Vietnamese-German University

Investments

Finance 2 - BFIN

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Lecture 10

Outline

- Derivatives II
  - Introduction to options
  - Option Payoffs
  - Put-Call Parity
  - Factors Affecting Option Prices

Introduction to Options

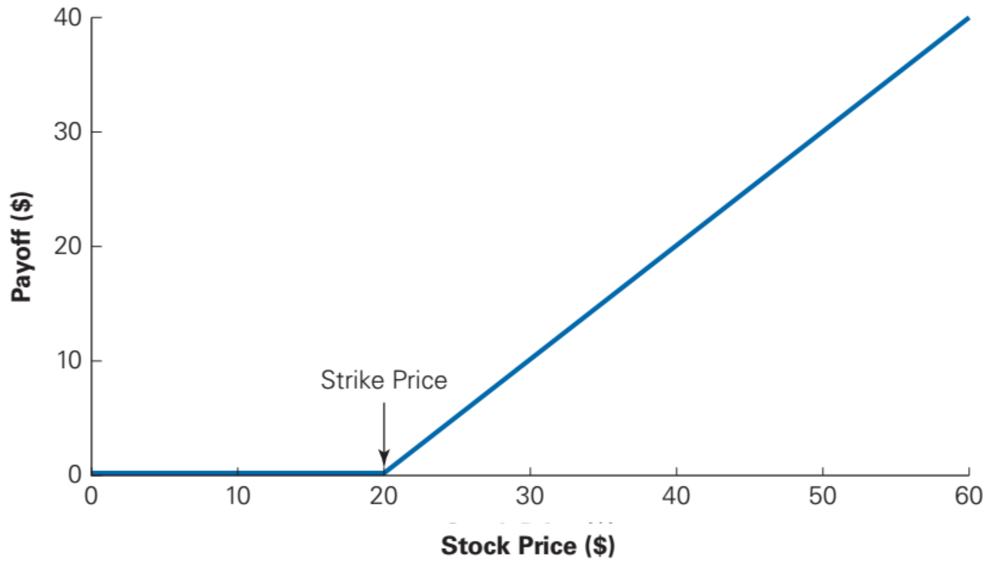
- Options are an example of a broader class of assets called contingent claims.
  - A contingent claim is any asset whose future payoff is contingent on the outcome of some uncertain event.
- A financial option contract gives its owner the right (but not the obligation) to purchase or sell an asset at a fixed price at some future date.
  - Call options
  - Put options
  - Option buyer (holder), Option seller (writer)
  - Long position (right), short position (obligation)
  - Option premium: The market price of the option.
    - Compensates the seller for the risk of loss in the event that the option holder chooses to exercise the option.

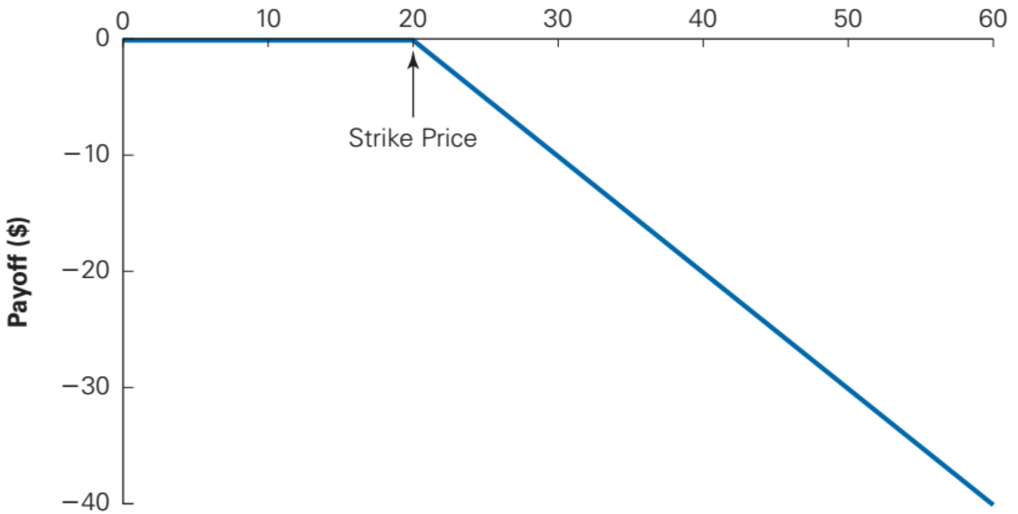
Introduction to Options

- The price at which the holder buys or sells the underlying asset when the option is exercised is called the strike price or exercise price.
  - <u>At-the-money:</u> When the exercise price of an option is equal to the current price of the underlying asset.
  - <u>In-the-money:</u> If the payoff from exercising an option immediately is positive.
  - <u>Out-of-the-money:</u> If the payoff from exercising the option immediately is negative.
- American options allow their holders to exercise the option on any date up to and including the expiration date.
- European options allow their holders to exercise the option only on the expiration date.

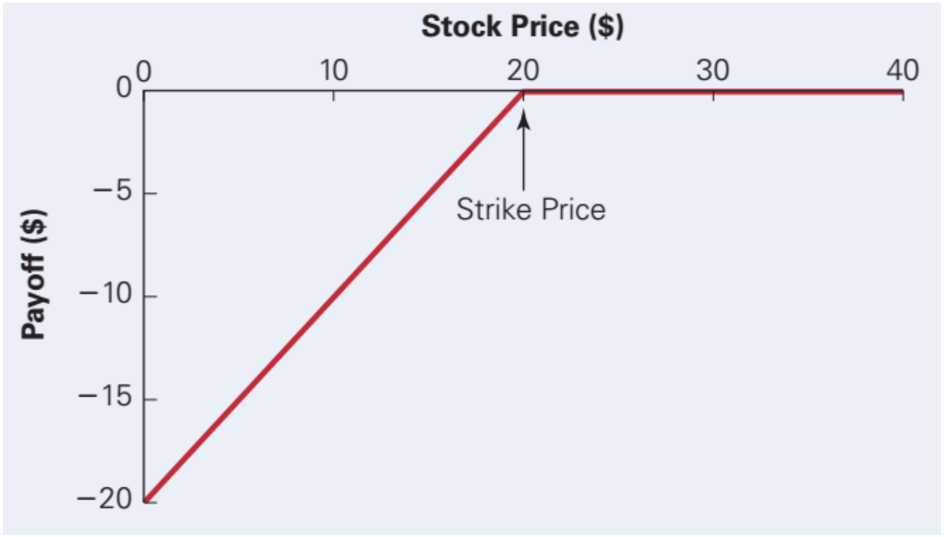
Option Payoffs

Analytical payoffs of a call: $max(S_T - K, 0)$





Analytical payoffs of a putt: $max(K - S_T, 0)$



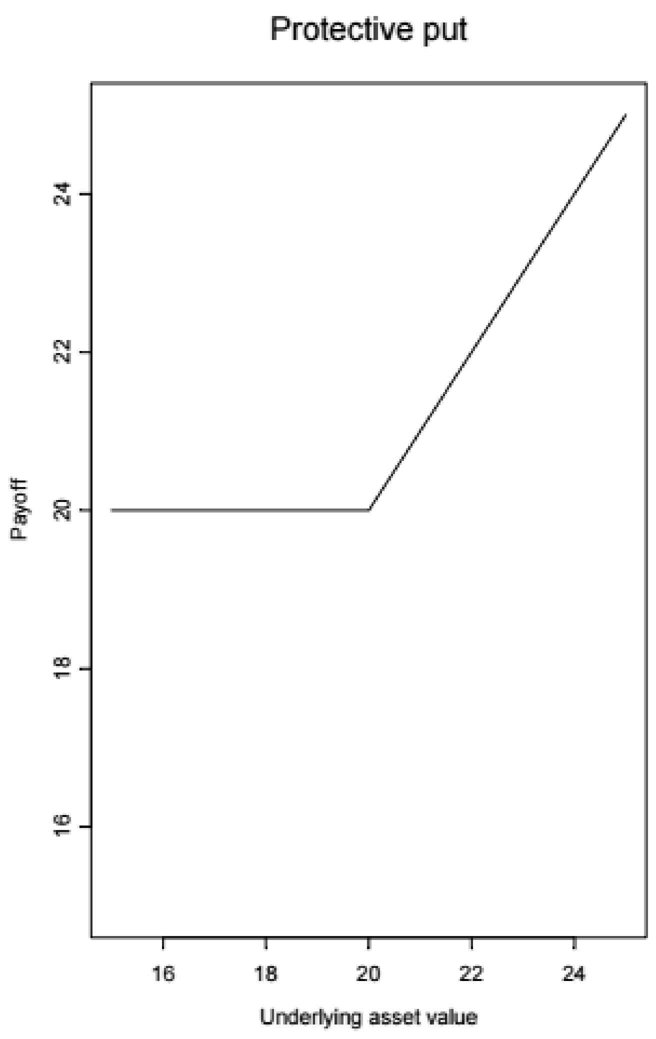
Option Strategies

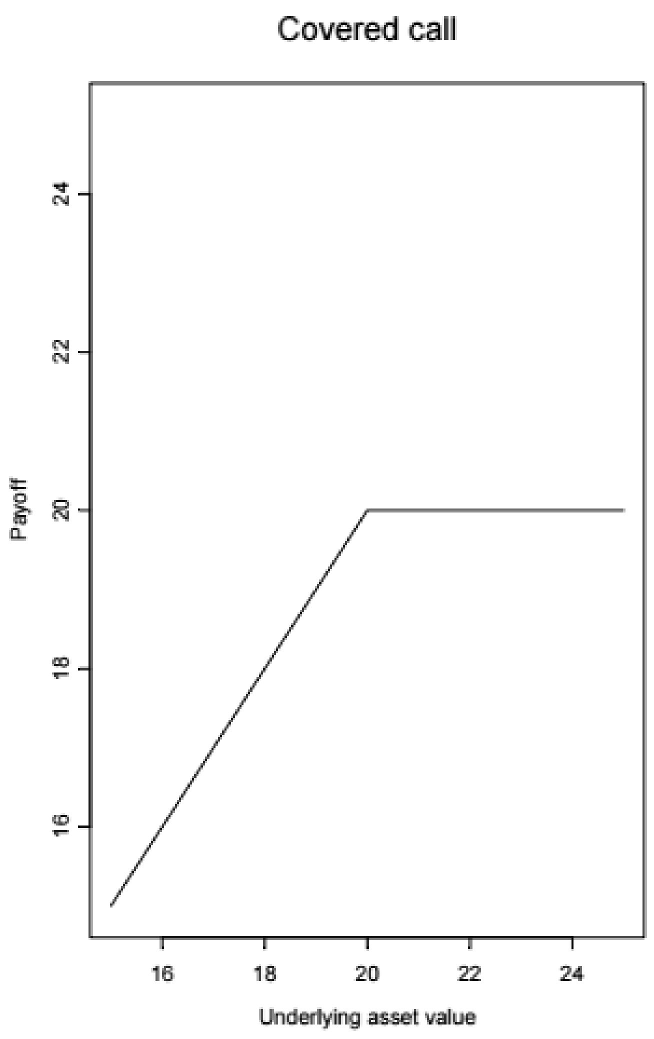
- Protective put positions: long asset and a long put.

	$S_T \leq K$	$S_T > K$
Stock	S_T	S_T
+Put	$K-S_T$	0
Total	K	S_T

- Covered call positions: long asset and a short call.

	$S_T \leq K$	$S_T > K$
Stock	S_T	S_T
-Call	0	$-(S_T-K)$
Total	S_T	K





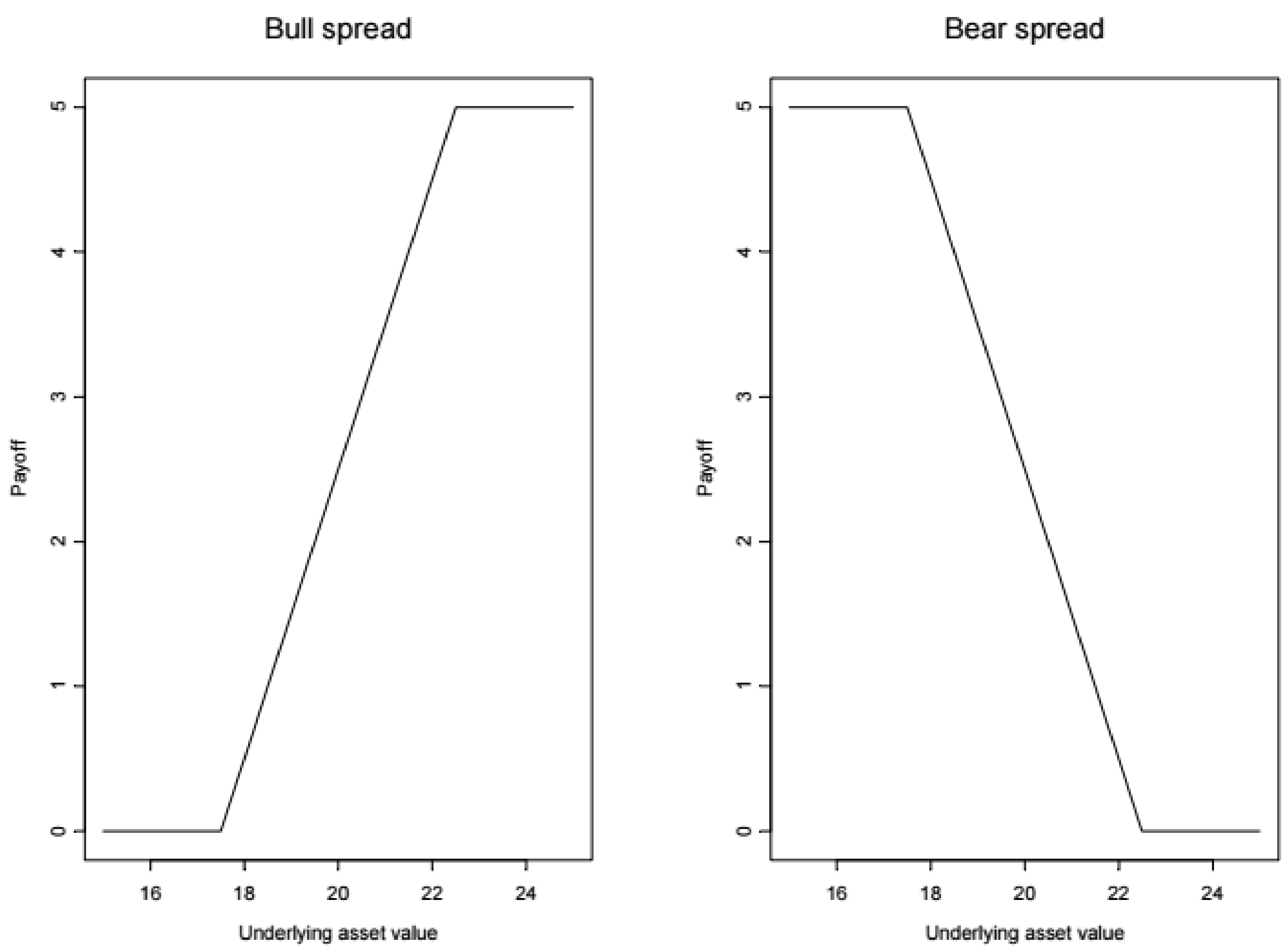
Option Strategies

- Bull spreads: long call K_1 and short call K_2 with $K_1 < K_2$.

	$S_T \le K_1$	$K_1 < S_T \le K_2$	$S_T \ge K_2$
+Call@K_1	0	$S_T - K_1$	$S_T - K_1$
-Call@K_2	0	0	$-(S_T - K_2)$
Total	0	$S_T - K_1$	$K_2 - K_1$

- Bear spreads: long put K_1 and short put K_2 with $K_1 > K_2$.

	$S_T \le K_2$	$K_2 < S_T \le K_1$	$S_T \ge K_1$
+Put@K_1	$K_1 - S_T$	$K_1 - S_T$	0
-Put@K_2	$-(K_2 - S_T)$	0	0
Total	$K_1 - K_2$	$K_1 - S_T$	0



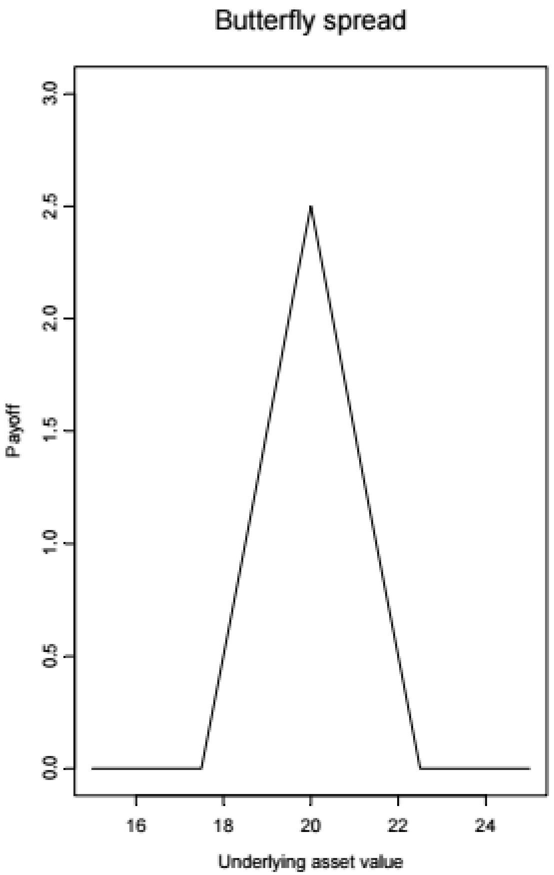
Option Strategies

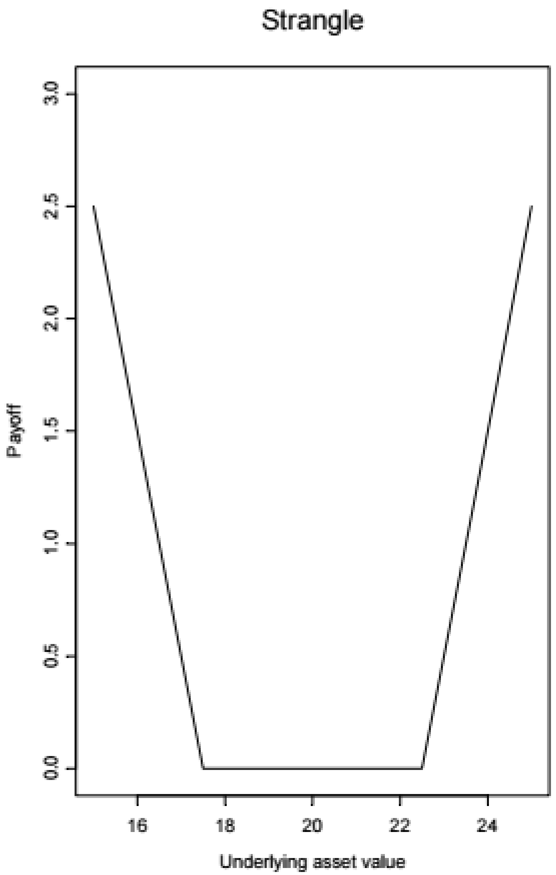
❖ Butterfly spreads: long call K_1, long call K_3 and short 2 calls with K_2 where $K_1 < K_2 < K_3$

$K_2 = 0.5(K_1 + K_3)$	$S_T \le K_1$	$K_1 < S_T \le K_2$	$K_2 < S_T \le K_3$	$S_T \ge K_3$
+Call@K_1	0	$S_T - K_1$	$S_T - K_1$	$S_T - K_1$
+Call@K_3	0	0	0	$S_T - K_3$
-2Call@K_2	0	0	$-2(S_T - K_2)$	$-2(S_T - K_2)$
Total	0	$S_T - K_1$	$K_3 - S_T$	0

❖ Strangles: buy call K_1 and buy put K_2, where $K_2 \le K_1$ (if K_1 = K2 it is called straddle).

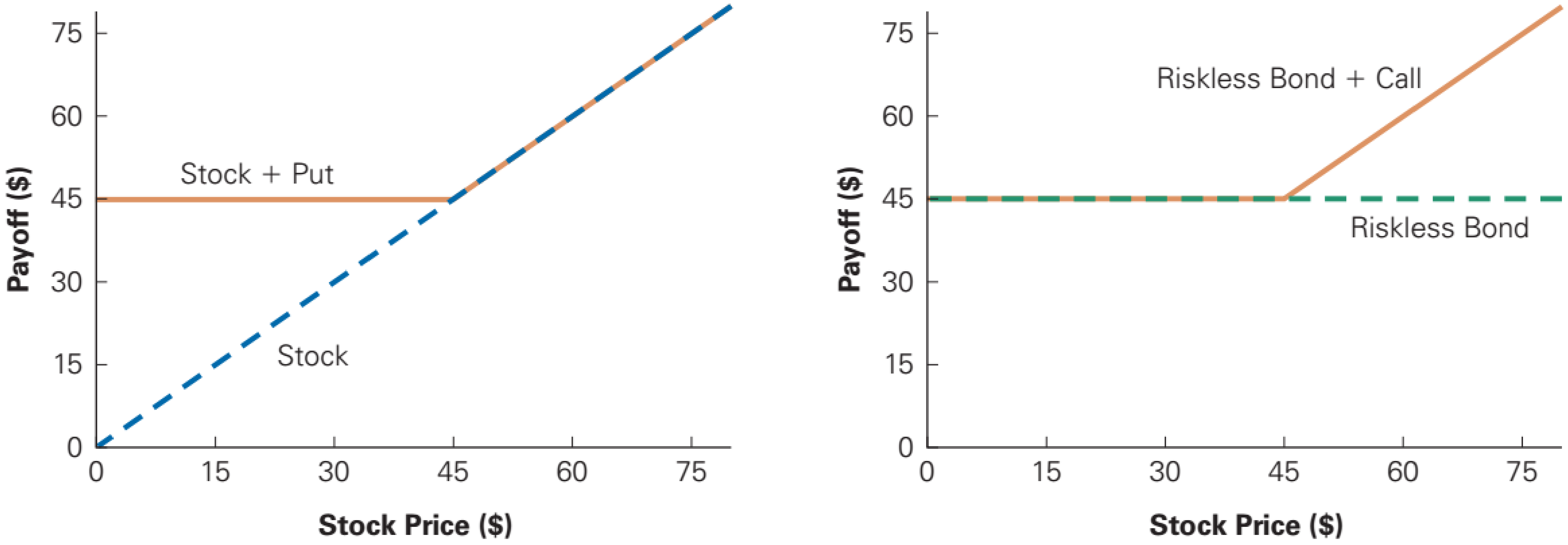
	$S_T \le K_2$	$K_2 < S_T \le K_1$	$S_T \ge K_1$
+Call@K_1	0	0	$S_T - K_1$
+Put@K_2	$K_2 - S_T$	0	0
Total	$K_2 - S_T$	0	$S_T - K_1$





Portfolio Insurance

- Insuring against a loss without relinquishing the upside.
  - Protective put
  - Purchasing a bond and a call option



Put-Call Parity

- Purchasing the underlying asset and a put provides exactly the same payoff from purchasing a bond and a call. Then, by the Law of One Price they must have the same price.

$$S + P = PV(K) + C \quad \Rightarrow \quad \textit{Put-call parity} \quad \boldsymbol{C + \frac{K}{(1+r_f)^T} = S_0 + P}$$

$$C = S_0 + P - \frac{K}{(1+r_f)^T}$$

Calls are equivalent to long positions in puts, buying the underlying asset and (partially) financing the purchases by borrowing the PV of the strike price of the options.

$$P = C - S_0 + \frac{K}{(1+r_f)^T}$$

Puts are equivalent to long positions in calls, shorting the underlying asset and using the proceeds to lend the PV of the strike price of the options.

Put-Call Parity

- It is possible to buy three-month call options and three-month puts on stock Q. Both options have an exercise price of \$60 and both are worth \$10. If the interest rate is 5% a year, what is the stock price?

$$C = S_0 + P - \frac{K}{\left(1 + r_f\right)^T} \qquad C = P, \quad K = 60 \qquad r_f = 5\%, \quad T = 0.25 \qquad S_0 = \frac{K}{\left(1 + r_f\right)^T} = \frac{60}{(1 + 0.05)^{0.25}} = \$59.27$$

- A stock is currently selling for \$47 per share. A call option with an exercise price of \$50 sells for \$3.80 and expires in three months. If the risk-free rate of interest is 2.6 percent per year, compounded continuously, what is the price of a put option with the same exercise price?

$$C = S_0 + P - Ke^{-rT} \qquad 3.80 = 47 + P - 50e^{-0.026*0.25} \qquad P = \$6.48$$

- A put option that expires in six months with an exercise price of \$75 sells for \$4.89. The stock is currently priced at \$72, and the risk-free rate is 3.6 percent per year, compounded continuously. What is the price of a call option with the same exercise price?

$$C = 72 + 4.89 - 75e^{-0.036*0.5} = \$3.23$$

Factors Affecting Option Prices

- Strike Price and Stock Price
  - The value of an otherwise identical call option is higher if the strike price the holder must pay to buy the stock is lower. $max(S_T - K, 0)$
  - Puts with a lower strike price are less valuable. $max(K - S_T, 0)$
- Arbitrage Bounds on Option Prices
  - A put option cannot be worth more than its strike price.
  - A call option cannot be worth more than the stock itself.
  - An American option cannot be worth less than its European counterpart.
  - An American option cannot be worth less than its intrinsic value.
    - ✓ The intrinsic value of an option is the value it would have if it expired immediately.
  - An American option cannot have a negative time value.
    - ✓ The time value of an option is the difference between the current option price and its intrinsic value.

Factors Affecting Option Prices

- Option Prices and the Exercise Date
  - An American option with a later exercise date cannot be worth less than an otherwise identical American option with an earlier exercise date.
  - A European option with a later exercise date may potentially trade for less than an otherwise identical option with an earlier exercise date.
- Option Prices and Volatility
  - The value of an option generally increases with the volatility of the stock.
    - ✓ An increase in volatility increases the likelihood of very high and very low returns for the stock.
      - The holder of a call option benefits from a higher payoff when the stock goes up and the option is in-the-money, but earns the same (zero) payoff no matter how far the stock drops once the option is out-of-the-money.
      - Insurance is more valuable when there is higher volatility—hence put options on more volatile stocks are also worth more.

Exercising Options Early

❖ Non-Dividend-Paying Stocks

$$C = P + S - PV(K)$$

If $PV(K) = K - dis(K)$

where *dis(K)* is the amount of the discount from face value to account for interest.

$$C = \underbrace{S - K}_{Intrinsic\,Value} + \underbrace{dis(K) + P}_{Time\,Value}$$

- The price of any call option on a non-dividend-paying stock always exceeds its intrinsic value.
  - ✓ It is never optimal to exercise a call option on a non-dividend paying stock early.
  - ✓ Always better off just selling the option.
  - ✓ An American call on a non-dividend-paying stock has the same price as its European counterpart.
  - ✓ Under certain circumstances, exercising an American put option on a non-dividend-paying stock might make sense.

$$P = \underbrace{K - S}_{Intrinsic\,Value} + \underbrace{C - dis(K)}_{Time\,Value}$$

- ✓ The European put may sell for less than its intrinsic value.
- ✓ American put cannot sell for less than its intrinsic value.
- ✓ The American option can be worth more than an otherwise identical European option.

Exercising Options Early

❖ Dividend-Paying Stocks

$$C = P + S - PV(Div) - PV(K) \qquad \text{If} \quad PV(K) = K - dis(K)$$

where *dis(K)* is the amount of the discount from face value to account for interest.

$$C = \underbrace{S - K}_{Intrinsic\,Value} + \underbrace{dis(K) + P - PV(Div)}_{Time\,Value}$$

- If PV(Div) is large enough, the time value of a European call option can be negative.
- The price of the American option can exceed the price of a European option.
  - ✓ When a company pays a dividend, investors expect the price of the stock to drop to reflect the cash paid out.
  - ✓ This price drop hurts the owner of a call option because the stock price falls.
  - ✓ By exercising early, the owner of the call option can capture the value of the dividend.

$$P = \underbrace{K - S}_{Intrinsic\,Value} + \underbrace{C - dis(K) + PV(Div)}_{Time\,Value}$$

- ✓ Dividends reduce the likelihood of early exercise of a put.
- ✓ The likelihood of early exercise increases whenever the stock goes ex-dividend.

Exercising Options Early

- The stock of Harford Inc. is about to pay a \$0.30 dividend. It will pay no more dividends for the next month. Consider call options that expire in one month. If the interest rate is 6% APR (monthly compounding), what is the maximum strike price where it could be possible that early exercise of the call option is optimal?

$$C = \underbrace{S - K}_{Intrinsic\,Value} + \underbrace{dis(K) + P - PV(Div)}_{Time\,Value}$$

$$dis(K) - PV(Div) < dis(K) + P - PV(Div) < 0$$

$$K - K(1 + 0.005)^{-1} < 0.30 \quad \rightarrow \quad K < 60.30$$

- Suppose the S&P 500 is at 2700, and it will pay a dividend of \$90 at the end of the year. Suppose also that the interest rate is 2%. If a one-year European put option has a negative time value, what is the lowest possible strike price it could have?

$$P = \underbrace{K - S}_{Intrinsic\,Value} + \underbrace{C - dis(K) + PV(Div)}_{Time\,Value}$$

$$PV(Div) - dis(K) < C - dis(K) + PV(Div) < 0$$

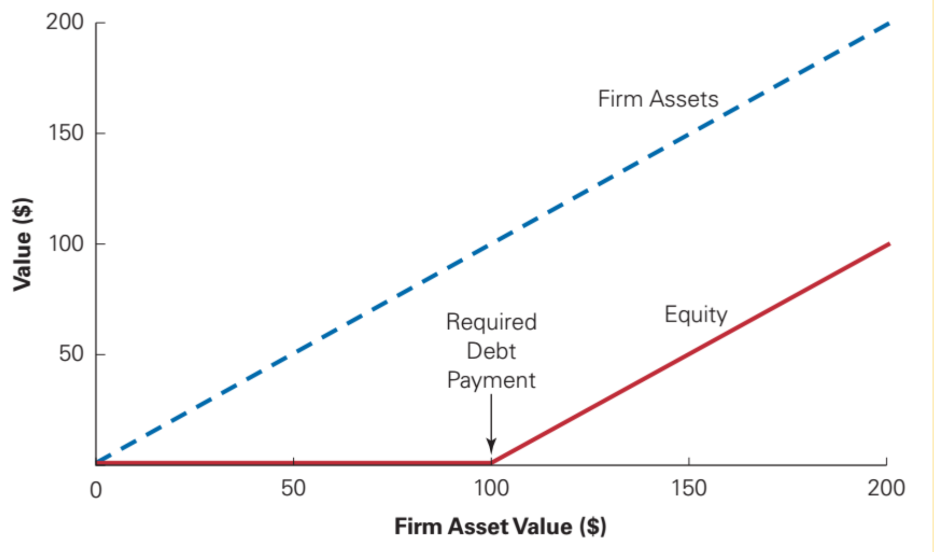
$$\frac{90}{(1.02)} < K - K(1 + 0.02)^{-1} \quad \rightarrow \quad K > 4{,}500$$

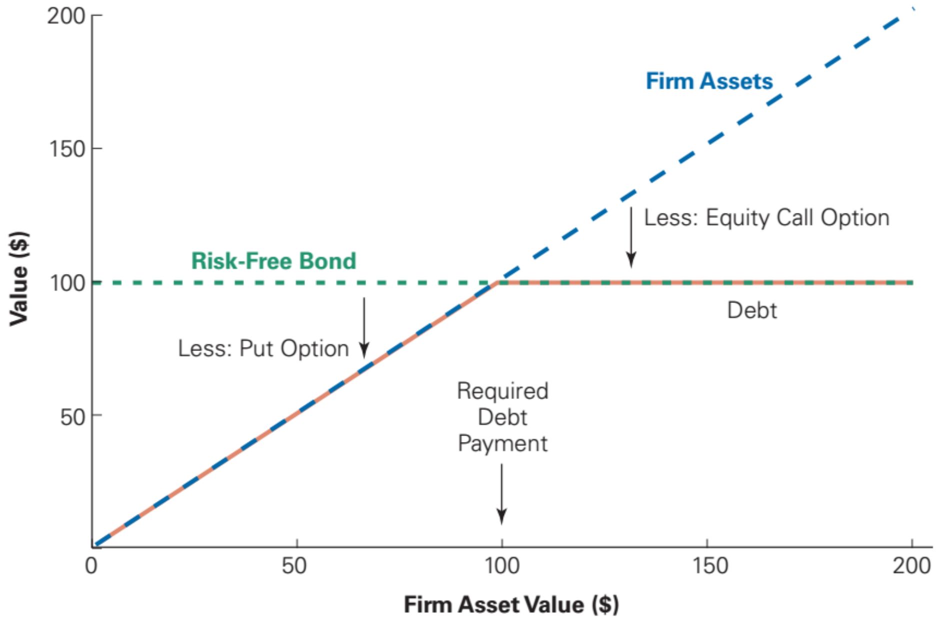
Options and Corporate Finance

- Equity as a Call Option
  - Residual claim
    - ✓ Value of the firm's assets exceeds the required debt payment.
    - ✓ Value of the firm's assets is less than the required debt payment.
- Debt as an Option Portfolio
  - The payoff to debt can be viewed as,
    - ✓ The firm's assets, less the equity call option.
    - ✓ A risk-free bond, less a put option on the assets with a strike price equal to the required debt payment.

$$Risky\ debt = Risk{-}free\ debt - Put\ option\ on\ firm\ assets$$

$$Risk{-}free\ debt = Risky\ debt + Put\ option\ on\ firm\ assets$$





Pricing Risky Debt

- As of September 2012, Google (GOOG) had no debt. Suppose the firm's managers consider recapitalizing the firm by issuing zero-coupon debt with a face value of $163.5 billion due in January of 2014, and using the proceeds to pay a special dividend. Suppose too that Google had 327 million shares outstanding, trading at $700.77 per share, implying a market value of $229.2 billion. The risk-free rate is 0.25%. The current value of a call option with a strike price of $500 is $222.05 per share. Estimate the credit spread Google would have to pay on the debt assuming perfect capital markets.
  - Assuming perfect capital markets, the total value of Google's equity and debt should remain unchanged after the recapitalization.
  - The $163.5 billion face value of the debt is equivalent to a claim of $163.5 billion/(327 million shares) = $500 per share on Google's current assets.
  - Total value of the equity after recap
    - ✓ P(Call option with K=500) * Shares Outstanding
    - ✓ $222.05 * 327 million shares = $72.6 billion
  - Value of the new debt=Total Value-Equity Value
    - ✓ 229.2 - 72.6 = $156.6

$$PV(Debt) = \frac{FV(Debt)}{(1+r)^t} \qquad 156.6 = \frac{163.5}{(1+r)^{16/12}} \qquad r = \left(\frac{163.5}{156.6}\right)^{12/16} - 1 = 3.29\% \quad Spread = 3.29\% - 0.25\%$$

Exercises

- You own a call option on Intuit stock with a strike price of $40. The option will expire in exactly three months' time.
  - If the stock is trading at $55 in three months, what will be the payoff of the call?
  - If the stock is trading at $35 in three months, what will be the payoff of the call?
  - Payoff diagram?

$$Payoff = \begin{cases} max(S_T - K, 0) & if\ S_T = 55 \\ max(S_T - K, 0) & if\ S_T = 35 \end{cases}$$

$$Payoff = \begin{cases} max(55 - 40, 0) = 15 \\ max(35 - 40, 0) = 0 \end{cases}$$

Exercises

- You wrote a call option on Intuit stock with a strike price of $40. The option will expire in exactly three months' time.
  - If the stock is trading at $55 in three months, what will be the payoff of the call?
  - If the stock is trading at $35 in three months, what will be the payoff of the call?
  - Payoff diagram?

$$Payoff = \begin{cases} -max(S_T - K, 0) & if\ S_T = 55 \\ -max(S_T - K, 0) & if\ S_T = 35 \end{cases}$$

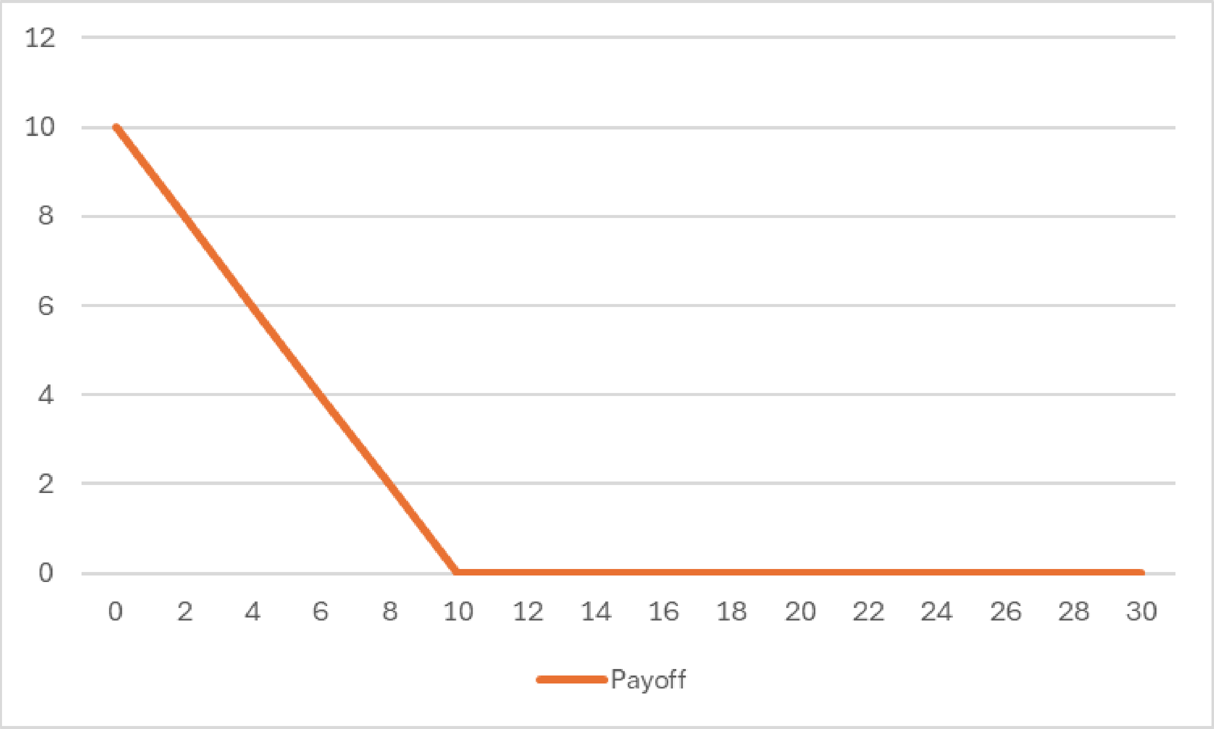
$$Payoff = \begin{cases} -max(55 - 40, 0) = -15 \\ -max(35 - 40, 0) = \quad 0 \end{cases}$$

Exercises

- You own a put option on Ford stock with a strike price of \$10. The option will expire in exactly six months' time.
  - If the stock is trading at \$8 in six months, what will be the payoff of the put?
  - If the stock is trading at \$20 in six months, what will be the payoff of the put?
  - Payoff diagram?

$$Payoff = \begin{cases} max(K - S_T, 0) \ if \ S_T = \ 8 \\ max(K - S_T, 0) \ if \ S_T = 20 \end{cases}$$

$$Payoff = \begin{cases} max(10 - \ 8, 0) = 2 \\ max(10 - 20, 0) = \ 0 \end{cases}$$



Exercises

- You wrote a put option on Ford stock with a strike price of $10. The option will expire in exactly six months' time.
  - If the stock is trading at $8 in six months, what will be the payoff of the put?
  - If the stock is trading at $20 in six months, what will be the payoff of the put?
  - Payoff diagram?

$$Payoff = \begin{cases} -max(K - S_T, 0) \; if \; S_T = 8 \\ -max(K - S_T, 0) \; if \; S_T = 20 \end{cases}$$

$$Payoff = \begin{cases} -max(10 - 8, 0) = -2 \\ -max(10 - 20, 0) = 0 \end{cases}$$

Exercises

❖ What position has more downside exposure: a short position in a call or a short position in a put?

$$-max(S_T - K, 0) \qquad vs. \qquad -max(K - S_T, 0)$$

Exercises

- Dynamic Energy Systems stock is currently trading for \$31 per share. The stock pays no dividends. A one-year European put option on Dynamic with a strike price of \$41 is currently trading for \$10.24. If the risk-free interest rate is 3% per year, what is the price of a one-year European call option on Dynamic with a strike price of \$41?

$$C = S_0 + P - \frac{K}{\left(1 + r_f\right)^T} = 31 + 10.24 - \frac{41}{(1 + 0.03)^1} = \$1.4342$$

- A put option and a call option with an exercise price of \$55 expire in two months and sell for \$2.65 and \$5.32, respectively. If the stock is currently priced at \$57.30, what is the annual continuously compounded rate of interest?

$$C = 5.32 = S_0 + P - Ke^{-rT}$$

$$5.32 = 57.30 + 2.65 - 55e^{-r*(\frac{2}{12})}$$

$$r = 4.05\%$$

Exercises

- The current stock price of Intrawest is \$20 per share and the one-year risk-free interest rate is 8%. A one-year put on Intrawest with a strike price of \$18 sells for \$3.33, while the identical call sells for \$7.
  - Is there an arbitrage opportunity?
  - What must you do to exploit this arbitrage opportunity?

$$C = S_0 + P - \frac{K}{\left(1 + r_f\right)^T} = 20 + 3.33 - \frac{18}{(1 + 0.08)^1} = \$6.67 < \$7$$

  - Calls are equivalent to long positions in puts, buying the underlying asset and (partially) financing the purchases by borrowing the PV of the strike price of the options.
  - So, sell a call @\$7, borrow PV(K=18), buy the stock @\$20, buy a put @\$3.33

	Today	@Expiration if $S_T \leq K$	@Expiration if $S_T > K$
Sell Call	+7	0	$-(S_T - K)$
Borrow PV(K)	+16.67	$-K$	$-K$
Buy Stock @ \$20	-20	S_T	S_T
Buy Put	-3.33	$(K - S_T)$	0
Overall	**0.3367**	**0**	**0**

Exercises

- LNUX's stock is currently trading for \$4.59. There are puts and calls traded on LNUX. In particular, you know that a call option with a strike of \$4.25 which matures one year from today is trading in the market for \$0.85. The risk-free rate is 5% (in annual terms).
  - What should the put trade at if there are no arbitrage opportunities?
  - If the put was trading at \$0.20, how could you construct an arbitrage strategy? Assume you can take long or short positions at the given prices, as well as lend or borrow at the risk-free rate.
  - If the put was trading at \$0.40, how could you construct an arbitrage strategy?

$$P = C - S_0 + \frac{K}{\left(1 + r_f\right)^T} = 0.85 - 4.59 + \frac{4.25}{(1 + 0.05)^1} = \$0.3076$$

If the put was trading at \$0.20, then it is cheap.

	Today	@Expiration if $S_T \leq$ K	@Expiration if $S_T >$ K
Buy Put	-0.20	$(K - S_T)$	0
Borrow PV(K)	+4.0476	$-K$	$-K$
Buy Stock @ \$4.59	-4.59	S_T	S_T
Sell Call	+0.85	0	$-(S_T - K)$
Overall	**0.1076**	**0**	**0**

Exercises

- LNUX's stock is currently trading for $4.59. There are puts and calls traded on LNUX. In particular, you know that a call option with a strike of $4.25 which matures one year from today is trading in the market for $0.85. The risk-free rate is 5% (in annual terms).
  - What should the put trade at if there are no arbitrage opportunities?
  - If the put was trading at $0.20, how could you construct an arbitrage strategy? Assume you can take long or short positions at the given prices, as well as lend or borrow at the risk-free rate.
  - If the put was trading at $0.40, how could you construct an arbitrage strategy?

$$P = C - S_0 + \frac{K}{\left(1 + r_f\right)^T} = 0.85 - 4.59 + \frac{4.25}{(1 + 0.05)^1} = \$0.3076$$

If the put was trading at $0.40, then it is expensive.

	Today	@Expiration if $S_T \leq$ K	@Expiration if $S_T >$ K
Sell Put	+0.40	$-(\mathrm{K} - \mathrm{S_T})$	0
Sell Stock @ $4.59	+4.59	$-\mathrm{S_T}$	$-\mathrm{S_T}$
Lend PV(K)	−4.0476	+K	+K
Buy Call	−0.85	0	$(\mathrm{S_T} - \mathrm{K})$
Overall	**0.0924**	**0**	**0**

Exercises

- AGE Corp shares are currently trading at \$83. Risk-free rate is 6% per year, compounded continuously. Consider options on AGE Corp stock with an exercise price of \$80 and 6 month to maturity. Current price of a put is \$8.13.
  - What is the intrinsic value of the call option? Of the put option?
  - What is the time value of the call option? Of the put option?

$$C = \underbrace{S - K}_{Intrinsic\ Value} + \underbrace{dis(K) + P}_{Time\ Value}$$

$$P = \underbrace{K - S}_{Intrinsic\ Value} + \underbrace{C - dis(K)}_{Time\ Value}$$

$$C_{Intrinsic} = Max[S - K, 0] = 3$$

$$P_{Intrinsic} = Max[K - S, 0] = 0$$

$$C = S_0 + P - Ke^{-rT} = 83 + 8.13 - 80 * e^{-0.06*0.5} = \$13.49$$

$$C_{Time} = dis(K) + P = K - Ke^{-rT} + P = \$10.49$$

$$P_{Time} = P - P_{Intrinsic} = \$8.13$$

- Suppose the S&P 500 is at 889, and a one-year European call option with a strike price of \$429 has a negative time value. If the interest rate is 6%, what can you conclude about the dividend yield of the S&P 500? (Assume all dividends are paid at the end of the year.)

$$C = \underbrace{S - K}_{Intrinsic\ Value} + \underbrace{dis(K) + P - PV(Div)}_{Time\ Value}$$

$$dis(K) - PV(Div) < dis(K) + P - PV(Div) < 0$$

$$429 - 429(1.06)^{-1} < PV(Div) \rightarrow PV(Div) > \$24.28$$

$$Dividend\ Yield > {}^{(24.28 * 1.06)}/_{889} = 2.90\%$$

Exercises

❖ Use the option data from July 13, 2009, in the following table to determine the rate Google would have paid if it had issued $128 billion in zero-coupon debt due in January 2011. Suppose Google currently had 320 million shares outstanding, implying a market value of $135.1 billion. The risk-free rate is 1.00%. (Assume perfect capital markets.)

GOOG **422.27 +7.87**

Jul 13 2009 @ 13:10 ET **Vol** 2177516

Calls	Bid	Ask	Open Int
11 Jan 150.0 (OZF AJ)	273.60	276.90	100
11 Jan 160.0 (OZF AL)	264.50	267.20	82
11 Jan 200.0 (OZF AA)	228.90	231.20	172
11 Jan 250.0 (OZF AU)	186.50	188.80	103
11 Jan 280.0 (OZF AX)	162.80	165.00	98
11 Jan 300.0 (OZF AT)	148.20	150.10	408
11 Jan 320.0 (OZF AD)	133.90	135.90	63
11 Jan 340.0 (OZF AI)	120.50	122.60	99
11 Jan 350.0 (OZF AK)	114.10	116.10	269
11 Jan 360.0 (OZF AM)	107.90	110.00	66
11 Jan 380.0 (OZF AZ)	95.80	98.00	88
11 Jan 400.0 (OZF AU)	85.10	87.00	2577
11 Jan 420.0 (OUP AG)	74.60	76.90	66
11 Jan 450.0 (OUP AV)	61.80	63.30	379

- Assuming perfect capital markets, the total value of Google's equity and debt should remain unchanged after the recapitalization.
- The $128.0 billion face value of the debt is equivalent to a claim of $128.0 billion/(320 million shares) = $400 per share on Google's current assets.
- Total value of the equity after recap
  - ✓ P(Call option with K=400) * Shares Outstanding
  - ✓ $86.05 * 320 million shares = $27.54 billion
- Value of the new debt=Total Value-Equity Value
  - ✓ 135.1 - 27.54 = $107.56

$$PV(Debt) = \frac{FV(Debt)}{(1+r)^t} \rightarrow 107.56 = \frac{128.0}{(1+r)^{18/12}}$$

$$r = (\frac{128}{107.56})^{12/18} - 1 = 12.3\% \qquad Spread = 12.3\% - 1.00\%$$

What is next?

- Derivatives III
  - Binomial Option Pricing Model
  - Black-Scholes Option Pricing Model
  - Reading(s):
    - ✓ **BKM: Ch. 21**
  - Suggested Problems
    - ✓ **Chapter 20:** 6, 7, 8, 10, 20, 30

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Lecture 10