<u>I'll show that does not always exist an EF1 allocation for the setting of chores with capacity constraints</u>. (?)

Let x, y and z be **3** agents.
Suppose that there are **5** chores.
They all need to be divided between the agents.
The chores are divided into **2** categories: the pink one, and the orange one.
The capacity constraint on each category is **1 -** that is, an agent cannot get more than one item from one category.

Agent preferences are represented in the following table:

	chore 1	chore 2	chore 3	chore 4	chore 5
agent x	-4	-2	-1	-2	-1
agent y	-2	-4	-2	-1	-1
agent z	-4	-2	-1	-2	-1

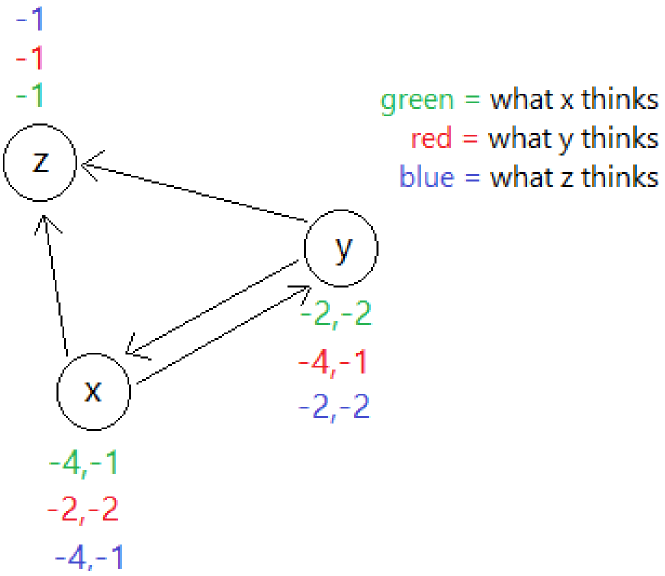
Note that the following allocations are the only possible ones given the capacity constraints:
$\{\{1,3\},\{2,4\},\{5\}\}$
$\{\{1,3\},\{2,5\},\{4\}\}$
$\{\{1,4\},\{2,3\},\{5\}\}$
$\{\{1,4\},\{2,5\},\{3\}\}$
$\{\{1,5\},\{2,3\},\{4\}\}$
$\{\{1,5\},\{2,4\},\{3\}\}$

- An agent cannot get 2 chores from the same category, which leads to the fact that allocations containing the {1,2},{3,4},{3,5},{4,5} groups are not possible.
  In addition, because all objects must be divided, the single chore will always be a chore from the orange group.
- Assume without loss of generality that the first sub-allocation is always given to agent x, the second - to y, and the third - to z.

Now for each of the above allocations, I will show one of the following:

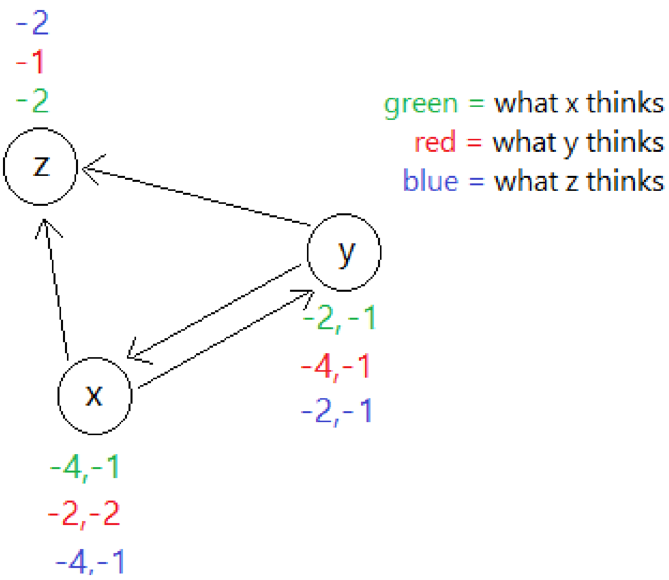
1. The allocation is not EF1.
2. Does not exist a circle in the envy-graph that we can "break" without spoiling the EF1.

$\{\{1,3\},\{2,4\},\{5\}\}$



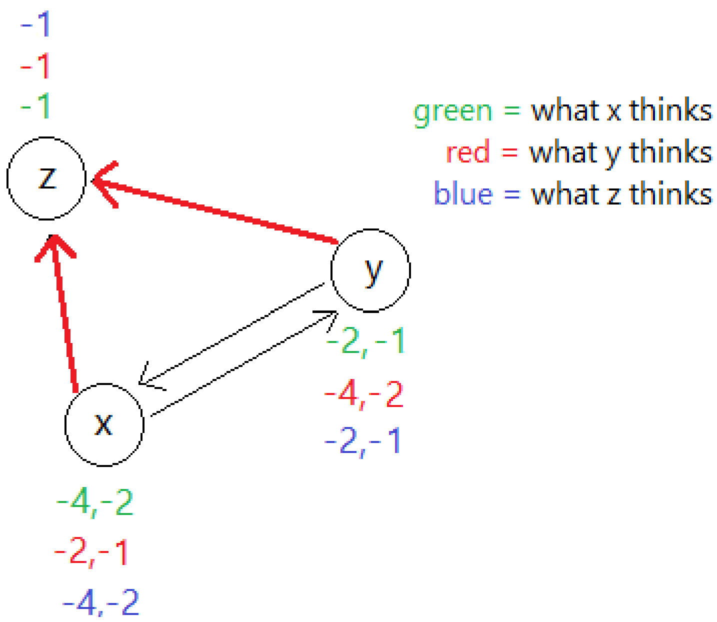
This is the envy-graph, which all the envy displayed in it is at most in one item.
The only circle on this graph is {x,y}.
Note that if we'll "break" it, x and y will envy z in more than one item!

$\{\{1,3\},\{2,5\},\{4\}\}$



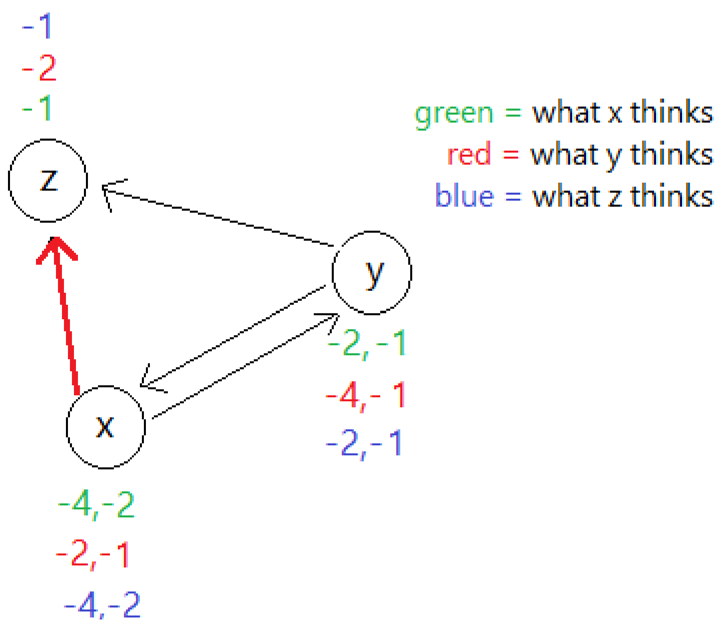
Again, the only circle here is {x,y}, but if we'll "break" it, y will start to envy z in more than one item (z will see his allocation's utilities as {-2,-2}, and z's as {-1}).

$\{\{1,4\},\{2,3\},\{5\}\}$



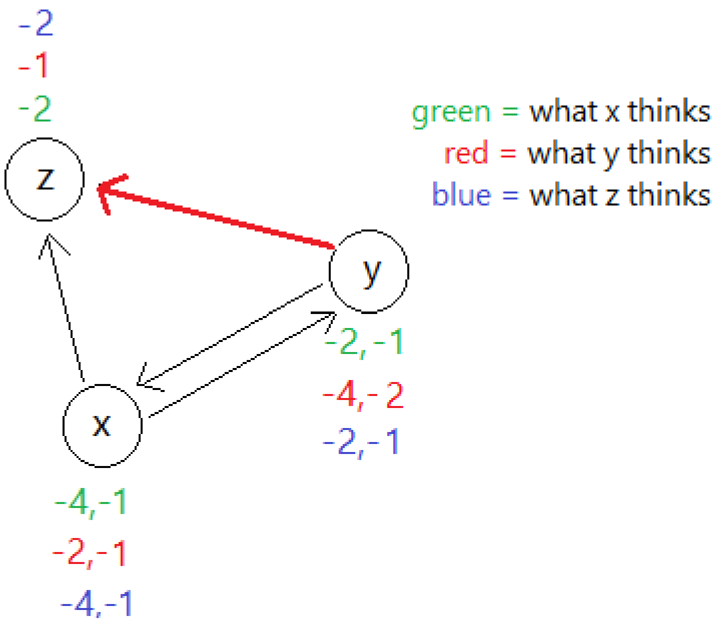
Both x and y envy z in more than one item!! (the red arrows).

$\{\{1,4\},\{2,5\},\{3\}\}$



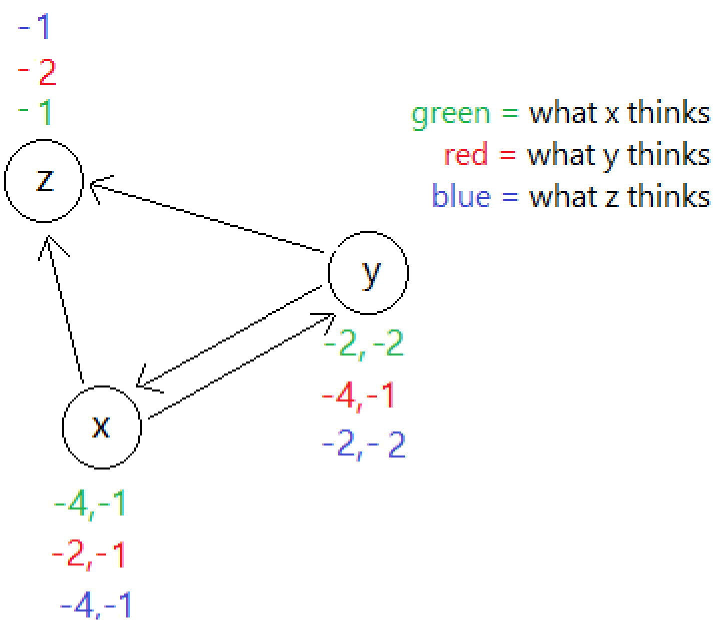
Here x envies z in more than one item!

$\{\{1,5\},\{2,3\},\{4\}\}$



Here y envies z in more than one item!

$\{\{1,5\},\{2,4\},\{3\}\}$



This allocation is EF1, but if we'll "break" the {x,y} circle, then x will envy z in more than one item!

Done.