Vietnamese-German University

Investments

Finance 2 - BFIN

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Lecture 7

Outline

- Index Models and The Capital Asset Pricing Model
  - The Single-Index Model
  - The Capital Asset Pricing Model

A Single-Factor Market

- Advantages
  - Reduces the number of inputs for diversification
    - ✓ Suppose your security analysts can thoroughly analyze 50 stocks. This means that your input list will include the following:

$$\begin{aligned} n &= \quad 50 \text{ estimates of expected returns} \\ n &= \quad 50 \text{ estimates of variances} \\ (n^2 - n)/2 &= \underline{1{,}225} \text{ estimates of covariances} \\ & \; 1{,}325 \text{ total estimates} \end{aligned}$$

  - Easier for security analysts to specialize
- Model

$$r_i = E(r_i) + \textit{unanticipated surprise}$$

$$r_i = E(r_i) + \beta_i m + e_i$$

A Single-Factor Market

❖Regression equation: $R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$

❖Expected return-beta relationship: $E(R_i) = \alpha_i + \beta_i E(R_M)$

❖Variance = Systematic risk + Firm-specific risk: $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$

❖Covariance = Product of betas × Market risk: $\mathrm{Cov}(r_i, r_j) = \beta_i \beta_j \sigma_M^2$

❖Correlation =

$$\mathrm{Corr}(r_i, r_j) = \frac{\beta_i \beta_j \sigma_M^2}{\sigma_i \sigma_j} = \frac{\beta_i \sigma_M^2 \beta_j \sigma_M^2}{\sigma_i \sigma_M \sigma_j \sigma_M}$$

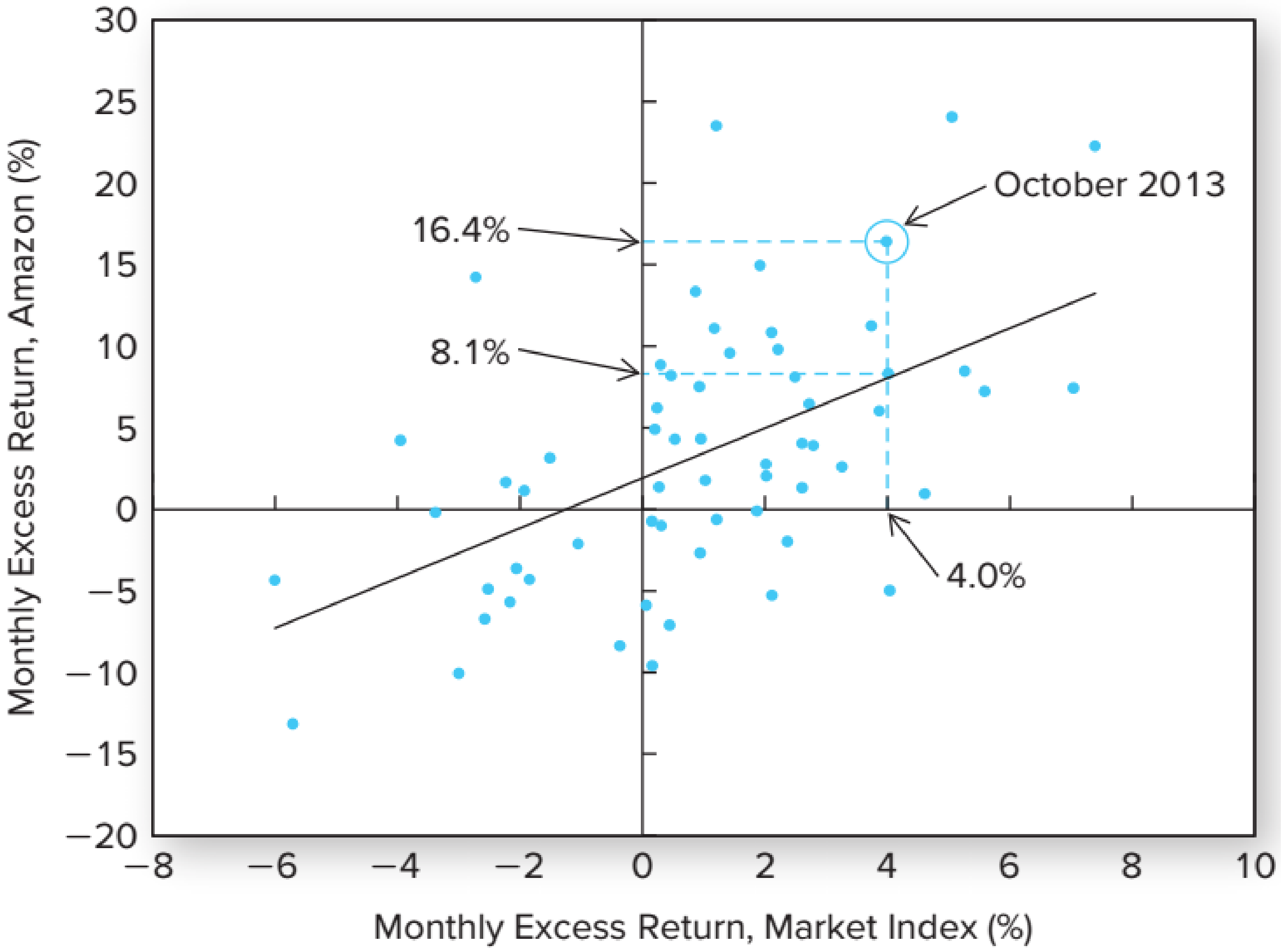
$$= \mathrm{Corr}(r_i, r_M) \times \mathrm{Corr}(r_j, r_M)$$

A Single-Factor Market

- Some securities will be more sensitive than others to macroeconomic shocks.
- The variance of returns attributable to the marketwide factor is called the systematic risk of the security.
- The index model assumes that firm-specific surprises are mutually uncorrelated:
  - The only source of covariance between any pair of securities is their common dependence on the market return.

$$\mathrm{Cov}(r_i, r_j) = \beta_i \beta_j \sigma_M^2 \qquad \mathrm{Corr}(r_i, r_j) = \mathrm{Corr}(r_i, r_M) \times \mathrm{Corr}(r_j, r_M)$$

A Single-Factor Market



$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

$$E(R_i) = \alpha_i + \beta_i E(R_M)$$

- α is the security's expected excess return when the market excess return is zero. It is the vertical intercept.
- The slope of the line in the figure is the security's beta coefficient, β_i.

Opportunity Set of Risky Assets

The data below describe a three-stock financial market that satisfies the single-index model.

Stock	Capitalization	Beta	Mean Excess Return	Standard Deviation
A	$3,000	1.0	10%	40%
B	1,940	0.2	2	30
C	1,360	1.7	17	50

The standard deviation of the market-index portfolio is 25%.

a. What is the mean excess return of the index portfolio?

b. What is the covariance between stock *A* and stock *B*?

c. What is the covariance between stock *B* and the index?

d. Break down the variance of stock *B* into its systematic and firm-specific components.

$$\mathrm{Cov}(r_i, r_j) = \beta_i \beta_j \sigma_M^2$$

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$$

Index Model Regression Equation

Excess return of security i

$$R_i(t) = \alpha_i + \beta_i R_{S\&P500}(t) + e_i(t)$$

Zero-mean, firm-specific surprise in security i's return in month t.
(the *residual*)

Expected excess return when the market excess return is zero

Sensitivity of security i's return to changes in the return of the market

Expected excess return of the market

	Symbol
1. The stock's expected return if the market is neutral, that is, if the market's excess return, $r_M - r_f$, is zero	α_i
2. The component of return due to movements in the overall market in any period; β_i is the security's responsiveness to market movements	$\beta_i(r_M - r_f)$
3. The unexpected component of return in any period due to unexpected events that are relevant only to this security (firm specific)	e_i
4. The variance attributable to the uncertainty of the common macro-economic factor	$\beta_i^2\sigma_M^2$
5. The variance attributable to firm-specific uncertainty	$\sigma^2(e_i)$

Index Model Regression Equation

$$R_i(t) = \alpha_i + \beta_i R_{S\&P500}(t) + e_i(t)$$

Regression statistics for Amazon

Regression Statistics				
Multiple *R*	0.5351			
R-Square	0.2863			
Adjusted *R*-Square	0.2742			
Standard Error	0.0686			
Observations	60			
	Coefficients	**Standard Error**	***t*-statistic**	***p*-value**
Intercept	0.0192	0.0093	2.0645	0.0434
Market index	1.5326	0.3150	4.8648	0.0000

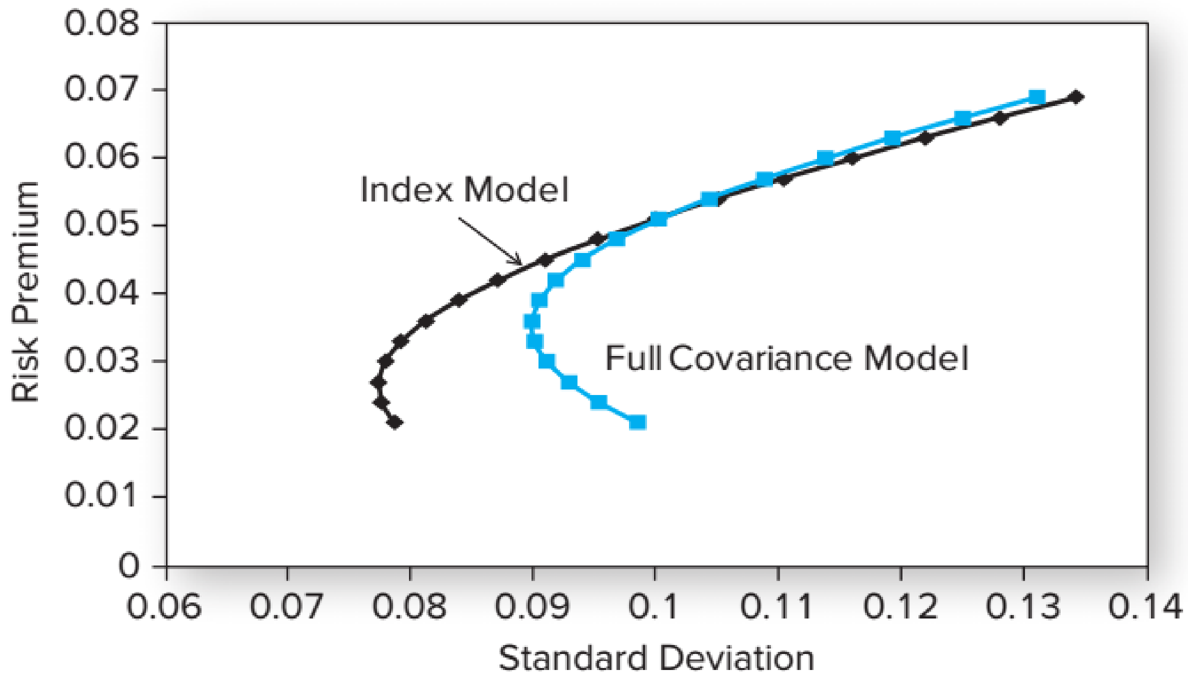
- The adjusted R-square corrects for an upward bias in R-square that arises because we use the estimated values of two parameters, the slope (beta) and intercept (alpha), rather than their true, but unobservable, values.
- The standard error of the regression is the standard deviation of the residual, e.
  - Higher the standard errors greater the impact of firm-specific events.

Index Model Regression Equation

Ticker	Company	Beta	Alpha	*R*-Square	Residual Std Dev	Standard Error Beta	Standard Error Alpha	Adjusted Beta
CPB	Campbell Soup	0.247	0.001	0.012	0.064	0.292	0.009	0.498
NEM	Newmont Mining	0.430	0.005	0.011	0.117	0.538	0.016	0.620
MCD	McDonald's	0.563	0.006	0.165	0.036	0.165	0.005	0.709
SBUX	Starbucks	0.581	0.004	0.112	0.046	0.212	0.006	0.721
KO	Coca-Cola	0.663	–0.001	0.258	0.032	0.146	0.004	0.776
UNP	Union Pacific	0.859	0.005	0.212	0.047	0.216	0.006	0.906
PFE	Pfizer	0.895	–0.000	0.367	0.033	0.153	0.005	0.930
XOM	ExxonMobil	0.920	–0.006	0.342	0.036	0.166	0.005	0.946
MSFT	Microsoft	0.923	0.012	0.193	0.054	0.246	0.007	0.948
INTC	Intel	0.934	0.007	0.187	0.055	0.254	0.007	0.956
GOOG	Alphabet (Google)	0.960	0.008	0.209	0.053	0.243	0.007	0.973
DIS	Walt Disney	1.288	–0.001	0.496	0.037	0.169	0.005	1.192
BAC	Bank of America	1.357	0.003	0.270	0.063	0.291	0.009	1.238
BA	Boeing	1.368	0.011	0.330	0.055	0.254	0.007	1.246
AMZN	Amazon	1.533	0.019	0.286	0.069	0.315	0.009	1.355
MRO	Marathon Oil	2.632	–0.023	0.314	0.110	0.506	0.015	2.088
	AVERAGE	1.010	0.003	0.235	0.057	0.260	0.008	1.006
	STD DEVIATION	0.560	0.009	0.127	0.025	0.115	0.003	0.374

Index Model Regression Equation

	Index Model	Full-Covariance Model
A. Weights in Optimal Risky Portfolio		
Market index	0.82	0.90
WMT (Walmart)	0.13	0.17
TGT (Target)	−0.07	−0.14
VZ (Verizon)	−0.05	−0.18
T (AT&T)	0.10	0.19
F (Amazon)	0.07	0.08
GM (General Motors)	0.01	−0.03
B. Portfolio Characteristics		
Risk premium	0.0605	0.0639
Standard deviation	0.1172	0.1238
Sharpe ratio	0.5165	0.5163



❖Is the index model inferior to the full-blown Markowitz model?

➢It imposes additional assumptions that may not be fully accurate.

➢The Markowitz model allows far more flexibility in modeling asset covariance structure.

➢Estimating the covariances with a sufficient degree of accuracy is an issue.

The Capital Asset Pricing Model

❖ Portfolio Theory vs. CAPM

- Mean-Variance Analysis
  - ✓ Tells us how to select a portfolio given expected returns, variances, and covariances.
  - ✓ Does not tell us anything about what prices should be.
- CAPM
  - ✓ An equilibrium model that characterizes risk-return combinations of securities that occur when investors are mean-variance optimizers.
  - ✓ The term equilibrium refers to a situation where no investor wants to do anything differently.
  - ✓ It is the equilibrium model that underlies all modern financial theory.

The Capital Asset Pricing Model

- The CAPM can be derived by asking the question:
  - If everybody in the economy holds an efficient portfolio, what do prices need to be so that markets clear?
  - Suppose that based on the prices/expected returns our model comes up with, we find that IBM does not enter into any maximizing investor's portfolio.
    - ✓ Then IBM must be priced too high
    - ✓ Since nobody will hold IBM shares at that price, the price will fall to a point where the aggregate demand of investors equals the number of IBM shares outstanding.

The Capital Asset Pricing Model

- Modern Portfolio Theory tells us that every investor holds a combination of just two portfolios
  - The risk-free asset
  - The tangency portfolio
- Thus, the tangency portfolio must be the market portfolio
  - The market portfolio consists of all risky assets held in proportion to their market value
  - Stocks, bonds, real estate, human capital, etc.

The Capital Asset Pricing Model

❖ The CAPM Assumptions

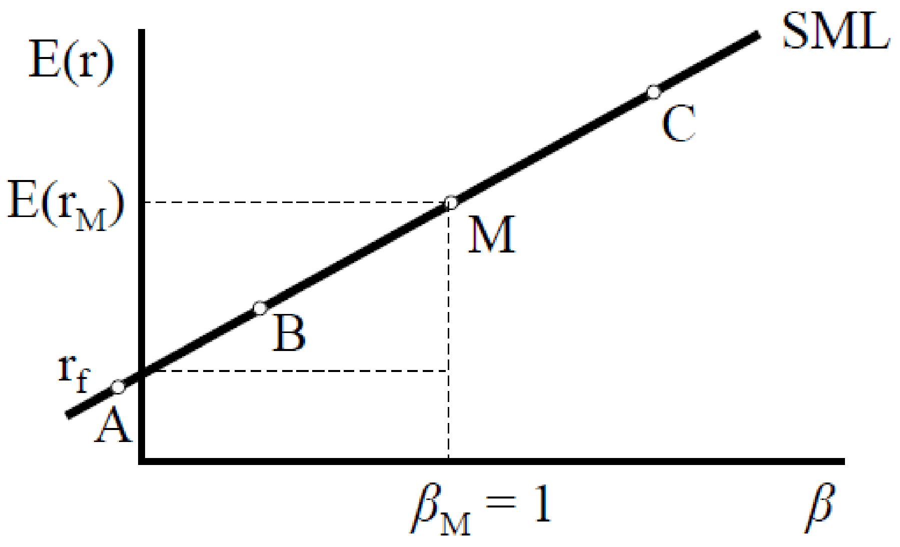
- All investors are rational mean-variance optimizers with identical planning horizons.
- Investors can borrow or lend any amount at a fixed, risk-free rate.
- Perfect competition: no individual investor can affect security prices.
- All assets are tradable and perfectly divisible.
- No taxes, transaction costs, or short-sale constraints.
- Investors have homogeneous expectations.
  - ✓ They agree on the estimates of expected returns, variances, and covariances.

The Capital Asset Pricing Model

❖ The CAPM –Main Result

$$E[R_i] = r_i = r_f + \underbrace{\beta_i \times (E[R_{Mkt}] - r_f)}_{\text{Risk premium for security } i}$$

$$\beta_i = \frac{\overbrace{SD(R_i) \times Corr(R_i, R_{Mkt})}^{\text{Volatility of } i \text{ that is common with the market}}}{SD(R_{Mkt})} = \frac{Cov(R_i, R_{Mkt})}{Var(R_{Mkt})}$$



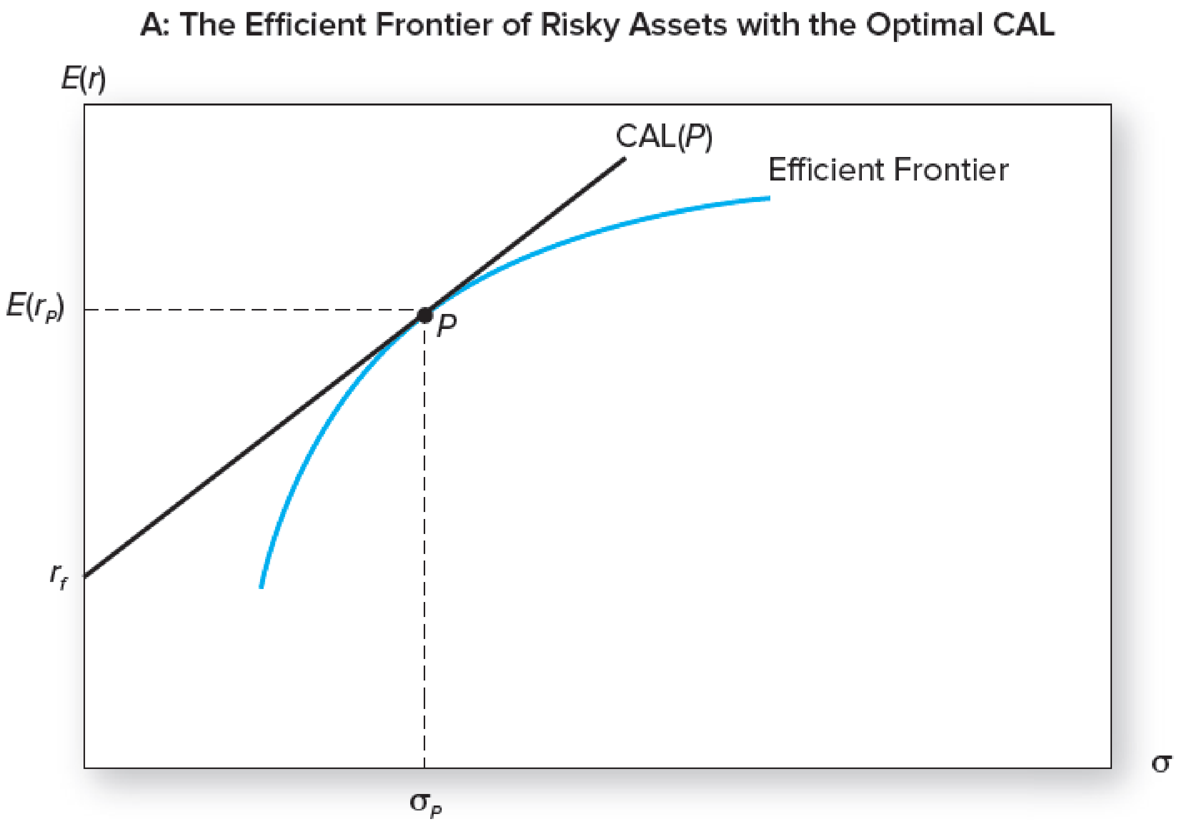
❖ Security Market Line (SML)

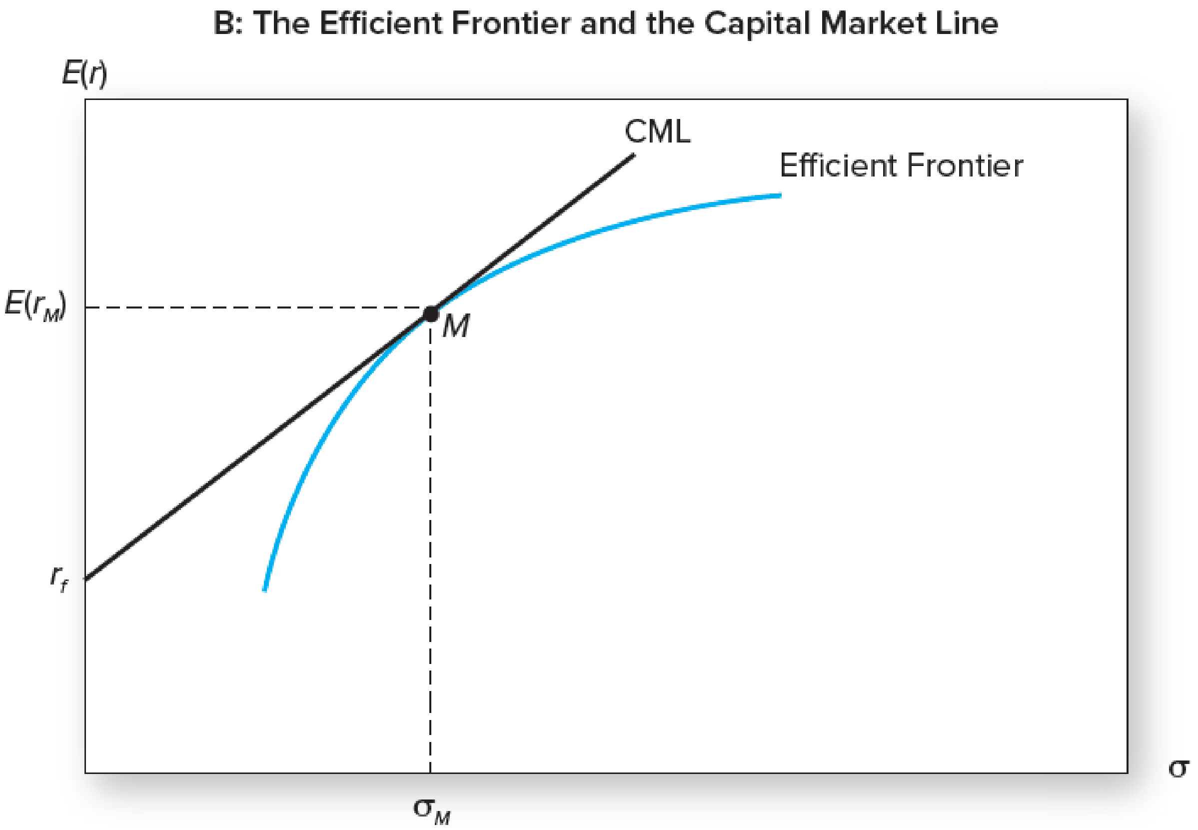
- ➢ Plots the expected return of assets against their beta.
- ➢ The CAPM states that all assets must be on the SML.

❖ SML vs CML

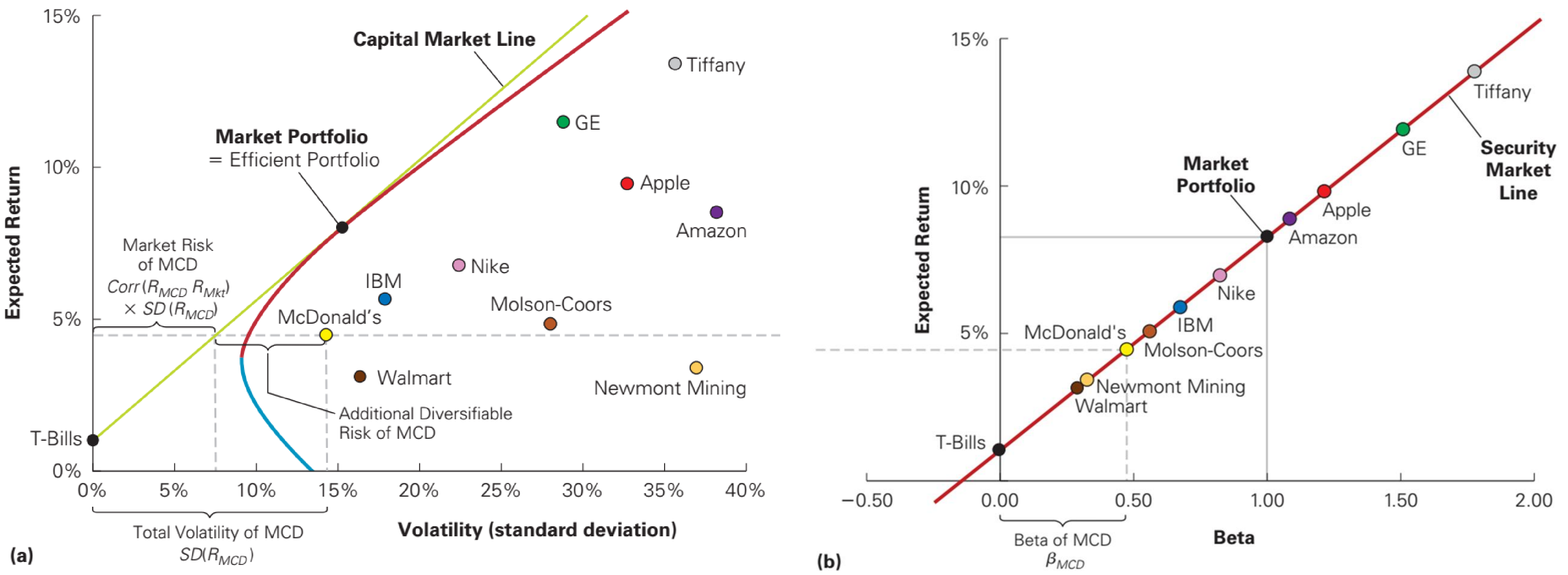
- ➢ SML
  - ✓ Shows systematic risk only
  - ✓ Every security / portfolio lies on the SML
- ➢ CML
  - ✓ Shows the total risk (systematic + unsystematic)
  - ✓ Only two securities lie on the CML
    - Risk-free asset and the market portfolio

The Capital Asset Pricing Model





The Capital Asset Pricing Model



The Capital Asset Pricing Model

- The risk premium on the market portfolio is proportional to its risk and the degree of risk aversion:

$$\boxed{E(R_M) = \bar{A}\sigma_M^2}$$

- An individual security's risk premium is a function of:
  - Its contribution to the risk of the market portfolio
  - The covariance of returns with the assets that make up the market portfolio

$$\sum_{i=1}^{n} w_i Cov(R_i, R_{GE}) = Cov\left(\sum_{i=1}^{n} w_i R_i, R_{GE}\right)$$

$$\frac{\text{GE's contribution to risk premium}}{\text{GE's contribution to variance}} = \frac{w_{GE} E(R_{GE})}{w_{GE} \text{Cov}(R_{GE}, R_M)} = \frac{E(R_{GE})}{\text{Cov}(R_{GE}, R_M)}$$

$$\frac{\text{Market risk premium}}{\text{Market variance}} = \frac{E(R_M)}{\sigma^2(R_M)}$$

$$\Rightarrow \frac{E(R_{GE})}{Cov(R_{Ge}, R_M)} = \frac{E(R_M)}{\sigma^2(R_M)}$$

$$E(R_{GE}) = \frac{Cov(R_{Ge}, R_M)}{\sigma^2(R_M)} E(R_M) \Rightarrow E(r_{GE}) = r_f + \beta_{GE}\left[E(r_M) - r_f\right]$$

Expected return–beta relationship

The Capital Asset Pricing Model

- CAPM holds for the overall portfolio because:

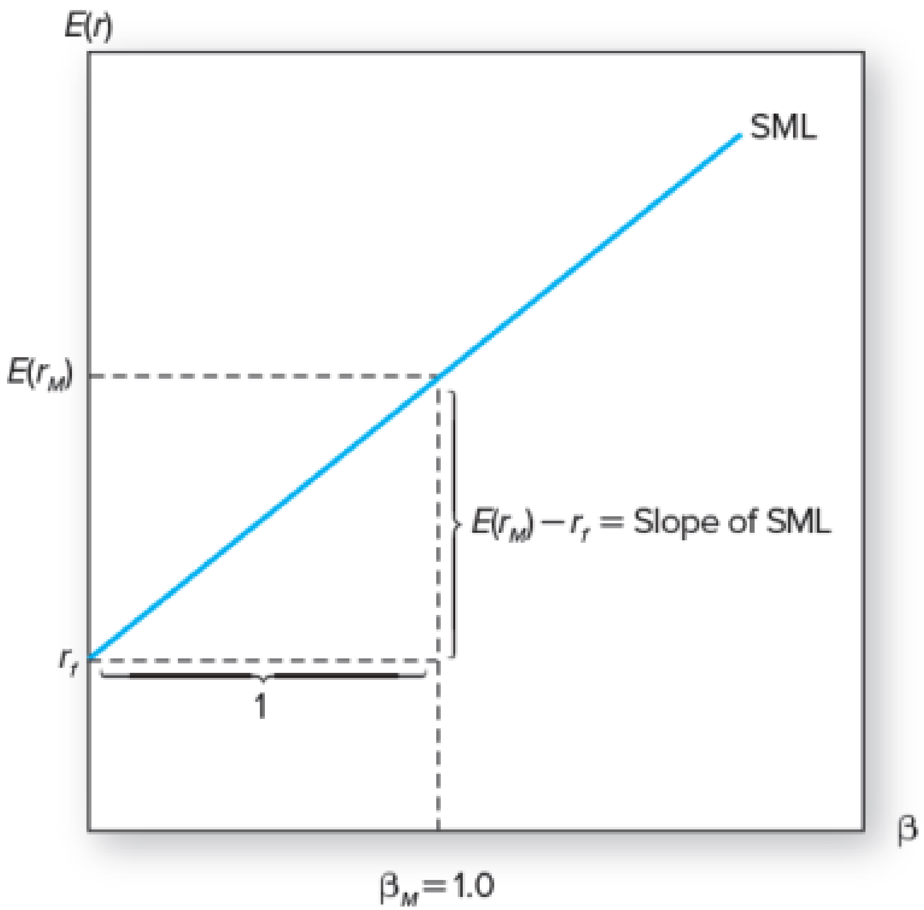
$$E(r_P) = \sum_k w_k E(r_k) \text{ and}$$

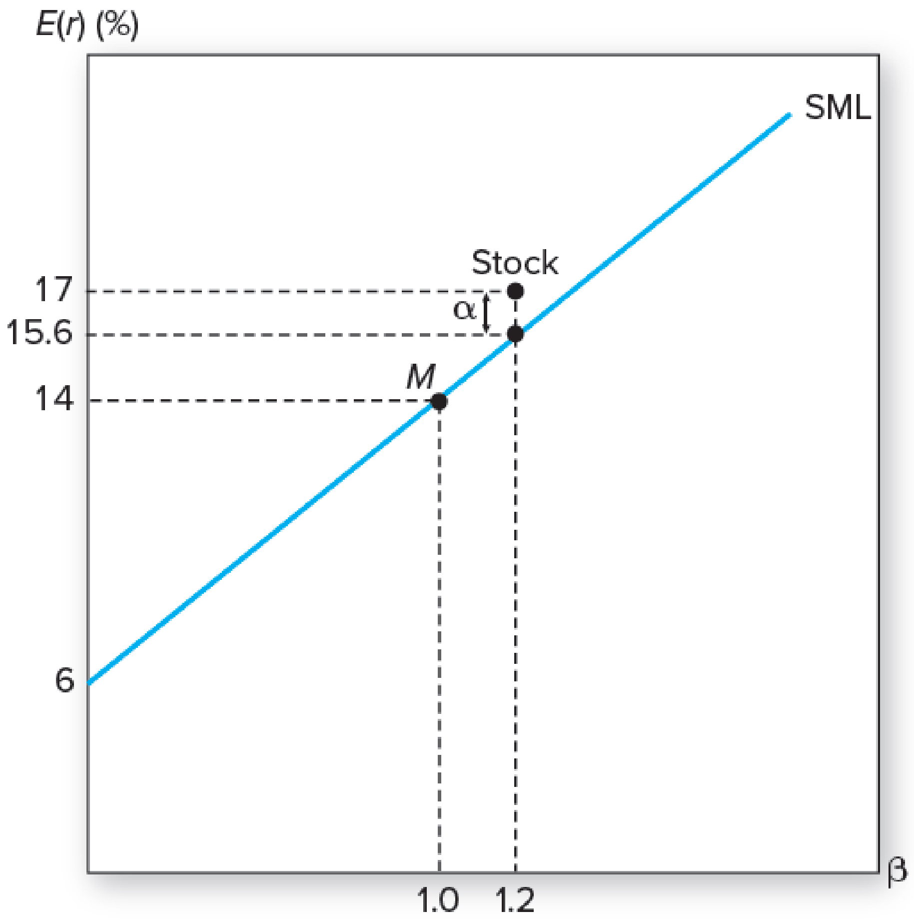
$$\beta_{\mathrm{P}} = \sum_k w_k \beta_k$$

- This also holds for the market portfolio:

$$E(r_M) = r_f + \beta_M \left[E(r_M) - r_f \right]$$

The Capital Asset Pricing Model





- An asset that is not priced according to the CAPM will not line up on the SML.
- The difference between its actual risk premium and its risk premium predicted by the CAPM is called the asset's alpha:

$$E(r_i) - r_f = \alpha_i + \beta_i[E(r_M) - r_f]$$

The Capital Asset Pricing Model

- The Equity Cost of Capital
  - Best expected return available in the market on investments with similar risk.
  - Under the CAPM?
    - ✓ Investments have similar risk if they have the same sensitivity to market risk, as measured by their beta with the market portfolio.
- The Market Portfolio
  - Value-weighted vs. equal-weighted
  - S&P 500, DJIA, NDQ, S&P 1500 Composite, Russel 2000, Wilshire 5000.
- The Market Risk Premium
  - Risk-Free Rate
    - ✓ Maturity vs. investment horizon
  - The Risk Premium
    - ✓ Historical
    - ✓ Expected

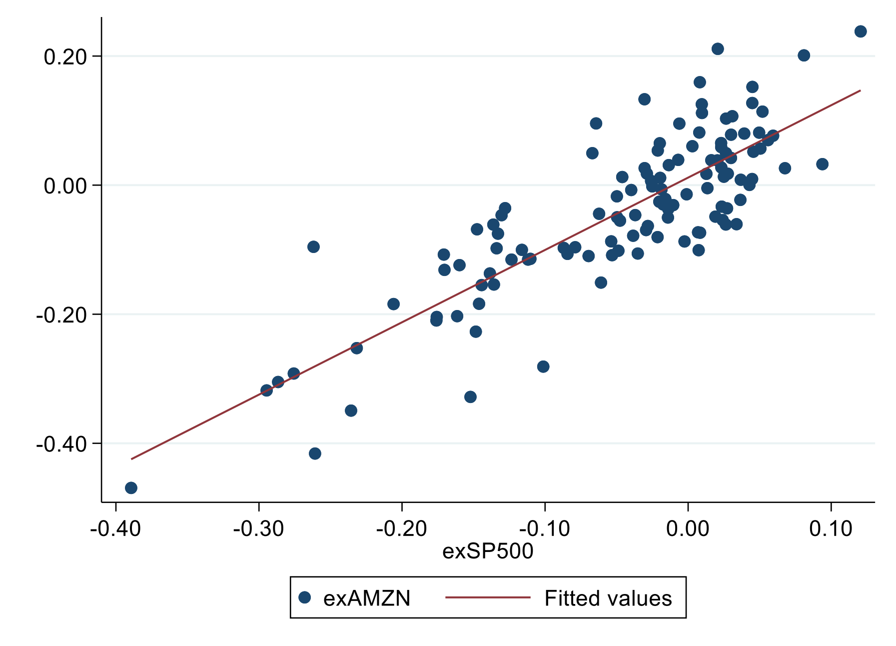
	Period	
S&P 500 Excess Return Versus	1926–2015	1965–2015
One-year Treasury	7.7%	5.0%
Ten-year Treasury*	5.9%	3.9%

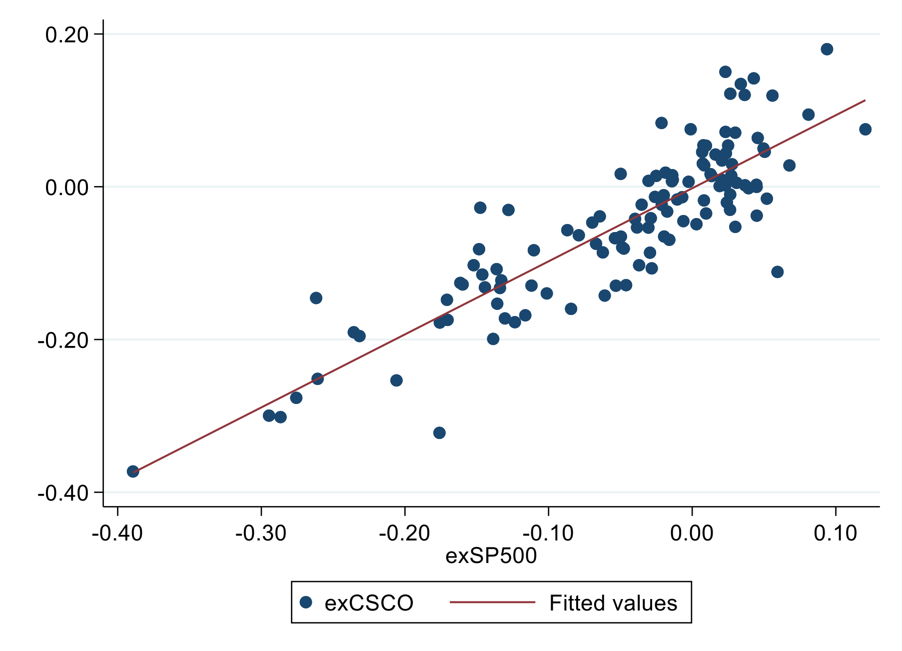
The Capital Asset Pricing Model

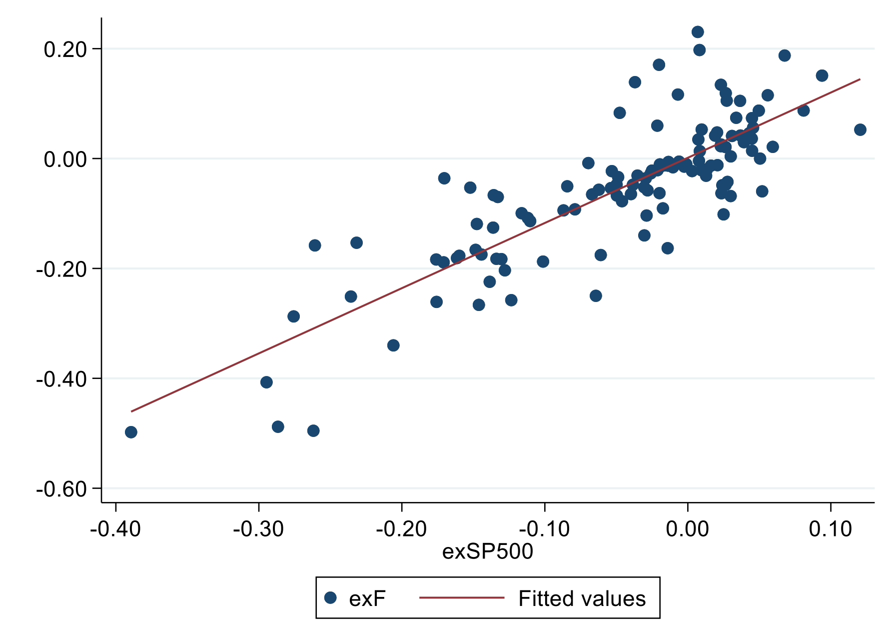
- Beta Estimation
  - Using Historical Returns
    - ✓ What is the assumption?
  - Best-fitting line
    - ✓ Beta corresponds to the slope of the best-fitting line in the plot of the security’s excess returns vs. the market excess return.
  - Linear Regression

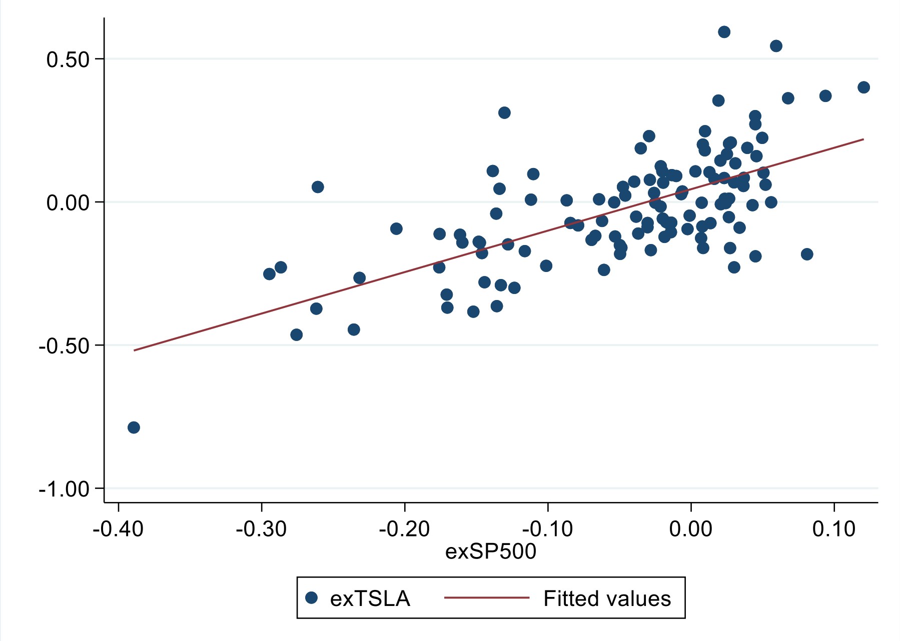
$$(R_i - r_f) = \alpha_i + \beta_i (R_M - r_f) + e_i$$

The Capital Asset Pricing Model









The Capital Asset Pricing Model

❖ Historical Betas

$$r_{i,t}^{e} = \alpha_i + \beta_i \, r_{M,t}^{e} + \varepsilon_{i,t}$$

$$S.E.(\hat{\beta}_i) = \frac{\sqrt{1-R^2}}{\sqrt{T}} \frac{\hat{\sigma}_i}{\hat{\sigma}_M}$$

❖ The precision of an OLS beta estimate can be increased by

- Increasing the number of observations (T),
- By using portfolios instead of individual securities,
- By increasing the frequency of return observations

The Capital Asset Pricing Model

- Should idiosyncratic risk matter for pricing?
- Would you ever buy a stock with negative expected returns?
- How do the betas add up?
- Can you arbitrage stocks that are on the SML?
- Can you arbitrage stocks that are not on the SML?

What is next?

- Multifactor Models
- Arbitrage Pricing Theory
- Reading(s): BKM Ch. 10
- Suggested Problems
  - **Ch.8:** 6, 9-12
  - **Ch.9:** 2, 4, 8, 17-19, 21.
  - **Ch.9-CFA Problems:** 1-2, 8-9,12.

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