



Vietnamese-German University

Investments

Finance 2 - BFIN

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Lecture 3

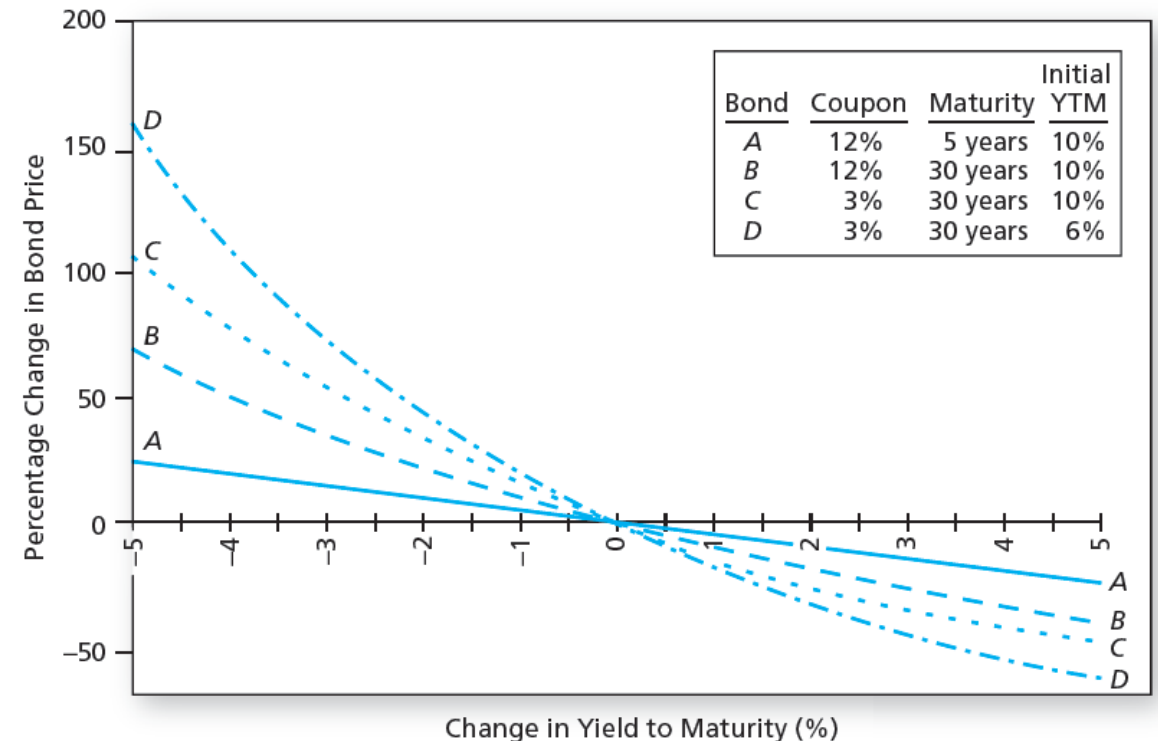
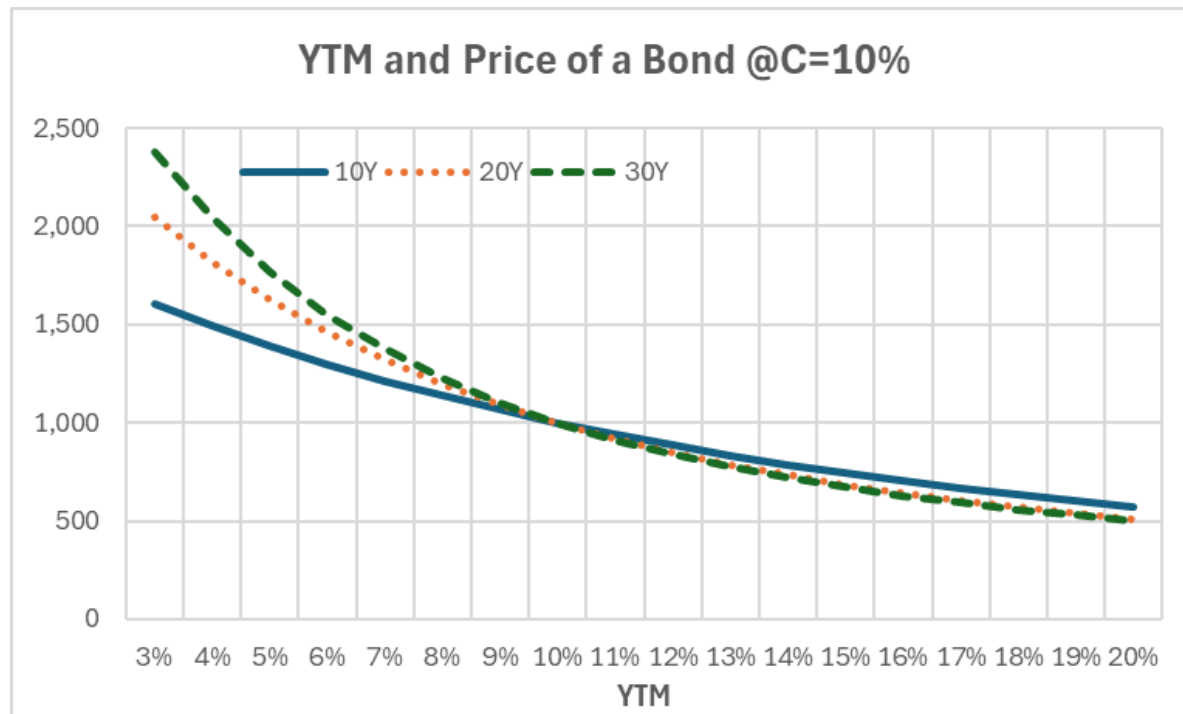
❖ Fixed-Income Securities II

➤ Managing Bond Portfolios

- ✓ Interest rate risk
 - Interest rate sensitivity of bond prices
 - Duration and its determinants
- ✓ Convexity
- ✓ Passive and active management strategies

Interest Rate Risk

- ❖ Bond prices and yields are inversely related
- ❖ An increase in a bond's yield to maturity causes smaller price change than a decrease of equal magnitude
- ❖ Long-term bonds tend to be more price sensitive than short-term bonds



Interest Rate Risk

❖ Prices of 8% Coupon Bond (Coupons Paid Semiannually)

Yield to Maturity (APR)	$T = 1$ Year	$T = 10$ Years	$T = 20$ Years
8%	1,000.00	1,000.00	1,000.00
9%	990.64	934.96	907.99
Fall in price (%)*	0.94%	6.50%	9.20%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

❖ Prices of Zero-Coupon Bond(Semiannual Compounding)

Yield to Maturity (APR)	$T = 1$ Year	$T = 10$ Years	$T = 20$ Years
8%	924.56	456.39	208.29
9%	915.73	414.64	171.93
Fall in price (%)*	0.96%	9.15%	17.46%

*Equals value of bond at a 9% yield to maturity divided by value of bond at (the original) 8% yield, minus 1.

Interest Rate Risk

❖ Duration

- A measure of the effective maturity of a bond
- The weighted average of the times until each payment is received
- The weights are proportional to the present value of the payment
 - ✓ Duration = Maturity for zero coupon bonds
 - ✓ Duration < Maturity for coupon bonds

$$D = \sum_{t=1}^T t \times w_t$$

$$w_t = \frac{CF_t / ((1 + y))^t}{P}$$

CF_t : Cash flow at time t

P : Price of the bond

y : Yield to maturity

❖ Duration is a key concept in fixed-income portfolio management.

- A simple summary statistic of the effective average maturity of the portfolio.
- An essential tool in immunizing portfolios from interest rate risk.
- A measure of the interest rate sensitivity of a portfolio.

Interest Rate Risk

❖ Duration-Price Relationship

- Price change is proportional to duration.
- The percentage change in bond price is the product of modified duration and the change in the bond's yield to maturity.

$$\frac{\Delta P}{P} = -D \times \left[\frac{\Delta(1+y)}{1+y} \right]$$

$$\frac{\Delta P}{P} = -D_M \Delta y$$

$$D_M = \frac{D}{(1+y)}$$

D_M : Modified duration

12% annual coupon, 5y bond

t	PV(CF)	w _t	t*w _t
0.0			
1.0	1,071.43	0.11	0.1071
2.0	956.63	0.10	0.1913
3.0	854.14	0.09	0.2562
4.0	762.62	0.08	0.3050
5.0	6,355.18	0.64	3.1776
Price	10,000.00	1.00	
D			4.0373
D _M			3.6048

t	PV(CF)	w _t	t*w _t
0.0			
0.5	566.04	0.06	0.0283
1.0	534.00	0.05	0.0534
1.5	503.77	0.05	0.0756
2.0	475.26	0.05	0.0951
2.5	448.35	0.04	0.1121
3.0	422.98	0.04	0.1269
3.5	399.03	0.04	0.1397
4.0	376.45	0.04	0.1506
4.5	355.14	0.04	0.1598
5.0	5,918.98	0.59	2.9595
Price	10,000.00	1.00	
D			3.9008
D _M			3.6800

Interest Rate Risk

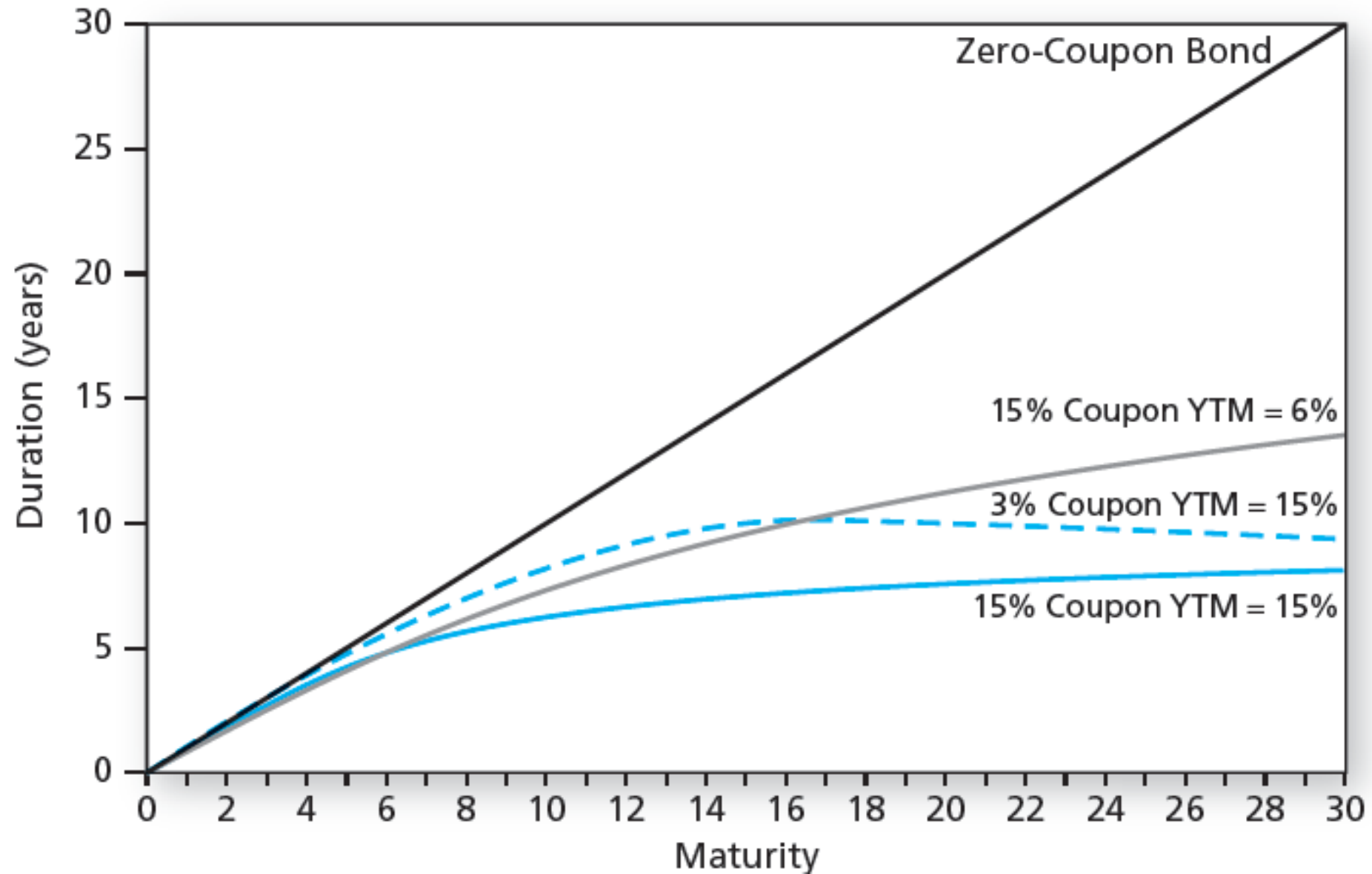
- ❖ Rule 1: The duration of a zero-coupon bond equals its time to maturity.
- ❖ Rule 2: Holding maturity constant, a bond's duration is higher when the coupon rate is lower.
- ❖ Rule 3: Holding the coupon rate constant, a bond's duration generally increases with its time to maturity.
- ❖ Rule 4: Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower.
- ❖ Rule 5: The duration of a level perpetuity is equal to:

$$\frac{1+y}{y}$$

Maturity (years)	Coupon Rates (per Year)				
	2%	4%	6%	8%	10%
1	0.995	0.990	0.985	0.981	0.976
5	4.742	4.533	4.361	4.218	4.095
10	8.762	7.986	7.454	7.067	6.772
20	14.026	11.966	10.922	10.292	9.870
Infinite (perpetuity)	13.000	13.000	13.000	13.000	13.000

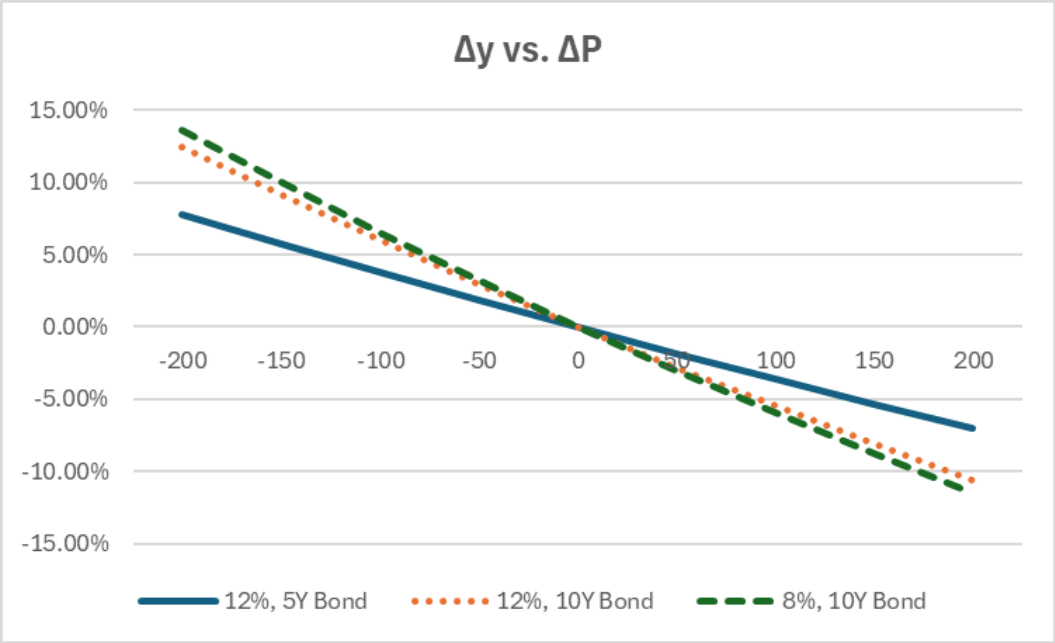
Bond durations
(YTM = 8% BEY; semiannual coupons)

Interest Rate Risk

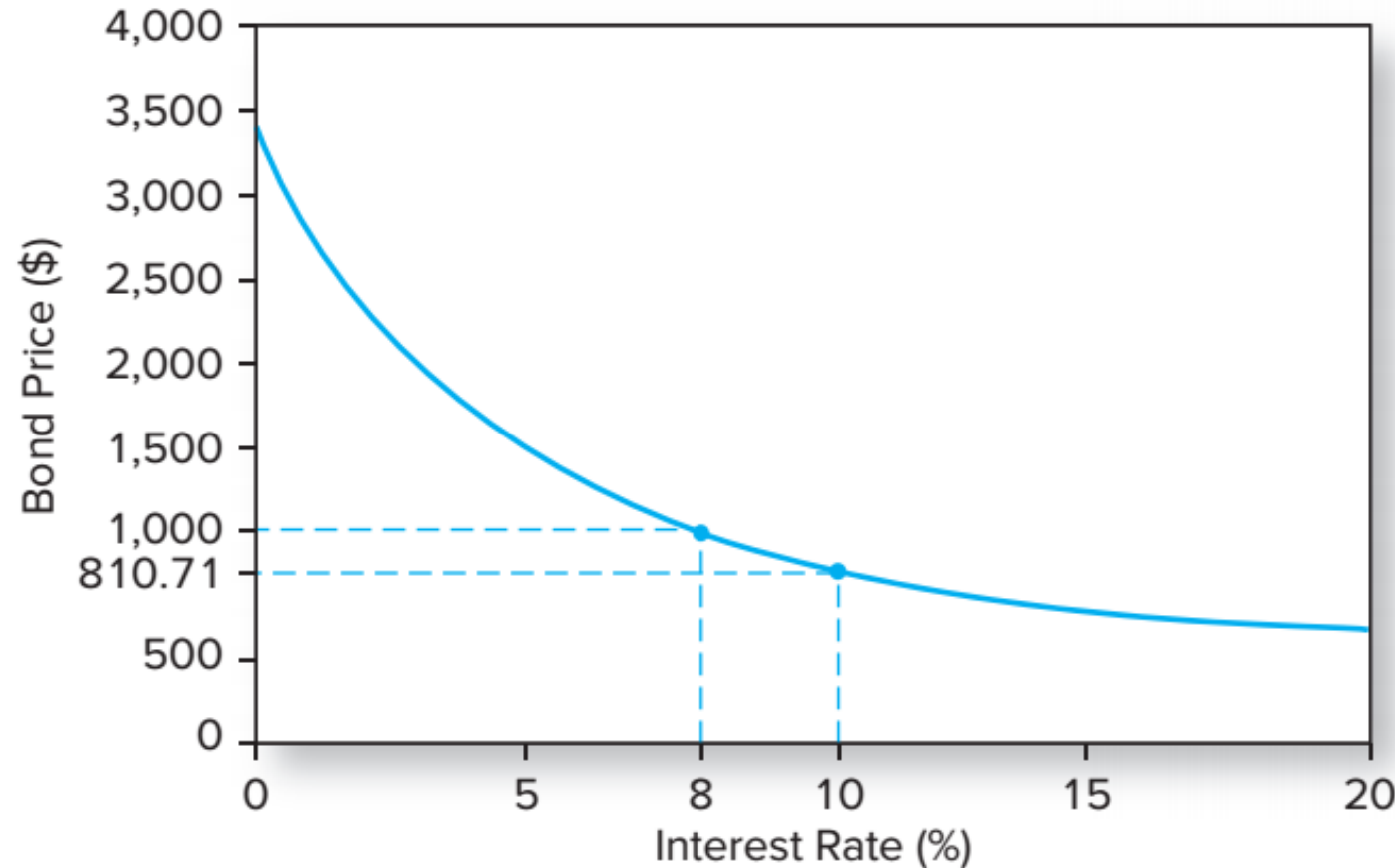


Interest Rate Risk

		12%, 5Y Bond				12%, 10Y Bond				8%, 10Y Bond			
y'	Δy	P'	Δ Price	D	D _M	P'	Δ Price	D	D _M	P'	Δ Price	D	D _M
10.00%	-200	10,772.17	7.72%	3.95	3.76	11,246.22	12.46%	6.31	6.01	8,753.78	13.60%	6.84	6.51
10.50%	-150	10,572.16	5.72%	3.93	3.74	10,915.17	9.15%	6.25	5.94	8,474.72	9.98%	6.78	6.44
11.00%	-100	10,376.88	3.77%	3.92	3.72	10,597.52	5.98%	6.20	5.87	8,207.44	6.51%	6.72	6.37
11.50%	-50	10,186.20	1.86%	3.91	3.70	10,292.66	2.93%	6.14	5.80	7,951.38	3.18%	6.67	6.30
12.00%	0	10,000.00	0.00%	3.90	3.68	10,000.00	0.00%	6.08	5.73	7,706.02	0.00%	6.61	6.23
12.50%	50	9,818.16	-1.82%	3.89	3.66	9,718.98	-2.81%	6.02	5.67	7,470.84	-3.05%	6.55	6.16
13.00%	100	9,640.56	-3.59%	3.88	3.64	9,449.07	-5.51%	5.96	5.60	7,245.37	-5.98%	6.49	6.09
13.50%	150	9,467.09	-5.33%	3.87	3.62	9,189.77	-8.10%	5.90	5.53	7,029.17	-8.78%	6.43	6.02
14.00%	200	9,297.64	-7.02%	3.85	3.60	8,940.60	-10.59%	5.85	5.46	6,821.80	-11.47%	6.37	5.95



Interest Rate Risk



- ❖ Modified duration is proportional to the derivative of the bond's price with respect to changes in the bond's yield.
- ❖ For small changes in yield,

$$D_M = -\frac{1}{P} \frac{dP}{dy}$$

$$P = \sum_{t=1}^T \frac{CF_t}{(1+y)^t}$$

$$\text{Money Duration} = D_M * (-P) = \frac{dP}{dy}$$

- ❖ Money (dollar) duration is the negative of the first derivative of a bond's price with respect to its yield.
- ❖ The money duration of a portfolio is just the sum of the dollar duration of the various instruments in the portfolio.
- ❖ DV01 is the dollar value of a basis point.

Interest Rate Risk

- ❖ A bond with a par value of €10,000, a maturity of 4 years, semiannual coupon payments, has a price of €9,326.72 and a yield to maturity of 8%.
 - What is the annual coupon rate?
 - What is the duration of the bond at the current price?
 - What is the modified duration?
 - Estimate the price of the bond given a 50 basis points decline in the YTM.
 - What is the actual price of the bond when the YTM declines to 7.5%?

$$€9,326.72 = \sum_{t=1}^8 \frac{C_t}{(1 + 4\%)^t} + \frac{Par\ Value}{(1 + 4\%)^8} \quad C = €300$$

$$Annual\ coupon\ rate = \frac{€300 * 2}{€10,000} = 6\%$$

t	PV(CF)	w _t	t*w _t	PV(CF)	w _t	t*w _t
0.0						
0.5	288.46	0.03	0.0155	289.16	0.03	0.0152
1.0	277.37	0.03	0.0297	278.71	0.03	0.0294
1.5	266.70	0.03	0.0429	268.63	0.03	0.0425
2.0	256.44	0.03	0.0550	258.92	0.03	0.0546
2.5	246.58	0.03	0.0661	249.56	0.03	0.0657
3.0	237.09	0.03	0.0763	240.54	0.03	0.0760
3.5	227.98	0.02	0.0856	231.85	0.02	0.0855
4.0	7,526.11	0.81	3.2278	7,672.42	0.81	3.2340
	9,326.73	1.00	3.60	9,489.79	1.00	3.60

$$D_M = \frac{D}{(1 + y/2)} = \frac{3.60}{(1 + 0.04)} = 3.4615$$

$$\frac{\Delta P}{P} = -D_M \Delta y = -3.4615 * -0.5\% = 1.73\%$$

$$P' = \sum_{t=1}^8 \frac{300}{(1 + 3.75\%)^t} + \frac{10,000}{(1 + 3.75\%)^8} = €9,489.79$$

$$\frac{\Delta P}{P} = \frac{9,489.79}{9,326.72} - 1 = 1.75\%$$

Interest Rate Risk

- ❖ A bond with a par value of €10,000, a maturity of 7 years, coupon payments twice a year, an annual coupon of 10%, and a yield to maturity of 8%.
 - What is the current price of the bond?
 - What is the duration of the bond at the current price?
 - What is the modified duration?
 - Estimate the bond price given a 50 basis points increase in the YTM.
 - What is the actual price of the bond when the YTM increases to 8.5%?

t	PV(CF)	w _t	t*w _t	PV(CF)	w _t	t*w _t
0.0						
0.5	480.77	0.04	0.0217	479.62	0.04	0.0222
1.0	462.28	0.04	0.0418	460.06	0.04	0.0427
1.5	444.50	0.04	0.0603	441.31	0.04	0.0614
2.0	427.40	0.04	0.0773	423.32	0.04	0.0785
2.5	410.96	0.04	0.0929	406.06	0.04	0.0942
3.0	395.16	0.04	0.1072	389.51	0.04	0.1084
3.5	379.96	0.03	0.1203	373.63	0.03	0.1213
4.0	365.35	0.03	0.1322	358.39	0.03	0.1330
4.5	351.29	0.03	0.1430	343.78	0.03	0.1435
5.0	337.78	0.03	0.1528	329.77	0.03	0.1530
5.5	324.79	0.03	0.1616	316.32	0.03	0.1614
6.0	312.30	0.03	0.1695	303.43	0.03	0.1689
6.5	300.29	0.03	0.1765	291.06	0.03	0.1755
7.0	6,063.49	0.55	3.8389	5,863.06	0.54	3.8074
	11,056.31	1.00	5.30	10,779.32	1.00	5.27

$$P_0 = \sum_{t=1}^{14} \frac{500}{(1 + 4\%)^t} + \frac{10,000}{(1 + 4\%)^{14}} \quad P_0 = \text{€}11,056.30$$

$$D_M = \frac{D}{(1 + y/2)} = \frac{5.30}{(1 + 0.04)} = 5.10$$

$$\frac{\Delta P}{P} = -D_M \Delta y = -5.10 * 0.5\% = -2.55\%$$

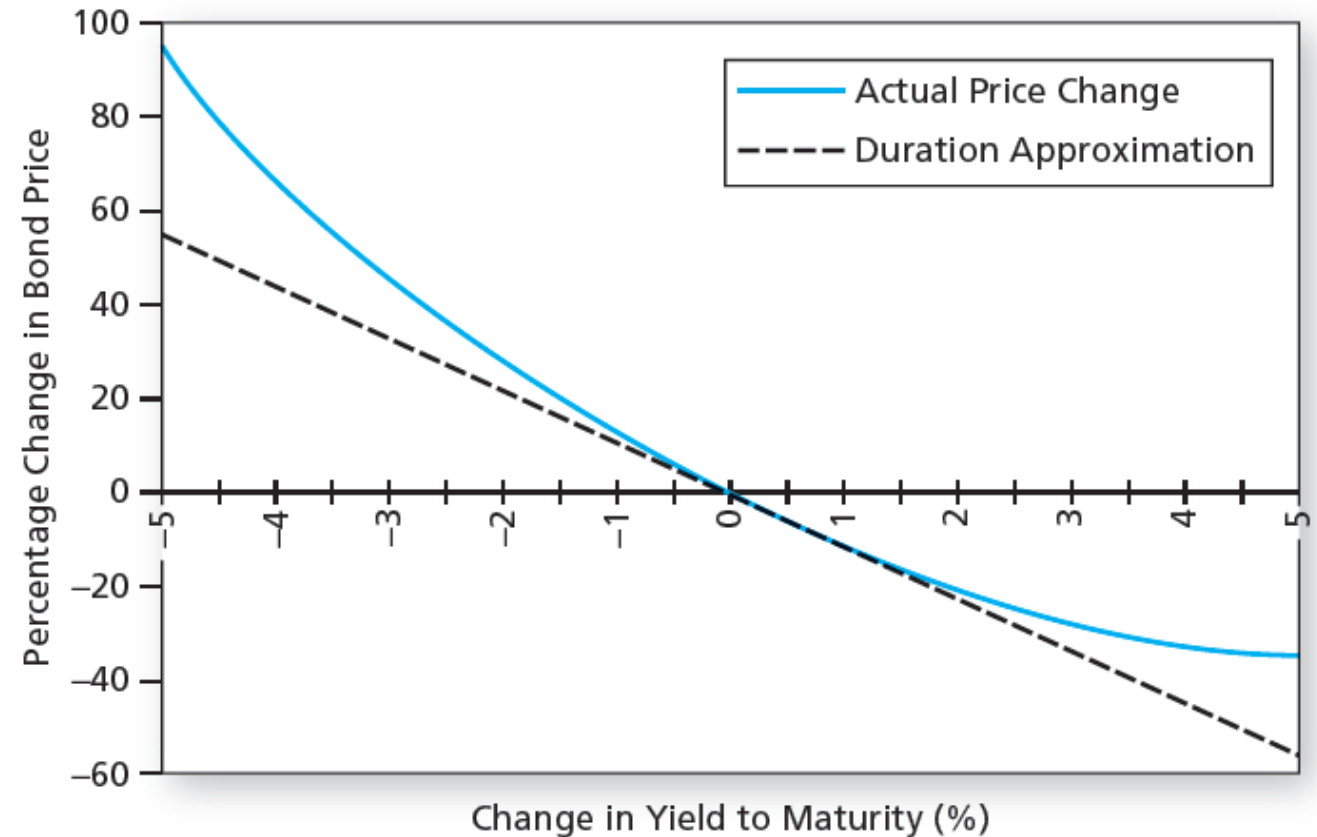
$$P' = \sum_{t=1}^{14} \frac{500}{(1 + 4.25\%)^t} + \frac{10,000}{(1 + 4.25\%)^{14}} = \text{€}10,779.30$$

$$\frac{\Delta P}{P} = \frac{10,779.30}{11,056.30} - 1 = -2.51\%$$

Interest Rate Risk

❖ Convexity

- Duration rule for the impact of interest rates on bond prices is only an approximation.
- A good approximation for small changes in bond yield, but it is less accurate for larger changes.
- Duration approximation always understates the value of the bond;
 - ✓ Underestimates the increase in bond price when the yield falls,
 - ✓ Overestimates the decline in price when the yield rises.
- The curvature of the price-yield curve is called the convexity of the bond.
 - ✓ Convexity is measured as the rate of change of the slope of the price-yield curve, expressed as a fraction of the bond price.



$$Convexity = \frac{1}{P \times (1 + y)^2} \sum_{t=1}^T \left[\frac{CF_t}{(1 + y)^t} (t^2 + t) \right]$$

$$\frac{\Delta P}{P} = -D_M * \Delta y + \frac{1}{2} [Convexity \times (\Delta y)^2]$$

Interest Rate Risk

- ❖ A bond with a par value of €10,000, a maturity of 8 years, annual coupon payments, an annual coupon rate of 6%, and a yield to maturity of 8%.

- Duration of the bond is 6.48. What is the modified duration?
- Estimate the bond price given a 50 basis points decline in the YTM.
- What is the actual price of the bond when the YTM declines to 7.5%?
- What is the convexity of the bond at the current price?

$$Convexity = \frac{1}{P \times (1 + y)^2} \sum_{t=1}^T \left[\frac{CF_t}{(1 + y)^t} (t^2 + t) \right]$$

$$P = \sum_{t=1}^8 \frac{600}{(1 + 8.0\%)^t} + \frac{10,000}{(1 + 8.0\%)^8} = €8,850.67$$

$$D_M = \frac{D}{(1 + y)} = \frac{6.48}{(1 + 0.08/1)} = 6.00$$

$$\frac{\Delta P}{P} = -D_M \Delta y = -6.00 * -0.5\% = 3.00\%$$

$$P' = \sum_{t=1}^8 \frac{600}{(1 + 7.5\%)^t} + \frac{10,000}{(1 + 7.5\%)^8} = €9,121.40$$

$$\frac{\Delta P}{P} = \frac{9,121.40}{8,850.67} - 1 = 3.06\%$$

t	PV(CF@8.0%)	w _t	t*w _t	Convexity	PV(CF@7.5%)
0.0					
1.0	555.56	0.06	0.0628	0.11	558.14
2.0	514.40	0.06	0.1162	0.30	519.20
3.0	476.30	0.05	0.1614	0.55	482.98
4.0	441.02	0.05	0.1993	0.85	449.28
5.0	408.35	0.05	0.2307	1.19	417.94
6.0	378.10	0.04	0.2563	1.54	388.78
7.0	350.09	0.04	0.2769	1.90	361.65
8.0	5,726.85	0.65	5.1764	39.94	5,943.44
	8,850.67	1.00	6.48	46.38	9,121.40
		D_M	6.00		

Initial Price	8,850.67	
Duration Estimate	3.00%	9,116.20
Convexity Adjustment	0.06%	9,121.33

Interest Rate Risk

- ❖ A bond with a par value of €10,000, a maturity of 8 years, semiannual coupon payments, an annual coupon rate of 6%, and a yield to maturity of 8%.
 - Estimate the bond price given a 50 basis points decline in the YTM.
 - What is the actual price of the bond when the YTM declines to 7.5%?

$$P = \sum_{t=1}^{16} \frac{300}{(1 + 4.0\%)^t} + \frac{10,000}{(1 + 4.0\%)^{16}} = €8,834.77$$

$$P' = \sum_{t=1}^{16} \frac{300}{(1 + 3.75\%)^t} + \frac{10,000}{(1 + 3.75\%)^{16}} = €9,109.74$$

$$\frac{\Delta P}{P} = \frac{9,109.74}{8,834.77} - 1 = 3.112\%$$

$$D_M = \frac{D}{(1 + y)} = \frac{6.35}{(1 + 0.08/2)} = 6.11$$

$$\frac{\Delta P}{P} = -D_M \Delta y = -6.11 * -0.5\% = 3.055\%$$

$$\begin{aligned} \frac{\Delta P}{P} &= -D_M * \Delta y + \frac{1}{2} [\text{Convexity} \times (\Delta y)^2] = 3.055\% + \frac{1}{2} [45.98 \times (0.005)^2] \\ &= \mathbf{3.055\% + 0.575\% = 3.112\%} \end{aligned}$$

Par Value	10,000.00	t	PV(CF@8.0%)	w _t	t*w _t	Convexity	PV(CF@7.5%)
C (Annual)	6.00%	0.0					
Frequency	2	1.0	288.46	0.03	0.0327	0.03	289.16
YTM_0	8.00%	2.0	277.37	0.03	0.0628	0.09	278.71
YTM_1	7.50%	3.0	266.70	0.03	0.0906	0.17	268.63
		4.0	256.44	0.03	0.1161	0.27	258.92
		5.0	246.58	0.03	0.1395	0.39	249.56
		6.0	237.09	0.03	0.1610	0.52	240.54
		7.0	227.98	0.03	0.1806	0.67	231.85
		8.0	219.21	0.02	0.1985	0.83	223.47
		9.0	210.78	0.02	0.2147	0.99	215.39
		10.0	202.67	0.02	0.2294	1.17	207.61
		11.0	194.87	0.02	0.2426	1.35	200.10
		12.0	187.38	0.02	0.2545	1.53	192.87
		13.0	180.17	0.02	0.2651	1.72	185.90
		14.0	173.24	0.02	0.2745	1.90	179.18
		15.0	166.58	0.02	0.2828	2.09	172.70
		16.0	5,499.25	0.62	9.9593	78.27	5,715.15
			8,834.77	1.00	6.35	45.98	9,109.74
				D_M	6.11		

Initial Price	8,834.77	
Duration Estimate	3.05%	9,104.59
Convexity Adjustment	0.06%	9,109.67

Interest Rate Risk

$$\text{Bond Convexity Approximation} = \frac{P_{up} + P_{Down} - 2 * P}{P * (\Delta y)^2}$$

- ❖ A bond with a par value of €10,000, a maturity of 8 years, semiannual coupon payments, an annual coupon rate of 6%, and a yield to maturity of 8%.
 - Estimate current bond price and the bond prices given a 50 basis points decline and increase in the YTM.
 - Calculate the approximate convexity of the bond.

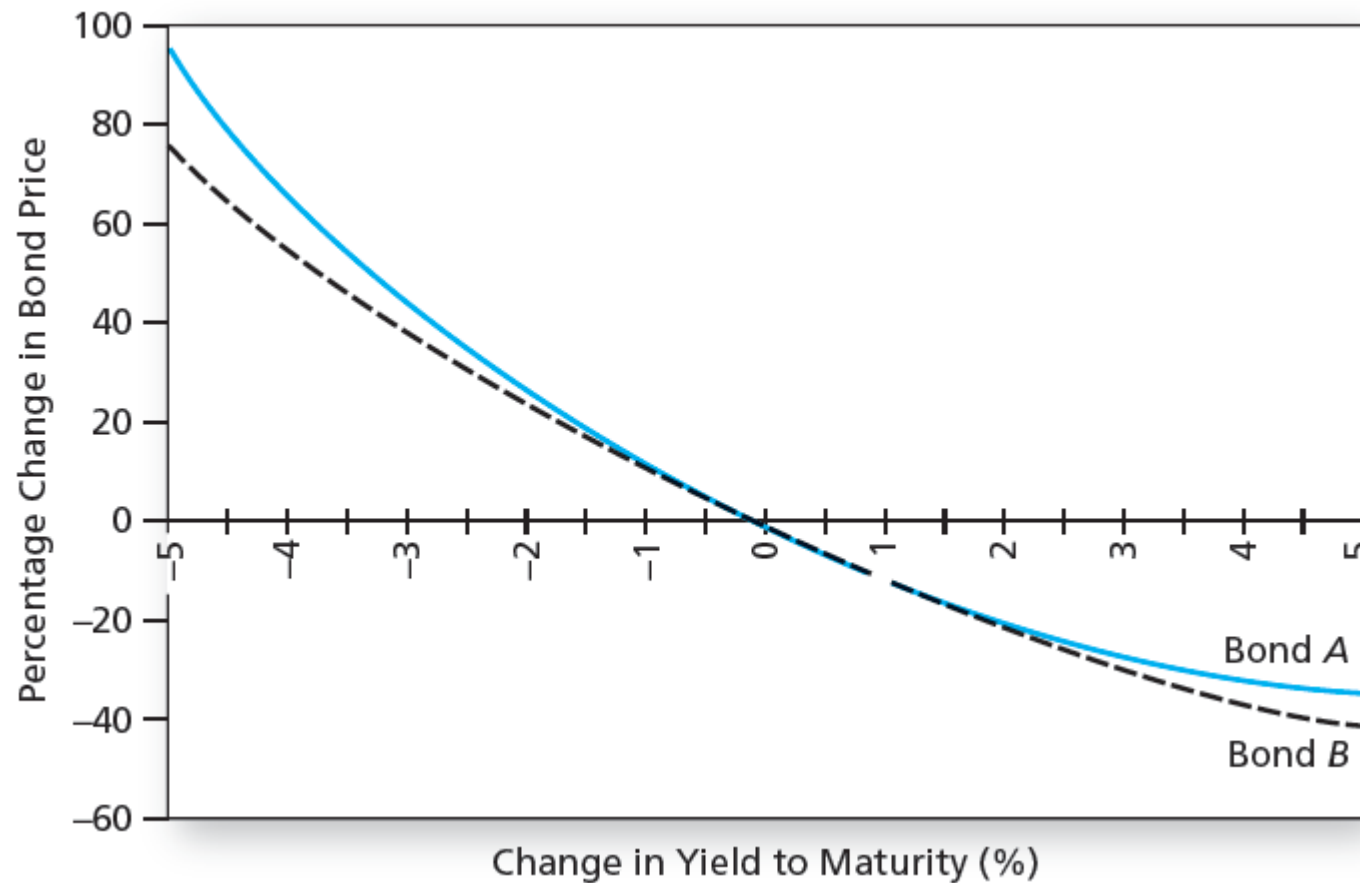
$$P = \sum_{t=1}^{16} \frac{300}{(1 + 4.0\%)^t} + \frac{10,000}{(1 + 4.0\%)^{16}} = €8,834.77$$

$$P_{Up} = \sum_{t=1}^{16} \frac{300}{(1 + 3.75\%)^t} + \frac{10,000}{(1 + 3.75\%)^{16}} = €9,109.74$$

$$P_{Down} = \sum_{t=1}^{16} \frac{300}{(1 + 4.25\%)^t} + \frac{10,000}{(1 + 4.25\%)^{16}} = €8,569.96$$

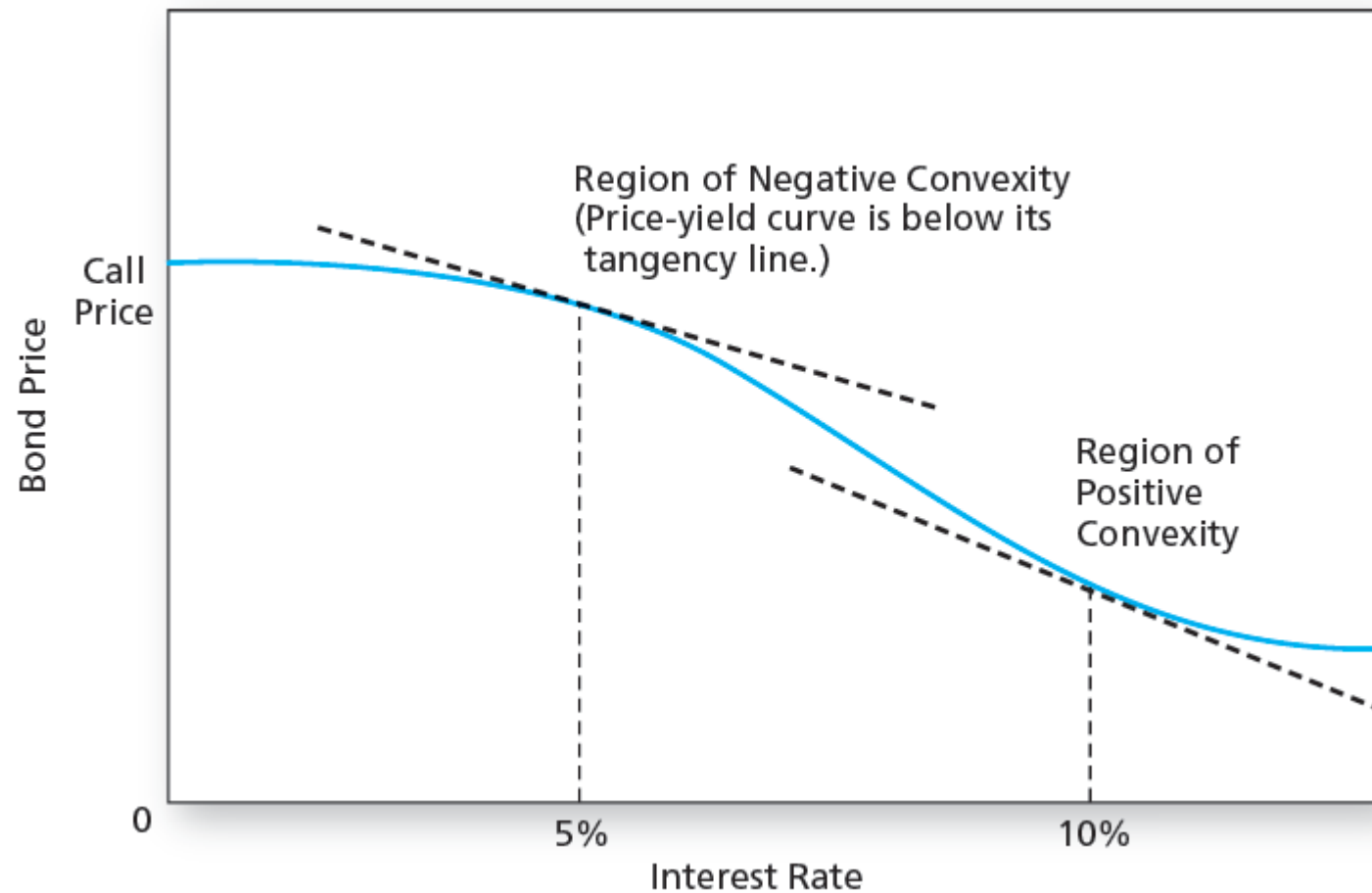
$$\text{Bond Convexity Approximation} = \frac{9,109.74 + 8,569.96 - 2 * 8,834.77}{8,834.77 * (0.005)^2} = \frac{9,109.74 + 8,569.96 - 2 * 8,834.77}{0.220869} = 46.00$$

Interest Rate Risk



- ❖ Higher Convexity → Bigger price increases when yields fall than the price declines when yields rise.
- ❖ The more volatile interest rates, the more attractive this asymmetry.
- ❖ Bonds with greater convexity → higher prices and/or lower yields, all else equal.

Interest Rate Risk



❖ Price–Yield Curve for a Callable Bond

- As rates fall, there is a ceiling on the bond's market price, which cannot rise above the call price.
- Negative convexity

Passive Management

❖ Two passive bond portfolio strategies:

➤ Indexing

- ✓ Attempts to replicate the performance of a given bond index.

➤ Immunization

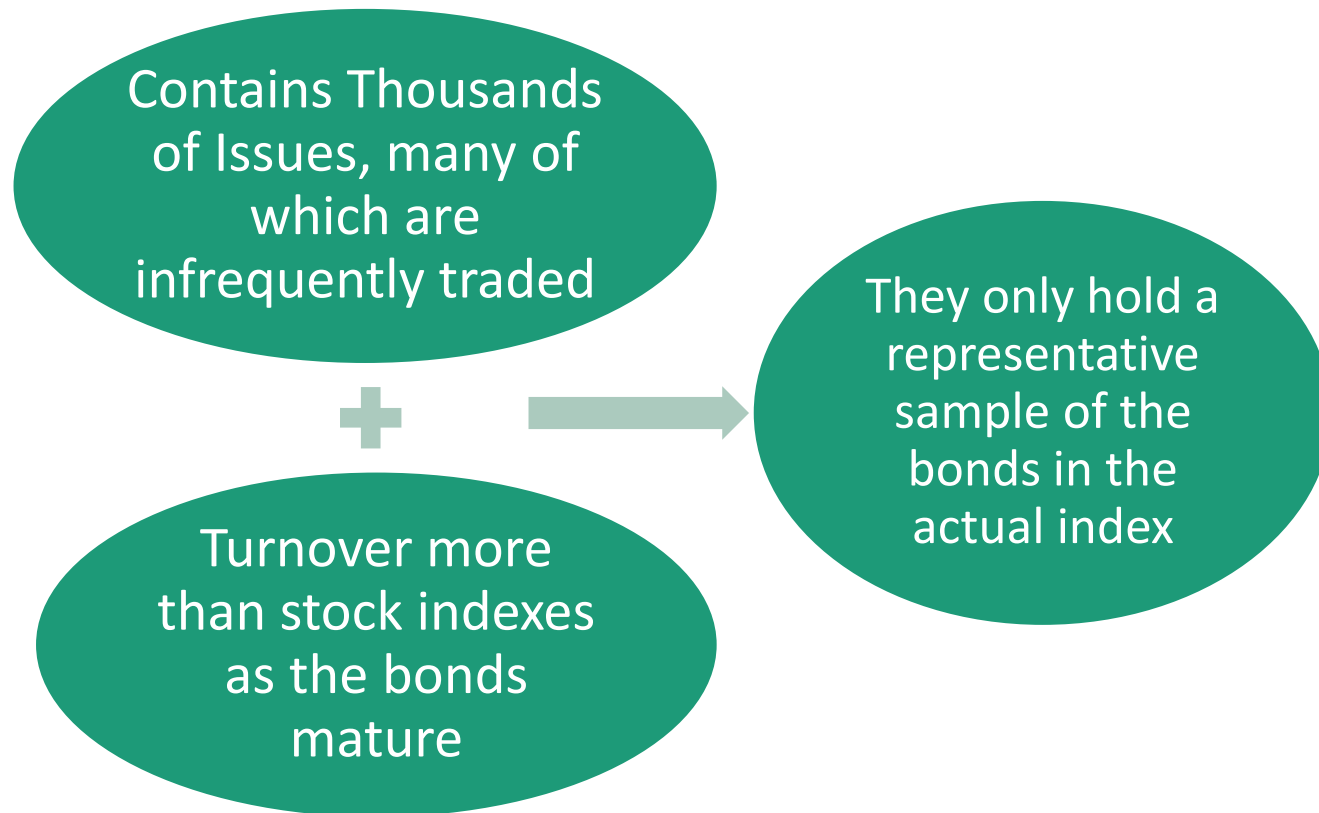
- ✓ Used widely by financial institutions such as insurance companies and pension funds to shield overall financial status from exposure to interest rate fluctuations.

❖ Both see market prices as being correct

❖ Differ greatly in terms of risk

- A bond-index portfolio will have the same risk-reward profile as the bond market index to which it is tied.
- Immunization strategies seek to establish a virtually zero-risk profile
 - ✓ Interest rate movements have no impact on the value of the portfolio.

Bond Index Funds

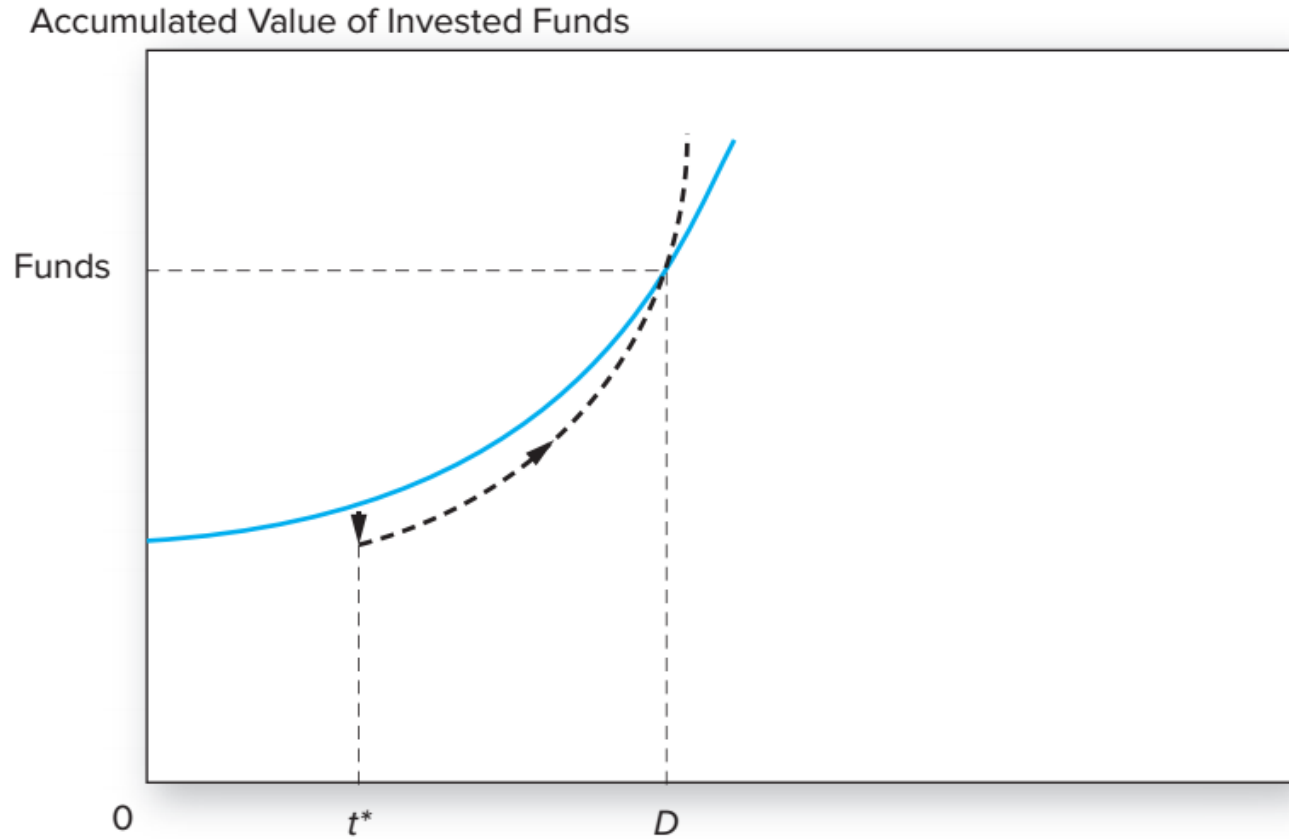


- ❖ The idea is to create a portfolio that mirrors the composition of an index that measures the broad market.
 - Broad market indexes
 - Government, Agencies, Supras
 - Corporate,
 - Mortgage-backed securities
 - Yankee bonds
- ❖ Challenges in constructing an indexed bond portfolio
 - Indexes include thousands of securities
 - Many bonds are very thinly traded
 - Rebalancing problems
 - ✓ Bonds are continually dropped from the index as they approach maturity.
 - ✓ New bonds are added to the index as they are issued.
 - Bonds generate considerable interest income that must be reinvested.

Passive Management: Immunization

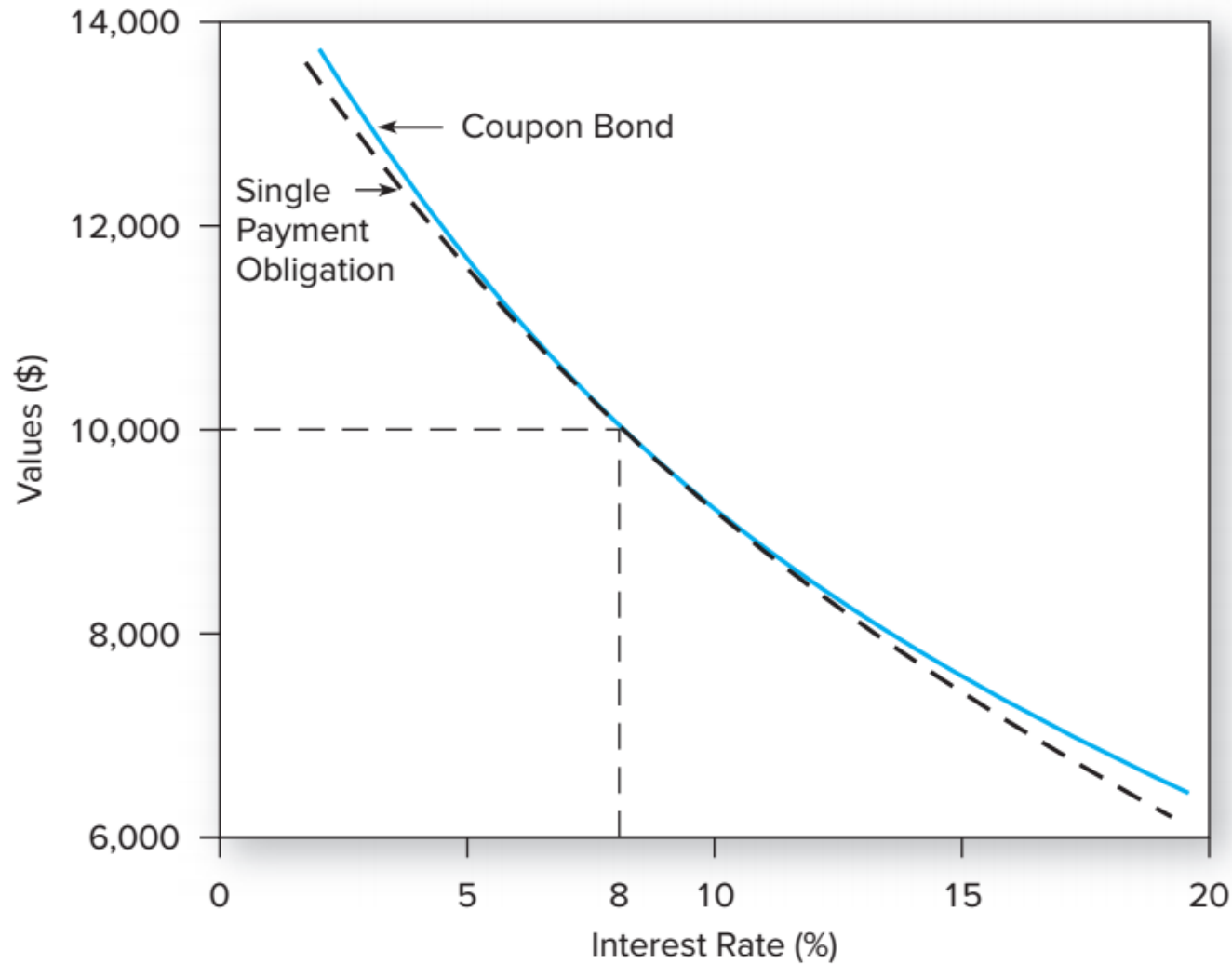
- ❖ Control interest rate risk
- ❖ Widely used by pension funds, insurance companies, and banks
- ❖ A natural mismatch between asset and liability maturity structures.
 - Bank liabilities are primarily the deposits owed to customers, most of which are short-term and, consequently, have low duration.
 - Bank assets by contrast are composed largely of outstanding commercial and consumer loans or mortgages, which have longer duration.
 - **What happens when interest rates rise unexpectedly?**
 - **What about the risks pension funds are exposed to?**
- ❖ The interest rate exposure of assets and liabilities is matched in the portfolio.
 - Match the duration of the assets and liabilities.
 - Price risk and reinvestment rate risk exactly cancel out.
 - ✓ When the portfolio duration equals the investor's horizon date, the accumulated value of the investment fund at the horizon date will be unaffected by interest rate fluctuations.
 - Value of assets match liabilities whether rates rise/fall.
 - Duration-matched assets and liabilities let the asset portfolio meet the firm's obligations despite interest rate movements.

Passive Management: Immunization



- ❖ Duration matching balances the difference between the accumulated value of the coupon payments (reinvestment rate risk) and the sale value of the bond (price risk).
 - When interest rates fall, the coupons grow less than in the base case, but the higher value of the bond offsets this.
 - When interest rates rise, the value of the bond falls, but the coupons more than make up for this loss because they are reinvested at the higher rate.

Passive Management: Immunization



- ❖ Coupon paying bond and single-payment obligation.
 - For small changes in interest rates, the change in value of both the asset and the obligation is equal, so the obligation remains fully funded.
 - For greater changes in the interest rate, the present value curves diverge.
 - Convexity
 - Rebalancing
 - ✓ Interest rate changes cause mismatch.
 - ✓ Asset durations will change with time.
 - ✓ Without rebalancing, durations will become unmatched.

Passive Management: Cash Flow Matching and Dedication

❖ Cash Flow Matching

- Buy a zero-coupon bond with a face value equal to the projected cash outlay.
- The cash flow from the bond and the obligation exactly offset each other, no interest rate risk.

❖ Dedication strategy

- Cash flow matching on a multiperiod basis.
- Either zero-coupon or coupon bonds with total cash flows in each period that match a series of obligations.
- There is no need for rebalancing once the cash flows are matched.

❖ Some Issues

- Cash flow matching imposes constraints on bond selection.
- Immunization or dedication strategies are appealing to firms/investors that do not wish to bet on general movements in interest rates.
- Sometimes, cash flow matching is simply not possible b/c of the availability of assets with required maturities.

❖ Other Problems

- Duration is calculated using YTM. The implicit assumption is a flat yield curve.
- What if the yield curve is not flat as is the case most of the time in the real world?
 - Discount using the appropriate spot interest rate from the zero-coupon yield curve corresponding to the date of the particular cash flow.
 - Even with this modification, duration matching will immunize portfolios only for parallel shifts in the yield curve.

What is next?

❖ Portfolio Theory and Practice I

- Risk, Return, and the Historical Record
- Capital Allocation to Risky Assets
 - ✓ Readings: Ch. 5 & 6
- Suggested Problems
 - ✓ **Ch. 16:** 4, 5, 9, 11, 12, 16, 21, 23
 - ✓ **Ch 16 – CFA Problems:** 7, 12



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Investments

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Lecture 3