#1

Mathematical Foundation-2

Hikmatallah Nasiri 1181610 Date........ /...... /..........

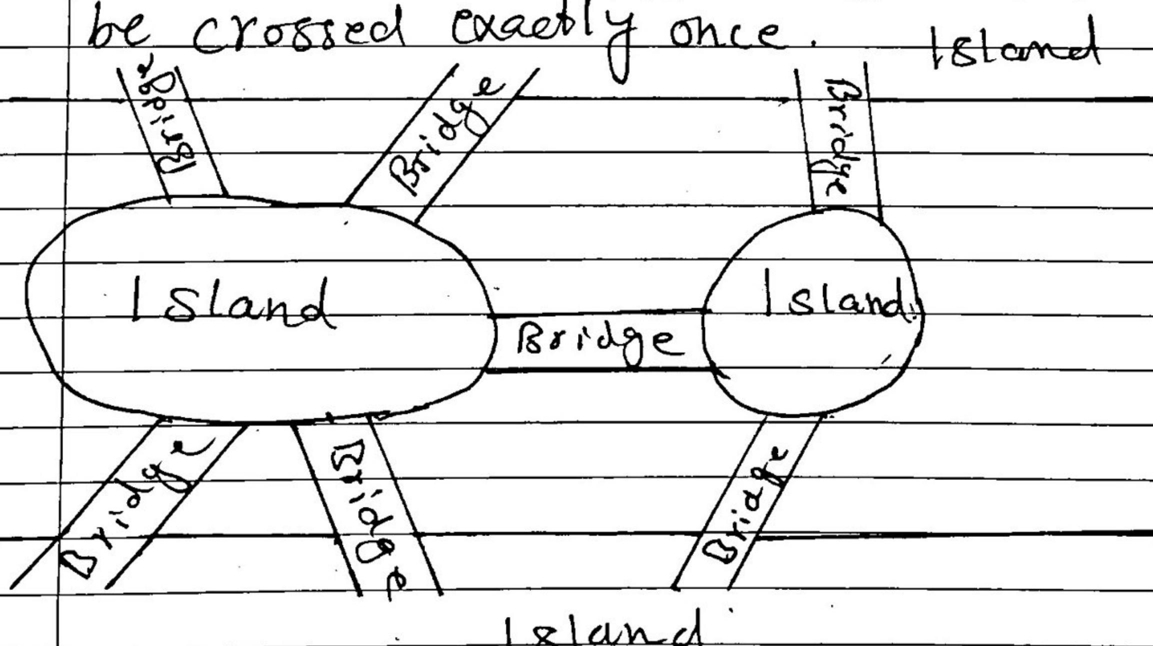
Answer-1 Konigsberg Problem:

Konigsberg bridge problem, a recreational mathematical puzzle, set in the old Prussian city of Konigsberg (now Kaliningard, Russia), that led to the development of the branches of mathematics known as topology and graph theory. In the early 18 century, the citizens of Konigsberg spent their days walking on the intricate arrangement of bridges across the waters of the Pregel (Pregolya) River, which surrounded two central landmass (an Island) was connected by two bridges to the bank of the Pregel and also by two bridges to the upper bank, while the

#2

Date......../......./..........

other landmass (which split the Pregel into branches) was connected to the lower bank by one bridge and to the upper bank by one bridge for total of seven bridges (7). The question arose of whether a citizen could take a walk through the town in such a way that each bridge wold be crossed exactly once.

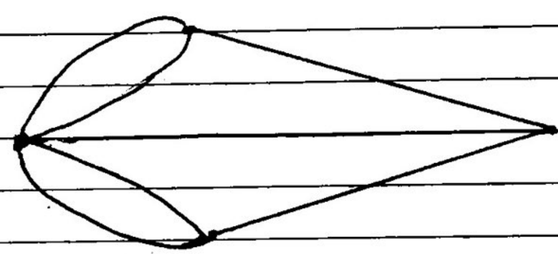


In 1735 the swiss mathematician Leonhard Euler presented a

#3

Date......../......./.........

solution to this problem, concluding that such a walk was impossible. To decide the path euler assumes the 'bridges' as 'edges' and 2 land as 'vertex' which is connected with which vertex is connected with which bridge.



This kind of graph came in result after that 'Euler' observed that whenever one enters a vertex by a bridge, then leaves the vertex by a bridge. So if every brige is tobe walked once, it follows that, for each land mass, the

#4

Date......../......./..........

number of bridges touching land mass would be even. However all the four landmasses in Konigsberg problem are touched by an odd no. of bridges. So it proved that the problem has no solution.

In modern graph theory, an Eulerian path traverses each edge of a graph once and only once. Thus, Euler's claim that a graph possessing such a path has at most two vertices of odd degree was the first theorm in graph theory.

Evergreen

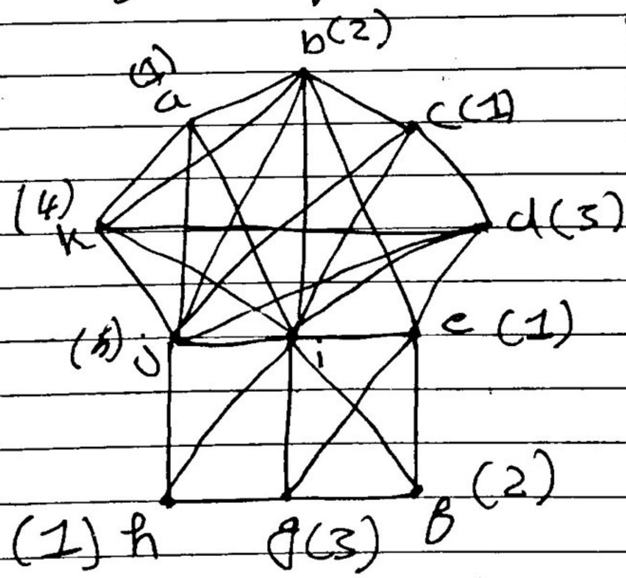
#5

Date......../......./..........

Answer 2: Chromatic Number of the Graph:

Chromatic no. of graph is known by colouring all the vertex of a graph in which no adjacent vertices of a graph are of same colour.

The minimum number of colour used to colour all the vertex by following the above condition is chromatic number of the graph.



Colours (1 – 5)

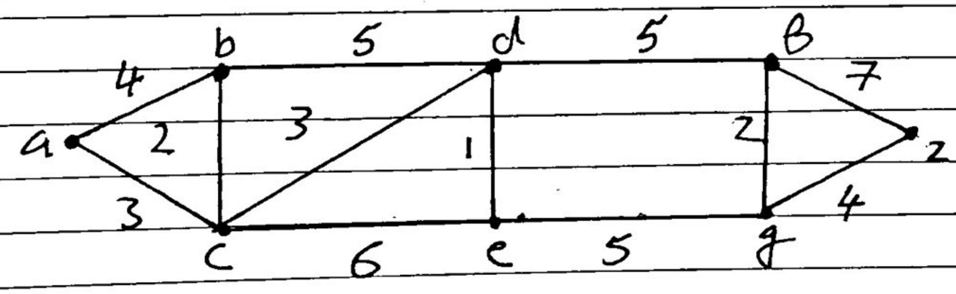
#6

Date........../......./...........

vertex	Colour
a	(1) (Black)
b	(2) (Blue)
c	(1) (Black)
d	(3) (Green)
e	(1) (Black)
f	(2) (Blue)
g	(3) (Green)
h	(1) (Black)
i	(4) (red)
j	(5) (purple)
k	~~t'~~ (4) (red)

Total 5 colours are used to colour all vertices of the graph.

<u>Answer 3</u>- Lenght of shortest path a-z



Shortest path from 'a to z' starting from 'a' by Dijkstra algorithm

#7

Date......../....../.........

	a	b	c	d	e	f	g	z
on start ⇒	0	∞	∞	∞	∞	∞	∞	∞
{a}	$\boxed{0}$	4	3	∞	∞	∞	∞	∞
{a, b}	0	4	$\boxed{3}$	9	∞	∞	∞	∞
{a, b, c}			3	$\boxed{6}$	9	∞	∞	∞
{a, b, c, d}				6	7	11	∞	∞
{a, b, c, d, e}					$\boxed{7}$	11	14	18
{a, b, c, d, e, f}						$\boxed{11}$	13	18
{a, b, c, d, e, f, g}							12	16
{a, b, c, d, e, f, g, z}								16

The shortest distance from a to z is (a, c, d, e, g, z) i.e 16

Answer 4 – Given → G is connected graph
connected simple planar graph

∴ By Euler formula ⇒ $v - e + r = 2$ – (i)

where r = region
v = vertices
e = edges

∵ According to handshaking lemma ~~[illegible]~~
sum of degree of all regions = 2e

$\Sigma(r) = 2 \times e \longrightarrow$ (ii)

8

Date........ /....... /..........

In the question It is given that $v \geq 3$ hence :-

degree of each region ≥ 3

sum of degree all region ≥ 3

$$\text{i.e } \sum \deg(r) \geq 3r$$

But we know that according to - (ii) ...

$$\sum \deg(r) = 2e$$

$$2e \geq 3r$$

$$\frac{2}{3}e \geq r \quad \text{or} \quad \boxed{r \leq \frac{2}{3}e}$$

Put in $\longrightarrow$ (i)

$$2 = v - e + r$$

$$2 = v - e + \frac{2e}{3} \qquad 3e = 3v - 6 + 2e$$

$$\boxed{e \leq 3v - 6} \text{ proved}$$

Answer-5 Euler Circuit: graph will contain an euler circuit if all the vertices have even degree & starting & ending point must be same.

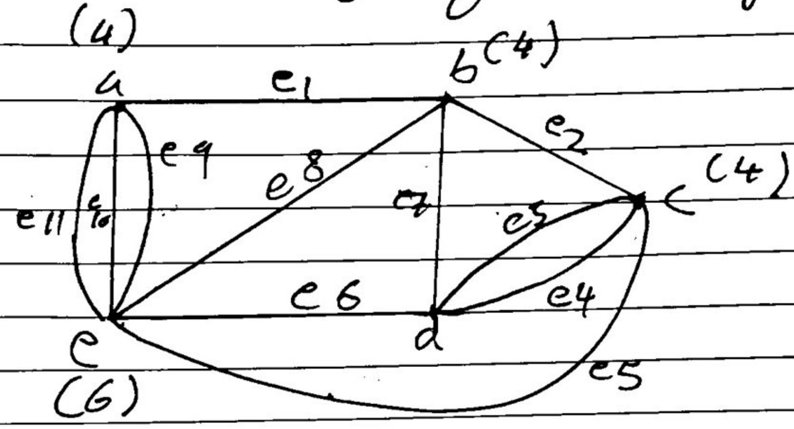
Euler Path: graph will contain

#9

Date........ /....... /..........

an Euler path if it contains at most two vertices of odd degree.

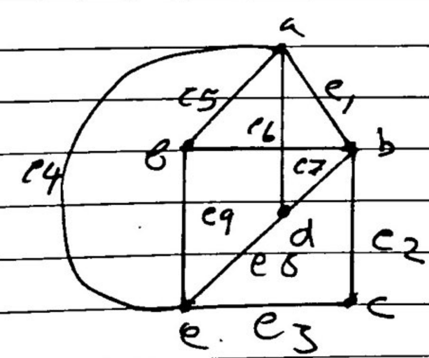
(i)



This graph contain Euler circuit. as each vertex has even degree.

Path : $a \to e_1 \to b \to e_2 \to c \to e_3 \to d \to e4 \to c \to e5 \to e \to e8 \to d \to e7 \to b \to e8 \to e \to e9 \to a \to e10 \to e \to e_{11} \to a$

(ii)



This graph conteins Euler's path because there are exactly two vertices 'f' and 'd' that are odd and all others are even.

#10

Date........../......./...........

Path: $f \to e5 \to a \to e1 \to b \to e2 \to c \to e3 \to e \to e4 \to a \to e7 \to d \to e8 \to e \to e9 \to f \to e6 \to b \to e8 \to d$

Answer 6: From Euler formula we know that

$$v - e + r = 2 \to (1)$$

According to the handshaking lemma

$$\sum deg(v) = 2e$$

$$4 \times 25 = 2e$$

$$100 = 2e$$

$$e = 50$$

Put $v=25$, $e=50$ in $\to (1)$

$$v - e + r = 2$$

$$r = 2 + e - v$$

$$r = 2 + 50 - 25$$

$$\boxed{r = 27}$$

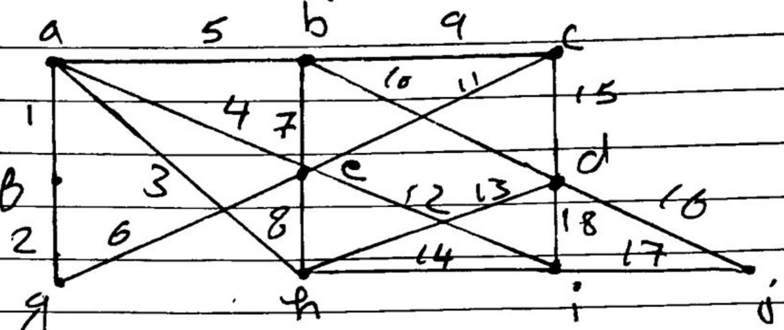
$\Rightarrow$ There are 27 regions

#11

Date....../...../......

Answer 7: Euler's formula

$$r = e - v + 2 \quad \text{——} \quad (1)$$



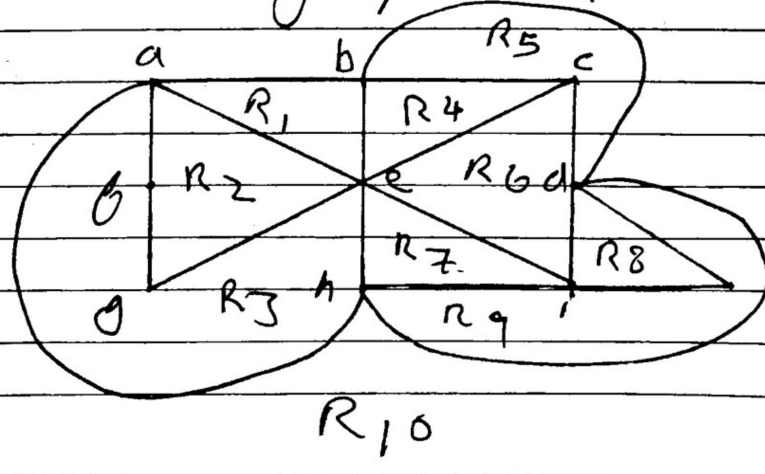
Total edges = 18
Total vertices = 10

Now put in (1)
$r = 18 - 10 + 2$
(b) — $r = 10 \Rightarrow$ Total regions $\Rightarrow 10$

Now for the above graph to proof as planar $e \leq 3v - 6$

$18 \leq 3(10) - 6$
$18 \leq 30 - 6$
$18 \leq 24$

Yes the graph is planner.



Evergreen

12

Date....../....../......

Total region = 10 —— (a)

from (a) & (b)
$a = b$
Hence proved

The graph is planner & has 10 regions and Euler formula applies to the graph.

Now the proof of Euler's formula

$$R = e - v + 2$$

$L \cdot H \cdot S = r$ | $R \cdot H \cdot S = e - v + 2$

By method of Induction

step 1: Basic step, $n = 1$, $e = 1$
$v = 2$, $r = 1$

By putting $R \cdot H \cdot S$

$$R \cdot H \cdot S = 1 - 2 + 2 = 1 = L \cdot H \cdot S$$

(i) Base → True i.e $P(1)$ True
(ii) Induction step → k → True
$k+1$ → True (assume)

step 2: Induction step
Assume for 'k' = True

$G_k \to r_k = e_k - v_k + 2$ —— (i)

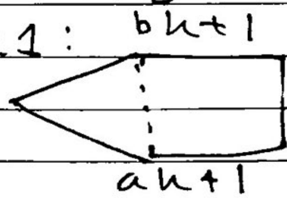
#13

Date....../....../......

let (a_{k+1}, b_{k+1}) be the edge that is added to G_k

result $\Rightarrow G_{k+1}$

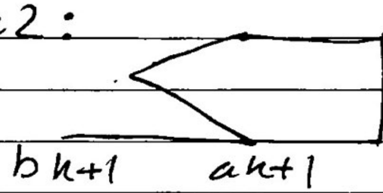
Case 1:



$r_{k+1} = r_k + 1$
$e_{k+1} = e_k + 1$
$v_{k+1} = v_k$

Thus $r_{k+1} = e_{k+1} - v_{k+1} + 2$
$\quad r_k + \cancel{1} = e_k + \cancel{1} - v_k + 2 \Rightarrow$
$\quad r_k = e_k - v_k + 2$
Hence the result is true

Case 2:



$r_{k+1} = r_k,\ e_{k+1} = e_k + 1$
$v_{k+1} = v_k + 1$

Now $r_{k+1} = e_{k+1} - (v_k + 1) + 2$

$r_k = e_k - v_k + 2$
As we assumed for n is True
for all n values the result is True.