EE5321, Spring 2018
Homework Assignment 5: Min Time Constrained Input
Due 4/12/2018

Problem 1: **Linear Bang-Bang Control (50 points)**

Implement the system $\dot{x}_1 = x_2$, $\dot{x}_2 = x_3$, $\dot{x}_3 = -x_1 - x_2 - x_3 + u$ in Simulink using a fixed solver with time step of 0.025 seconds. Impose a control (input) constraint of +/-1. Optimize the minimum time performance index $J = \int_0^{t_f} 1\, dt$ with the final constraint of zero for all three states. (i.e. this should result in a bang-bang type solution, but note that the switches might not be 'crisp'). You can use the options as options = optimset('Display', 'iter', 'TolCon', 1e-5,'TolFun', 1e-5, 'PlotFcns', 'optimplotx', 'MaxFunEvals', 4400.'Algorithm','interior-point'); and an initial control of 0.01. Note you may have to adjust the maximum number of function evaluations to make the final control look reasonable. You get to pick the initial guess at the optimal final time, but keep it in the range of 1 to 6. Note you may have to try a few different final time guesses to ensure the state constraints are satisfied at the final time.

a) Take the initial state to be $(x_1(0), x_2(0), x_3(0)) = (1,0,0)$ and find the optimal control and optimal final time. Plot the final state and control time histories and show the optimal final time. How many switches were needed?
b) Take the same initial states as a), but impose control constrained to +/- 0.75 and find the optimal control and optimal final time. Plot the final state and control time histories and show the optimal final time. How many switches were needed and did the switch times change?
c) Take the same initial states as a), but impose control constrained to +/- 0.5 and find the optimal control and optimal final time. Plot the final state and control time histories and show the optimal final time. How many switches were needed? Did the approximate switch times change and, if so, by how much?
d) Take the initial state to be $(x_1(0), x_2(0), x_3(0)) = (1,1,1)$ and find the optimal control and optimal final time with the control constrained to +/- 1.0. Plot the final state and control time histories and show the optimal final time. Did the number of switches change?

Problem 2: Nonlinear Van der Pol (50 points)

For the Van der Pol system, implement the following in Simulink:

$$\dot{x}_1 = x_2 \qquad x_1(0) = 0 \qquad x_1(t_f) = 3$$
$$\dot{x}_2 = \mu(1 - x_1^2)x_2 - x_1 + u \qquad x_2(0) = 0 \qquad x_2(t_f) = 0$$

Minimize the performance index $J = \int_0^{t_f} 1\, dt$ I suggest a fixed step integration of 0.02 seconds. I suggest you have the initial control, *u*, to be 0.01 for all time 0:0.02:1. Also, I suggest you use the following options:

```
options = optimset('Display','iter','TolCon',1e-4,'TolFun',1e-
4,'PlotFcns','optimplotx', 'MaxFunEvals',5000,'Algorithm','interior-
point');
```

a) Set the upper and lower bounds of the control to be +/-10, find the optimal final time and plot the optimal state and control time histories when $\mu = 5$.
b) Set the bounds to be +/-7 and rerun, plotting the optimal state and control time histories and note the optimal final time.
c) Set the bounds to be +/-5 and rerun, plotting the optimal state and control time histories and note the optimal final time.
d) Compare the optimal final times found for these three situations and note the trend. Make comments about how the control time histories changed as well as the qualatative differences between the three state time histories.