Mínimos Quadrados

Joao Felipe Bianchi Curcio
Jonas Edward Tashiro
Luan Lopes Barbosa de Almeida
Rafael Melloni Chacon Arnone

Contents

1 Dependências

O programa foi escrito na linguagem C++ e faz utilização de API's e bibliotecas de Computação Gráfica para mostrar ao usuário, na forma de um plano cartesiano, a saída do problema dos Mínimos Quadrados, sendo que a biblioteca para "plotar" o gráfico foi a biblioteca ImGui.

A API utilizada para renderizar o plano cartesiano foi o Open-GL, pois é uma API multiplataforma, podendo ser executado em ambientes Linux e Windows. Além disto, algumas bibliotecas foram utilizadas para inicializar a API e facilitar a criação de janelas no Sistema Operacional, estas são, respectivamente, GLEW e GLFW.

Abaixo fornecemos links de tutoriais para instalação das dependências.

1.1 Windows:

Segue um vídeo demonstrando os passos necessários para instalação na plataforma.

https://www.youtube.com/watch?v=OR4fNpBjmq8

1.2 Linux(Distro-Ubuntu):

Primeiramente instale o ppa da biblioteca mesa para adquirir versões mais recentes da Biblioteca Mesa que possui o OpenGL.

https://itsfoss.com/install-mesa-ubuntu/

E siga os passos a seguir para instalar GLEW e GLFW:

https://medium.com/geekculture/a-beginners-guide-to-setup-opengl-in-linux-debian-2bfe02ccd1e

Após isto entre no terminal na pasta do projeto e escreva o comando make para compilar as bibliotecas e gerar o executável do programa.

1.3 Linux(Distro-Arch):

A instalação para distribuição Arch é elementar, basta fazer os seguintes comandos:

```
$ sudo pacman -Sy mesa
$ sudo pacman -Sy glfw
$ sudo pacman -Sy make
```

Após isto entre no terminal na pasta do projeto e escreva o comando make para compilar as bibliotecas e gerar o executável do programa.

2 Organização de Pastas

```
/src/
├─/imgui/
├─App.cpp
├─lsq.h
└─lsq.cpp
```

3 Conteúdo dos Arquivos

Listing 1: Makefile

```
EXE = App
IMGUI_DIR = ./imgui
SOURCES = App.cpp lsq.cpp
SOURCES += $(IMGUI_DIR)/imgui.cpp $(IMGUI_DIR)/imgui_demo.cpp
SOURCES += $(IMGUI_DIR)/imgui_draw.cpp $(IMGUI_DIR)/imgui_tables.cpp $(IMGUI_DIR)/imgui_widgets.cpp
SOURCES += $(IMGUI_DIR)/implot_items.cpp $(IMGUI_DIR)/implot.cpp
SOURCES += $(IMGUI_DIR)/imgui_impl_glfw.cpp $(IMGUI_DIR)/imgui_impl_opengl3.cpp
SOURCES += $(IMGUI_DIR)/imgui_stdlib.cpp
OBJS = $(addsuffix .o, $(basename $(notdir $(SOURCES))))
UNAME_S := $(shell uname -s)
LINUX_GL_LIBS = -lGL -lGLEW

CXXFLAGS = -std=c++11 -I$(IMGUI_DIR)
CXXFLAGS += -g -Wall -Wformat -pthread
LIBS =

##---------------------------------------------------------------------
## BUILD FLAGS PER PLATFORM
##---------------------------------------------------------------------

ifeq ($(UNAME_S), Linux) #LINUX
   ECHO_MESSAGE = "Linux"
      LIBS += $(LINUX_GL_LIBS) `pkg-config --static --libs glfw3`

   CXXFLAGS += `pkg-config --cflags glfw3`
      CFLAGS = $(CXXFLAGS)
endif

ifeq ($(OS), Windows_NT)
   ECHO_MESSAGE = "MinGW"
      LIBS += -lglfw3 -lgdi32 -lopengl32 -limm32

   CXXFLAGS += -L$(shell pwd)
   CXXFLAGS += `pkg-config --cflags glfw3`
      CFLAGS = $(CXXFLAGS)
endif

##---------------------------------------------------------------------
## BUILD RULES
##---------------------------------------------------------------------

%.o:%.cpp
      $(CXX) $(CXXFLAGS) -c -o $@ $<
```

```
%.o:$(RENDER)/%.cpp
	$(CXX) $(CXXFLAGS) -c -o $@ $<

%.o:$(IMGUI_DIR)/%.cpp
	$(CXX) $(CXXFLAGS) -c -o $@ $<

%.o:$(STBI)/%.cpp
	$(CXX) $(CXXFLAGS) -c -o $@ $<

all: $(EXE)
	@echo Build complete for $(ECHO_MESSAGE)

$(EXE): $(OBJS)
	$(CXX) -o $@ $^ $(CXXFLAGS) $(LIBS)

clean:
	rm -f $(EXE) $(OBJS)
```

Listing 2: App.cpp

```cpp
#include <GL/glew.h>
#include <GLFW/glfw3.h>
#include "imgui/imgui.h"
#include "imgui/imgui_stdlib.h"
#include "imgui/imgui_impl_glfw.h"
#include "imgui/imgui_impl_opengl3.h"
#include "imgui/implot.h"
#include "imgui/implot_internal.h"
#include <array>
#include <cstddef>
#include <string>
#include <utility>
#include <vector>
#include <iostream>
#include <algorithm>
#include "lsq.h"
#define glsl_version "#version 420"

void Initialize_GLFW();
void Initialize_GLEW();
void Initialize_ImGui();
void Cleanup_OpenGL();
void Cleanup_ImGui();
void display();

int main()
{
    Initialize_GLFW();
    Initialize_ImGui();

    GLFWwindow* window = glfwCreateWindow(1920, 1080, "Least Squares Method", NULL, NULL);
    if(window == NULL)
    {
        std::cout<<"Failed to Initialize Window\n";
        glfwTerminate();
    }

    glfwMakeContextCurrent(window);
    glfwSwapInterval(1);
    ImGui_ImplGlfw_InitForOpenGL(window, true);
    ImGui_ImplOpenGL3_Init(glsl_version);
    Initialize_GLEW();

    glClearColor(0.07f, 0.13f, 0.17f, 1.0f);

    while (!glfwWindowShouldClose(window))
    {
        glClear(GL_COLOR_BUFFER_BIT);
        ImGui_ImplOpenGL3_NewFrame();
        ImGui_ImplGlfw_NewFrame();
        ImGui::NewFrame();
        display();
        ImGui::Render();
        ImGui_ImplOpenGL3_RenderDrawData(ImGui::GetDrawData());
        glfwPollEvents();
        glfwSwapBuffers(window);
    }

    Cleanup_ImGui();
    Cleanup_OpenGL();

    return 0;
}

void Initialize_GLFW()
{
    if (!glfwInit())
    {
        std::cout << "Failed to Initialize GLFW\n";
        exit(EXIT_FAILURE);
    }

    glfwWindowHint(GLFW_CONTEXT_VERSION_MAJOR, 4);
    glfwWindowHint(GLFW_CONTEXT_VERSION_MINOR, 2);
    glfwWindowHint(GLFW_OPENGL_PROFILE, GLFW_OPENGL_CORE_PROFILE); // 3.2+ only
}

void Initialize_GLEW()
{
    /*Initialize GLEW Library*/
    if (glewInit() != GLEW_OK)
    {
        glfwTerminate();
        exit(EXIT_FAILURE);
    }
}

void Initialize_ImGui()
{
    /*ImGUI Initialization*/
    IMGUI_CHECKVERSION();
    ImGui::CreateContext();
    ImPlot::CreateContext();
    ImGuiIO &io = ImGui::GetIO();
    (void)io;
    ImGui::StyleColorsDark();
}
```

```

void Cleanup_OpenGL()
{
    glfwTerminate();
}

void Cleanup_ImGui()
{
    // Cleanup
    ImGui_ImplOpenGL3_Shutdown();
    ImGui_ImplGlfw_Shutdown();
    ImPlot::DestroyContext();
    ImGui::DestroyContext();
}

void display()
{
    static int precision = 3;
    static int number = 5;
    static std::string ids[20];
    static std::string expression;
    static std::string info;
    static std::array<float, 2> x_axis;
    static std::array<float, 2> y_axis;
    static std::vector<float> points_x_axis;
    static std::vector<float> points_y_axis;
    static std::vector<std::pair<float, float>> coord_pairs =
        {{2.0f,5.0f},{4.0f,4.0f},{6.0f,8.0f},{8.0f,6.0f},{10.0f,12.0f}};
    static lsq solver(coord_pairs);
    static bool setup = true; // used only once

    if(setup)
    {
        for (int i = 0; i < 20; i++)
        {
            ids[i] = 'P';
            ids[i] += std::to_string(i + 1);
            ids[i] += "(x,y)";
        }
        solver.get_fuction_expr(expression, precision);
        solver.generate_graph(x_axis, y_axis, points_x_axis, points_y_axis, coord_pairs);
        solver.get_info(info, precision);
        setup = false;
    }

    ImGui::Begin("Opcoes", NULL, ImGuiWindowFlags_NoCollapse);
    ImGui::SliderInt("Precisao", &precision, 2, 10);
    ImGui::SliderInt("Numero de Pares", &number, 3, 20);

    while (coord_pairs.size() < (std::size_t) number)
        coord_pairs.push_back(std::make_pair(1.0f, 1.0f));
    if ((std::size_t) number < coord_pairs.size())
        coord_pairs.resize(number);

    ImGui::Spacing();
    if(ImGui::TreeNode("Coordenadas"))
    {
        for (int i = 0; i < number; i++)
            ImGui::InputFloat2(ids[i].data(), (float*) &coord_pairs[i]);
        ImGui::TreePop();
```

```cpp
    }

    if(ImGui::Button("Atualizar Dados"))
    {
        solver.associate_data(coord_pairs);
        solver.generate_graph(x_axis, y_axis, points_x_axis, points_y_axis, coord_pairs);
        solver.get_fuction_expr(expression, precision);
        solver.get_info(info, precision);
    }

    if(ImPlot::BeginPlot("Grafico"))
    {
        ImPlot::PlotLine(expression.data(), x_axis.data(), y_axis.data(),x_axis.size());
        ImPlot::PlotScatter("Pares de Pontos", points_x_axis.data(),points_y_axis.data(),
                            points_x_axis.size());
        ImPlot::EndPlot();
    }

    ImGui::InputTextMultiline("Info", &info, ImVec2(0,0), ImGuiInputTextFlags_ReadOnly);

    ImGui::End();
}
```

Listing 3: lsq.h

```cpp
#include <array>
#include <cstddef>
#include <string>
#include <utility>
#include <vector>

using pair_iter = std::vector<std::pair<float, float>>::iterator;

struct lsq
{
    /* data */
        std::array<float, 2> line;
        float pearson;
        float pearson_sqr;
        float sum_x;
        float sum_y;
        float sum_xx;
        float sum_xy;
        std::size_t n_pairs;
    /* methods */
        lsq();
        lsq(std::vector<std::pair<float, float>> &point_data);
        void generate_graph(std::array<float,2> &x_axis, std::array<float,2> &y_axis,
                        std::vector<float> &x_axis_points, std::vector<float> &y_axis_points,
                        std::vector<std::pair<float, float>> &point_data);
        void associate_data(std::vector<std::pair<float, float>> &point_data);
        void get_fuction_expr(std::string& expression, int precision);
        void get_info(std::string& lsq_info, int precision);

        private:
        pair_iter find_max_x(const pair_iter& begin, const pair_iter&end);
        pair_iter find_min_x(const pair_iter& begin, const pair_iter&end);
        pair_iter find_max_y(const pair_iter& begin, const pair_iter&end);
```

```
	pair_iter find_min_y(const pair_iter& begin, const pair_iter&end);
};
```

Listing 4: lsq.cpp

```
#include "lsq.h"
#include <ios>
#include <sstream>
#include <string>
#include <type_traits>
#include <utility>
#include <cmath>
#include <iomanip>


lsq::lsq()
{
    /* f(x) = ax + b, na qual a: line[1] e b: line[0]*/
    line[0] = 1.0f;
    line[1] = 0.0f;
    pearson = pearson_sqr = sum_x = sum_y
    = sum_xx = sum_xy = n_pairs = 0.0f;
}

lsq::lsq(std::vector<std::pair<float, float>> &point_data)
{
    associate_data(point_data);
}

void lsq::associate_data(std::vector<std::pair<float, float>> &point_data)
{
    #define x first
    #define y second

    float mean_x{}, mean_y{}, desv_x, desv_y, sum_yy{};
    sum_x = sum_y = sum_xx = sum_xy = 0.0f;

    for (auto elem : point_data)
    {
        sum_x += elem.x;
        sum_y += elem.y;
        sum_xy += elem.x * elem.y;
        sum_xx += elem.x * elem.x;
        sum_yy += elem.y * elem.y;
    }

    n_pairs = point_data.size();
    mean_x = sum_x / static_cast<float>(n_pairs);
    mean_y = sum_y / static_cast<float>(n_pairs);
    pearson = sum_xy / static_cast<float>(n_pairs);
    pearson = pearson - mean_x * mean_y; // Cov(mean_x, mean_y)
    desv_x = sum_xx / static_cast<float>(n_pairs) - mean_x * mean_x;
    desv_y = sum_yy / static_cast<float>(n_pairs) - mean_y * mean_y;
    pearson /= std::sqrt(desv_x * desv_y);
    pearson_sqr = pearson * pearson;
    float denum = static_cast<float>(n_pairs) * sum_xx - sum_x * sum_x;

    line[0] = (static_cast<float>(n_pairs) * sum_xy - sum_x * sum_y) / denum;
    line[1] = (sum_y * sum_xx - sum_x * sum_xy) / denum;
}

void lsq::get_fuction_expr(std::string& expression, int precision)
{
    std::stringstream stream;
    stream << std::fixed << std::setprecision(precision) << line[0];
    expression.clear();
    expression = stream.str();
    expression += " * x + ";
    stream.str("");
    stream << line[1];
    expression += stream.str();
}

void lsq::get_info(std::string &lsq_info, int precision)
{
    std::stringstream stream;
    lsq_info.clear();
    stream << std::fixed << std::setprecision(precision) << pearson;
    lsq_info = "Coeficiente de correlacao de Pearson = ";
    lsq_info += stream.str();
    stream.str("");
    stream << pearson_sqr;
    lsq_info += "\nCoeficiente de determinacao de Pearson = ";
    lsq_info += stream.str();
    stream.str("");
    stream << sum_x;
    lsq_info += "\nsum_x = ";
    lsq_info += stream.str();
    stream.str("");
    stream << sum_y;
    lsq_info += "\nsum_y = ";
    lsq_info += stream.str();
    stream.str("");
    stream << sum_xy;
    lsq_info += "\nsum_xy = ";
    lsq_info += stream.str();
    stream.str("");
    stream << sum_xx;
    lsq_info += "\nsum_xx = ";
    lsq_info += stream.str();
    lsq_info += "\nNumero de Pares = ";
    lsq_info += std::to_string(n_pairs);
}

void lsq::generate_graph(std::array<float,2> &x_axis, std::array<float,2> &y_axis,
                         std::vector<float> &x_axis_points, std::vector<float> &y_axis_points,
                         std::vector<std::pair<float, float>> &point_data)
{
    float max_x = (*find_max_x(point_data.begin(), point_data.end())).first;
    float min_x = (*find_min_x(point_data.begin(), point_data.end())).first;

    x_axis[0] = min_x - 10.0f;
    y_axis[0] = line[0] * x_axis[0] + line[1];
    x_axis[1] = max_x + 10.0f;
    y_axis[1] = line[0] * x_axis[1] + line[1];

    x_axis_points.clear();
    y_axis_points.clear();
    for (auto elem : point_data)
    {
        x_axis_points.push_back(elem.first);
        y_axis_points.push_back(elem.second);
    }
}

pair_iter lsq::find_max_x(const pair_iter& begin, const pair_iter&end)
{
    pair_iter found = begin;
    pair_iter b_iter = begin;
    pair_iter e_iter = end;
    while (b_iter != e_iter)
    {
        if((*found).first < (*b_iter).first)
            found = b_iter;
        b_iter++;
    }
    return found;
}

pair_iter lsq::find_min_x(const pair_iter& begin, const pair_iter&end)
{
    pair_iter found = begin;
    pair_iter b_iter = begin;
    pair_iter e_iter = end;
    while (b_iter != e_iter)
    {
        if((*found).first > (*b_iter).first)
            found = b_iter;
        b_iter++;
    }
    return found;
}

pair_iter lsq::find_max_y(const pair_iter& begin, const pair_iter&end)
{
    pair_iter found = begin;
    pair_iter b_iter = begin;
    pair_iter e_iter = end;
    while (b_iter != e_iter)
    {
        if((*found).second < (*b_iter).second)
            found = b_iter;
        b_iter++;
    }
    return found;
}

pair_iter lsq::find_min_y(const pair_iter& begin, const pair_iter&end)
{
    pair_iter found = begin;
    pair_iter b_iter = begin;
    pair_iter e_iter = end;
    while (b_iter != e_iter)
    {
        if((*found).second > (*b_iter).second)
            found = b_iter;
        b_iter++;
    }
    return found;
}
```

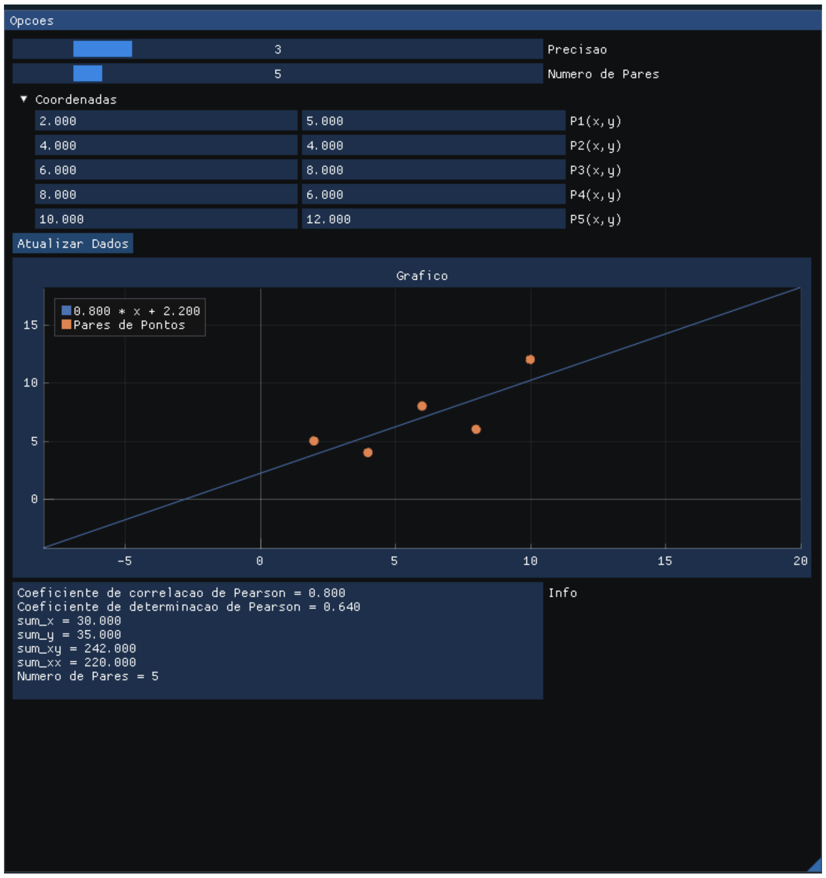
4 Exemplo

A seguir demonstramos um exemplo extraído do 2 Volume do livro do Um Curso de Cálculo, capítulo 17 página 373, do autor Guidorizzi.

Table 1: Entrada

x	2.0	4.0	6.0	8.0	10.0
y	5.0	4.0	8.0	6.0	12.0

Exemplo de execução do programa



```
Opcoes
3          Precisao
5          Numero de Pares
▼ Coordenadas
2.000      5.000      P1(x,y)
4.000      4.000      P2(x,y)
6.000      8.000      P3(x,y)
8.000      6.000      P4(x,y)
10.000     12.000     P5(x,y)
Atualizar Dados
```

```
Coeficiente de correlacao de Pearson = 0.800
Coeficiente de determinacao de Pearson = 0.640
sum_x = 30.000
sum_y = 35.000
sum_xy = 242.000
sum_xx = 220.000
Numero de Pares = 5
```
Info