rightarrow \tau_2}$$
$$\frac{\Gamma(x) = \tau}{\Gamma \vdash x : \tau}$$

Type systems



Compiler

```
*A.java
public class A {
    static int x;

    int plus(int y) {
        return y + x;
    }
}
```

IDE

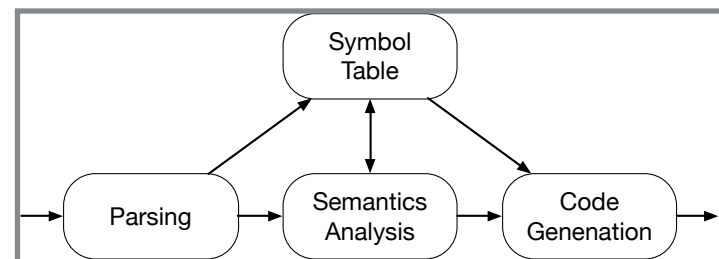
Name Resolution Concerns

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Type systems



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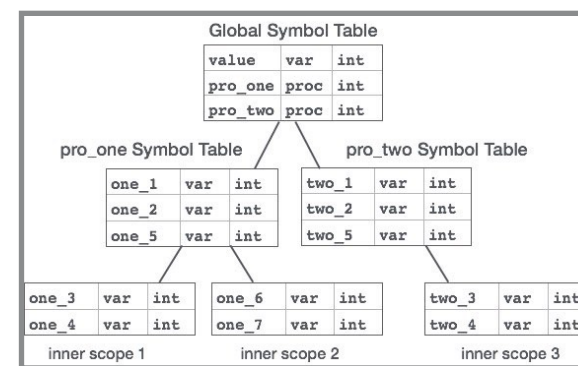
    int plus(int y) {
        return y + x;
    }
}
  
```

IDE

... resulting in different encodings of name resolution

$x:\text{int}, \Gamma$

$[3/x].\sigma$



$\text{lookup}(x_i)$

What about Syntax?

What about Syntax?

*A standard
formalism*

**Context-Free
Grammars**

What about Syntax?

*A unique
definition*

*A standard
formalism*

```
program = decl*
decl   = module id { decl* }
      | import qid
      | def id = exp
exp    = qid
      | fun id { exp }
      | fix id { exp }
      | let bind* in exp
      | letrec bind* in exp
      | letpar bind* in exp
      | exp exp
      | exp ⊕ exp
      | int
qid    = id
      | id . qid
bind   = id = exp
```

**Context-Free
Grammars**

What about Syntax?

*A unique
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formalism*

Provides

```
program = decl*
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**Context-Free
Grammars**

Parser

AST

Pretty-Printer

Highlighting

A Theory of Name Resolution

A Theory of Name Resolution

*For **statically lexically scoped** languages*

A Theory of Name Resolution

*For **statically lexically scoped** languages*

***A standard
formalism***



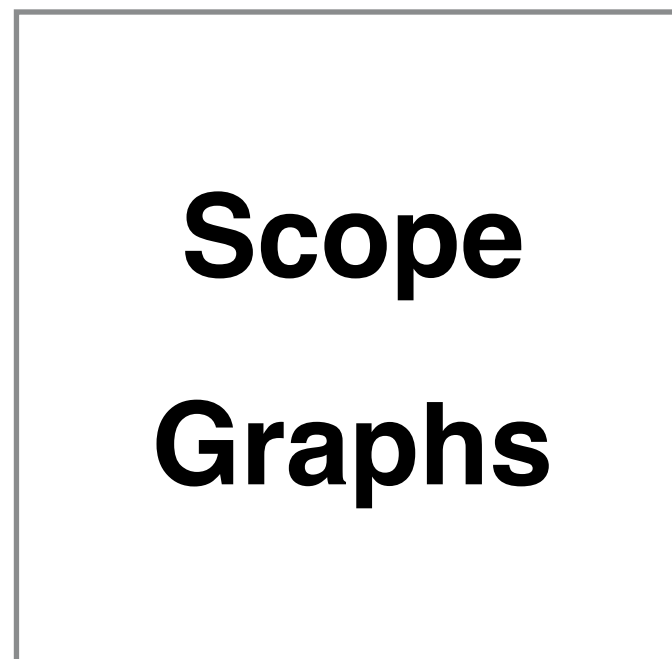
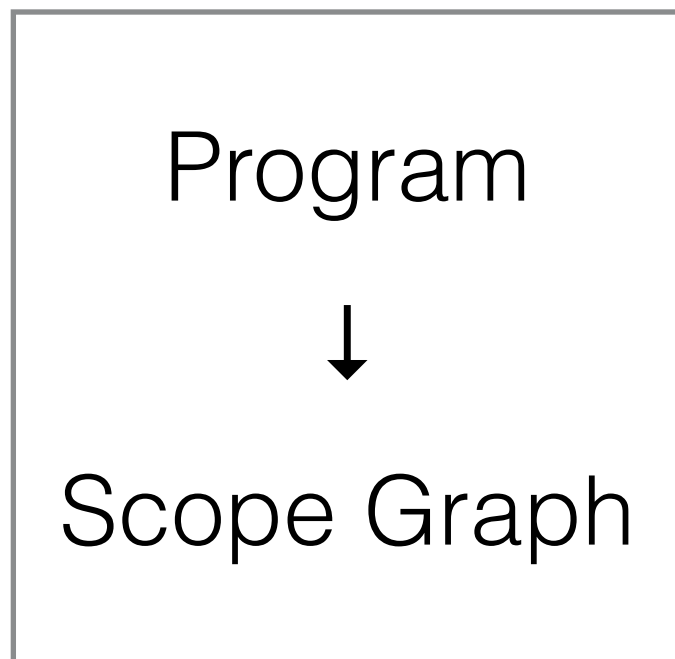
**Scope
Graphs**

A Theory of Name Resolution

*For **statically lexically scoped** languages*

***A unique
definition***

***A standard
formalism***



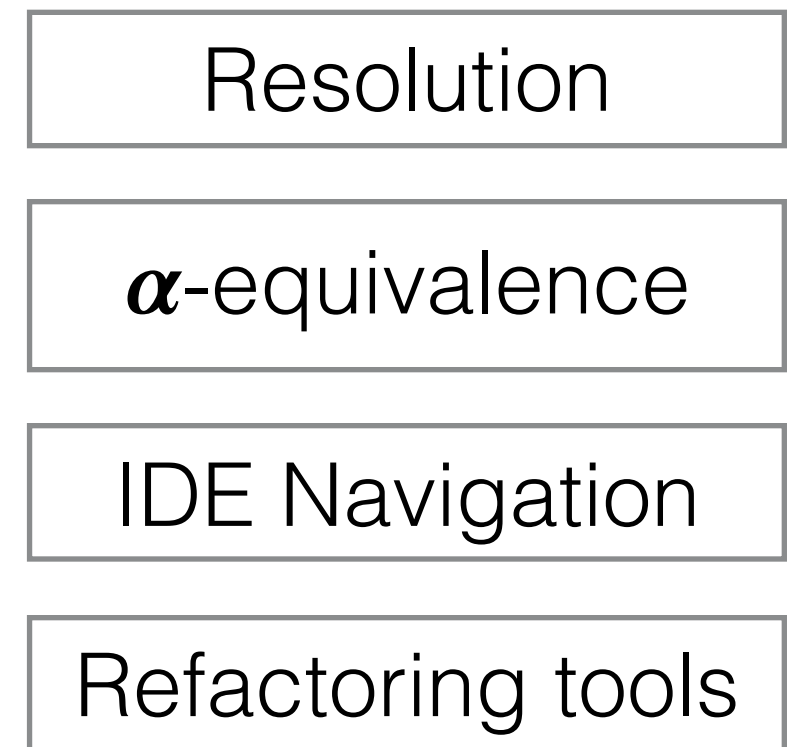
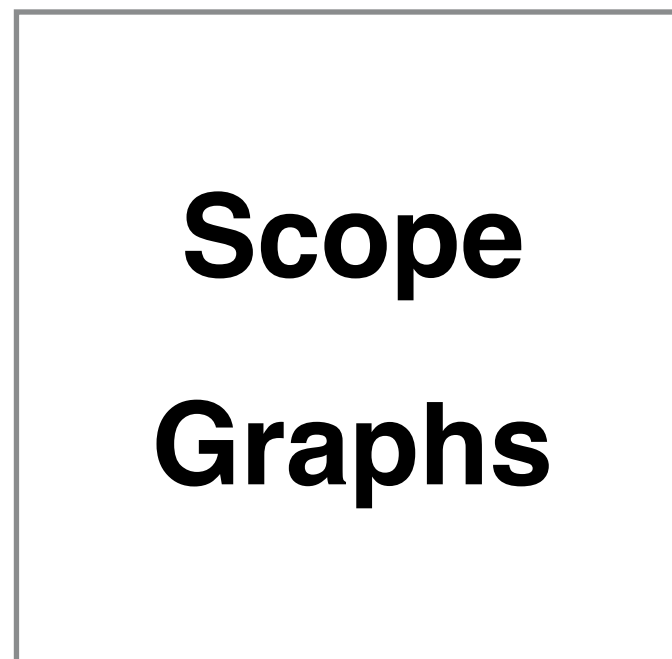
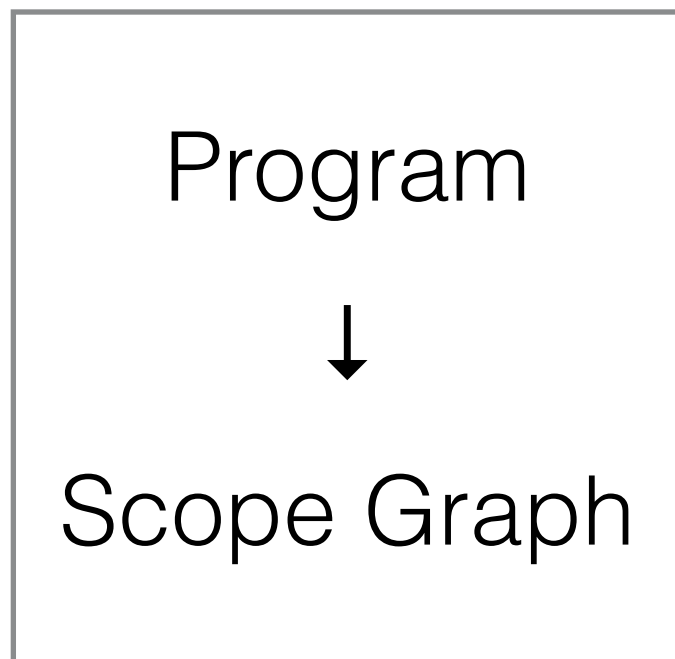
A Theory of Name Resolution

For ***statically lexically scoped*** languages

***A unique
definition***

***A standard
formalism***

Provides



Resolution Scheme

**Language
Dependent**



Scope
Graph

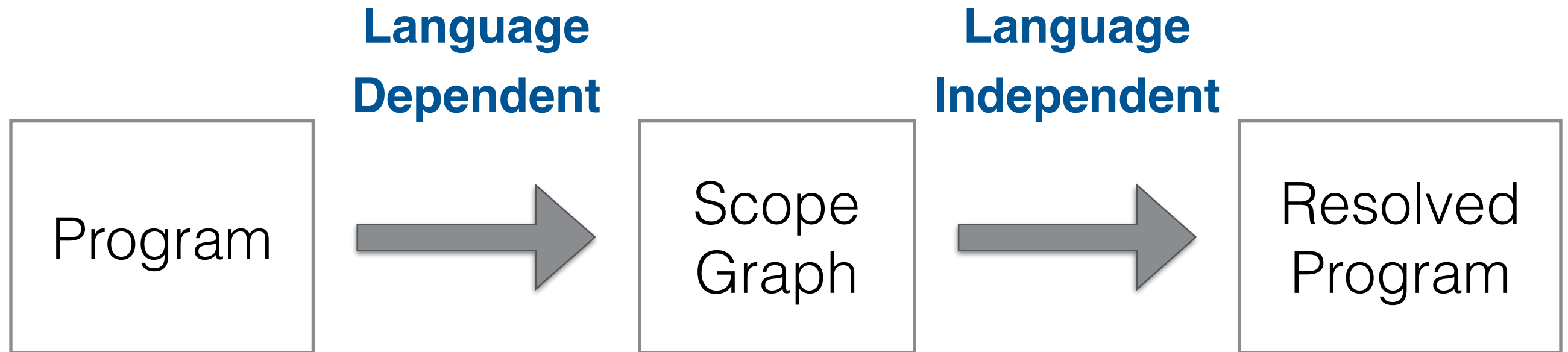
**Language
Independent**



Resolved
Program

Program

Resolution Scheme



Resolution of a reference in a scope graph:

Building a **path**
from a **reference** node
to a **declaration** node
following path construction **rules**

Scope Graph

```
def y = x + 1  
def x = 5
```

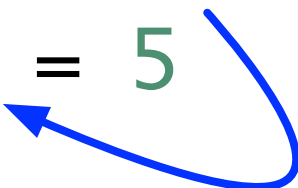
Scope Graph

```
def  $y_1 = x_2 + 1$   
def  $x_1 = 5$ 
```

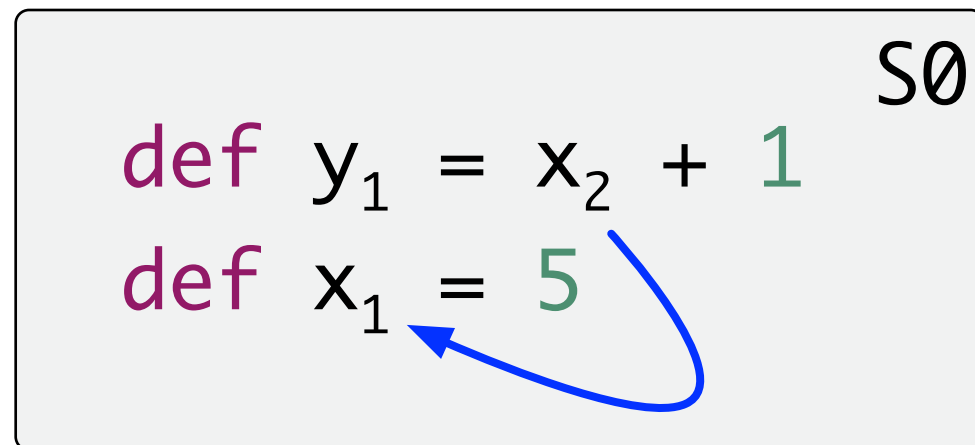
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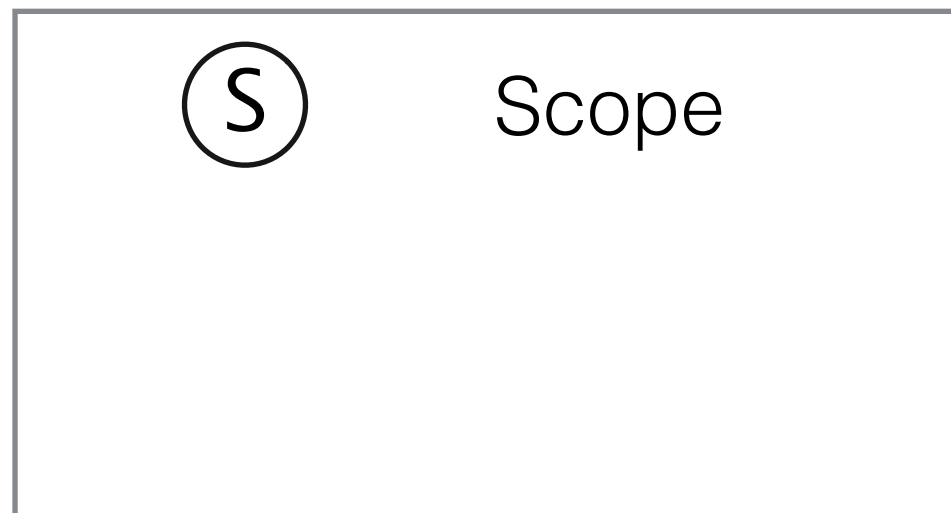
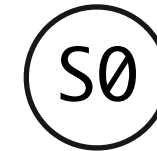
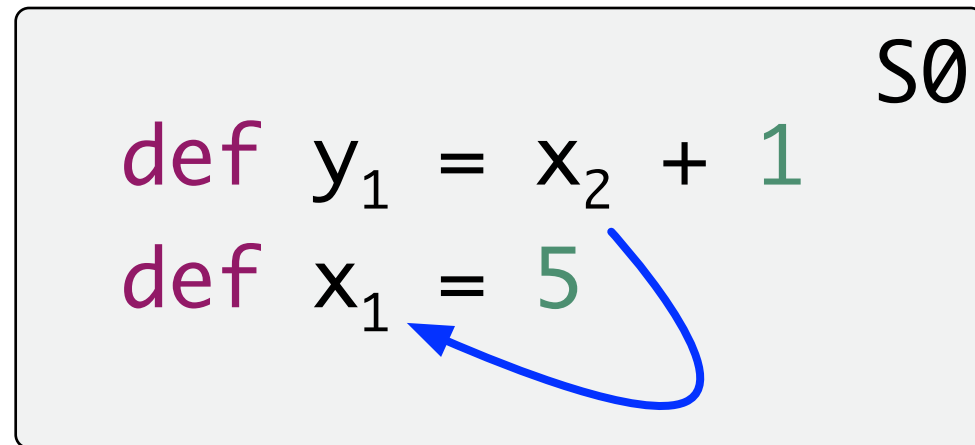
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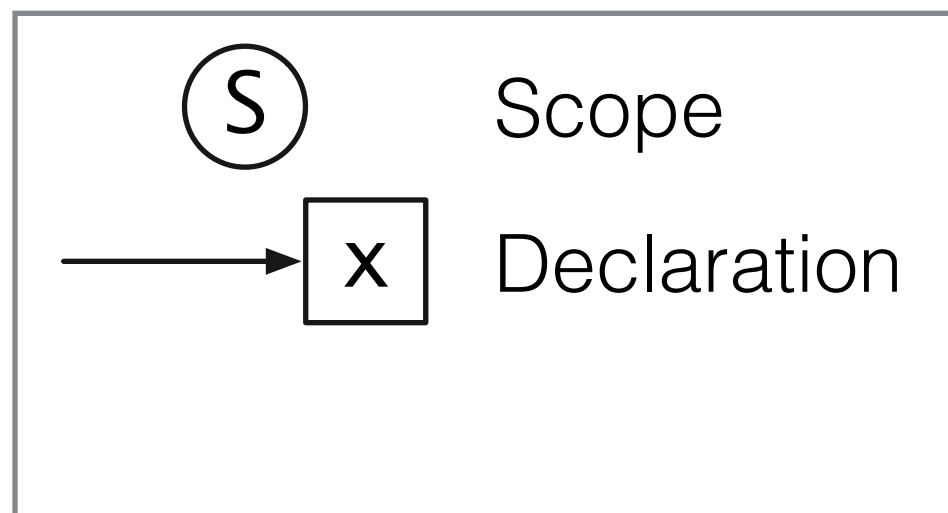
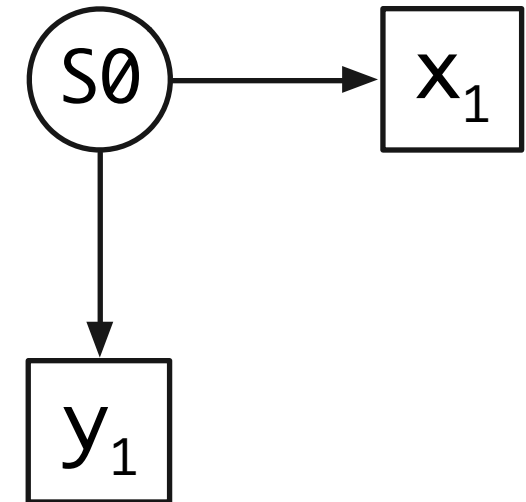
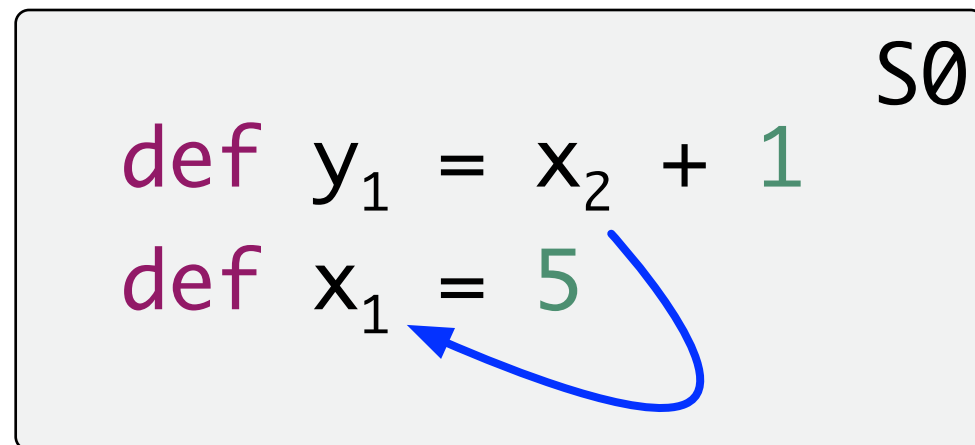
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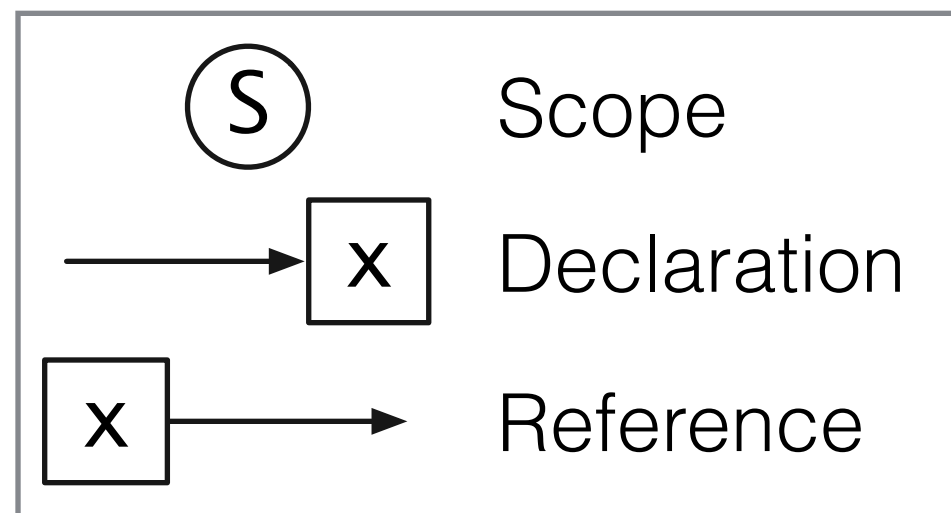
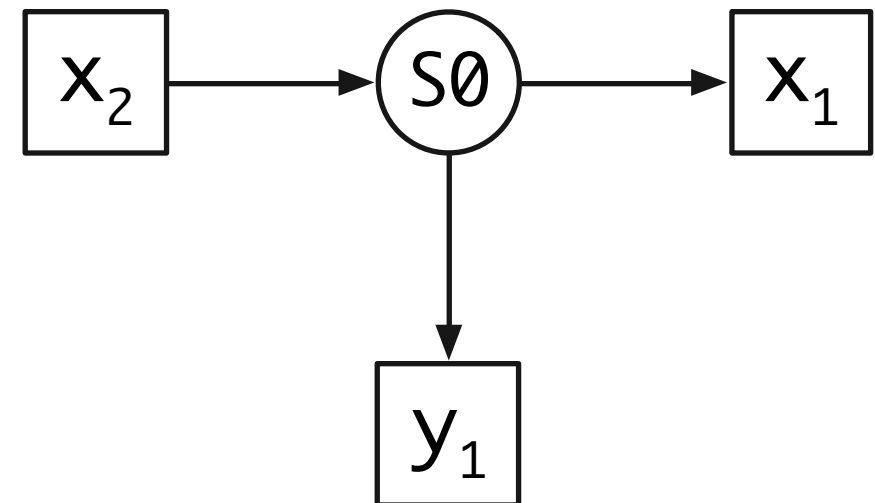
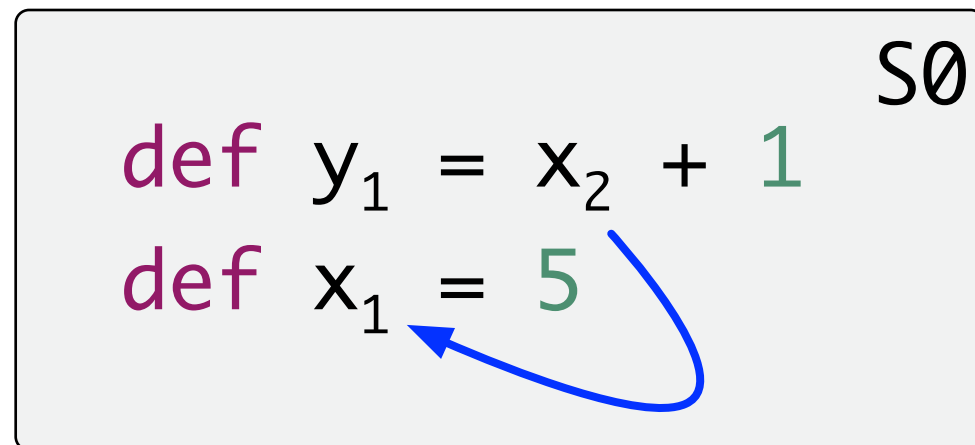
Scope Graph



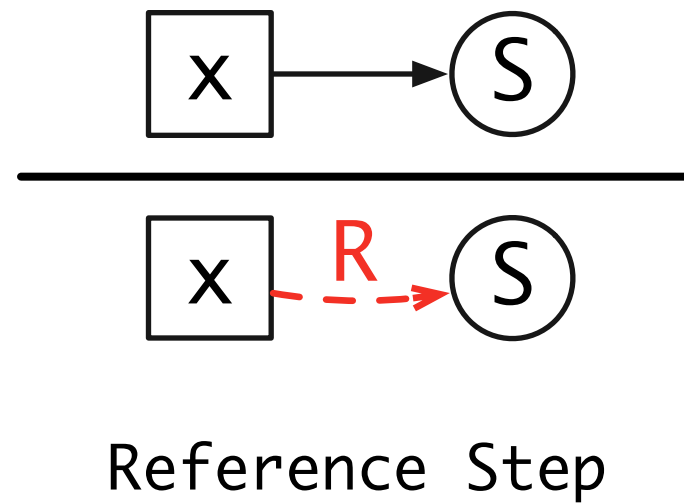
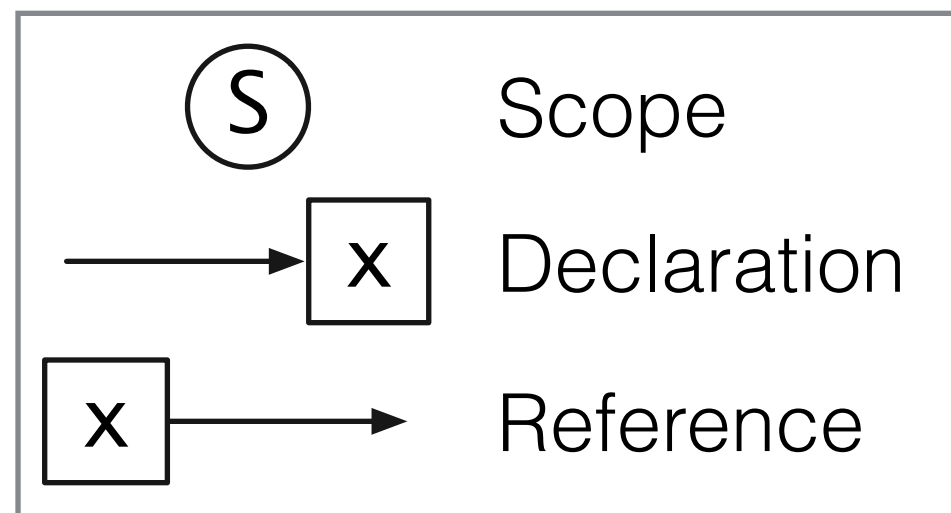
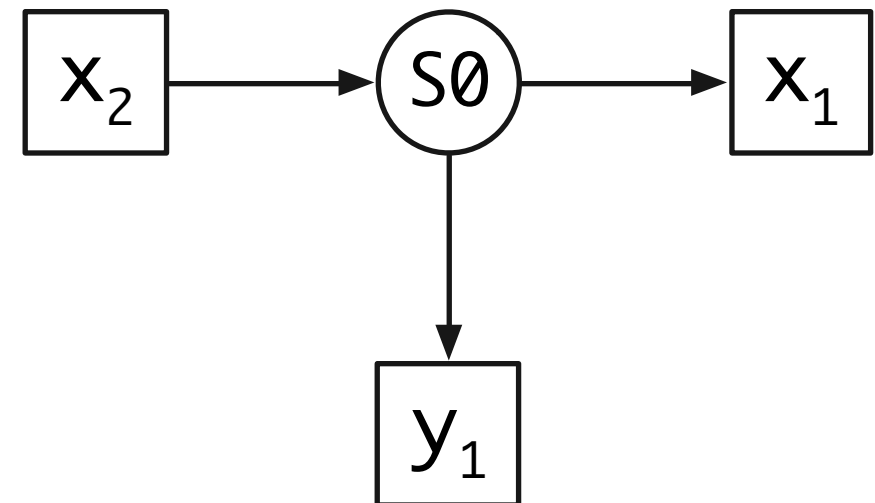
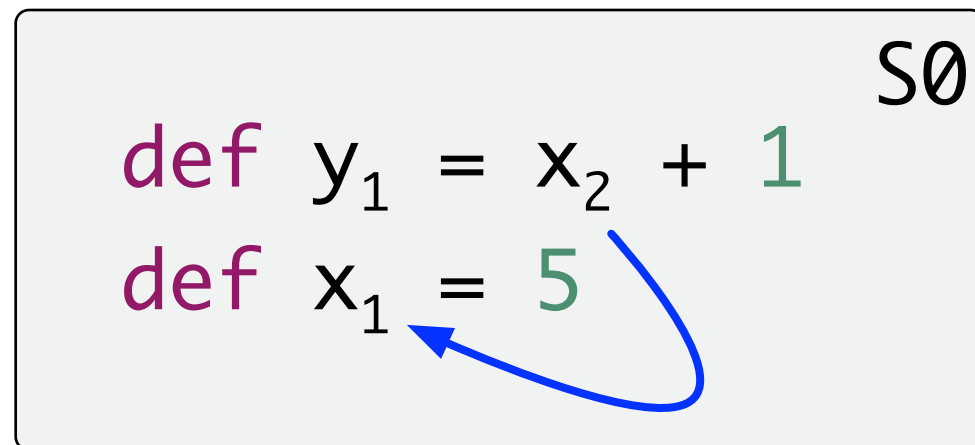
Scope Graph



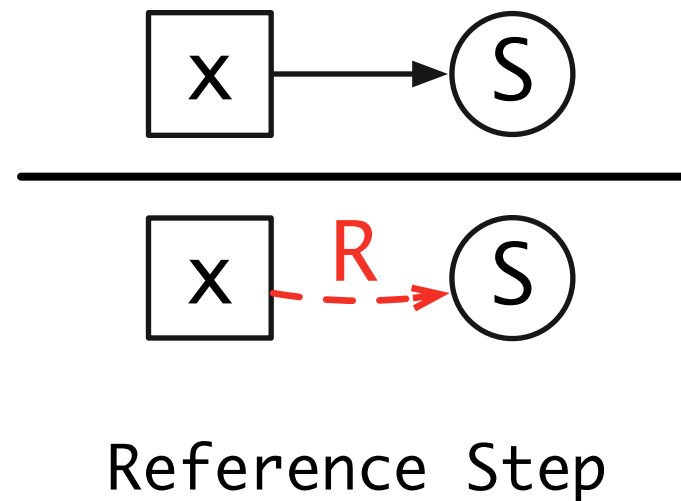
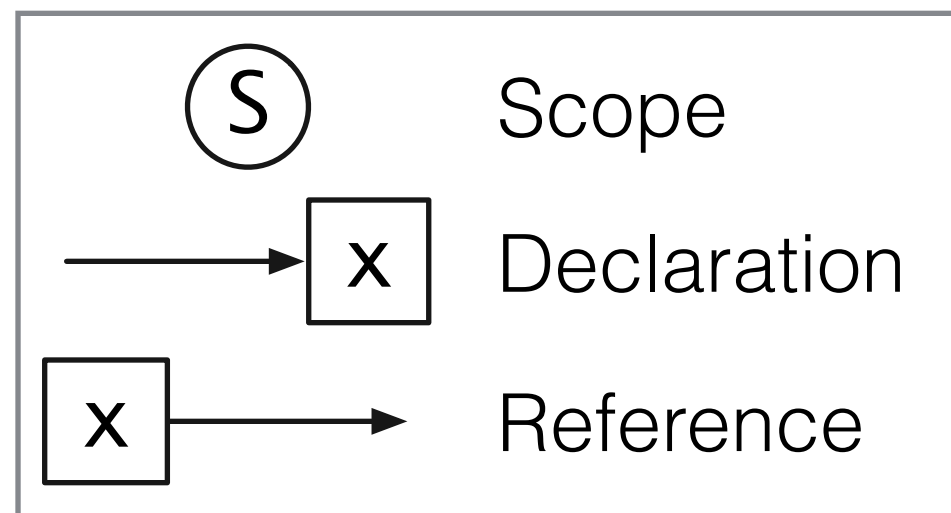
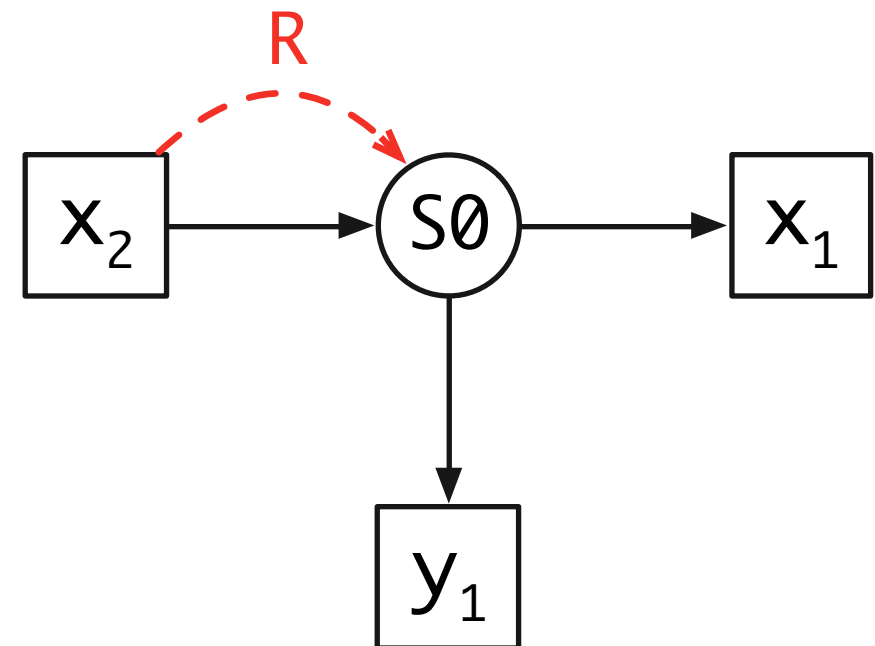
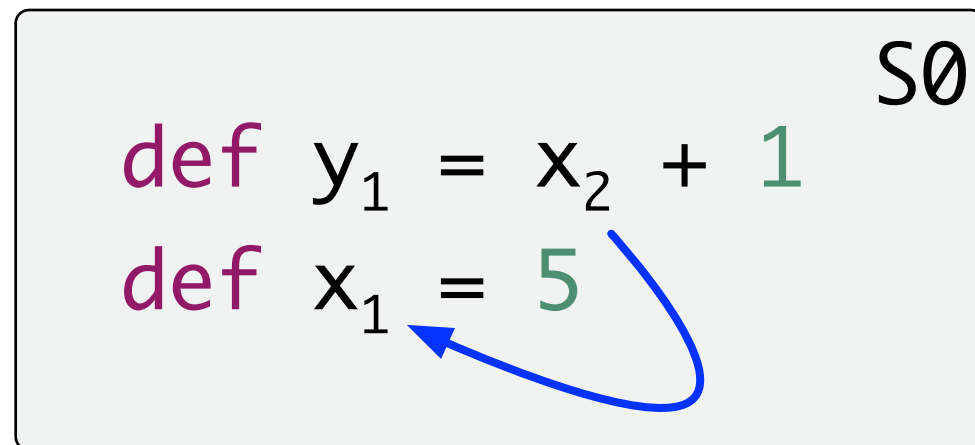
Scope Graph



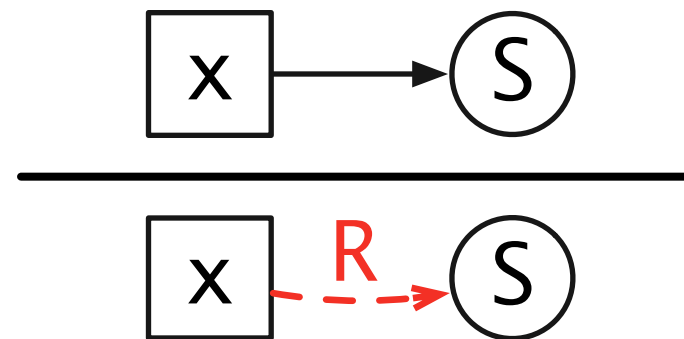
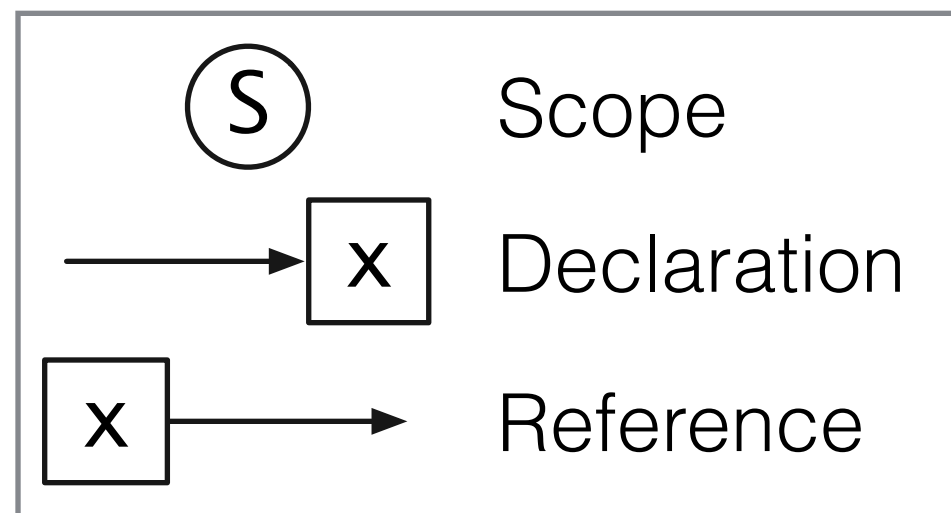
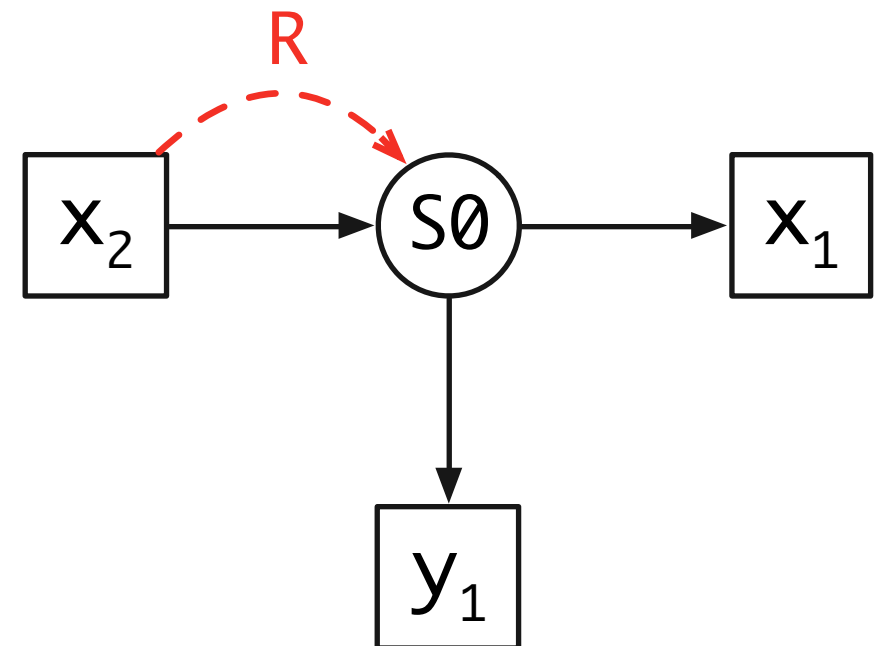
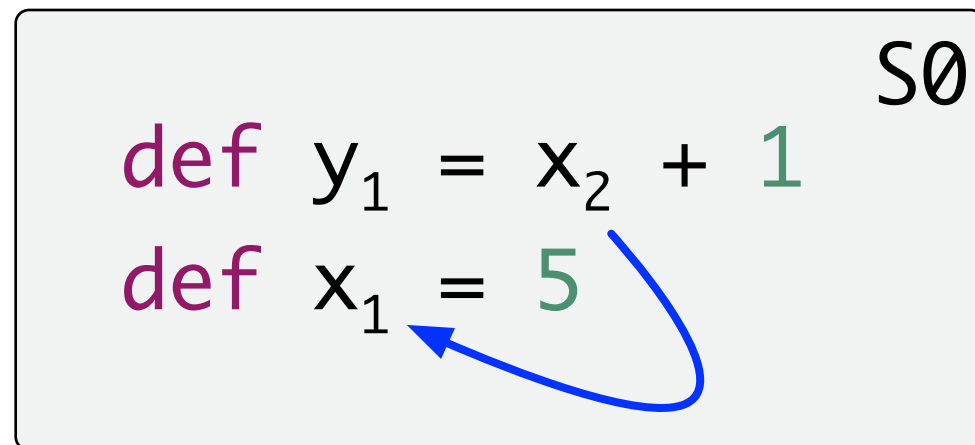
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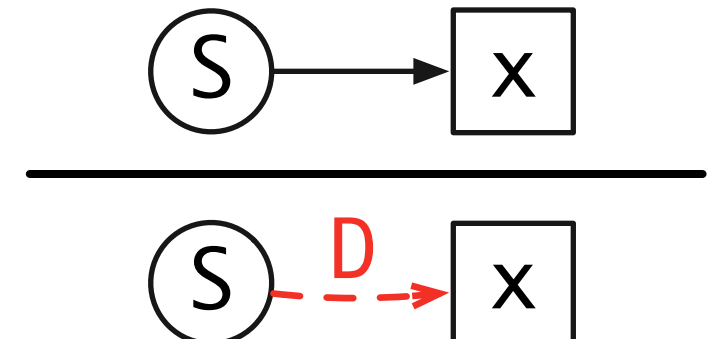
Scope Graph



Scope Graph

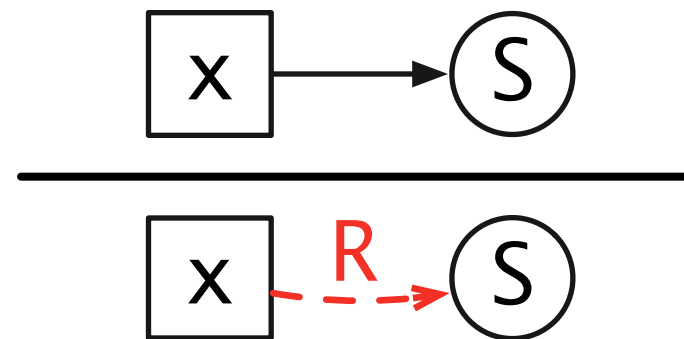
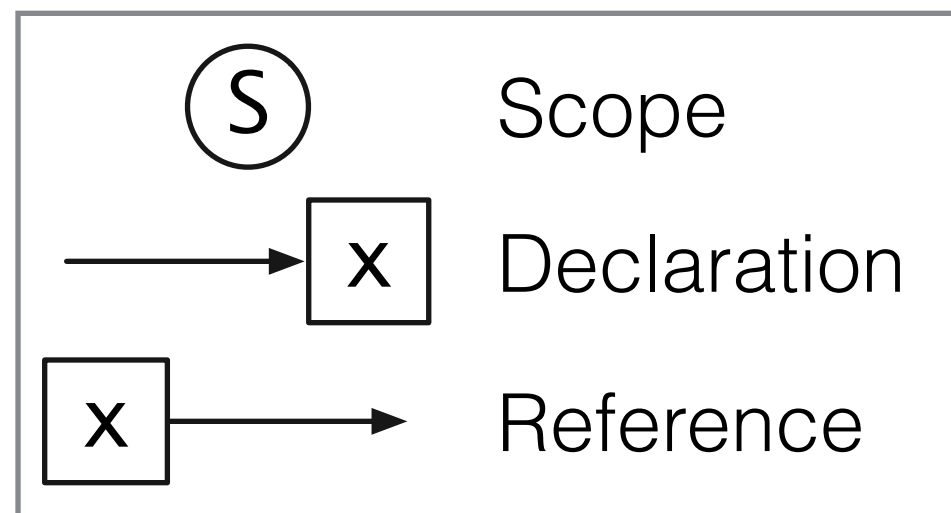
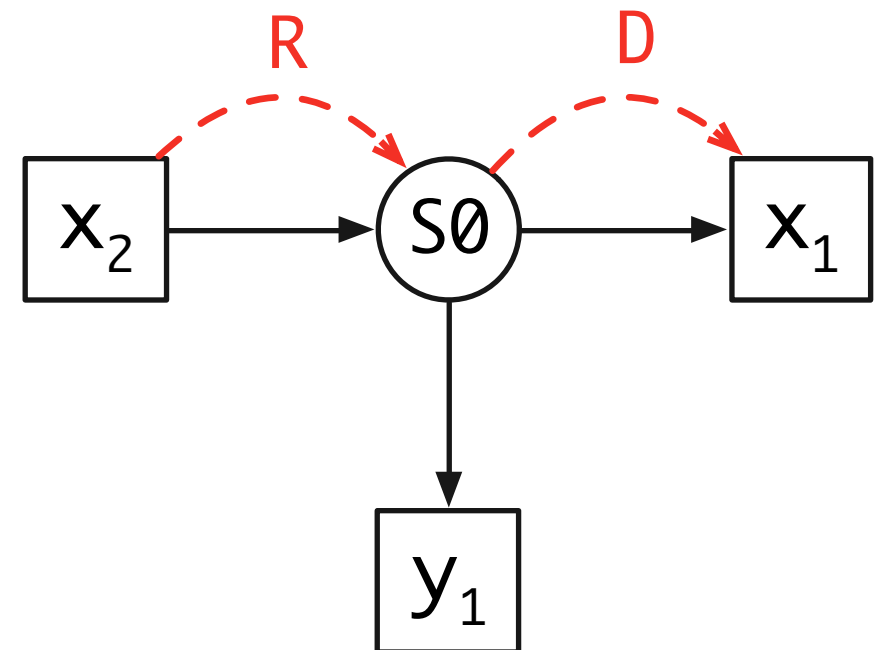
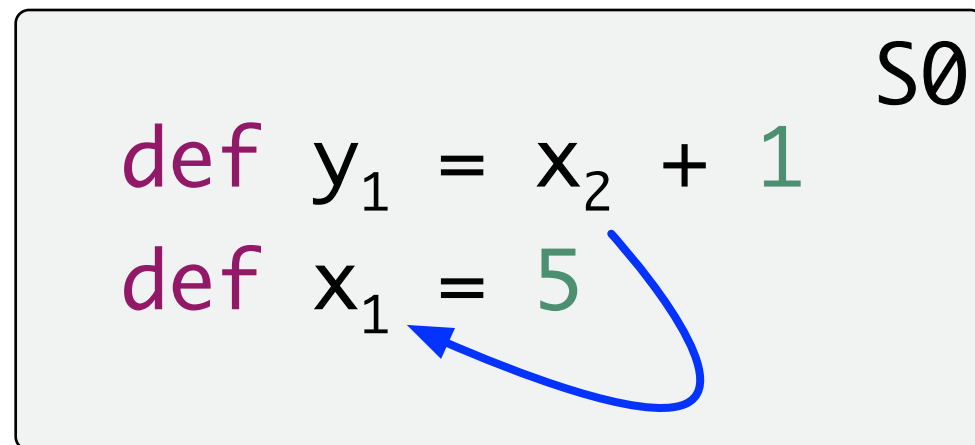


Reference Step

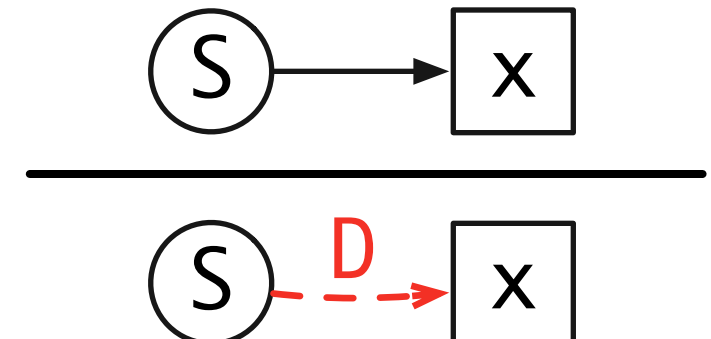


Declaration Step

Scope Graph



Reference Step



Declaration Step

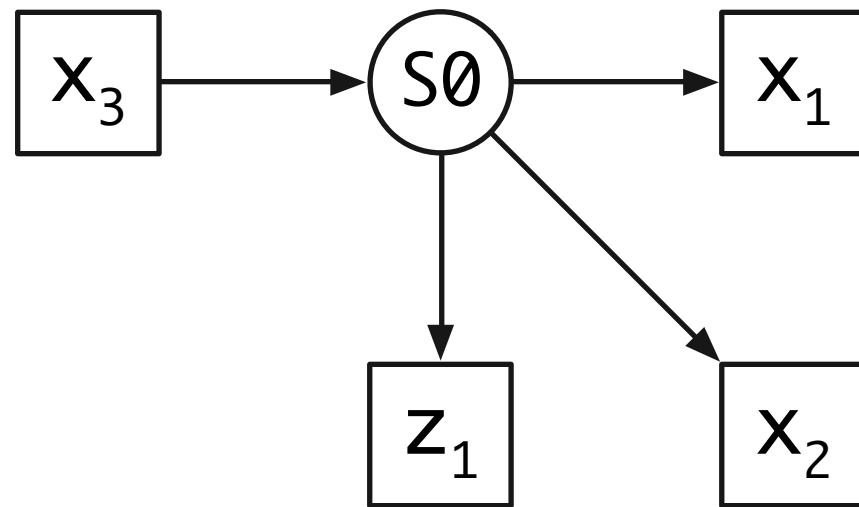
Ambiguous Resolutions

def $x_1 = 5$

def $x_2 = 3$

def $z_1 = x_3 + 1$

Ambiguous Resolutions

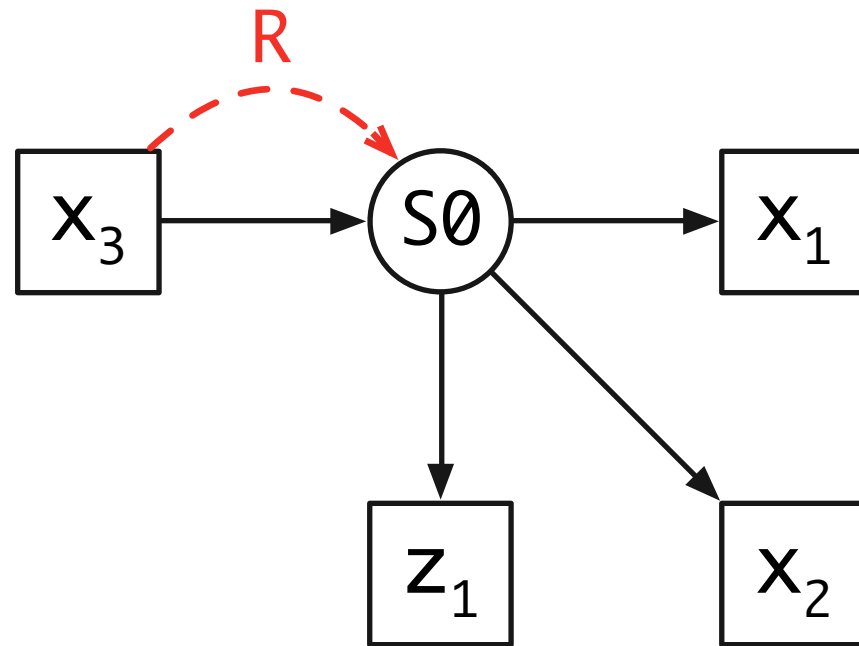


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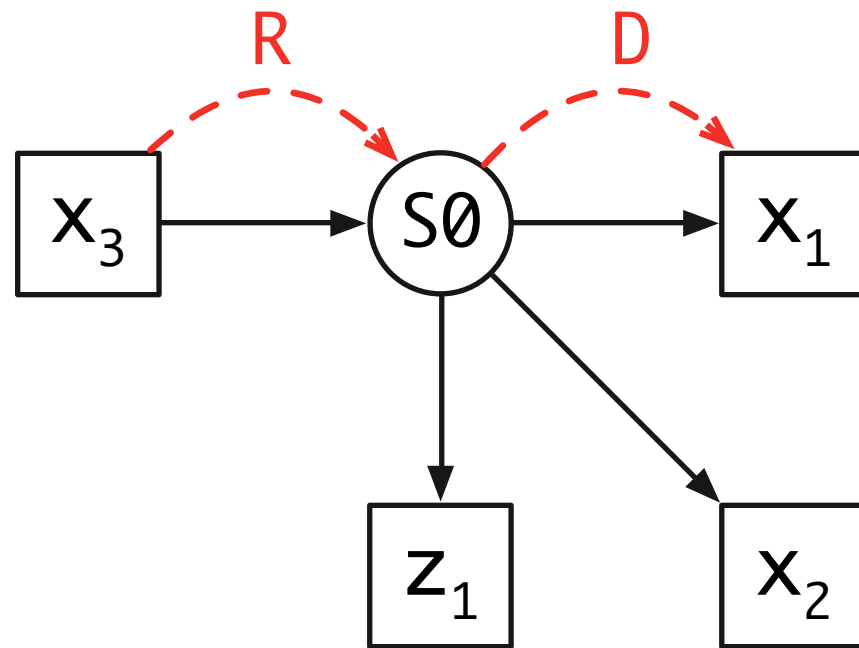


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def x1 = 5
```

```
def x2 = 3
```

```
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```

Ambiguous Resolutions

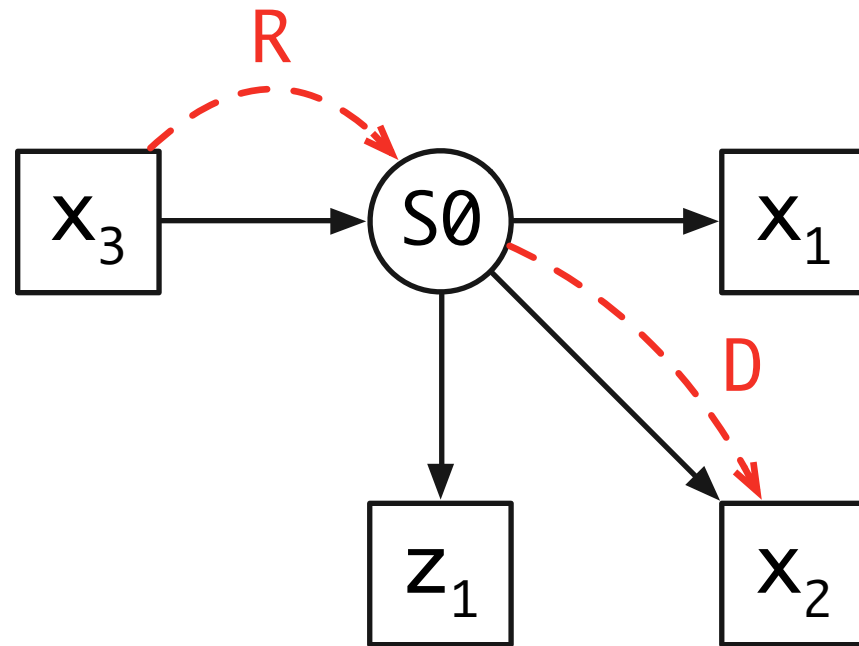


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Ambiguous Resolutions

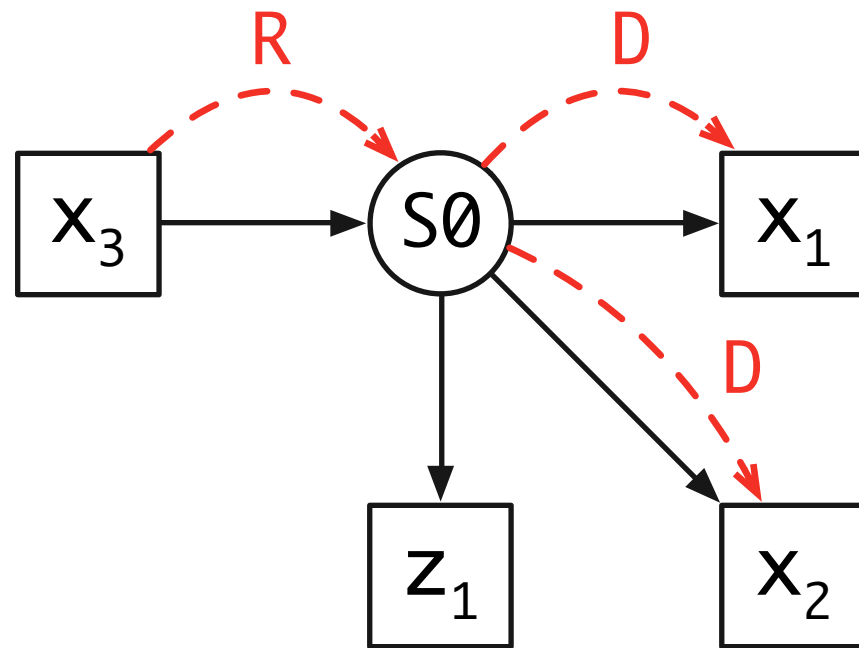


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Ambiguous Resolutions

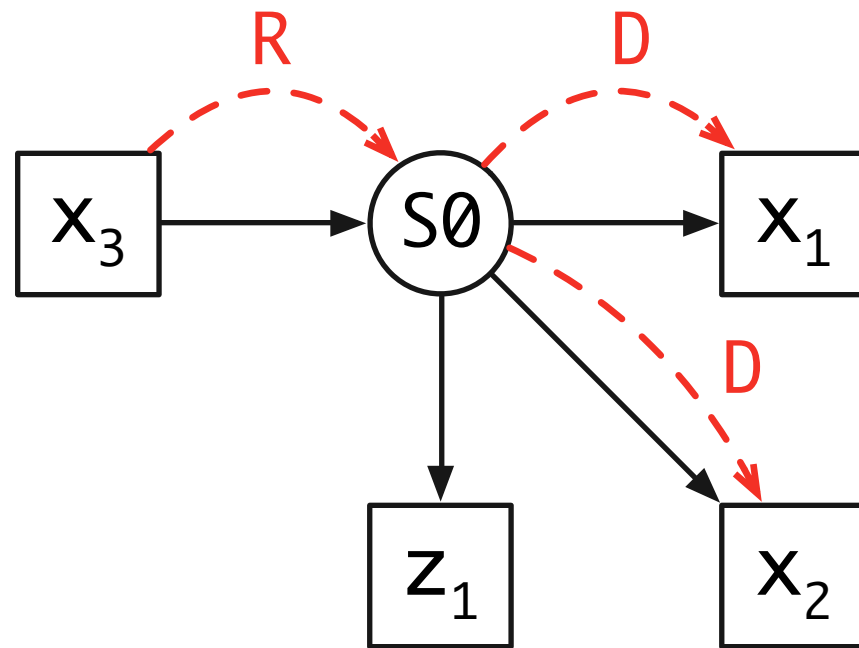


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```
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```

```
def z1 = x3 + 1
```

Ambiguous Resolutions



```
def x1 = 5  
def x2 = 3  
def z1 = x3 + 1
```

```
match t with  
| A x | B x => ...
```

Lexical Scoping

```
def x1 = z2 5
```

```
def z1 =  
  fun y1 {  
    x2 + y2  
  }
```

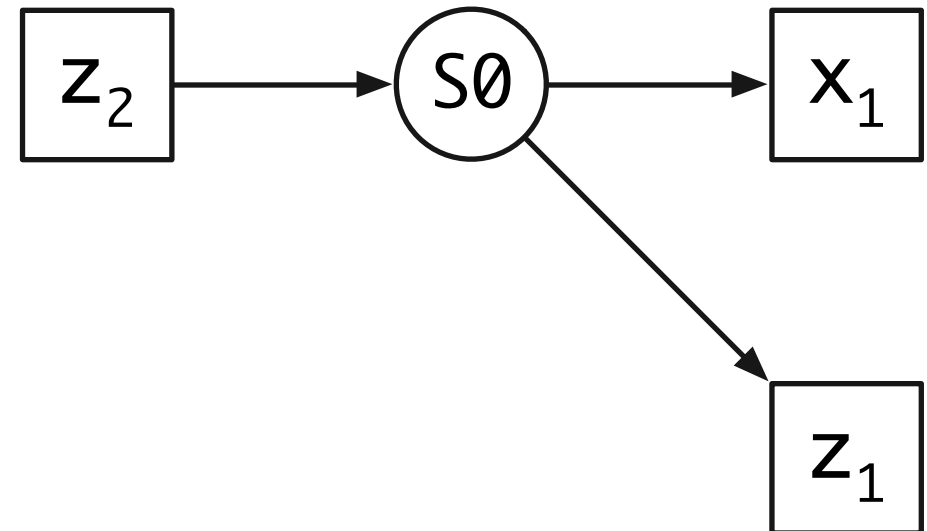
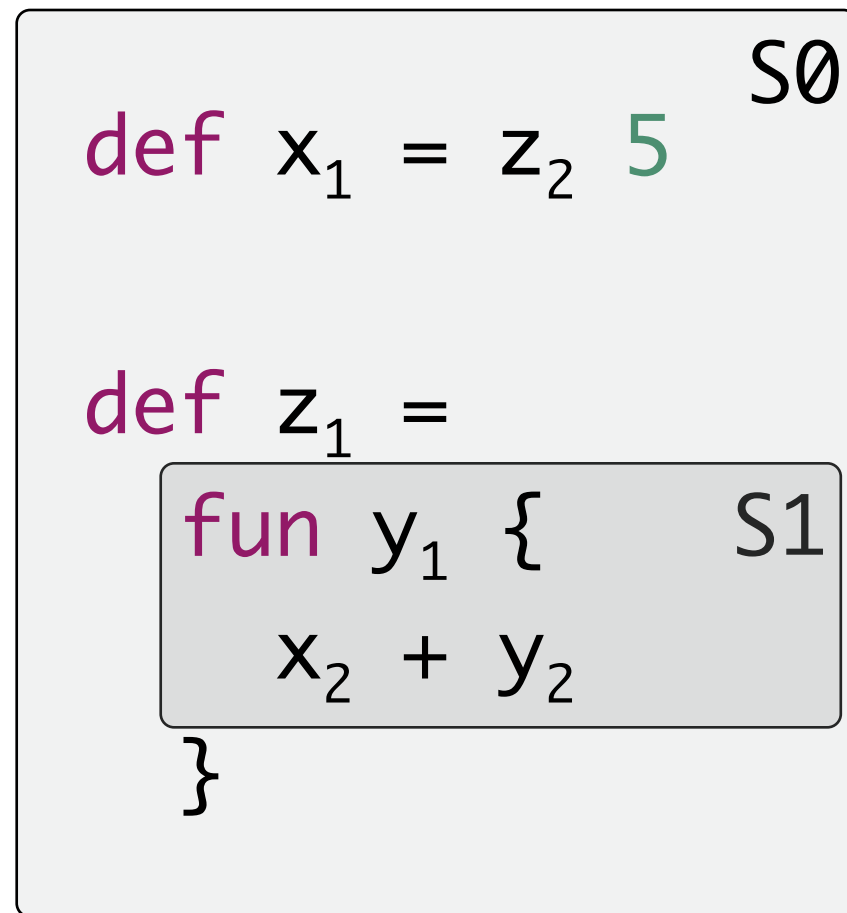

Lexical Scoping

```
def x1 = z2 5 S0  
  
def z1 =  
  fun y1 {  
    x2 + y2  
  }
```

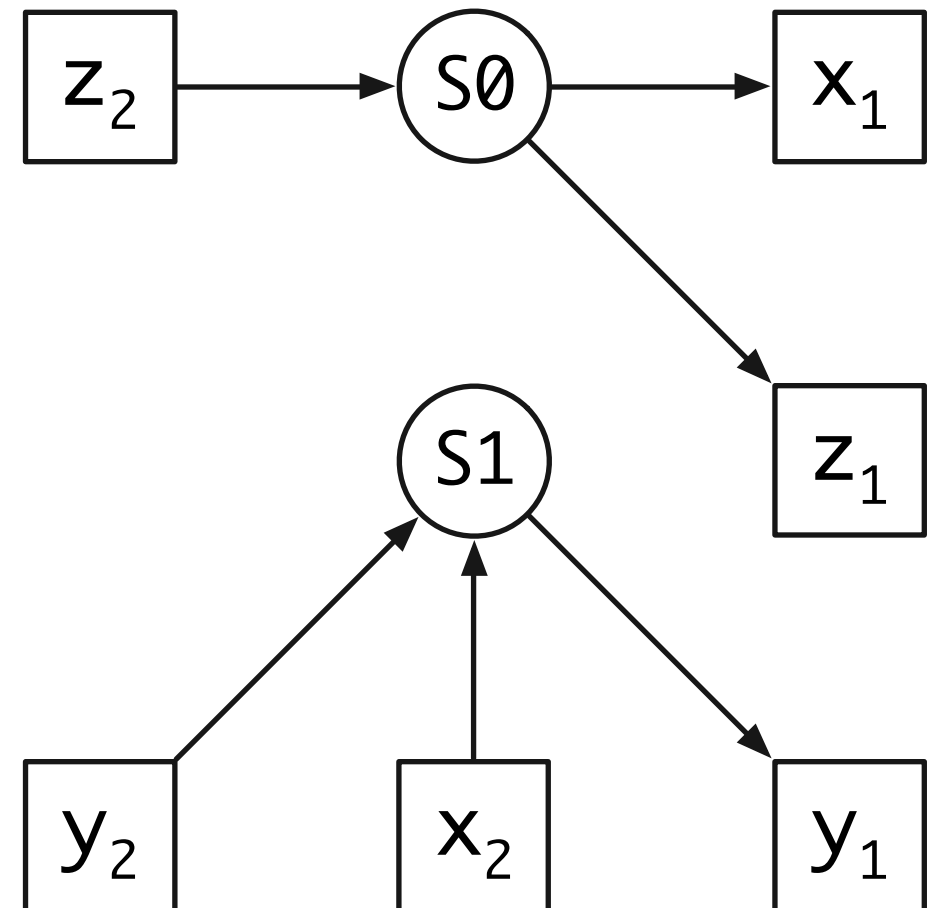
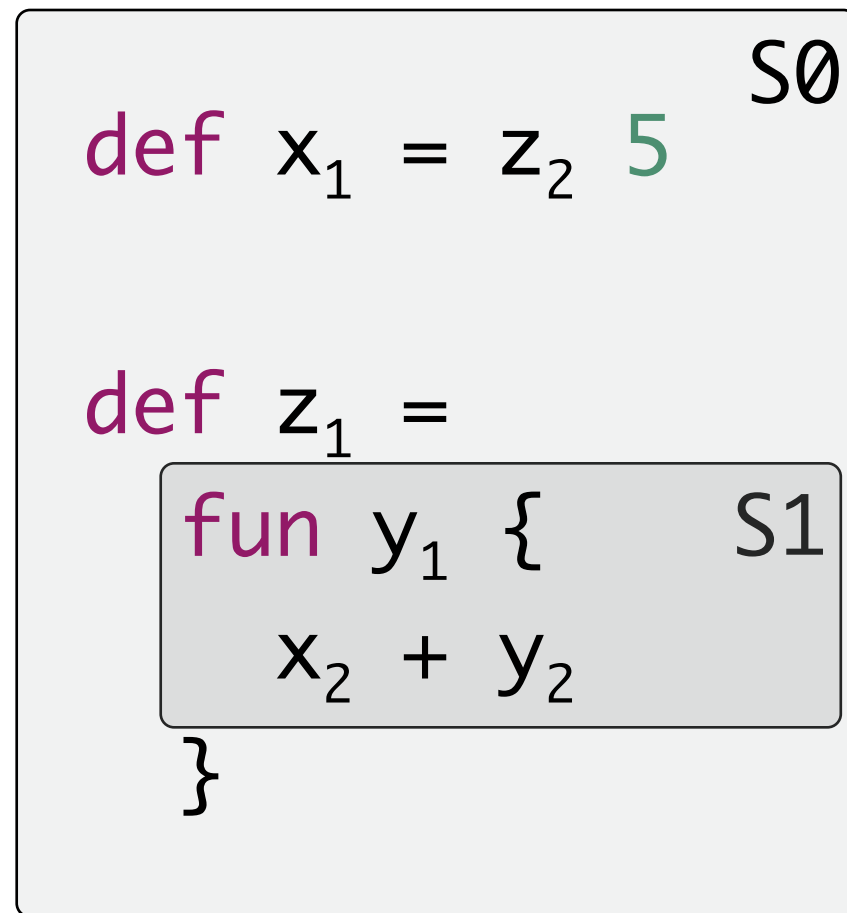
Lexical Scoping

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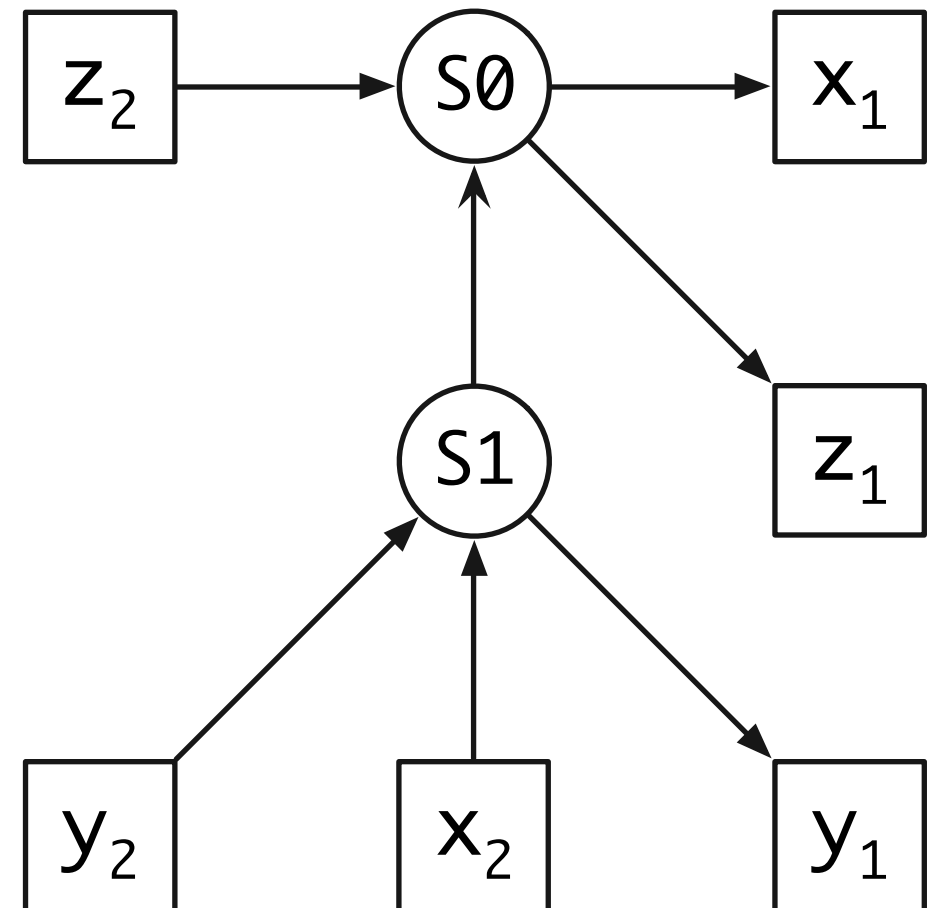
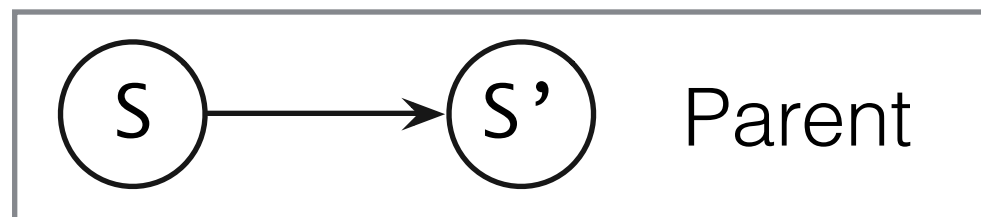
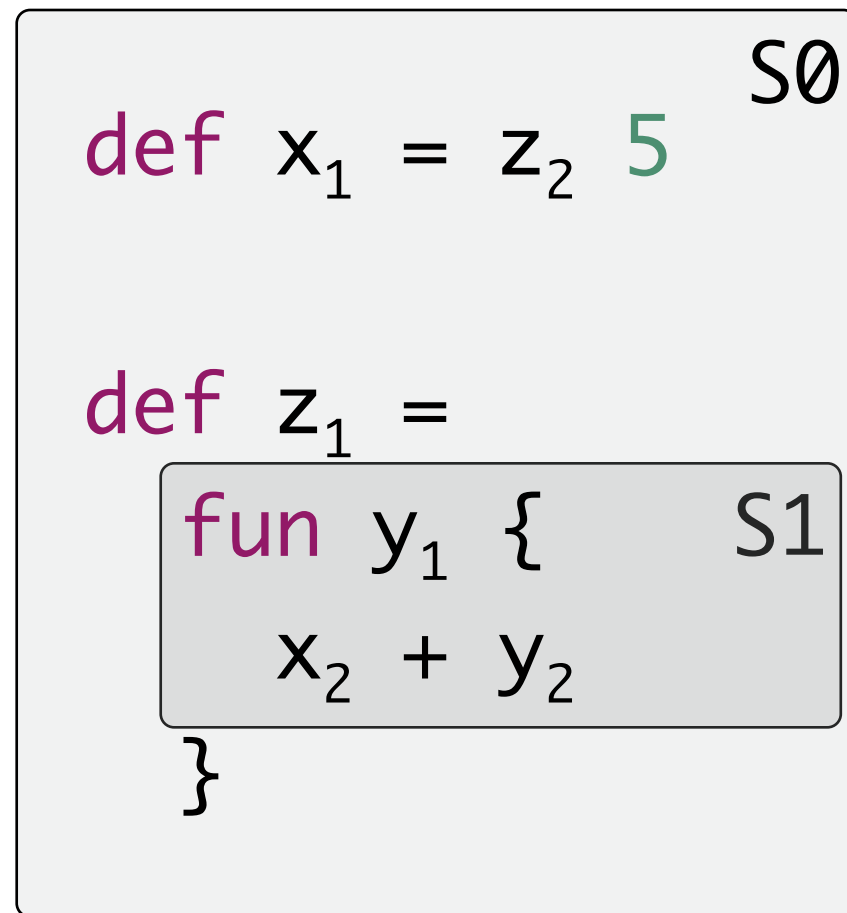
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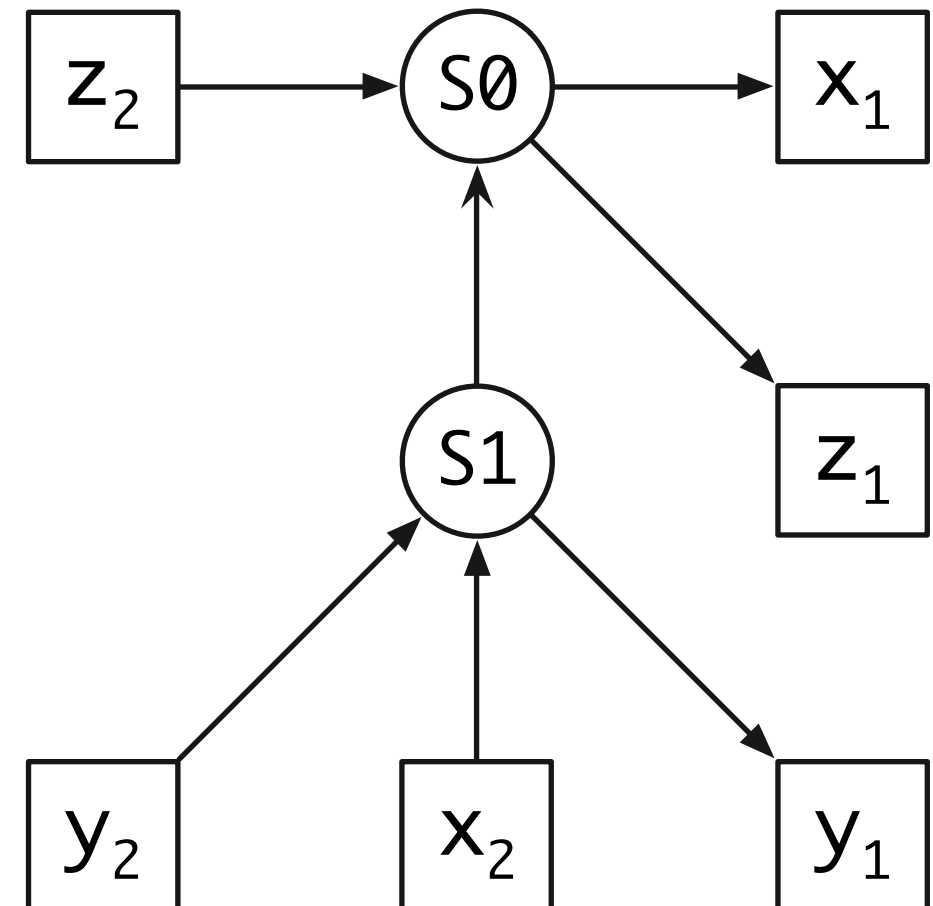
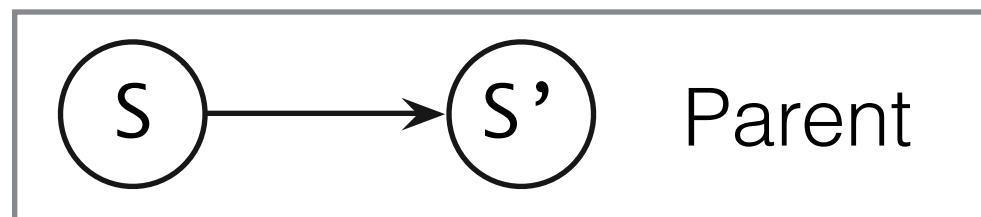
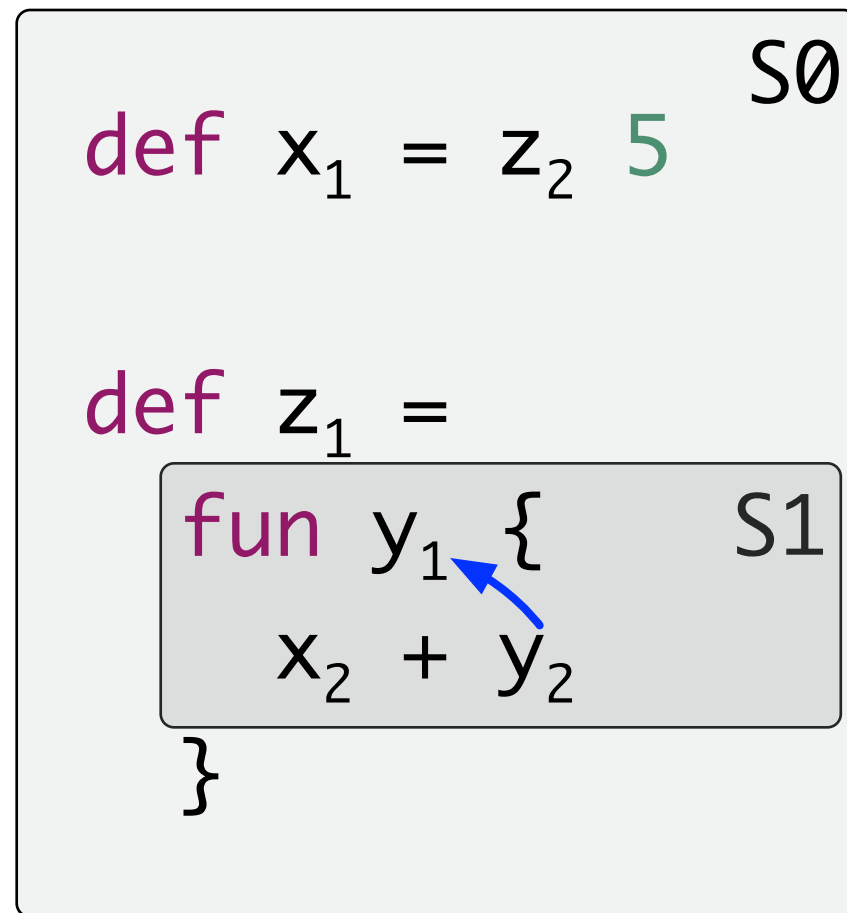
Lexical Scoping



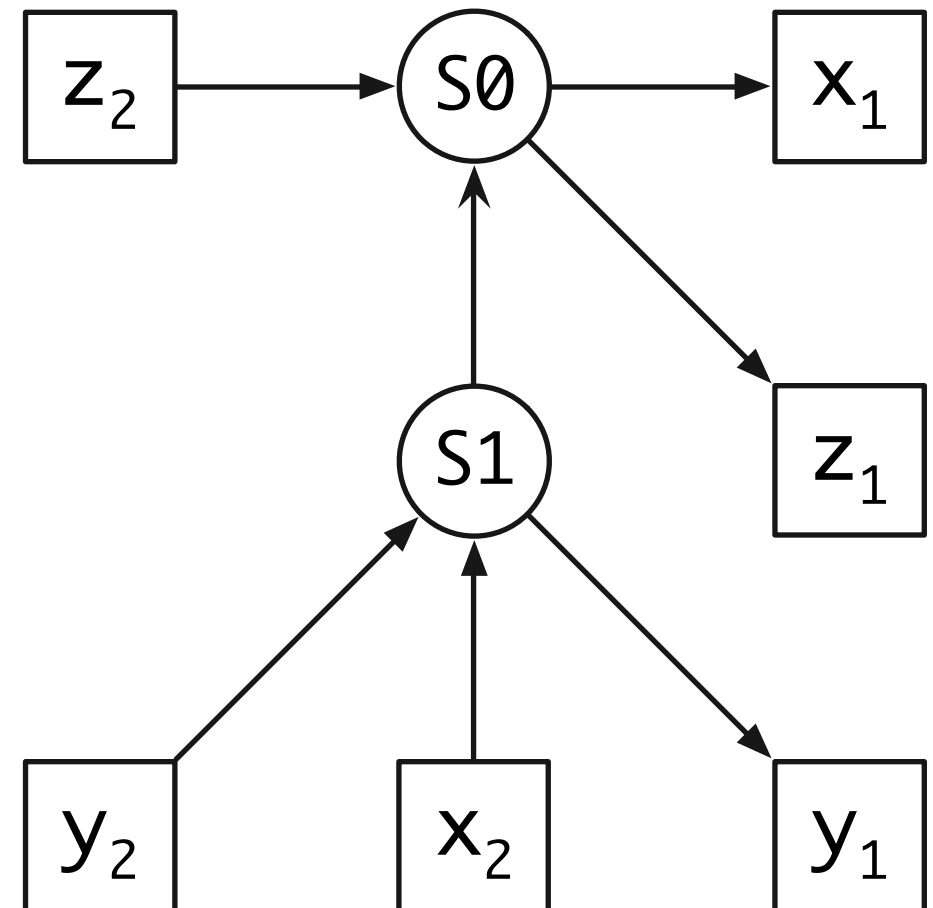
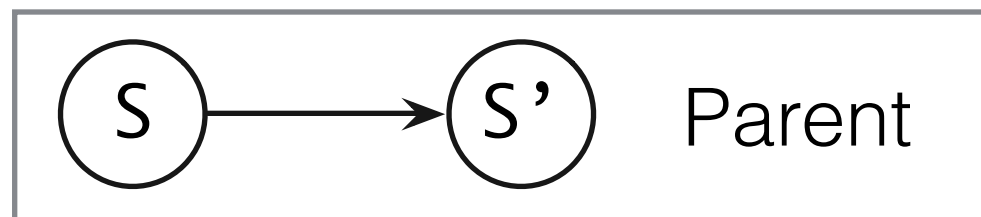
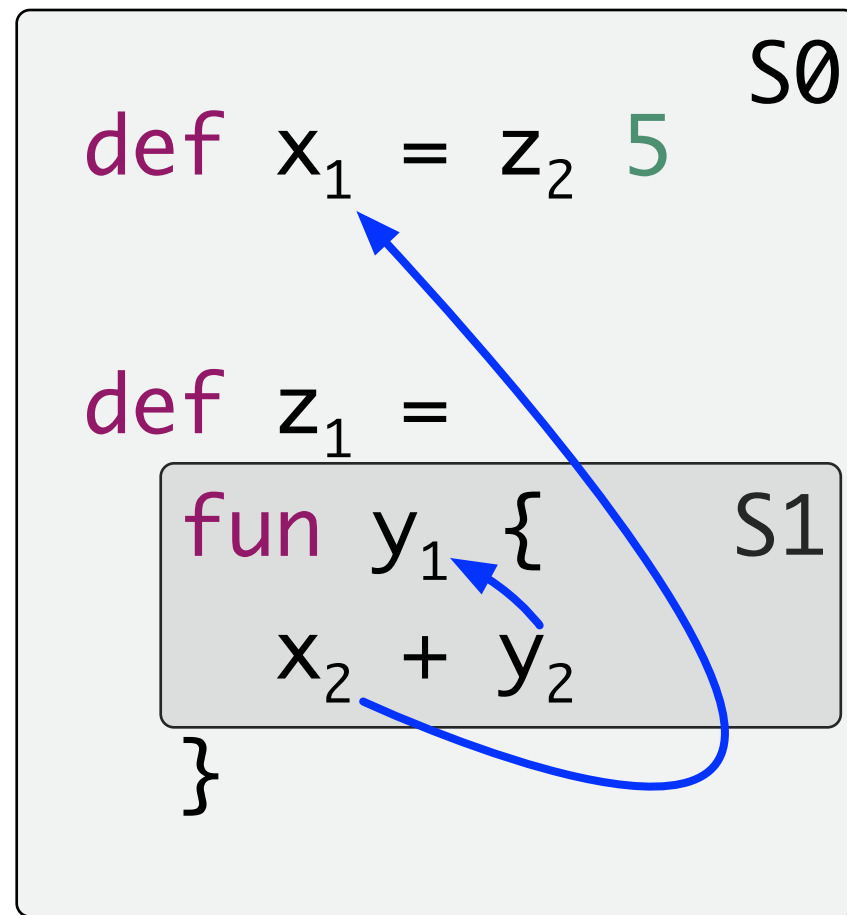
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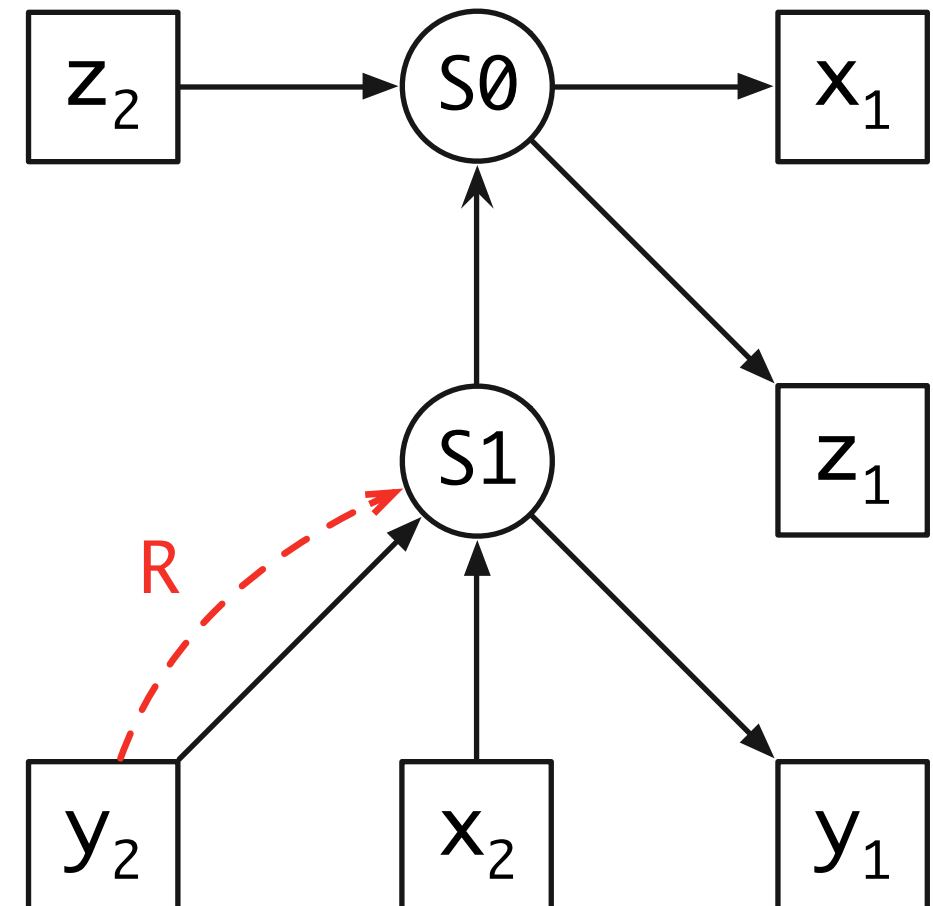
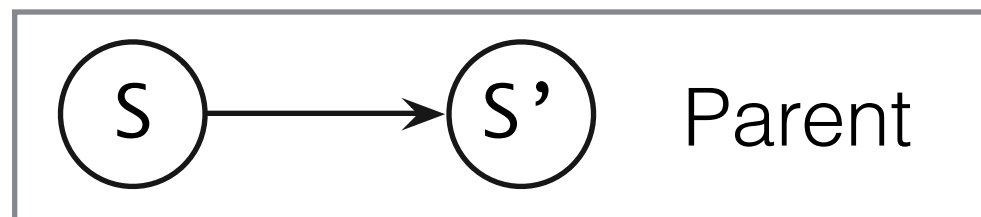
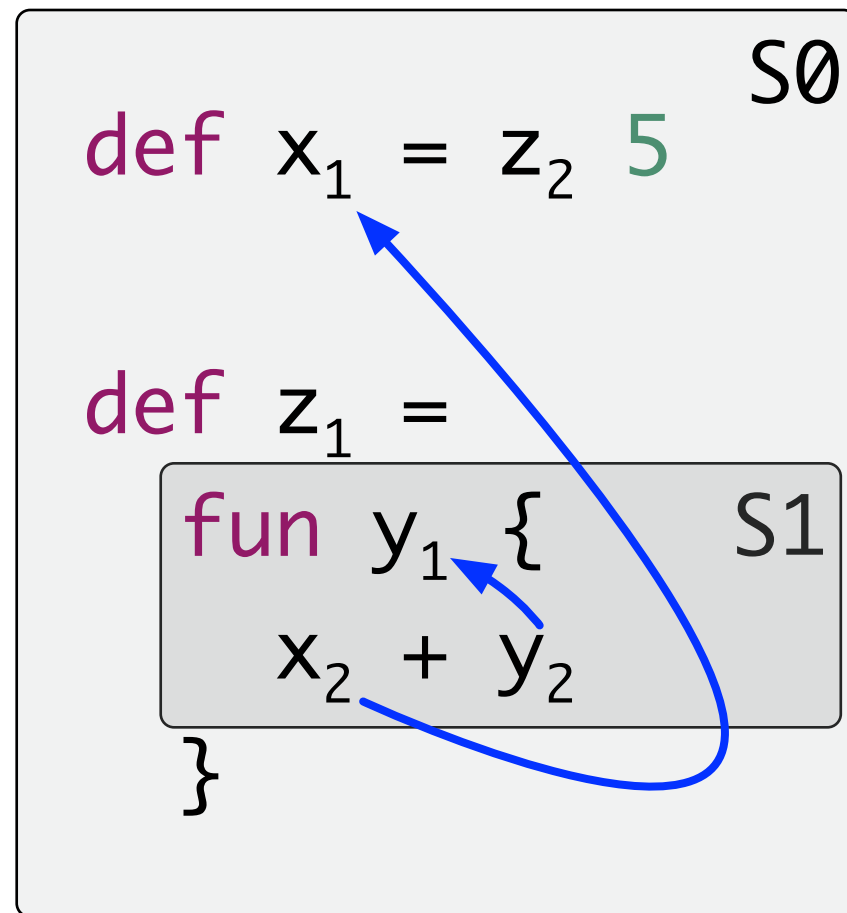
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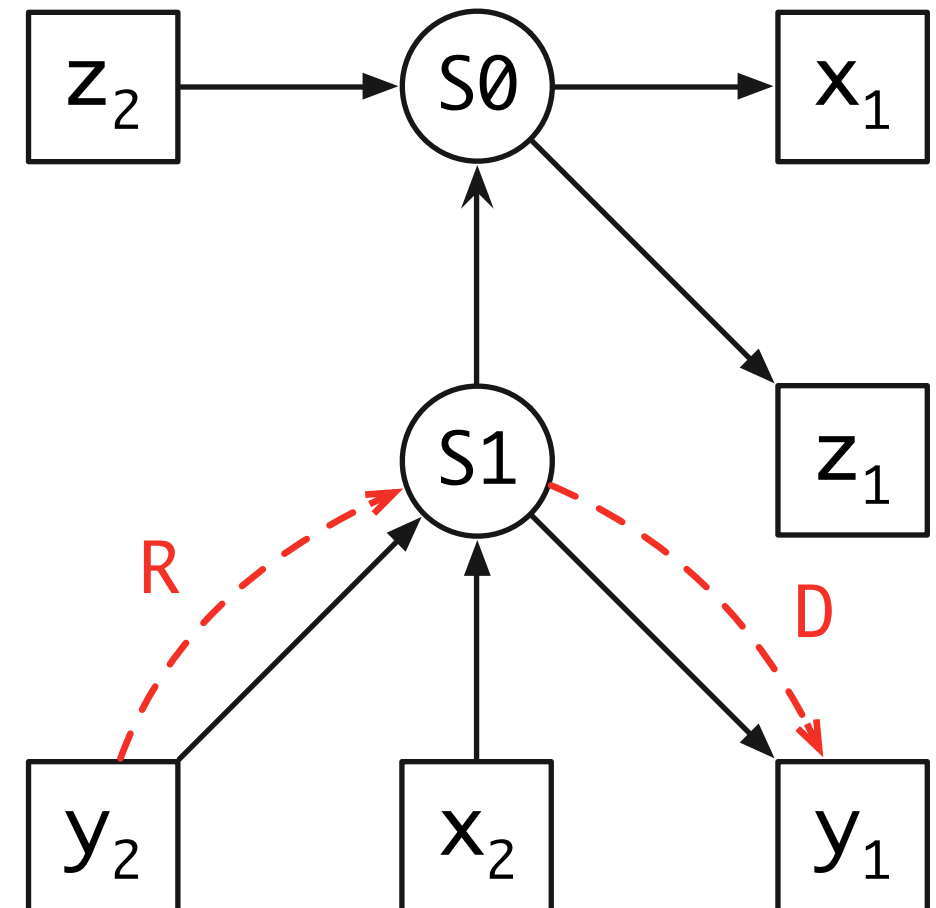
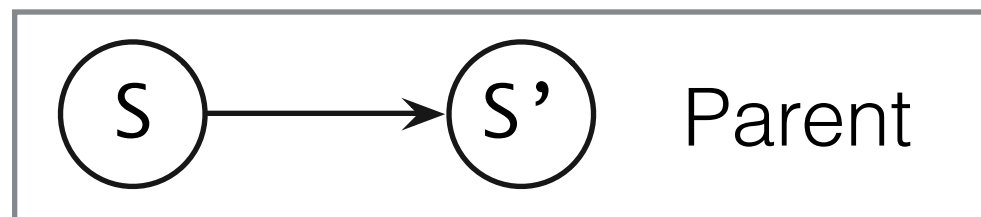
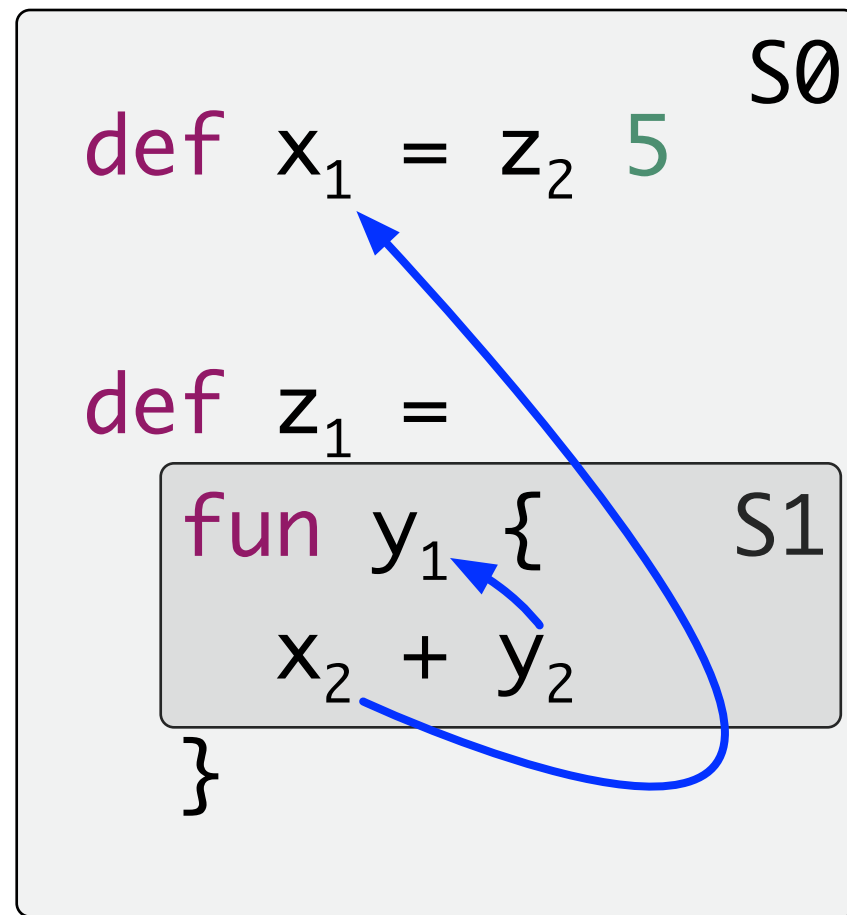
Lexical Scoping



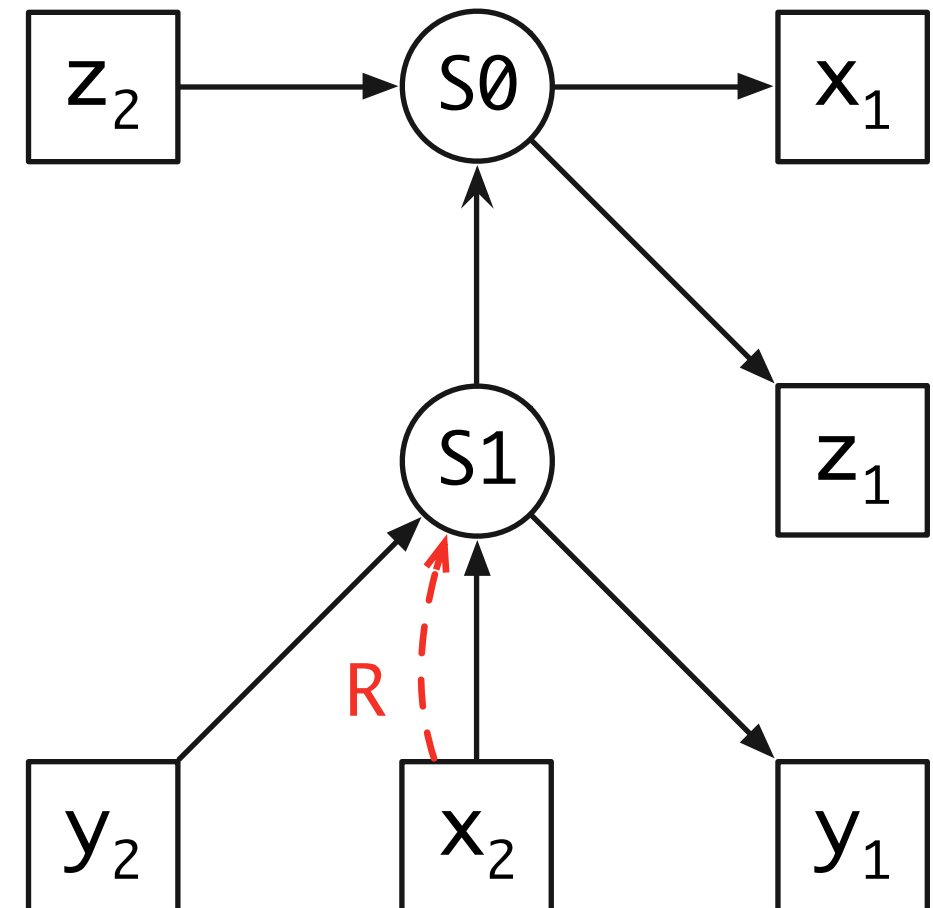
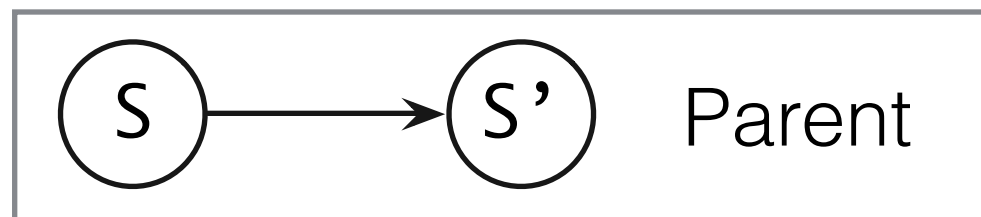
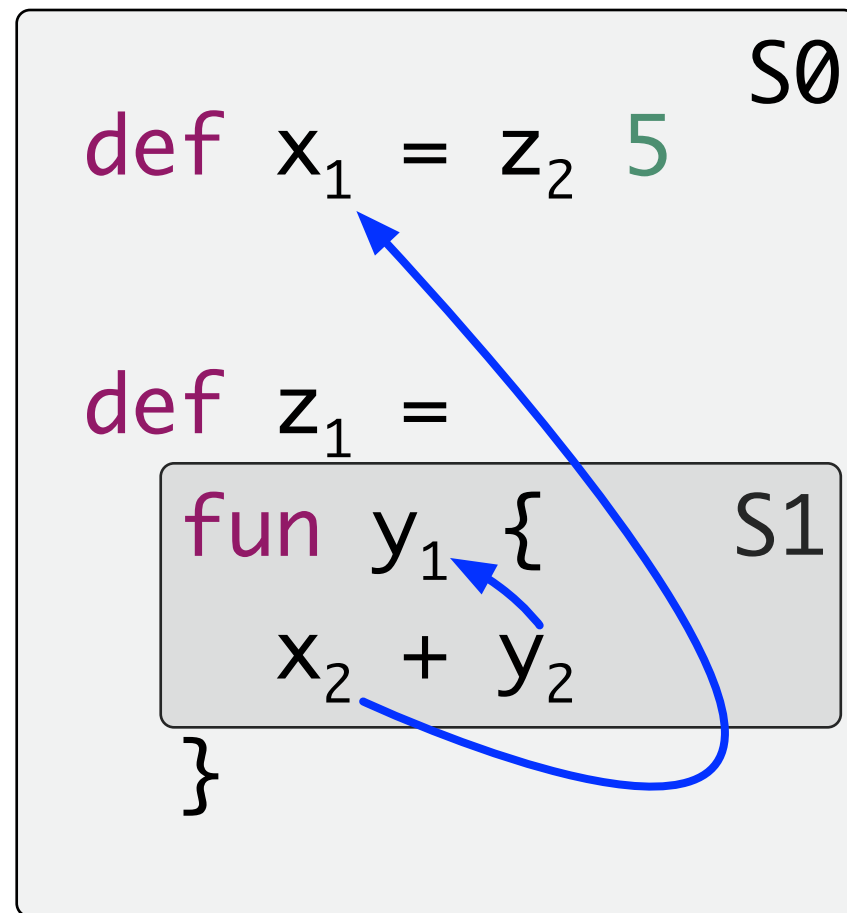
Lexical Scoping



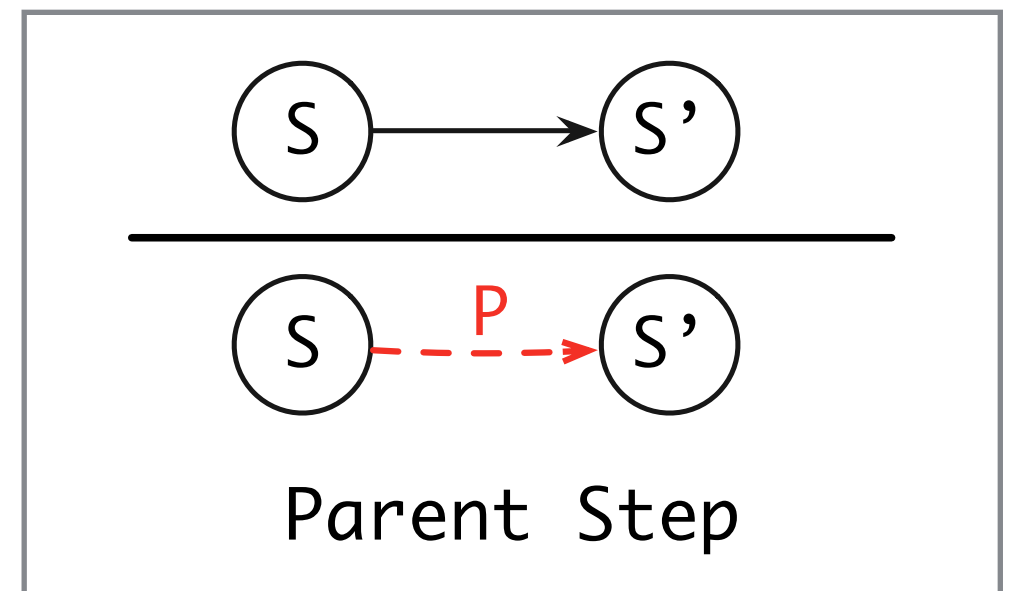
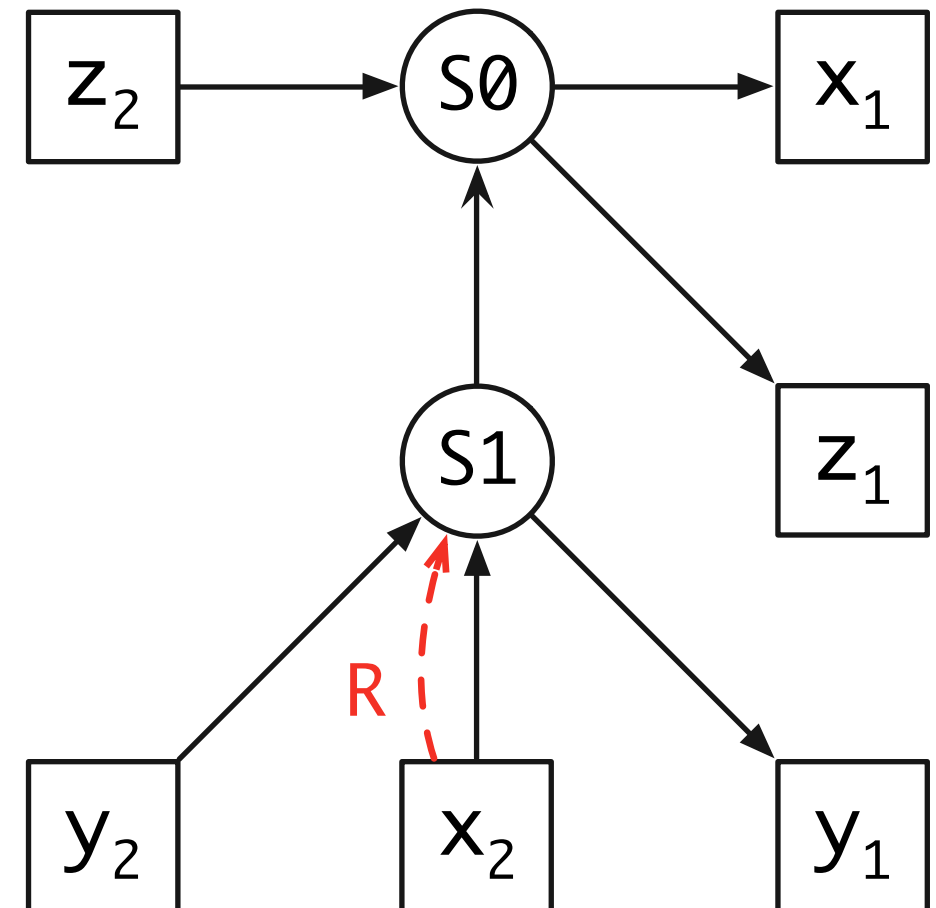
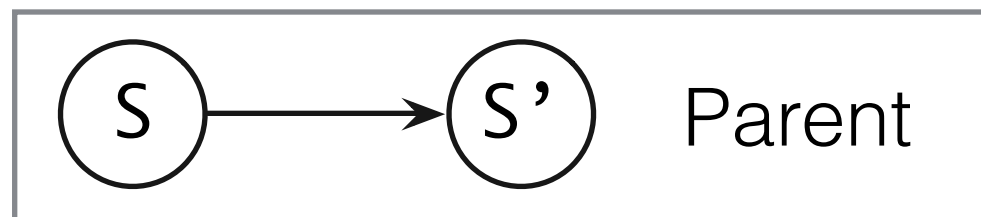
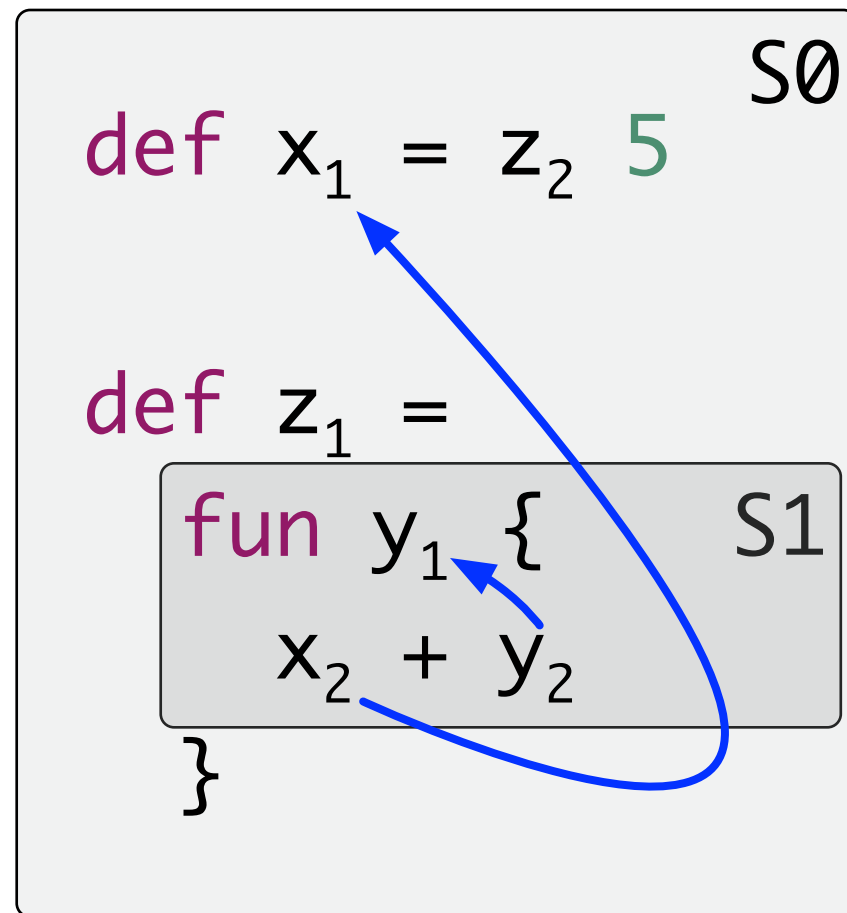
Lexical Scoping



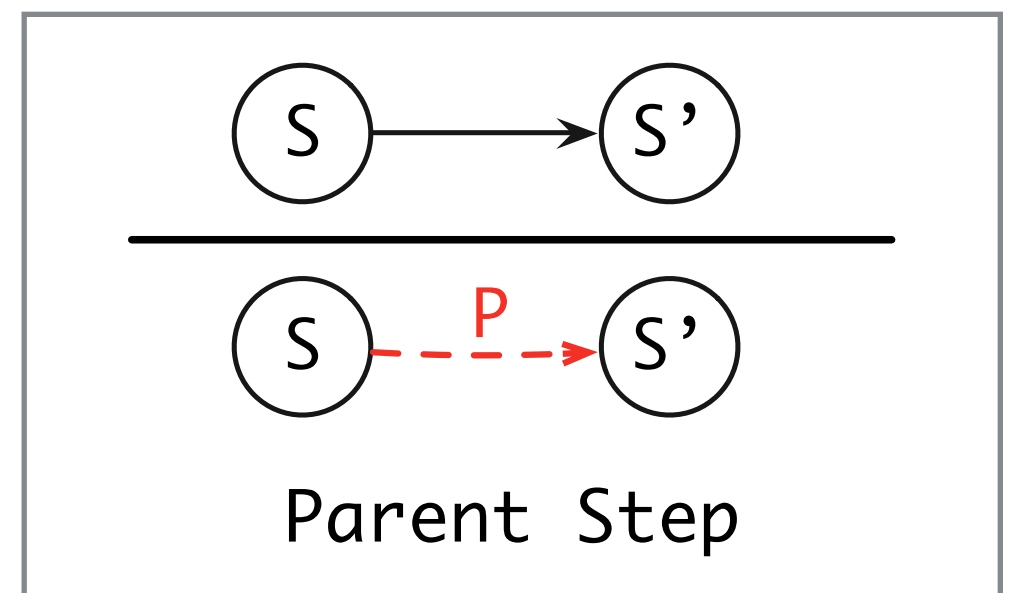
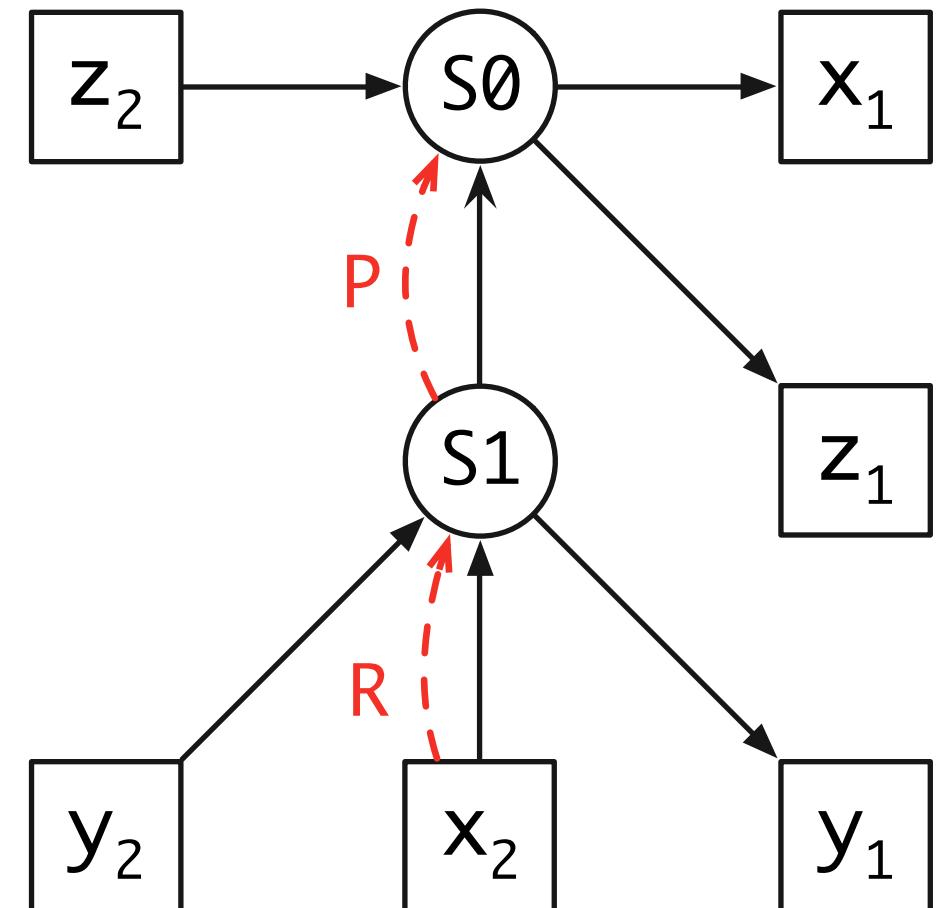
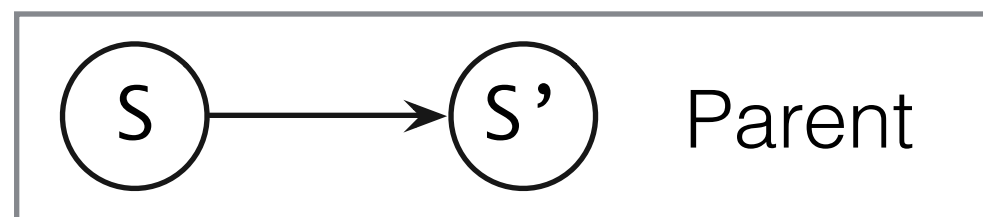
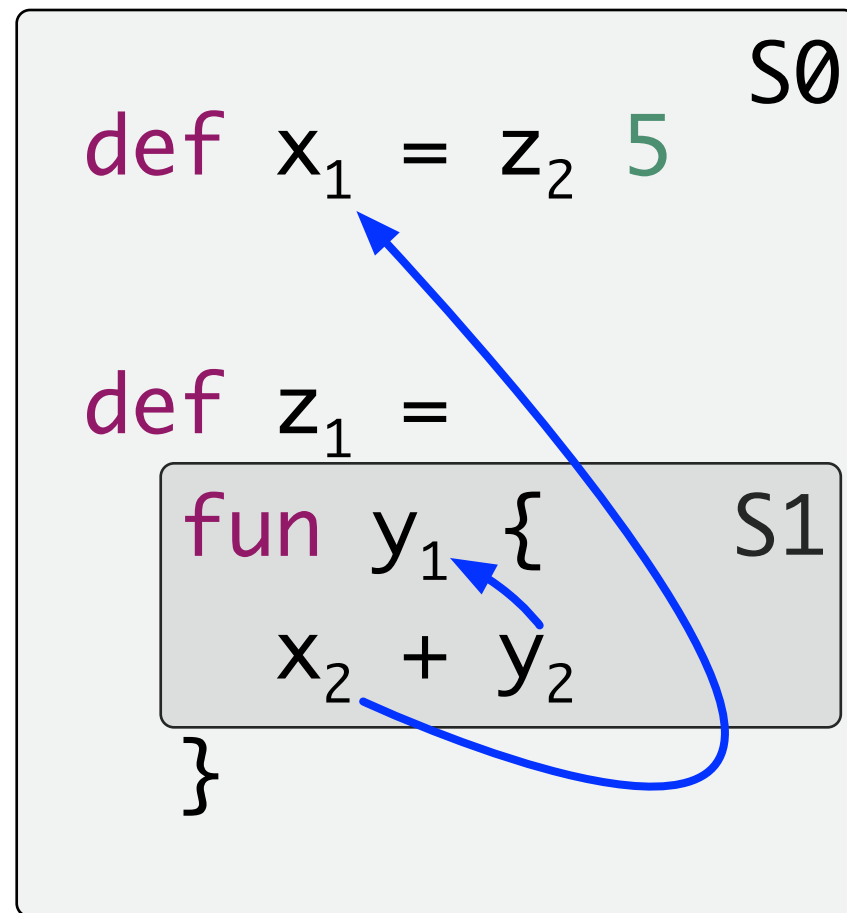
Lexical Scoping



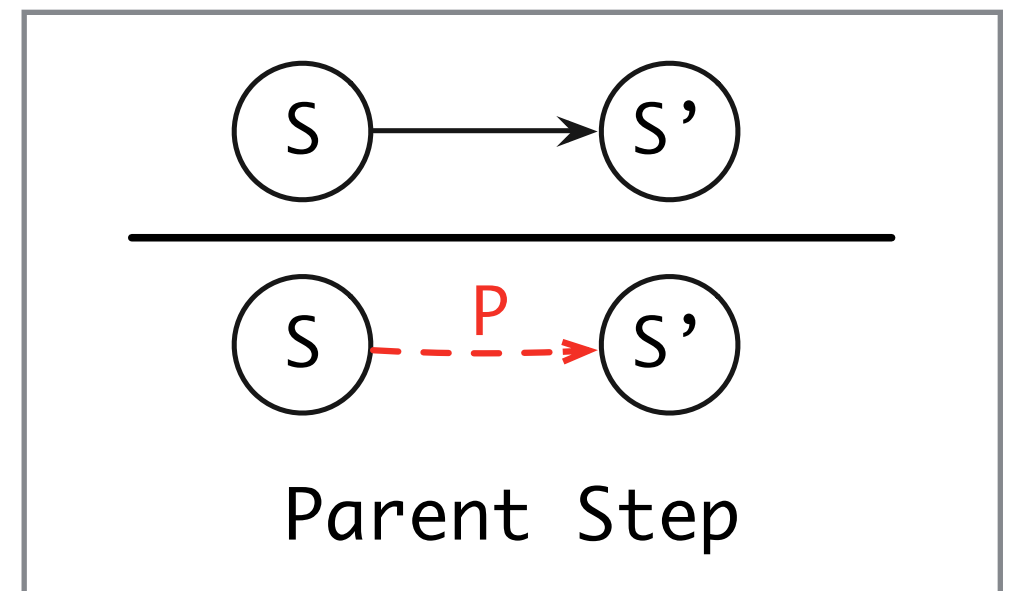
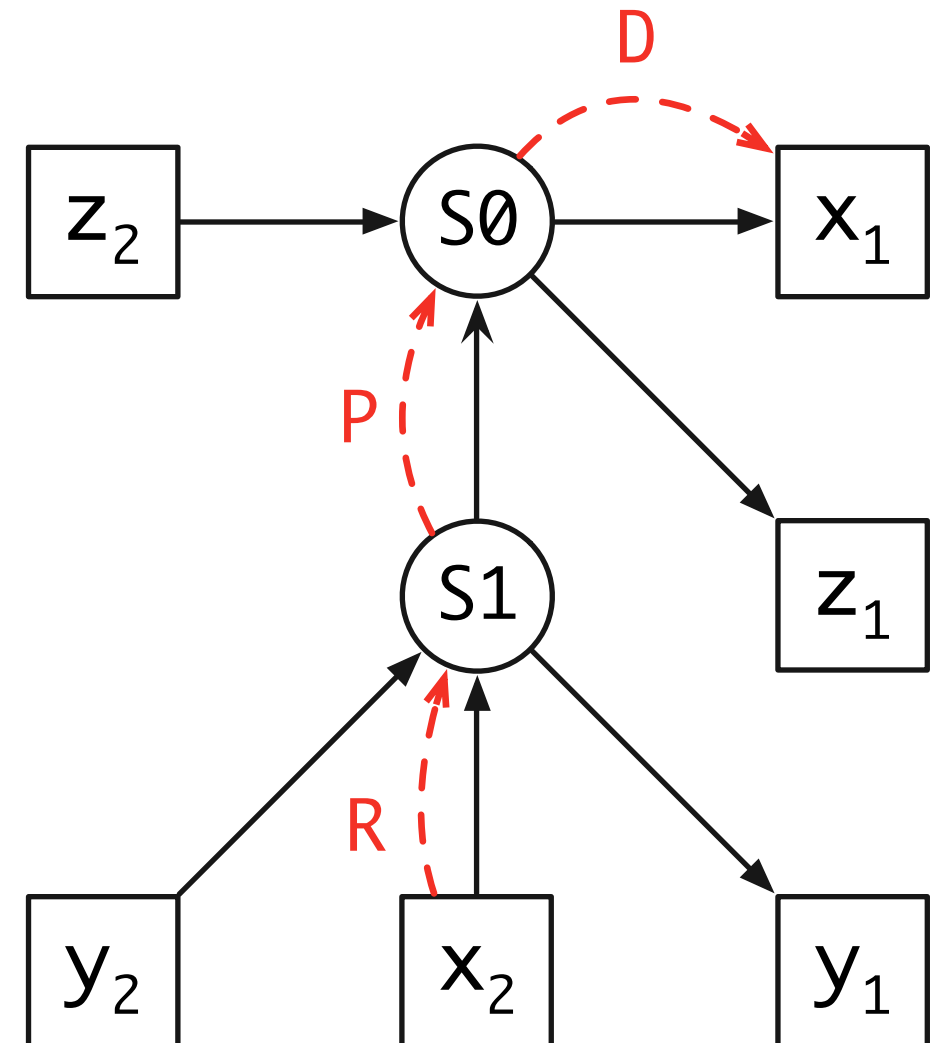
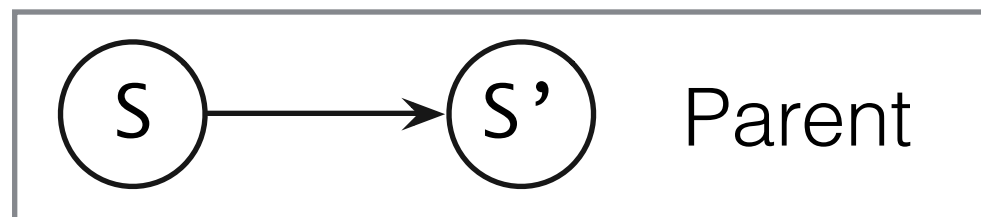
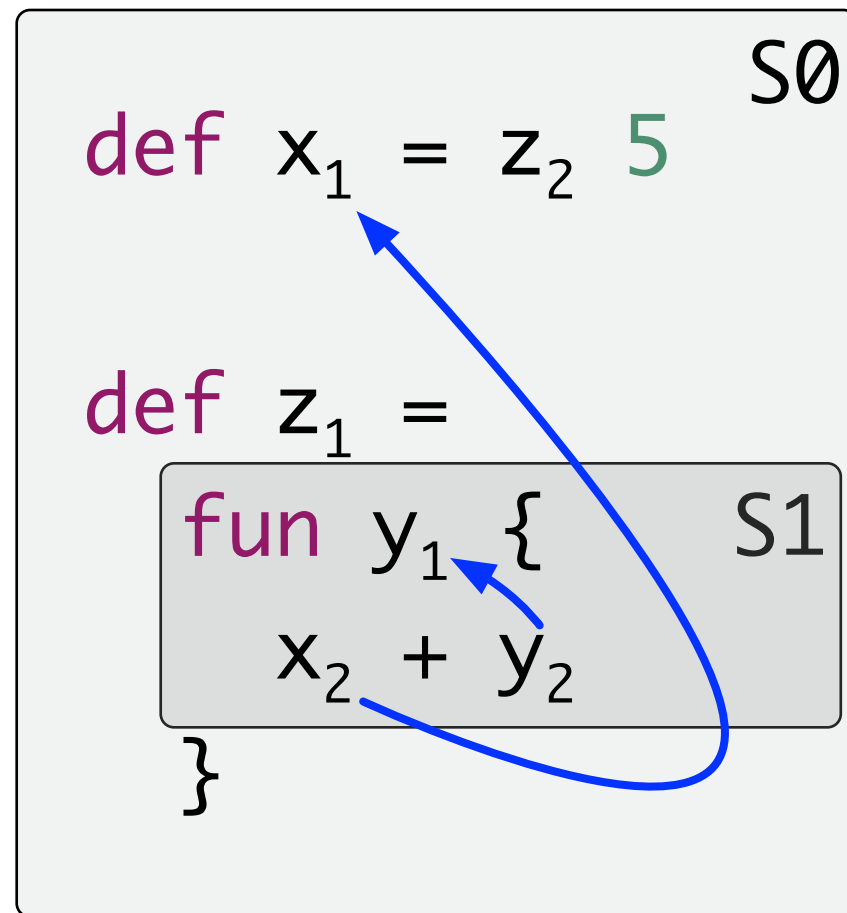
Lexical Scoping



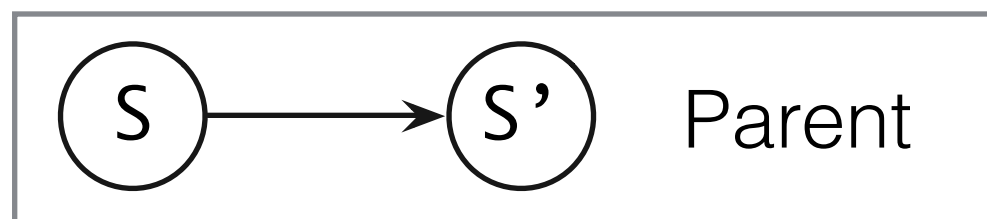
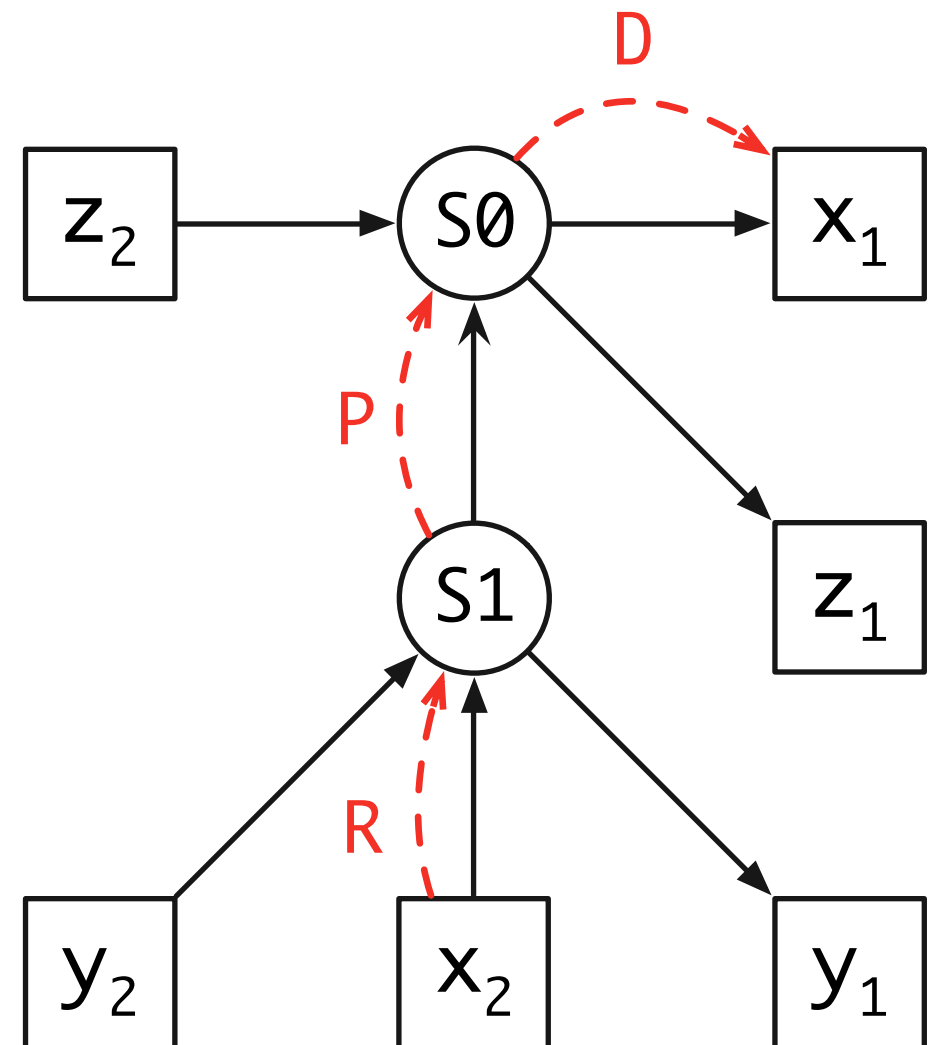
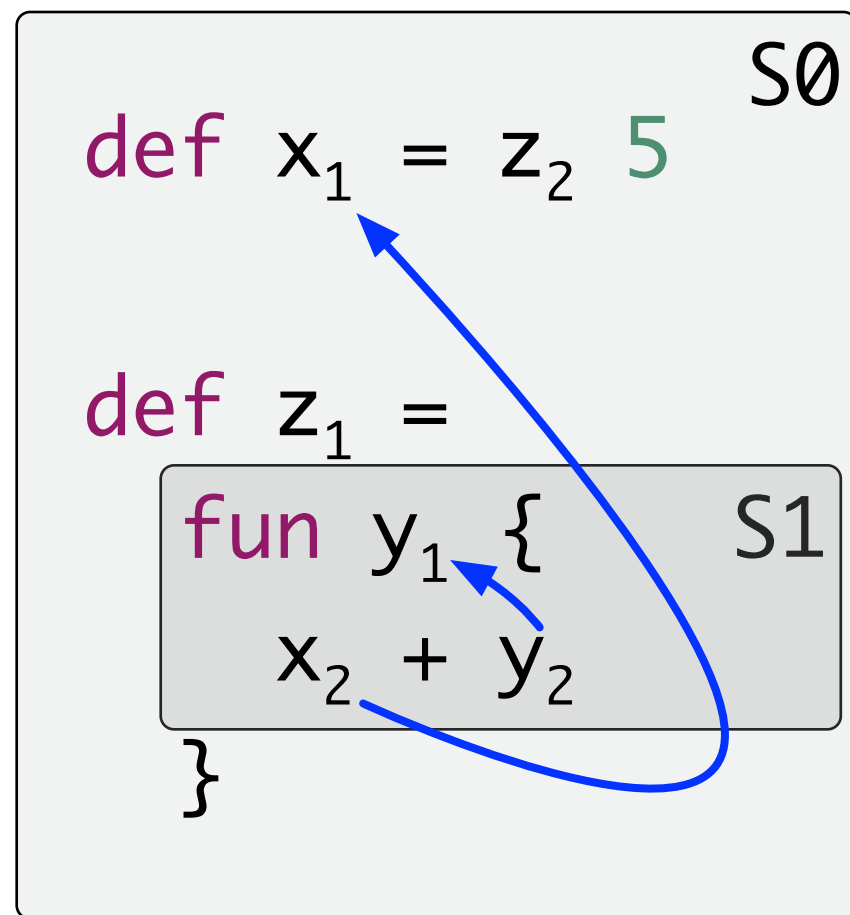
Lexical Scoping



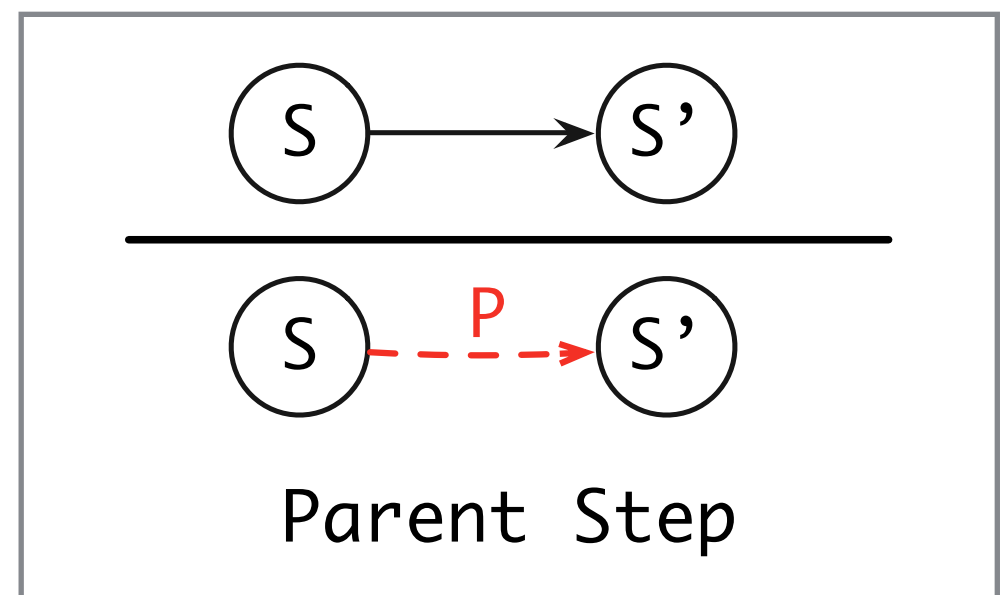
Lexical Scoping



Lexical Scoping



Well formed path: **R.P*.D**



Shadowing

```
def x3 = z2 5 7
```

```
def z1 =  
  fun x1 {  
    fun y1 {  
      x2 + y2  
    }  
  }
```

Shadowing

```
def x3 = z2 5 7 S0
```

```
def z1 =
```

```
  fun x1 { S1
```

```
    fun y1 { S2
```

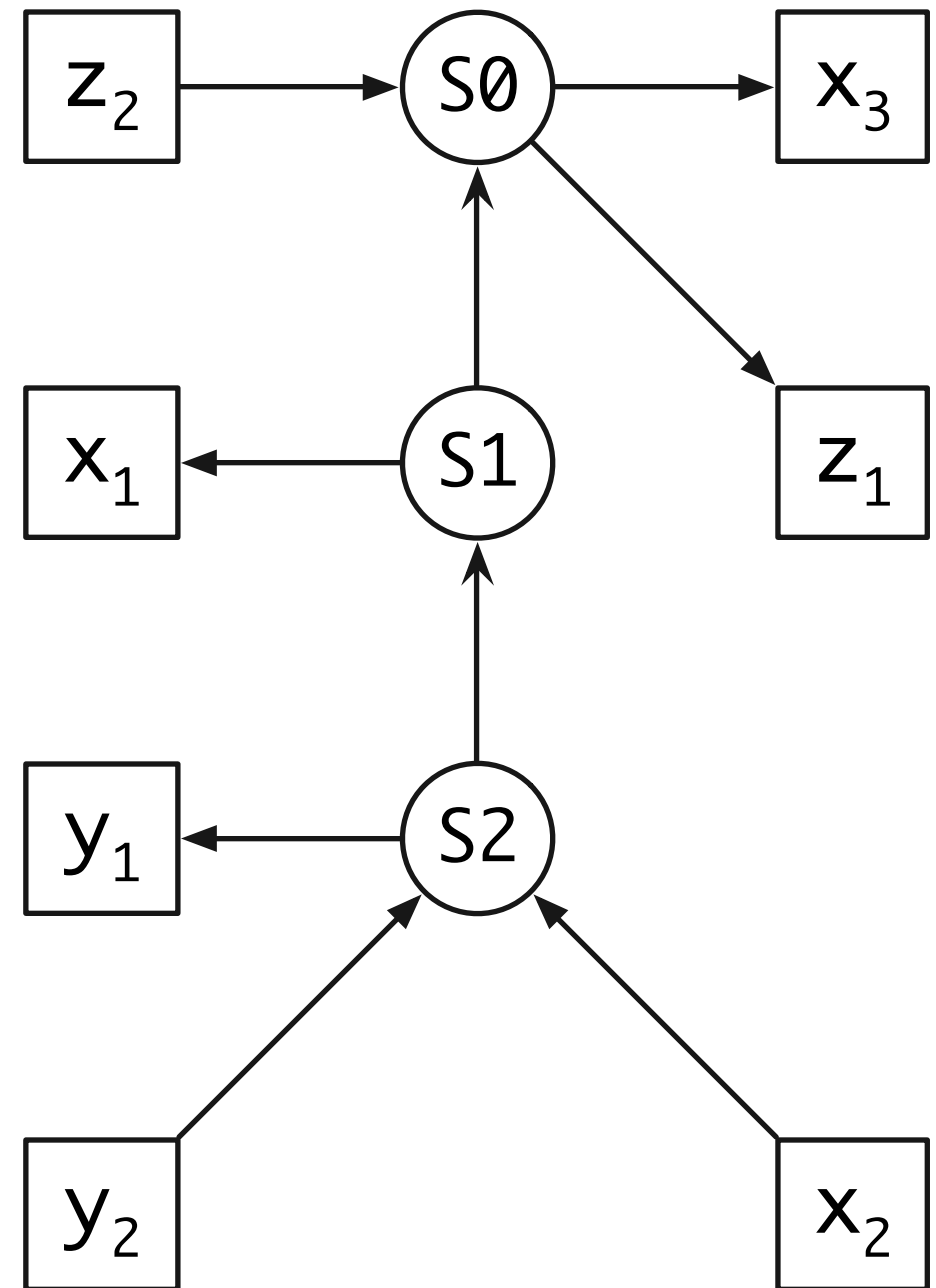
```
      x2 + y2
```

```
    }
```

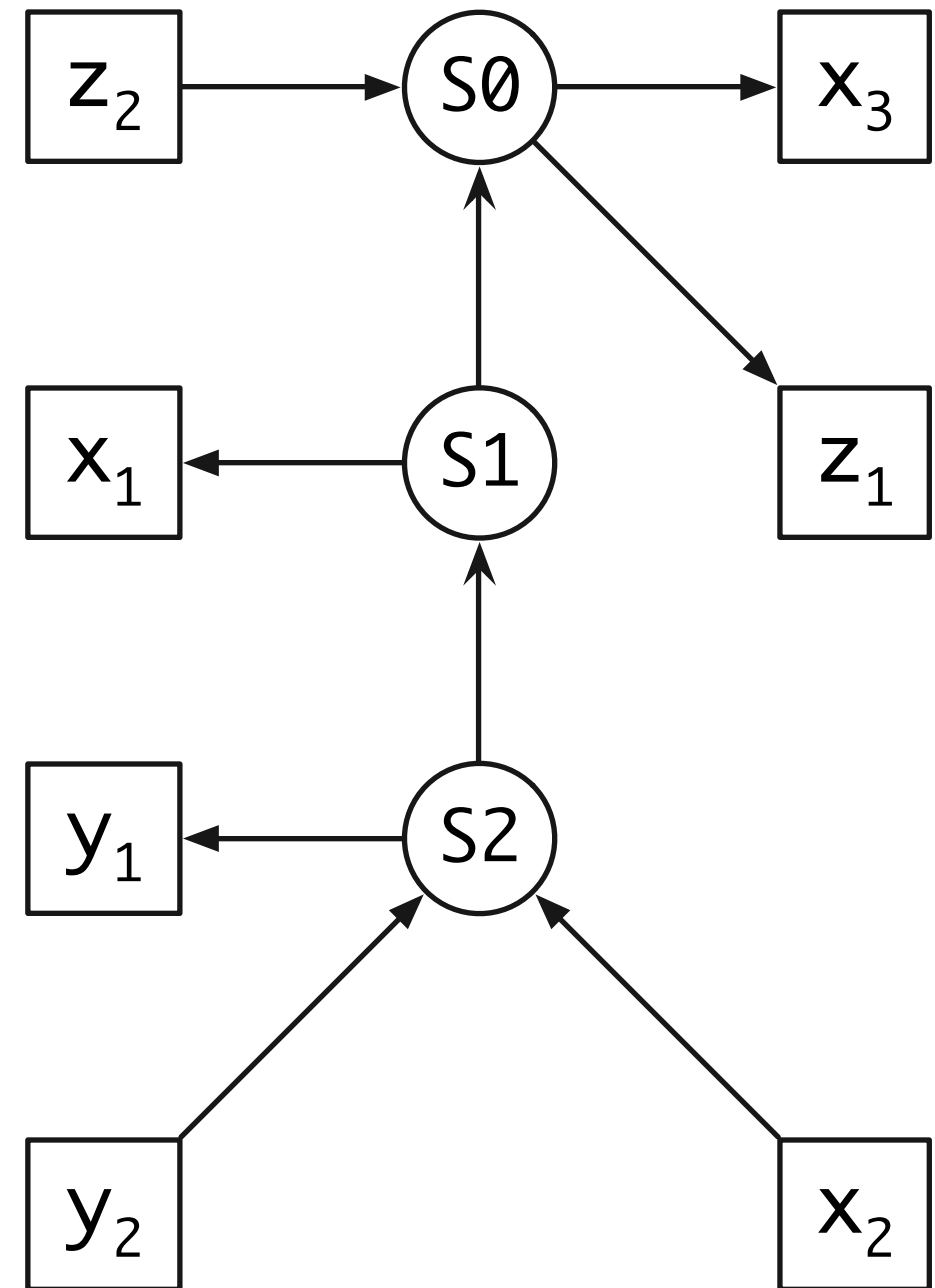
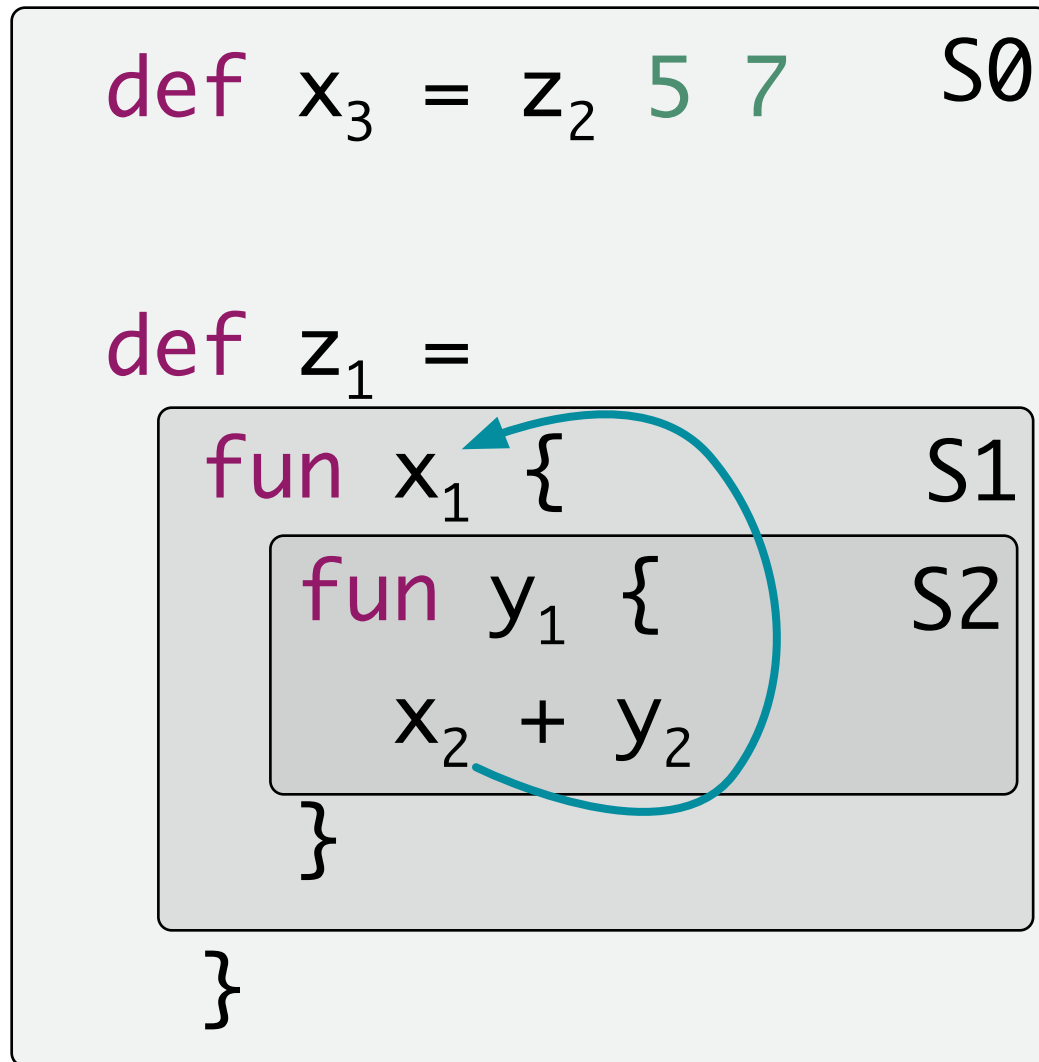
```
  }
```


Shadowing

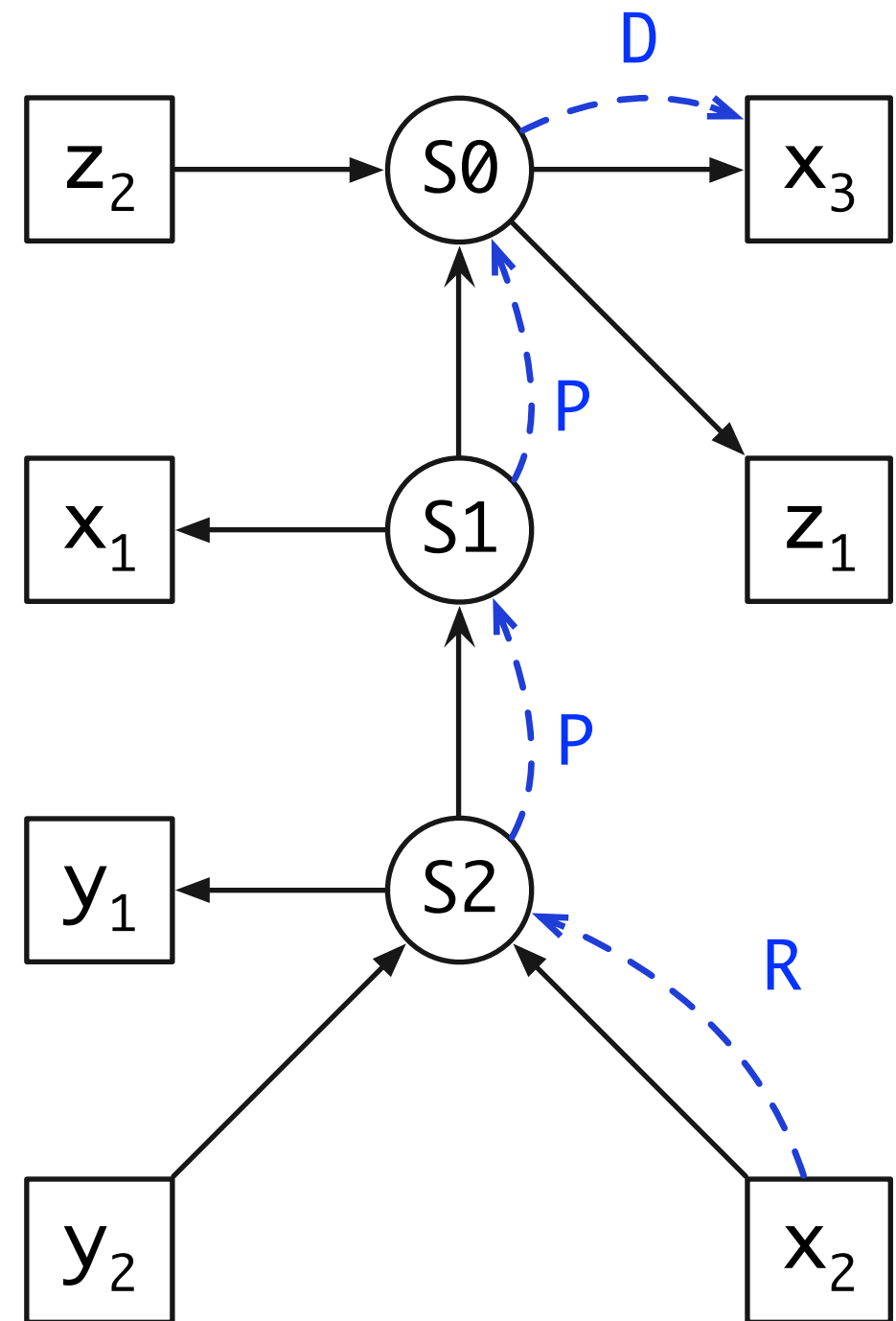
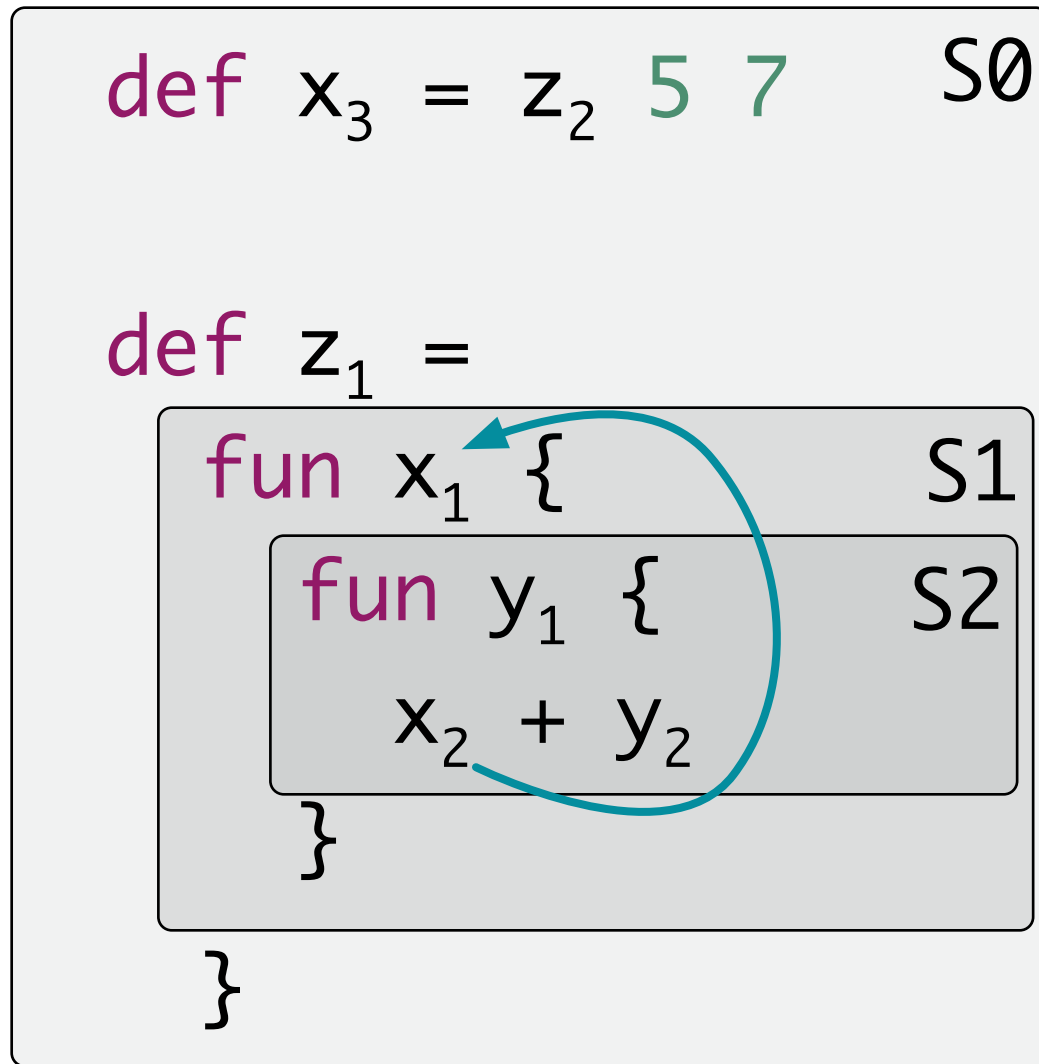
```
def x3 = z2 5 7 S0  
  
def z1 =  
  fun x1 { S1  
    fun y1 { S2  
      x2 + y2  
    }  
  }
```



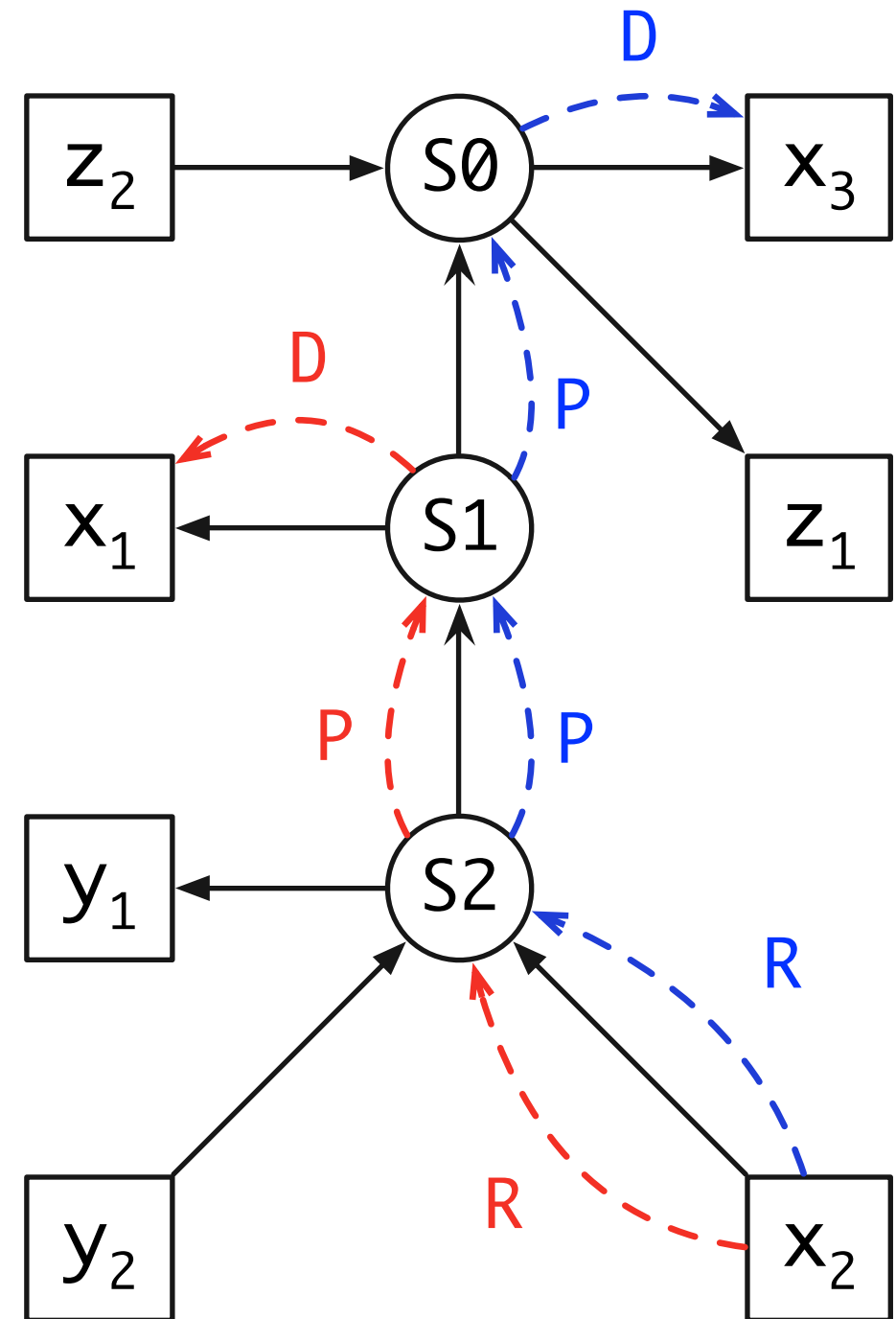
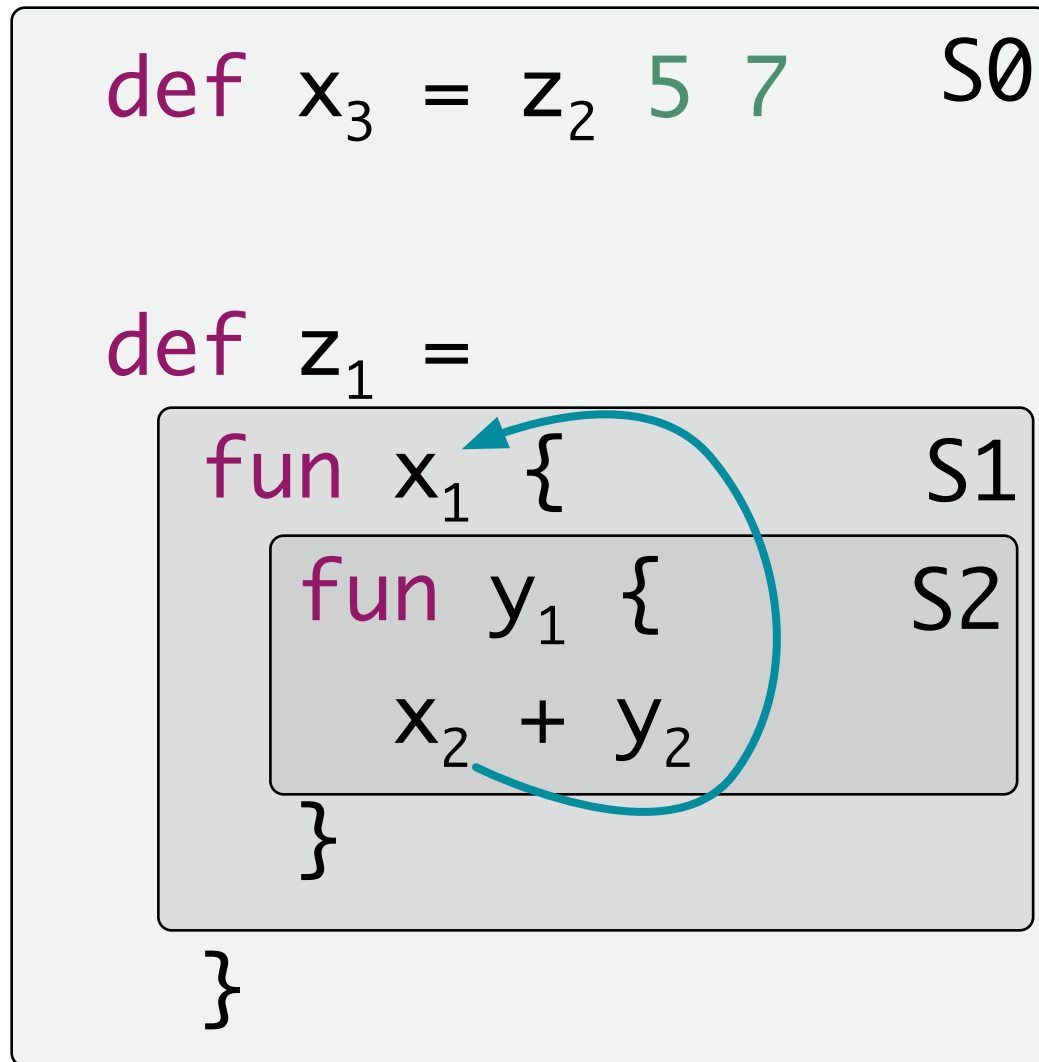
Shadowing



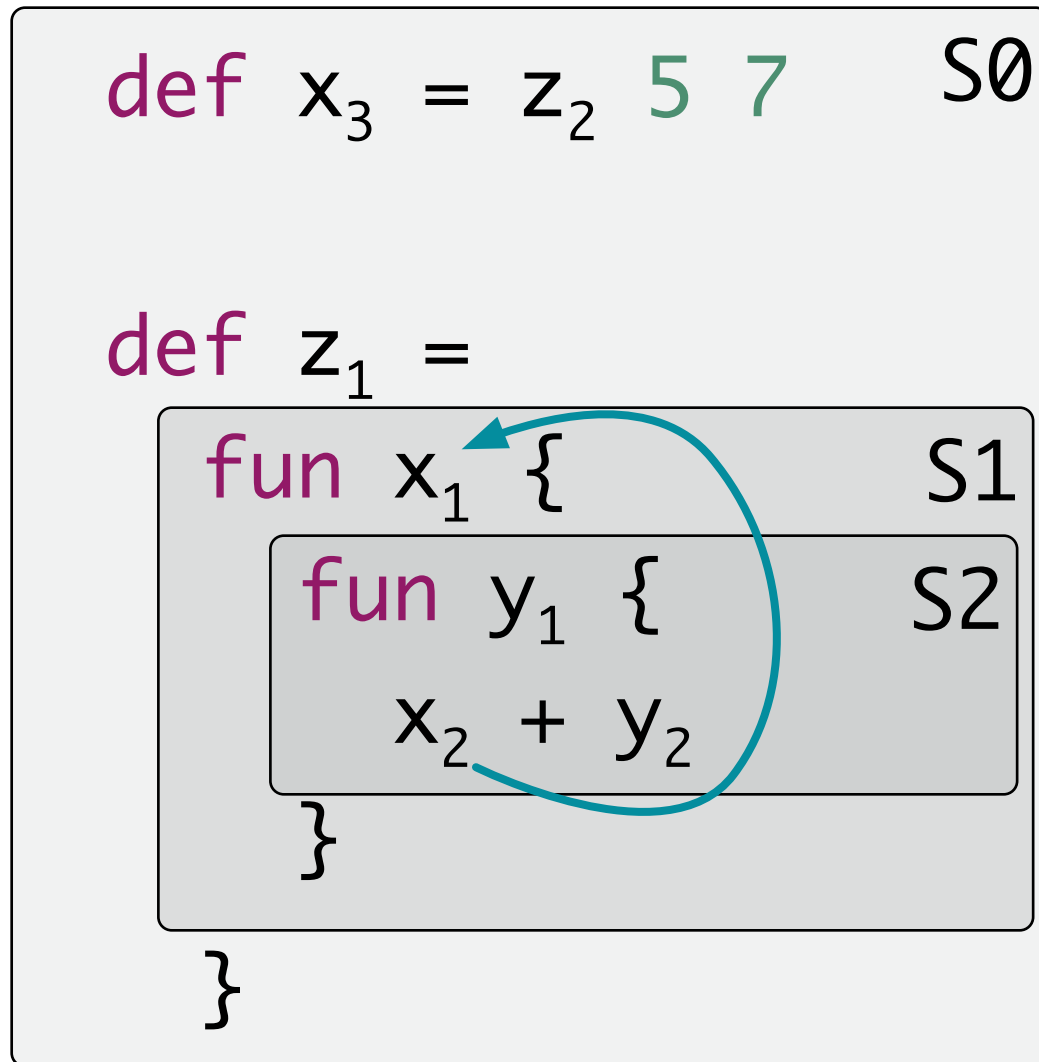
Shadowing



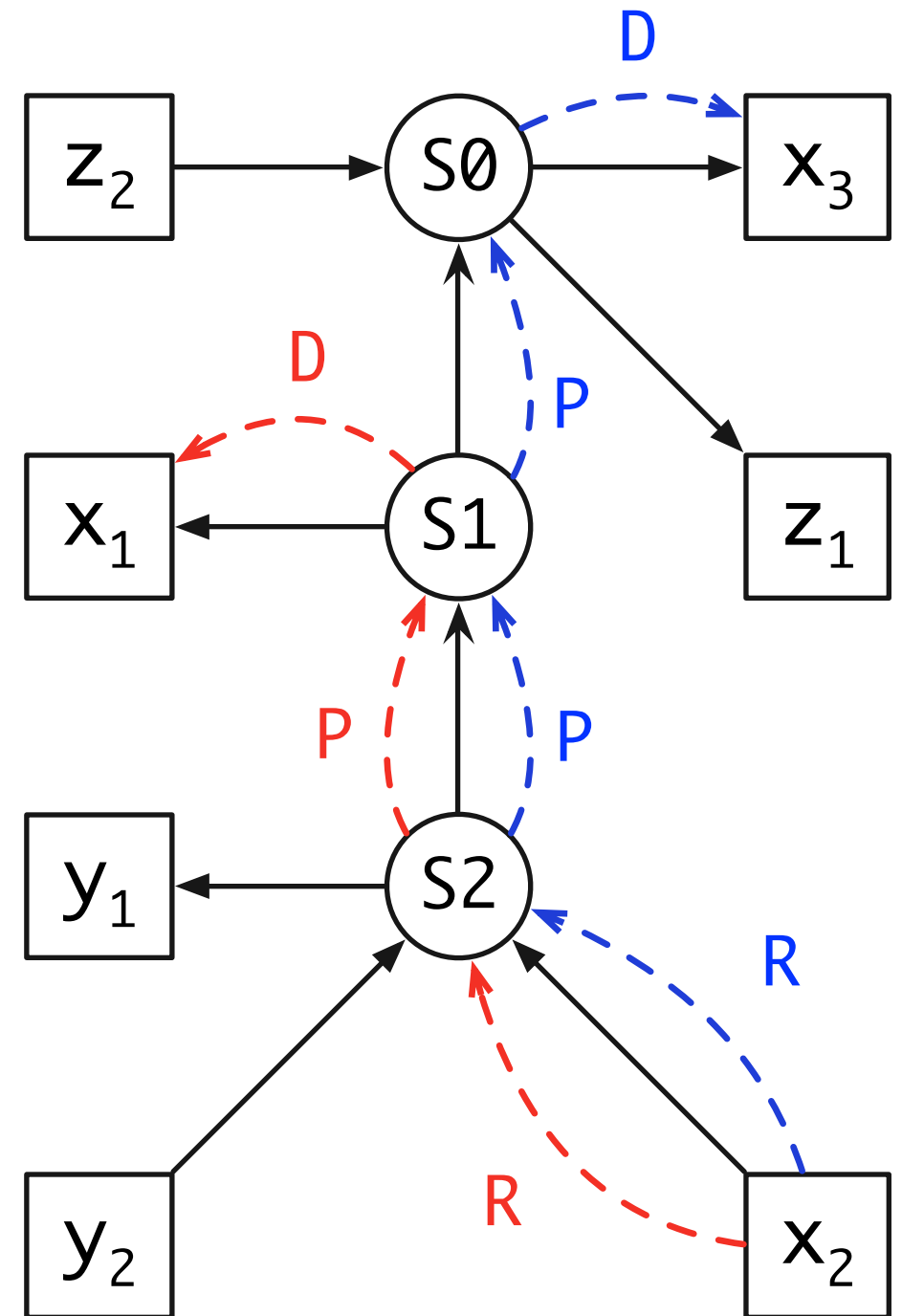
Shadowing



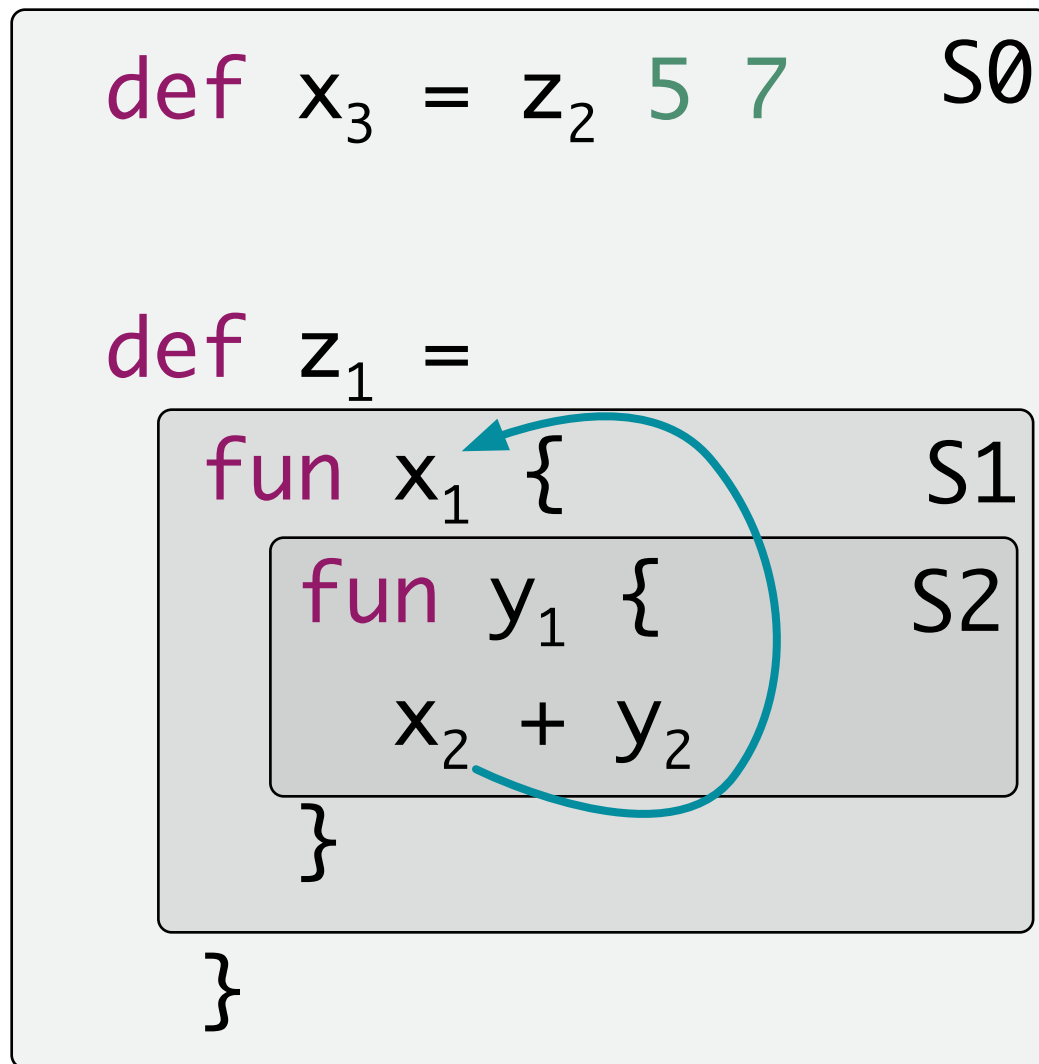
Shadowing



$D < P.p$

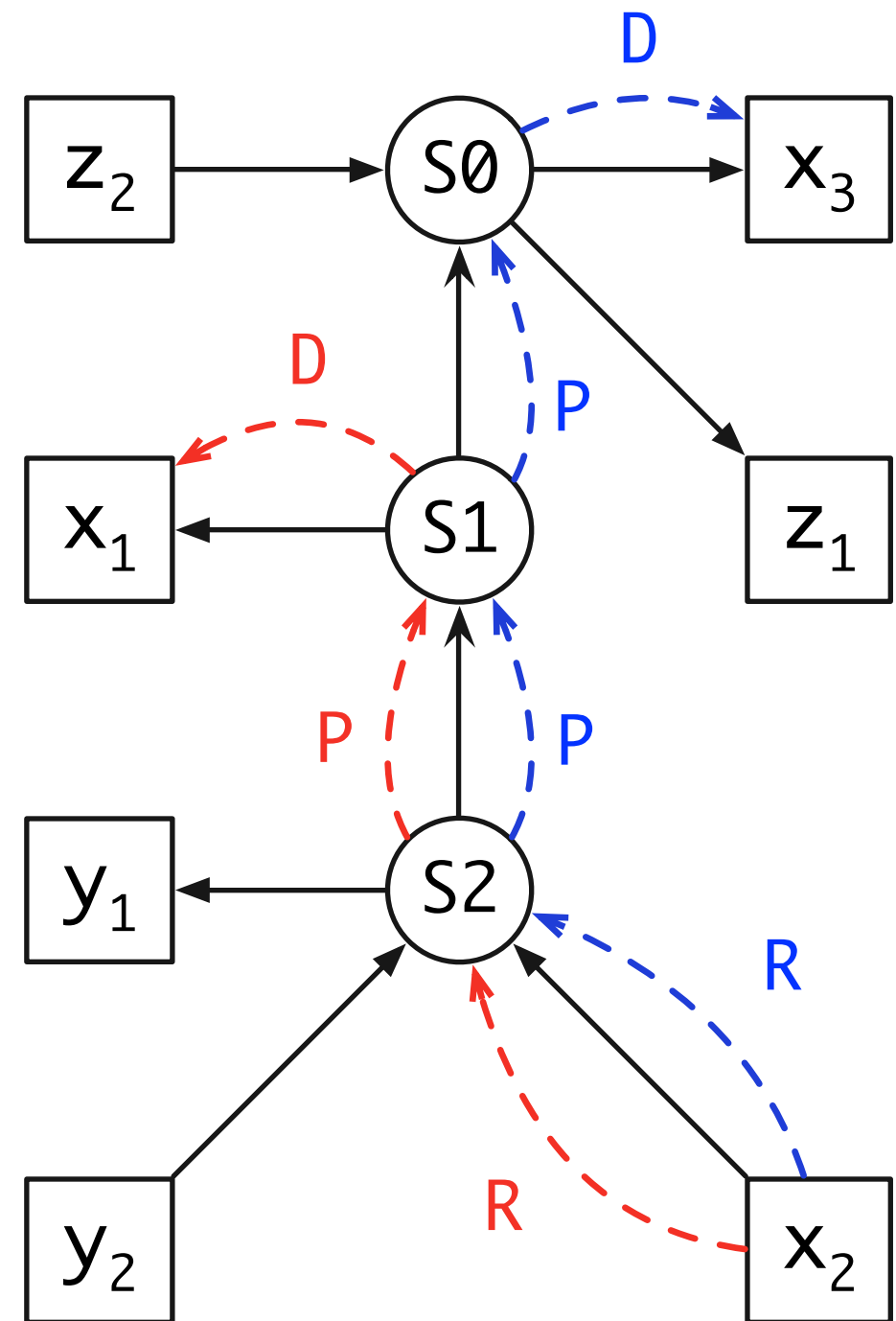


Shadowing

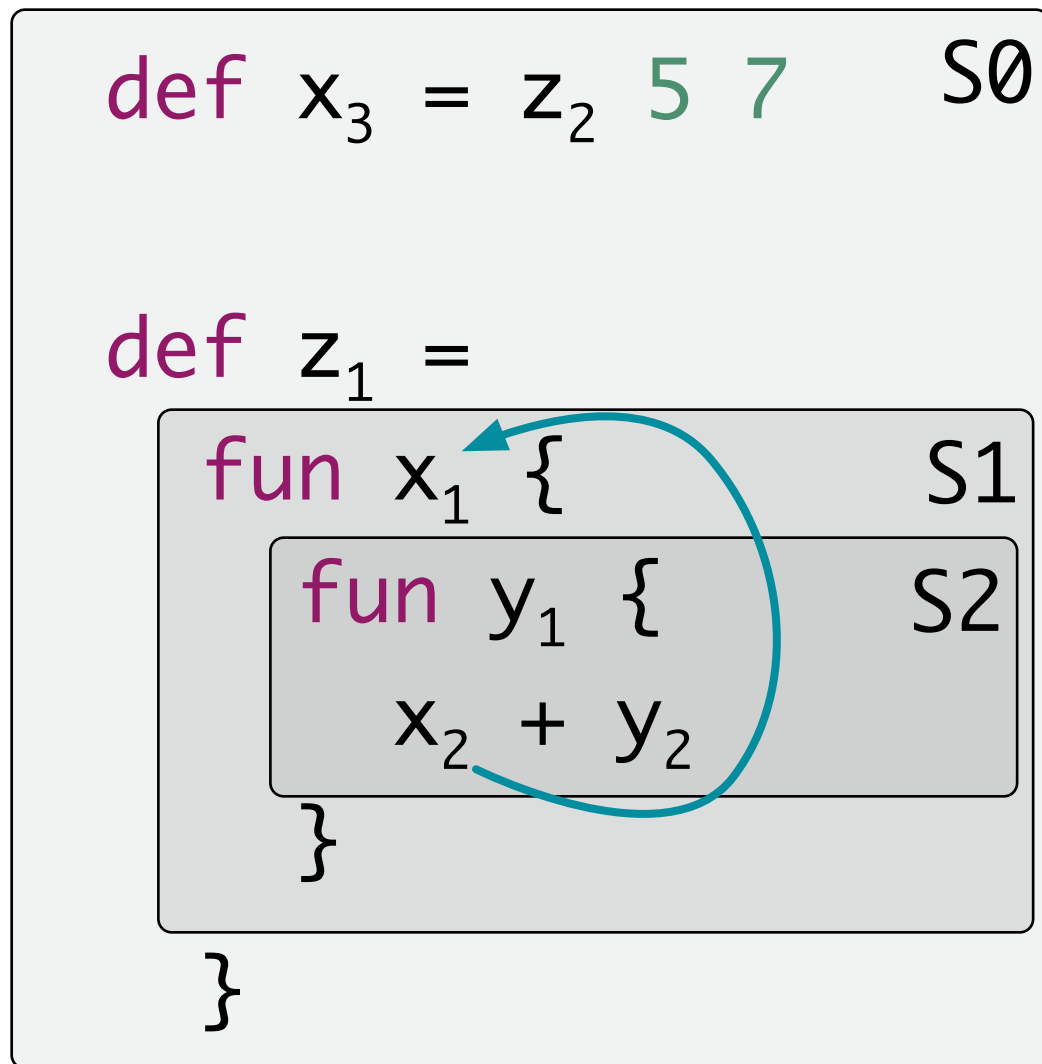


$$\frac{D < P.p}{p < p'}$$

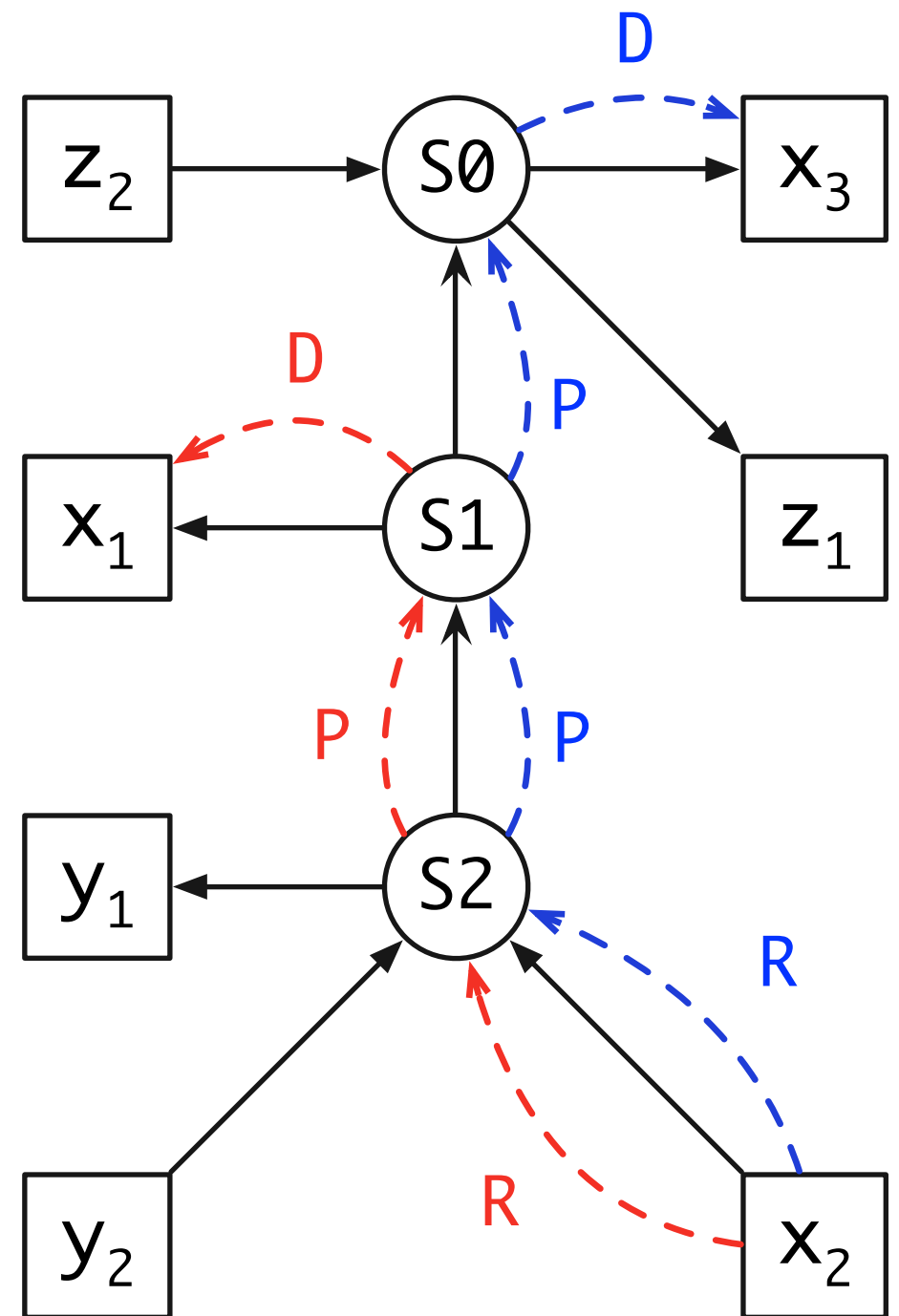
$$\frac{p < p'}{s.p < s.p'}$$



Shadowing



$$\frac{D < P.p}{\frac{p < p'}{s.p < s.p'}}$$

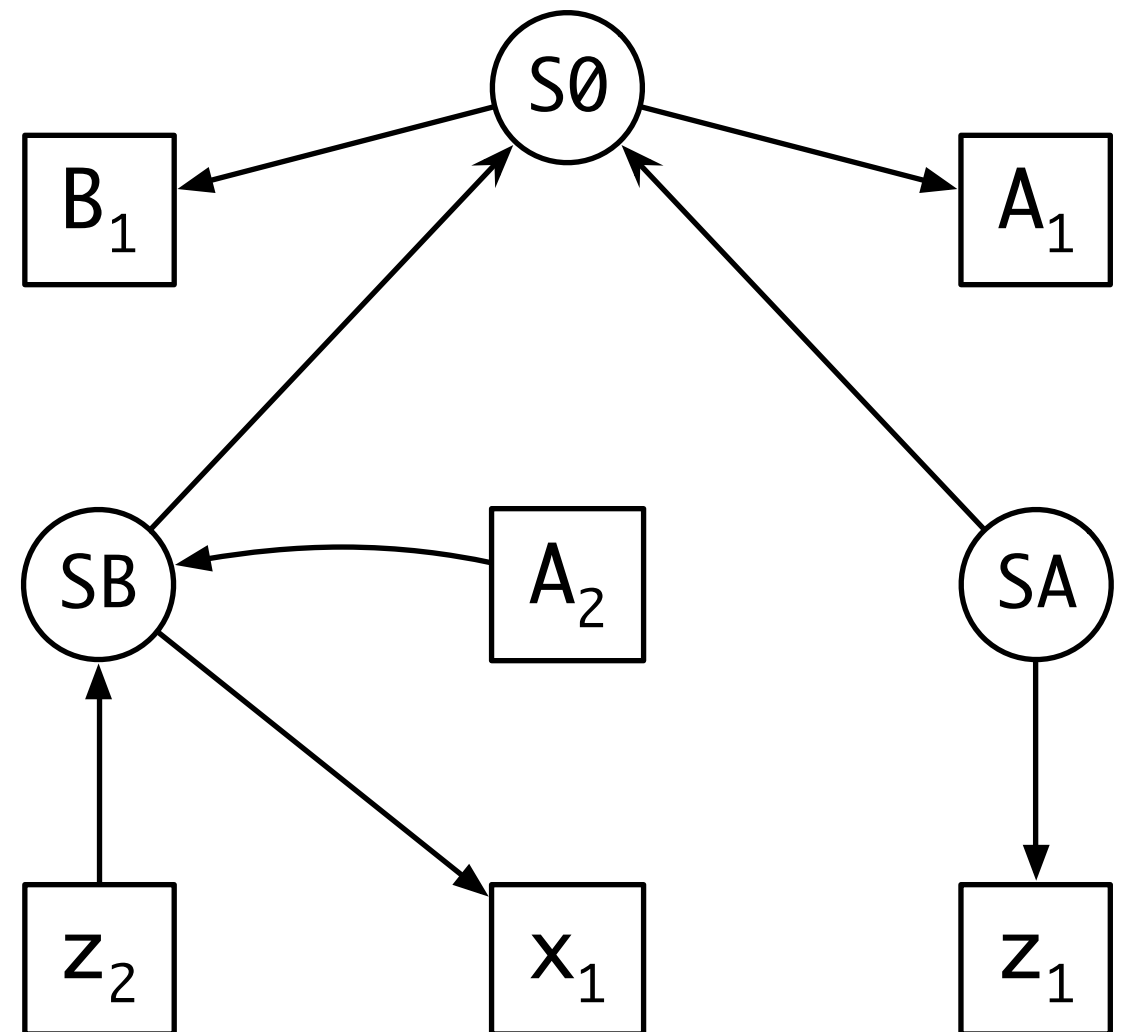
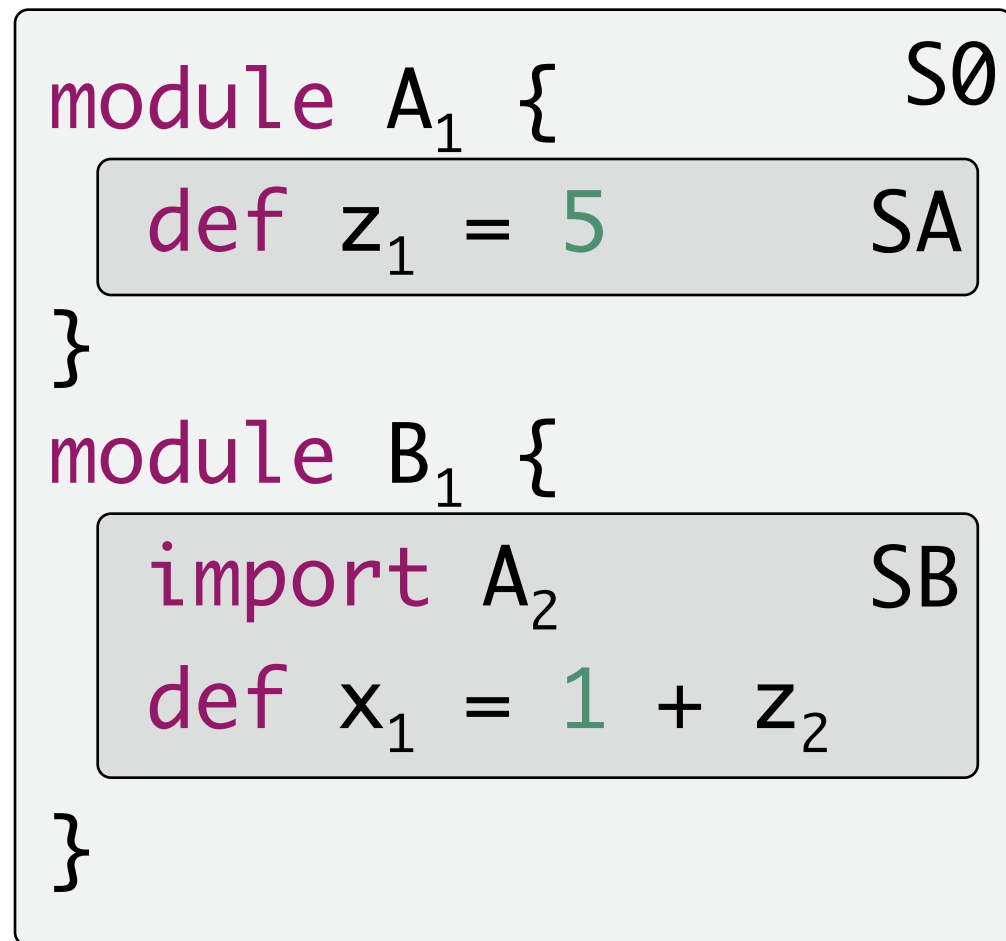


$$R.P.D < R.P.P.D$$

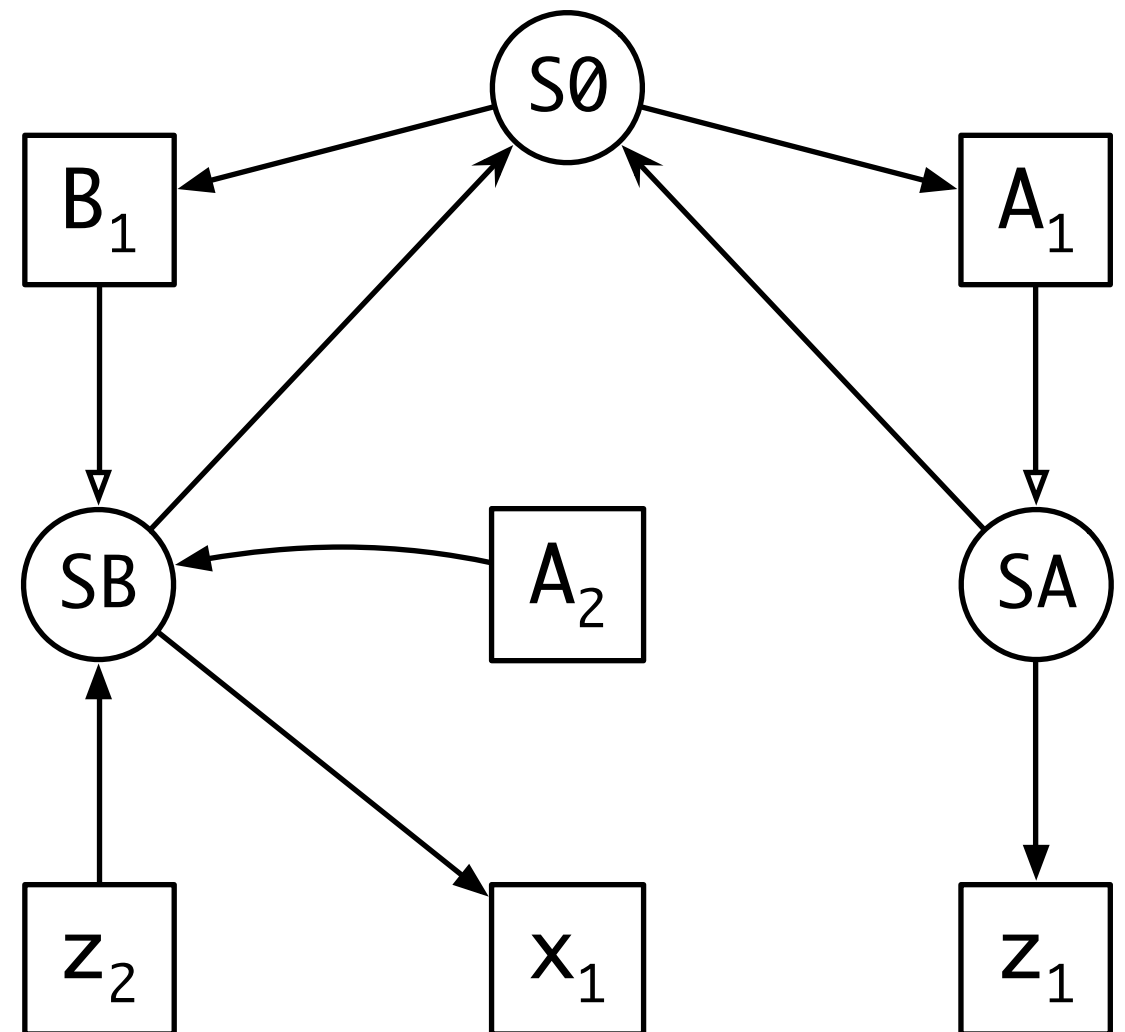
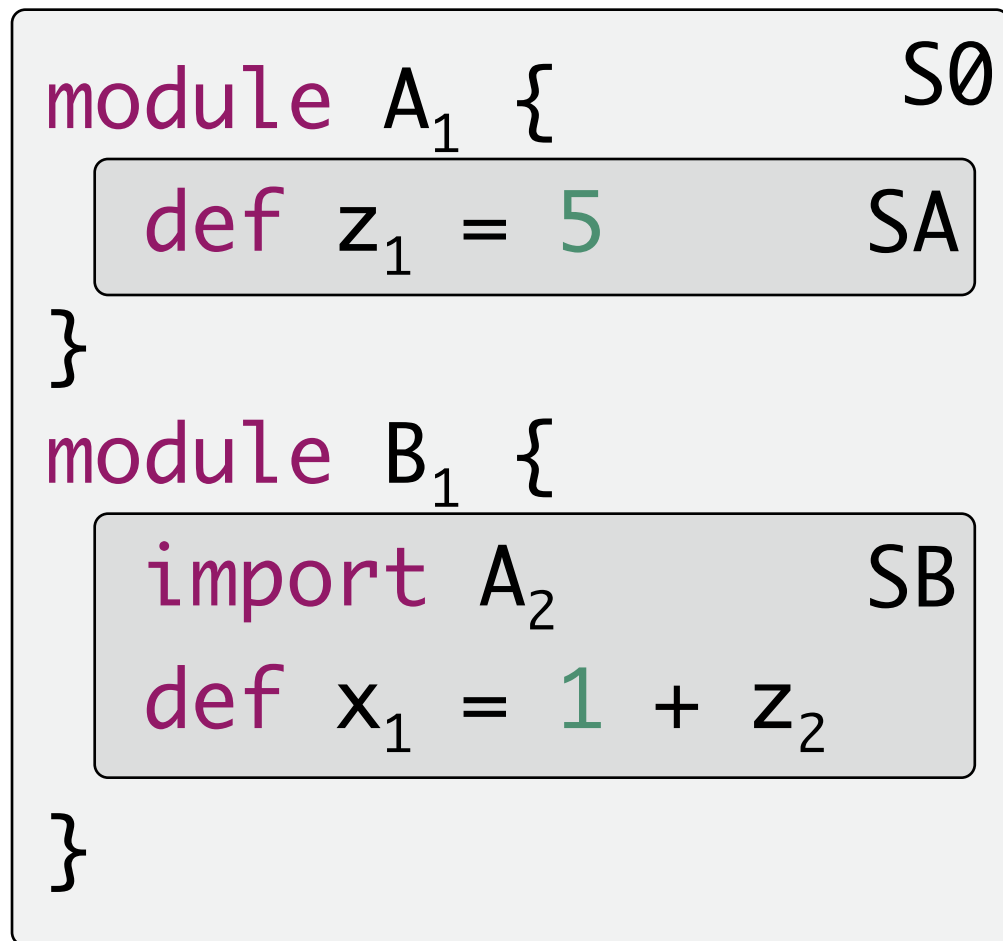
Imports

```
module A1 {                               S0
  def z1 = 5                               SA
}
module B1 {
  import A2                               SB
  def x1 = 1 + z2
}
```

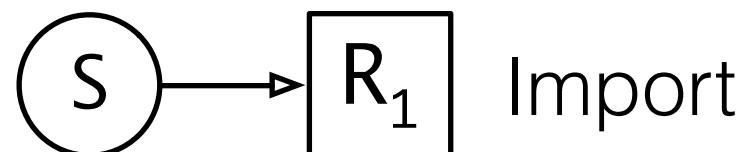
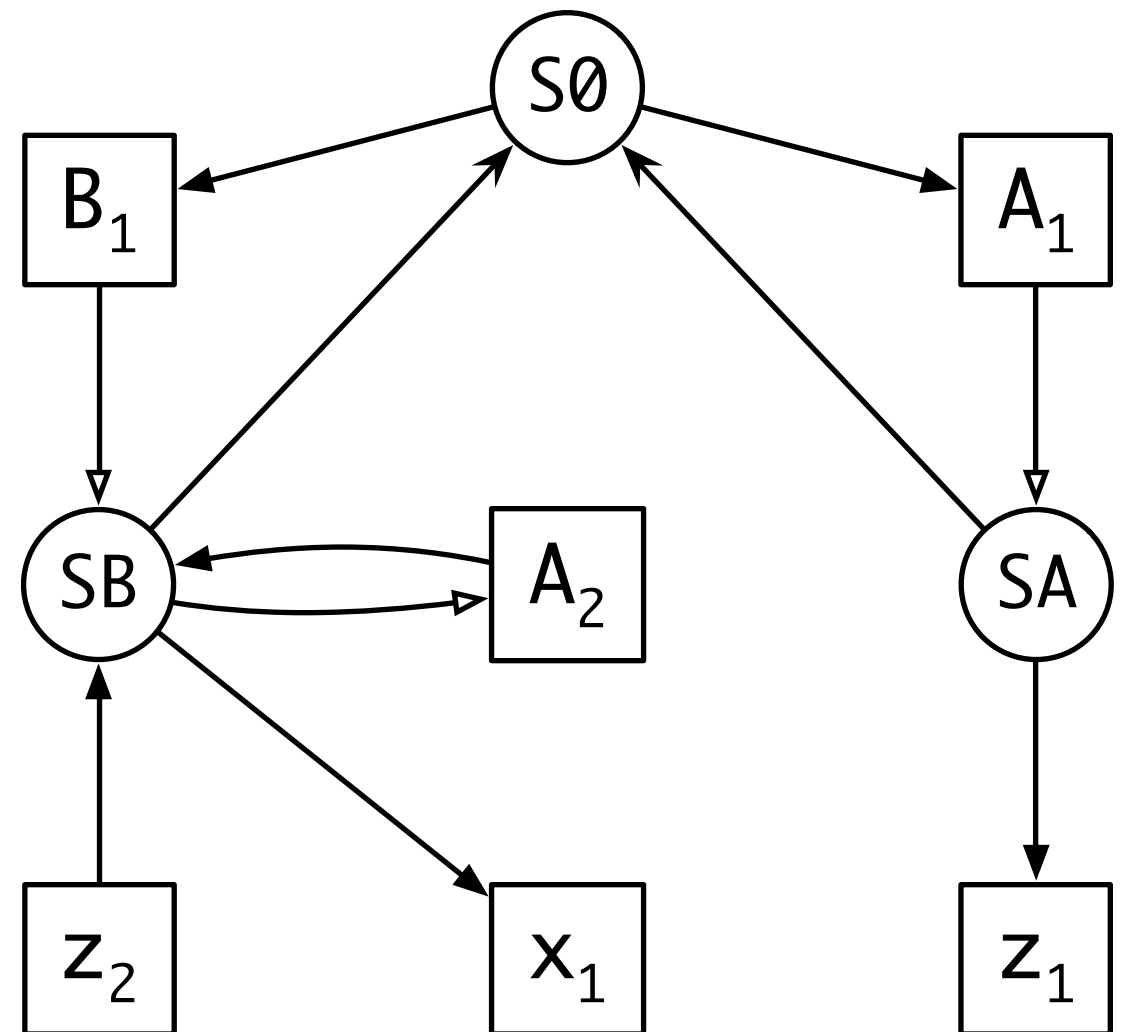
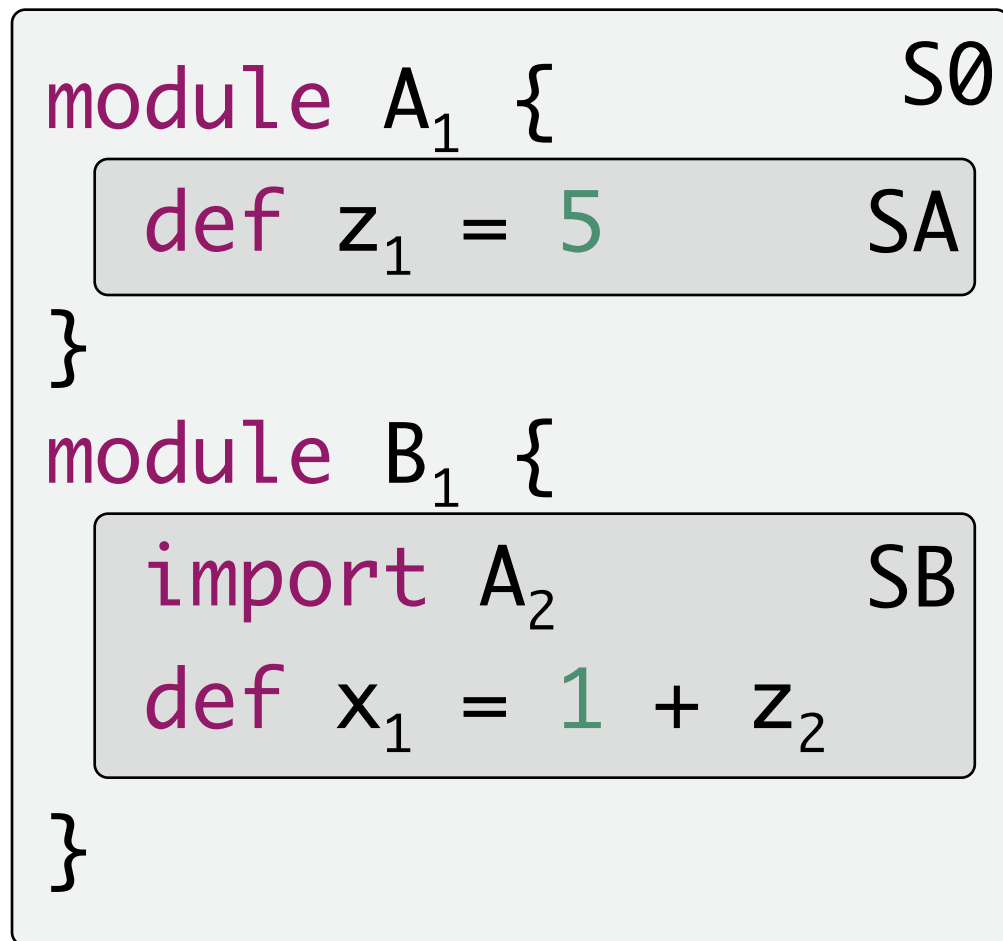

Imports



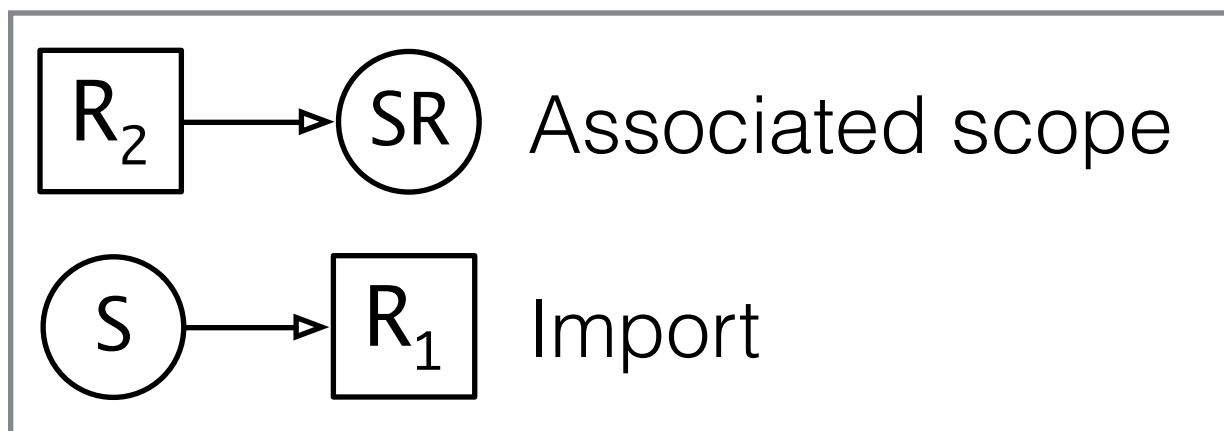
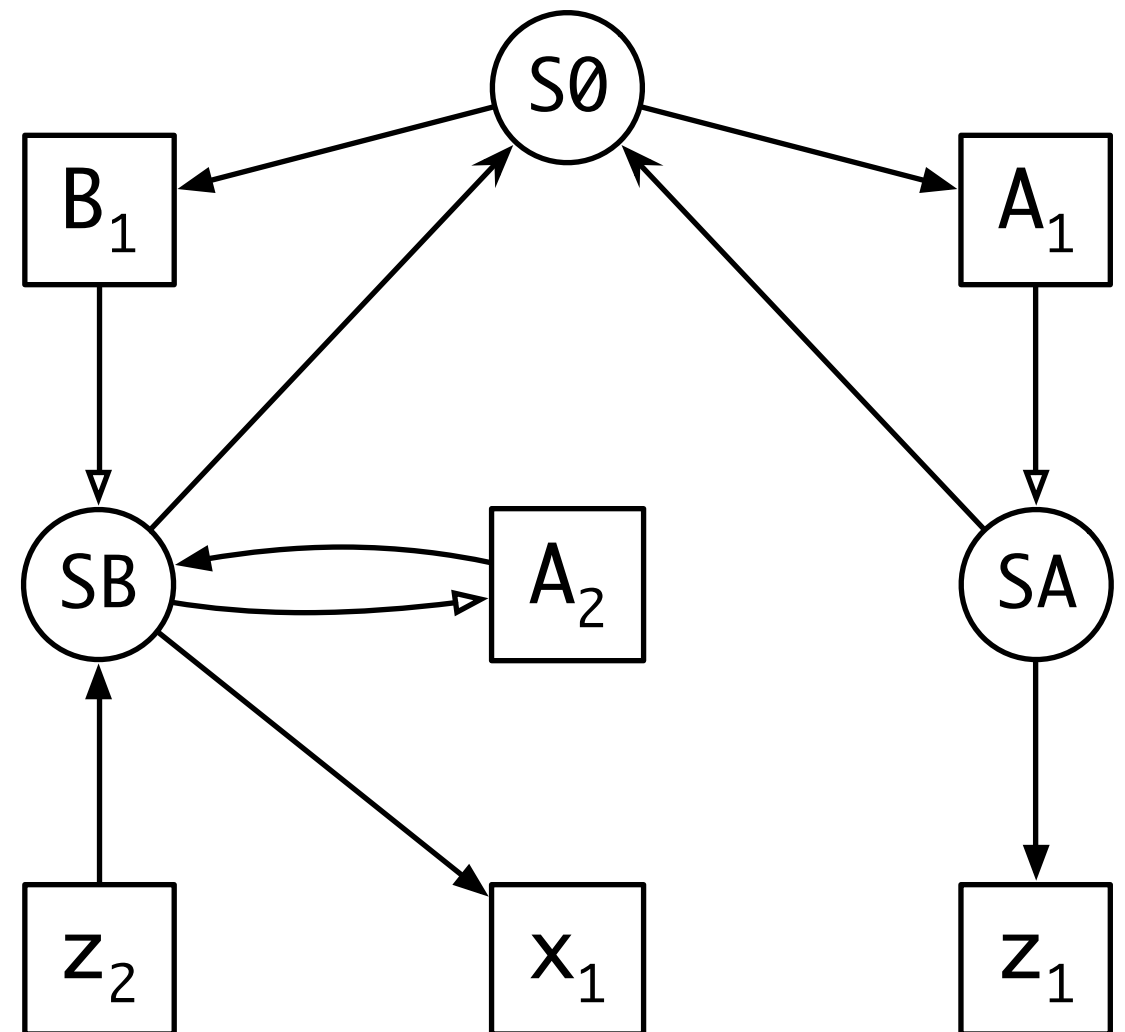
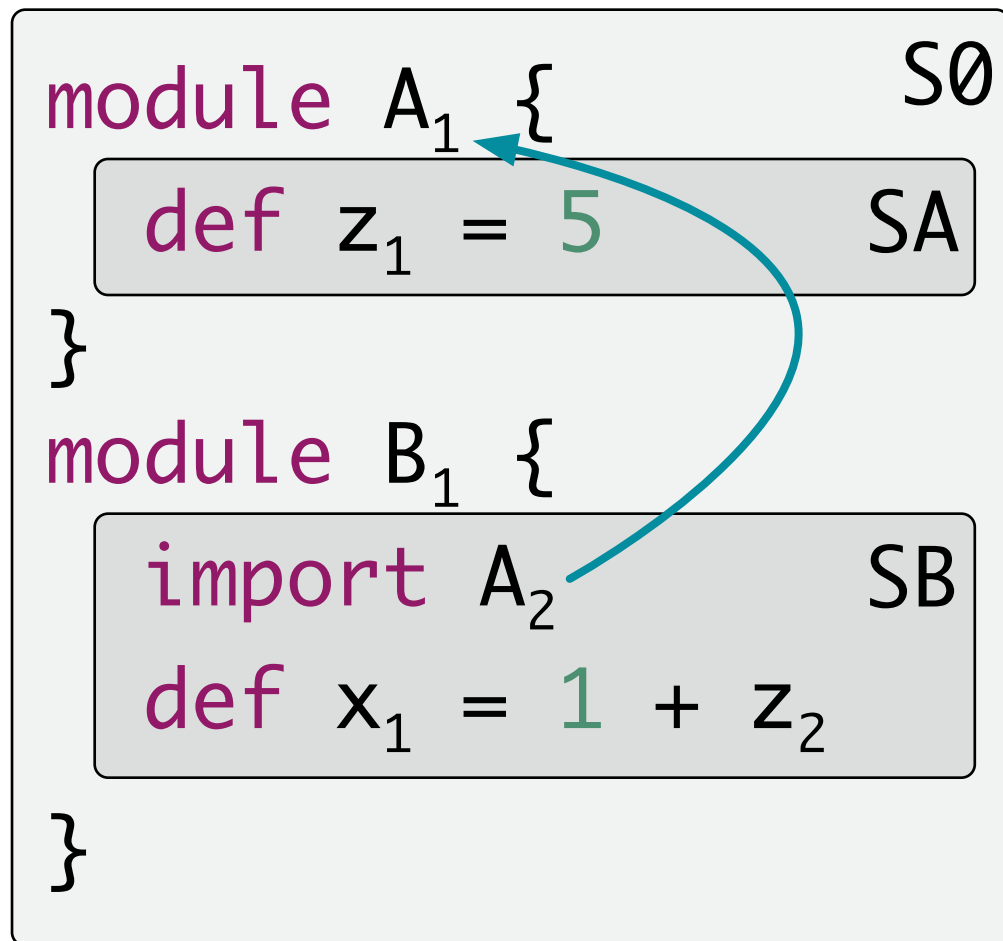
Imports



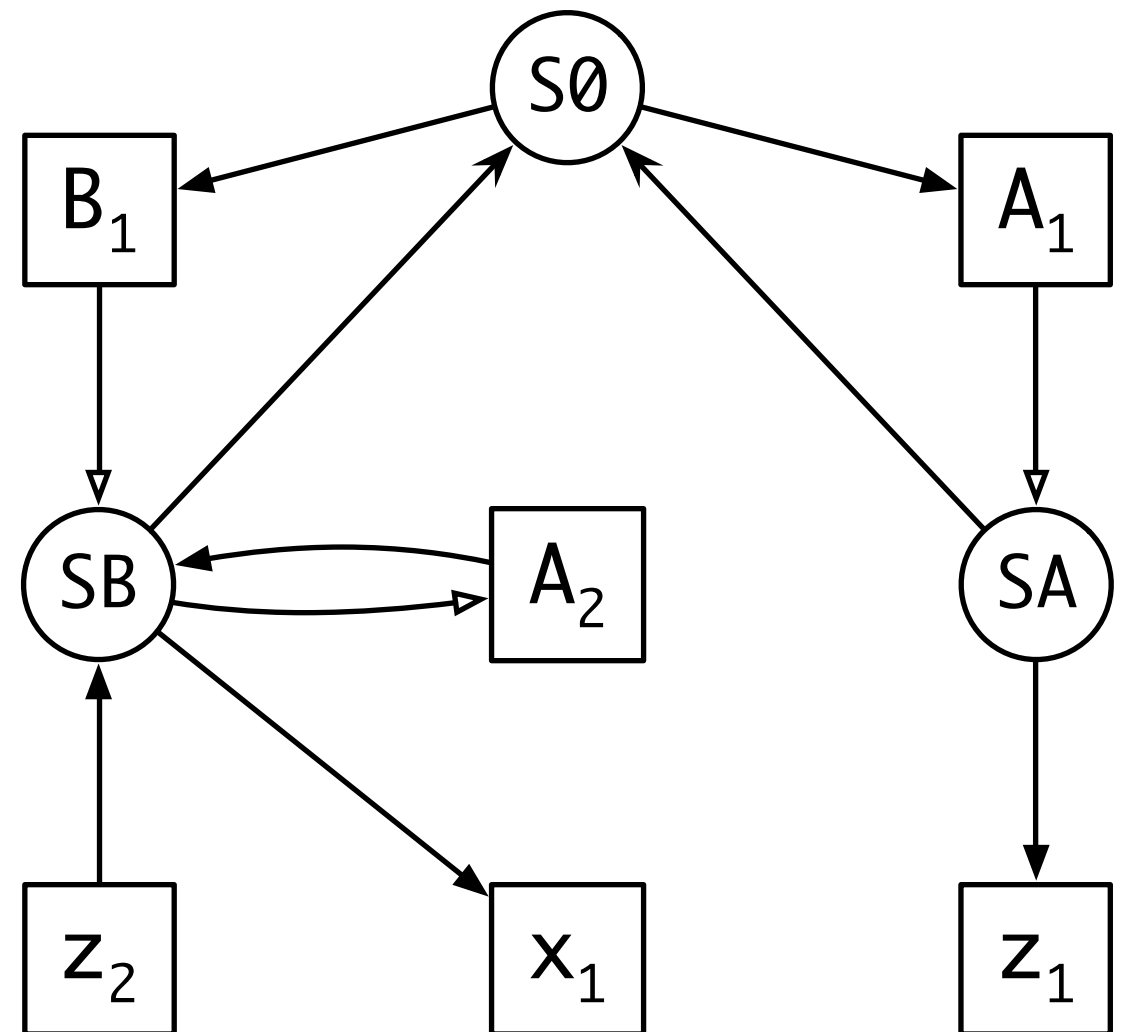
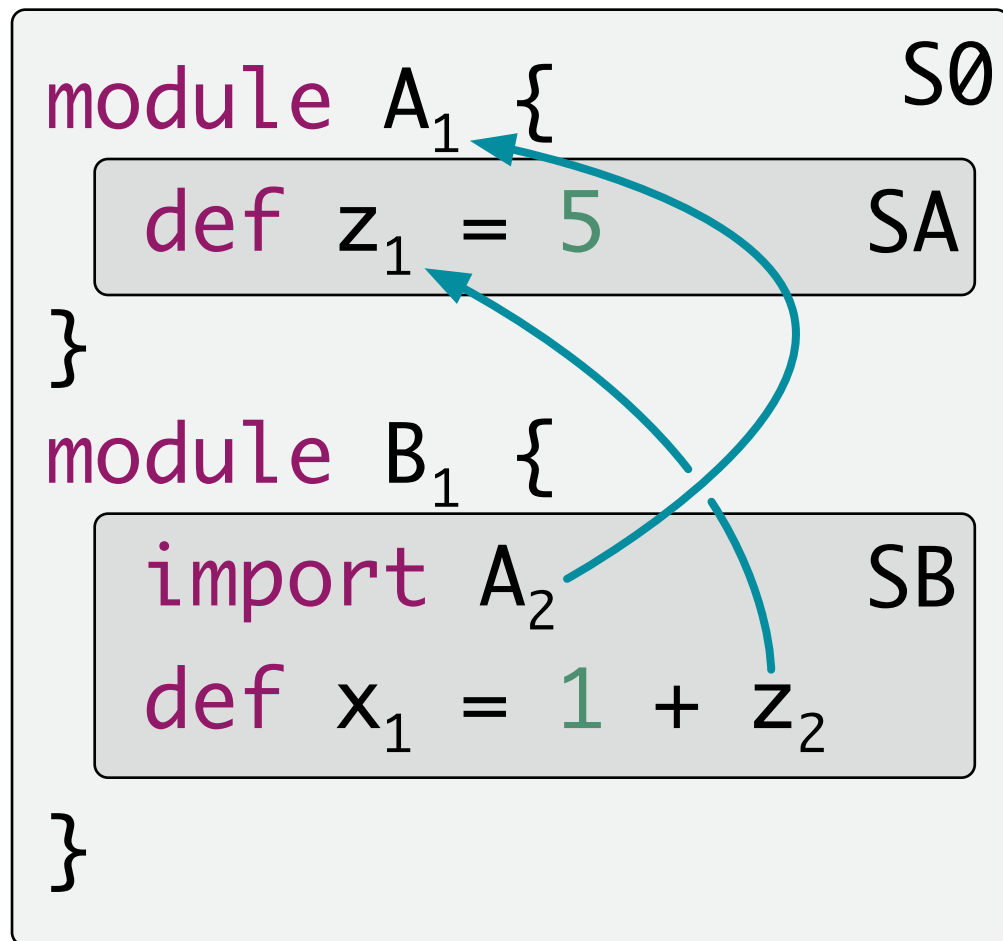
Imports



Imports



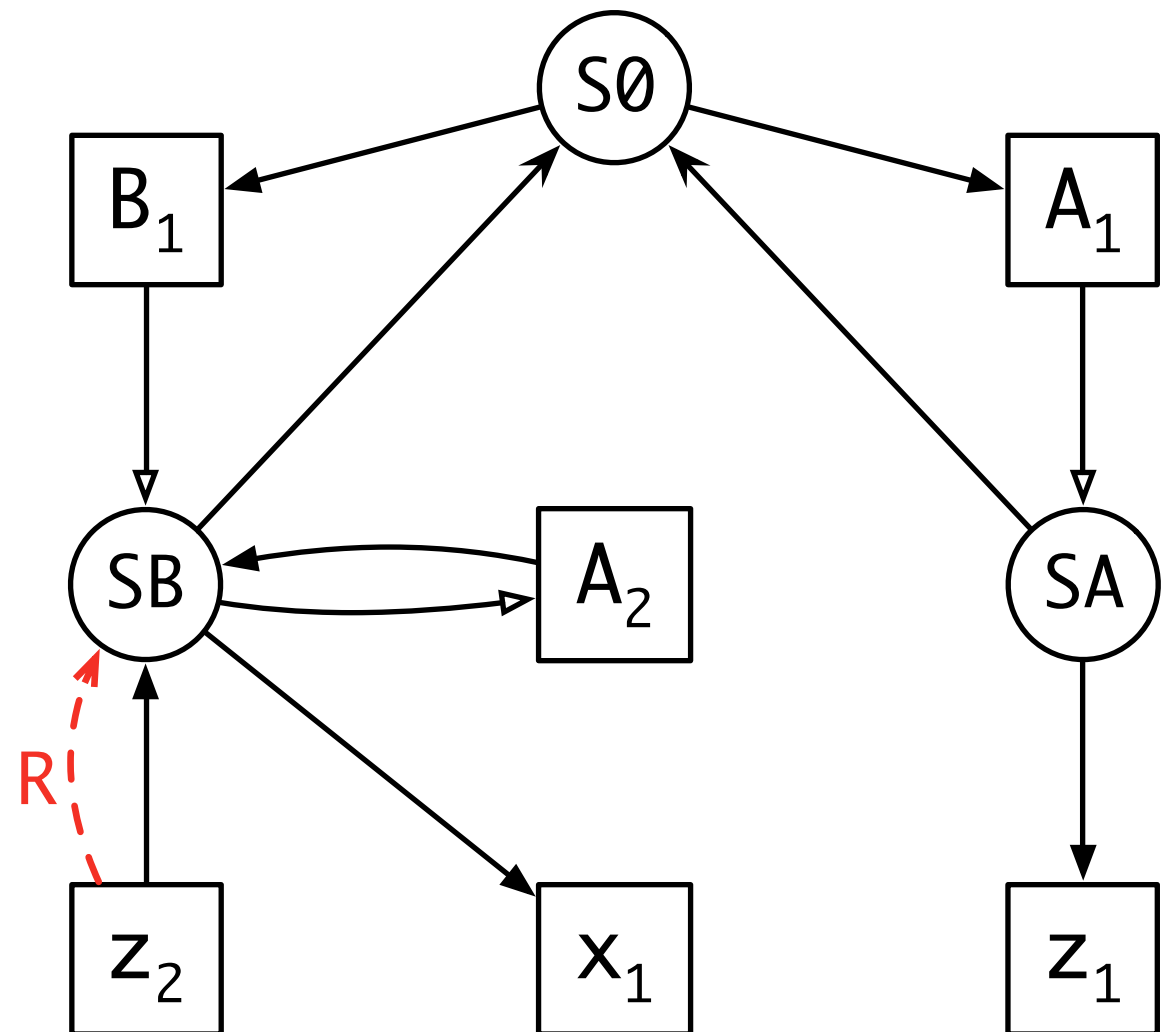
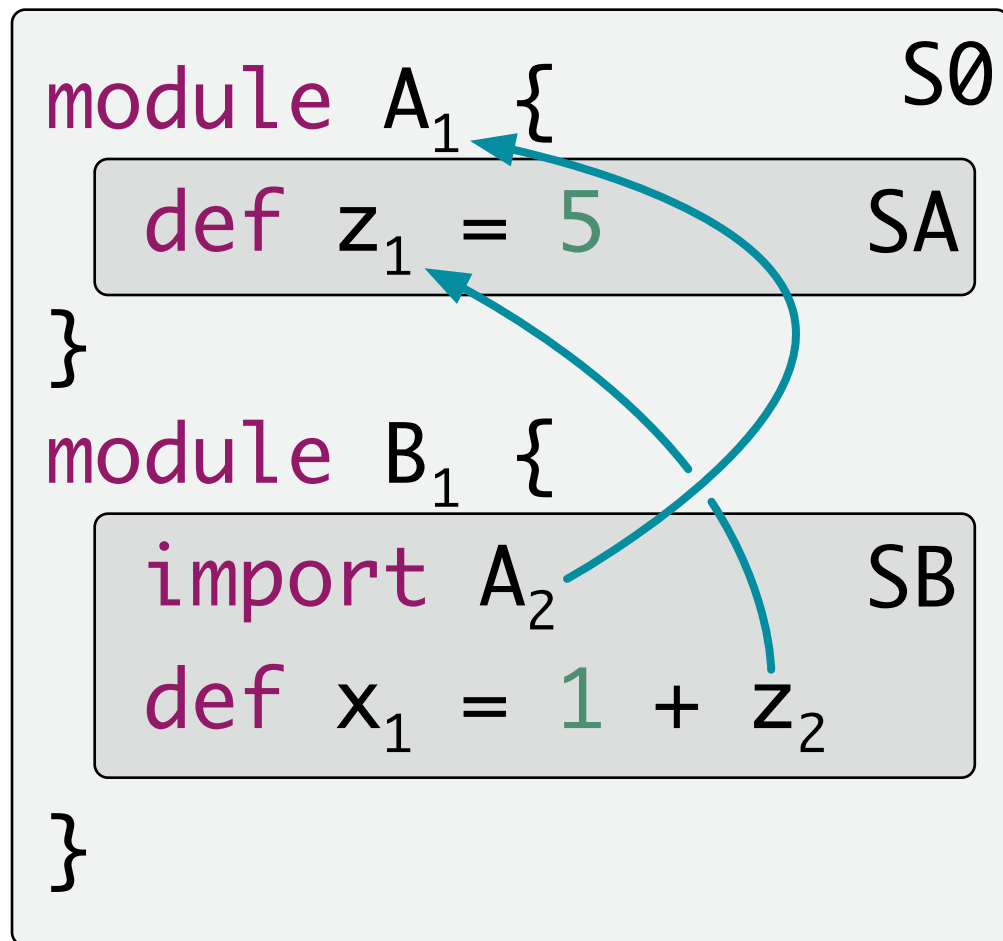
Imports



$R_2 \rightarrow SR$ Associated scope

$S \rightarrow R_1$ Import

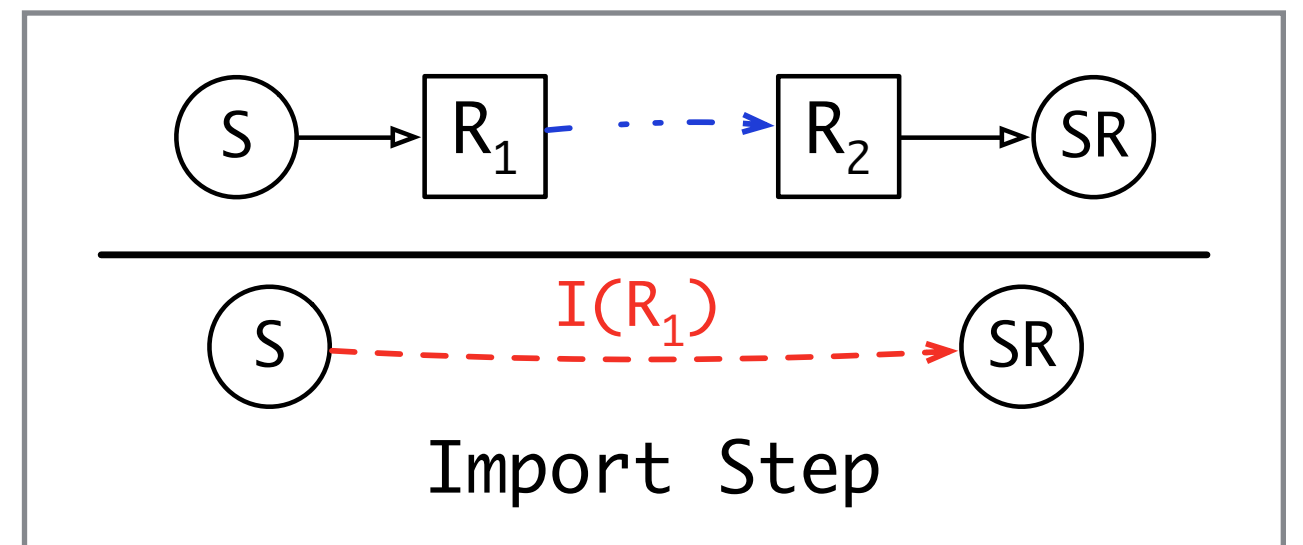
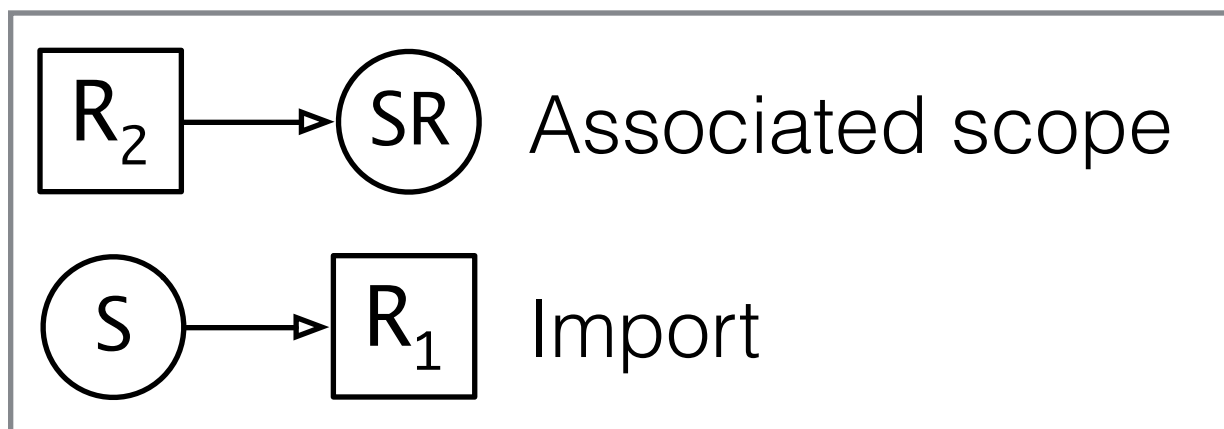
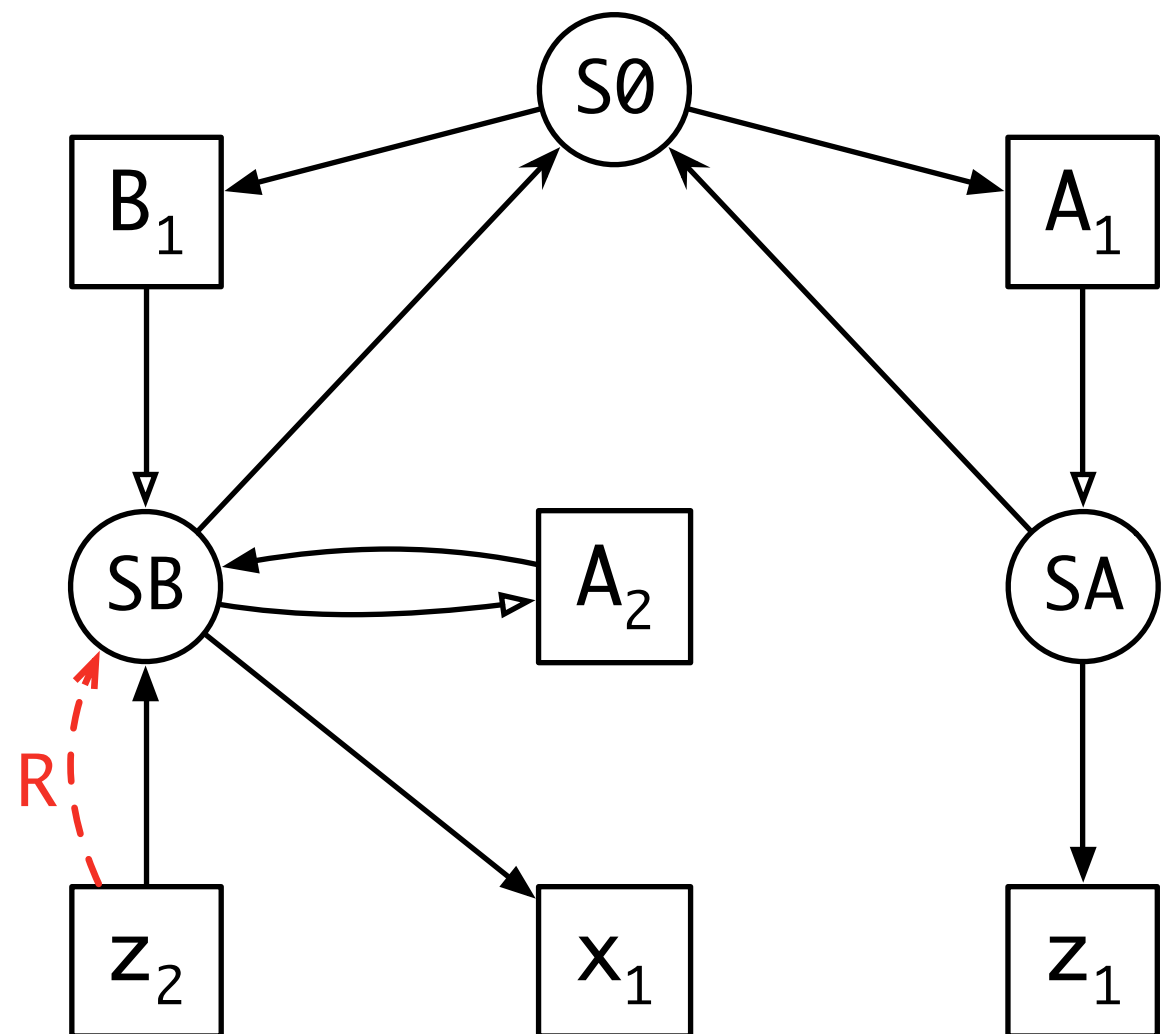
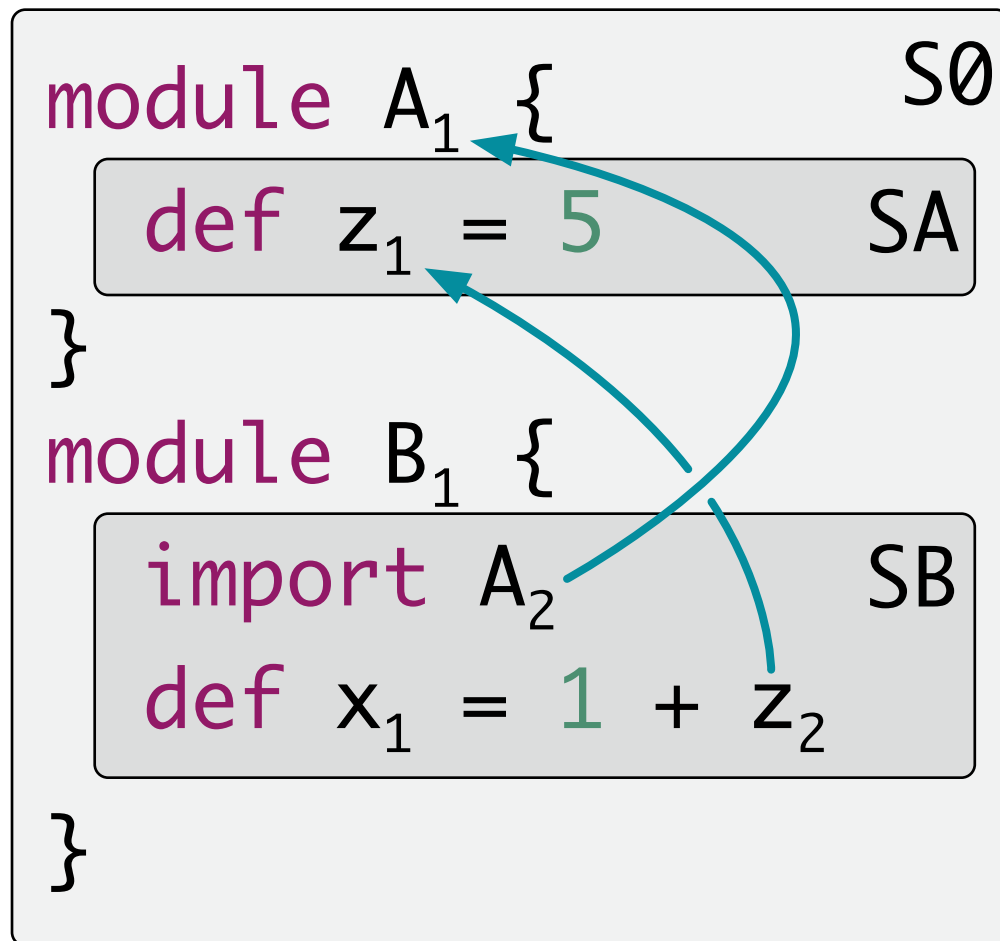
Imports



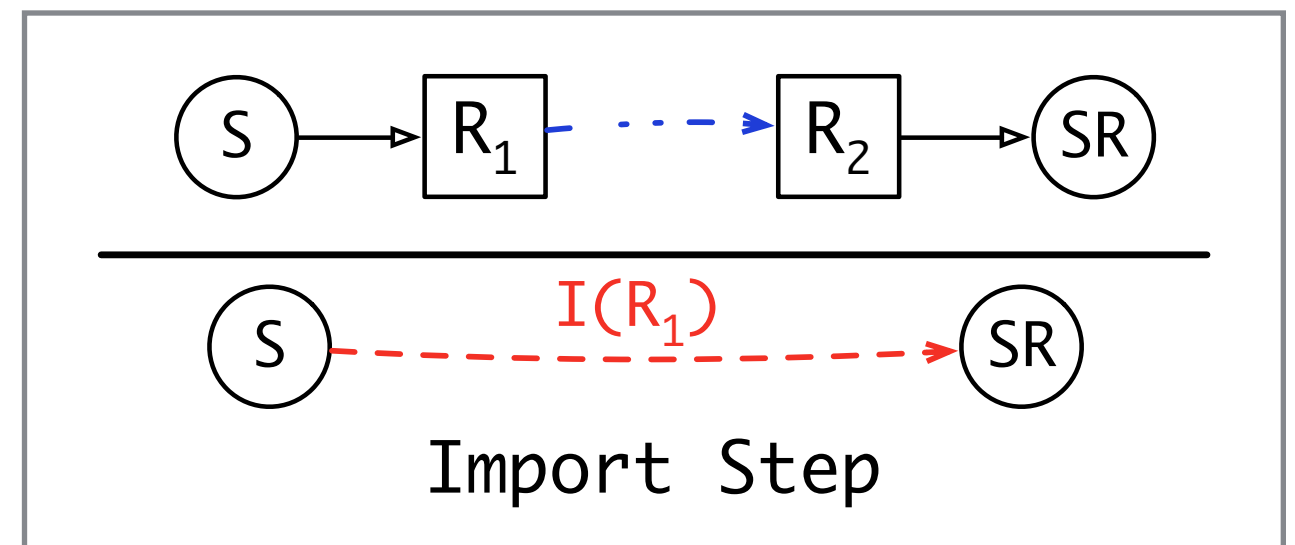
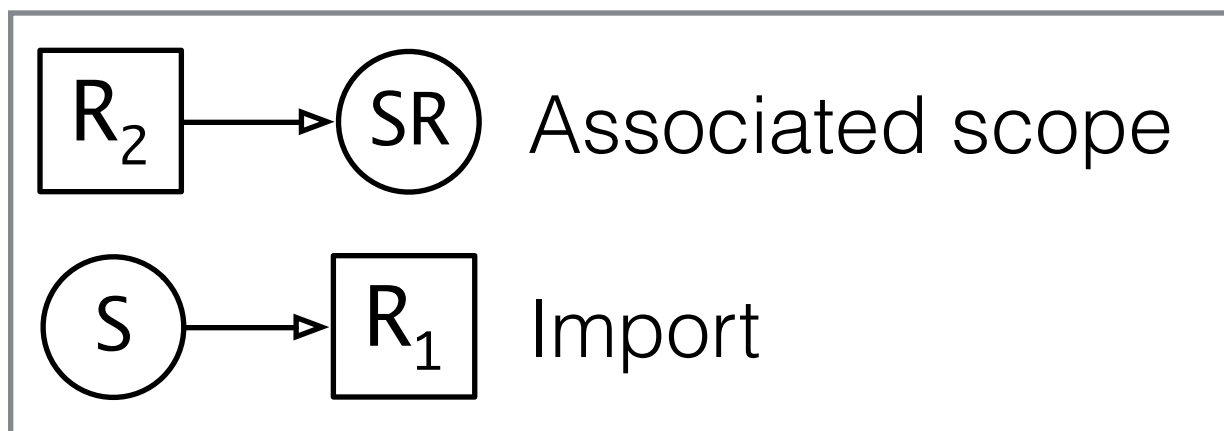
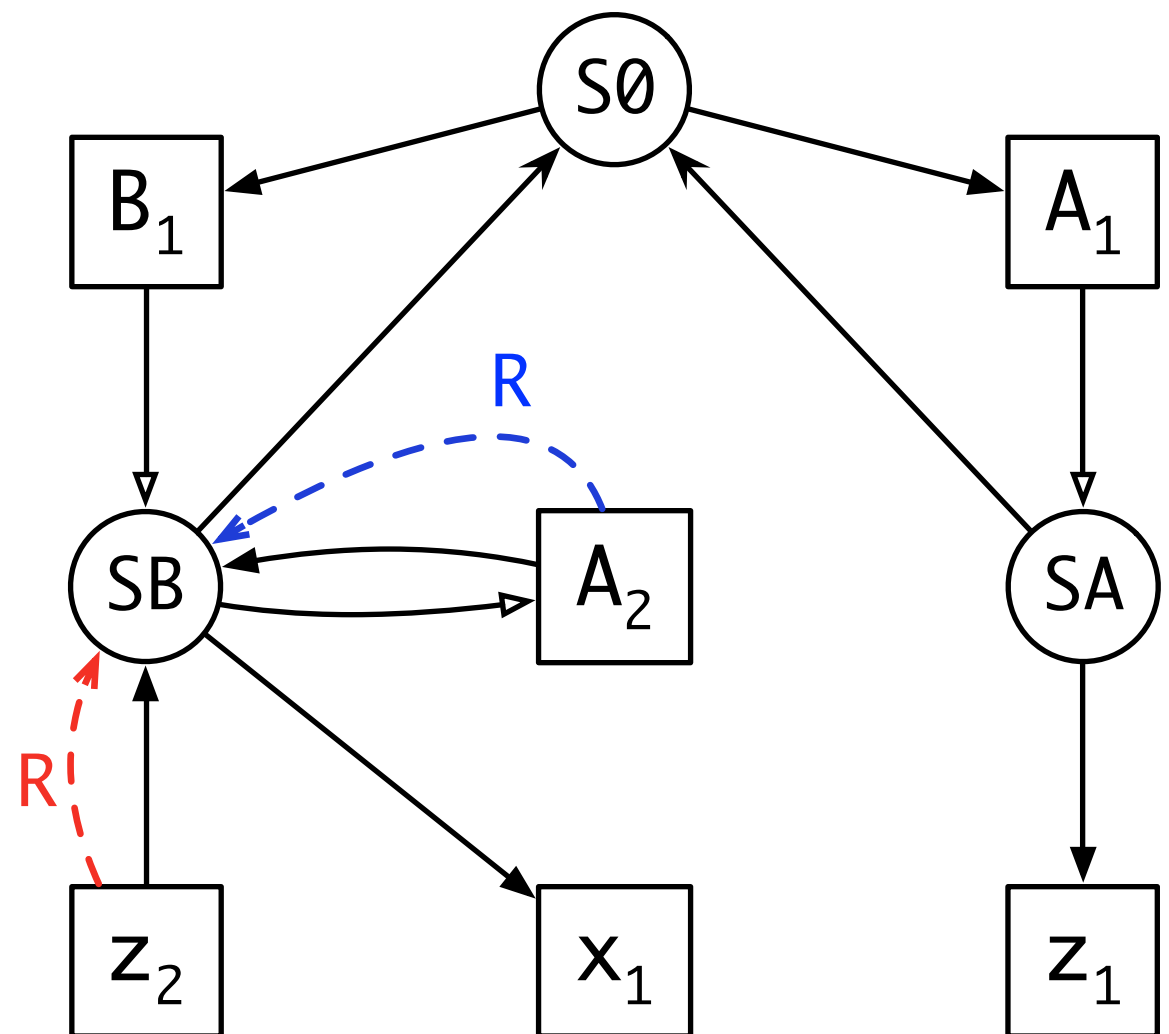
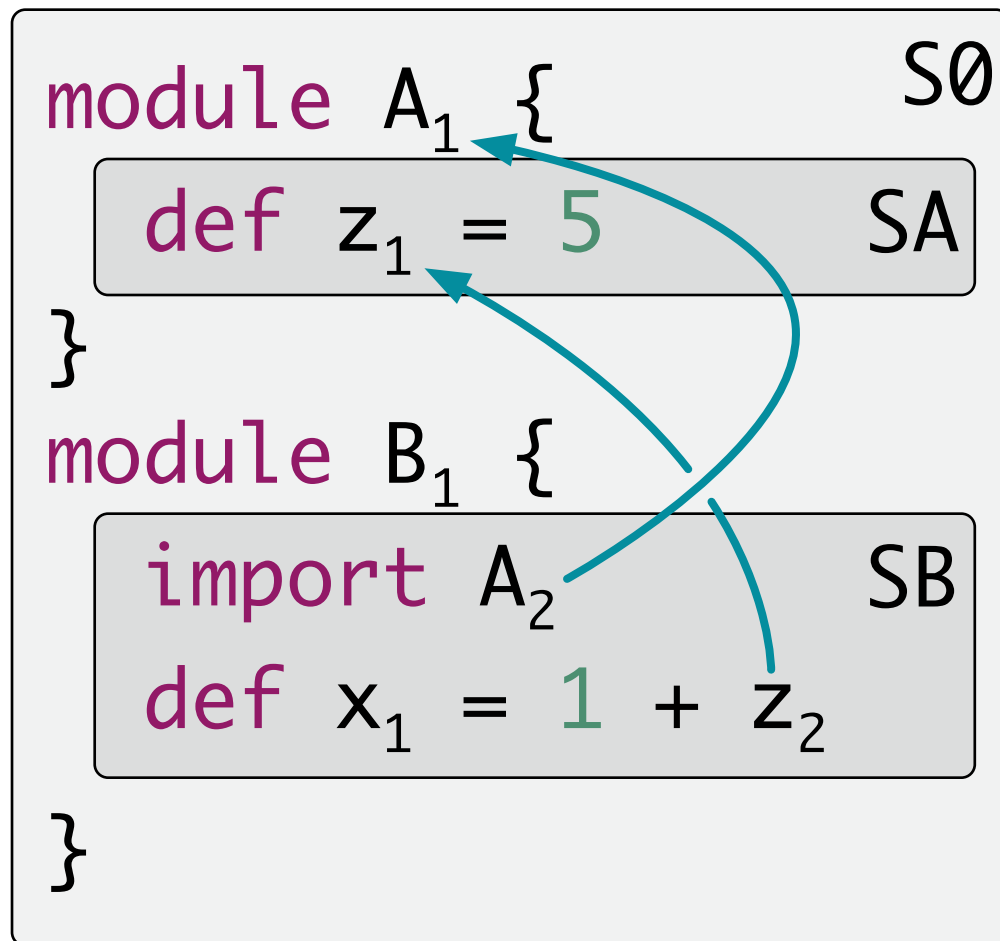
$R_2 \rightarrow SR$ Associated scope

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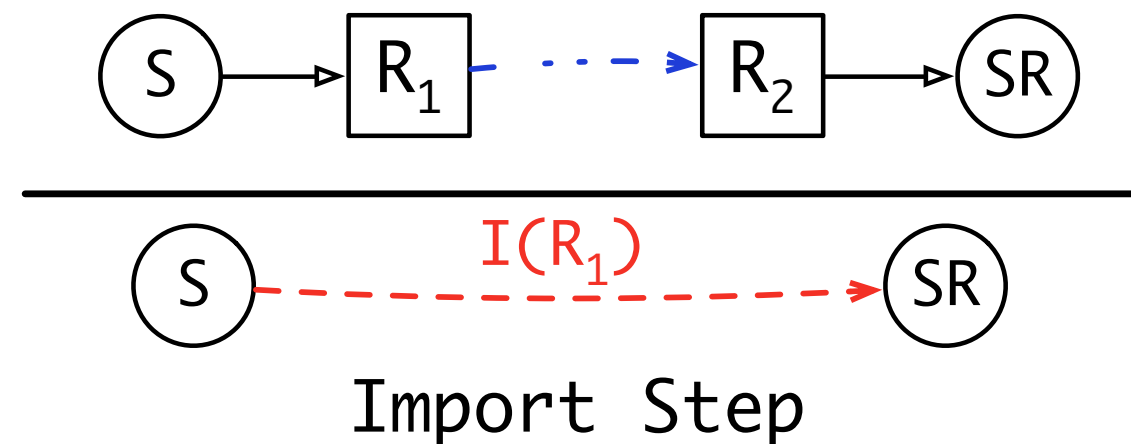
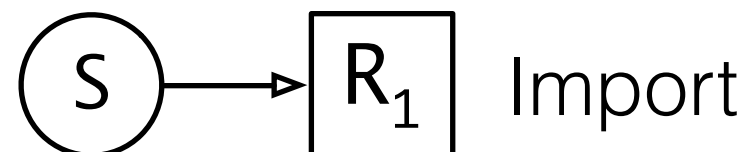
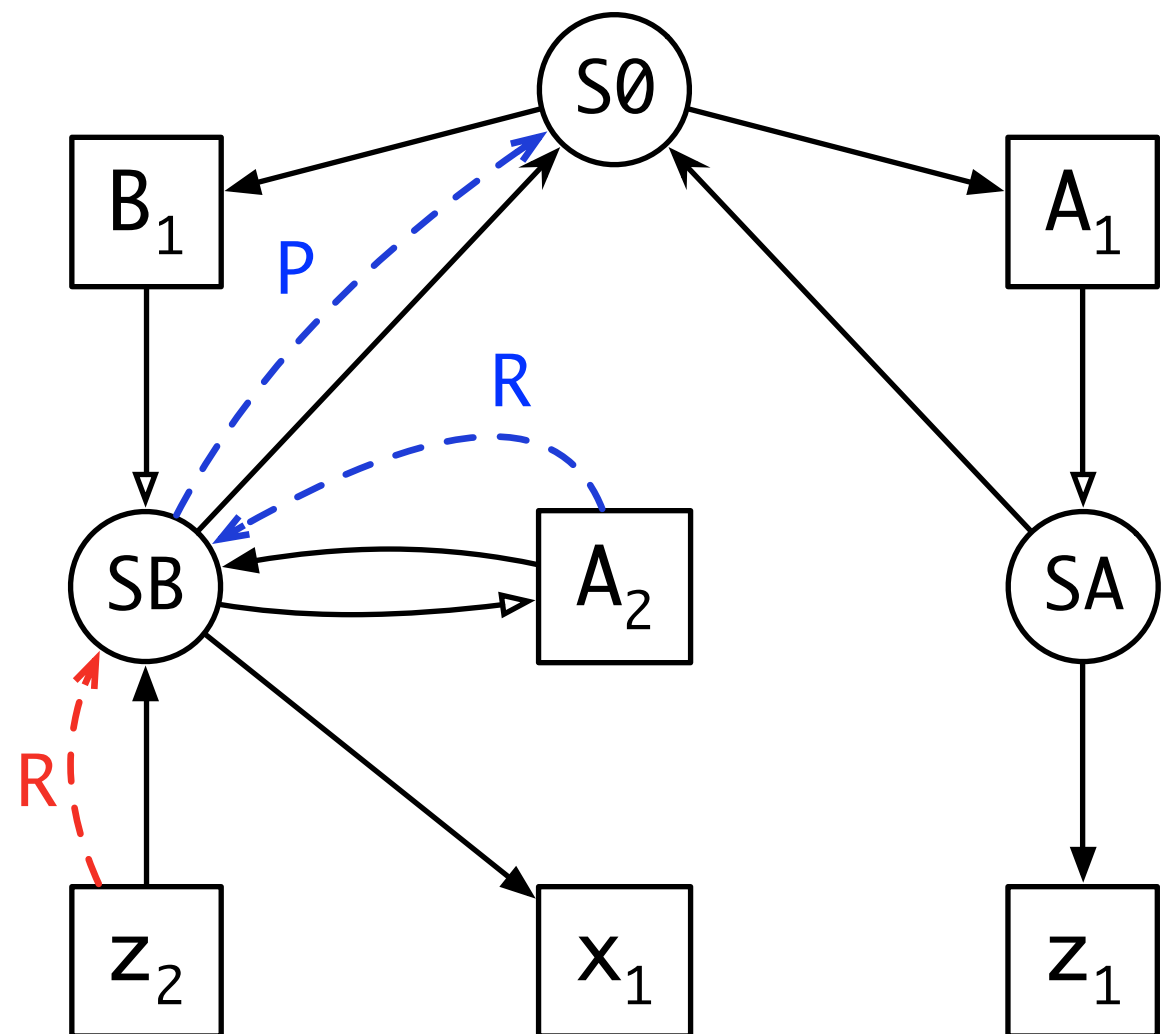
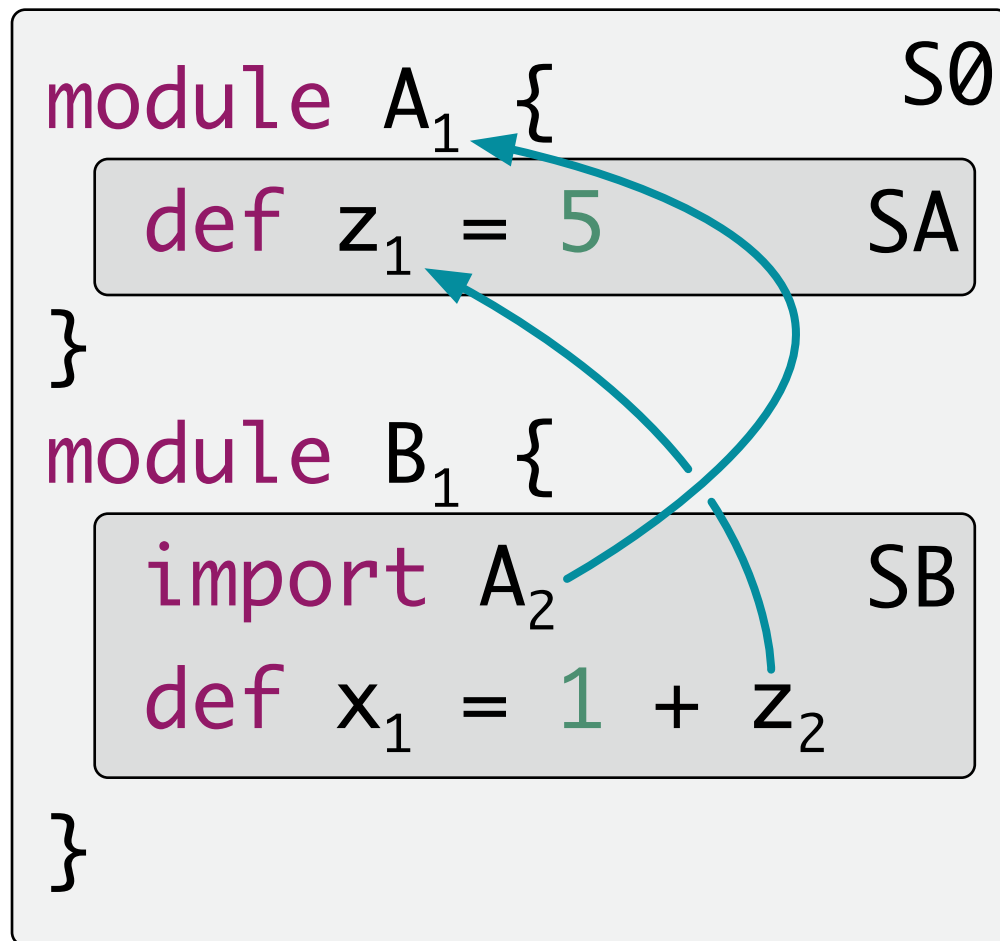
Imports



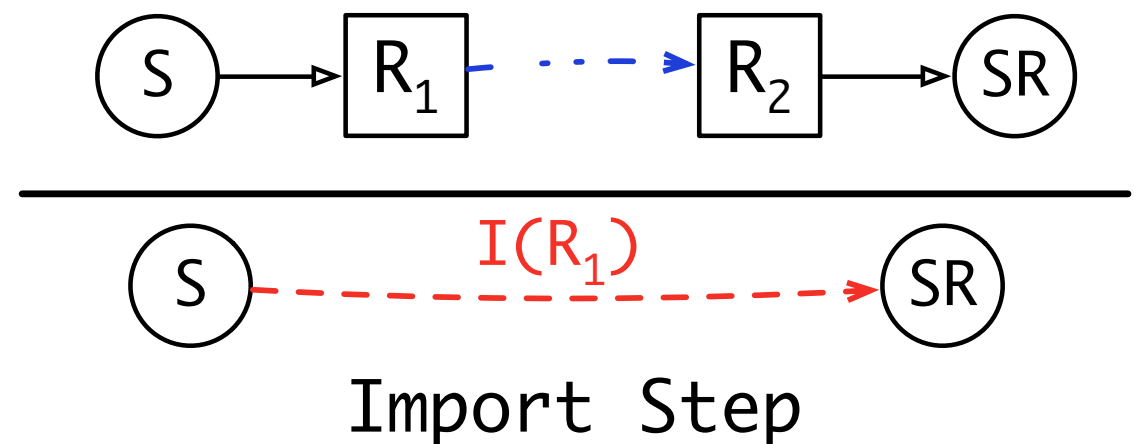
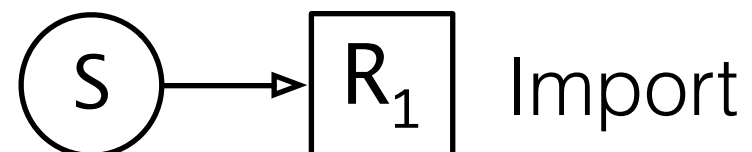
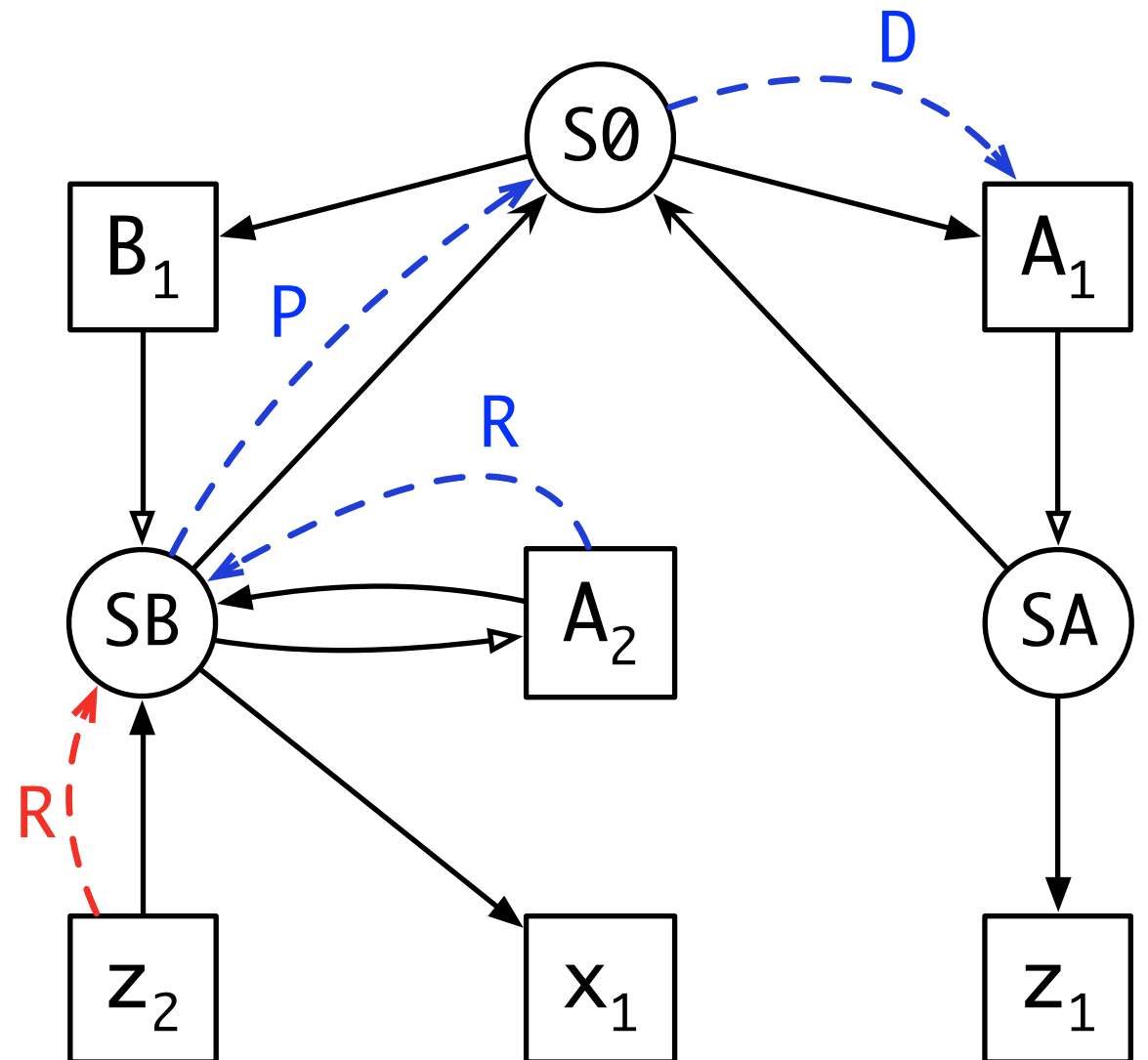
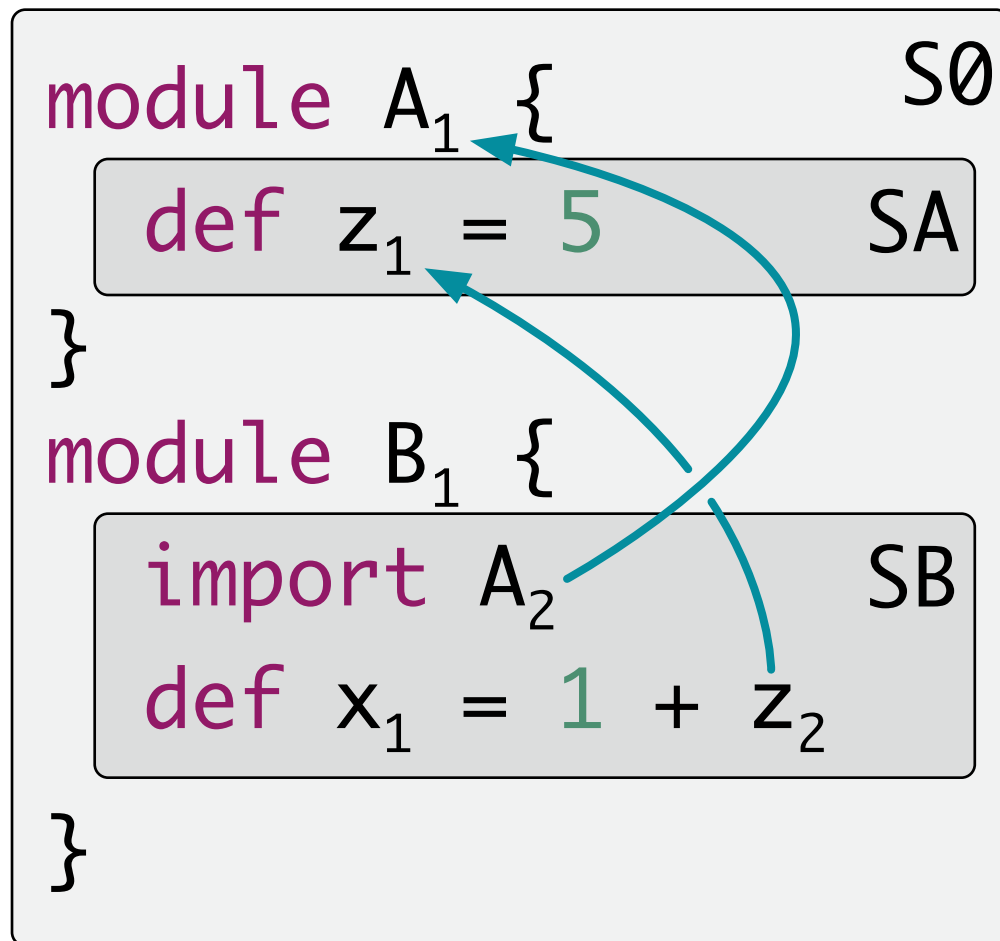
Imports



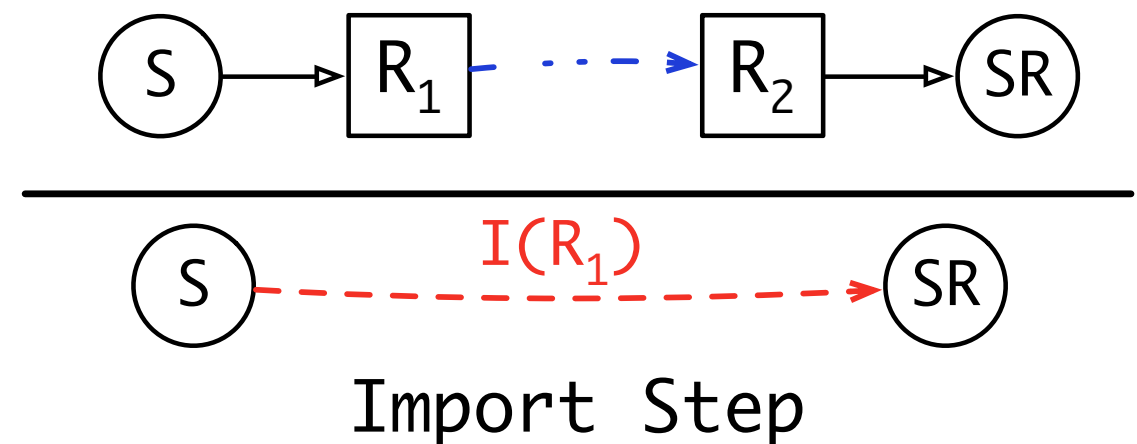
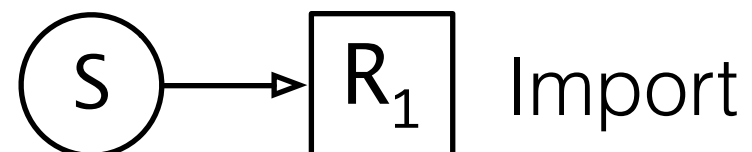
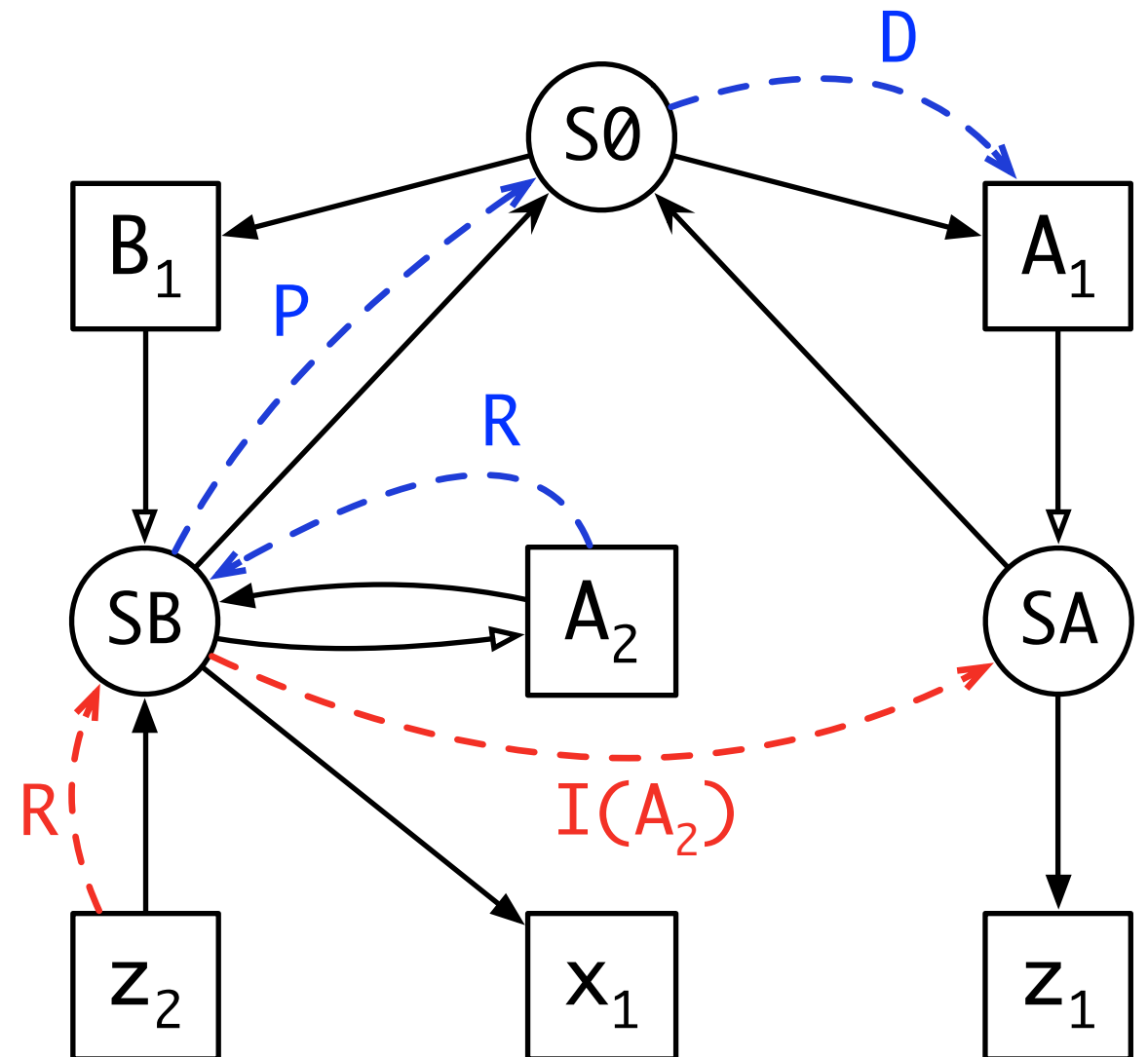
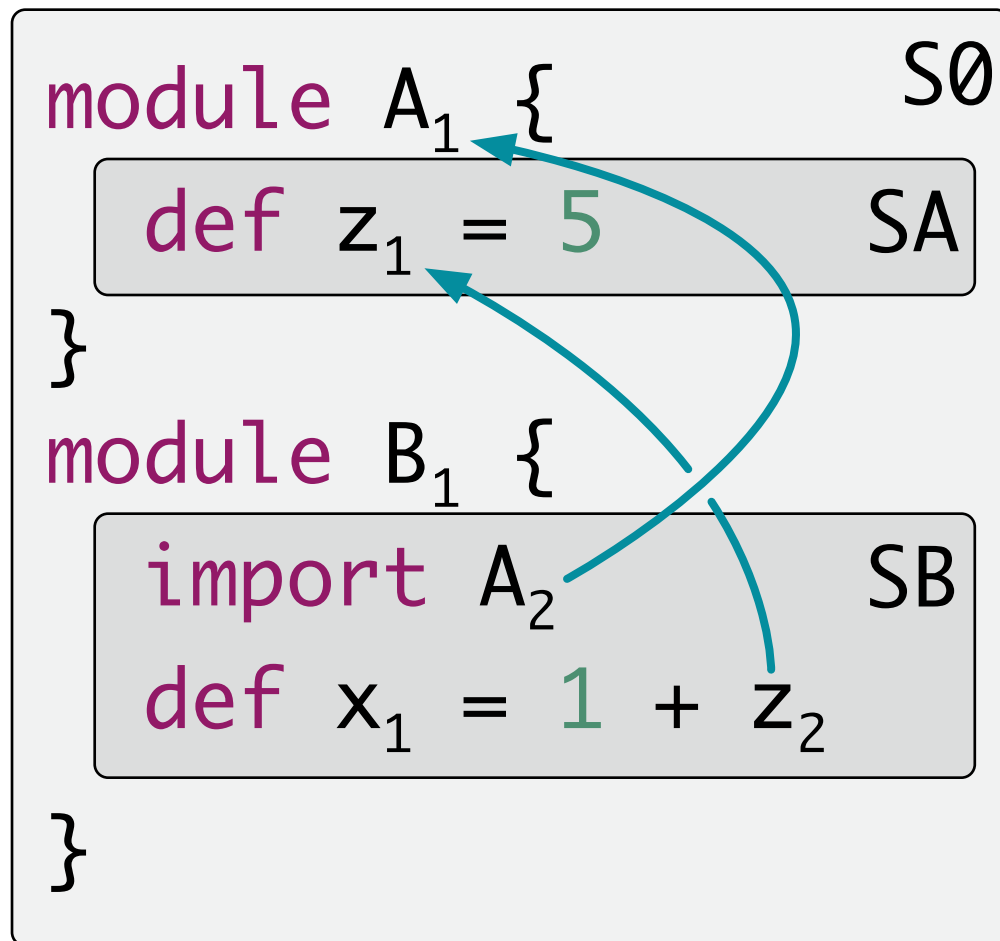
Imports



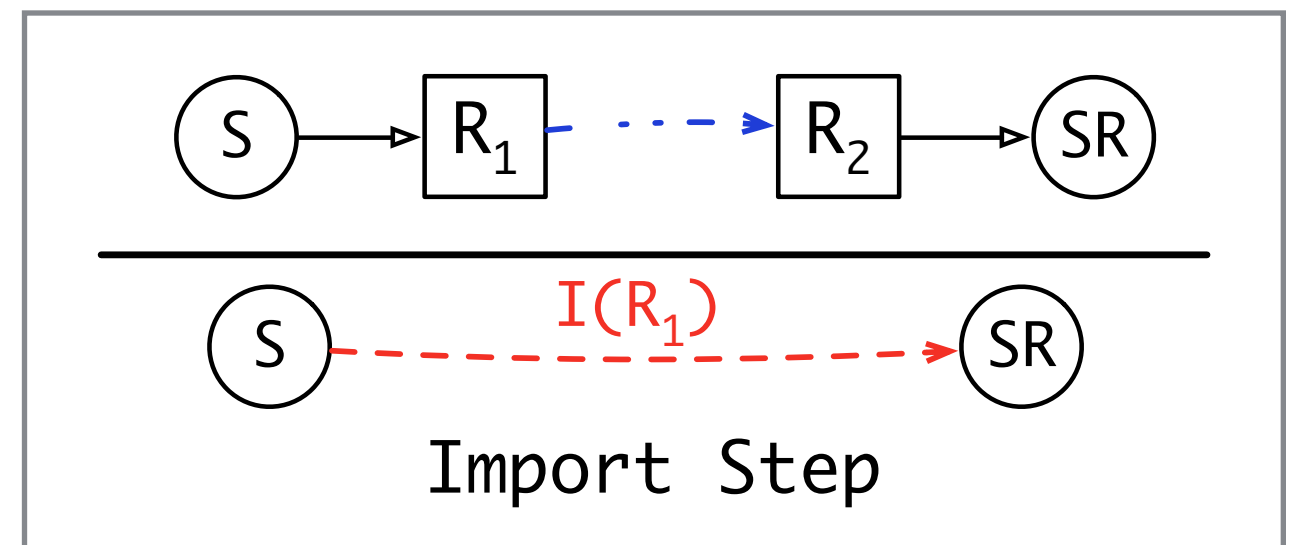
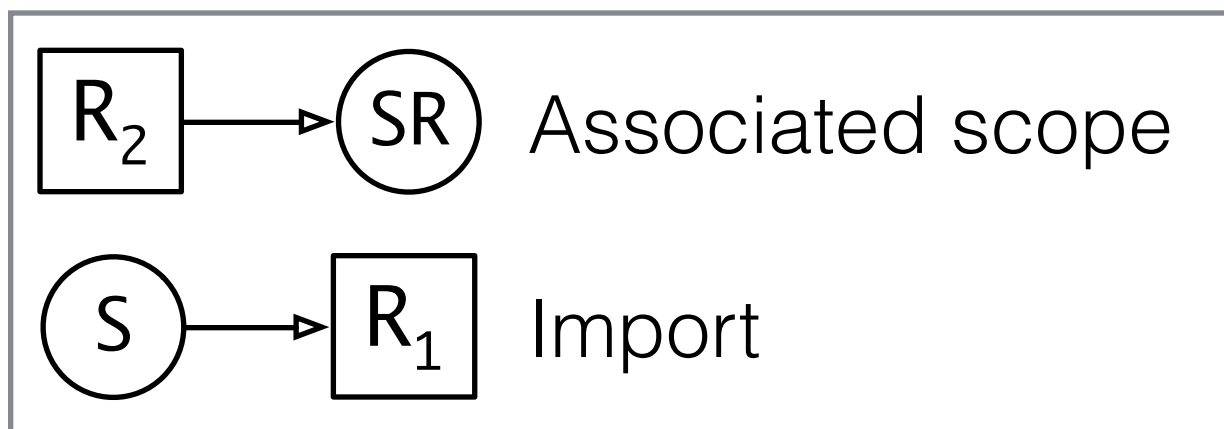
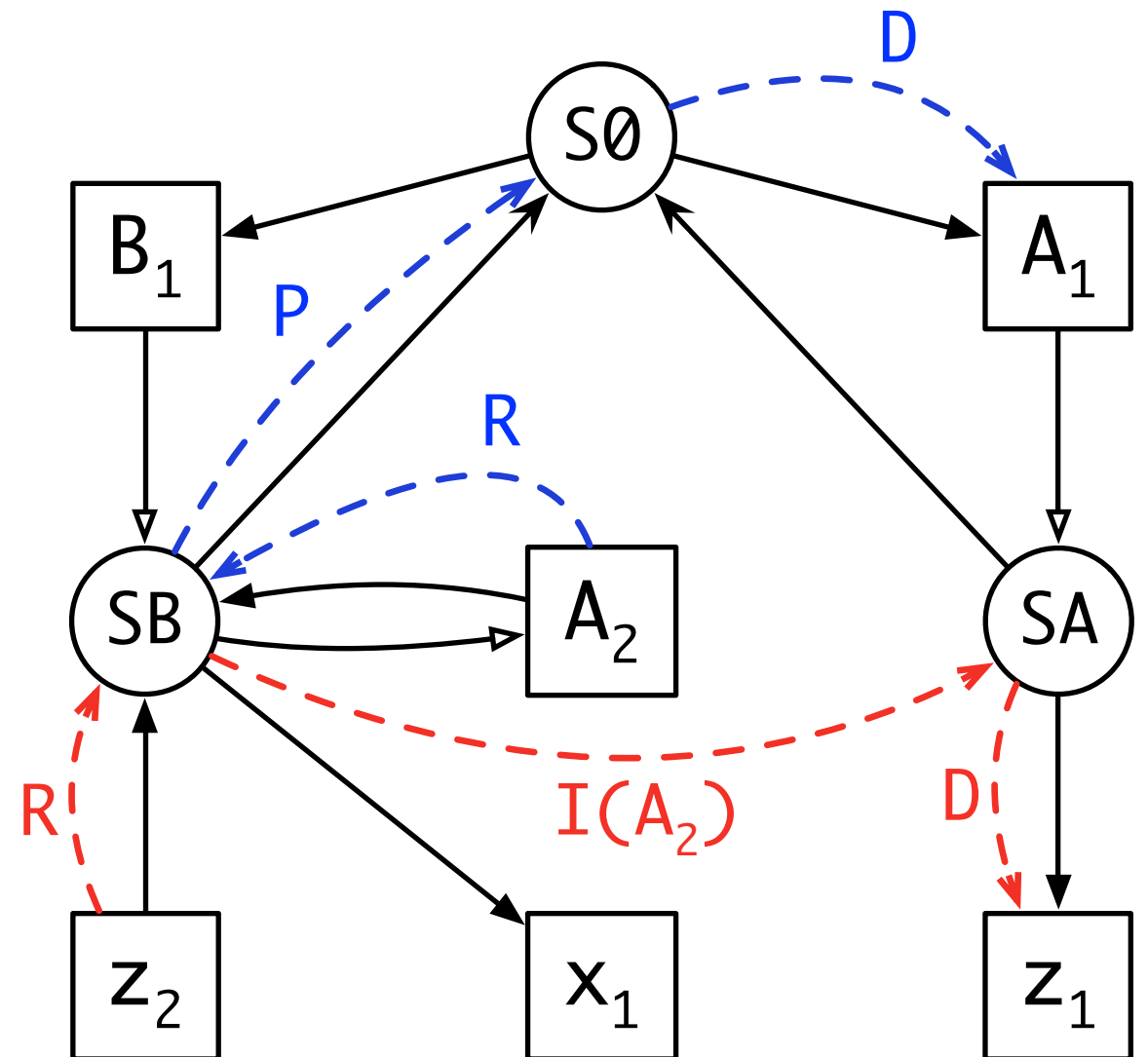
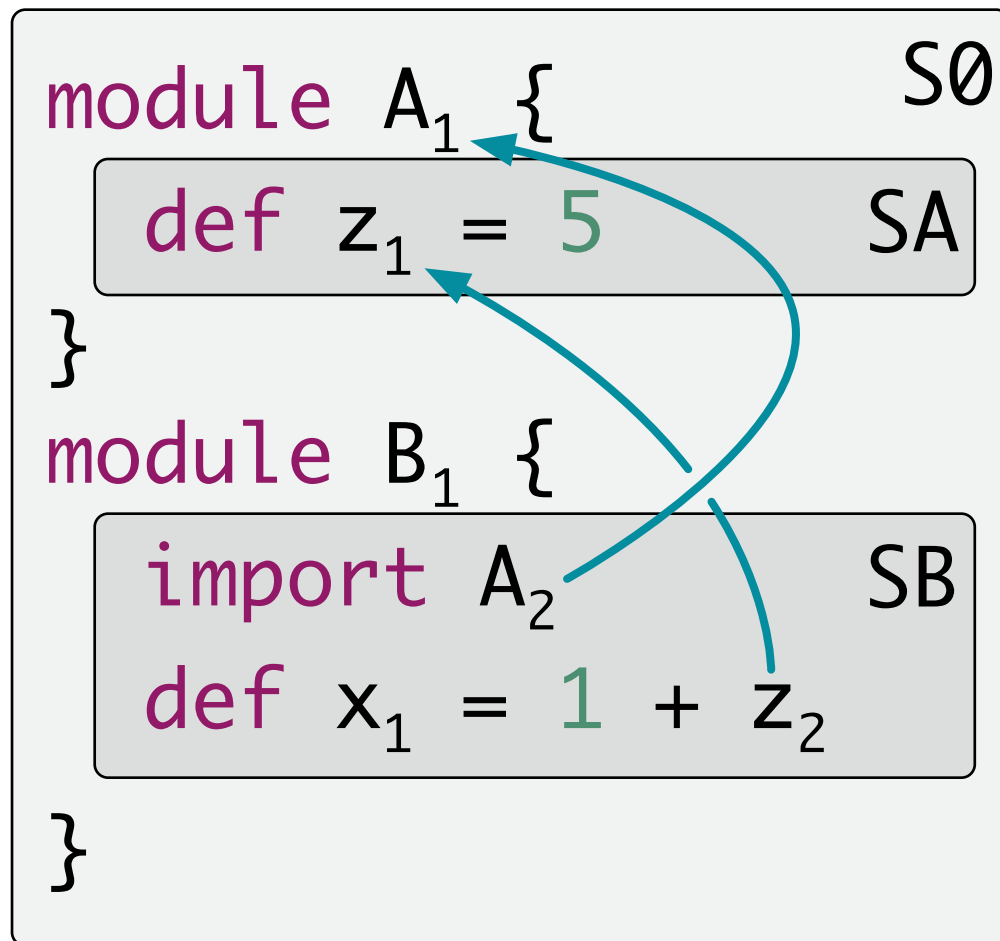
Imports



Imports



Imports



Imports over Parents

```
def z3 = 2
```

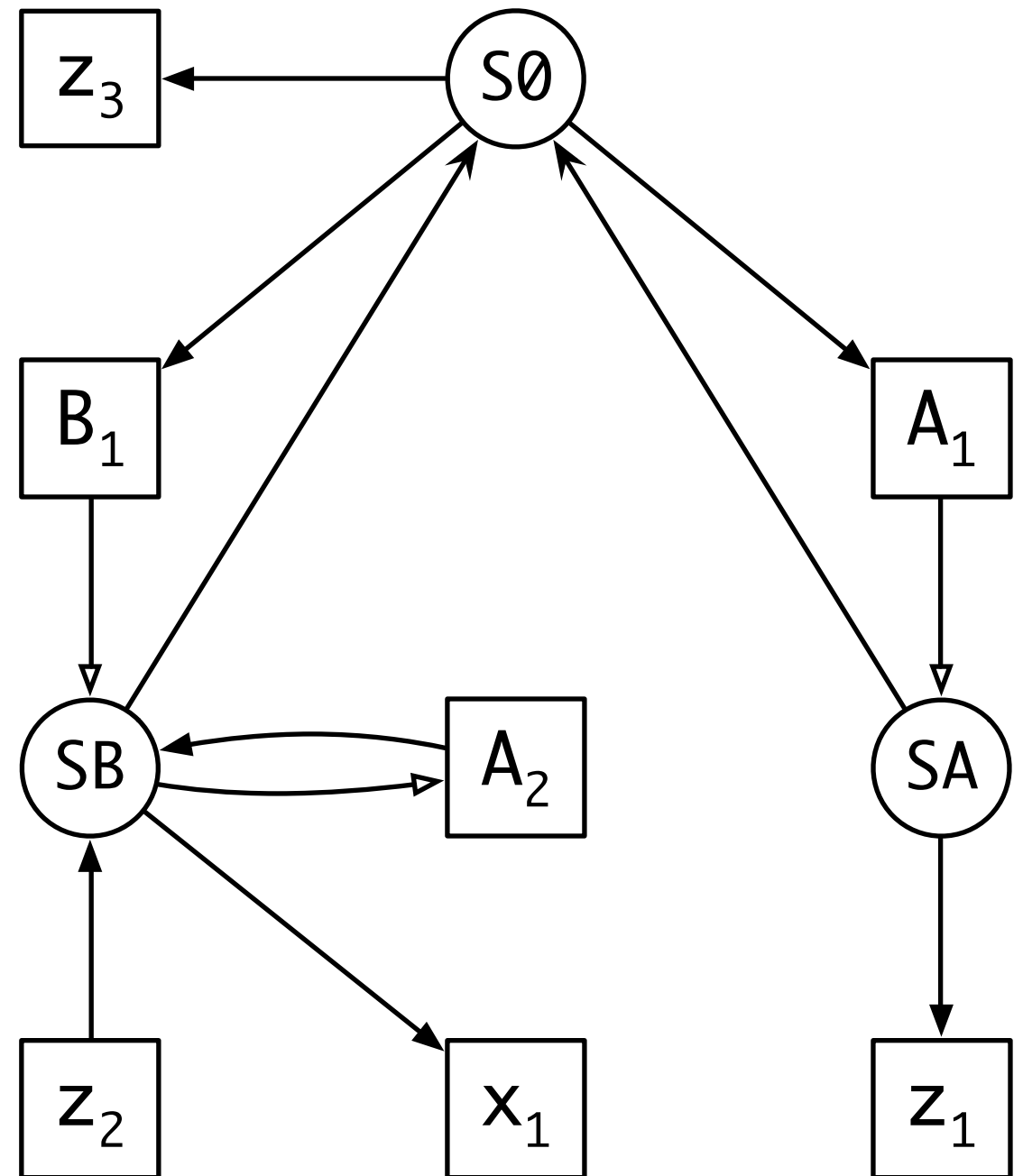
```
module A1 {  
  def z1 = 5  
}
```

```
module B1 {  
  import A2  
  def x1 = 1 + z2  
}
```

Imports over Parents

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def z3 = 2

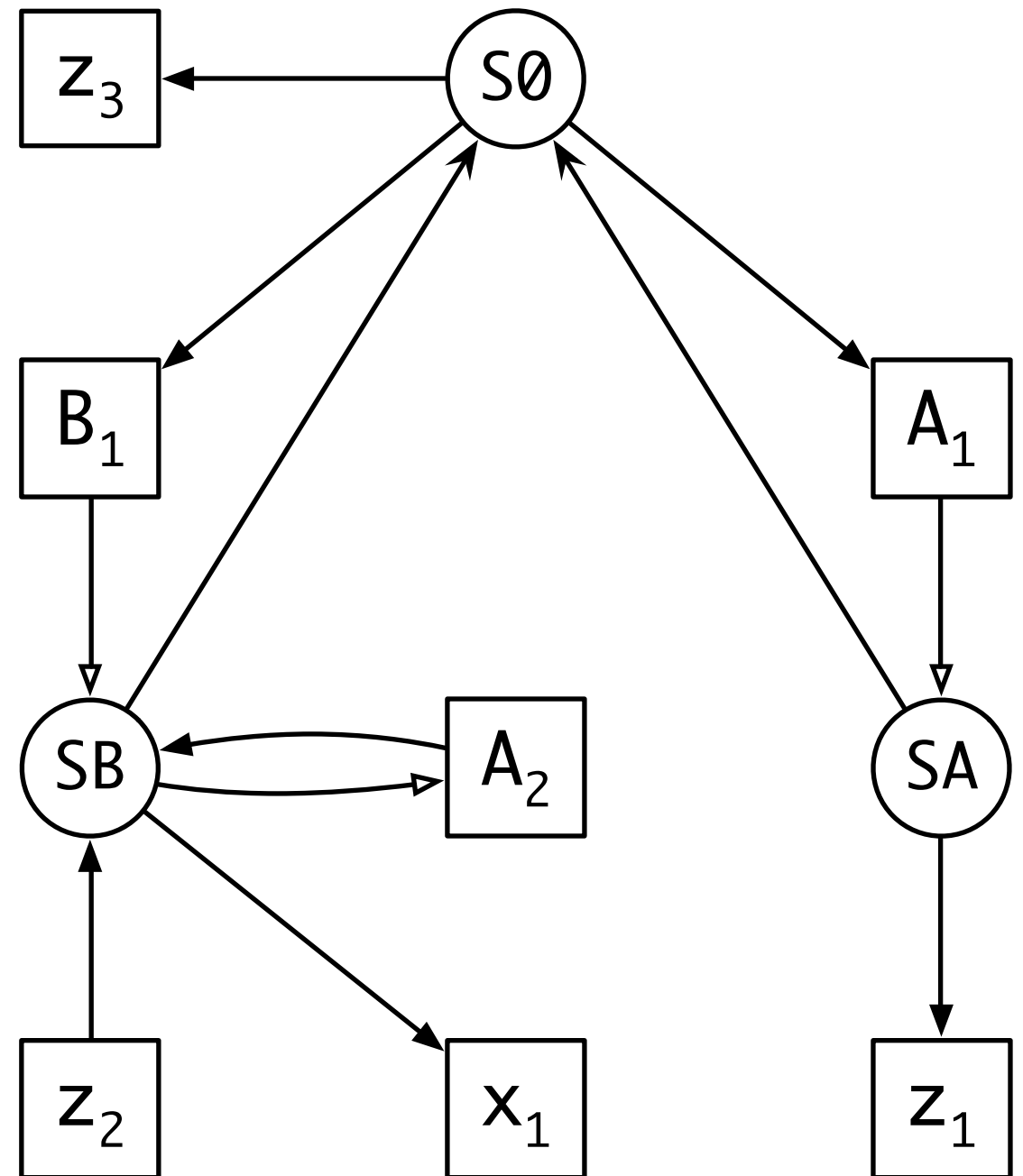
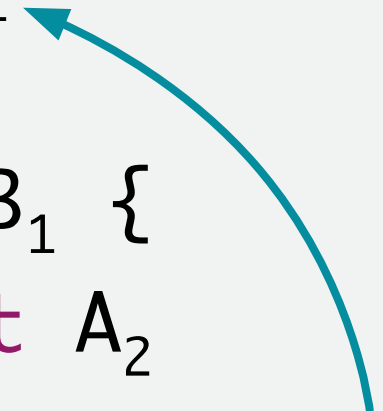
module A1 {
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Imports over Parents

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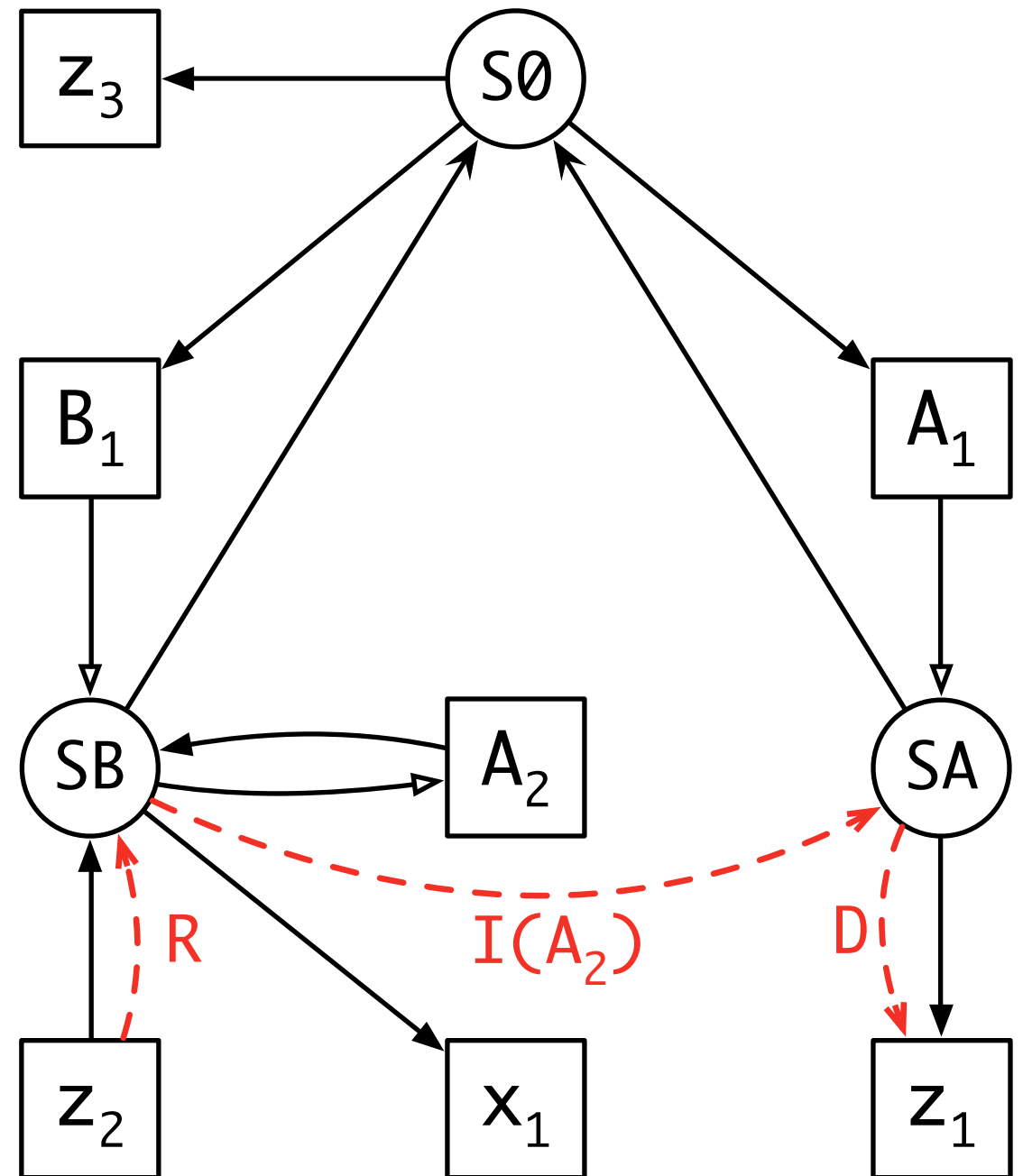
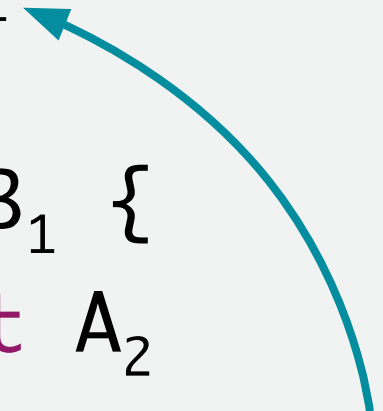
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Imports over Parents

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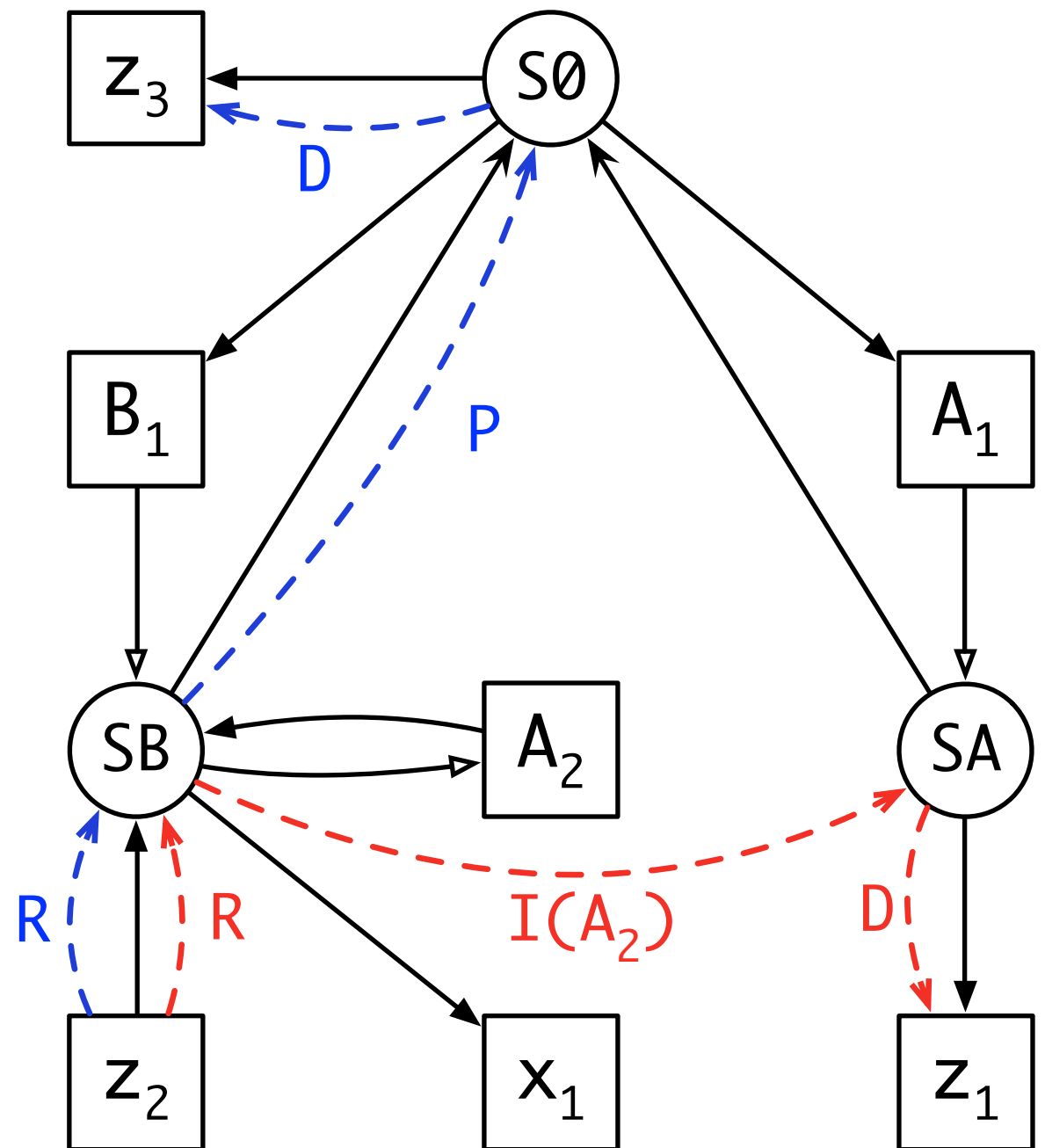
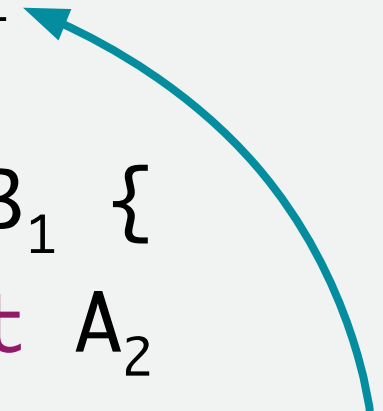
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Imports over Parents

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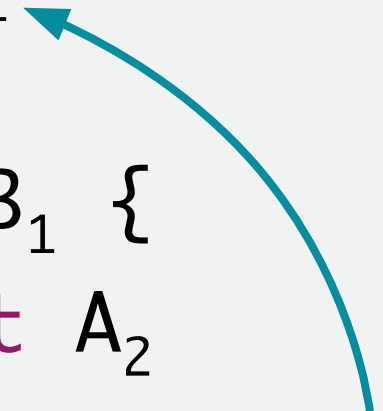
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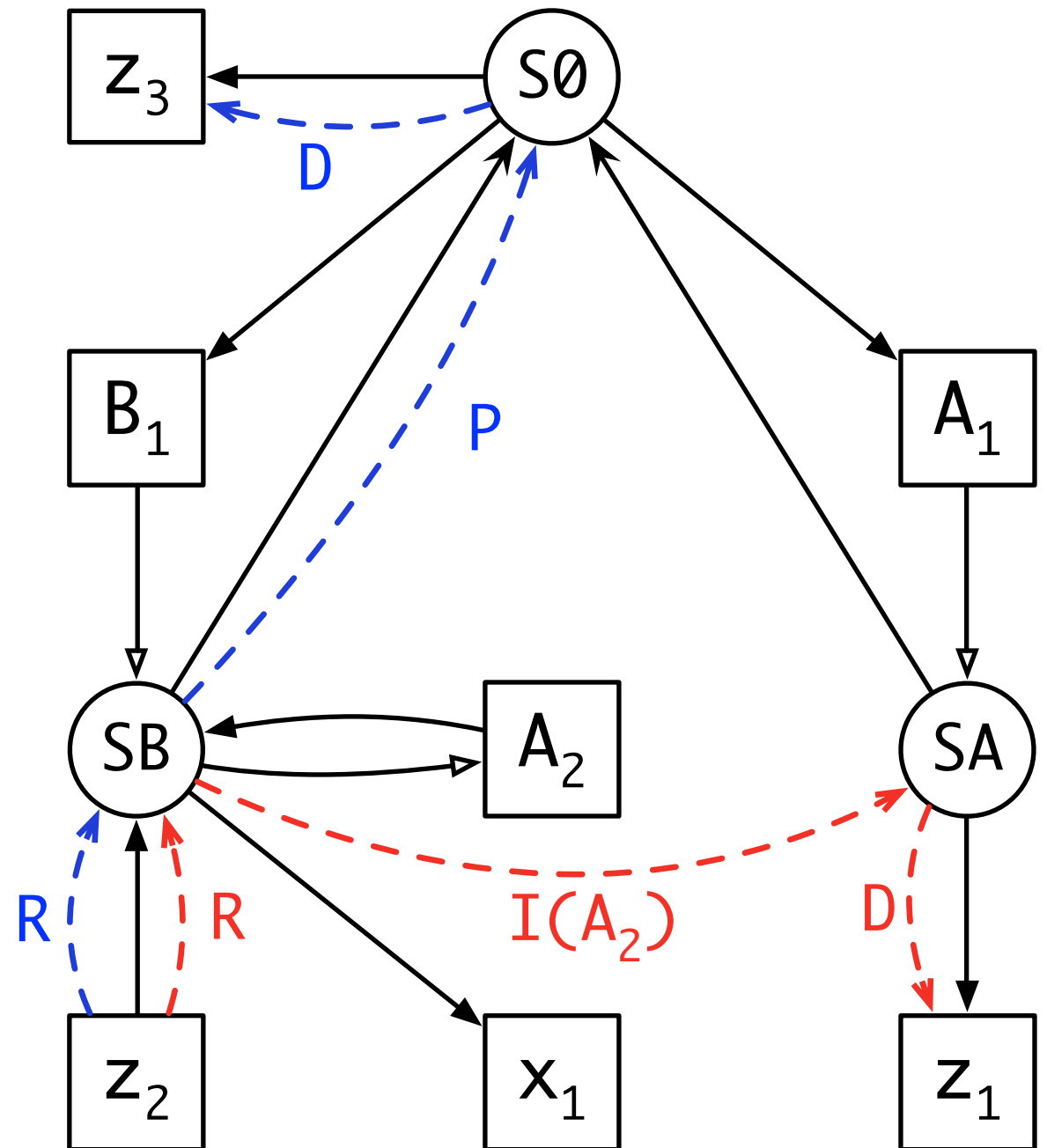
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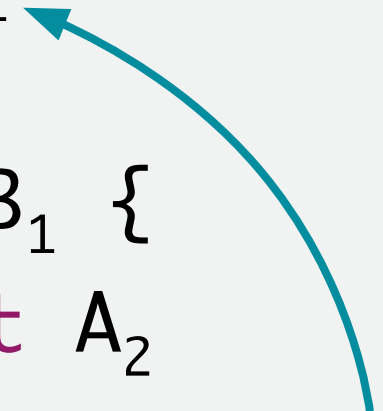
$I(_).p' < P.p$



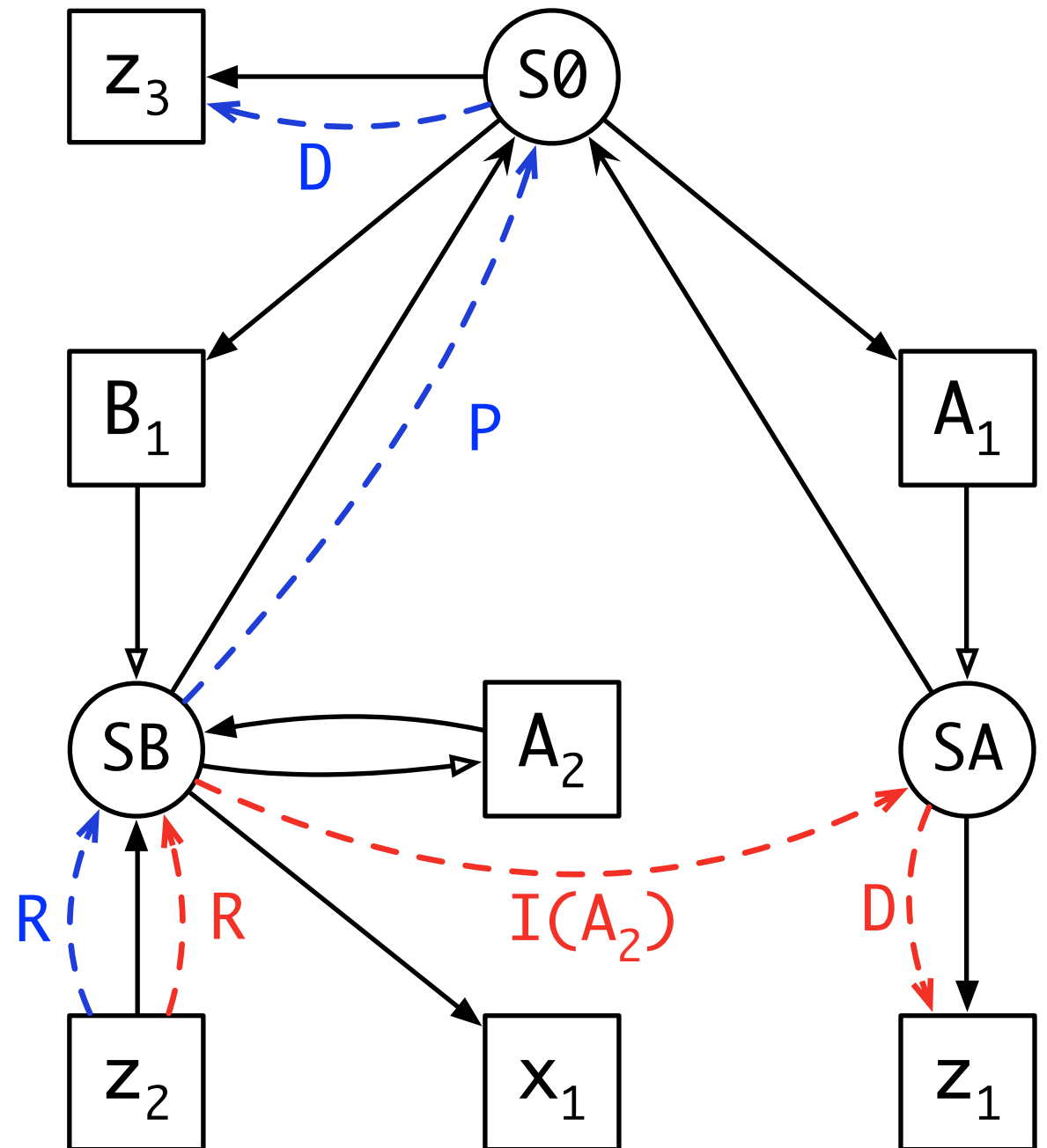
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}
```



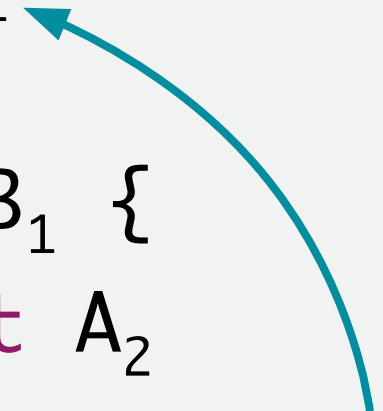
$$I(_).p' < P.p$$

$$D < I(_).p'$$


Imports over Parents

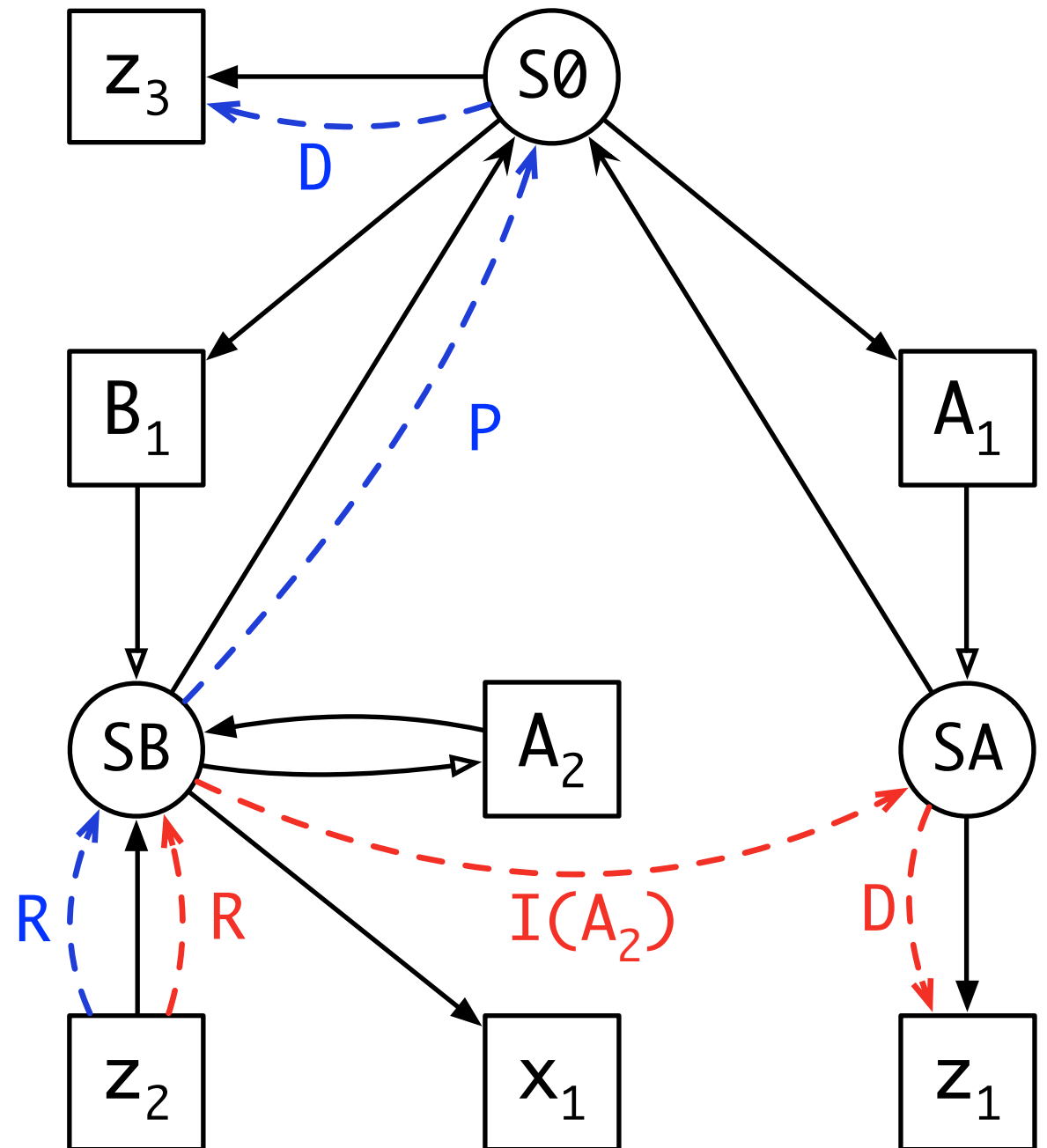
```
def z3 = 2

module A1 {
  def z1 = 5
}
module B1 {
  import A2
  def x1 = 1 + z2
}
```



$$I(_).p' < P.p$$

$$D < I(_).p'$$




$$\Rightarrow R.I(A_2).D < R.P.D$$

Transitive / Non-Transitive

```
module A1 {  
    def z1 = 5  
}  
module B1 {  
    import A2  
}  
module C1 {  
    import B2  
    def x1 = 1 + z2  
}
```


Transitive / Non-Transitive

```
module A1 {  
  def z1 = 5  
}  
module B1 {  
  import A2  
}  
module C1 {  
  import B2  
  def x1 = 1 + z2  
}
```



Transitive / Non-Transitive

```
module A1 {  
  def z1 = 5  
}  
module B1 {  
  import A2  
}  
module C1 {  
  import B2  
  def x1 = 1 + z2  
}
```

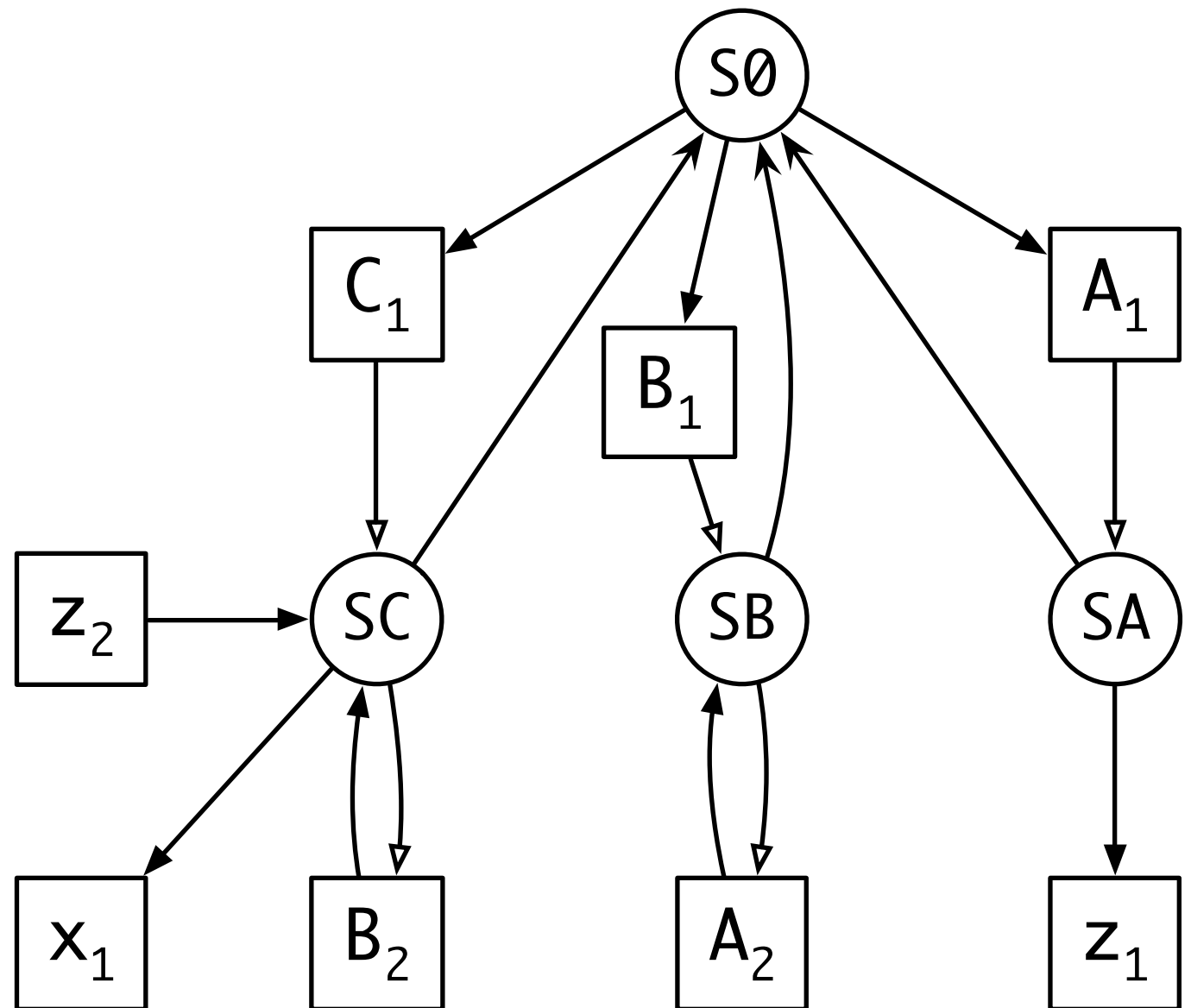


??

Transitive / Non-Transitive

```
module A1 {  
  def z1 = 5  
}  
module B1 {  
  import A2  
}  
module C1 {  
  import B2  
  def x1 = 1 + z2  
}
```

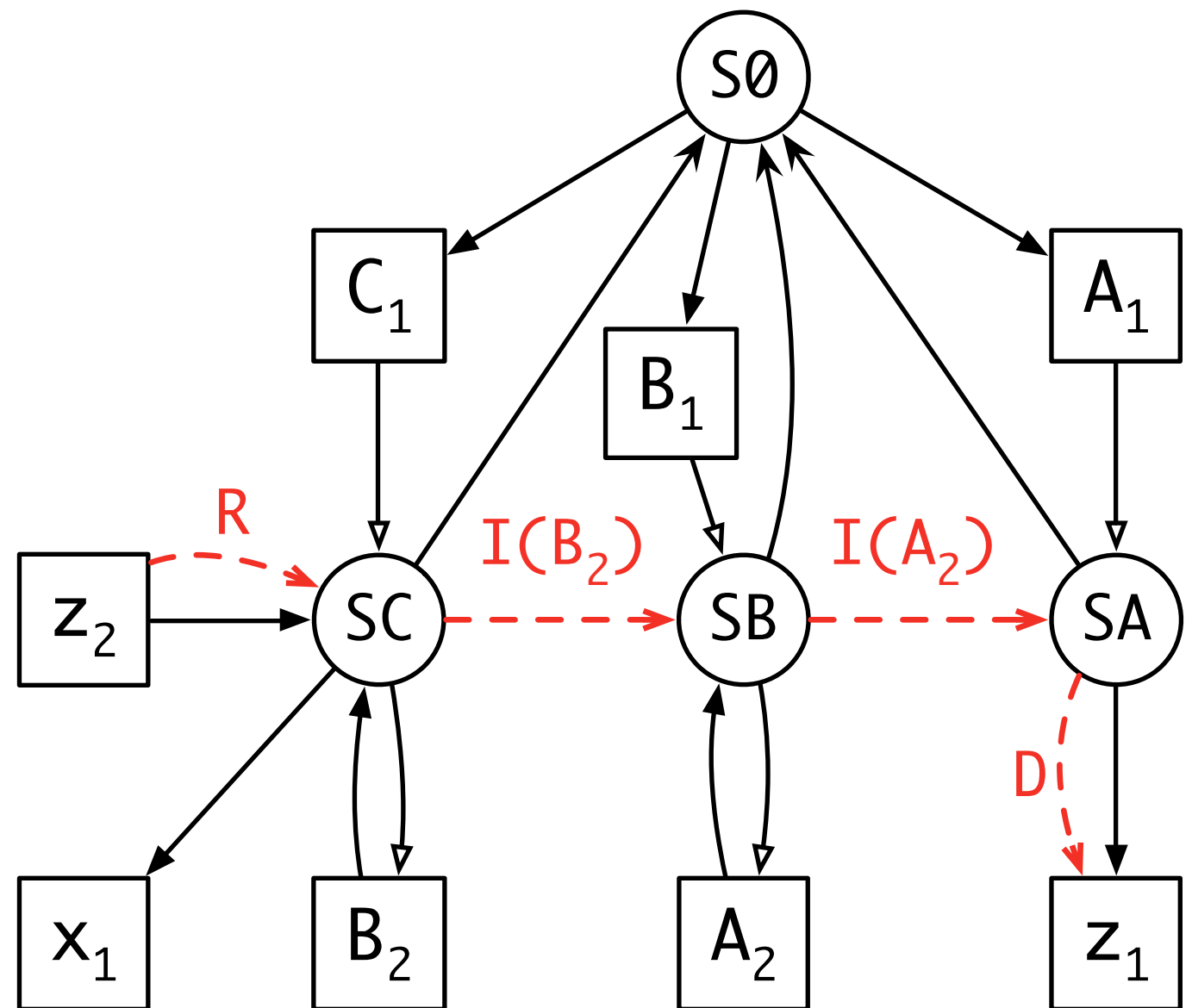
??



Transitive / Non-Transitive

```
module A1 {  
  def z1 = 5  
}  
module B1 {  
  import A2  
}  
module C1 {  
  import B2  
  def x1 = 1 + z2  
}
```

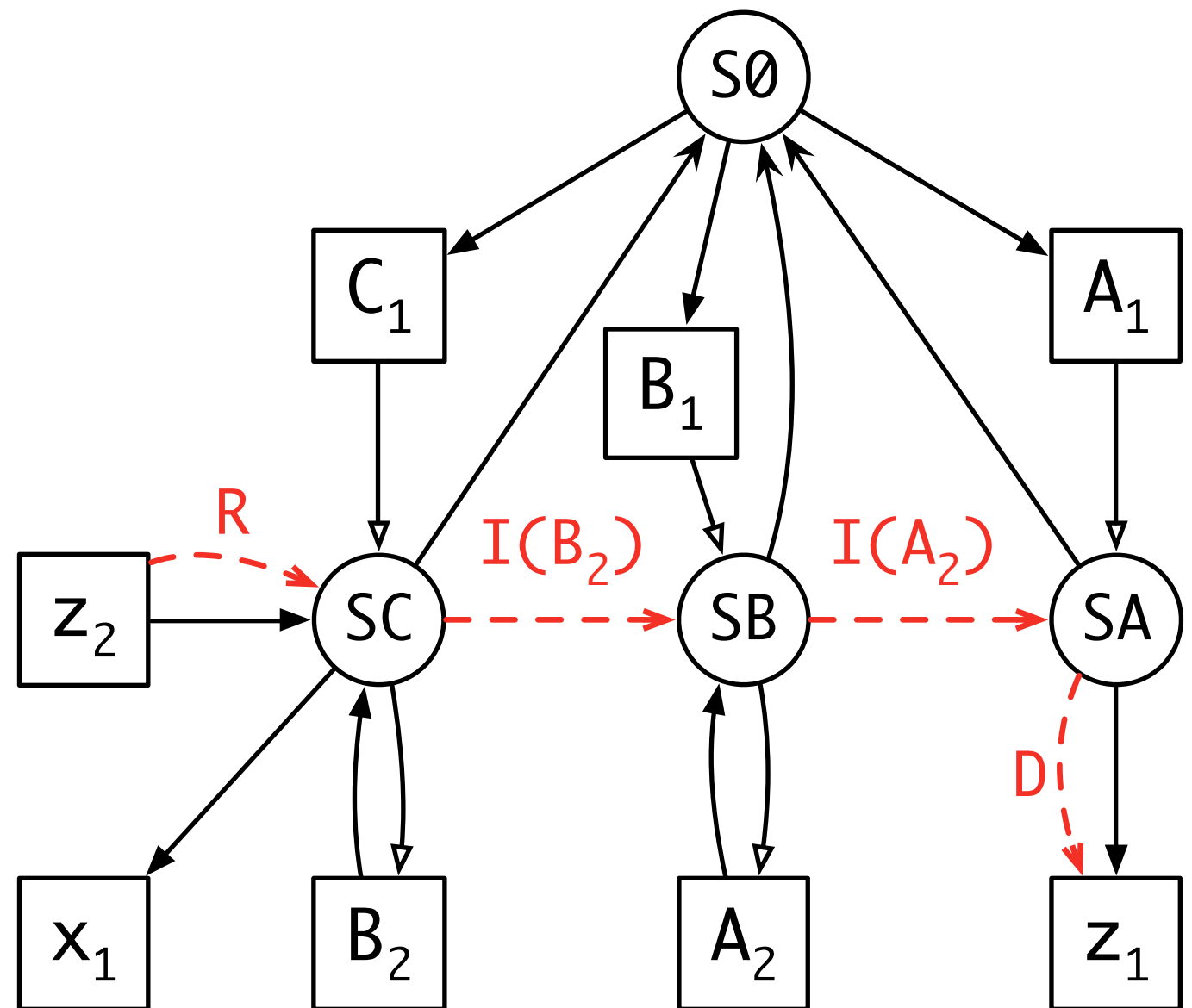
??



Transitive / Non-Transitive

```
module A1 {  
  def z1 = 5  
}  
module B1 {  
  import A2  
}  
module C1 {  
  import B2  
  def x1 = 1 + z2  
}
```

??

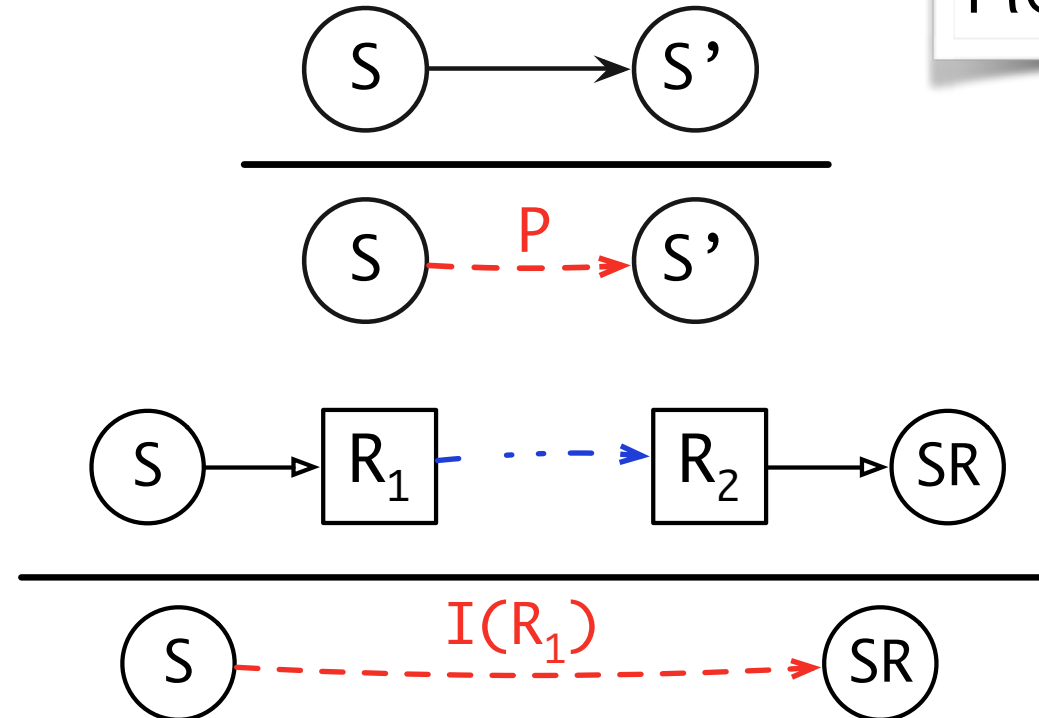
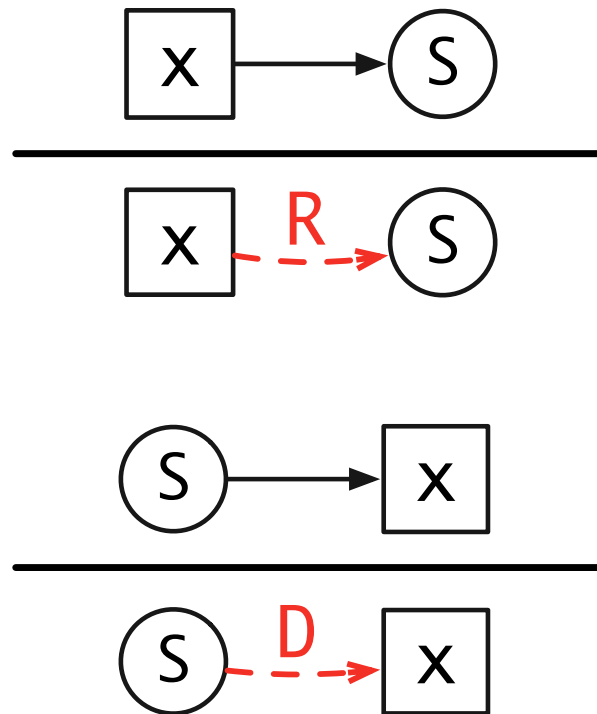


With transitive imports, a well formed path is $R.P^*.I(_)*.D$

With non-transitive imports, a well formed path is $R.P^*.I(_)?.D$

A Calculus for Name Resolution

Reachability



Disambiguation

$$\frac{}{D < P.p}$$

$$\frac{p < p'}{s.p < s.p'}$$

$$\frac{}{I(_).p' < P.p}$$

$$\frac{}{D < I(_).p'}$$

Well formed path: $R.P^*.I(_)^*.D$

In the Paper

- Formalization of the resolution calculus

Edges in scope graph

$$\frac{\mathcal{P}(S_1) = S_2}{\mathbb{I} \vdash \mathbf{P} : S_1 \longrightarrow S_2} \quad (P)$$

$$\frac{y_i^R \in \mathcal{I}(S_1) \setminus \mathbb{I} \quad \mathbb{I} \vdash p : y_i^R \longmapsto y_j^D : S_2}{\mathbb{I} \vdash \mathbf{I}(y_i^R, y_j^D : S_2) : S_1 \longrightarrow S_2} \quad (I)$$

Transitive closure

$$\overline{\mathbb{I} \vdash [] : A \twoheadrightarrow A} \quad (N)$$

$$\frac{\mathbb{I} \vdash s : A \longrightarrow B \quad \mathbb{I} \vdash p : B \twoheadrightarrow C}{\mathbb{I} \vdash s \cdot p : A \twoheadrightarrow C} \quad (T)$$

Reachable declarations

$$\frac{x_i^D \in \mathcal{D}(S') \quad \mathbb{I} \vdash p : S \twoheadrightarrow S' \quad WF(p)}{\mathbb{I} \vdash p \cdot \mathbf{D}(x_i^D) : S \twoheadrightarrow x_i^D} \quad (R)$$

Visible declarations

$$\frac{\mathbb{I} \vdash p : S \twoheadrightarrow x_i^D \quad \forall j, p' (\mathbb{I} \vdash p' : S \twoheadrightarrow x_j^D \Rightarrow \neg(p' < p))}{\mathbb{I} \vdash p : S \longmapsto x_i^D} \quad (V)$$

Reference resolution

$$\frac{x_i^R \in \mathcal{R}(S) \quad \{x_i^R\} \cup \mathbb{I} \vdash p : S \longmapsto x_j^D}{\mathbb{I} \vdash p : x_i^R \longmapsto x_j^D} \quad (X)$$

Well-formed paths

$$WF(p) \Leftrightarrow p \in \mathbf{P}^* \cdot \mathbf{I}(-, -)^*$$

Specificity ordering on paths

$$\overline{\mathbf{D}(-) < \mathbf{I}(-, -)} \quad (DI) \qquad \overline{\mathbf{I}(-, -) < \mathbf{P}} \quad (IP) \qquad \overline{\mathbf{D}(-) < \mathbf{P}} \quad (DP)$$

$$\frac{s_1 < s_2}{s_1 \cdot p_1 < s_2 \cdot p_2} \quad (Lex1) \qquad \frac{p_1 < p_2}{s \cdot p_1 < s \cdot p_2} \quad (Lex2)$$

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples
 - definition before use

In the Paper

- Formalization of the resolution calculus
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 - definition before use
 - different let binding flavors

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 - modules

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 - definition before use
 - different let binding flavors
 - modules
 - imports
 - qualified names
 - inheritance
 - partial classes
 - ...

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples
- Sound and complete resolution algorithm

$$\begin{aligned} Res[\mathbb{I}](x_i^R) &:= \{x_j^D \mid \exists S \text{ s.t. } x_i^R \in \mathcal{R}(S) \wedge x_j^D \in Env_V[\{x_i^R\} \cup \mathbb{I}, \emptyset](S)\} \\ Env_V[\mathbb{I}, \mathbb{S}](S) &:= Env_L[\mathbb{I}, \mathbb{S}](S) \triangleleft Env_P[\mathbb{I}, \mathbb{S}](S) \\ Env_L[\mathbb{I}, \mathbb{S}](S) &:= Env_D[\mathbb{I}, \mathbb{S}](S) \triangleleft Env_I[\mathbb{I}, \mathbb{S}](S) \\ Env_D[\mathbb{I}, \mathbb{S}](S) &:= \begin{cases} \emptyset & \text{if } S \in \mathbb{S} \\ \mathcal{D}(S) & \end{cases} \\ Env_I[\mathbb{I}, \mathbb{S}](S) &:= \begin{cases} \emptyset & \text{if } S \in \mathbb{S} \\ \bigcup \{ Env_L[\mathbb{I}, \{S\} \cup \mathbb{S}](S_y) \mid y_i^R \in \mathcal{I}(S) \setminus \mathbb{I} \wedge y_j^D : S_y \in Res[\mathbb{I}](y_i^R) \} & \end{cases} \\ Env_P[\mathbb{I}, \mathbb{S}](S) &:= \begin{cases} \emptyset & \text{if } S \in \mathbb{S} \\ Env_V[\mathbb{I}, \{S\} \cup \mathbb{S}](\mathcal{P}(S)) & \end{cases} \end{aligned}$$

$$\forall \mathbb{I}, x_i^R, j, (x_j^D \in Res[\mathbb{I}](x_i^R)) \iff (\exists p \text{ s.t. } \mathbb{I} \vdash p : x_i^R \longmapsto x_j^D).$$

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples
- Sound and complete resolution algorithm
- Language-independent definition of alpha-equivalence

$$\begin{array}{c}
 \frac{\vdash p : x_i^R \mapsto x_{i'}^D}{i \stackrel{P}{\sim} i'} \quad
 \frac{i' \stackrel{P}{\sim} i}{i \stackrel{P}{\sim} i'} \quad
 \frac{i \stackrel{P}{\sim} i' \quad i' \stackrel{P}{\sim} i''}{i \stackrel{P}{\sim} i''} \quad
 \frac{}{i \stackrel{P}{\sim} i}
 \end{array}$$

$$P1 \stackrel{\alpha}{\approx} P2 \triangleq P1 \simeq P2 \wedge \forall i \ i', \ i \stackrel{P1}{\sim} i' \Leftrightarrow i \stackrel{P2}{\sim} i'$$

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples
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Future Work

- Interaction with types

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples
- Sound and complete resolution algorithm
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Future Work

- Interaction with types
- Resolution-sensitive program transformations

In the Paper

- Formalization of the resolution calculus
- Validation by an extensive set of examples
- Sound and complete resolution algorithm
- Language-independent definition of alpha-equivalence

Future Work

- Interaction with types
- Resolution-sensitive program transformations
- And more ...

The Future of Language Design

A universal formalism to describe the name binding patterns in programming languages

A single representation of the name binding structure of programs to reuse across name-sensitive language artifacts

Qualified Names

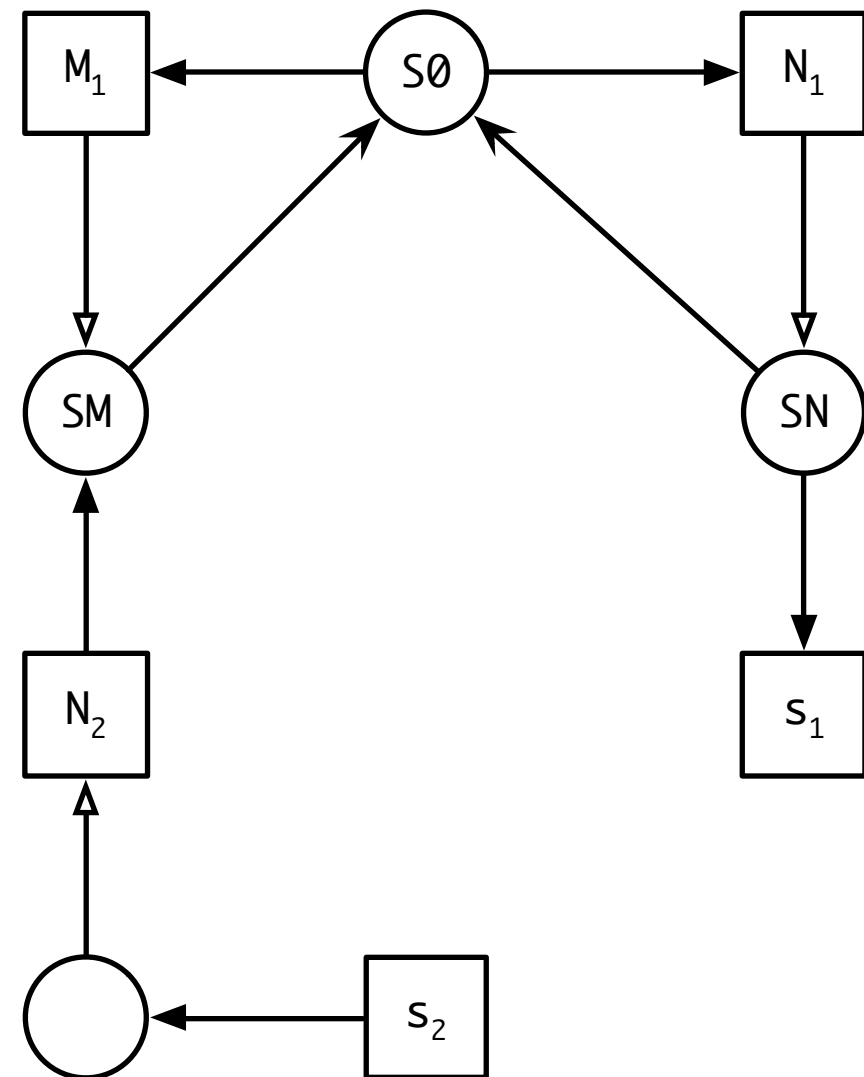
```
module N1 {  
    def s1 = 5  
}
```

```
module M1 {  
    def x1 = 1 + N2.s2  
}
```

Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

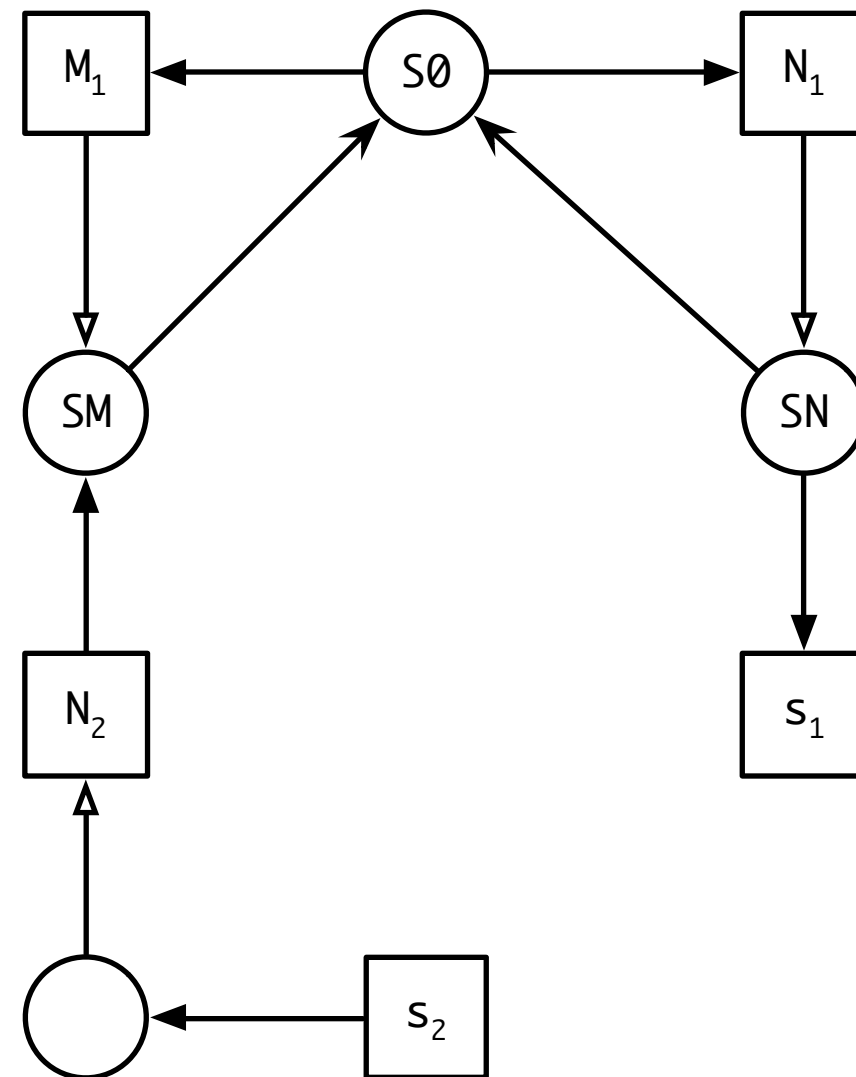
```
module M1 {  
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}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

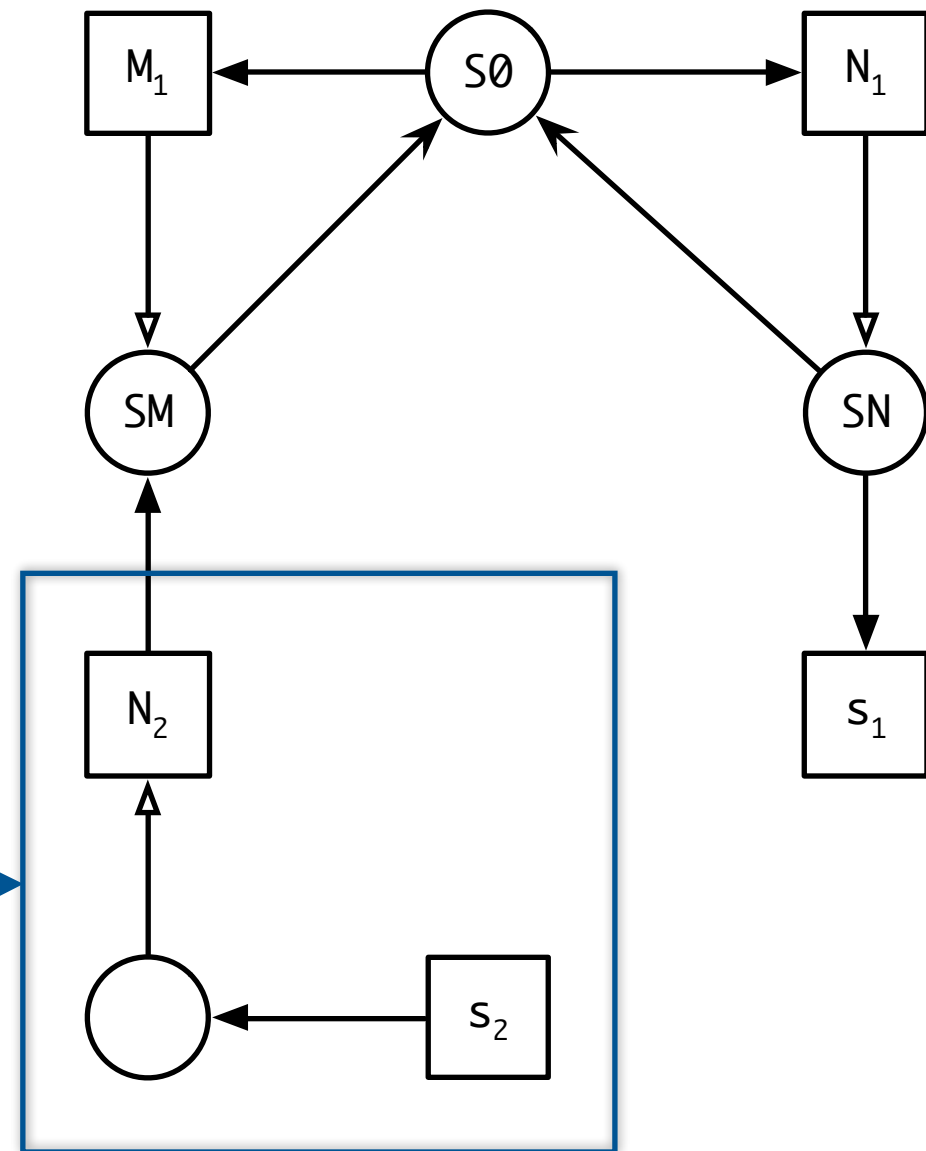
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

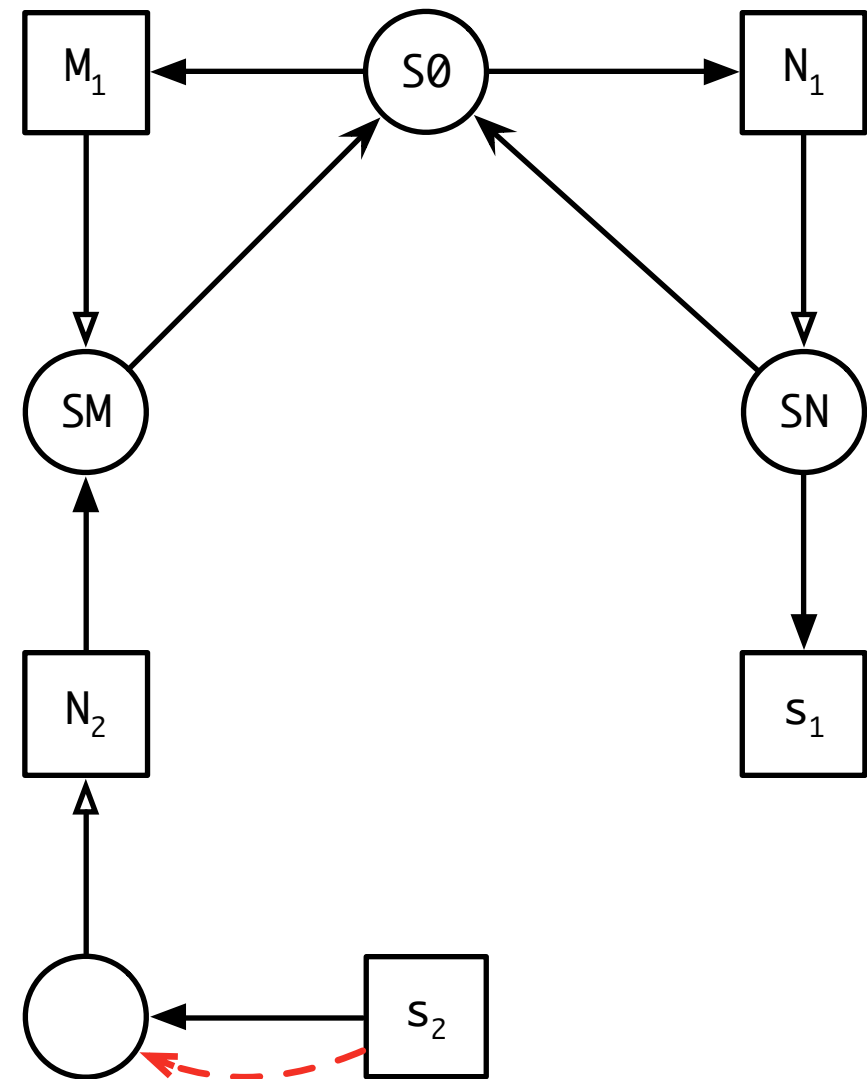
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

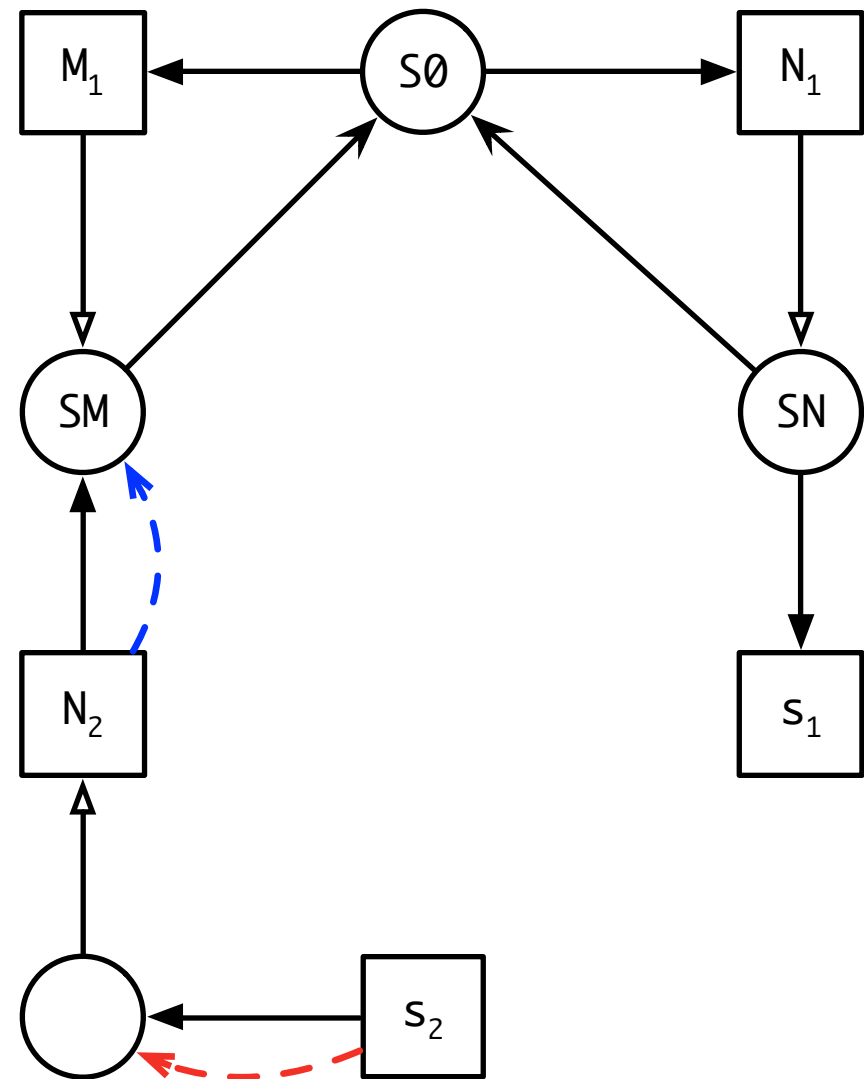
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

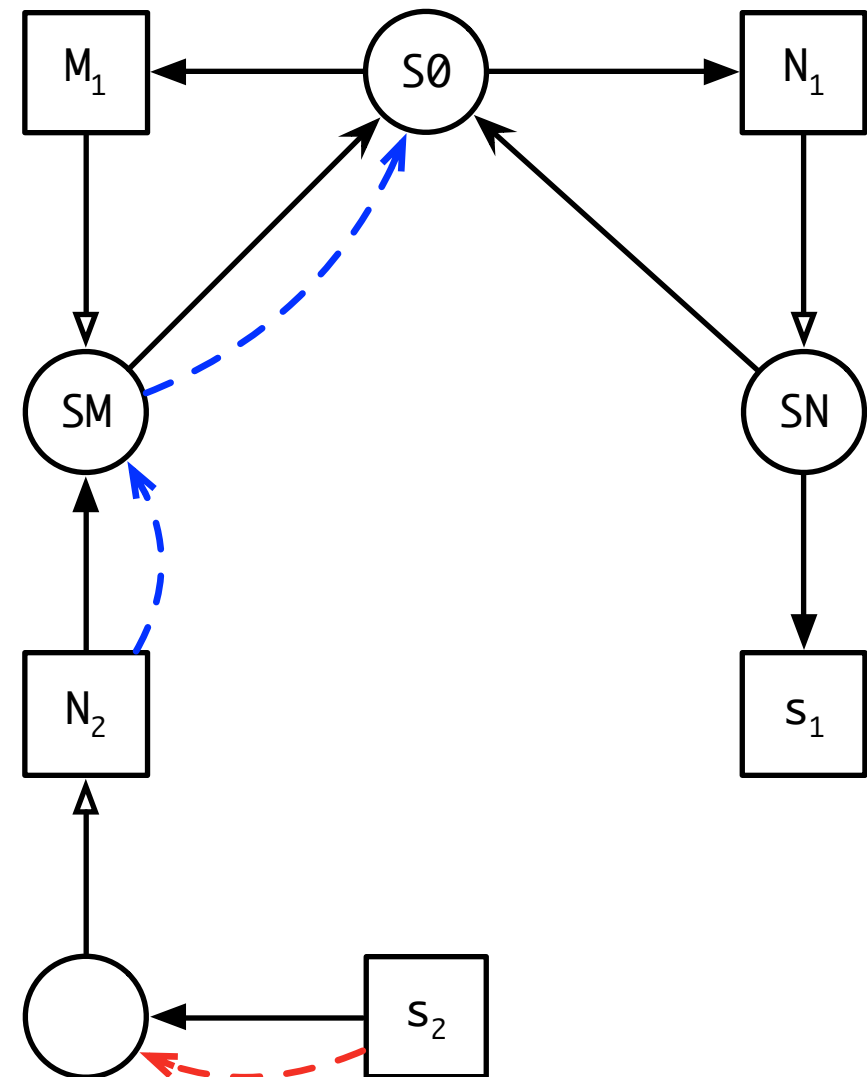
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
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}
```

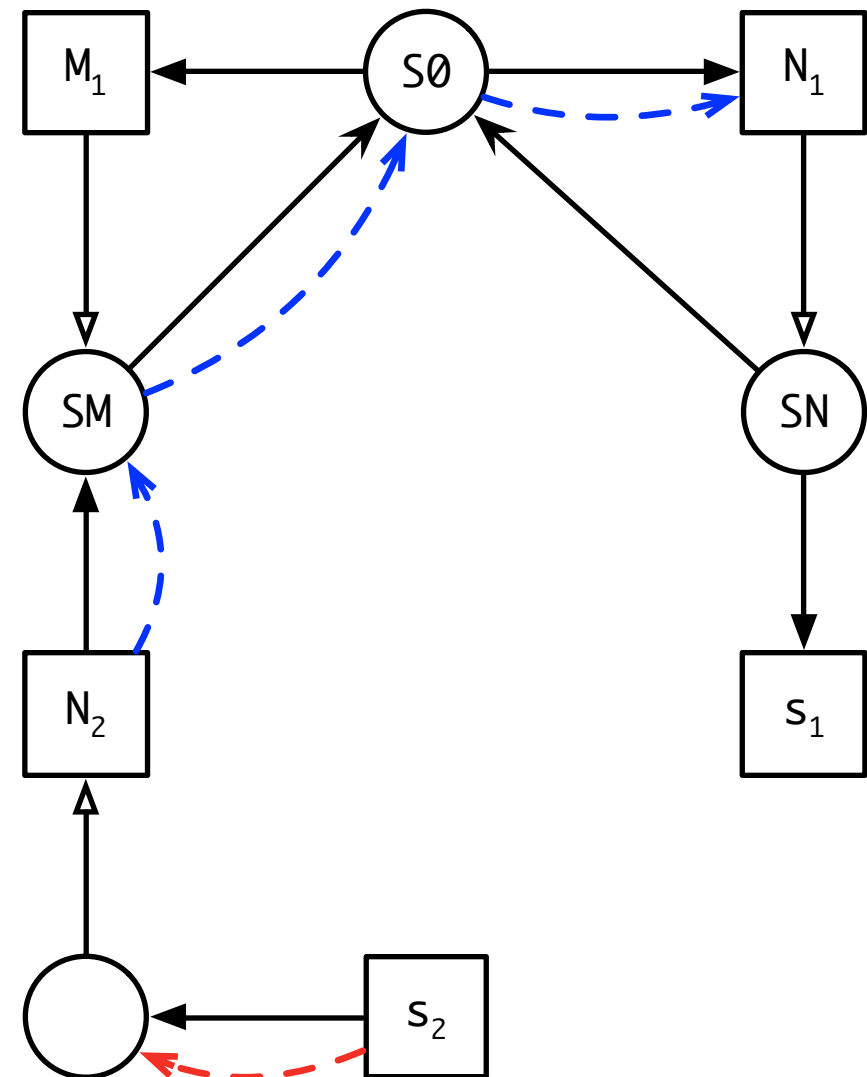
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

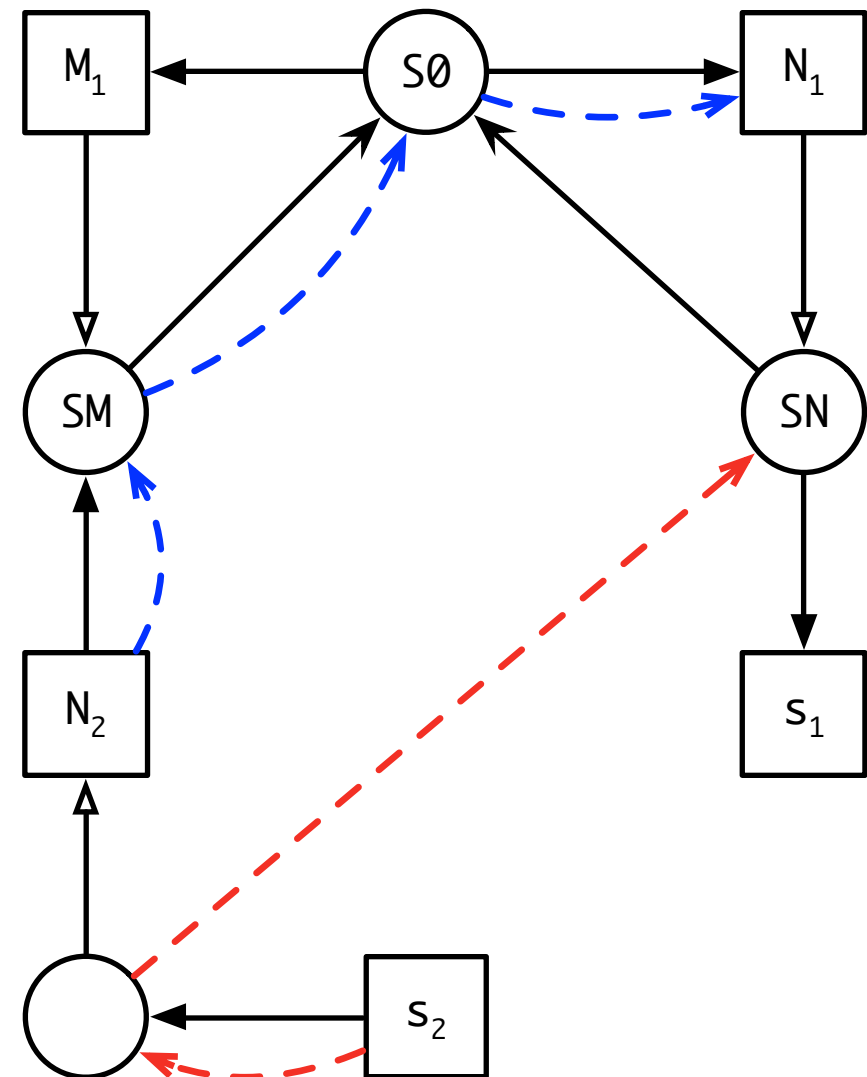
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

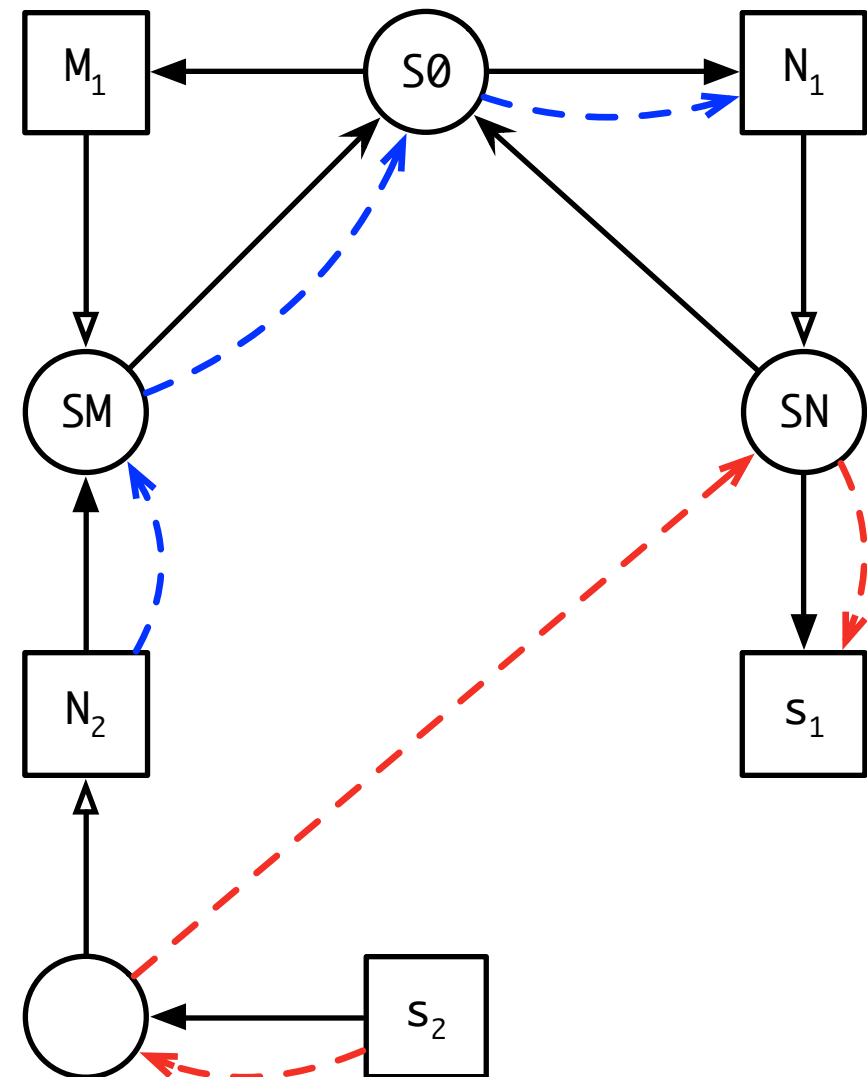
```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Qualified Names

```
module N1 {  
  def s1 = 5  
}
```

```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Well-Formedness

```
def s3 = 6
```

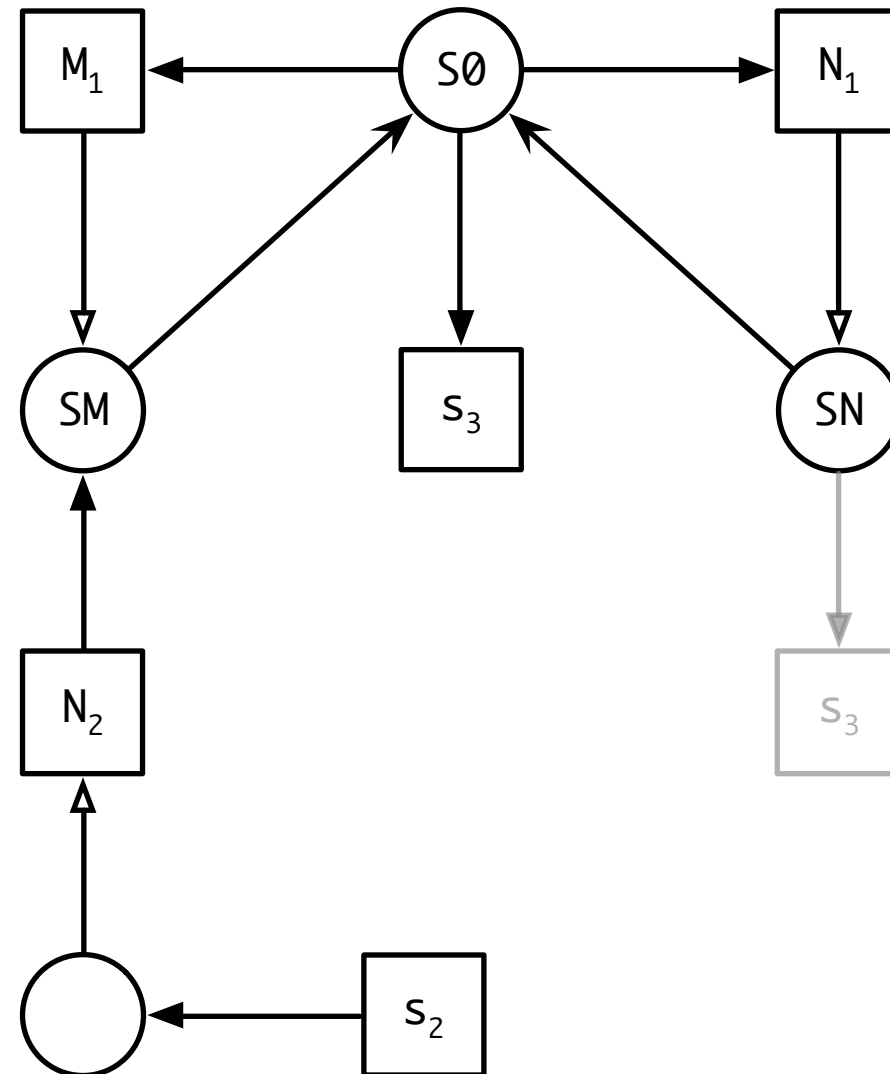
```
module N1 {
```

```
}
```

```
module M1 {
```

```
  def x1 = 1 + N2.s2
```

```
}
```

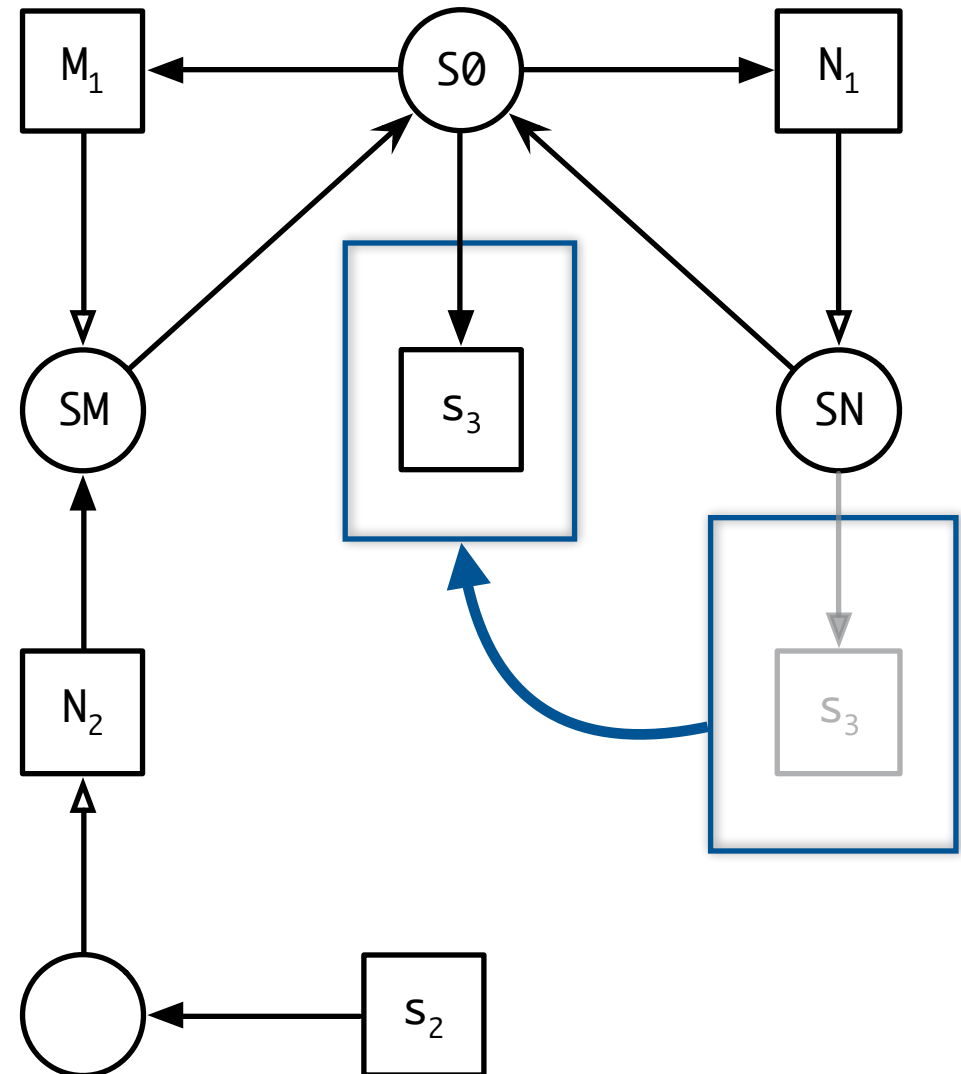


Well-Formedness

```
def s3 = 6
```

```
module N1 {  
}
```

```
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Well-Formedness

```
def  $s_3 = 6$ 
```

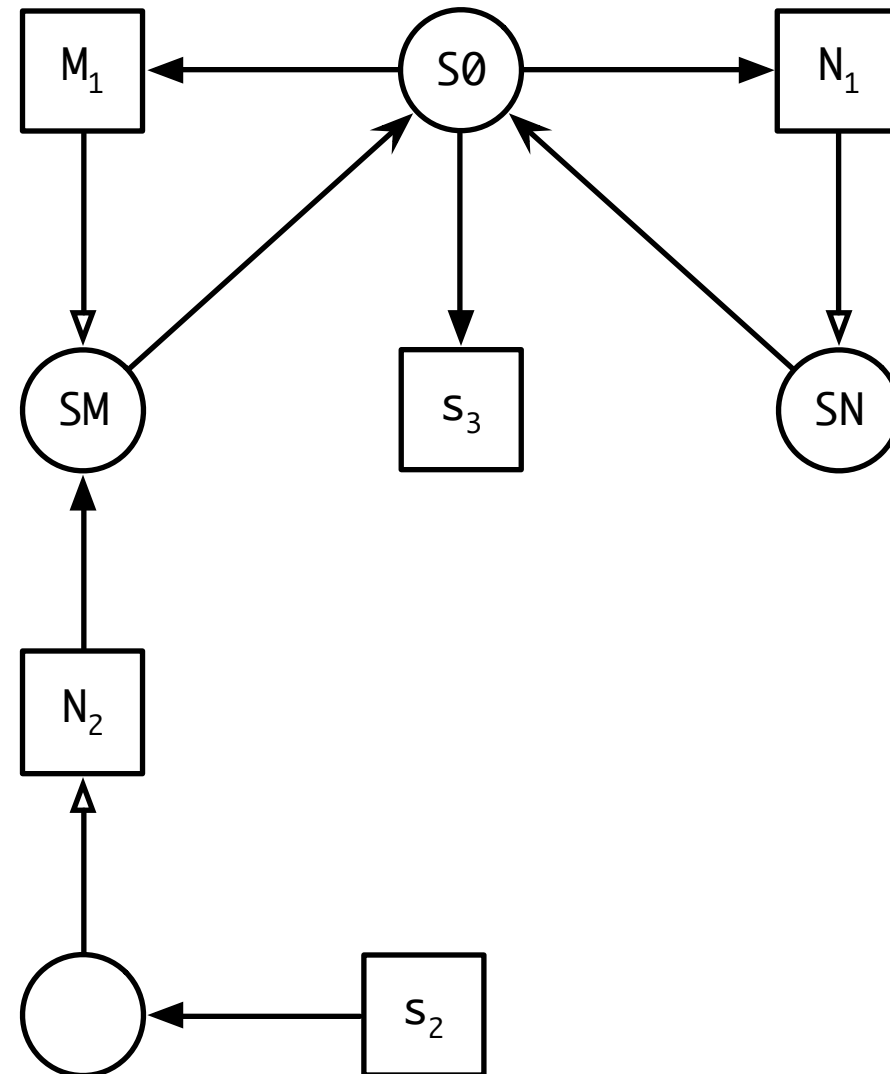
```
module  $N_1$  {
```

```
}
```

```
module  $M_1$  {
```

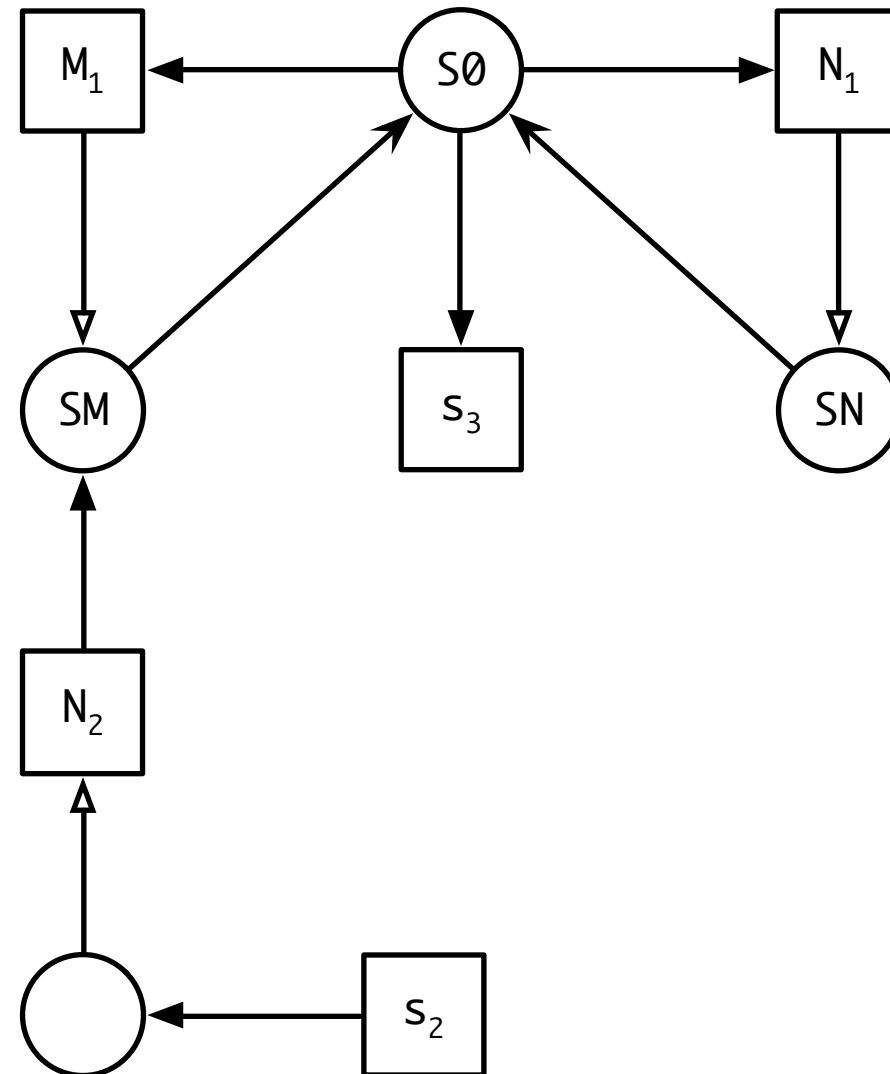
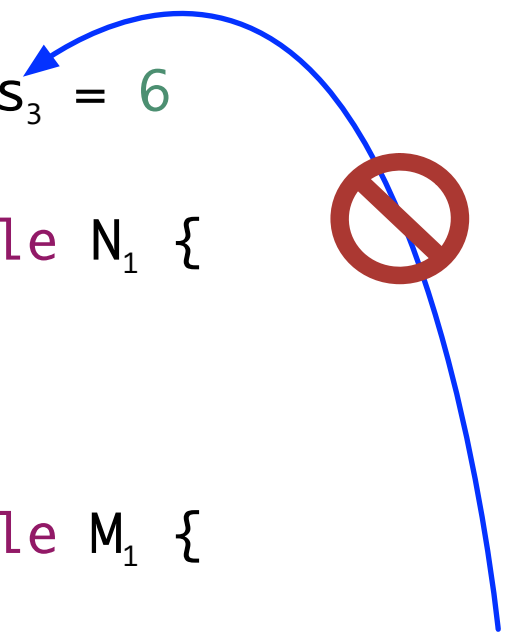
```
    def  $x_1 = 1 + N_2.s_2$ 
```

```
}
```



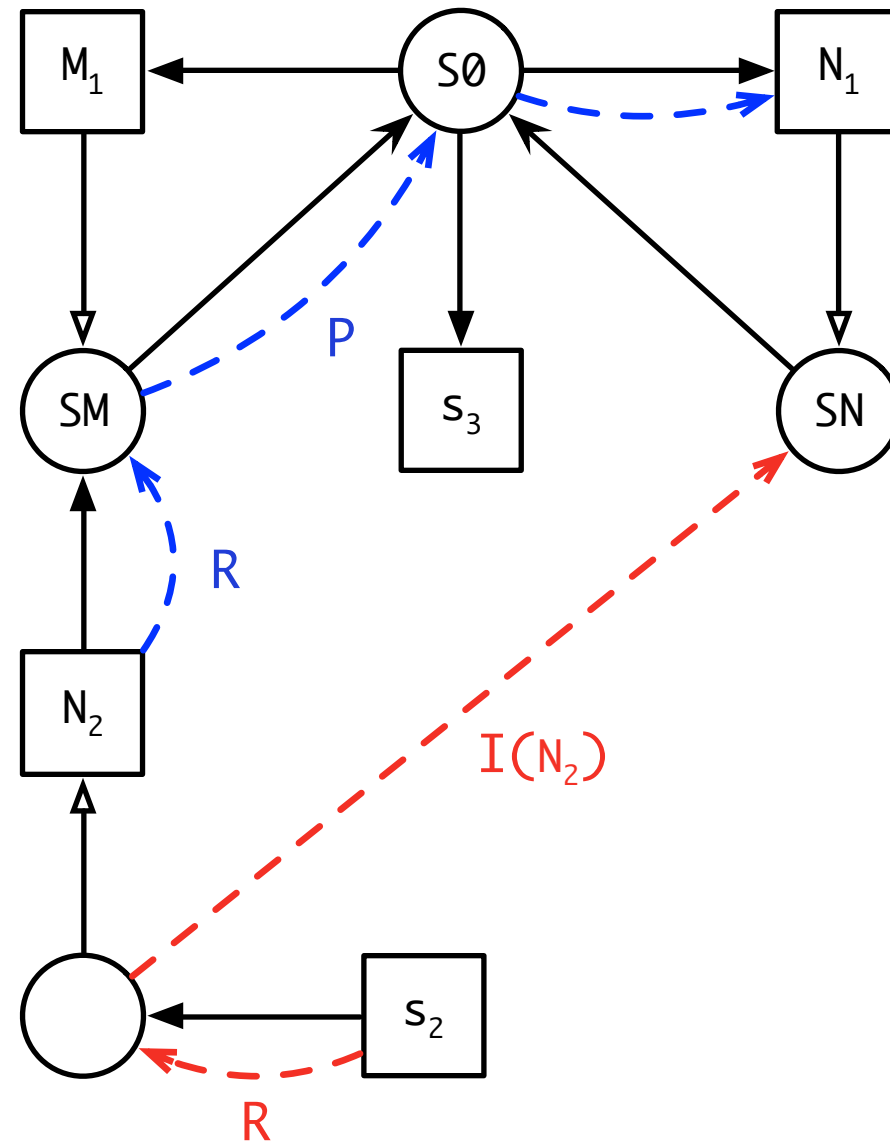
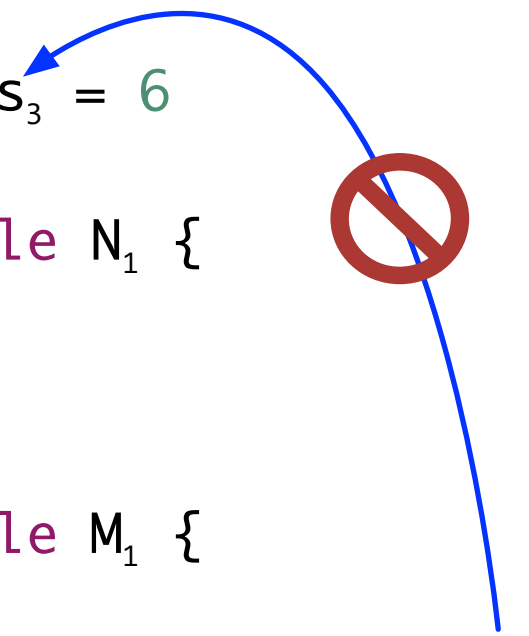
Well-Formedness

```
def s3 = 6  
module N1 {  
}  
module M1 {  
  def x1 = 1 + N2.s2  
}
```



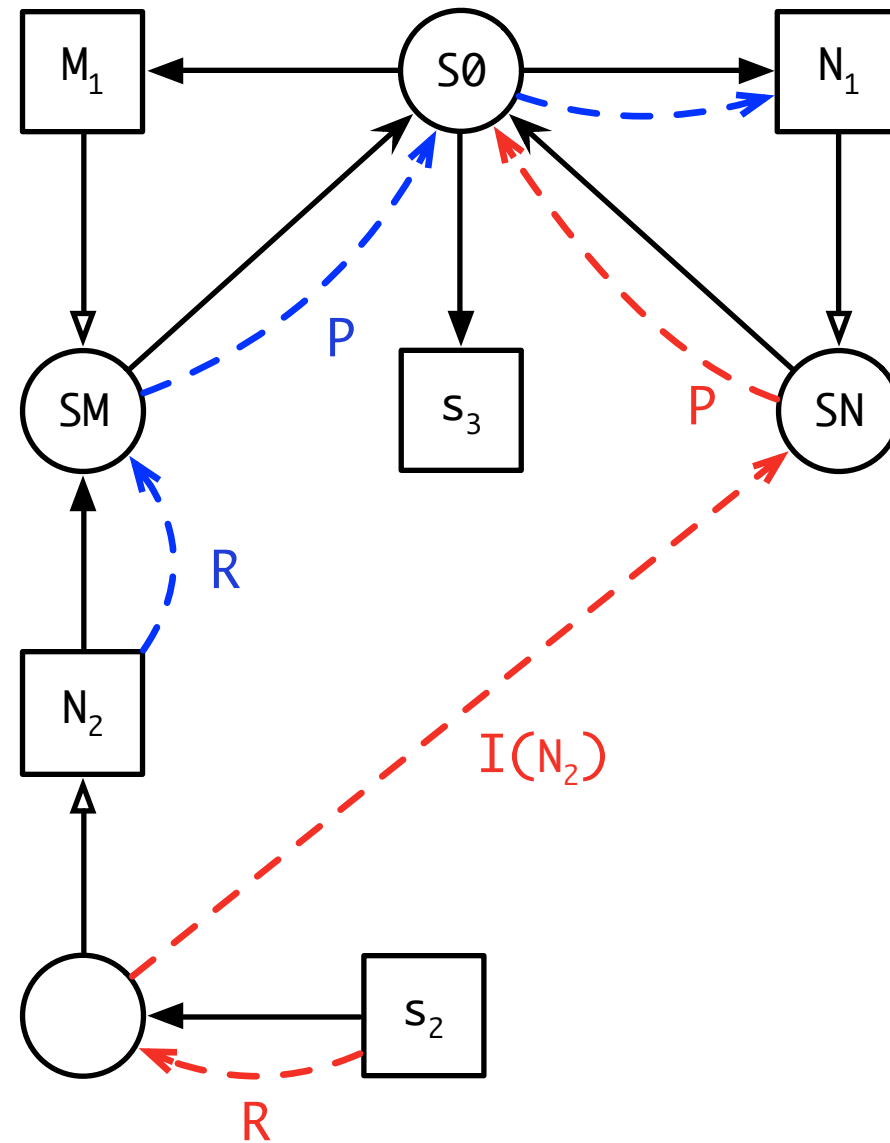
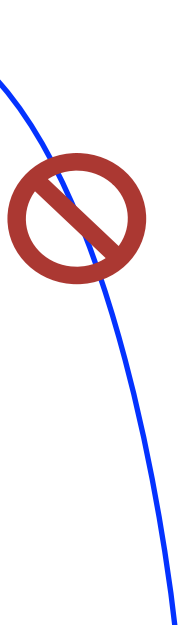
Well-Formedness

```
def s3 = 6  
module N1 {  
}  
module M1 {  
  def x1 = 1 + N2.s2  
}
```



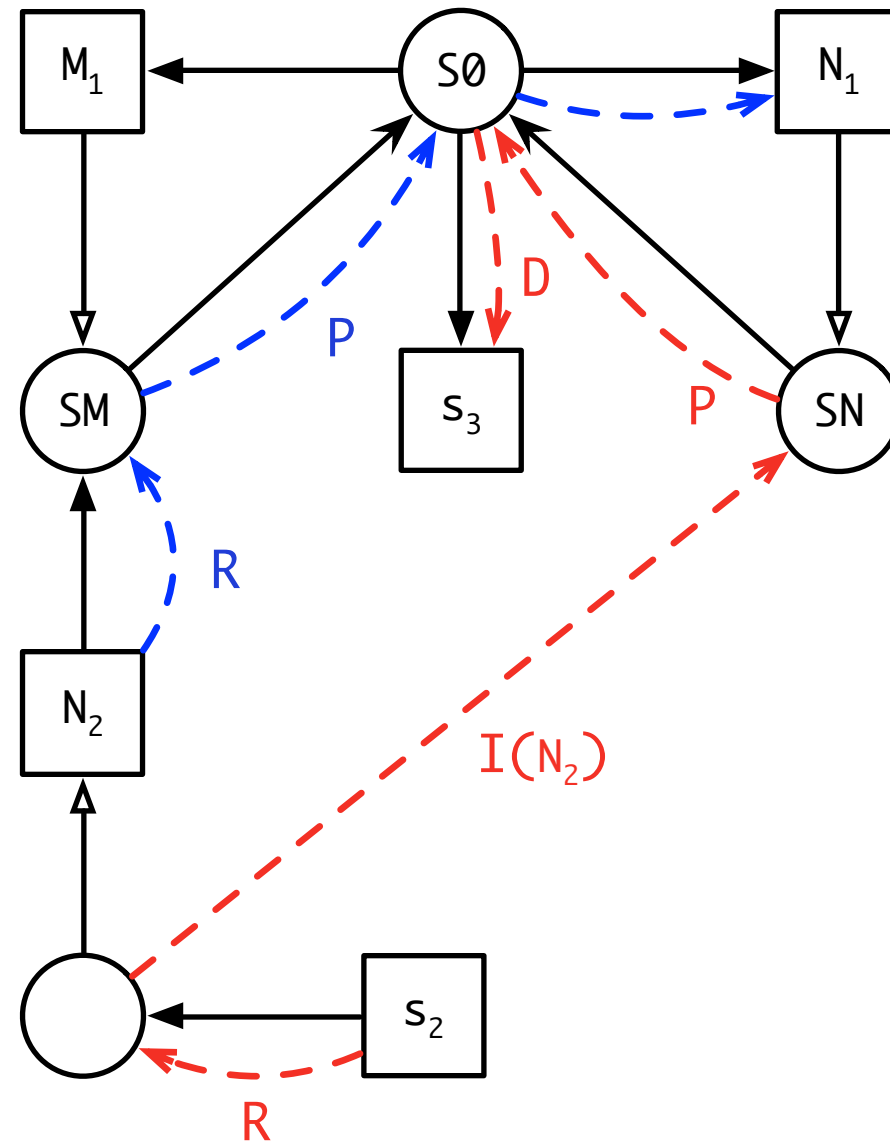
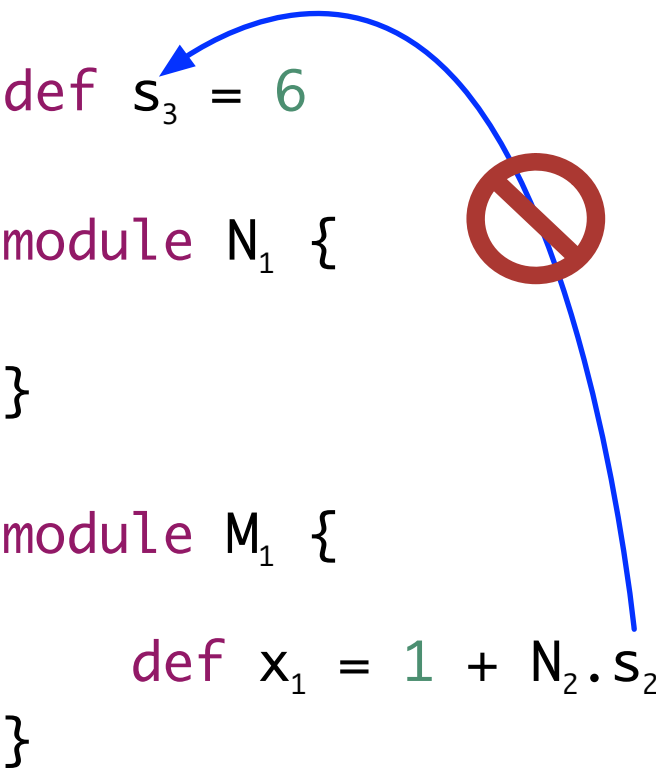
Well-Formedness

```
def s3 = 6  
module N1 {  
}  
module M1 {  
  def x1 = 1 + N2.s2  
}
```



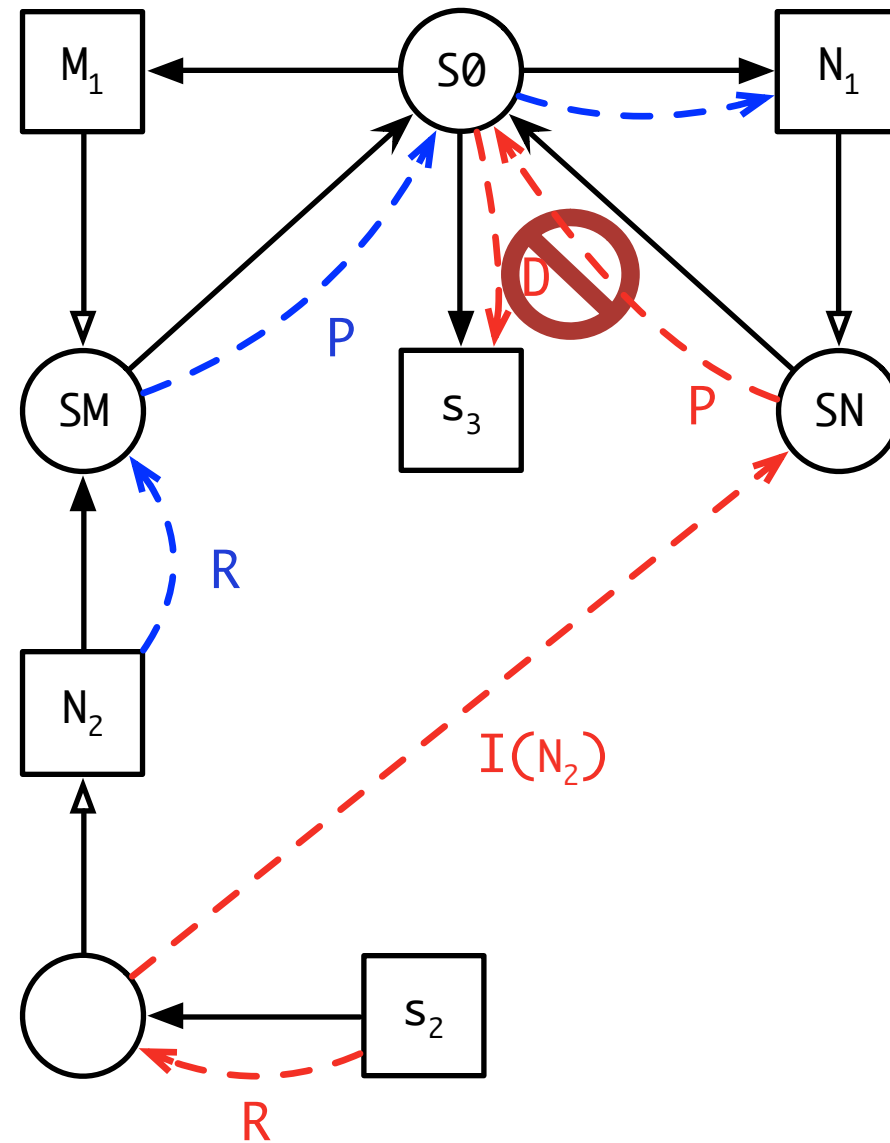
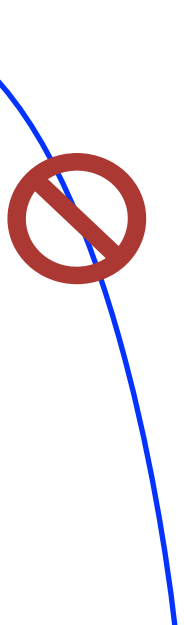
Well-Formedness

```
def s3 = 6  
module N1 {  
}  
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Well-Formedness


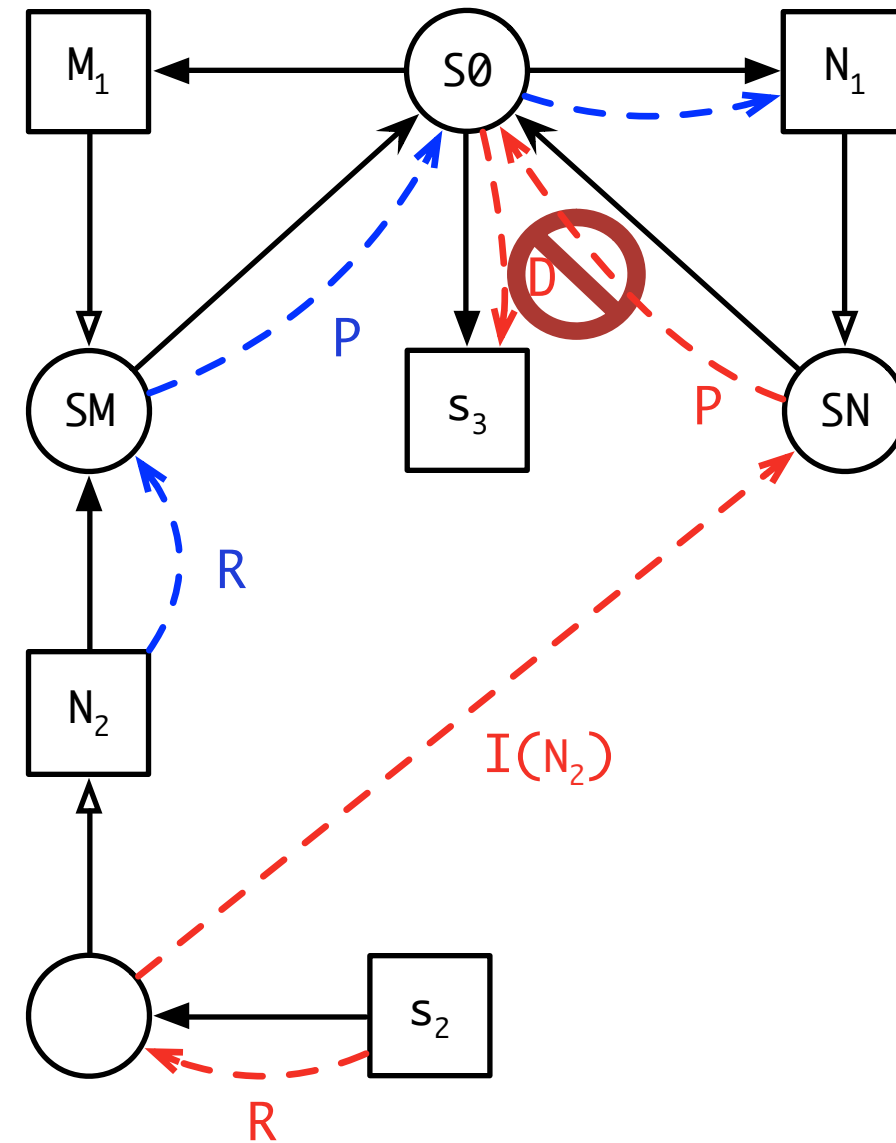
```
def s3 = 6  
module N1 {  
}  
module M1 {  
  def x1 = 1 + N2.s2  
}
```



Well-Formedness

```

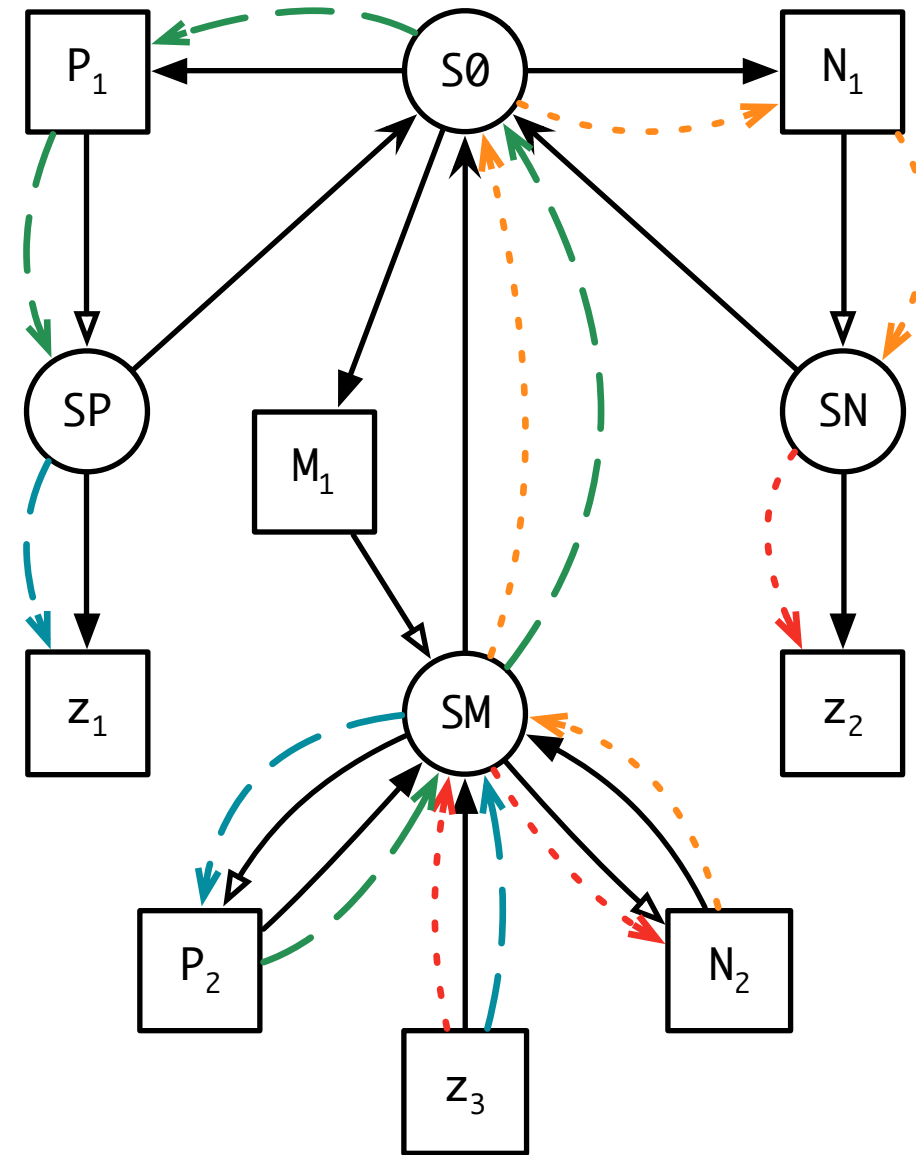
def s3 = 6
module N1 {
}
module M1 {
  def x1 = 1 + N2.s2
}
    
```

Well formed path: $R.P^*.I(_)*.D$

Duplicate Resolution (imports)

```
module P1 {  
  def z1 = 5  
}  
  
module N1 {  
  def z2 = 5  
}  
  
module M1 {  
  import N2  
  import P2  
  def x1 = 1 + z3  
}
```



Nested Imports

```
module P1 {  
    module N1 {  
        def z2 = 5  
    }  
}
```

```
module M1 {  
  
    import N2  
    import P2  
  
    def x1 = 1 + z3  
}
```