Vietnamese-German University

Investments

Finance 2 - BFIN

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Lecture 4

Outline

- Portfolio Theory and Practice I
  - Risk, Return, and the Historical Record
  - Capital Allocation to Risky Assets

Return on Investments

- Total Return (Holding Period Return)
  - The total return realized on an investment in a given period and has two components:
    - ✓ Capital gains in the relevant period,
    - ✓ Income generated during the period (Dividend, coupon, rent, etc.)

$$TR = \frac{P_1 - P_0 + I}{P_0}$$

TR	*Total return*
$\boldsymbol{P_1}$	*The price of the asset at the end of the period*
$\boldsymbol{P_0}$	*The price of the asset at the beginning of the period*
I	*Income generated during the holding period*

  - Total return calculations do not take the return on reinvestment of the income earned during the period into account.
  - The holding period return is not a standardized value in terms of time.

Return on Investments

Expected Ending Price	=	\$110
Beginning Price	=	\$100
Expected Dividend	=	\$4

$$\text{HPR} = \frac{E(P_1) - P_0 + E(D_1)}{P_0} = \frac{\$110 - \$100 + \$4}{\$100}$$

$$= \frac{\$110 - \$100}{\$100} + \frac{\$4}{\$100}$$

$$= \underbrace{10\%}_{\text{Capital Gains Yield}} + \underbrace{4\%}_{\text{Dividend Yield}} = 14\% \longleftarrow \text{Holding Period Return}$$

Return on Investments

$$TR_{Annual} = [1 + TR]^{1/N} - 1$$

❖ An investor bought stocks of AGE, Inc. at a price of €25 On January 01, 2022. The company paid a dividend of €1 per share on 30 June 2022. The investor sold the stock at a price of €30 on January 31, 2023. Calculate the holding period return and the annualized total return.

$$TR = \frac{P_1 - P_0 + \mathrm{D}}{P_0} \qquad TR = \frac{30-25+1}{25} \qquad TR = \frac{6}{25} = \%24$$

$$TR_{\mathrm{Annual}} = [1 + TR]^{1/N} - 1$$

$$TR_{\mathrm{Annual}} = [1 + 0{,}24]^{\frac{1}{13/12}} - 1 = \%21{,}97$$

Return on Investments

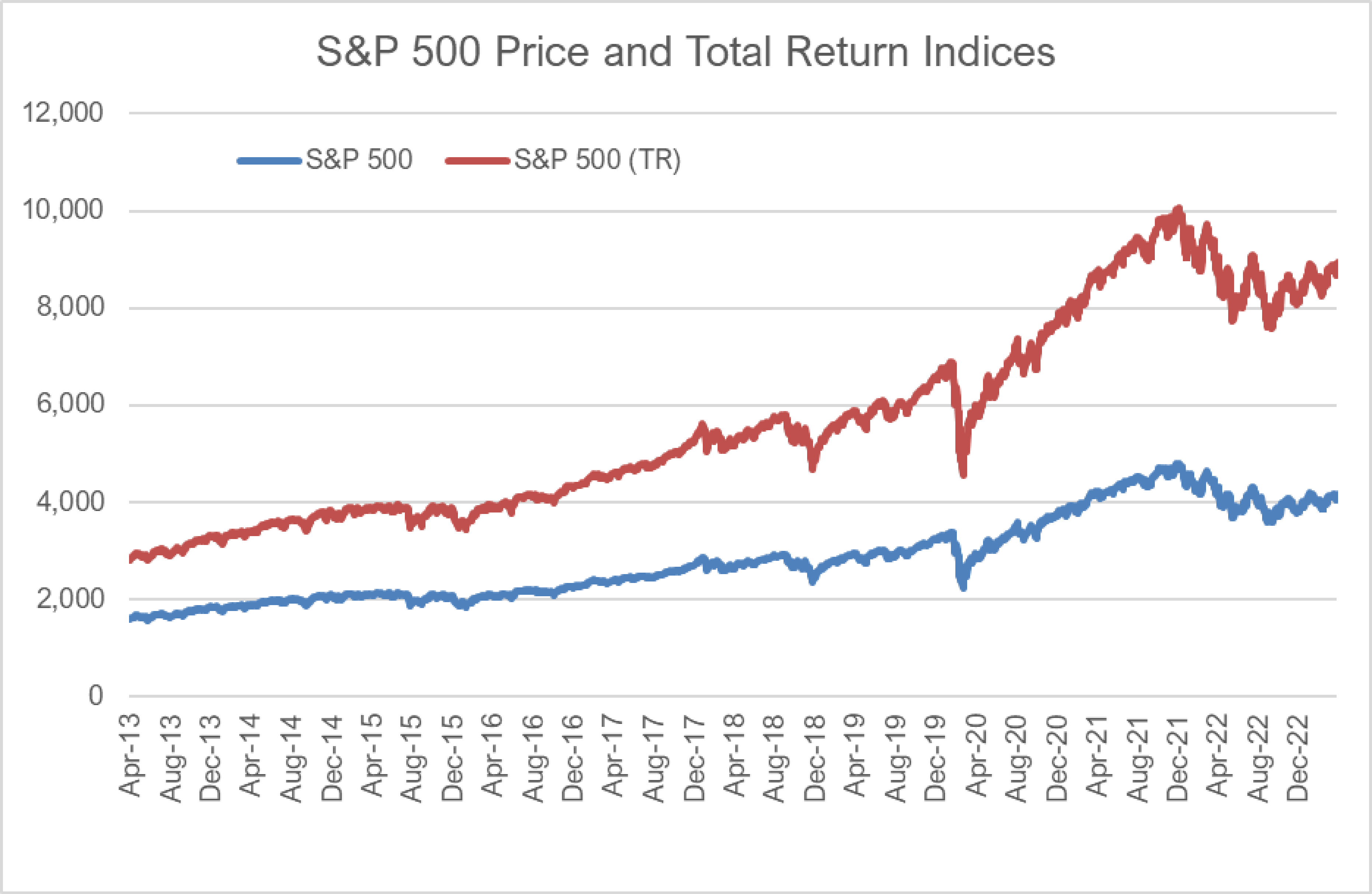
- On January 01, 2021, an investor purchased government bond with a face value of €1,000 and a maturity of 5 years at a price of €1,100. The bond makes coupon payments of €60 on 30 June and 31 December. The investor sold the bonds on 30 June 2022 at a price of €1,058. What is the investor's holding period return and the annual total return?

$$TR = \frac{P_1 - P_0 + C}{P_0} \qquad TR = \frac{1058-1100+(60*3)}{1100} \qquad TR = \frac{138}{1100} = \%12{,}55$$

$$TR_{\text{Annual}} = [1 + TR]^{1/N} - 1 \qquad TR_{\text{Annual}} = [1 + 0{,}1255]^{\frac{1}{3/2}} - 1$$

$$TR_{\text{Annual}} = [1 + 0{,}1255]^{\frac{2}{3}} - 1 \qquad TR_{\text{Annual}} = 8{,}20\%$$

Return on Investments



Return on Investments

❖ Expected Return

- When investors purchase an asset, the level of total return they will realize during their holding period involves uncertainty.
- The expected rate of return is the probability-weighted average of the rates of return that will occur in each scenario.

$$E(r) = \sum_{s} p(s) \times r(s)$$

Where,

$p(s)$ = Probability of a state
$r(s)$ = Return if a state occurs
s = State

P_0 (€)	20

Market Conditions	Probability	Stock Price	Dividend	Total Return	TR*Prob.
Perfect	10%	29	1.50	0.5250	0.053
Good	25%	26	1.25	0.3625	0.091
Flat	35%	23	1.00	0.2000	0.070
Bad	25%	16	0.25	-0.1875	-0.047
Crisis	5%	10	0.00	-0.5000	-0.025
	100%		**Expected Return**		**14.13%**

Return on Investments

- Excess Return
  - The return differential between a risky asset and a risk-free asset in any given period.
  - Also called the risk premium.
  - The risk-free rate is the rate of interest on an instrument that has no default risk and reinvestment risk.
  - The risk-free interest rate in real and nominal terms:

$$r_{f,real} = \frac{1 + r_{f,nominal}}{1 + e} - 1$$

P_0	20
r_f	0.08

Market Conditions	Probability	Stock Price	Dividend	Total Return	Excess Return	$(TR-r_f)$*Prob.
Perfect	10%	29	1.50	0.5250	0.4450	0.045
Good	25%	26	1.25	0.3625	0.2825	0.071
Flat	35%	23	1.00	0.2000	0.1200	0.042
Bad	25%	16	0.25	-0.1875	-0.2675	-0.067
Crisis	5%	10	0.00	-0.5000	-0.5800	-0.029
	100%			**Expected Excess Return**		**6.13%**

Risk in Investments

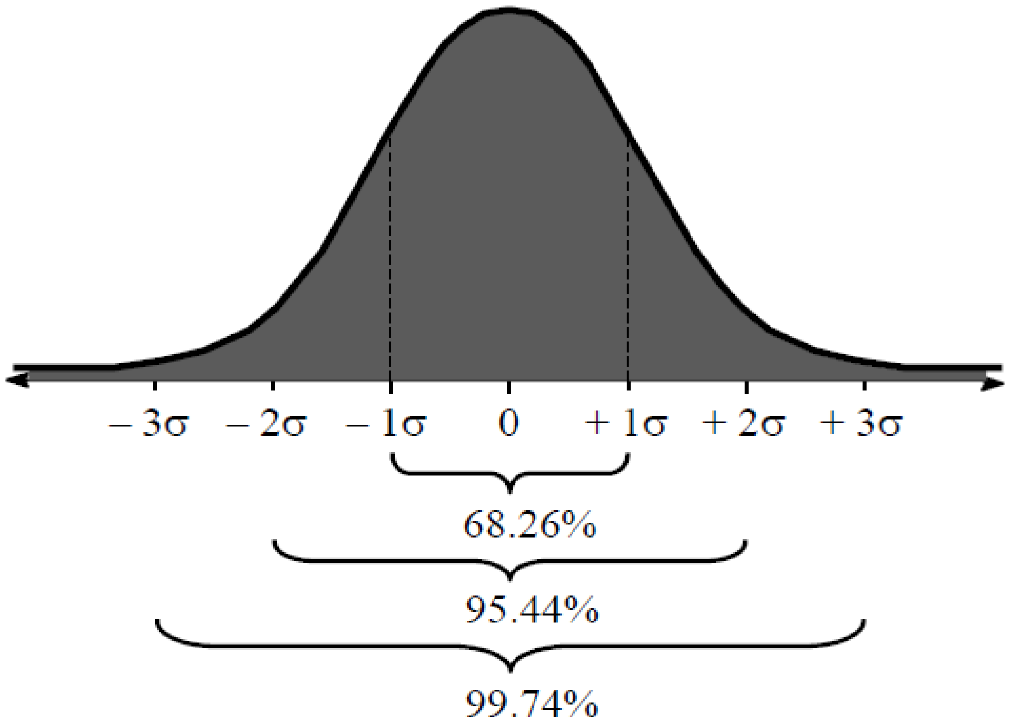
- Two of the fundamental questions of investments:
  - What is risk and how can we measure it?
  - How much additional return should we earn for bearing an additional unit of risk?

Risk in Investments

- One of the main functions of financial markets is to enable economic agents to transfer their risks to willing counterparties or to assume the risks of others.
- Financial risks are generally associated with uncertainty about the outcome of an investment.
- Accordingly, volatility (variance) in returns is the main risk criterion.
- Risk measures are not limited to volatility:
  - VaR (Value at Risk)
    - ✓ Measures the potential loss in value of a risky asset or portfolio over a defined period for a given confidence interval.
  - Expected Shortfall or CVaR (Conditional Value at Risk)
    - ✓ The expected loss given that the loss is greater than or equal to the VaR.
    - ✓ Quantifies the losses that might be encountered in the tail.
  - Semivariance
    - ✓ Measures the dispersion of all observations that fall below the mean or target value of a set of data.
  - Sortino Ratio
    - ✓ The ratio of average excess returns to semivariance

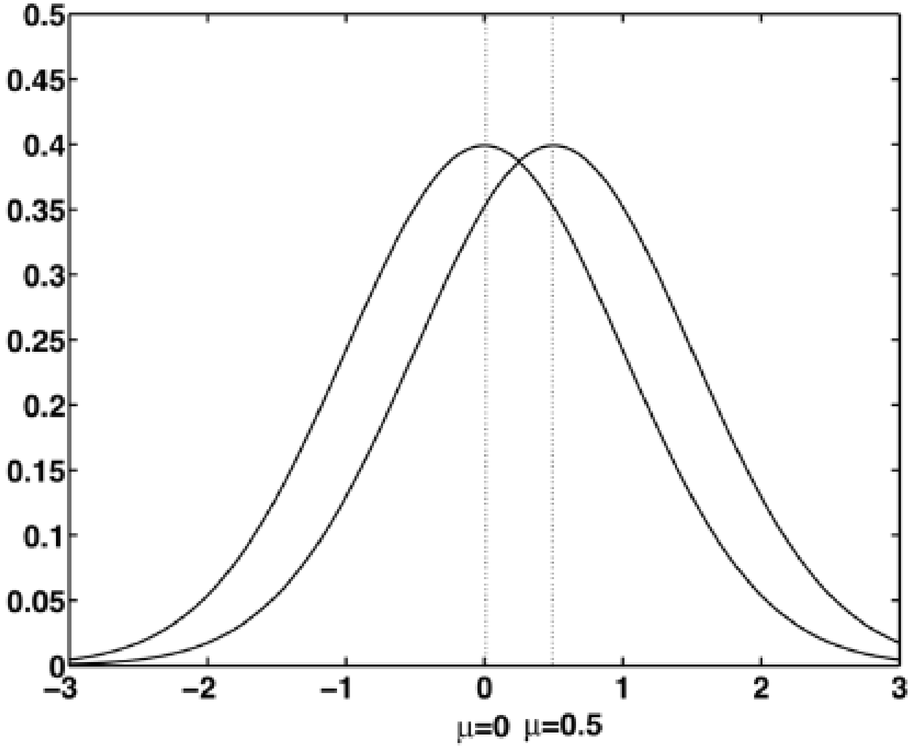
Risk in Investments

- Modern portfolio theory and asset pricing models often begin with the assumption of a normal distribution of returns.
- The normality assumption ensures that the standard deviation is an exact measure of risk.

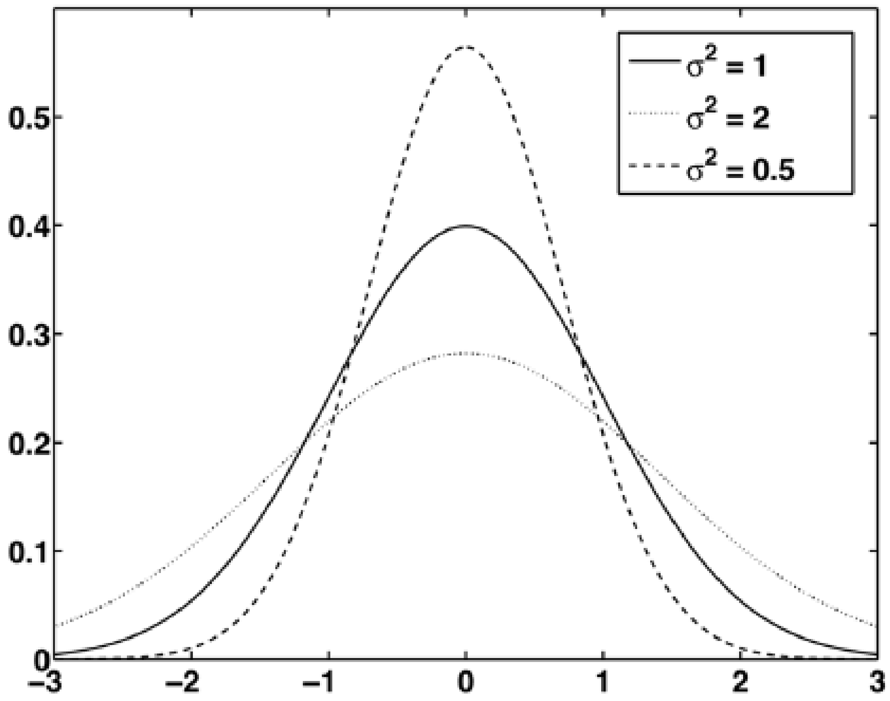


- A normal distribution;
  - Fully defined by its mean (μ) and variance (σ^2).
  - Symmetrical, bell-shaped curve.
- The mean of the standard normal distribution is 0 and the variance is 1.

Risk in Investments

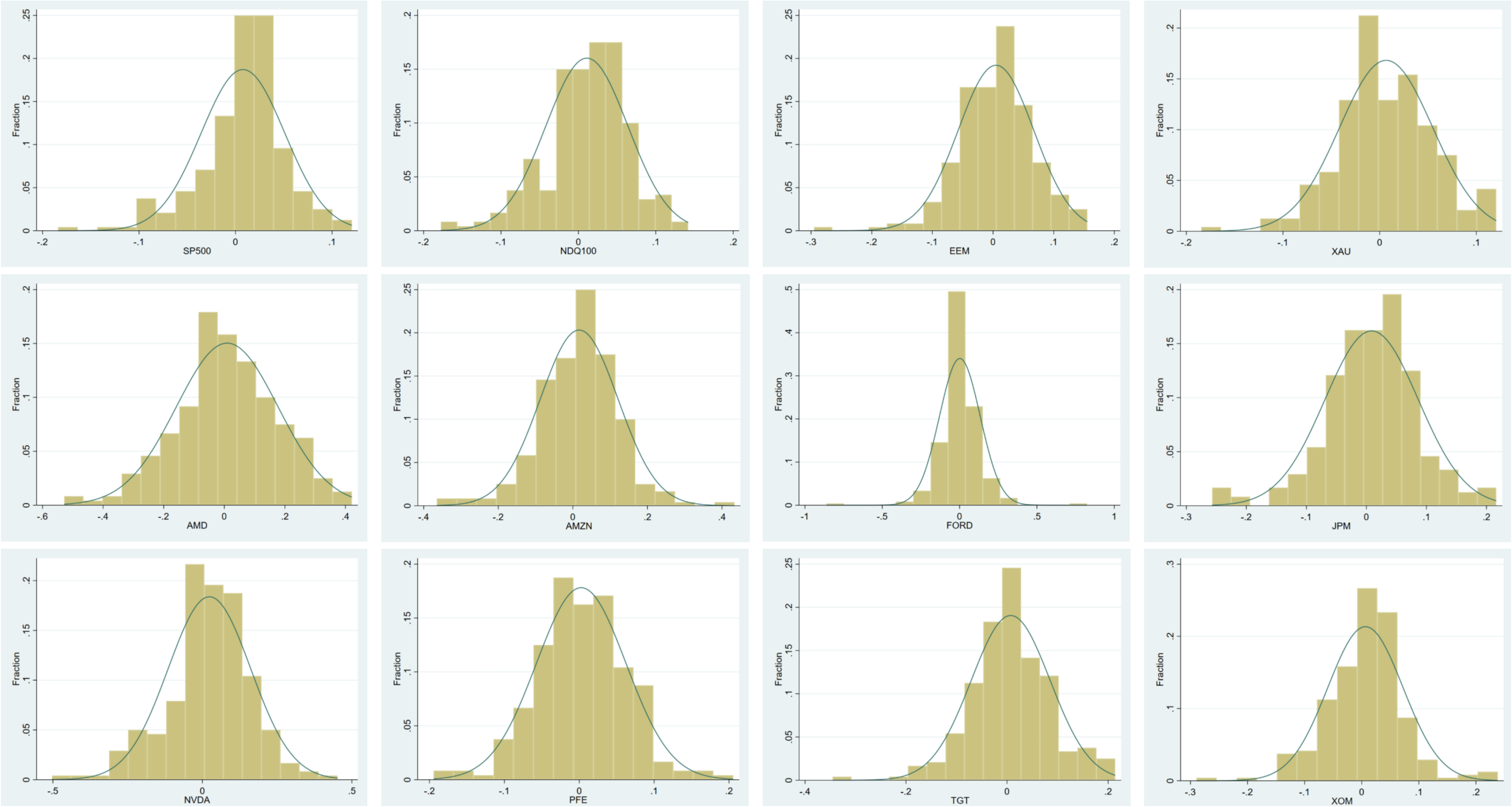


- The mean or expected value (μ) of the distribution is known as the location parameter.
- A normal distribution with μ≠0 can be easily converted to a standard normal distribution (μ=0, σ=1).



- The standard deviation (σ) of the distribution is known as the distribution parameter.
- When μ is constant, changes in distribution parameter affect the shape of the distribution.
- Kurtosis is a measure of the "tailedness" of the probability distribution. A standard normal distribution has kurtosis of 3.

Risk in Investments



Risk in Investments

	SP500	NDQ100	EEM	FORD	AMZN	NVDA	TGT	AMD	XOM	PFE	JPM	XAU
Mean	0.77%	1.09%	0.49%	0.15%	1.69%	2.34%	0.73%	0.95%	0.64%	0.26%	0.87%	0.67%
Standard Error	0.28%	0.34%	0.40%	0.85%	0.67%	0.89%	0.50%	1.08%	0.42%	0.39%	0.50%	0.31%
Median	1.38%	1.79%	0.77%	-0.18%	2.25%	3.18%	0.42%	0.35%	0.70%	0.33%	1.24%	0.35%
Standard Deviation	4.33%	5.30%	6.24%	13.17%	10.45%	13.82%	7.81%	16.80%	6.57%	5.98%	7.79%	4.82%
Sample Variance	0.0019	0.0028	0.0039	0.0174	0.0109	0.0191	0.0061	0.0282	0.0043	0.0036	0.0061	0.0023
CoV	0.3624	0.3142	0.8300	5.7714	0.3990	0.3811	0.6953	1.1359	0.6589	1.4931	0.5792	0.4661
Kurtosis	1.80	0.67	2.18	13.25	1.92	0.98	1.59	0.16	2.68	0.68	1.47	0.60
Skewness	-0.78	-0.57	-0.52	-0.15	-0.21	-0.42	-0.26	-0.18	-0.12	-0.03	-0.55	-0.16
Range	0.30	0.32	0.45	1.68	0.80	0.96	0.56	0.95	0.53	0.40	0.47	0.31
Minimum	-18.39%	-17.77%	-29.52%	-86.11%	-36.49%	-50.31%	-34.54%	-52.85%	-28.92%	-19.46%	-25.70%	-18.45%
Maximum	12.06%	14.18%	15.54%	82.37%	43.35%	45.20%	21.42%	42.13%	23.83%	20.57%	21.64%	12.06%
Count	240	240	240	240	240	240	240	240	240	240	240	240

- A distribution with no skew, such as a normal distribution, is symmetrical, with the mean, median, and mode all being equal.
- A positively skewed distribution is a type of data distribution where most of the data points are clustered towards the lower end of the spectrum, with a smaller number of outliers near the higher end.
- A negatively skewed distribution is a distribution where the majority of values are concentrated on the right side of the distribution graph, while the left tail is longer.
- Coefficient of variation (CoV): Quantifies the relative variability in relation to the mean.

Risk in Investments

Variance (VAR): $\sigma^2 = \sum_s p(s) \times [r(s) - E(r)]^2$

Standard Deviation (STD): $\mathrm{STD} = \sqrt{\sigma^2}$

State	Prob. of State	*r* in State
Excellent	.25	0.3100
Good	.45	0.1400
Poor	.25	-0.0675
Crash	.05	-0.5200

$E(r) = (.25) \times (.31) + (.45) \times (.14) + (.25) \times (-.0675) + (0.05) \times (-0.52)$ $E(r) = 9.76\%$

$$\sigma^2 = .25 \times (.31 - 0.0976)^2 + .45 \times (.14 - .0976)^2 + .25 \times (-0.0675 - 0.0976)^2 + .05 \times (-.52 - .0976)^2$$
$$= \mathbf{.038}$$

$\sigma = \sqrt{.038} = .1949$

Risk in Investments

$$E(r_i) = p_1 r_1 + p_2 r_2 + \cdots + p_n r_n$$

$$\boldsymbol{\sigma^2(r_i)} = p_1 * (r_1 - E(r_i))^2 + p_2 * (r_2 - E(r_i))^2 + \ \ldots + p_n * (r_n - E(r_i))^2$$

$$\sigma = \sqrt{\sigma^2(r_i)}$$

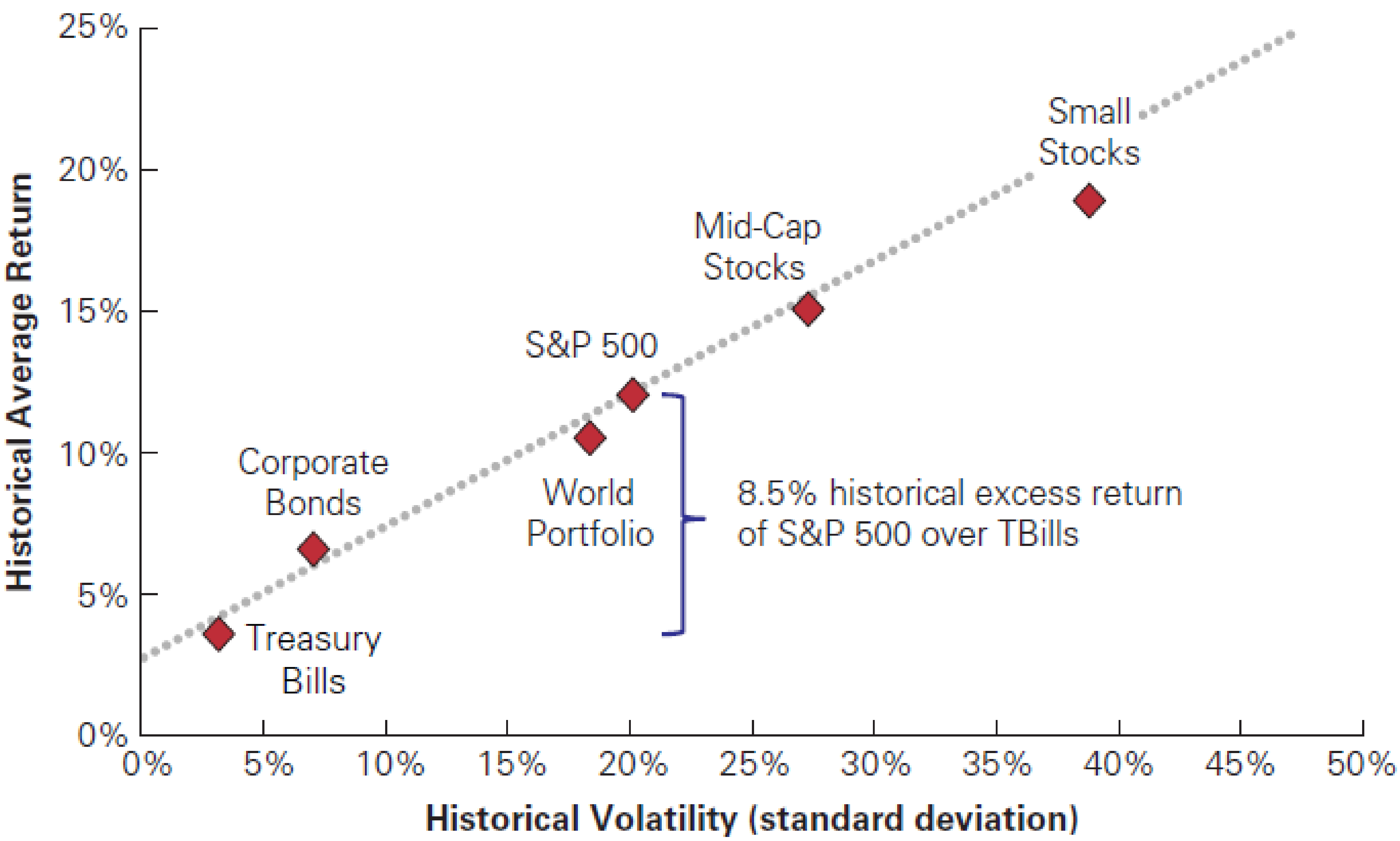
Scenario	Prob.	Return	Deviations
	p_n	r_n	$p_n * [r_n - E(r_i)]^2$
1	15%	9.0%	0.000005
2	20%	8.0%	0.000048
3	10%	6.0%	0.000126
4	30%	10.0%	0.000006
5	25%	12.0%	0.000150
	100%	**9.55%**	**0.000335**
		$E(r_i)$	σ^2

Risk in Investments

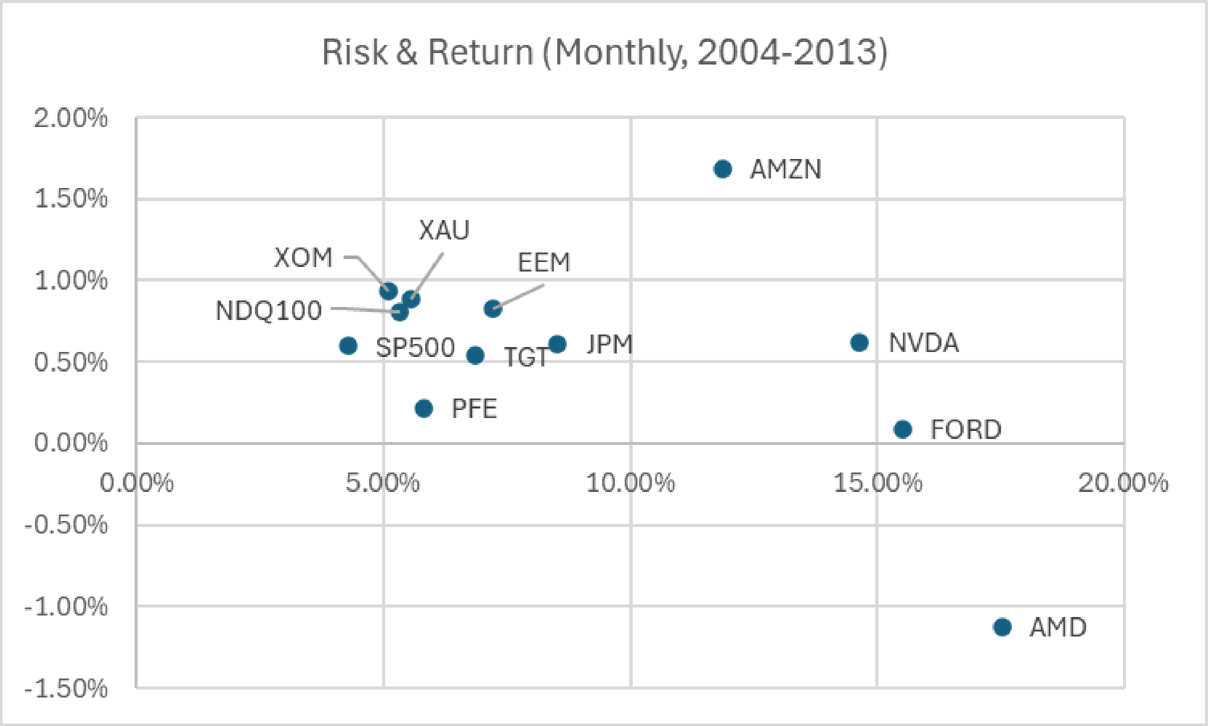
P_0	20
r_f	0.08

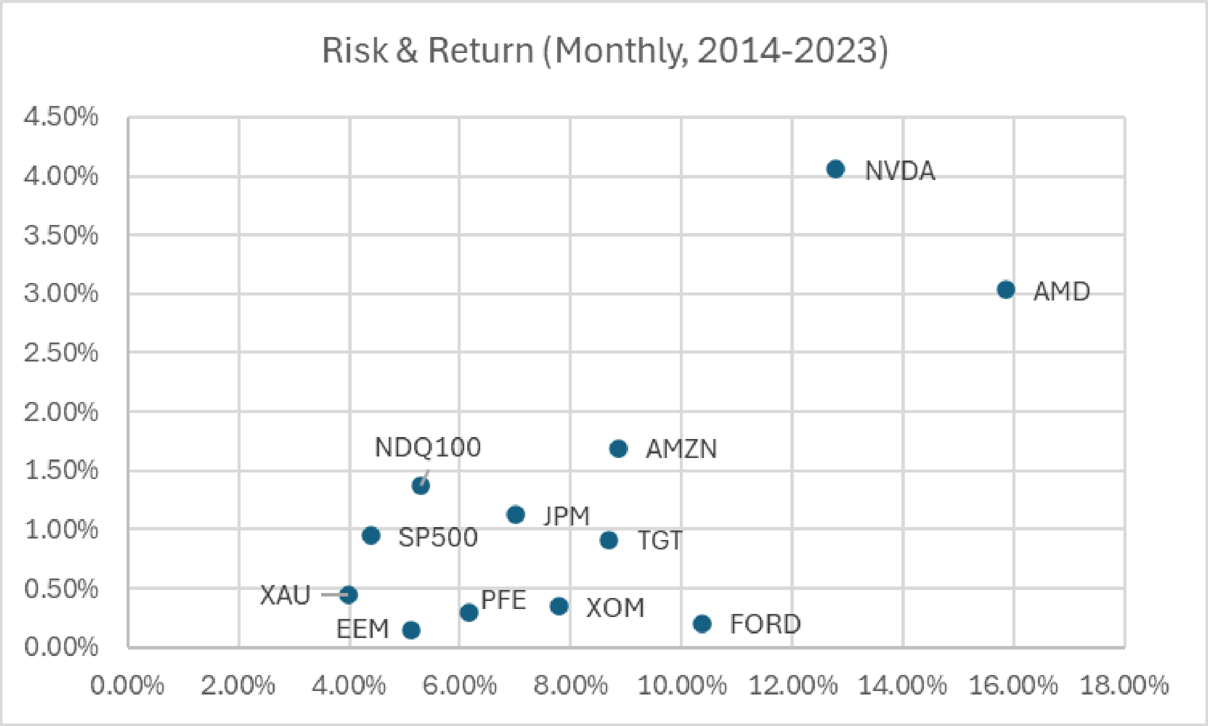
Market Conditions	Prob.	Stock Price	Dividend	Total Return	TR*Prob.	TR Deviations	Excess Return	$(TR-r_f)$*Prob.	Excess Return Deviations
Perfect	10%	29	1.50	0.5250	0.0525	0.0147	0.4450	0.045	0.0147
Good	25%	26	1.25	0.3625	0.0906	0.0122	0.2825	0.071	0.0122
Flat	35%	23	1.00	0.2000	0.0700	0.0012	0.1200	0.042	0.0012
Bad	25%	16	0.25	-0.1875	-0.0469	0.0270	-0.2675	-0.067	0.0270
Crisis	5%	10	0.00	-0.5000	-0.0250	0.0206	-0.5800	-0.029	0.0206
	100%				**14.13%**	**0.0758**		**6.13%**	**0.0758**
					$E(r_i)$	**σ^2**		**$E(r_i) - r_f$**	**σ^2**

Risk in Investments

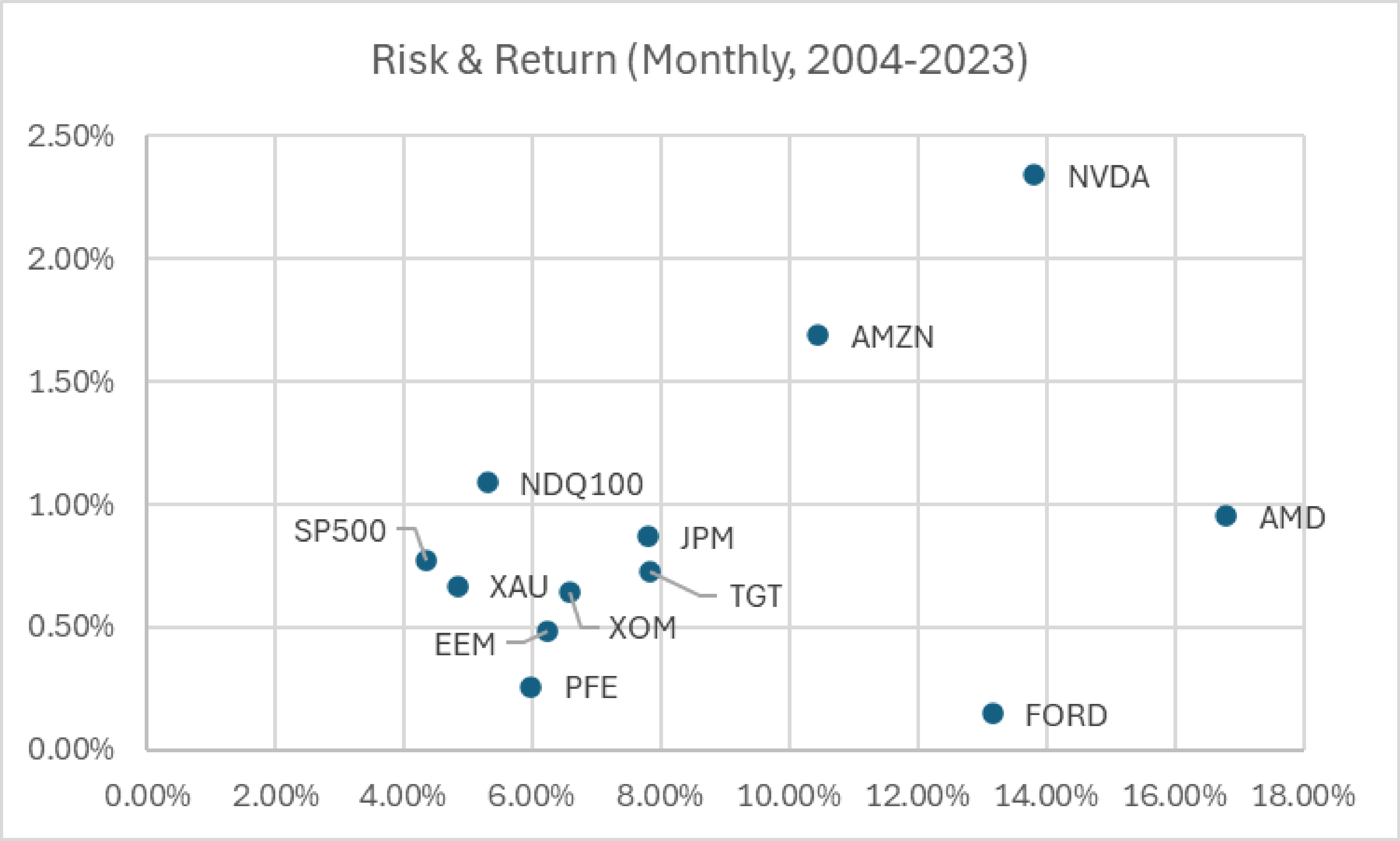


Risk in Investments





Risk in Investments



Risk in Investments

- True means and variances are unobservable
  - Possible scenarios like the ones we used in the examples are unknown
- Means and variances must be estimated
- Arithmetic Average

$$E(r) = \sum_{s=1}^{n} p(s)r(s) = \frac{1}{n}\sum_{s=1}^{n} r(s)$$

- Geometric (Time-Weighted) Average
  - Terminal value of the investment:

$$TV_n = (1 + r_1)(1 + r_2)\ldots(1 + r_n)$$

  - Geometric Average:

$$g = TV_n^{1/n} - 1$$

Risk in Investments

- Estimated Variance
  - Expected value of squared deviations

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{s=1}^{n}[r(s) - \bar{r}]^2$$

  - Unbiased estimated standard deviation

$$\hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}[r(s) - \bar{r}]^2}$$

- Sharpe Ratio

$$\frac{\text{Risk premium}}{\text{SD of excess returns}}$$

Risk in Investments

- Symmetric Returns
  - Standard deviation is a good measure of risk
  - Portfolio returns will also be symmetric
  - Only mean and standard deviation needed to estimate future scenarios
  - Pairwise correlation coefficients summarize the dependence of returns across securities
- What if excess returns are not normally distributed?
  - STD is no longer a complete measure of risk
    - ✓ Skewness:

$$Skew = Average\left[\frac{(R-\bar{R})^3}{\hat{\sigma}^3}\right]$$

    - ✓ Kurtosis:

$$Kurtosis = Average\left[\frac{(R-\bar{R})^4}{\hat{\sigma}^4}\right]-3$$

  - Sharpe ratio is not a complete measure of portfolio performance

Risk in Investments

- Normal distribution is generally a good approximation of returns
  - VaR indicates no greater tail risk than equivalent normal
  - ES $\leq$ 0.41 of monthly SD → no evidence against normality
- However
  - Negative skew is present in some portfolios some of the time
  - Positive kurtosis is present in all portfolios all the time

Risk and Risk Aversion

- Speculation
  - Taking a considerable risk for a commensurate gain
  - Parties have heterogeneous expectations
- Gambling
  - Bet on an uncertain outcome for enjoyment
  - Parties assign the same probabilities to the possible outcomes
- Utility Values
  - Investors are willing to consider:
    - ✓ Risk-free assets
    - ✓ Speculative positions with positive risk premiums
  - Portfolio attractiveness
    - ✓ Increases with expected return
    - ✓ Decreases with risk
    - ✓ What happens when return increases with risk?

Risk and Risk Aversion

❖ Utility Function

$$U = E(r) - \frac{1}{2} A\sigma^2$$

- ✓ U = Utility
- ✓ E(r) = Expected return on the asset or portfolio
- ✓ A = Coefficient of risk aversion
- ✓ σ^2 = Variance of returns
- ✓ ½ = A scaling factor

➢ Risk Averse Investors: $A > 0$

➢ Risk-Neutral Investors: $A = 0$

➢ Risk Lovers: $A < 0$

Risk and Risk Aversion

$$U = E(r) - \frac{1}{2}A\sigma^2$$

Portfolio	Risk Premium	Expected Return	Risk (SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

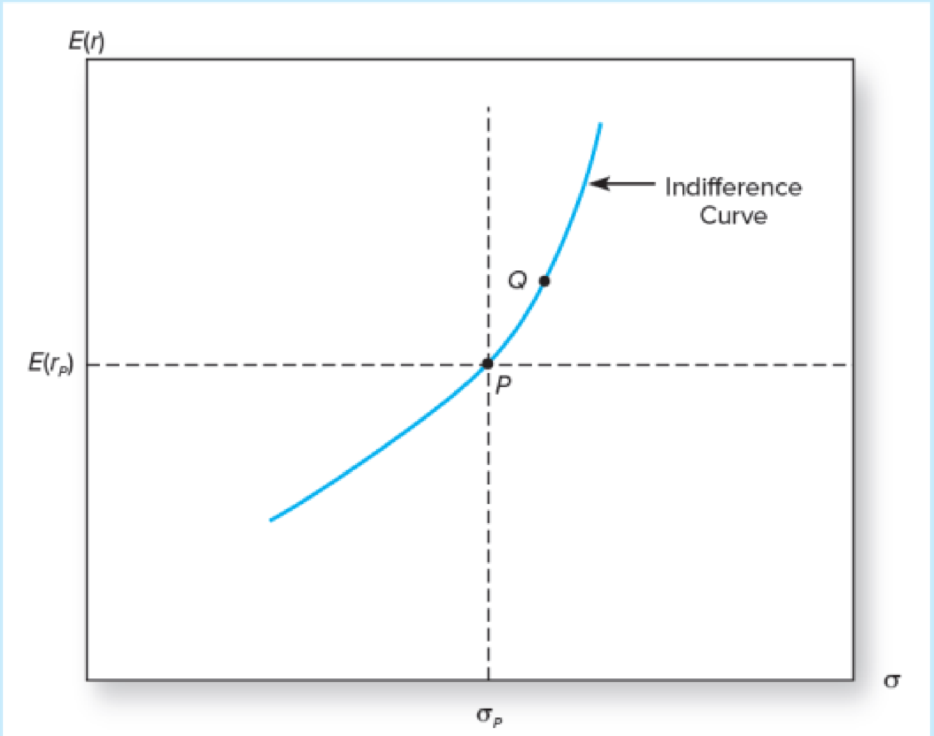
Investor Risk Aversion (*A*)	Utility Score of Portfolio *L* [$E(r) = .07; \sigma = .05$]	Utility Score of Portfolio *M* [$E(r) = .09; \sigma = .10$]	Utility Score of Portfolio *H* [$E(r) = .13; \sigma = .20$]
2.0	$.07 - ½ \times 2 \times .05^2 = .0675$	$.09 - ½ \times 2 \times .1^2 = .0800$	**$.13 - ½ \times 2 \times .2^2 = .09$**
3.5	$.07 - ½ \times 3.5 \times .05^2 = .0656$	**$.09 - ½ \times 3.5 \times .1^2 = .0725$**	$.13 - ½ \times 3.5 \times .2^2 = .06$
5.0	$.07 - ½ \times 5 \times .05^2 = .0638$	**$.09 - ½ \times 5 \times .1^2 = .0650$**	$.13 - ½ \times 5 \times .2^2 = .03$

Risk and Risk Aversion

- Mean-Variance (M-V) Criterion
  - Portfolio X dominates portfolio Y if:

$E(r_X) \geq E(r_Y)$ and $\sigma_X \leq \sigma_Y$ and at least one inequality is strict

- Indifference Curves
  - Equally preferred portfolios will lie in the mean–standard deviation plane on an indifference curve
  - All portfolio points with the same utility value



Portfolio Return

- In a portfolio of multiple assets, the return of the portfolio over a given period is the weighted average of the returns of each asset.

$$r_p = w_1 r_1 + w_2 r_2 + \cdots + w_N r_N$$

$$r_p = \sum_{n=1}^{N} w_n r_n$$

Asset	Price	Units	Value (t=0)	Weight in Portfolio	TR	Value (t=1)
A	120.00	5.00	600.00	20.0%	12.0%	672.00
B	5.00	150.00	750.00	25.0%	30.0%	975.00
C	90.00	10.00	900.00	30.0%	5.0%	945.00
D	30.00	15.00	450.00	15.0%	10.0%	495.00
E	12.00	25.00	300.00	10.0%	-10.0%	270.00
			3,000.00	**100.0%**	**11.9%**	**3,357.00**
			Portfolio Value (t=0)		**TR on the Portfolio**	**Portfolio Value (t=1)**

Portfolio Return

❖ In a portfolio with more than one asset, the expected return in a given period is the weighted average of the expected returns on the assets in the portfolio.

$$E(r_p) = w_1 E(r_1) + w_2 E(r_2) + \cdots + w_N E(r_N)$$

$$E(r_p) = \sum_{n=1}^{N} w_n E(r_n)$$

		Asset A (w=0.40)		Asset B (w=0.60)	
Market Conditions	**Prob.**	**TR**	**TR*Prob.**	**TR**	**TR*Prob.**
1	25%	12.0%	3.0%	6.0%	1.5%
2	15%	11.0%	1.7%	8.0%	1.2%
3	35%	10.0%	3.5%	10.0%	3.5%
4	15%	9.0%	1.4%	12.0%	1.8%
5	10%	6.0%	0.6%	14.0%	1.4%
	100%		**10.10%**		**9.40%**

$$E(r_p) = w_A E(r_A) + w_B E(r_B)$$

$$E(r_p) = 0{,}40 * 0{,}1010 + 0{,}60 * 0{,}0940 = 9{,}68\%$$

Portfolio Risk

$$E(r_i) = p_1 r_1 + p_2 r_2 + \cdots + p_n r_n$$

$$\boldsymbol{\sigma^2(r_i)} = p_1 * (r_1 - E(r_i))^2 + p_2 * (r_2 - E(r_i))^2 + \ \ldots + p_n * (r_n - E(r_i))^2$$

$$\sigma = \sqrt{\sigma^2(r_i)}$$

		Asset A					Asset B				
Market Conditions	**Prob.**	**TR**	**TR*Prob.**	**$(r_n\text{-}E(r_n))$**	**$(r_n\text{-}E(r_n))^2$**	**Prob.* $(r_n\text{-}E(r_n))^2$**	**TR**	**TR*Prob.**	**$(r_n\text{-}E(r_n))$**	**$(r_n\text{-}E(r_n))^2$**	**Prob.* $(r_n\text{-}E(r_n))^2$**
1	25%	12.0%	3.0%	1.9%	0.0004	0.00009	6.0%	1.5%	-3.4%	0.0012	0.00029
2	15%	11.0%	1.7%	0.9%	0.0001	0.00001	8.0%	1.2%	-1.4%	0.0002	0.00003
3	35%	10.0%	3.5%	-0.1%	0.0000	0.00000	10.0%	3.5%	0.6%	0.0000	0.00001
4	15%	9.0%	1.4%	-1.1%	0.0001	0.00002	12.0%	1.8%	2.6%	0.0007	0.00010
5	10%	6.0%	0.6%	-4.1%	0.0017	0.00017	14.0%	1.4%	4.6%	0.0021	0.00021
	100%		**10.10%**		**σ^2=**	**0.00029**		**9.40%**		**σ^2=**	**0.00064**
					σ=	**0.0170**				**σ=**	**0.0254**

Portfolio Risk

		Asset A					Asset B				
Market Conditions	Prob.	TR	TR*Prob.	$(r_n\text{-}E(r_n))$	$(r_n\text{-}E(r_n))^2$	Prob.* $(r_n\text{-}E(r_n))^2$	TR	TR*Prob.	$(r_n\text{-}E(r_n))$	$(r_n\text{-}E(r_n))^2$	Prob.* $(r_n\text{-}E(r_n))^2$
1	25%	12.0%	3.0%	1.9%	0.0004	0.00009	6.0%	1.5%	-3.4%	0.0012	0.00029
2	15%	11.0%	1.7%	0.9%	0.0001	0.00001	8.0%	1.2%	-1.4%	0.0002	0.00003
3	35%	10.0%	3.5%	-0.1%	0.0000	0.00000	10.0%	3.5%	0.6%	0.0000	0.00001
4	15%	9.0%	1.4%	-1.1%	0.0001	0.00002	12.0%	1.8%	2.6%	0.0007	0.00010
5	10%	6.0%	0.6%	-4.1%	0.0017	0.00017	14.0%	1.4%	4.6%	0.0021	0.00021
	100%		**10.10%**		**σ^2=**	**0.00029**		**9.40%**		**σ^2=**	**0.00064**
					σ=	**0.0170**				**σ=**	**0.0254**

- For an asset, the variance is a measure of the dispersion of possible returns on the asset around the expected return.
- Similarly, portfolio variance measures the dispersion of possible portfolio returns around the expected portfolio return.
- How should we calculate the standard deviation (risk) of the portfolio consisting of these two assets?

Portfolio Risk

- When calculating the expected return of the portfolio, we take the weighted average of the expected returns of the assets.
- We cannot use the same method for variance or standard deviation.
- Covariance;
  - The relationship between the changes in the returns of two assets.
  - When viewed at the portfolio level, the sign and size of the covariance will have an impact on the final volatility of the portfolio.
- For two assets;

$$
\begin{aligned}
\operatorname{cov}(r_i, r_j) = p_1 * [r_{i1} - E(r_i)] * [r_{j1} - E(r_j)] &+ \\
p_2 * [r_{i2} - E(r_i)] * [r_{j2} - E(r_j)] &+ \cdots \\
+ p_N * [r_{iN} - E(r_i)] * [r_{jN} - E(r_j)]&
\end{aligned}
$$

Portfolio Risk

- Covariance;
  - The relationship between the changes in the returns of two assets.

		Asset A		Asset B		
Market Conditions	**Prob.**	**TR_A**	**$(r_A-E(r_A))$**	**TR_B**	**$(r_B-E(r_B))$**	**$p * (r_A-E(r_A)) * (r_B-E(r_B))$**
1	25%	12.0%	1.9%	6.0%	-3.4%	-0.000162
2	15%	11.0%	0.9%	8.0%	-1.4%	-0.000019
3	35%	10.0%	-0.1%	10.0%	0.6%	-0.000002
4	15%	9.0%	-1.1%	12.0%	2.6%	-0.000043
5	10%	6.0%	-4.1%	14.0%	4.6%	-0.000189
	100%	**10.10%**		**9.40%**	**$Cov(r_A, r_B)$=**	**-0.000414**

Portfolio Risk

- A concept closely related to covariance is the correlation.
  - Covariance is not a standardized measure.
  - Correlation is a function of covariance and expresses the relationship between the returns of two assets in terms of both direction and strength.
  - The correlation is much easier to interpret since it is standardized and takes values between -1 and +1.

$$\rho_{i,j} = \frac{cov(r_i, r_j)}{\sigma_i \sigma_j}$$

- In our example,
  - Cov $(r_A, r_B) = -0{,}000414$, $\sigma_A = \%1{,}70$ and $\sigma_B = \%2{,}54$

$$\rho_{A,B} = \frac{-0{,}000414}{0.017 * 0.0254} = -0{,}9588$$

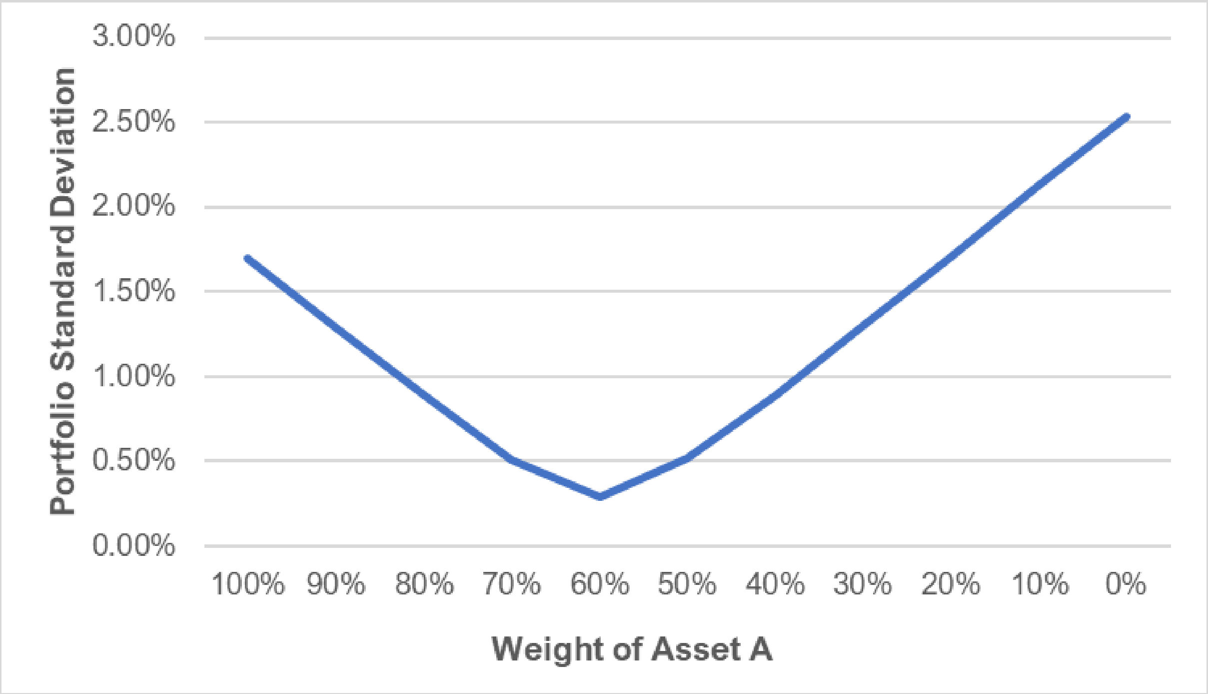
Portfolio Risk

❖ For a two-asset portfolio:

$$\boldsymbol{\sigma^2(r_p)} = w_1^2 * \sigma^2(r_1) + w_2^2 * \sigma^2(r_2) + 2 * w_1 w_2 Cov(r_1, r_2)$$

r_A	10.10%	r_B	9.40%	$Cov(r_A, r_B)$	-0.000414
σ_A	1.70%	σ_B	2.54%		
σ_A^2	0.000289	σ_B^2	0.00064516		

w_A	w_B	r_p	σ_P^2	σ_P
100%	0%	10.100%	0.000289	1.70%
90%	10%	10.030%	0.000166	1.29%
80%	20%	9.960%	0.000078	0.88%
70%	30%	9.890%	0.000026	0.51%
60%	40%	9.820%	0.000009	0.29%
50%	50%	9.750%	0.000027	0.52%
40%	60%	9.680%	0.000080	0.89%
30%	70%	9.610%	0.000168	1.30%
20%	80%	9.540%	0.000292	1.71%
10%	90%	9.470%	0.000451	2.12%
0%	100%	9.400%	0.000645	2.54%



Portfolio Risk

- Large portfolio variance

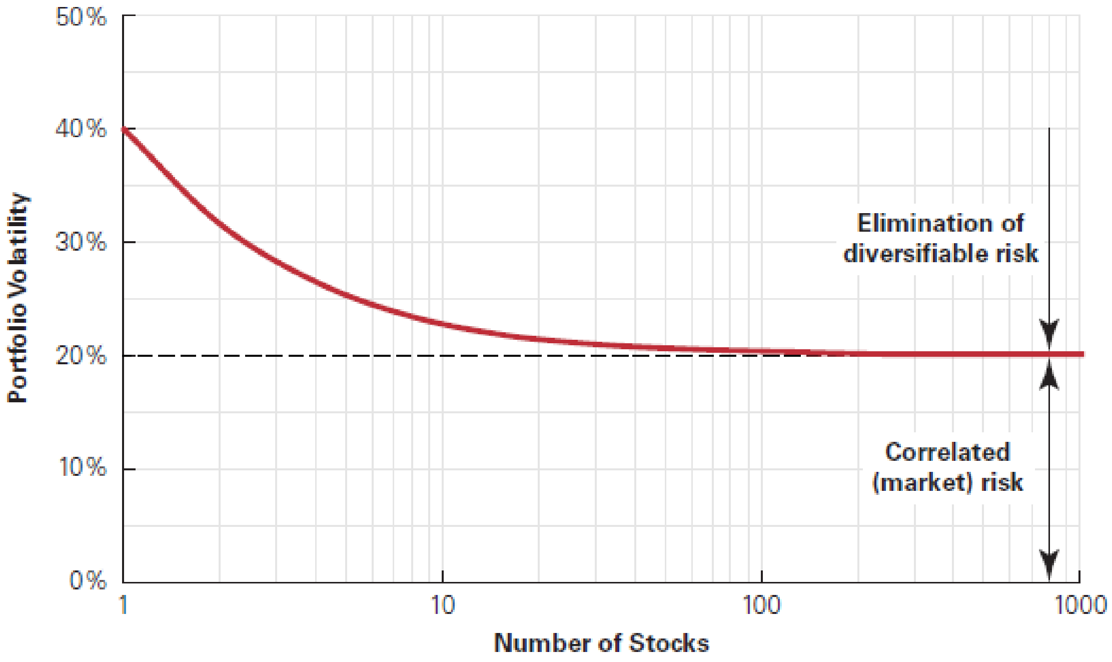
$$E(r_p) = \sum_{i=1}^{n} w_i E(r_i) \qquad \sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{Cov}(r_i, r_j)$$

- Diversification with an equally weighted portfolio

$$\sigma_p^2 = \underbrace{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{n}\sigma_i^2}_{\text{n variance terms}} + \underbrace{\sum_{\substack{j=1\\ j\neq i}}^{n}\sum_{i=1}^{n}\frac{1}{n^2}\operatorname{Cov}(r_i, r_j)}_{\text{n(n-1) covariance terms}}$$

n
variance terms

n(n-1)
covariance terms



Opportunity Set of Risky Assets

- Minimum-variance frontier shows the risk–return opportunities available to the investor.
- Efficient frontier
  - There is an efficient portfolio for each risk level.
  - Portfolios on the efficient frontier represent trade-offs in terms of risk and return.

r_A	10.10%	r_B	9.40%	$Cov(r_A, r_B)$	-0.000414
σ_A	1.70%	σ_B	2.54%		
σ_A^2	0.000289	σ_B^2	0.00064516		

w_A	w_B	r_p	σ_P^2	σ_P
100%	0%	10.100%	0.000289	1.70%
90%	10%	10.030%	0.000166	1.29%
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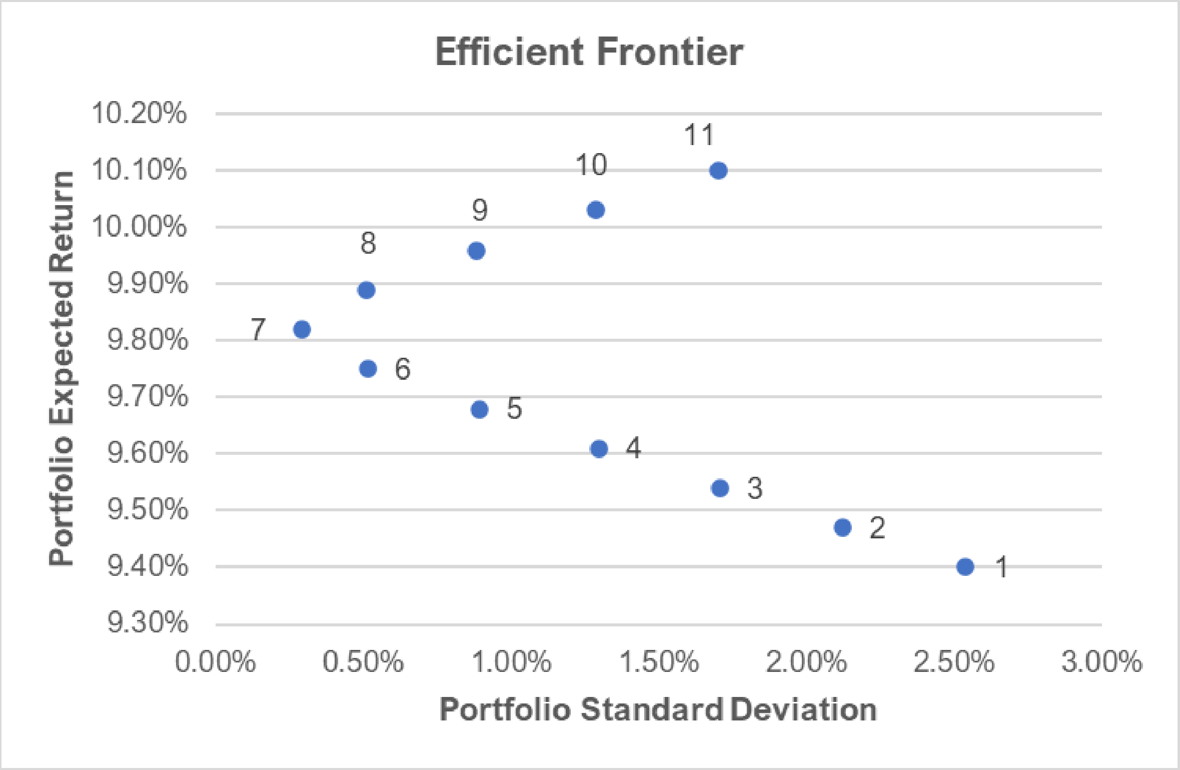

What is next?

- Portfolio Theory and Practice II
  - Efficient Diversification
    - ✓ Readings: Ch. 7
  - Suggested Problems
    - ✓ **Chapter 5:** 7, 13
    - ✓ **Chapter 5 CFA Problems:** 1, 2, 3, 4, 5, 6.
    - ✓ **Chapter 6:** 4, 13, 14, 15, 16, 17, 18.
    - ✓ **Chapter 6 CFA Problems:** 1, 2, 3, 8

Vietnamese-German University

Investments

Finance 2 - BFIN

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Lecture 4