NAME – NISHU SHARMA SECTION – D1914

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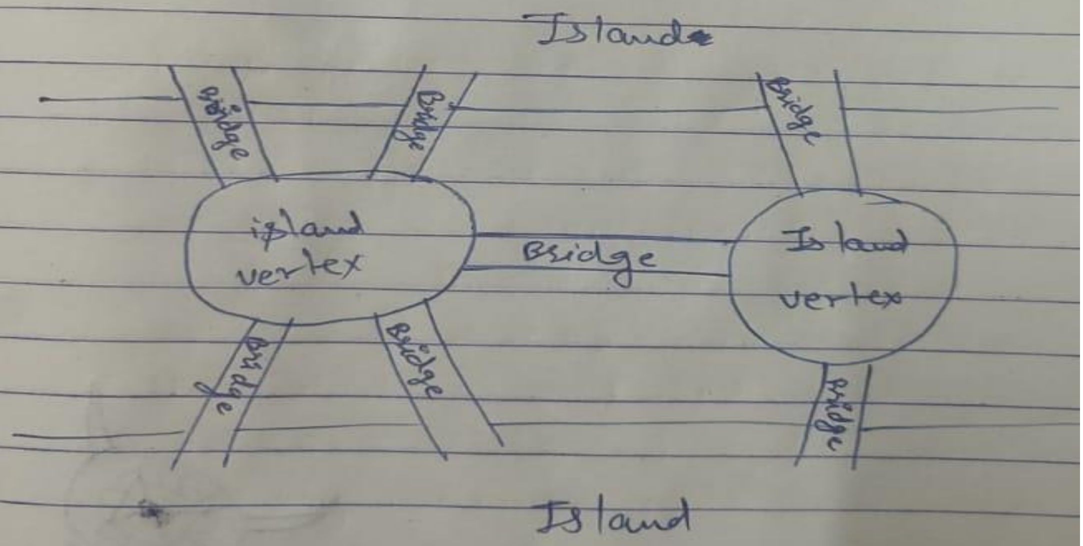
Math CA

Ans-1 Koningsberg problem :

The city of Konigsberg in prussia. Included 2 large island which were connected to each, or to the two main portions of the city by seven bridges.



The problem was to walk through the city that would cross each of the 7 bridges once & only once.

So famous Mathematician Euler decided to find the answer

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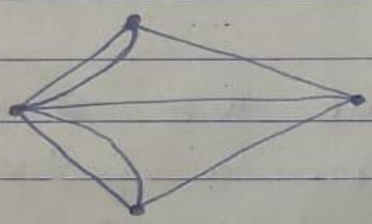
of the problem after analysing the problem. So to decide the path euler assumed the 'bridges' as 'edges' and 2 land lands as 'vertex' which in result told that which vertex is connected with which bridge



So this kind of graph came in result after the assumption of famous mathematician 'Euler'. After that Euler observed that whenever one enters a vertex by a bridge, it leaves the vertex by a bridge. So if every bridge is to be walked once, it follows that, for each land mass, the number of bridges touching the land mass would be even. However all the four landmasses in original problem are touched by an odd no. of

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bridges. So he proved that the problem has no solution. But he gave the alternate solution which was unacceptable :-

1 reaching an island or mainland bank other than one of the bridges
2 accessing any bridge without crossing to its other end.

So Euler gave the conclusion & said that for a walk a necessary condition is that the graph be connected & have exactly zero or two nodes of odd degree. which was known as "Eulerian Path", which also concluded that if there are nodes of odd degree, then any Eulerian path will start at one of them & end at the other.

Another alternate solution was emerged that asked for a path that transverse all bridges & also has same starting &

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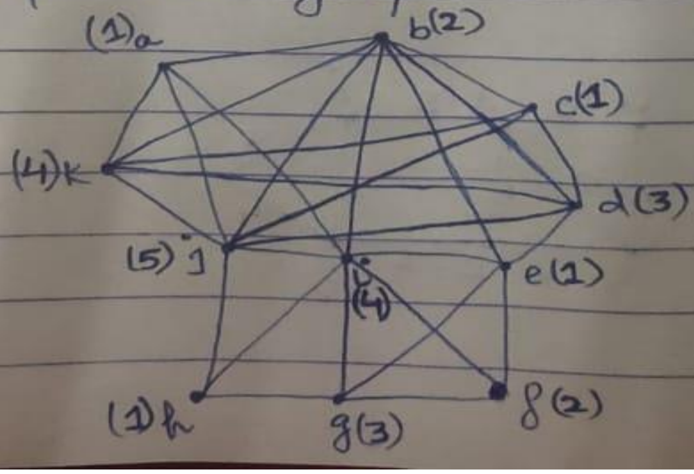
ending point is known as "Eulerian circuit". And this circuit exists if there are no nodes of odd degree & the graph is connected graph

Ans - 2 Chromatic no. of the graph :-

Chromatic no. of graph is known by colouring all the vertex of a graph in which no ~~____~~ adjacent vertices of a graph are of same colour. The minimum number of colour used to colour all the vertex by following the above condition is the chromatic number of the graph.

$\boxed{\text{Colours } (1-5)}$



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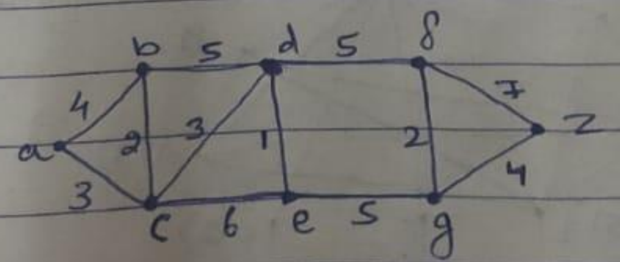
vertex	Colour
a	(1) (Black)
b	(2) (Blue)
c	(1) (Black)
d	(3) (green)
e	(1) (Black)
f	(2) (Blue)
g	(3) (green)
h	(1) (Black)
i	(4) (red)
j	(5) (purple)
k	(4) (red)

Total 5 colours are used to colour all vertices of the graph

So Chromatic no. of the graph is '5'

Ans-3



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Shortest path from 'a to z'.
Starting from 'a' by Dijkshtra's algorithm

	a	b	c	d	e	f	g	z
On start ⇒	0	∞	∞	∞	∞	∞	∞	∞
{a}	$\boxed{0}$	4	3	∞	∞	∞	∞	∞
{a,b}	0	4	$\boxed{3}$	9	∞	∞	∞	∞
{a,b,c}			3	$\boxed{6}$	9	∞	∞	∞
{a,b,c,d}				6	$\boxed{7}$	11	∞	∞
{a,b,c,d,e}					7	$\boxed{11}$	14	18
{a,b,c,d,e,f}						11	13	18
{a,b,c,d,e,f,g}							12	16
{a,b,c,d,e,f,g,z}								16

The shortest distance from a to z is (a, c, d, e, g, z) i.e 16 units.

Ans-4 Given → G is connected graph
connected simple planar graph.

∴ By Euler formula ⇒ $v - e + r = 2$ — ①

where r = region

v = vertices

e = edges

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$\therefore$ Acc to handshaking lemma
sum of degree of all regions $= 2e$.

$$\Sigma \deg(r) = 2e \quad — \quad (2)$$

As the ques says $v \geq 3$ so the
deg of each region ≥ 3
sum of deg of all region ≥ 3

i.e $\Sigma \deg(r) \geq 3r$

But according to (2)

$$\Sigma \deg(r) = 2e$$

$$2e \geq 3r$$

$$\frac{2}{3}e \geq r \quad \text{or} \quad r \leq \frac{2}{3}e \quad — A$$

put in (1)

$$2 = v - e + r$$

$$2 + e - v = r$$

$$2 + e - v \geq \frac{2}{3}e \quad — \text{from } A$$

$$\frac{e}{3} \leq v - 2.$$

$$e \leq 3v - 6$$

Hence proved

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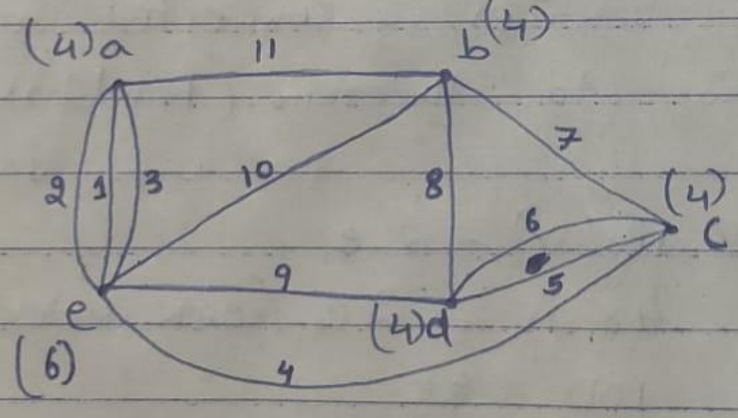
Ans-5 Euler circuit :- graph will contain an Euler circuit if all the vertices have even degree & starting & ending point must be same.

Euler path :- graph will contain an Euler path if it contains atmost two vertices of odd degree.

(i)



This graph contains euler circuit

path :- a → e → a → e → c → d → c → b → d → e → b → a

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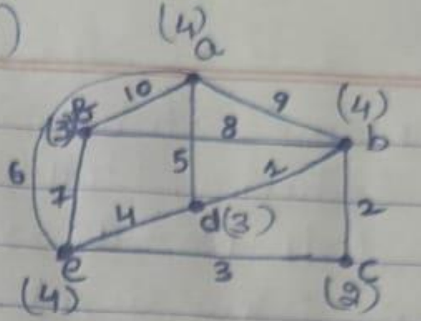
(ii)



This graph contains euler path.

path → d → b → c → e → d → a → e

↓

f ← a ← b ← f

Ans-6 Euler formula ⇒ $v - e + r = 2$

25 vertices, connected graph
According to theorem (handshaking lemma)

$\Sigma \deg(v) = 2e$

$4 \times 25 = 2e$ (each vertex of deg 4)

$100 = 2e$

$50 = e$

Now $v - e + r = 2$

$r = 2 + e - v$

$= 2 + 50 - 25$

$= 27$

$\therefore \boxed{r = 27}$

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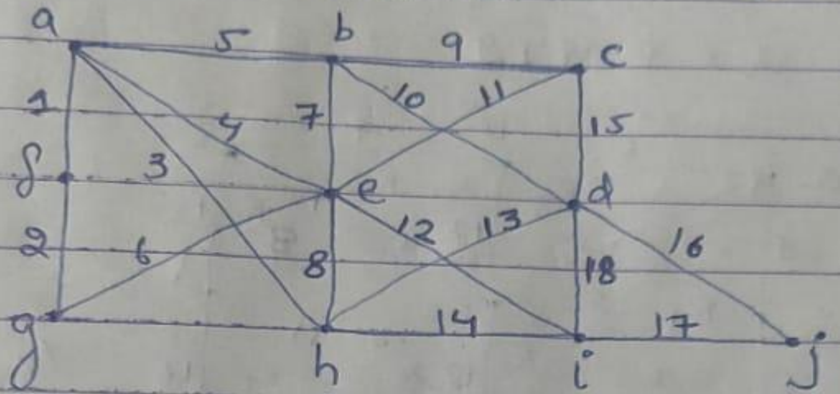
Ans-7 Euler's formula.

$r = e - v + 2$ ——— ①



Total edges = 18
Total vertices = 10
Now put in ①
$r = 18 - 10 + 2$

(b) ——— $r = 10$ $\Rightarrow$ Total regions $\Rightarrow 10$

Now for the above graph to proof as planar

$e \leq 3v - 6$

$18 \leq 3(10) - 6$

$18 \leq 30 - 6$

$18 \leq 24$

Yes the graph is planar

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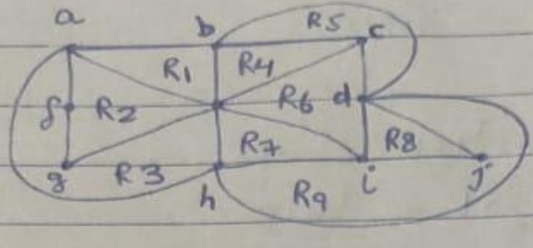

R10

Total region = 10 —— (a)

from (a) & (b)
a = b
Hence proved

The graph is planar & has 10 regions and Euler formula applies to the graph.

Now The proof of Euler's formula
formula = $r = e - v + 2$
$L \cdot H \cdot S = r$ { $R \cdot H \cdot S = e - v + 2$
By Method of Induction.

Step 1: Basic step, $n = 1$
$e = 1, v = 2, r = 1$ v_1 —— v_2
By putting in R.H.S.
$R \cdot H \cdot S = 1 - 2 + 2 = 1 = L \cdot H \cdot S$

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Method of Induction

(i) Base => True i.e P(i) → True
(ii) Induction step → K → True (assume)
K+1 → True (verify)

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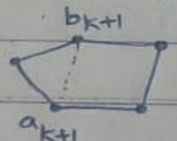
Step 1 = True

Step 2 : Induction Step
Assume for 'k' = True.

$G_k \Rightarrow r_k = e_k - v_k + 2$ — (i)

Let (a_{k+1}, b_{k+1}) be the edge that is added to G_k
result $\Rightarrow$ $G_k + 1$

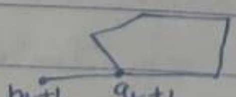
Case 1 :



$r_{k+1} = r_k + 1$
$e_{k+1} = e_k + 1$
$v_{k+1} = v_k$

Thus $r_{k+1} = e_{k+1} - v_{k+1} + 2$
$r_{k+\cancel{1}} = e_{k+\cancel{1}} - v_k + 2 \Rightarrow \boxed{r_k = e_k - v_k + 2}$
Hence the result is true

Case : 2



$r_{k+1} = r_k$, $e_{k+1} = e_k + 1$
$v_{k+1} = v_k + 1$

Now $r_{k+1} = e_{k+1} - v_{k+1} + 2$

$r_k = e_k + 1 - (v_k + 1) + 2$

as we have assumed for k is True

$\boxed{r_k = e_k - v_k + 2}$

for all n values the result is True.