Vietnamese-German University

Investments

Finance 2 - BFIN

Dr. Omer CAYIRLI

Lecture 5

Outline

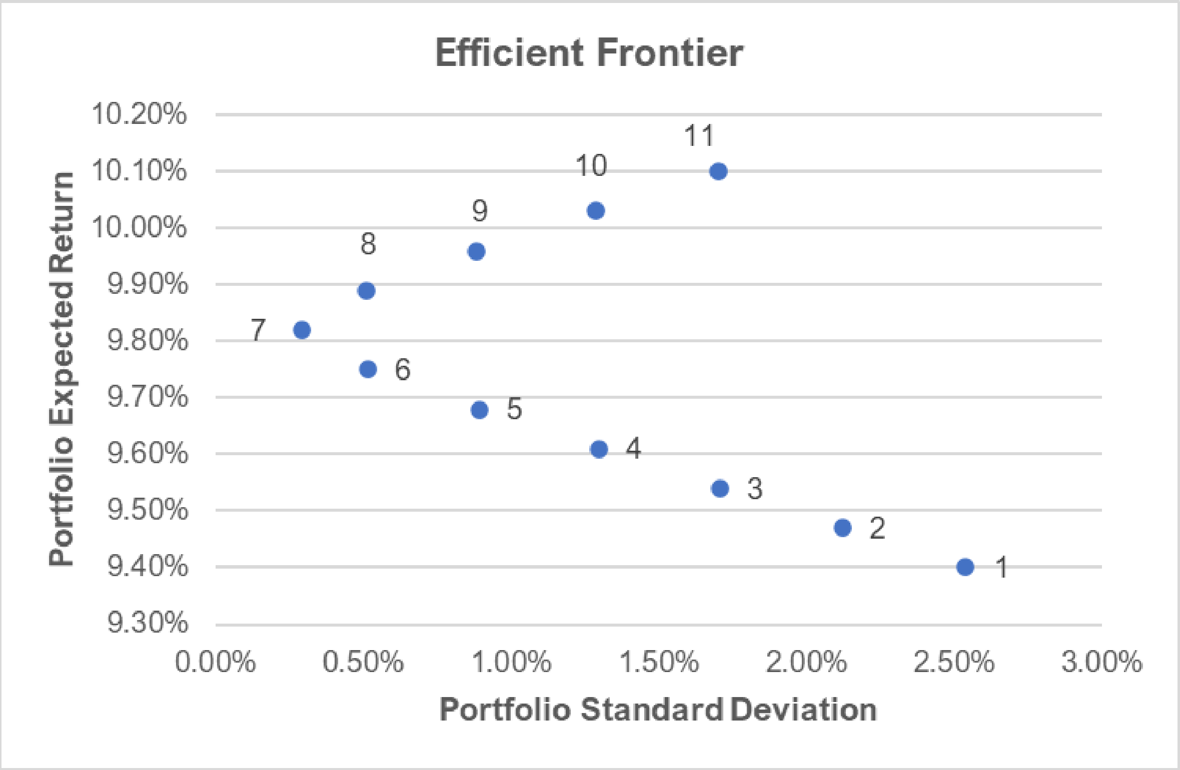
- Portfolio Theory and Practice I
  - Efficient Diversification

Opportunity Set of Risky Assets

- Minimum-variance frontier shows the risk–return opportunities available to the investor.
- Efficient frontier
  - There is an efficient portfolio for each risk level.
  - Portfolios on the efficient frontier represent trade-offs in terms of risk and return.

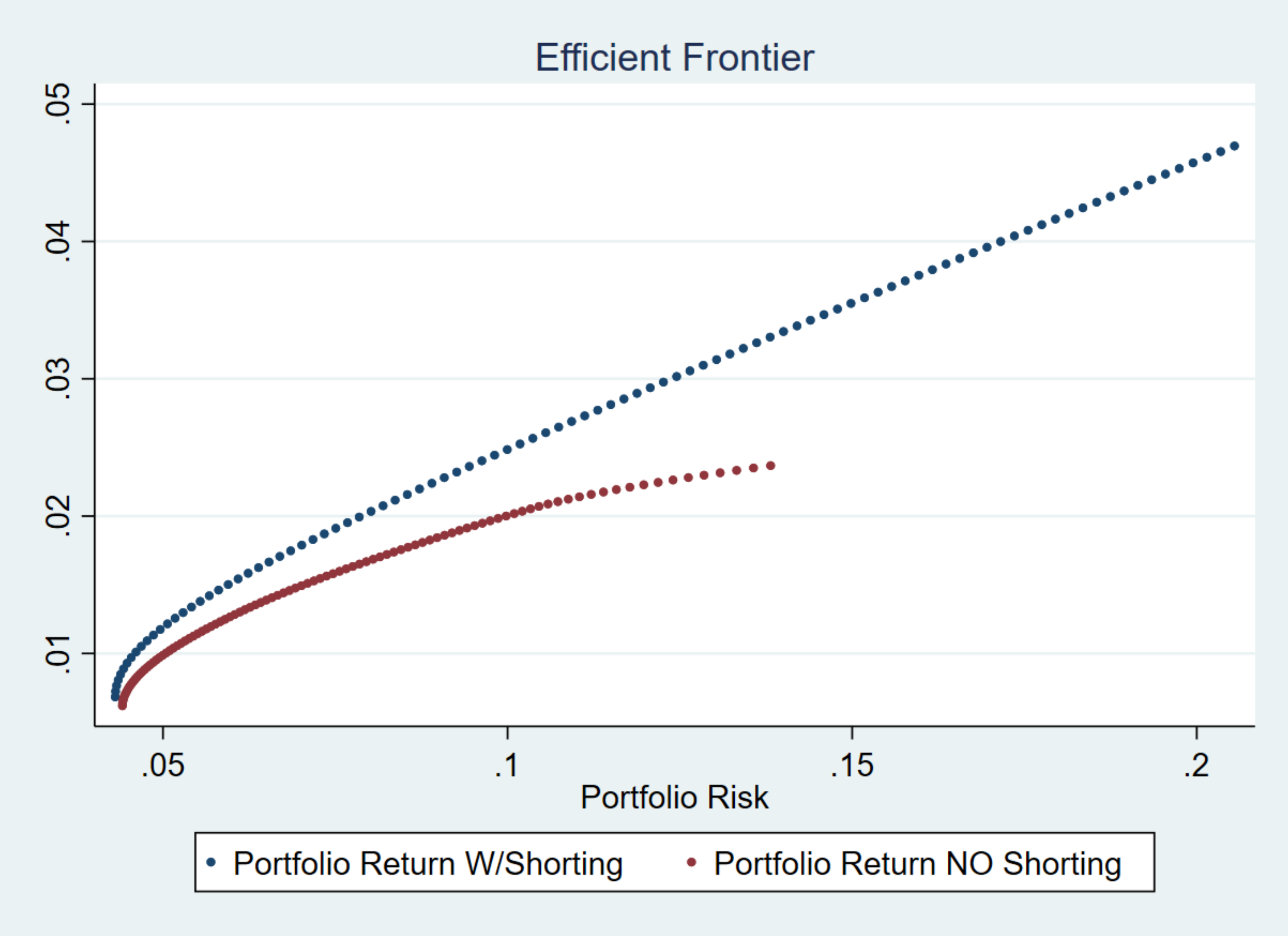
r_A	10.10%	r_B	9.40%	$Cov(r_A, r_B)$	-0.000414
σ_A	1.70%	σ_B	2.54%		
σ_A^2	0.000289	σ_B^2	0.00064516		

w_A	w_B	r_p	σ_P^2	σ_P
100%	0%	10.100%	0.000289	1.70%
90%	10%	10.030%	0.000166	1.29%
80%	20%	9.960%	0.000078	0.88%
70%	30%	9.890%	0.000026	0.51%
60%	40%	9.820%	0.000009	0.29%
50%	50%	9.750%	0.000027	0.52%
40%	60%	9.680%	0.000080	0.89%
30%	70%	9.610%	0.000168	1.30%
20%	80%	9.540%	0.000292	1.71%
10%	90%	9.470%	0.000451	2.12%
0%	100%	9.400%	0.000645	2.54%



Opportunity Set of Risky Assets

	Global MV Portfolios	
	No Short	With Short
AMD	0.00%	-5.53%
AMZN	9.39%	12.12%
FORD	0.00%	-4.49%
JPM	2.79%	7.59%
NVDA	0.00%	1.04%
PFE	38.25%	38.18%
TGT	18.82%	20.74%
XOM	30.75%	30.35%
$E(r_p)$	0.62%	0.68%
σ_p	4.41%	4.31%
r_f	0.41%	0.41%
Sharpe Ratio	0.0480	0.0641



Capital Allocation to Risk-free and Risky Assets

- Complete portfolio consists of the risky asset and the risk-free asset.
  - A portfolio risk closer to the desired level can be achieved by changing the weights of risk-free asset and risky assets in the complete portfolio.

$$w_{rf} + w_{risky} = 1 \qquad w_{rf} = 1 - w_{risky}$$

$$E(r_P) = (1 - w_{risky}) * r_f + w_{risky} * E(r_{risky})$$

$$E(r_P) = r_f + w_{risky} * [E(r_{risky}) - r_f]$$

$$\boldsymbol{\sigma^2(r_p)} = w_{rf}^2 * \sigma^2(r_f) + w_{risky}^2 * \sigma^2(r_{risky}) + 2 * w_{rf} w_{risky} Cov(r_f, r_{risky})$$

Since $\sigma^2(r_f) = 0$ and $Cov(r_f, r_{risky}) = 0$,

$$\boldsymbol{\sigma^2(r_p)} = w_{risky}^2 * \sigma^2(r_{risky})$$

Capital Allocation to Risk-free and Risky Assets

<u>If</u> $\sigma^2(r_p) = w_{risky}^2 * \sigma^2(r_{risky})$ <u>then</u> $\sigma(r_p) = w_{risky} * \sigma(r_{risky})$

Thus, $w_{risky} = \dfrac{\sigma(r_p)}{\sigma(r_{risky})}$

$$E(r_P) = r_f + w_{risky} * [E(r_{risky}) - r_f]$$

$$E(r_P) = r_f + \frac{\sigma(r_p)}{\sigma(r_{risky})} * [E(r_{risky}) - r_f]$$

$$E(r_P) = r_f + \left[\frac{E(r_{risky}) - r_f}{\sigma(r_{risky})}\right] * \sigma(r_p)$$

<u>Capital Allocation Line</u>

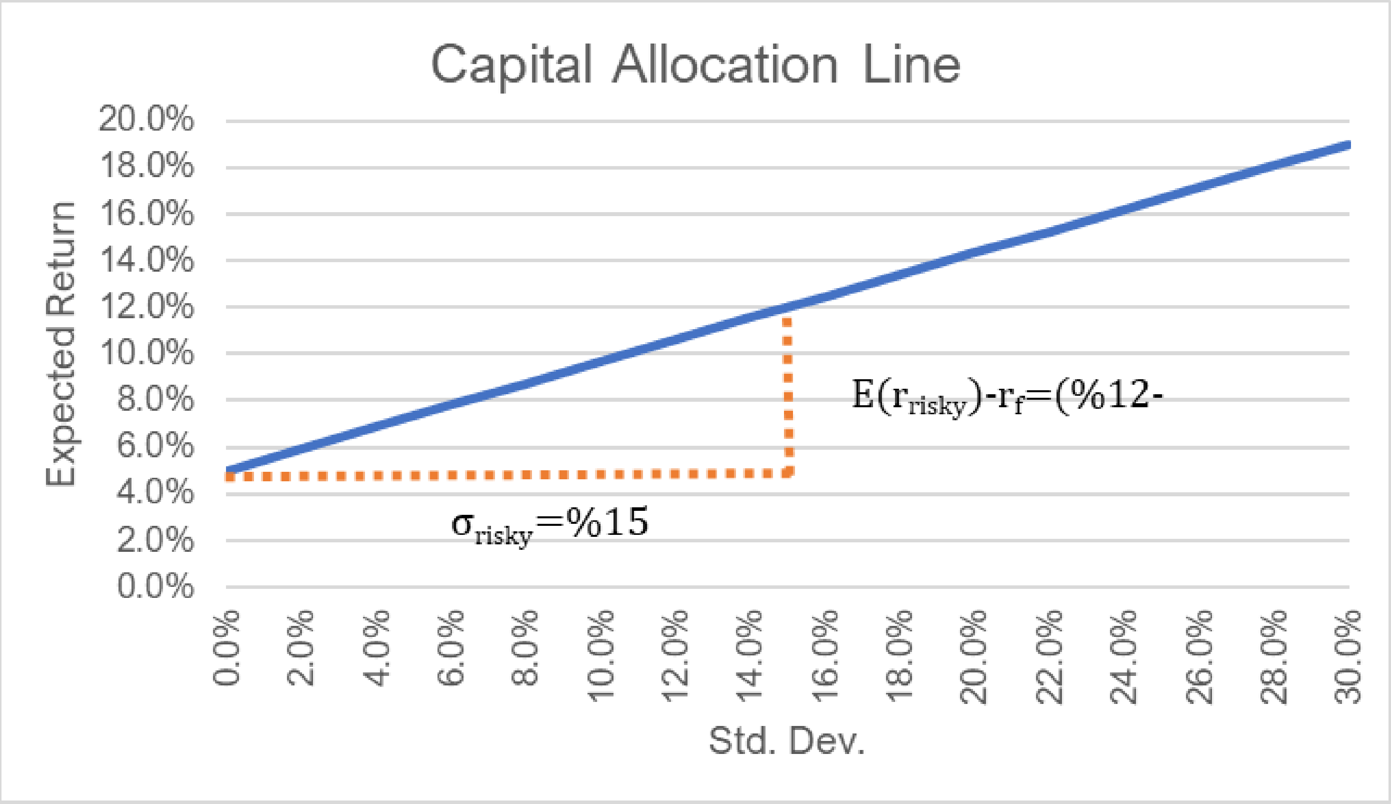
Capital Allocation to Risk-free and Risky Assets

- Suppose the risk-free interest rate is 5%, the expected return and the standard deviation of the risky portfolio are 12% and 15%, respectively. Then;

$$E(r_P) = r_f + \left[\frac{E(r_{risky}) - r_f}{\sigma(r_{risky})}\right] * \sigma(r_p)$$

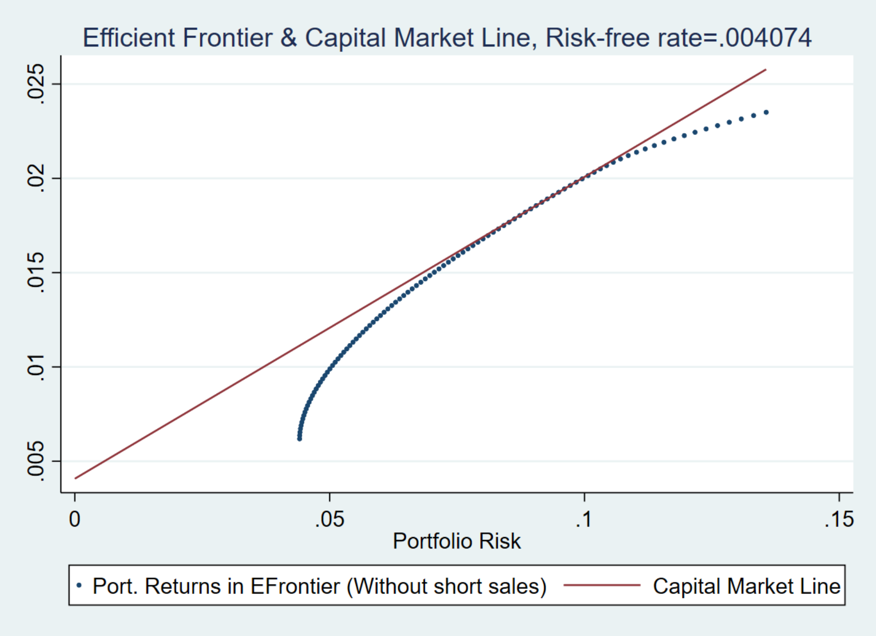
$$E(r_P) = 0{,}05 + \left[\frac{0{,}12 - 0{,}05}{0{,}15}\right] * \sigma(r_p)$$

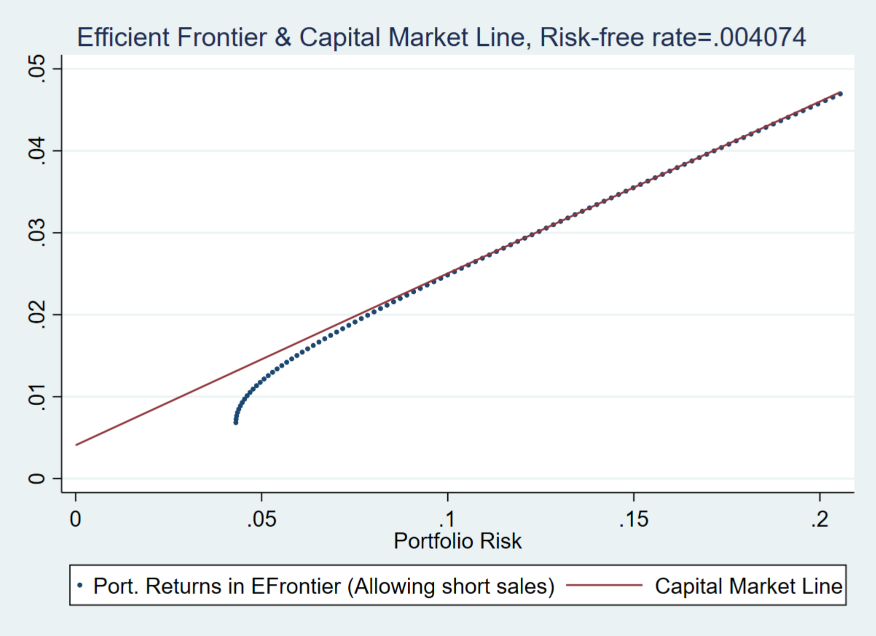
$$E(r_P) = 0{,}05 + 0{,}4667 * \sigma(r_p)$$



Opportunity Set of Risky Assets

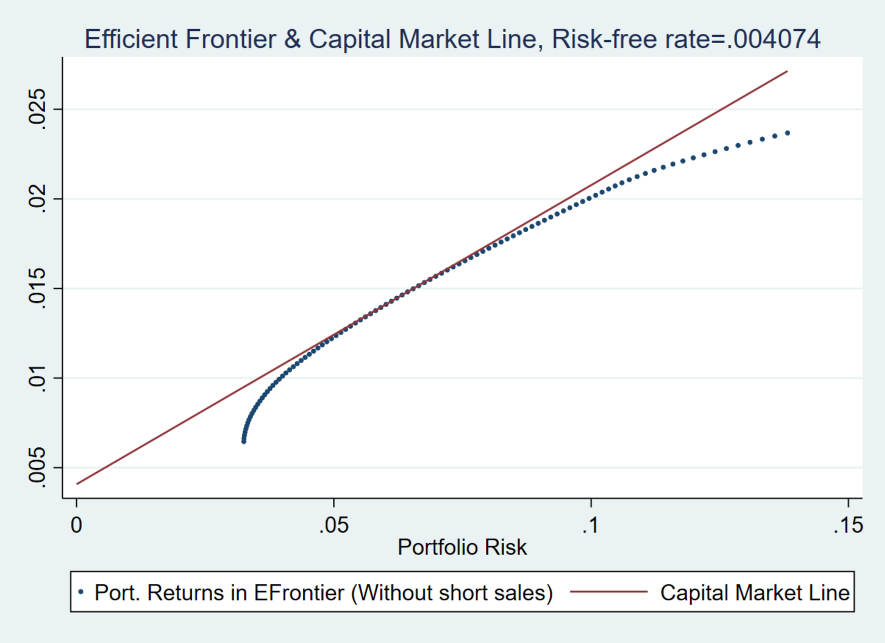
	Global MV Portfolios		Optimal Risk Portfolios	
	No Short	With Short	No Short	With Short
AMD	0.00%	-5.53%	0.00%	-34.75%
AMZN	9.39%	12.12%	43.48%	70.83%
FORD	0.00%	-4.49%	0.00%	-58.61%
JPM	2.79%	7.59%	11.42%	80.15%
NVDA	0.00%	1.04%	45.10%	76.25%
PFE	38.25%	38.18%	0.00%	-71.24%
TGT	18.82%	20.74%	0.00%	16.91%
XOM	30.75%	30.35%	0.00%	20.46%
$E(r_p)$	0.62%	0.68%	1.91%	3.36%
σ_p	4.41%	4.31%	9.38%	14.09%
r_f	0.41%	0.41%	0.41%	0.41%
Sharpe Ratio	0.0480	0.0641	0.1600	0.2097

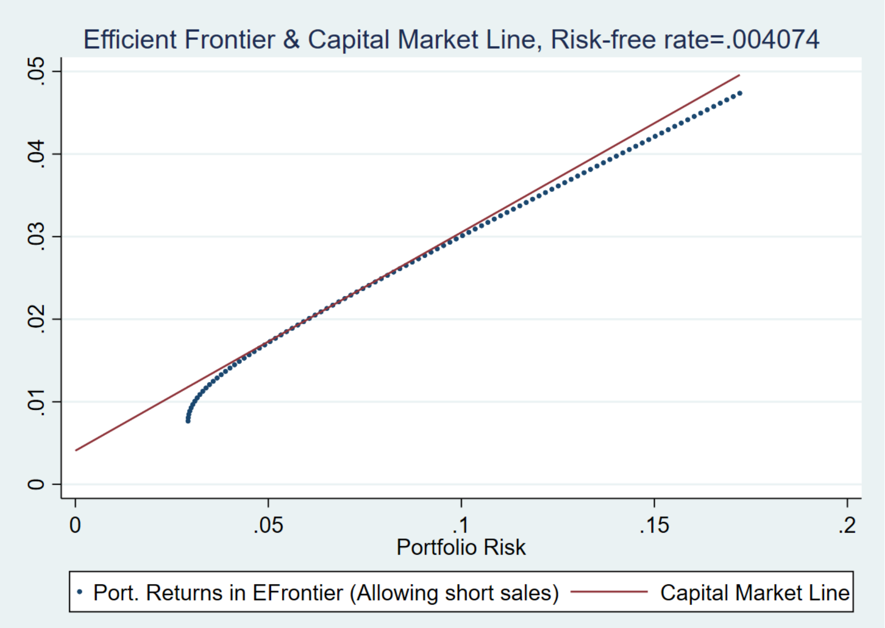




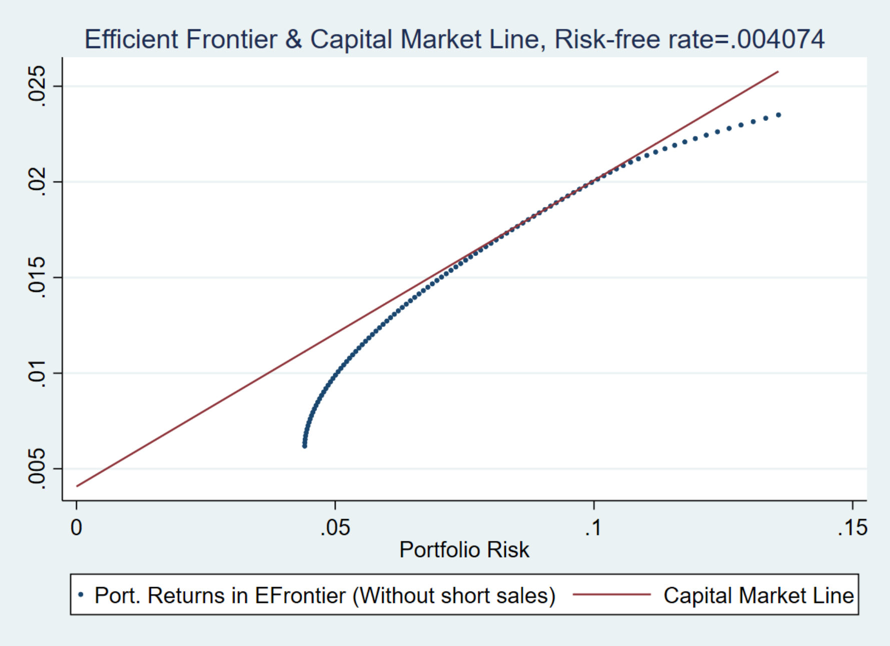
Opportunity Set of Risky Assets

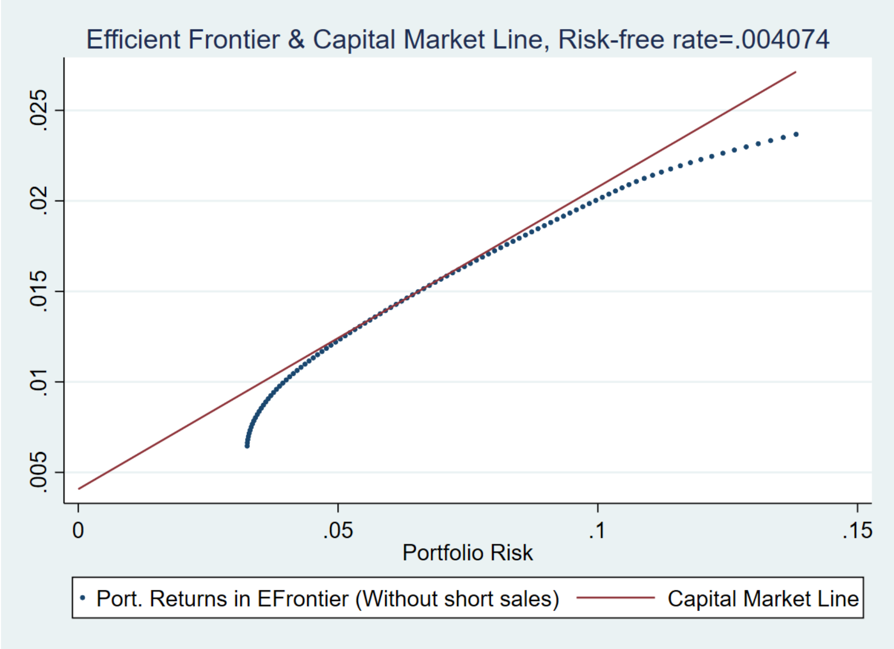
	Global MV Portfolios				Optimal Risk Portfolios			
	No Short	With Short	No Short	With Short	No Short	With Short	No Short	With Short
SP500			29.84%	57.82%			0.00%	-84.30%
NDQ100			0.00%	13.84%			0.00%	147.04%
EEM			0.00%	-21.19%			0.00%	-81.67%
XAU			41.40%	42.21%			36.33%	67.76%
AMD	0.00%	-5.53%	0.00%	-5.36%	0.00%	-34.75%	0.00%	-14.61%
AMZN	9.39%	12.12%	0.00%	-1.13%	43.48%	70.83%	26.28%	10.16%
FORD	0.00%	-4.49%	0.00%	-5.19%	0.00%	-58.61%	0.00%	-18.66%
JPM	2.79%	7.59%	0.00%	0.85%	11.42%	80.15%	8.75%	44.57%
NVDA	0.00%	1.04%	0.00%	-1.72%	45.10%	76.25%	28.63%	21.85%
PFE	38.25%	38.18%	13.72%	8.34%	0.00%	-71.24%	0.00%	-21.84%
TGT	18.82%	20.74%	5.43%	3.88%	0.00%	16.91%	0.00%	4.36%
XOM	30.75%	30.35%	9.60%	7.64%	0.00%	20.46%	0.00%	25.34%
$E(r_p)$	0.62%	0.68%	0.65%	0.77%	1.91%	3.36%	1.45%	2.07%
σ_p	4.41%	4.31%	3.25%	2.92%	9.38%	14.09%	6.22%	6.27%
r_f	0.41%	0.41%	0.41%	0.41%	0.41%	0.41%	0.41%	0.41%
Sharpe Ratio	0.0480	0.0641	0.0733	0.1231	0.1600	0.2097	0.1669	0.2644

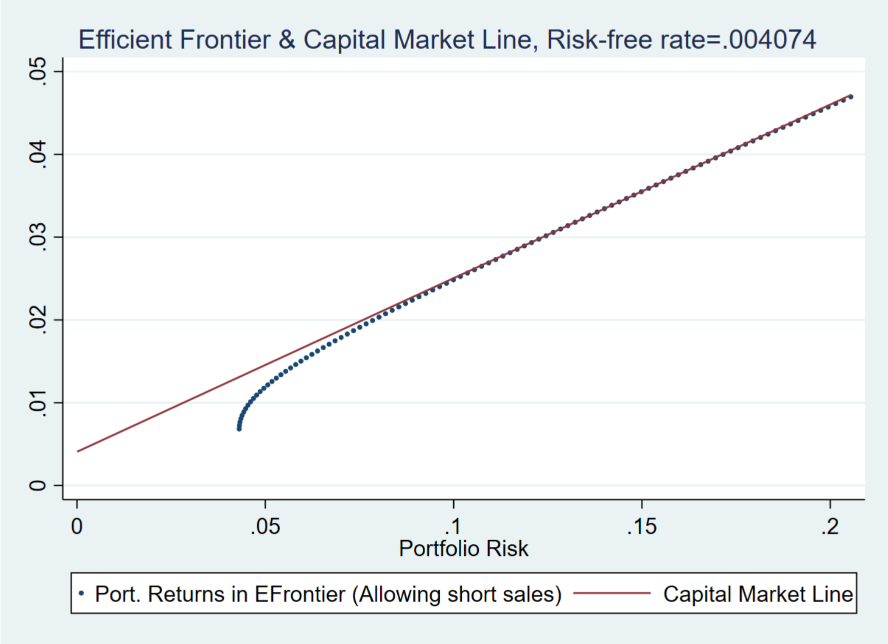


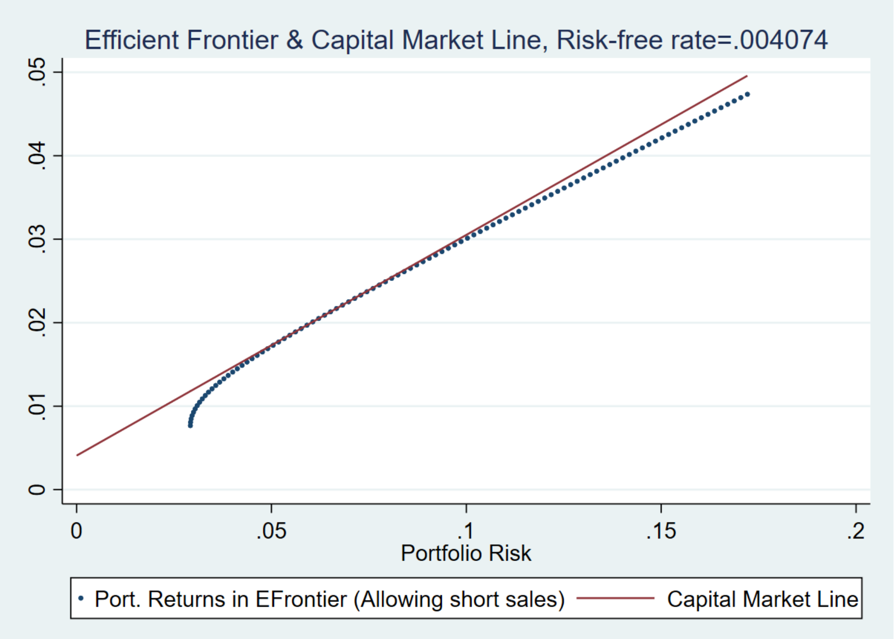


Opportunity Set of Risky Assets





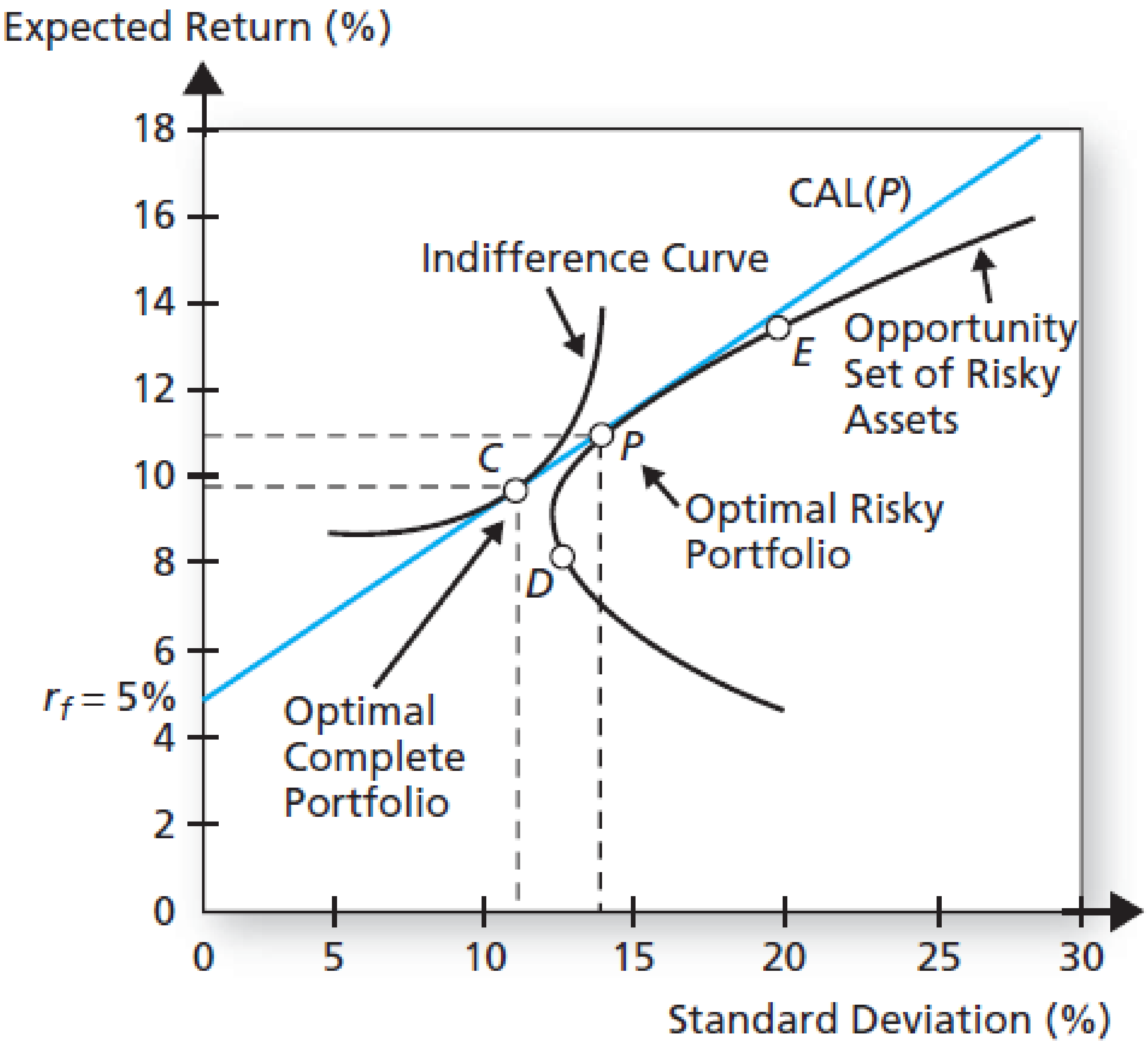




Capital Allocation to Risk-free and Risky Assets

- ❖ Each CAL is uniquely identified by its slope.
- ❖ The optimal risky portfolio is the tangency portfolio
  - ➢ The unique portfolio with the highest Sharpe-Ratio.
- ❖ The optimal risky portfolio does not involve the degree of risk aversion of any individual investor.
- ❖ Every investor, regardless of her/his level of risk aversion, will agree on the best CAL, and allocate her/his wealth between risk-free asset and the optimal risky portfolio.
- ❖ The portion invested in the optimal risky portfolio, however, will depend on each investor’s degree of risk.

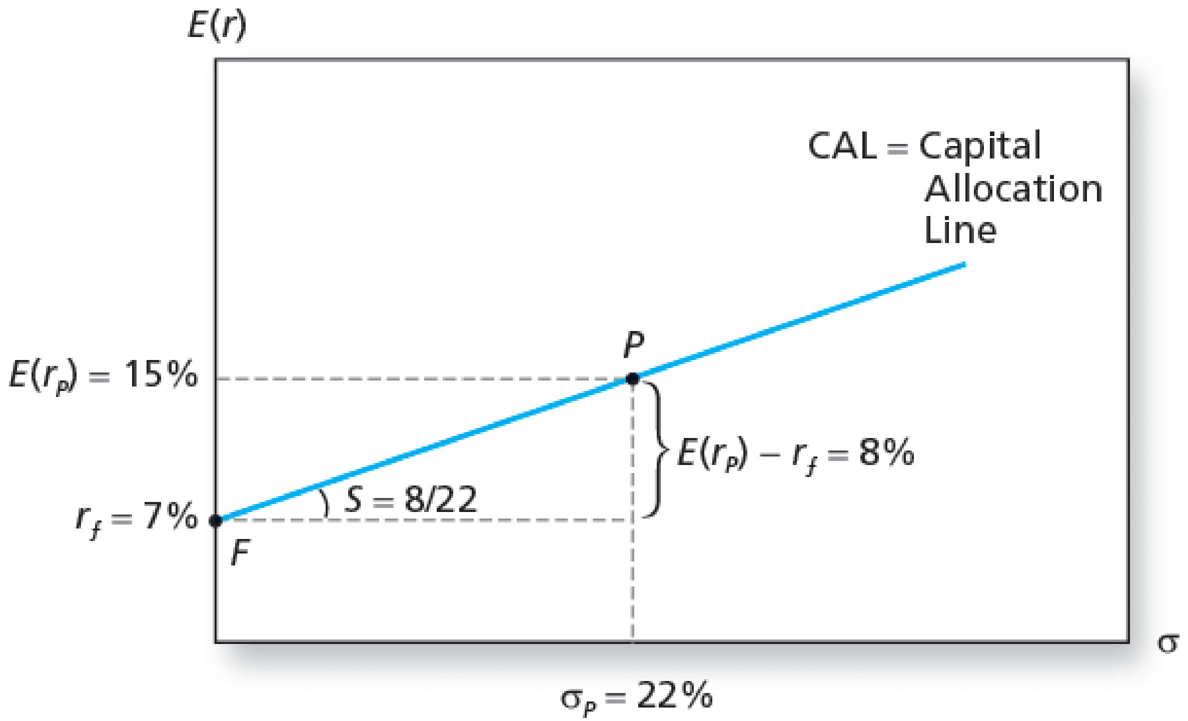
Capital Allocation to Risk-free and Risky Assets



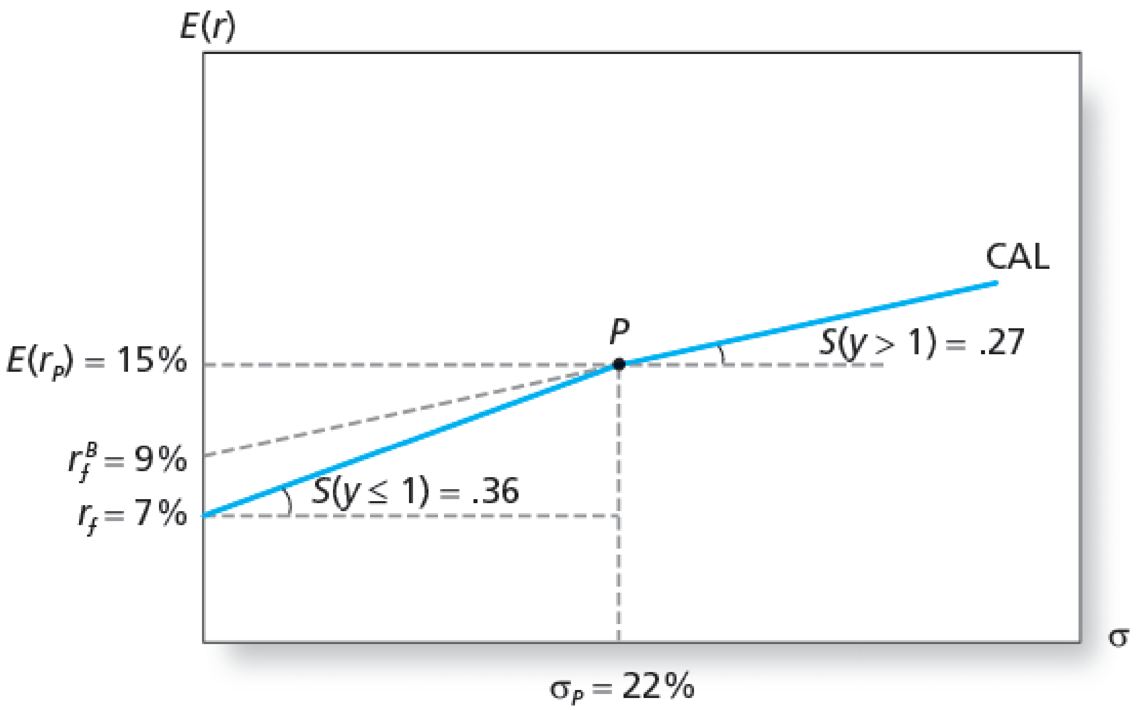
The Opportunity Set with Different Borrowing and Lending Rates

$r_f = 7\%$ $\sigma_{rf} = 0\%$

$E(r_p) = 15\%$ $\sigma_p = 22\%$



Lend at r_f = 7% and borrow at r_f = 9%



Lending range slope = 8/22 = 0.36

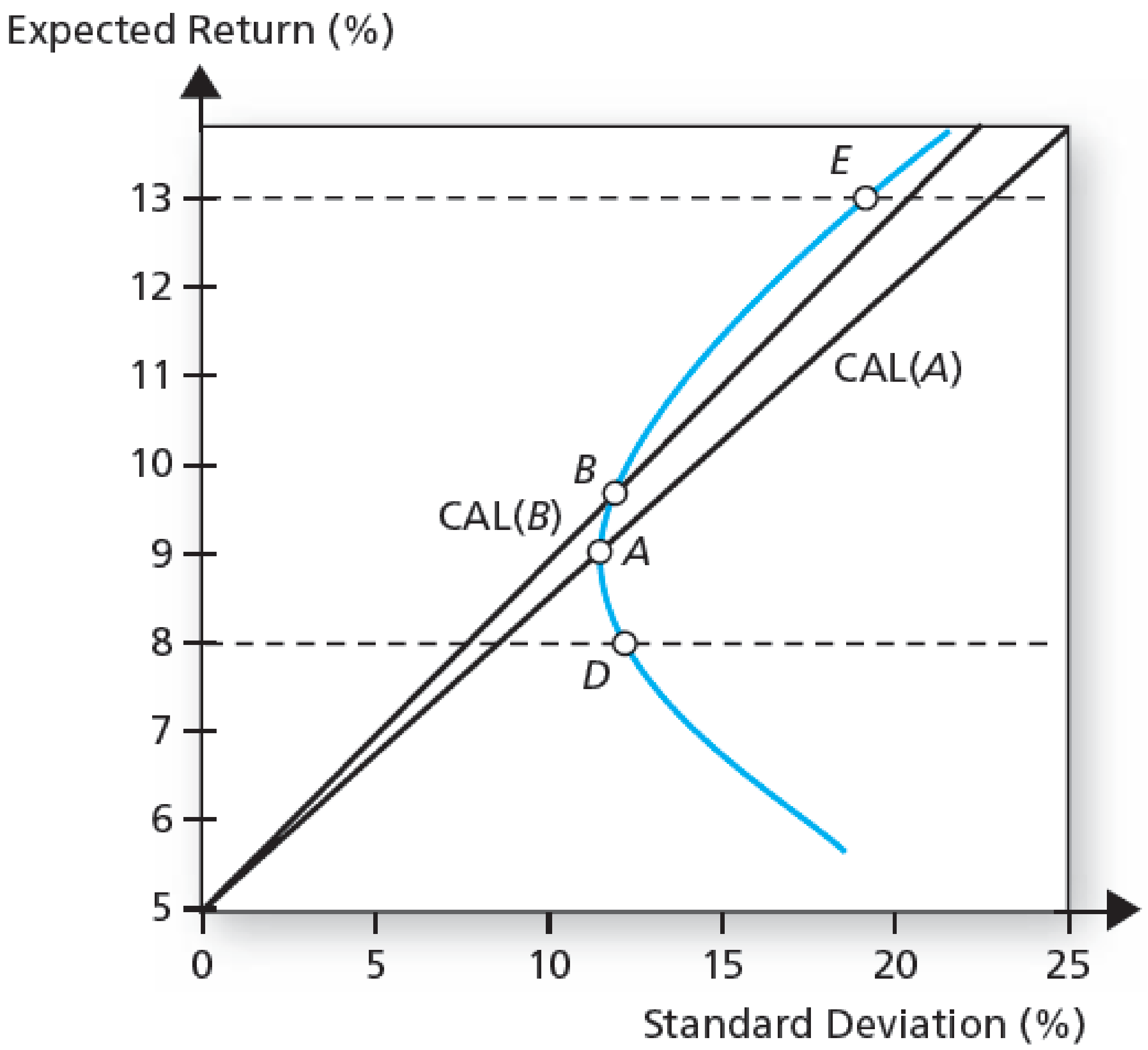
Borrowing range slope = 6/22 = 0.27

The Investment Decision

- Top-down process with 3 steps:
  - Capital allocation: risky portfolio and risk-free asset
  - Asset allocation: across broad asset classes
  - Security selection: individual assets within an asset class

The Investment Decision

❖ The Opportunity Set of the Debt and Equity Funds



Portfolio A: *82% in bonds and 18% in stocks*

$E(r_A) = 8.9\%$
$\sigma_A = 11.45\%$

Portfolio B: *70% in bonds and 30% in stocks*

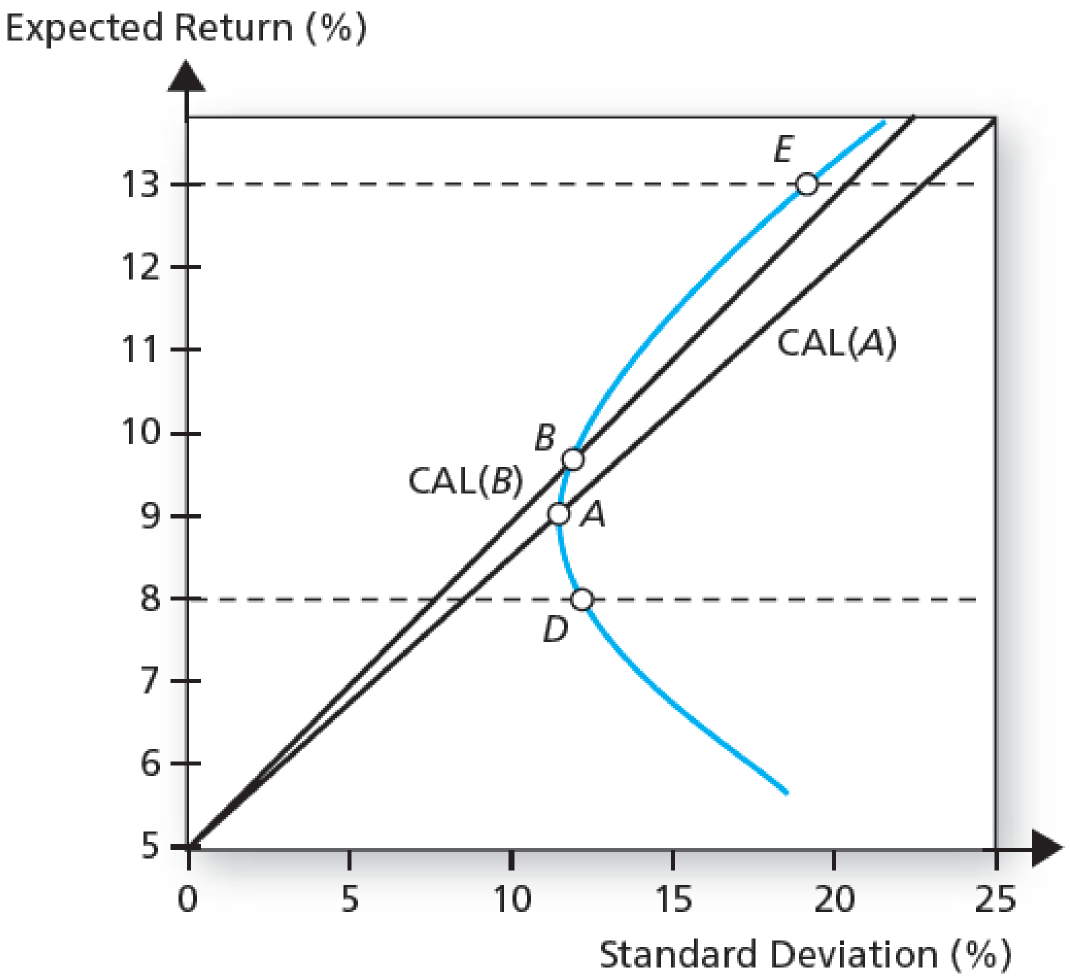
$E(r_B) = 9.5\%$
$\sigma_B = 11.70\%$

- Maximize the slope of the CAL for any possible portfolio, P
- The objective function is the slope:

$$S_p = \frac{E(r_p) - r_f}{\sigma_p}$$

The Investment Decision

❖ The Opportunity Set of the Debt and Equity Funds



$$\boldsymbol{Portfolio A}$$

$$E(r_A) = 8.9\%$$

$$\sigma_A = 11.45\%$$

$$S_A = \frac{E(r_A) - r_f}{\sigma_A} = \frac{8.9\% - 5\%}{11.45\%} = 0.34$$

$$\boldsymbol{Portfolio B}$$

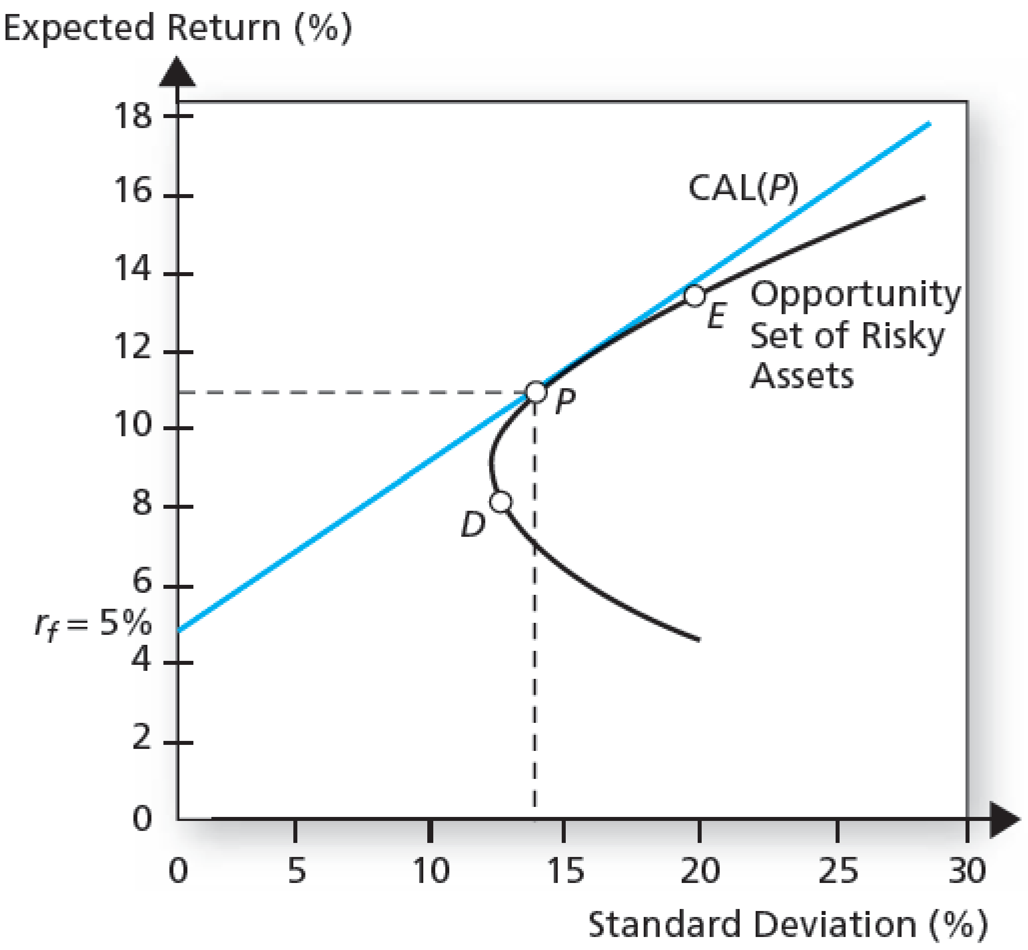
$$E(r_B) = 9.5\%$$

$$\sigma_B = 11.70\%$$

$$S_B = \frac{E(r_B) - r_f}{\sigma_B} = \frac{9.5\% - 5\%}{11.70\%} = 0.38$$

The Investment Decision

❖ The Opportunity Set of the Debt and Equity Funds



$$Optimal\ Risky\ Portfolio$$

$$E(r_P) = 11\%$$
$$\sigma_P = 14.2\%$$

$$S_P = \frac{E(r_P) - r_f}{\sigma_P}$$
$$= \frac{11\% - 5\%}{14.2\%} = 0.42$$

The Investment Decision

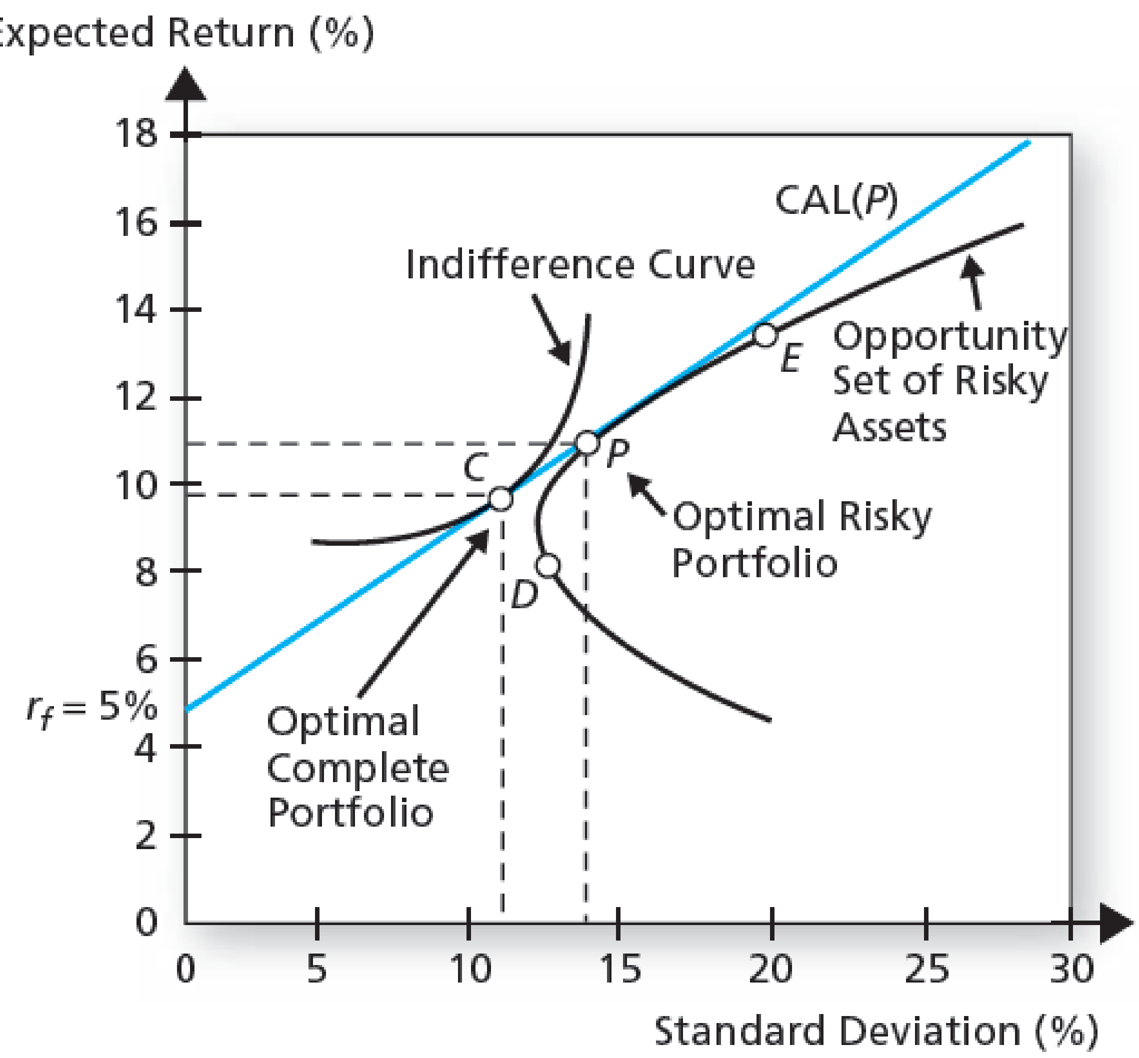
Port Mean	11.00%			Bonds	Equity
Port Variance	0.0202		Mean	8.00%	13.00%
Port Std Dev	14.20%		StdDev	12.00%	20.00%
RF	5.00%				
Sharpe Ratio	0.4226				
			Weights	40.00%	60.00%
Correlation	0.3				
			Constraints		
			1	1	1
			Portfolio Variance		
				Bonds	*Equity*
		40.00%	Bonds	0.002304	0.001728
		60.00%	Equity	0.001728	0.014400
			0.020160	0.004032	0.016128
			Covariance Matrix		
				Bonds	*Equity*
			Bonds	0.014400	0.007200
			Equity	0.007200	0.040000

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + \cdots + w_N E(r_N)$$

$$\boldsymbol{\sigma^2(r_p)} = w_1^2 * \sigma^2(r_1) + w_2^2 * \sigma^2(r_2) + 2 * w_1 w_2 Cov(r_1, r_2)$$

$$\rho_{i,j} = \frac{cov(r_i, r_j)}{\sigma_i \sigma_j}$$

The Investment Decision



$$U = E(r) - \frac{1}{2}A\sigma^2$$

Expected return on the complete portfolio:

$$E(r_C) = r_f + y[E(r_P) - rf]$$

$$\sigma_C = y \times \sigma_P$$

$$\underset{y}{\text{Max}}\, U = E(r_C) - \frac{1}{2}A\sigma_C^2$$

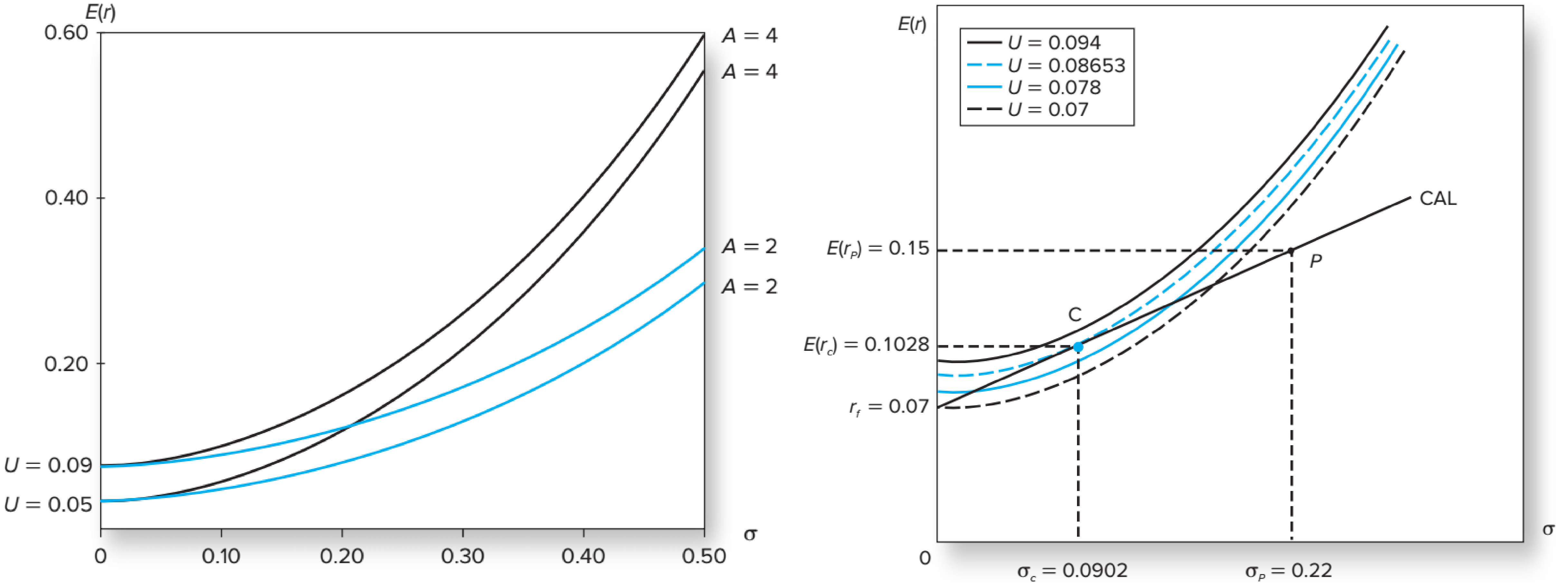
$$\underset{y}{\text{Max}}\, U = r_f + y[E(r_P) - rf] - \frac{1}{2}Ay^2\sigma_p^2$$

$$y^* = \frac{E(r_P) - rf}{A\sigma_p^2}$$

Optimal Allocation to P: A=4

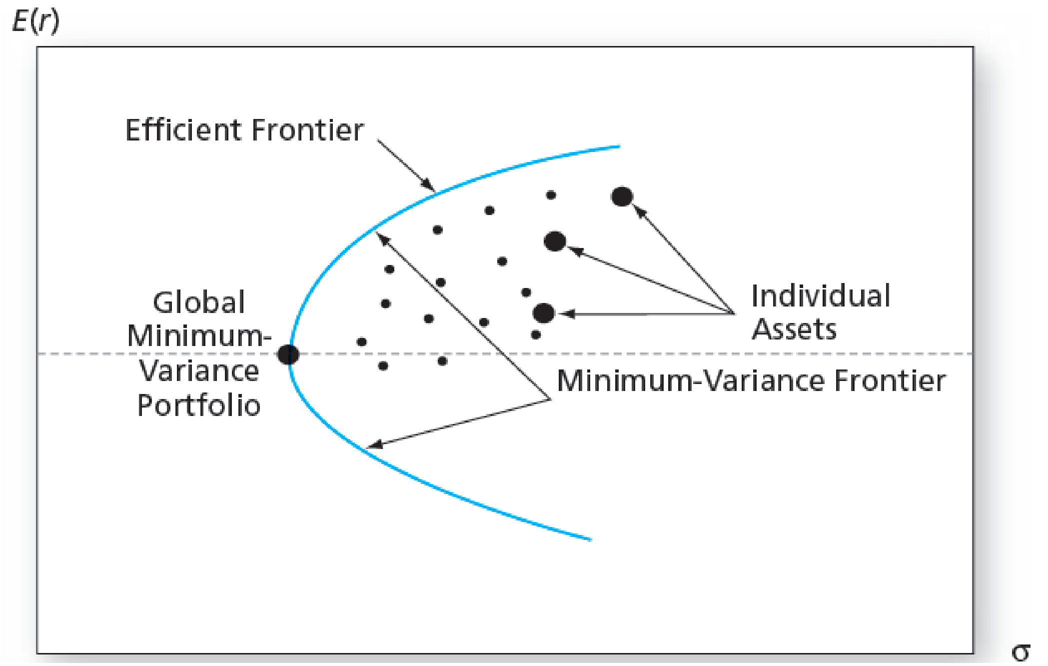
$$y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{11\% - 5\%}{4 \times (14.2\%)^2} = 0.7439$$

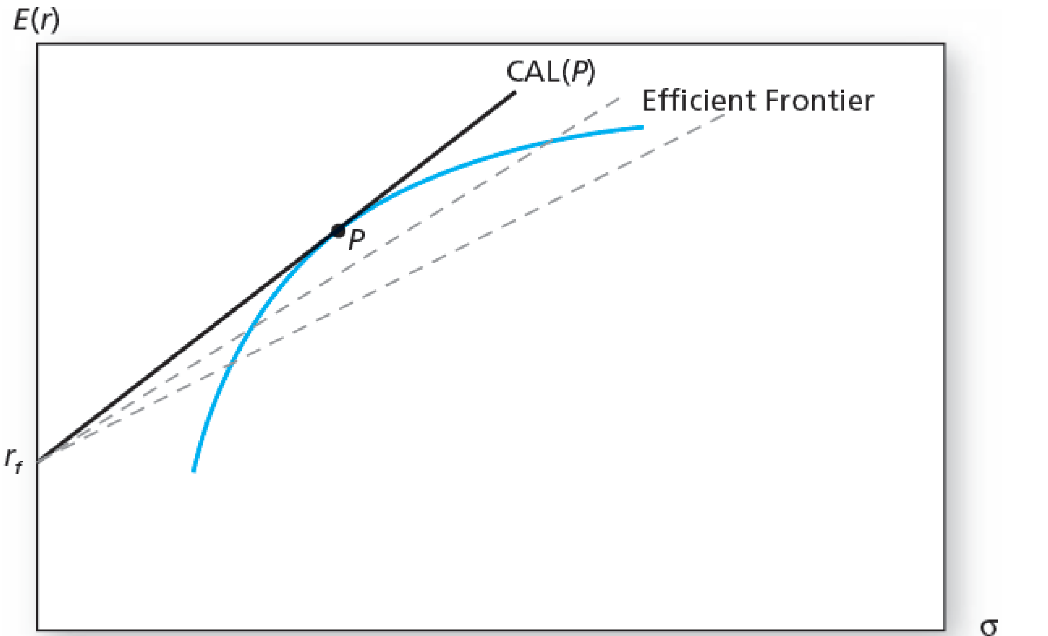
Markowitz Portfolio Optimization Model



Markowitz Portfolio Optimization Model

- ❖ Security selection
  - ➢ Determine the risk-return opportunities available
  - ➢ All portfolios that lie on the minimum-variance frontier from the global minimum-variance portfolio and upward provide the best risk-return combinations
- ❖ Search for the CAL with the highest reward-to-variability ratio
  - ➢ Everyone invests in P, regardless of their degree of risk aversion
    - ✓ More risk-averse investors put less in P
    - ✓ Less risk-averse investors put more in P





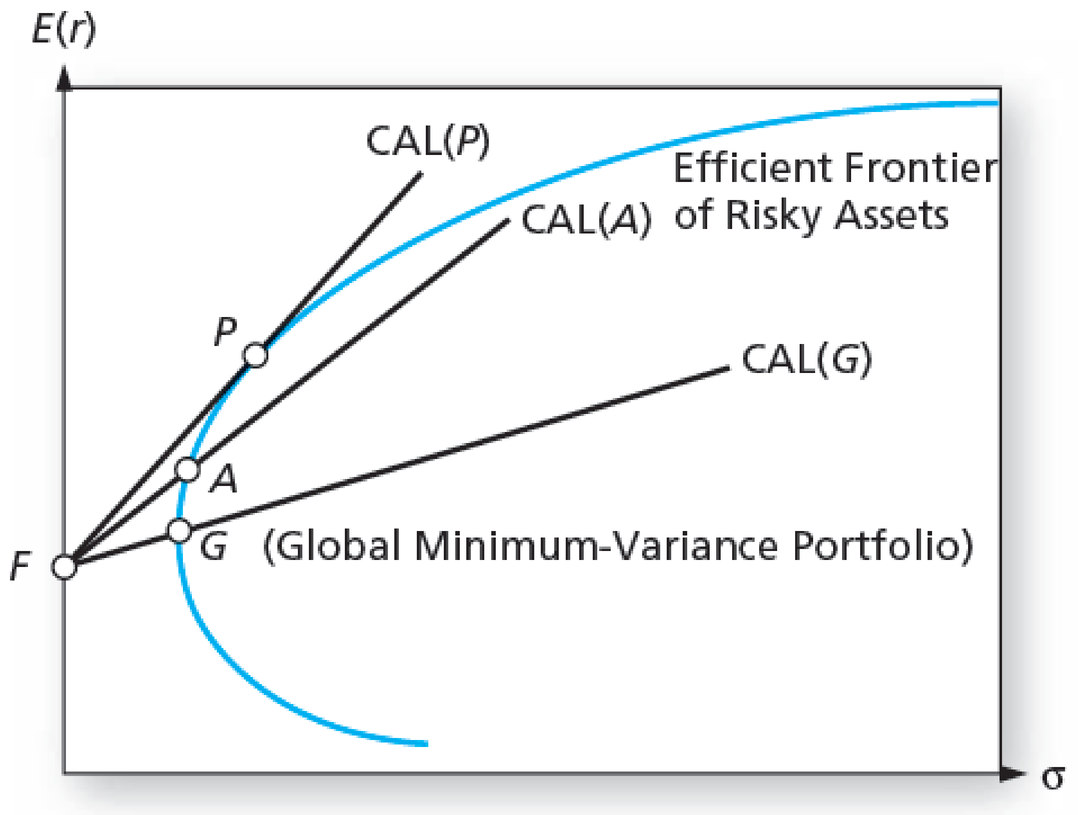
Markowitz Portfolio Optimization Model

❖ Capital Allocation and the Separation Property

➢ The portfolio choice problem may be separated into two independent tasks

✓ Determination of the optimal risky portfolio is purely technical

✓ Allocation of the complete portfolio to risk-free versus the risky portfolio depends on personal preference



Markowitz Portfolio Optimization Model

- Optimal Portfolios and Nonnormal Returns
  - Fat-tailed distributions can result in extreme values of VaR and ES.
  - If other portfolios provide sufficiently better VaR and ES values than the mean-variance efficient portfolio, we may prefer these when faced with fat-tailed distributions.

Markowitz Portfolio Optimization Model

- Passive Strategies: The Capital Market Line
  - Determination of the assets to include in P may result from a passive or an active strategy.
  - A passive strategy describes a portfolio decision that avoids any direct or indirect security analysis.
  - A natural candidate for a passively held risky asset would be a well-diversified portfolio of common stocks.

	Average Annual Returns		U.S. Equity Market		
Period	U.S. Equity Market	1-Month T-Bills	Excess Return	Standard Deviation	Sharpe Ratio
1927–2018	11.72	3.38	8.34	20.36	0.41
1927–1949	9.40	0.92	8.49	26.83	0.32
1950–1972	14.00	3.14	10.86	17.46	0.62
1973–1995	13.38	7.26	6.11	18.43	0.33
1996–2018	10.10	2.21	7.89	18.39	0.43

Markowitz Portfolio Optimization Model

	Monthly	Annualized
Port Mean	1.88%	25.05%
Port Variance	0.0087	0.1044
Port Std Dev	9.33%	32.30%
RF	0.41%	5.00%
Sharpe Ratio	0.1580	

	AMD	AMZN	FORD	JPM	NVDA	PFE	TGT	XOM
Mean	0.95%	1.69%	0.15%	0.87%	2.34%	0.26%	0.73%	0.64%
StdDev	16.80%	10.45%	13.17%	7.79%	13.82%	5.98%	7.81%	6.57%
Weights	0.00%	43.44%	0.00%	12.05%	44.51%	0.00%	0.00%	0.00%

Constraints

1	1	1	1	1	1	1	1	1

Portfolio Variance

		AMD	*AMZN*	*FORD*	*JPM*	*NVDA*	*PFE*	*TGT*	*XOM*
0.00%	AMD	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
43.44%	AMZN	0.000000	0.002060	0.000000	0.000086	0.001144	0.000000	0.000000	0.000000
0.00%	FORD	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
12.05%	JPM	0.000000	0.000086	0.000000	0.000088	0.000153	0.000000	0.000000	0.000000
44.51%	NVDA	0.000000	0.001144	0.000000	0.000153	0.003782	0.000000	0.000000	0.000000
0.00%	PFE	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.00%	TGT	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.00%	XOM	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
	0.008697	0.000000	0.003290	0.000000	0.000327	0.005079	0.000000	0.000000	0.000000

Covariance Matrix

	AMD	*AMZN*	*FORD*	*JPM*	*NVDA*	*PFE*	*TGT*	*XOM*
AMD	0.028238	0.007100	0.007415	0.004878	0.013188	0.002931	0.003786	0.001575
AMZN	0.007100	0.010915	0.003685	0.001647	0.005915	0.000905	0.001703	0.000677
FORD	0.007415	0.003685	0.017352	0.004850	0.005592	0.001378	0.003809	0.002817
JPM	0.004878	0.001647	0.004850	0.006061	0.002857	0.001776	0.002069	0.001799
NVDA	0.013188	0.005915	0.005592	0.002857	0.019092	0.001275	0.002959	0.001986
PFE	0.002931	0.000905	0.001378	0.001776	0.001275	0.003571	0.000867	0.000917
TGT	0.003786	0.001703	0.003809	0.002069	0.002959	0.000867	0.006099	0.000808
XOM	0.001575	0.000677	0.002817	0.001799	0.001986	0.000917	0.000808	0.004322

What is next?

- Index Models and The Capital Asset Pricing Model
  - The Single-Index Model
  - The Capital Asset Pricing Model
  - Reading(s): BKM Ch. 8 & 9
  - Assignment #1 (Due before Lecture 11)
  - Suggested Problems
    - ✓ **Ch. 7:** 4-10, 12, 22-27.
    - ✓ **Ch. 7 – CFA Problems:** 1-4, 12.

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Lecture 5

Vietnamese-German University