Proceedings of the 36th
Conference on Decision & Control
San Diego, California USA • December 1997

Closed-loop Model Validation for an Inverted Pendulum Experiment via a Linear Matrix Inequality Approach

Li Chen Roy Smith
Department of Electrical and Computer Engineering
University of California, Santa Barbara CA93106

Abstract

Recent sampled-data time domain model validation techniques are applied to the inverted-pendulum experiment. An LQG controller is first designed to stabilize the system. Based on the closed-loop finite length sampled datum, a robust control model is derived and model validation tests are run to check the model validity using the linear matrix inequality algorithm [1].

1 Introduction

In a robust control context, model validation is equivalent to asking whether or not there exist an unknown signal vector (noise and disturbance) and an unknown perturbation (modeling dynamic uncertainty), satisfying the specified assumptions, and accounting for the input-output datum. Based on the techniques developed in [1] [2], we present in this paper an experimental application of sampled-data time domain model validation with a closed-loop finite length datum. The model validation test can be formulated as an LMI EVP problem [3] and is implemented by an efficient method of centers algorithm in [1].

2 Application to Inverted-pendulum System

The setup for the inverted-pendulum experiment consists of a cart and a pendulum (Figure 1).

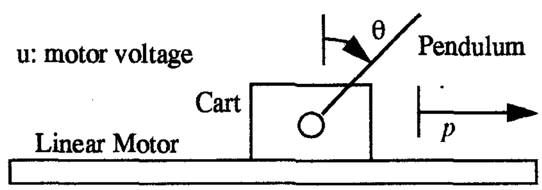

Figure 1: Inverted- pendulum experiment set-up

One end of the pendulum is pivoted on the cart, which enables the pendulum to rotate 360 degree in a vertical plane. The idea of the experiment is to design a controller that drives the cart to balance the pendulum in the upward position. The system can be analytically separated into two parts: cart and pendulum. The linearized models are as follows:

$$Cart(s) = \frac{p(s)}{u(s)} = \frac{0.039}{s(1 + 0.0245s)}$$

$$Pend(s) = \frac{\theta(s)}{p(s)} = \frac{-3.45s^2}{s^2 + 0.39s - 33.85}$$

where $u(s)$ is the linear motor voltage drive, $p(s)$ is the position of the cart and $\theta(s)$ is the angle of the pendulum. The coefficients for the two transfer functions are estimated experimentally.

It is noted that besides the coefficients estimation errors, there are also nonlinear effects from the power amplifier and static friction on the track (deadzone of about -3~3 volt). Two measurements, cart position and pendulum angle, are taken by an A/D board and the signals are corrupted by noises to some degree. Therefore, a robust control model including perturbation and disturbance is motivated to cover all of these degrading effects. Before proceeding, a preliminary LQG controller is designed to stabilize the system. The experiment results are plotted in Figure 2.

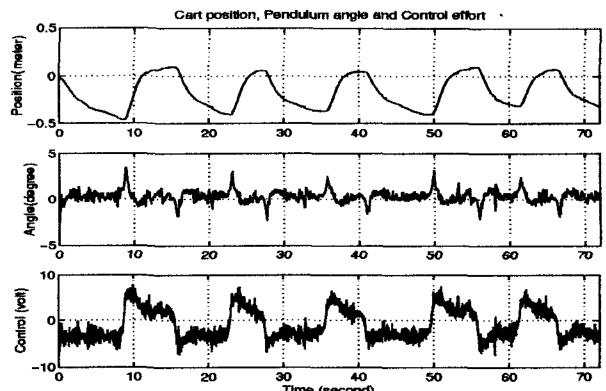

Figure 2: experimental results of closed-loop system (sampling period 0.03 sec.)

By analyzing the above datum, we have the following: to cover the uncertainty of the cart model at high frequencies (about 10~50 rad/sec.), an additive perturbation is introduced around the cart with weight

$$W_{\Delta} = \frac{0.4(s+1)}{(s+10)(s+50)};$$

a disturbance is also modeled at output of the cart with weight as follows to cover the drifting effect:

$$W_d = \frac{0.01}{s+1}.$$

The block diagram of the resulting robust control model is shown in Figure 3, where the bounds of $\Delta(s)$ and $\omega(t)$

are both assumed to be one.

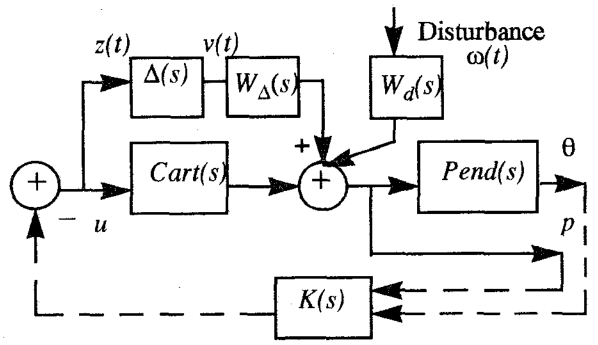

Figure 3: Block diagram of the robust control model (K(s) is the LQG controller)

3 Model Validation with Closed-loop Datum

We denote the measured cart position as p_m, nominal position as p_n and the difference between them as residue r. By linearity of the model in Figure 3, we have

$$r = p_m(t) - p_n(t) = W_\Delta v + W_d \omega$$

Therefore, the model validation conditions can be formulated as the following optimization problem:

$$\beta_{opt} = \inf_{v,\,\omega} \beta \text{ such that,}$$

$$\|v\| \le \beta \|z\|, \|\omega\| \le \beta, \; r = W_\Delta v + W_d \omega.$$

The robust model derived in Section 2 is invalidated if $\beta_{opt} > 1$,which means there does not exist any LTI perturbation operator and energy bounded disturbance signal in the assumed bounded sets to account for the experimental datum, i.e. our bounds assumption for perturbation and disturbance is invalidated.

In order to obtain the nominal cart position datum p_n in our application, we treat the closed-loop datum in an open-loop fashion, i.e. we feed the measured control signal $u(t)$ directly into the open-loop cart nominal model $Cart(s)$ to get p_n. For our testing, the bound of ω(t) is fixed to 1.

The experimental, nominal response and the residue are shown in Figure 4.

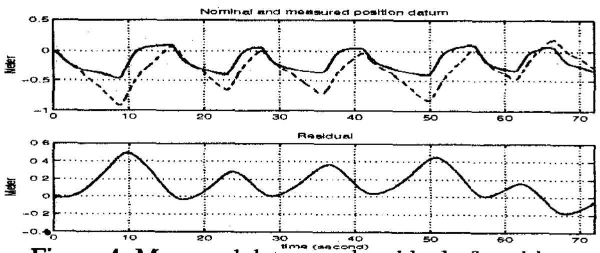

Figure 4: Measured datum and residual of position (solid: measured; dashed: nominal)

The above optimization problem can be reformulated into a computational tractable LMI[1], which is omitted here for brevity. The algorithm given in [1] is coded in both Matlab and C language.

To alleviate the burden of computation, the whole set of data are subsampled with relatively large sampling periods to conduct the validation tests (long-run test); for smaller subsampling periods, we truncate the first 16 sec. datum, which is about one period of the cart movement to run the tests (short-term test). The testing results are shown in Table 1.

Long	Subsampling	Dimension	β_{opt}
1	6.0 sec.	37	0.4586
2	4.5 sec.	49	0.5182
3	3.0 sec.	73	0.9475
4	2.4 sec.	91	1.2860
Short	Subsampling	Dimension	β_{opt}
1	1.2 sec.	37	0.3921
2	0.9 sec.	49	0.5267
3	0.6 sec.	73	0.8147

Table 1: Model validation test results

From Table 1, we conclude that the robust control model derived in section 2 is invalidated in the fourth long-run test. It is, therefore, suggested to increase the gain of the perturbation weight from 0.4 to at least 0.52 to account for the experimental datum.

4 Conclusion

In this paper we have completed a model validation experiment using LMI computational techniques. The closed-loop datum are treated in an open-loop fashion to fit in the model validation conditions test. The tests are run in both long-run and short-term fashions in order to reach a reasonable decision with relative less computational time.

References

[1] G. Dullerud and R. Smith, "Sampled-data model validation: an algorithm and experimental application," *Int. J. Robust & Nonlinear Control*, Vol. 6, 1996.

[2] R. Smith and G. Dullerud, "Continuous-time control model validation using finite experimental data," *IEEE Trans. Auto. Control*, Vol. 41, No. 8, August 1996.

[3] S. P. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia: SIAM, 1994.