Investments

Finance 2 - BFIN

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Lecture 11

Outline

- Derivatives III
  - Binomial Option Pricing Model
  - Black-Scholes Option Pricing Model

Binomial Option Pricing Model

- A 3-month call option on the stock has a strike price of 21. Risk–free rate is 12%.

Stock Price = \$22
Option Price = \$1

Stock price = \$20
Option Price=?

Stock Price = \$18
Option Price = \$0

Binomial Option Pricing Model

- Consider the portfolio: long Δ shares, short 1 call option

$22\Delta - 1$

18Δ

Portfolio is riskless when

$$22\Delta - 1 = 18\Delta$$
$$\Delta = 0.25$$

- The riskless portfolio is long 0.25 shares, short 1 call option
- The value of the portfolio in 3 months is $22 * 0.25 - 1 = 4.50$
- The value of the portfolio today is $4.5e^{-0.12*0.25} = 4.3670$
- The value of the shares is 5.00 ($= 0.25 * 20$)
- The value of the option is 0.633 ($= 5.00 - 4.367$)

Binomial Option Pricing Model

- ❖ Value of a security that gives \$1 in the up state, nothing in the down state.
  - ➢ Let synthetic portfolio have Δ units of stock and B in bonds.

$$
\begin{array}{lll}
 & \nearrow & \begin{array}{l}\Delta S_0 u + Br \\ f_u = 1\end{array} \\
\begin{array}{l}S_0 \\ f_0\end{array} & & \\
 & \searrow & \begin{array}{l}\Delta S_0 d + Br \\ f_d = 0\end{array}
\end{array}
$$

$$f_u = 1 = \Delta S_0 u + Br$$

$$f_d = 0 = \Delta S_0 d + Br$$

$$\Delta = \frac{1}{S_0(u-d)} \qquad B = \frac{-d}{r(u-d)}$$

  - ➢ Then, the cost of replicating portfolio is

$$\Delta S_0 + B = P_u = \frac{r-d}{r(u-d)}$$

- ❖ Value of a security that gives nothing in the up state, \$1 in the down state.

$$f_u = 0 = \Delta S_0 u + Br$$

$$f_d = 1 = \Delta S_0 d + Br$$

$$\Delta = \frac{-1}{S_0(u-d)}$$

$$B = \frac{u}{r(u-d)}$$

$$\Delta S_0 + B = P_d = \frac{u-r}{r(u-d)}$$

- ❖ **u, d and r and “1 plus rate of return.”**

Binomial Option Pricing Model

❖ Note that buying 1 u-security and 1 d-security gives you \$1 for certain. Therefore,

$$P_u + P_d = 1/r \Rightarrow \frac{r-d}{r(u-d)} + \frac{u-r}{r(u-d)} = 1/r \Rightarrow \underbrace{\frac{r-d}{(u-d)}}_{\rho_u} + \underbrace{\frac{u-r}{(u-d)}}_{\rho_d} = 1$$

❖ The parameters ρ_u and ρ_d should be interpreted as the probability of an up and down movement in a risk-neutral world.

❖ In a risk-neutral world, the expected return on a stock (or any other investment) is the risk-free rate.

$$E(S_T) = \rho_u S_0 u + (1-\rho_u) S_0 d \qquad E(S_T) = \rho_u S_0 (u-d) + S_0 d \Rightarrow E(S_T) = S_0 r$$

❖ Risk-neutral valuation provides a very important general result in the pricing of derivatives.

- ➢ It states that, when we assume the world is risk-neutral, we get the right price for a derivative in all worlds, not just in a risk-neutral one.
- ➢ Results are true regardless of the assumptions we make about the evolution of the stock price.
- ➢ To apply risk-neutral valuation to the pricing of a derivative,
  - ✓ First calculate what the probabilities of different outcomes would be if the world were risk-neutral.
  - ✓ Then, calculate the expected payoff from the derivative and discount that expected payoff at the risk-free rate of interest.

Binomial Option Pricing Model

❖ Consider a European call option that expires in one period and has an exercise price of \$50. Assume that the stock price today is equal to \$50. In one period, the stock price will either rise by \$10 or fall by \$10. The one-period risk-free rate is 6%.

$$\begin{array}{ccc} & & S_u = 60 \\ & \nearrow & f_u = 10 \\ S_0 = 50 & & \\ f_0 & \searrow & S_d = 40 \\ & & f_d = 0 \end{array}$$

$$u = {}^{60}/_{50} = 1.20$$

$$d = {}^{40}/_{50} = 0.80$$

$$f_u = 10 = \Delta S_0 u + Br$$

$$f_d = 0 = \Delta S_0 d + Br$$

$$\Delta = 10 \frac{1}{S_0(u-d)} = 0.50$$

$$B = 10 \frac{-d}{r(u-d)} = -18.868$$

➢ The replicating portfolio is long 0.50 shares, short 1 call option, borrows 18.868 for financing.

$$\Delta S_0 + B = P_u = 0.50 * 50 + (-18.868) = \$6.132$$

$$\Delta S_0 + B = P_u = 10 \frac{r-d}{r(u-d)} = 10 \frac{1.06 - 0.80}{1.06(1.20 - 0.80)} = \$6.132$$

$$\rho_u = \frac{r-d}{u-d} = 0.65$$

$$f_0 = [\rho_u f_u + (1 - \rho_u) f_d] r^{-T} = [0.65 * 10 + (1 - 0.65) * 0](1.06)^{-1} = \$6.132$$

Binomial Option Pricing Model

❖ The current price of Estelle Corporation stock is $25. In the next year, the stock price will either go up by 20% or go down by 20%. The stock pays no dividends. The one-year risk-free interest rate is 6%. Using the Binomial Model, calculate the price of a one-year put option on Estelle stock with a strike price of $25.

$$S_0 = 25 \qquad f_0$$

$$S_u = 30 \qquad f_u = 0$$

$$S_d = 20 \qquad f_d = 5$$

$$u = 1.20$$
$$d = 0.80$$

$$f_u = 0 = \Delta S_0 u + Br$$

$$f_d = 5 = \Delta S_0 d + Br$$

$$\Delta = 5\frac{-1}{S_0(u-d)} = -0.50$$

$$B = 5\frac{u}{r(u-d)} = +14.15$$

➢ The replicating portfolio is short 0.50 shares, long 1 put option, lends 14.15 for financing.

$$\Delta S_0 + B = P_d = -0.50 * 25 + 14.15 = \$1.65$$

$$\rho_d = \frac{u-r}{u-d} = 0.35$$

$$\begin{aligned} f_0 &= [(1-\rho_d)f_u + \rho_d f_d](1+r)^{-T} \\ &= [(1-0.65) * 0 + 0.35 * 5](1.06)^{-1} \\ &= \$1.65 \end{aligned}$$

Binomial Option Pricing Model

- Suppose a stock is currently trading for \$60, and in one period will either go up by 20% or fall by 10%. If the one-period risk-free rate is 3%, what is the price of a European put option that expires in one period and has an exercise price of \$60?

$$S_0 = 60 \qquad f_0$$

$$S_u = 72 \qquad f_u = 0$$

$$S_d = 54 \qquad f_d = 6$$

$$u = 1.20$$
$$d = 0.90$$

$$f_u = 0 = \Delta S_0 u + Br$$

$$f_d = 6 = \Delta S_0 d + Br$$

$$\Delta = 6\frac{-1}{S_0(u-d)} = -0.333$$

$$B = 6\frac{u}{r(u-d)} = +23.30$$

- The replicating portfolio is short 0.333 shares, long 1 put option, lends 23.30 for financing.

$$\Delta S_0 + B = P_d = -0.333 * 60 + 23.30 = \$3.32$$

$$\rho_d = \frac{u-r}{u-d} = 0.567$$

$$f_0 = [(1-\rho_d)f_u + \rho_d f_d]r^{-T}$$
$$= [(1-0.567) * 0 + 0.567 * 6](1.03)^{-1}$$
$$= \$3.30$$

Binomial Option Pricing Model

- We valued of a security that gives $1 in the up state, nothing in the down state.
- More generalized version would be a security that pays f_u in the up state and f_d in the down state.
  - Let synthetic portfolio have Δ units of stock and B in bonds.

$$\begin{array}{ccc} & & \Delta S_0 u + Br \\ & \nearrow & f_u \\ S_0 & & \\ f_0 & & \\ & \searrow & \Delta S_0 d + Br \\ & & f_d \end{array}$$

$$f_u = \Delta S_0 u + Br$$

$$f_d = \Delta S_0 d + Br$$

$$\Delta = \frac{f_u - f_d}{S_0(u-d)} \qquad B = \frac{uf_d - df_u}{r(u-d)}$$

  - Then, the cost of replicating portfolio is

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)}$$

- Suppose a stock is currently trading for $60, and in one period will either go up by 20% or fall by 10%. If the one-period risk-free rate is 3%, what is the price of a European put option that expires in one period and has an exercise price of $60?

$$\Delta = \frac{f_u - f_d}{S_0(u-d)} = \frac{0-6}{60(1.2-0.90)} = -0.333 \qquad B = \frac{uf_d - df_u}{r(u-d)} = \frac{1.20 * 6 - 0.90 * 0}{1.03(1.20-0.90)} = \$23.30$$

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)} = \frac{(1.03-0.90) * 0 - (1.03-1.20) * 6}{1.03(1.20-0.90)} = \$3.30$$

Binomial Option Pricing Model

- Eagletron's current stock price is \$10. Suppose that over the current year, the stock price will either increase by 100% or decrease by 50%. Also, the risk-free rate is 25% (EAR).
  - What is the value today of a one-year at-the-money European put option on Eagletron stock?
  - What is the value today of a one-year at-the-money European call option on Eagletron stock?
  - What is the value today of a one-year European put option on Eagletron stock with a strike price of \$20?
  - Suppose the put options in parts a and c could either be exercised immediately, or in one year. What would their values be in this case?

$$f_u = \max(K - S_T^u, 0) = \max(10 - 20, 0) = 0 \qquad f_d = \max\left(K - S_T^d, 0\right) = \max(10 - 5, 0) = 5$$

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)} = \frac{(1.25 - 0.50) * 5 - (1.25 - 2.00) * 0}{1.25(2.00 - 0.50)} = \$2.00$$

$$f_u = \max(S_T^u - K, 0) = \max(20 - 10, 0) = 10 \qquad f_d = \max\left(S_T^d - K, 0\right) = \max(5 - 10, 0) = 0$$

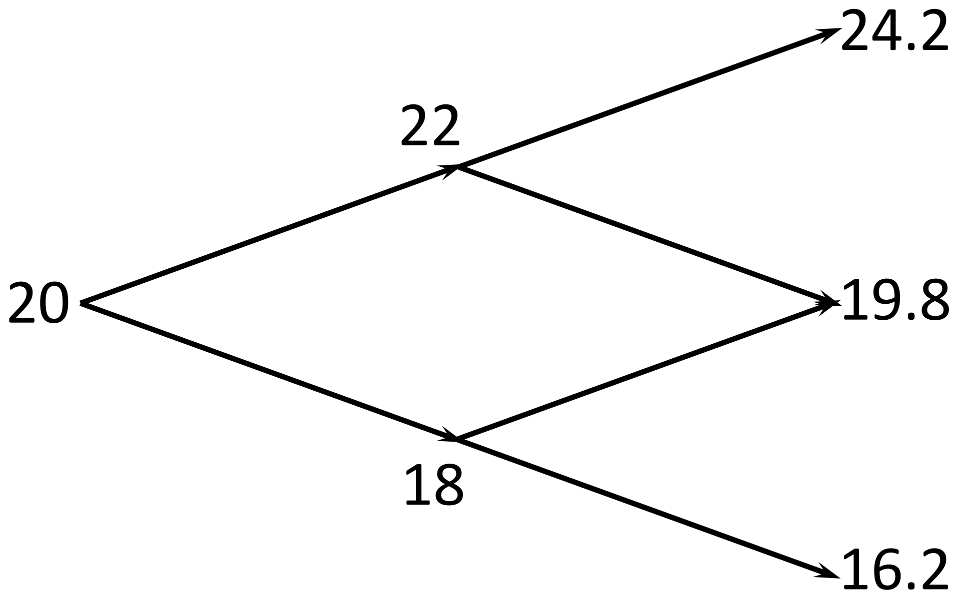
$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)} = \frac{(1.25 - 0.50) * 0 - (1.25 - 2.00) * 10}{1.25(2.00 - 0.50)} = \$4.00$$

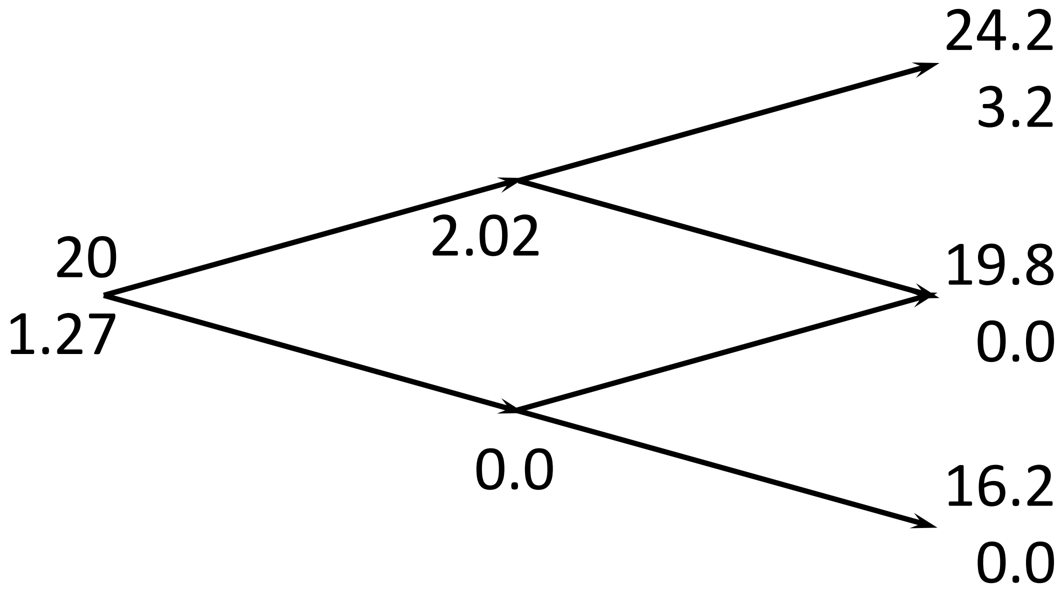
$$f_u = \max(K - S_T^u, 0) = \max(20 - 20, 0) = 0 \qquad f_d = \max\left(K - S_T^d, 0\right) = \max(20 - 5, 0) = 15$$

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)} = \frac{(1.25 - 0.50) * 15 - (1.25 - 2.00) * 0}{1.25(2.00 - 0.50)} = \$6.00$$

Binomial Option Pricing Model

- A 6-month call option on the stock has a strike price of 21. Risk–free rate is 12%. Stock goes up or down 10% for 2 periods (Each period is 3 months).



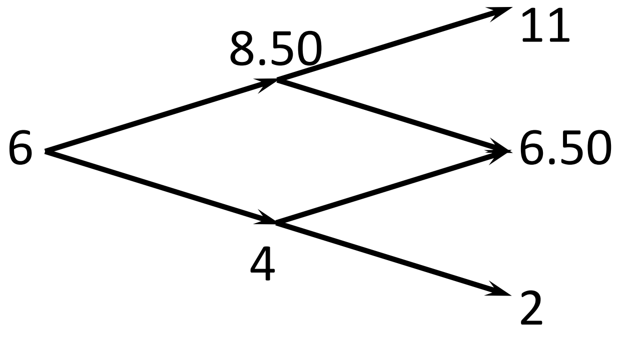


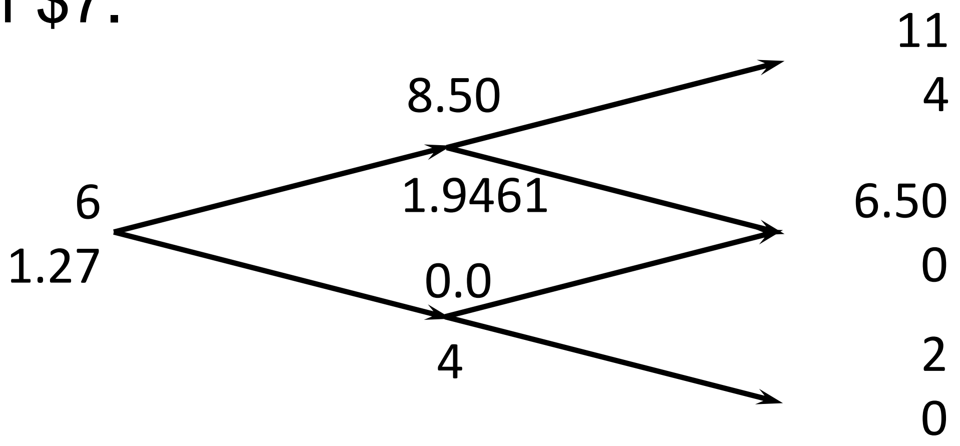
$$\Delta S_0 + B = P_1^u = \frac{(r-d)f_{uu} - (r-u)f_{ud}}{r(u-d)} = \frac{(1.03-0.90)*3.2-(1.03-1.10)*0}{1.03(1.10-0.90)} = \$2.02$$

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)d}{r(u-d)} = \frac{(1.03-0.90)*2.02-(1.03-1.10)*0}{1.03(1.10-0.90)} = \$1.27$$

Binomial Option Pricing Model

❖ The current price of Natasha Corporation stock is \$6. In each of the next two years, this stock price can either go up by \$2.50 or go down by \$2. The stock pays no dividends. The one-year risk-free interest rate is 3% and will remain constant. Using the Binomial Model, calculate the price of <u>a two-year call option</u> on Natasha stock with a strike price of \$7.



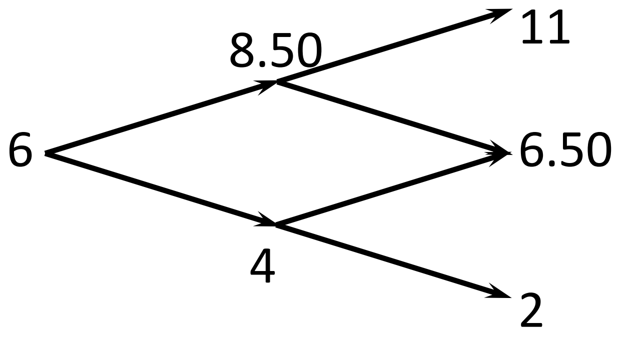


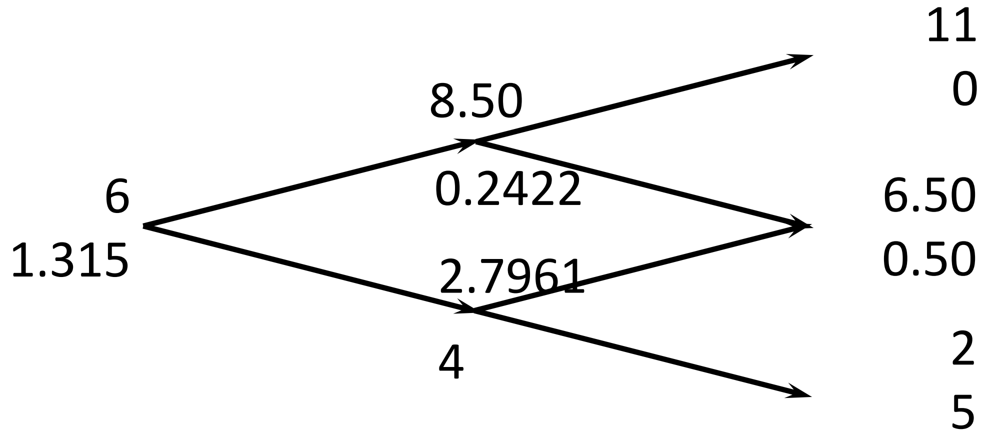
$$\Delta S_0 + B = P_1^u = \frac{(r-d)f_{uu} - (r-u)f_{ud}}{r(u-d)} = \frac{(1.03 - 0.7647) * 4 - (1.03 - 1.2941) * 0}{1.03(1.2941 - 0.7647)} = \$1.9461$$

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)} = \frac{(1.03 - 0.6667) * 1.9461 - (1.03 - 1.4167) * 0}{1.03(1.4167 - 0.6667)} = \$0.9152$$

Binomial Option Pricing Model

- The current price of Natasha Corporation stock is \$6. In each of the next two years, this stock price can either go up by \$2.50 or go down by \$2. The stock pays no dividends. The one-year risk-free interest rate is 3% and will remain constant. Using the Binomial Model, calculate the price of <u>a two-year put option</u> on Natasha stock with a strike price of \$7.





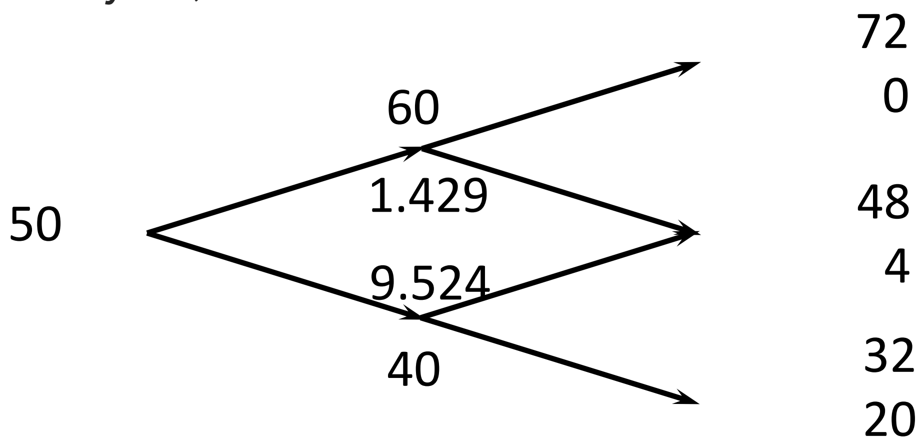
$$\Delta S_0 + B = P_1^d = \frac{(r-d)f_{du} - (r-u)f_{dd}}{r(u-d)} = \frac{(1.03-0.50)*0.50-(1.03-1.625)*5}{1.03(1.625-0.50)} = \$2.7961$$

$$\Delta S_0 + B = P_1^u = \frac{(r-d)f_{uu} - (r-u)f_{ud}}{r(u-d)} = \frac{(1.03-0.7647)*0-(1.03-1.2941)*0.50}{1.03(1.2941-0.7647)} = \$0.2422$$

$$\Delta S_0 + B = P_0 = \frac{(r-d)f_u - (r-u)f_d}{r(u-d)} = \frac{(1.03-0.6667)*0.2422-(1.03-1.4167)*2.7961}{1.03(1.4167-0.6667)} = \$1.514$$

Binomial Option Pricing Model

- European put option with a strike price of 52.
- Either up 20% or down 20%. Each time step is 1 year, r=5%.

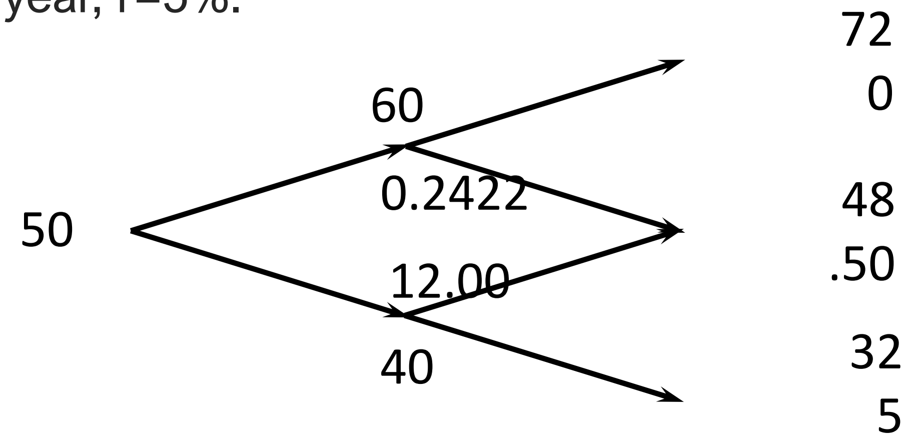


$$P_1^d = \frac{(1.05-0.80)*4-(1.05-1.20)*20}{1.05(1.20-0.80)} = \$9.524$$

$$P_1^u = \frac{(1.05-0.80)*0-(1.05-1.20)*4}{1.05(1.20-0.80)} = \$1.429$$

$$P_0 = \frac{(1.05-0.80)*1.429-(1.05-1.20)*9.524}{1.05(1.20-0.80)} = \$4.25$$

- American put option with a strike price of 52.
- Either up 20% or down 20%. Each time step is 1 year, r=5%.



$$P_1^d = \frac{(1.05-0.80)*4-(1.05-1.20)*20}{1.05(1.20-0.80)} = \$9.524 < (52-40)$$

$$P_1^u = \frac{(1.05-0.80)*0-(1.05-1.20)*4}{1.05(1.20-0.80)} = \$1.429$$

$$P_0 = \frac{(1.05-0.80)*1.429-(1.05-1.20)*12}{1.05(1.20-0.80)} = \$5.14$$

Black-Scholes Option Pricing Model

- Key assumptions
  - There are no arbitrage opportunities.
  - Underlying asset is a traded security, and investors can trade continuously.
  - The option to be priced is European.
  - Constant volatility (equal *u* and *d* moves through the tree), with no jumps.
  - No transaction costs or taxes, securities perfectly divisible, no portfolio restrictions, investors are price-takers.
- Continuous trading assumption
  - When markets are open, we can trade as we wish, so it is a good approximation.
  - But markets do close, so continuous trading is not ideal.

Black-Scholes Option Pricing Model

Black-Scholes Price of a Call Option on a Non-Dividend-Paying Stock

$$C_t = S_t * N(d_1) - PV(K) * N(d_2)$$

where

$$d_1 = \frac{\ln\left({S_t}/{PV(K)}\right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

Black-Scholes Price of a Put Option on a Non-Dividend-Paying Stock

$$P_t = PV(K)[1 - N(d_2)] - S_t[1 - N(d_1)]$$

- Default notation
  - S_t : Value of the underlying asset at date t;
  - K : Strike price of the option;
  - r_f : Risk-free rate (annual terms);
  - σ : Volatility (in annual terms), i.e. squared root of the variance;
  - T : Maturity, so at date t, T - t is time to maturity.
  - $N(d\ )$: The cumulative normal distribution

Black-Scholes Option Pricing Model

Black-Scholes Price of a Call Option on a Non-Dividend-Paying Stock

$$C_t = S_t * N(d_1) - PV(K) * N(d_2)$$

where

$$d_1 = \frac{\ln\left({S_t}/{PV(K)} \right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

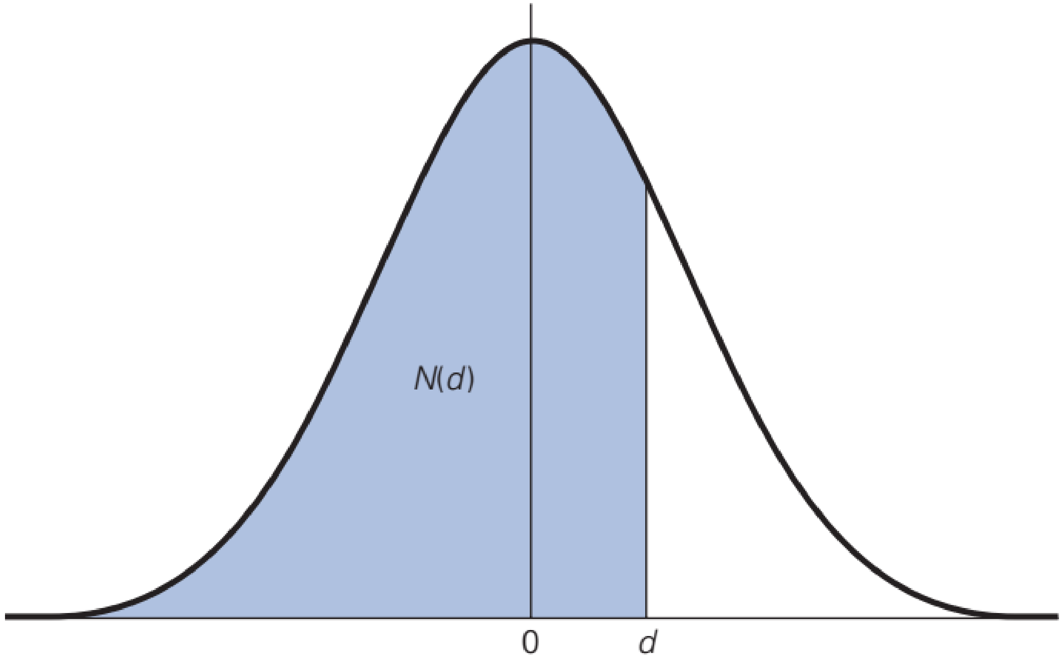
Black-Scholes Price of a Put Option on a Non-Dividend-Paying Stock

$$P_t = PV(K)[1 - N(d_2)] - S_t[1 - N(d_1)]$$

- ❖ The value of a call is a weighted difference of the present value of its potential benefits, S_t, and the present value of its potential costs $PV(K)$ where the weights N(d_1) and N($d_1 - \sigma\sqrt{T-t}$) are in (0, 1).
- ❖ The value of a put is a weighted difference of the present value of its potential costs $PV(K)$, and the present value of its potential benefits, S_t, where the weights $[1 - N(d_2)]$ and $[1 - N(d_1)]$ are in (0, 1).

$$C_t = S_t * \underbrace{N(d_1)}_{\Delta} - \underbrace{PV(K) * N(d_2)}_{Borrowing} \Rightarrow C_t = Asset\ Price * Delta + Cash\ Position$$

Black-Scholes Option Pricing Model

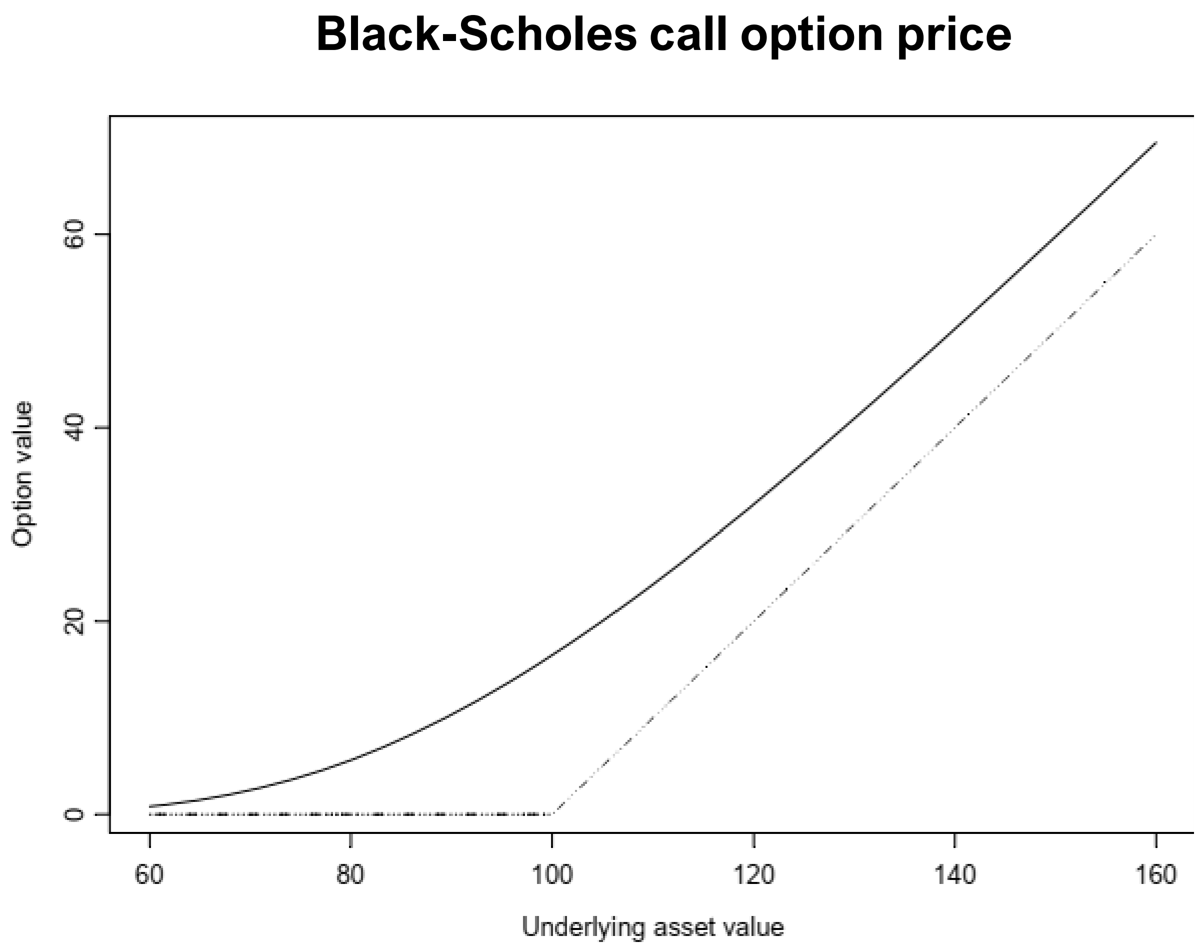


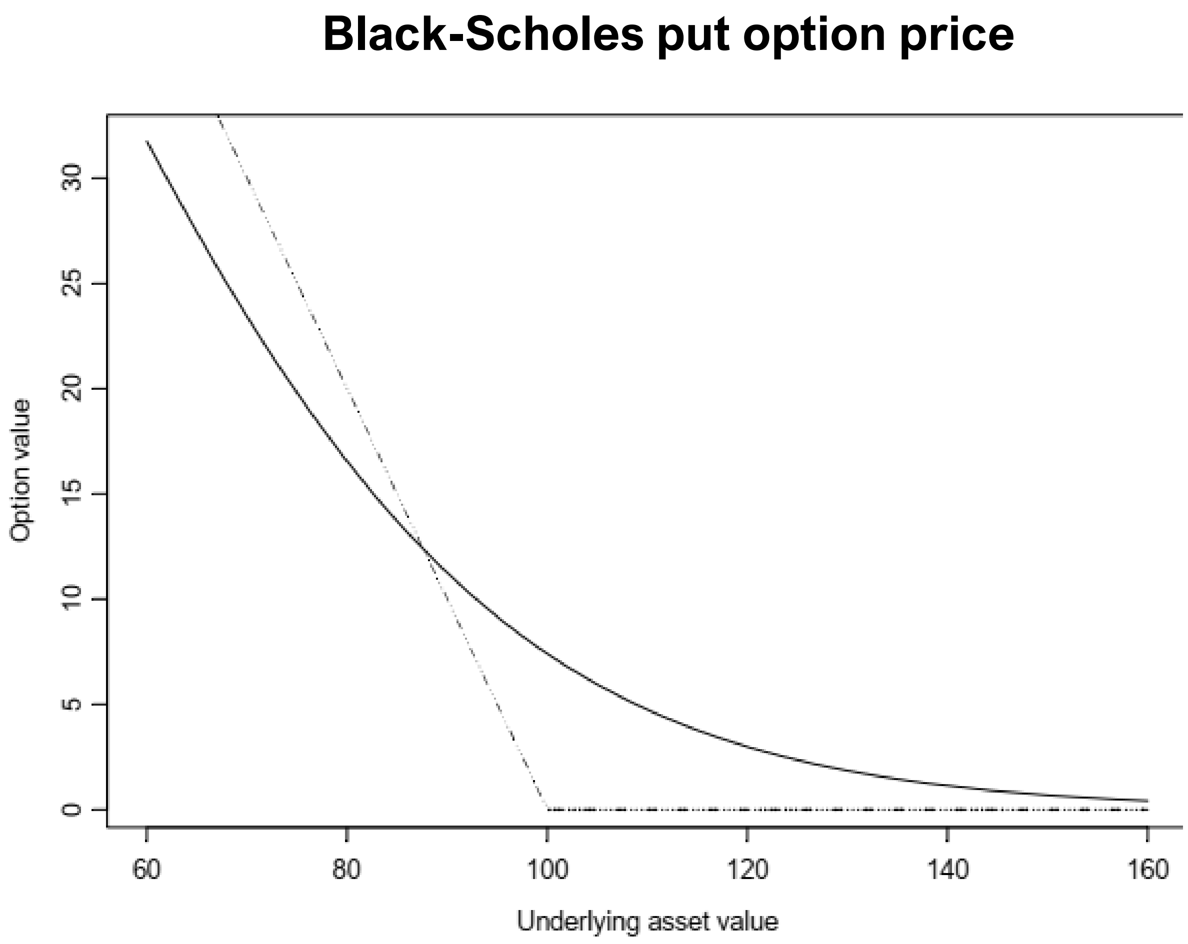
- The cumulative normal distribution [N(d)]
  - The probability that a normally distributed random variable will take on a value less than d.
  - Equal to the area under the normal distribution to the left of the point d.
  - N(d) has a minimum value of 0 and a maximum value of 1.
  - Can be calculated in Excel using the function NORMSDIST(d).

$$d_1 = \frac{\ln\left({S_t}/{PV(K)}\right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} \qquad d_2 = d_1 - \sigma\sqrt{T-t}$$

- Volatility (standard deviation) follows the squared-root rule.
  - Monthly volatility is σ_m, then the annual volatility is $\sigma_a = \sqrt{12\sigma_m}$
  - Weekly volatility is σ_w, then the annual volatility is $\sigma_a = \sqrt{52\sigma_w}$
  - Daily volatility is σ_d, then the annual volatility is $\sigma_a = \sqrt{250\sigma_d}$

Black-Scholes Option Pricing Model





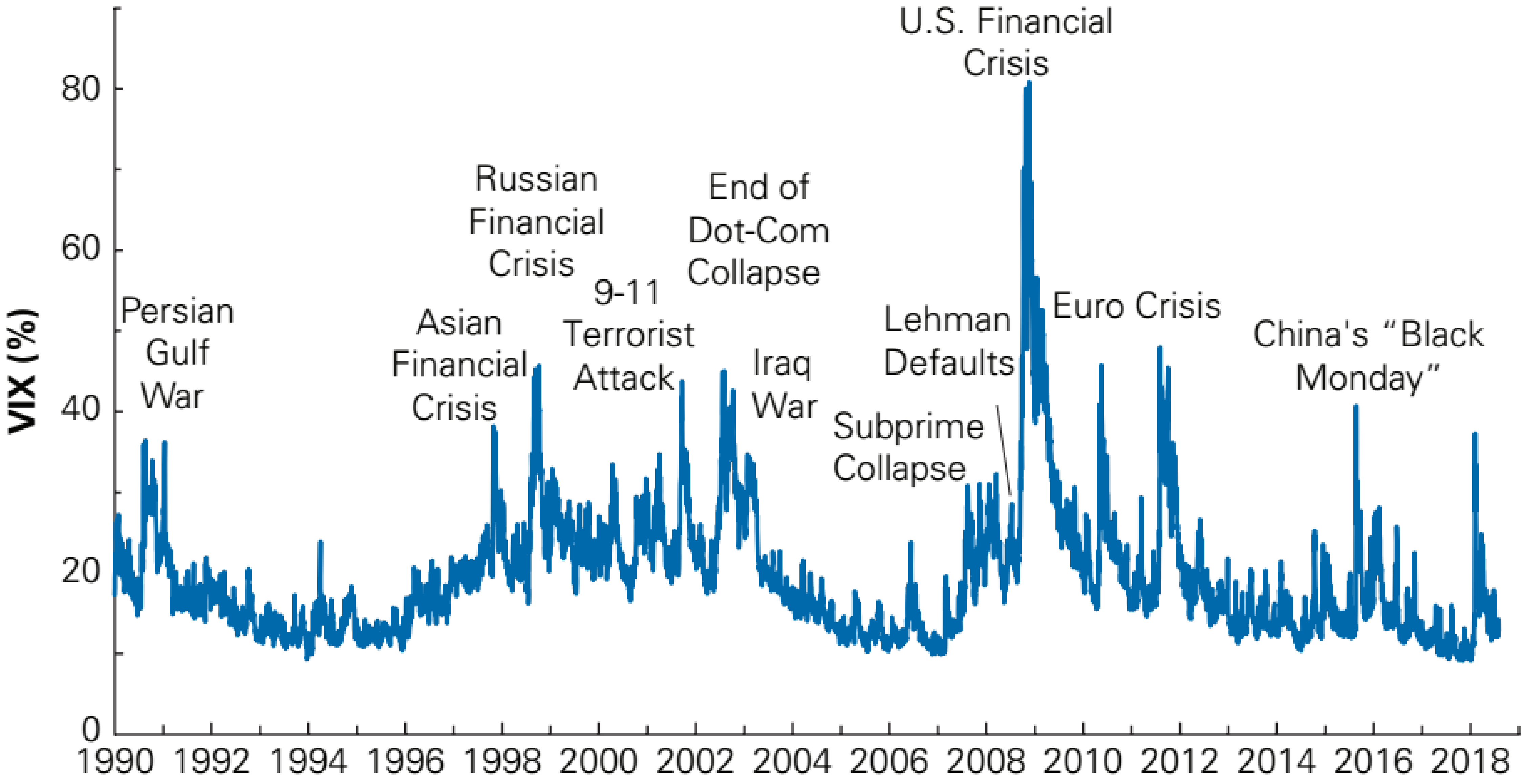
Black-Scholes Option Pricing Model

- Only non-observable variable in Black-Scholes Model is volatility(σ).
  - One option is to use historical data on daily stock returns to estimate the volatility of the stock over the past several months.
  - Implied volatility
    - ✓ The Black-Scholes formula can be expressed as

$$C_t = C(S_t, K, rf, T, \sigma)$$

    - ✓ If we know the price of a given option C_t, we can solve the Black-Scholes formula for σ.
    - ✓ In this case, σ is called the implied volatility of the option.

Black-Scholes Option Pricing Model

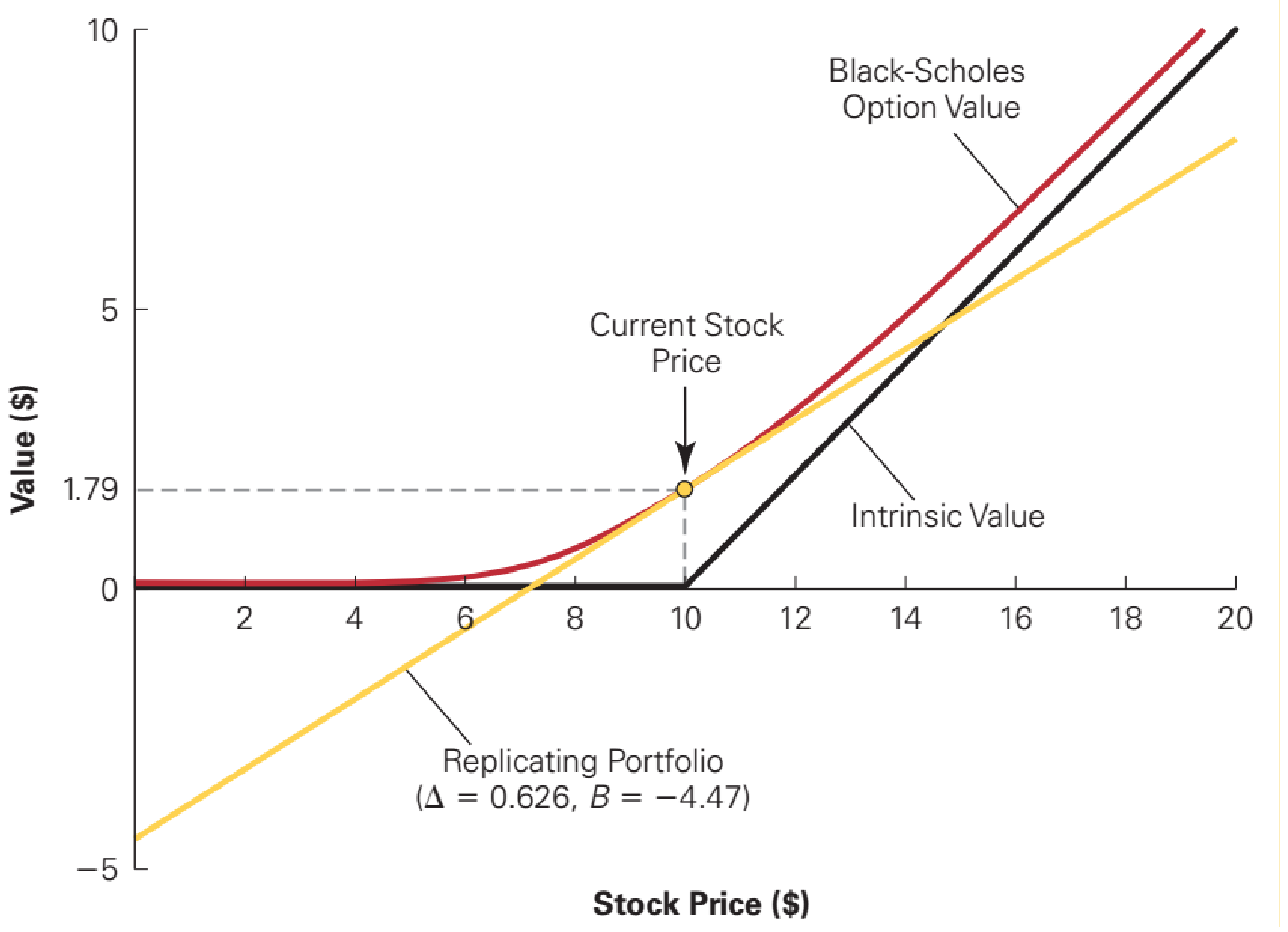


Black-Scholes Option Pricing Model

$$C_t = S_t * \underbrace{N(d_1)}_{\Delta} - \underbrace{PV(K) * N(d_2)}_{Borrowing}$$

$$P_t = \underbrace{PV(K) * [1 - N(d_2)]}_{Lending} \underbrace{-[1 - N(d_1)]}_{\Delta} * S_t$$

- The option delta, Δ, is the change in the price of the option given a 1-unit change in the price of the stock.
  - Because Δ is always less than 1, the change in the call price is always less than the change in the stock price.
  - When the price of the underlying asset changes, Δ also changes.
  - If we can update our portfolio continuously, we can replicate an option on the stock by constantly adjusting our portfolio to remain on a line that is tangent to the value of the option.



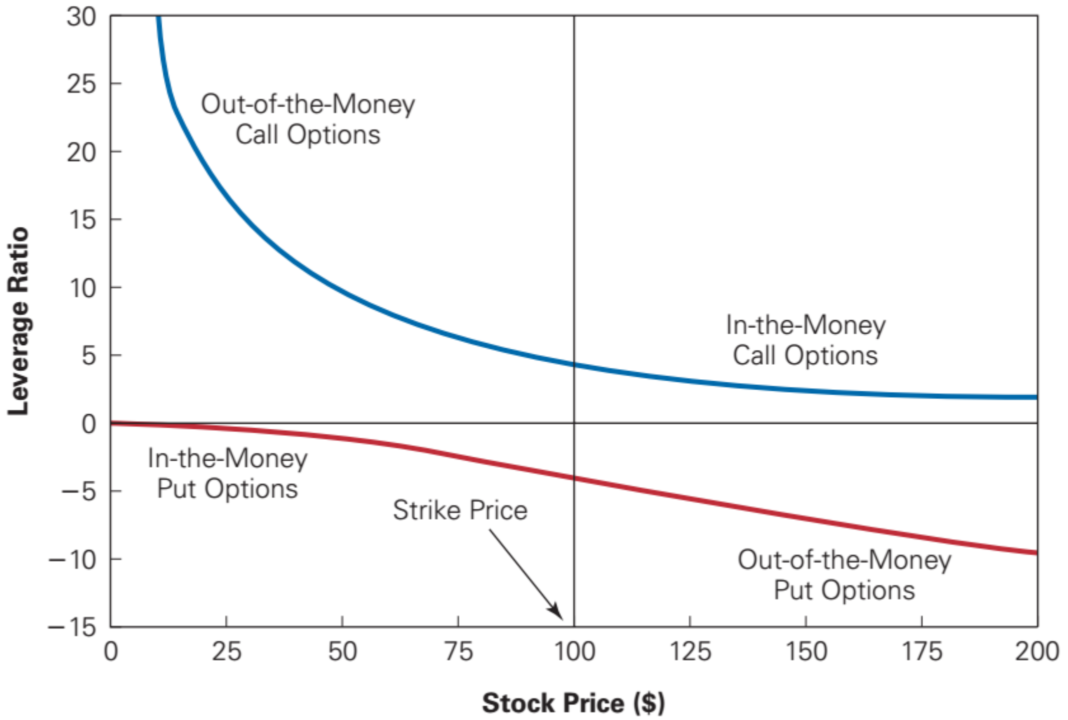
Black-Scholes Option Pricing Model

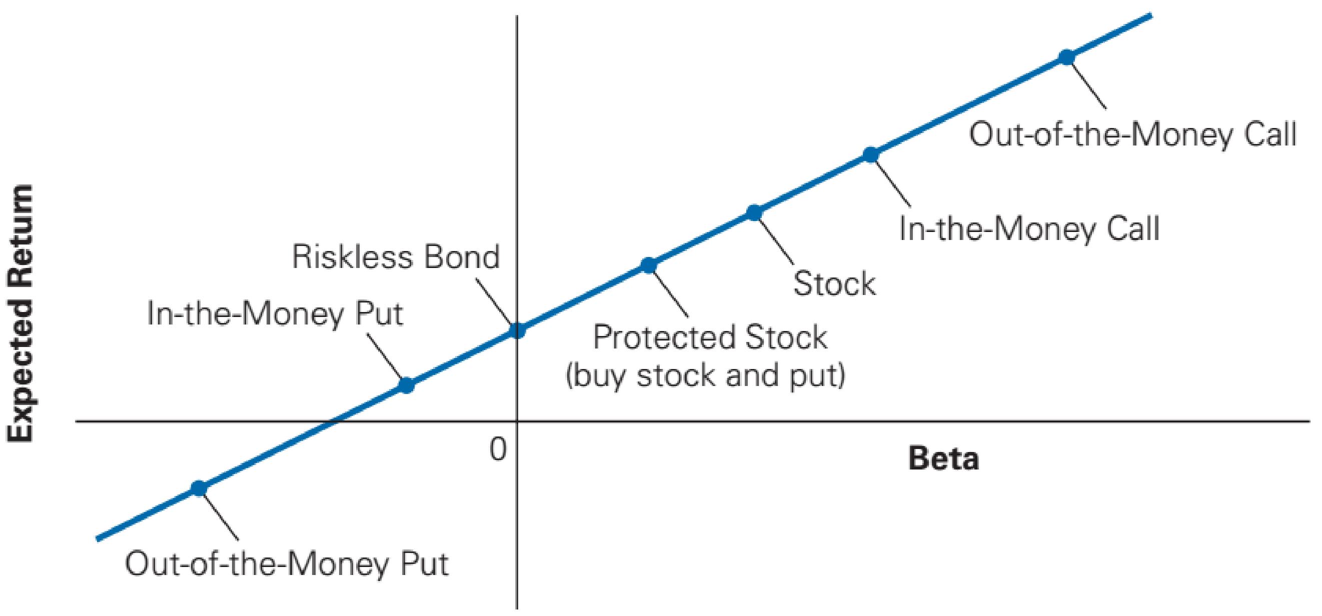
❖ Risk and Return of an Option

$$\beta_{Option} = \frac{S_t\Delta}{S_t\Delta + B}\beta_S + \frac{B}{S_t\Delta + B}\beta_B \qquad \text{Since } \beta_B = 0 \quad \beta_{Option} = \frac{S_t\Delta}{S_t\Delta + B}\beta_S$$

➢ ${S_t\Delta}/{S_t\Delta + B}$ is the options leverage ratio.

One-year options on a stock with a 30% volatility, given a risk-free interest rate of 5%.





Black-Scholes Option Pricing Model

- Roslin Robotics stock has a volatility of 30% and a current stock price of \$60 per share. Roslin pays no dividends. The risk-free interest is 5%. Determine the Black-Scholes value of a one-year, at-the-money call option on Roslin stock.

$$C_t = S_t * N(d_1) - PV(K) * N(d_2)$$

$$d_1 = \frac{\ln\left({}^{S_t}/_{PV(K)}\right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} = \frac{\ln\left({}^{60}/_{60*(1+0.05)^{-1}}\right)}{0.30\sqrt{1}} + \frac{0.30\sqrt{1}}{2} = 0.312634 \qquad N(d_1) = 0.622721$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = 0.312634 - 0.30 = 0.012634 \qquad N(d_2) = 0.50504$$

$$C_t = S_t * N(d_1) - PV(K) * N(d_2) = 60 * 0.622721 - 60 * (1+0.05)^{-1} * 0.50504 = \$8.5038$$

- Suppose the call option is not available for trade in the market. You would like to replicate a long position in call option. What portfolio should you hold today?

$$C_t = S_t * \underbrace{N(d_1)}_{\Delta} - \underbrace{PV(K) * N(d_2)}_{Borrowing} \qquad \Delta = N(d_1) = 0.622721 \qquad \Delta S_t = 0.622721 * 60 = \$37.3633$$

$$B = -PV(K) * N(d_2) = 60 * (1+0.05)^{-1} * 0.50504 = -28.8594$$

Black-Scholes Option Pricing Model

- Rebecca is interested in purchasing a European call on a hot new stock, Up, Inc. The call has a strike price of $100 and expires in 90 days. The current price of Up stock is $120, and the stock has a standard deviation of 40% per year. The risk-free interest rate is 6.18% per year.
  - Using the Black-Scholes formula, compute the price of the call.
  - Use put-call parity to compute the price of the put with the same strike and expiration date.

$$d_1 = \frac{\ln\left({}^{S_t}/_{PV(K)}\right)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2} = \frac{\ln\left({}^{120}/_{100 * (1.0618)^{-0.25}}\right)}{0.40\sqrt{0.25}} + \frac{0.40\sqrt{0.25}}{2} = 1.0866$$

$$N(d_1) = 0.8614$$

$$d_2 = d_1 - \sigma\sqrt{T-t} = 1.0866 - 0.40\sqrt{0.25} = 0.8866$$

$$N(d_2) = 0.8123$$

$$C_t = S_t * N(d_1) - PV(K) * N(d_2) = 120 * 0.8614 - 100 * (1.0618)^{-0.25} * 0.8123 = \$23.34$$

$$P_t = C_t - S_t + K\left(1 + r_f\right)^{-(T-t)} = 23.34 - 120 + 100(1 + 0.0618)^{-0.25} = \$1.8520$$

Black-Scholes Option Pricing Model

- Consider an at-the-money put option on AGE Inc stock that expires in 6 months. The stock of AGE Inc is currently trading at \$5, and you estimate the volatility of AGE Inc to be 50% per year and its beta is 0.90. The short-term risk-free rate of interest is 4% per year.
  - What is the put option's leverage ratio?
  - What is the beta of the put option?
  - If the expected risk premium of the market is 6%, what is the expected return of the put option based on the CAPM?
  - Given its expected return, why would an investor buy a put option?

$N(d_1) = 0.5918$ $\quad P_t = PV(K)[1 - N(d_2)] - S_t[1 - N(d_1)]$

$N(d_2) = 0.4517$ $\quad P_t = 5 * (1.04)^{-0.50}[1 - 0.4517] - 5[1 - 0.5918] = 0.6473$

$$P_t = \underbrace{PV(K) * [1 - N(d_2)]}_{Lending} \underbrace{-[1 - N(d_1)]}_{\Delta} * S_t \Rightarrow$$

$$\Delta = (-[1 - 0.5918]) = -0.4082$$

$$B = 5 * (1.04)^{-0.50} * [1 - 0.4517] = 2.6883$$

$$Option\ Leverage\ Ratio = \frac{5 * (-0.4082)}{5 * (-0.4082) + 2.6883} = -3.15$$

$$\beta_{Option} = -3.15 * 0.90 = -2.84$$

$$E[R_{Option}] = r_f + \beta_{Option}(E[R_{Mkt}] - r_f)$$

$$E[R_{Option}] = 0.04 + (-2.84)(0.06) = -13.04\%$$

Black-Scholes Option Pricing Model

- A stock is currently priced at $84. The stock will either increase or decrease by 17 percent over the next year. There is a call option on the stock with a strike price of $80 and one year until expiration. If the risk-free rate is 8 percent, what is the risk-neutral value of the call option?

$$C_{t=1}^{u} = \max(S_1^u - K, 0) = \max(84 * 1.17 - 80, 0) = \$18.28$$

$$C_{t=1}^{d} = \max\left(S_1^d - K, 0\right) = \max(84 * 0.83 - 80, 0) = \$0$$

$$\rho_u = \frac{r-d}{u-d} = \frac{1.08 - 0.83}{1.17 - 0.83} = 0.7353$$

$$\rho_d = \frac{u-r}{u-d} = \frac{1.17 - 1.08}{1.17 - 0.83} = 0.2647$$

$$C_0 = \frac{\rho_u * C_{t=1}^{u} + \rho_d * C_{t=1}^{d}}{(r)^1} = \frac{0.7353 * 18.28 + 0.2647 * 0}{1.08} = \$12.45$$

What is next?

- Solutions to all suggested problems
- Final exam on October 25, 2024
- Assignment 2 due by 16:00, October 30

Investments

Finance 2 - BFIN

Dr. Omer CAYIRLI

Lecture 11