**Section 1: Quarter Car Model**

**Part a)**

A polynomial least squares fit was performed on the given spring force and damping force data to determine the coefficients for the below spring and damping force equations. See the attached least squares fit MATLAB codes.

The coefficients were determined to be the following:



Figure 1. Plot of spring force data and its corresponding least squares fit



Figure 2. Plot of damping force data and its corresponding least squares fit

**Part b)**

The following 2-equation system of 2nd order ODEs describes the displacement, velocity, and acceleration of the quarter car model (QCM.

This system (equations 1 and 2) is converted into the following 4-equation system of 1st order ODEs:

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Final 4-equation system of 1st order ODEs:

**Part c)**

See the attached 4th order Runge-Kutta and simulation MATLAB codes.

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**Section 2: Simulation**

**Part a)**

See the attached 4th order Runge-Kutta and simulation MATLAB codes.

Characteristic time scale and timestep, h, used:

V = 10 km/hr: T = 1.8720 seconds and h = T/100 = 0.0187 seconds

V = 40 km/hr: T = 0.4680 seconds and h = T/50 = 0.0094 seconds

**Part b)**

NEED TO UPDATE ALL PART B PLOTS WITH THE CORRECT ONES FROM RUNNING THE MATLAB CODES

The plots of displacements and velocities for the sprung and unsprung masses at velocities of 10 km/hr and 40 km/hr are shown below in Figure 3 and Figure 4, respectively. In Figure 5 and Figure 6, respectively, the displacement and velocity of the sprung mass are compared for V = 10 km/hr and V = 40 km/hr, From Figures 5 and 6, observe that for the sprung mass displacement and velocity, the amplitudes are significantly greater for V = 40 km/hr than they are for V = 10 km/hr. Additionally, the periods of oscillation is shorter for V = 40 km/hr than they are for V = 10 km/hr.



Figure 3. Plot displacements and velocities of sprung and unsprung masses versus time using 4th order Runge-Kutta for V = 10 km/hr



Figure 4. Plot displacements and velocities of sprung and unsprung masses versus time using 4th order Runge-Kutta for V = 40 km/hr



Figure 5. Comparison of 4th order Runge-Kutta displacement of sprung mass for V = 10 km/hr and V = 40 km/hr



Figure 6. Comparison of 4th order Runge-Kutta velocity of sprung mass for V = 10 km/hr and V = 40 km/hr

**Part c)**

NEED TO UPDATE ALL PART C PLOTS WITH THE CORRECT ONES FROM RUNNING THE MATLAB CODES

In the forward Euler method, the following calculations are required:

See attached forward Euler and simulation MATLAB codes.



Figure 7. Plot displacements and velocities of sprung and unsprung masses versus time using forward Euler for V = 40 km/hr using a timestep, h, of T/50 = 0.0094 seconds (timestep used in Section 2, part a)



Figure 8. Comparison of displacements and velocities of sprung and unsprung masses between 4th order Runge-Kutta and forward Euler using a timestep, h, of T/50 = 0.0094 seconds (timestep used in Section 2, part a)

At time = 2T, the absolute difference between the 4th order Runge-Kutta and the forward Euler method for each of the four unknown variables is:

Sprung mass displacement: NaN

Sprung mass velocity: NaN

Unsprung mass displacement: NaN

Unsprung mass velocity: NaN

In order for the above absolute difference to drop below 10^-3 for all four unknown variables, the timestep, h, must decrease to approximately ….

The total number of timesteps run for this case is ….

….

CURRENTLY HAVE ISSUES WITH FORWARD EULER CODE. RESULTS ARE GOING TO INFINITY OR CLOSE TO INFINITY (~10^206).

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**Section 3. Data Analysis**

**Part a)**

O(h^2) centered finite difference scheme:



Figure 9. Plot of sprung mass acceleration versus time for V = 10 km/hr using the O(h^2) centered finite difference scheme



Figure 10. Plot of sprung mass acceleration versus time for V = 40 km/hr using the O(h^2) centered finite difference scheme

|  |  |
| --- | --- |
| V = 10 km/hr | V = 40 km/hr |
| Max acceleration = 1.1852 m/s^2 | Max acceleration = 12.1177 m/s^2 |
| Comfort level: Very uncomfortable | Comfort level: Extremely uncomfortable |

Table 1. Maximum acceleration and comfort level for each velocity level using the O(h^2) centered finite difference scheme

**Part b)**

O(h) centered finite difference scheme:



Figure 11. Plot of sprung mass acceleration versus time for V = 10 km/hr using the O(h) forward finite difference scheme



Figure 12. Plot of sprung mass acceleration versus time for V = 40 km/hr using the O(h) forward finite difference scheme

|  |  |
| --- | --- |
| V = 10 km/hr | V = 40 km/hr |
| Max acceleration = 1.1852 m/s^2 | Max acceleration = 12.1177 m/s^2 |
| Comfort level: Very uncomfortable | Comfort level: Extremely uncomfortable |

Table 2. Maximum acceleration and comfort level for each velocity level using the O(h) forward finite difference scheme

From Table 2, the maximum acceleration and comfort level is listed for each velocity. Notice that the acceleration values and the comfort levels using the O(h) forward finite difference scheme are the exact same as the values and levels using the O(h^2) centered finite difference scheme.

**Part c)**

**Part d)**