012_Folien

2018 年 11 月 23 日

```
In [1]: from matplotlib import pyplot as plt
        import numpy as np
        %matplotlib inline
```

1 Beschreibende Statistik

1.1 Übersicht

- Warum Statistik

LagemaSS

Streuung

Graphiken

Abhängigkeit Bei gleichem LagemaSS können sich die zugrundeliegenden Daten unterscheiden:

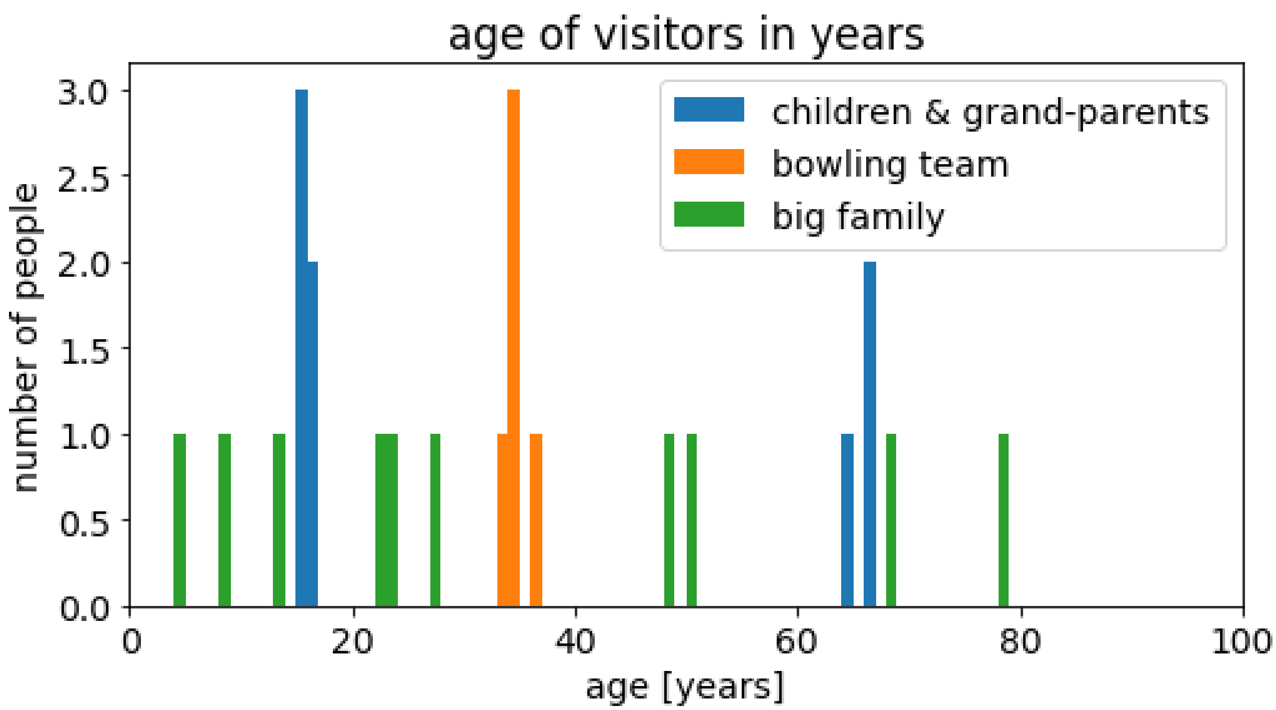
```
In [3]: '''survey for age of visitors in theme park'''
        a = np.array([15, 16, 64, 15, 66, 66, 15, 16])
        b = np.array([34, 33, 34, 34, 36])
        c = np.array([8, 68, 50, 48, 4, 23, 13, 27, 79, 22])
        print('average age in group A: {:.1f}  B: {:.1f}  C: {:.1f} years'
              .format(a.mean(), b.mean(), c.mean()))
```

```
average age in group A: 34.1  B: 34.2  C: 34.2 years
```

```
In [4]: '''same mean, different width'''
        print('average age in group A: {:.1f}  B: {:.1f}  C: {:.1f} years'
              .format(a.mean(), b.mean(), c.mean()))
        bins = range(80)
        plt.figure(figsize=(8, 4))
        plt.hist(a, bins=bins, label='children & grand-parents')
```

```
    plt.hist(b, bins=bins, label='bowling team')
    plt.hist(c, bins=bins, label='big family')
    plt.title('age of visitors in years')
    plt.xlabel('age [years]')
    plt.ylabel('number of people')
    plt.xlim(0, 100)
    plt.legend();
```

```
average age in group A: 34.1  B: 34.2  C: 34.2 years
```



2 StreuungsmaSSe

Spannweite Der Bereich vom kleinsten zum gröSSten Wert

$$R = x_{max} - x_{min}$$

```
In [5]: print('range of age in \ngroup A: {:2d}  \ngroup B: {:2d}  \ngroup C: {:2d}  '
              .format(a.max()-a.min(), b.max()-b.min(), c.max()-c.min()))
```

```
range of age in
group A: 51
group B:  3
group C: 75
```

2.1 Empirische Varianz

$$s^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

2.1.1 Verschiebungssatz

$$s^2 = \overline{x^2} - \overline{x}^2$$

Beweis: [ÜA]

```
In [6]: '''calculate mean and variance "on the fly"'''
        np.random.seed(654321)
        xx = 2*np.random.random(size=10) -1. # randomly sampled -1 .. +1
        print(xx)
        xxsum = 0                # initialize the sum
        xxqsum = 0               #            the square-sum
        n = 0                    #            the number of measures
        for xi in xx:            # after every step: tell me mean and standard deviation
            xxsum += xi
            xxqsum += xi**2
            n += 1               # to distinguish from i, even if nx = 1 + i
            print('{:2d} th step has mean {:5.2f} and std {:5.2f}'.
                  format(n, xxsum/n, np.sqrt(xxqsum/n-(xxsum/n)**2)))
```

```
[-0.53601194  0.46506068  0.92618017  0.38634326  0.71896458  0.67740375
 -0.84280437 -0.85675517 -0.93527115  0.93672658]
 1 th step has mean -0.54 and std  0.00
 2 th step has mean -0.04 and std  0.50
 3 th step has mean  0.29 and std  0.61
 4 th step has mean  0.31 and std  0.53
 5 th step has mean  0.39 and std  0.50
 6 th step has mean  0.44 and std  0.47
 7 th step has mean  0.26 and std  0.63
 8 th step has mean  0.12 and std  0.69
 9 th step has mean  0.00 and std  0.73
10 th step has mean  0.09 and std  0.75
```

2.1.2 Stichprobenvarianz

Die Stichprobenvarianz ist leicht unterschiedlich definiert durch

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$

Hintergrund:

- Die Anzahl der Freiheitsgrade ist $N-1$, da Nebenbedingung $\sum_{i=1}^{N}(x_i - \bar{x}) = 0$
- Für N=1 ist die Varianz nicht definiert
  - anstatt 0 bei der empirischen Varianz
- Für groSSe N ist sie asymptotisch gleich

2.1.3 Eigenschaften

Unter der linearen Abbildung

$$y_i = ax_i + b$$

ergibt sich

$$s_y^2 = a^2 s_x^2$$

2.2 Standardabweichung

$$s = +\sqrt{s^2}$$

Unter der linearen Abbildung

$$y_i = ax_i + b$$

ergibt sich

$$s_y = |a|\ s_x$$

```
In [7]: '''example with three groups visiting Volksfest:'''
        print('std. dev. of age in A: {:.1f}  B: {:.1f}  C: {:.1f}'
              .format(a.std(), b.std(), c.std()))
```

```
std. dev. of age in A: 24.2  B: 1.0  C: 24.4
```

3 Histogramme

Viele Daten: von der Strichliste zum Histogramm

3.0.1 Diskrete Ereignisse

Beispiel: Münze, Würfel, Karten, KörpergröSSe in 5cm-Schritten, ...

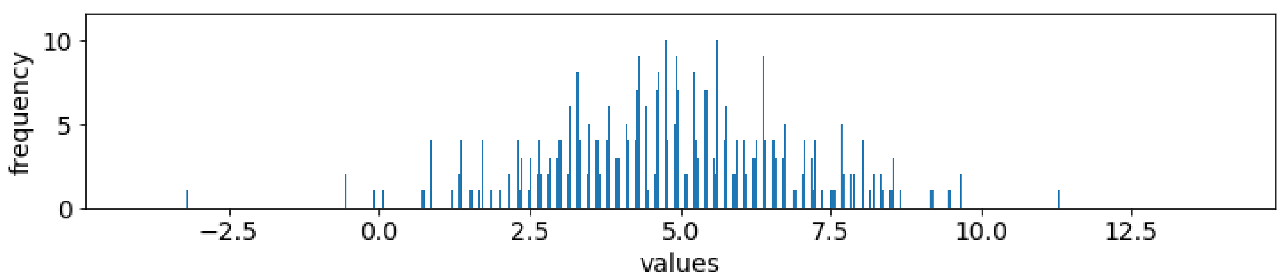
3.0.2 kontinuierliche Ereignisse

KörpergröSSe, Temperatur

Klasseneinteilung führt wieder zurück auf den diskreten Fall

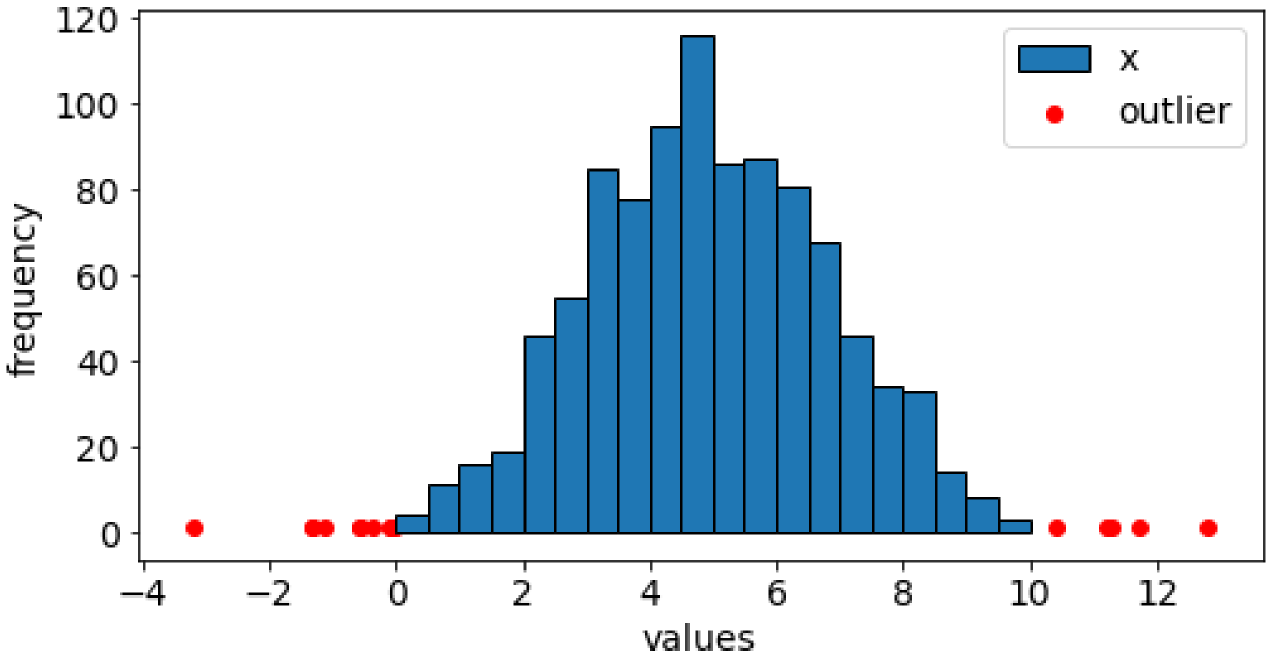
```
In [8]: '''thousand random numbers'''
        np.random.seed(9876543)
        xmany = np.random.normal(loc=5., scale=2., size=1000)
        xmany[0] = -3.2                          # extra values for later
        xmany[1] = 12.8
        plt.figure(figsize=(12,1))
        plt.plot(2*[xmany], [0, 1], 'k-')
        plt.yticks([]);
```


```
In [9]: plt.figure(figsize=(12, 2))
        bins = np.linspace(-4, 14, (14+4)*40+1)
        hist_full = plt.hist(xmany, bins, rwidth=0.4)
        plt.xlabel('values')
        plt.ylabel('frequency');
```




```
In [10]: '''information from histogram'''
         plt.figure(figsize=(8, 4))
         bins = np.linspace(0,10,21)
         hist_20 = plt.hist(xmany, bins, label='x', edgecolor='black')
         outlier = xmany[np.logical_or(xmany<0, xmany>10)]
```

```
plt.scatter(outlier, np.ones_like(outlier), color='red', label='outlier')
plt.xlabel('values')
plt.ylabel('frequency')
plt.legend();
```



3.0.3 Klasseneinteilung

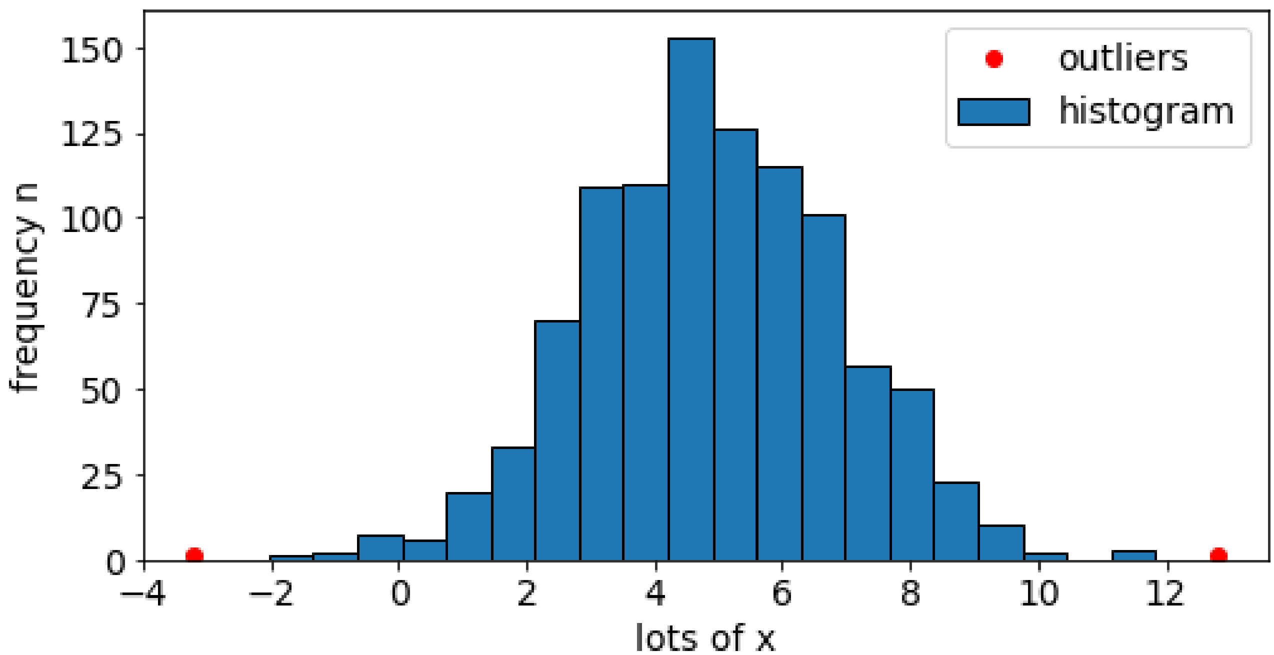
Sinnvoll für Informationsgehalt. Faustregel: - bis zu 20 Klassen - Anzahl Klassen $n_{\text{bins}} \equiv \sqrt{n}_{\text{data}}$ (für n_{data} bis 100 bzw. 400) - Klassenbreite $w_{\text{bins}} = \frac{3.49 \cdot \sigma}{\sqrt[3]{n_{\text{data}}}}$ (Scotts Regel) David W. Scott: On optimal and data-based histogram. Biometrika **3** 66, 1979, S. 605–610

Dabei **Breite** der Klassenbalken - möglichst konstant - sonst kodiere Anzahl in *Fläche* (nicht Höhe)

```
In [11]: '''optimal histogram according to Scott-s rule'''
         plt.figure(figsize=(8, 4))
         n = int(np.round(np.sqrt(min(400, xmany.shape[0]))))
         # print('optimal number of bins = {:.0f}'.format(n))
         w = 3.49*xmany.std()/xmany.shape[0]**0.3333333333333
         # print('optimal bin size = {:.3f}'.format(w))
         xstart = xmany.mean() - n/2.*w
         xend = xmany.mean() + n/2.*w
         print('Optimal histogram has {} bins, each {} wide in the range{} .. {}'
               .format(n, np.round(w, decimals=3), np.round(xstart, decimals=2),
                       np.round(xend, decimals=2)))
         bins_opt = np.linspace(xstart, xend, n+1) # number of bin-edges within range
```

```
hist_opt = plt.hist(xmany, bins_opt, label='histogram', edgecolor='black');

xoutliers = xmany[np.logical_or(xmany>xend, xmany<xstart)]
youtliers = np.ones_like(xoutliers)
plt.plot(xoutliers, youtliers, 'ro', label='outliers')
plt.legend(loc='upper right')
plt.xlabel('lots of x')
plt.ylabel('frequency n');
```

```
Optimal histogram has 20 bins, each 0.693 wide in the range-2.03 .. 11.83
```



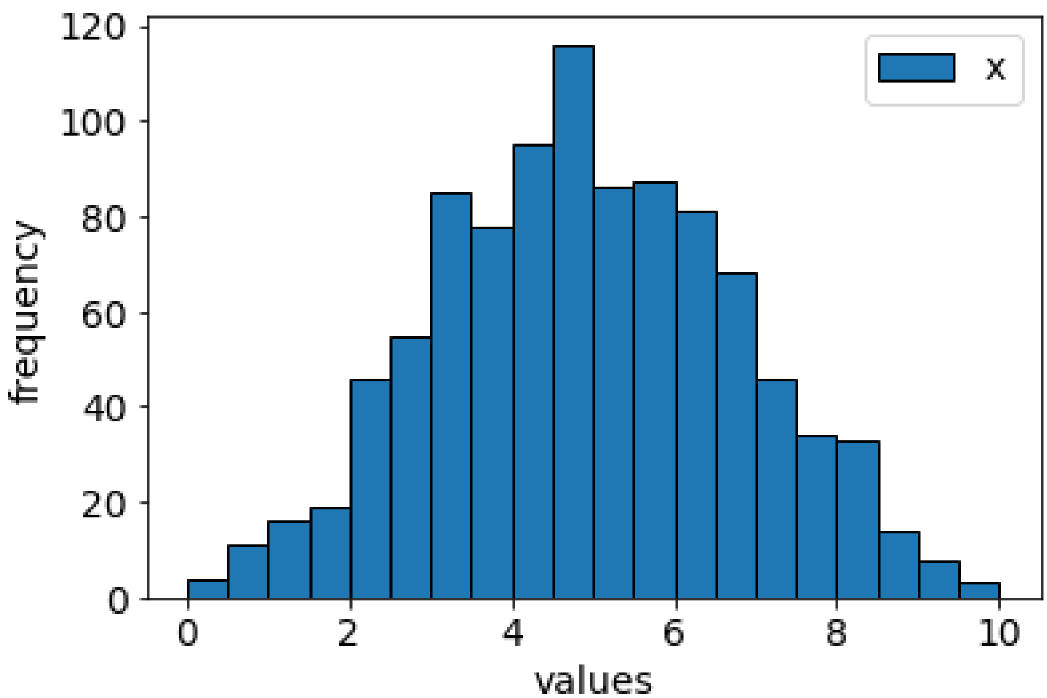
3.0.4 Information

- Häufigste Werte: "Modus"
- Ausreisser
- Mittelwert (=Schwerpunkt)
- Wertebereich
- Form der Verteilung

3.1 Modus

```
In [12]: '''once again the histogram with 20 bins'''
         bins = np.linspace(0, 10, 21)
         hist_20 = plt.hist(xmany, bins, edgecolor='black', label='x') # show histogram (again)
```

```
frequencies = hist_20[0]            # first sub-array contains frequencies
borders = hist_20[1]                # 2nd sub-array contains bin borders
fmax = max(frequencies)             # maximum frequency
imax = np.argmax(frequencies)       # ... and its location
xlower = borders[imax]              # start of max-bin
xhigher = borders[imax+1]           # ... and end
plt.xlabel('values')
plt.ylabel('frequency')
plt.legend(loc='upper right');
```



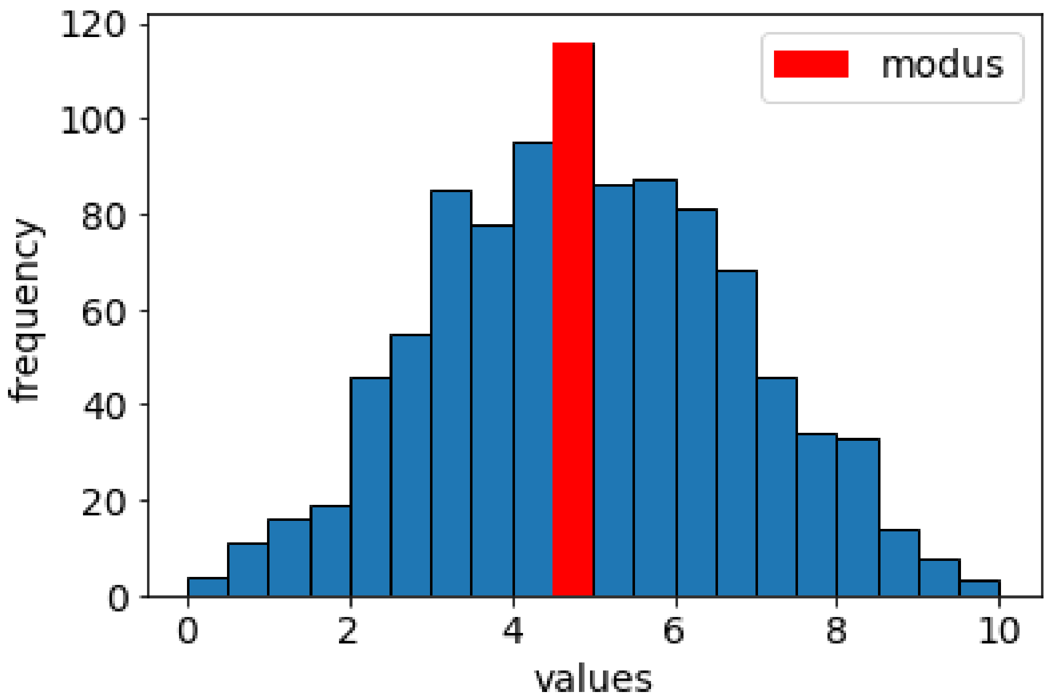

```
In [13]: print(hist_20)
```

```
(array([  4.,  11.,  16.,  19.,  46.,  55.,  85.,  78.,  95., 116.,  86.,
        87.,  81.,  68.,  46.,  34.,  33.,  14.,   8.,   3.]), array([ 0. ,  0.5,  1. ,  1.5,  2. ,
        5.5,  6. ,  6.5,  7. ,  7.5,  8. ,  8.5,  9. ,  9.5, 10. ]), <a list of 20 Patch objects>)
```

```
In [14]: '''show Modus'''
         hist_20 = plt.hist(xmany, bins, edgecolor='black') # show histogram (again)
         frequencies = hist_20[0]              # first sub-array contains frequencies
         borders = hist_20[1]                  # 2nd sub-array contains bin borders
         fmax = max(frequencies)               # maximum frequency
         imax = np.argmax(frequencies)         # ... and its location
         xlower = borders[imax]                # start of max-bin
```

```
xhigher = borders[imax+1]                # ... and end
print("The modus of 'hist20' is {} values in the bin number {}".format(fmax, imax))
print("  ranging from {:.2f} to {:.2f}".format(xlower, xhigher))  # highlight the modus-bi
plt.bar(x=xlower, width=(xhigher-xlower), bottom=0, height=fmax, color='red',
                                    label='modus', align='edge')
plt.xlabel('values')
plt.ylabel('frequency')
plt.legend(loc='upper right');
```

```
The modus of 'hist20' is 116.0 values in the bin number 9
  ranging from 4.50 to 5.00
```



In [15]:
```
from matplotlib import image as mpi          # image related routines

img = mpi.imread('data/gabor.png')           # read image data from file
imgvector3D = np.array(img)                  # convert colors to values in matrix
# select color red only,  round from 0..1 -> 0..255 integers
x = np.rint(255*imgvector3D[:,:,0].flatten()).astype(int)
```

In [16]:
```
'''Modus from Gabor-patch image'''
mybins = np.linspace(-0.5, 256.5, 257+1)
b, v = np.histogram(x, bins=mybins)
print(v[124:134])
```

```
print(b[124:133])
xm = np.argmax(b)
print('the modus of x is {} (with {} values between{} and {})'
                                   .format(xm, b[xm], v[xm], v[xm+1]))
```

```
[123.5 124.5 125.5 126.5 127.5 128.5 129.5 130.5 131.5 132.5]
[ 68  84 107 107 116  82  89  90  78]
the modus of x is 128 (with 116 values between127.5 and 128.5)
```

4 Quantile

Grenzwert, vor dem der Anteil (das Quantum) an Datenwerten liegt.

4.0.1 Quartile

- Unteres Quartil $x_{0.25}$: 25% und
- Oberes Quartil $x_{0.75}$: 75% der Werte
- Median entspricht dem 50% Quartil

Interquartils-Abstand $d_Q = x_{0.75} - x_{0.25}$
Ein StreuungsmaSS, robust gegen AusreiSSer

4.0.2 Perzentile

- 5%-Perzentil: niedrigste 5% der Werte
- 95%-Perzentil: ohne oberste 5% der Werte

Quantile sind einfach abzulesen aus der kumulativen Verteilungsfunktion:

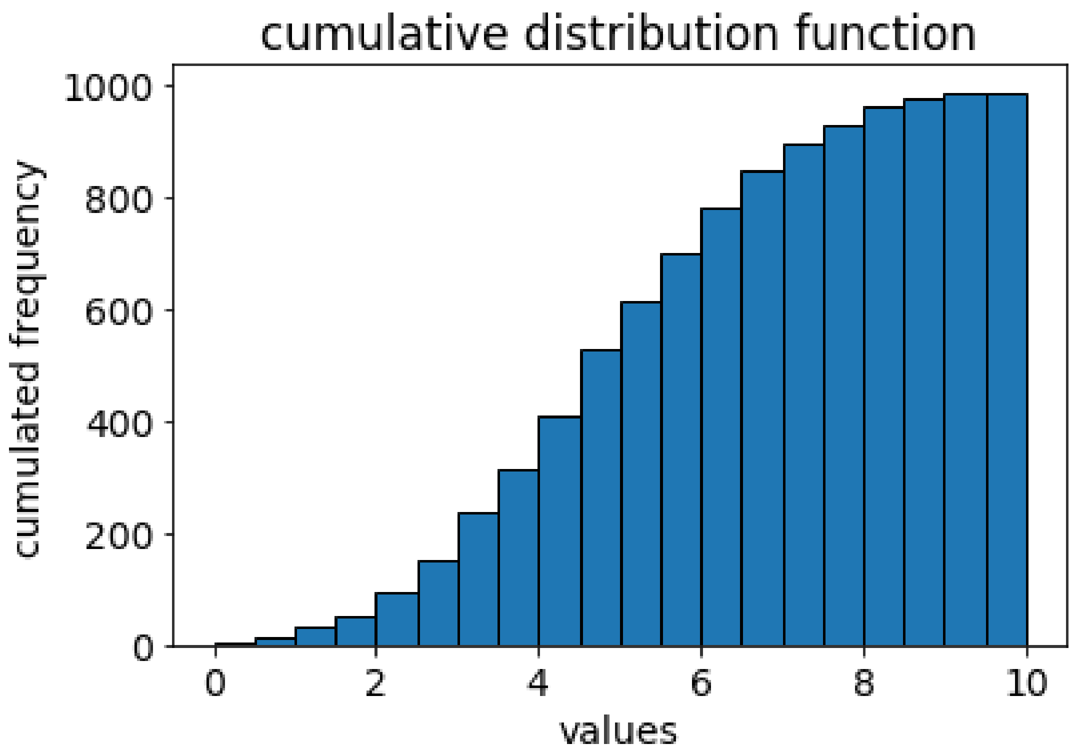
5 Empirische kumulative Verteilungsfunktion F(x)

- Relative Häufigkeiten $h_i = \frac{n_i}{N}$
  - beispielsweise aus einem gebinnten Histogramm
- Reihenfolge gemäSS Wert beibehalten
- aufstapeln

```
In [17]: '''make a cumulative histogram cdf'''
         hist_cumul = plt.hist(xmany, bins=np.linspace(0, 10, 21), cumulative=True,
                               edgecolor='black')
         plt.title('cumulative distribution function')
         print('histogram {} ... {}'.format(hist_20[0][:5], hist_20[0][-4:]))
         print('cumulated {} ... {}'.format(hist_cumul[0][:5], hist_cumul[0][-4:]))
```

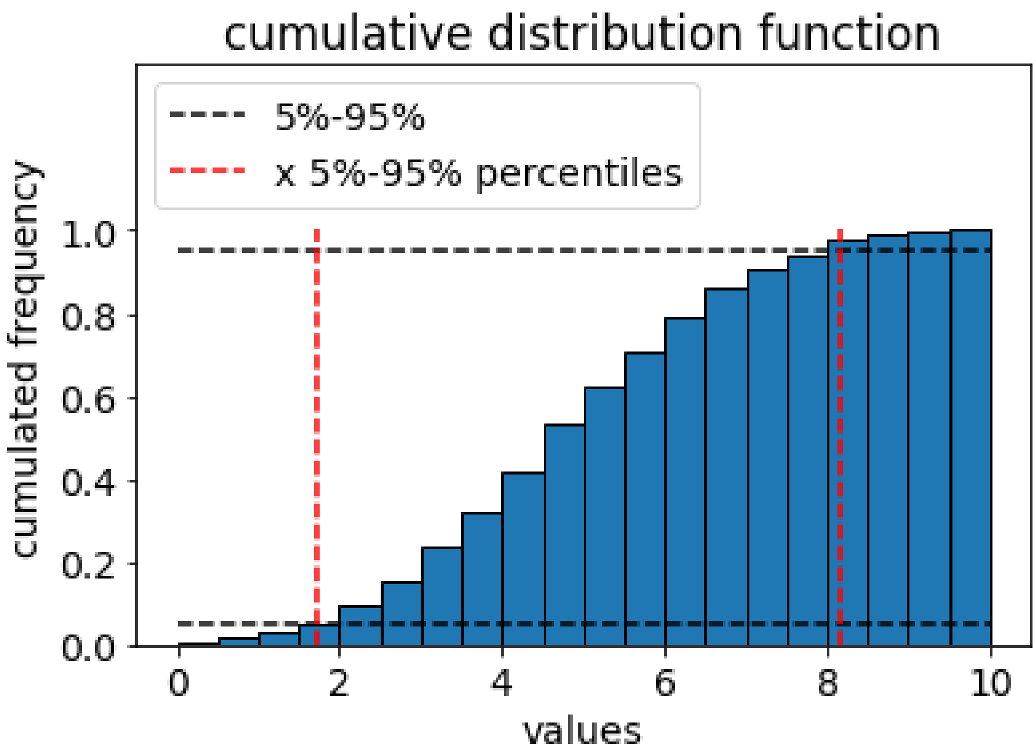
```
        plt.xlabel('values')
        plt.ylabel('cumulated frequency');
```

```
histogram [ 4. 11. 16. 19. 46.] ... [33. 14.  8.  3.]
cumulated [ 4. 15. 31. 50. 96.] ... [960. 974. 982. 985.]
```




```
In [18]: '''calculate quantiles'''
        hist_cumul = plt.hist(xmany, bins=np.linspace(0, 10, 21), cumulative=True,
                              density=True, edgecolor='black')
        plt.title('cumulative distribution function')
        plt.ylim(0.0, 1.4)
        plt.yticks(list(np.arange(0.0, 1.01, 0.2)))
        x05, x95 = np.percentile(xmany,  (5, 95))
        print(' 5% percentile = {:.2f} ... 95% percentile = {:.2f}'.format(x05, x95))
        plt.plot([0, 10], 2*[.05], 'k--', label='5%-95%')
        plt.plot([0, 10], 2*[.95], 'k--')
        plt.plot(2*[x05], [0, 1], 'r--', label='x 5%-95% percentiles')
        plt.plot(2*[x95], [0, 1], 'r--')
        plt.xlabel('values')
        plt.ylabel('cumulated frequency')
        plt.legend(loc='upper left');
```

```
5% percentile = 1.72 ... 95% percentile = 8.15
```



5.0.1 Schnelles Ablesen der Information:

- Lage: Median, (Modus)
- Bereich: Quartile, Perzentile
- Form der Verteilung: S-Kurve
    - wenn Verteilung *ein* Maximum hat ("unimodal")

  - Modus: 增长最大的
  - Media: 中位数

6 Kumulierter Mittelwert

6.1 Arithmetisches Mittel

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\approx \bar{x}' = \frac{1}{\sum_{k=1}^{N_k} h_k}\sum_{k=1}^{N_k} h_k(x_k)\cdot x_k$$

7 Varianz

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2$$
$$\approx \sigma'^2 = \frac{1}{\sum_{k=1}^{N_k} h_k - 1}\sum_{k=1}^{N_k} h_k(x_k)\cdot(x_k - \bar{x})^2$$

7.1 Standardabweichung

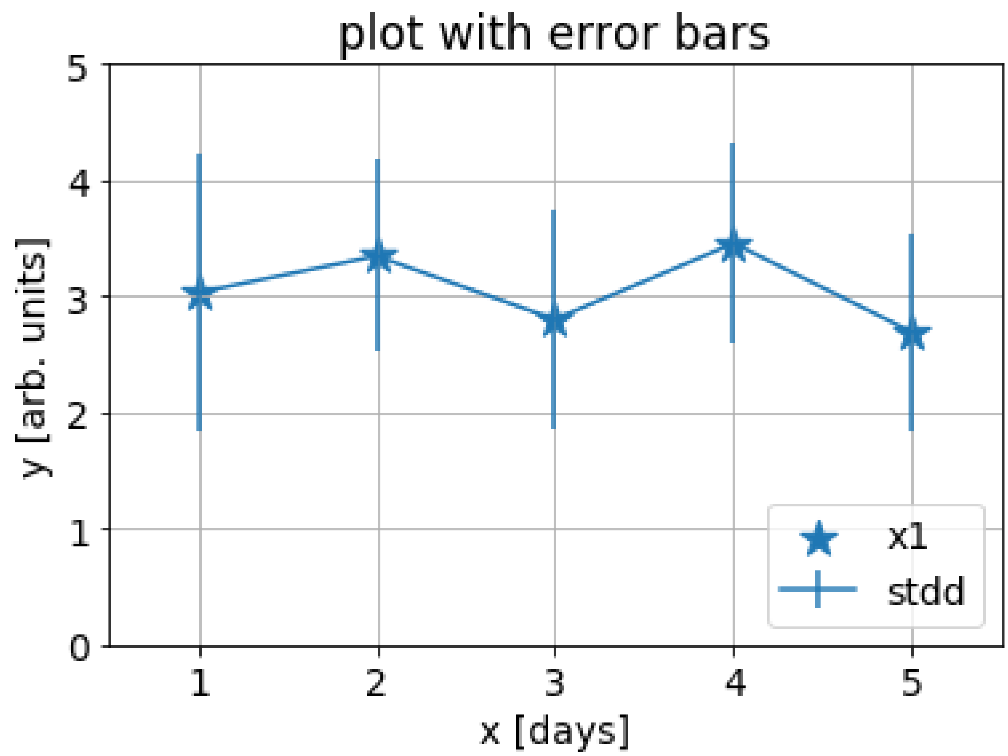
$$s = \sqrt{\sigma^2}$$
$$s' = \sqrt{\bar{\sigma}'^2}$$

8 Graphische Darstellung

8.1 Fehlerbalken

- arithmetischer Mittelwert als Symbol
  - Siehe http://matplotlib.org/api/markers_api.html
- Fehlerbalken +/-Standardabweichung σ
- AusreiSSer als Sterne

In [19]:
```
'''data graph typical in science
   error bars are standard deviations
'''
N = 5
x = 1 + np.arange(N)                          # fixed x values
y = 3 + np.random.randn(N, 20)                # 20 random y values for each x
plt.axis([0.5, 5.5, -0, 5.])                  # range of plot
plt.errorbar(x, np.mean(y, axis=1), yerr=np.std(y, axis=1), label='stdd')
plt.scatter(x, np.mean(y, axis=1), marker='*', s=200, label='x1');
plt.title('plot with error bars')
plt.xlabel('x [days]')
plt.ylabel('y [arb. units]')
plt.grid(b=True, which=u'major', axis=u'both')
plt.legend(loc='lower right');
```



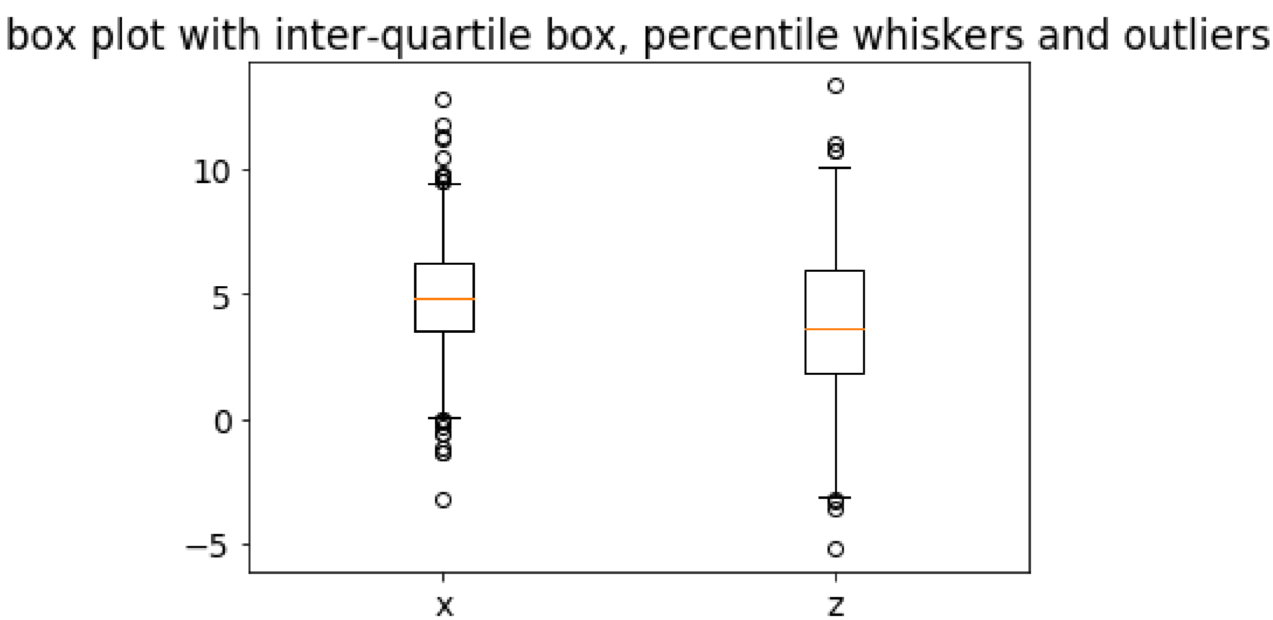
8.1.1 Gute Praxis

- Achsen beschriften: `xlabel`, `ylabel`
- *ehrliche* Achsen
  - 0 mit einbeziehen, wenn absolute Zahlen
  - sonst deutlich kennzeichnen (Lücke)
  - entsprechende Zahlen bzw. tickmarks: `xaxis.set_ticklabels`
  - sinnvollen Bereich: `axis`
- Überschrift: `title`
  - in Veröffentlichungen Bildunterschrift mit Text; keine Überschrift
- Graphen beschriften: `label` und `legend`
- deutlich unterscheiden (auch für schwarz-weiSS-Druck)
  - Symbole: `o` `.` `,` `p` `<` `...`
  - Linienstil: `-` `--` `..` `-.` `...`
  - Farbe wenn nötig
- nicht überfrachten
- manchmal hilfreich
  - Gitterlinien: `grid`
  - Spiegelstriche: `ax.tick_params(axis='y', direction='out')` und `ax.yaxis.tick_left()`

8.2 Box-Plot

- Median als Strich
- GröSSe der Box:
  - Quartile $Q_{1/4} \ldots Q_{3/4}$
- Länge der Striche:
  - $Q_{1/4} - \frac{3}{2} \cdot (Q_{3/4} - Q_{1/4}) \ \ldots \ Q_{3/4} + \frac{3}{2} \cdot (Q_{3/4} - Q_{1/4})$
  - Merkregel: Werte innerhalb "±100%"-Quartile
- AusreiSSer als Sterne $+$

```
In [20]: '''data graph with boxes and whiskers show inter-quartile-distances
         '''
         np.random.seed(9876543)
         zmany = np.random.normal(loc=4., scale=3., size=400)# another data set
         data = [xmany, zmany]                                # combine 2 data sets
         plt.boxplot(data, labels=['x', 'z'], whis=[1, 99])   # name labels; 1%-99% percentiles
         plt.title('box plot with inter-quartile box, percentile whiskers and outliers');
```

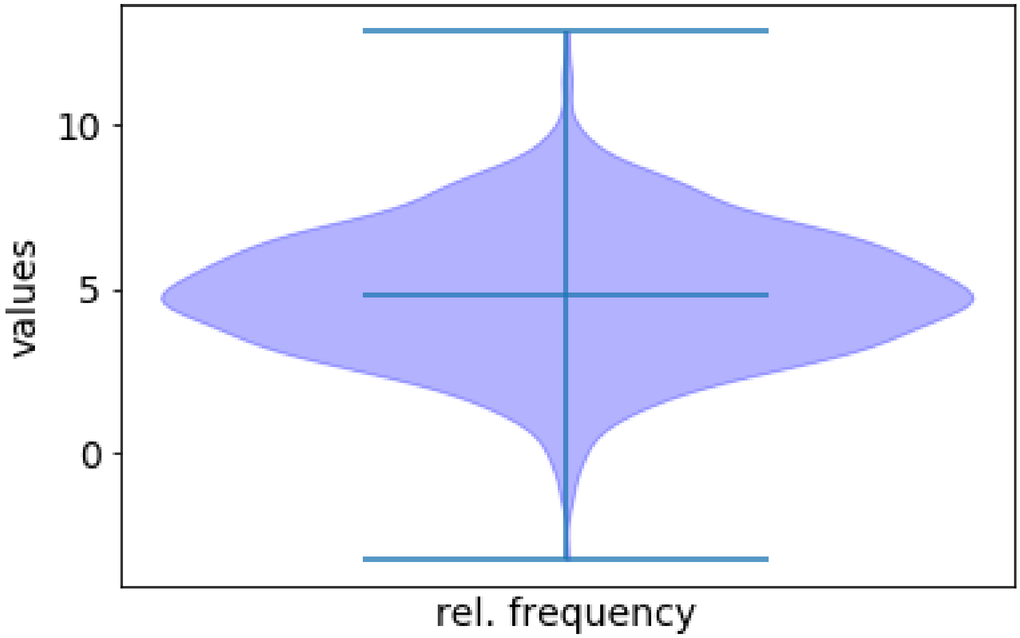



```
plt.boxplot?
```

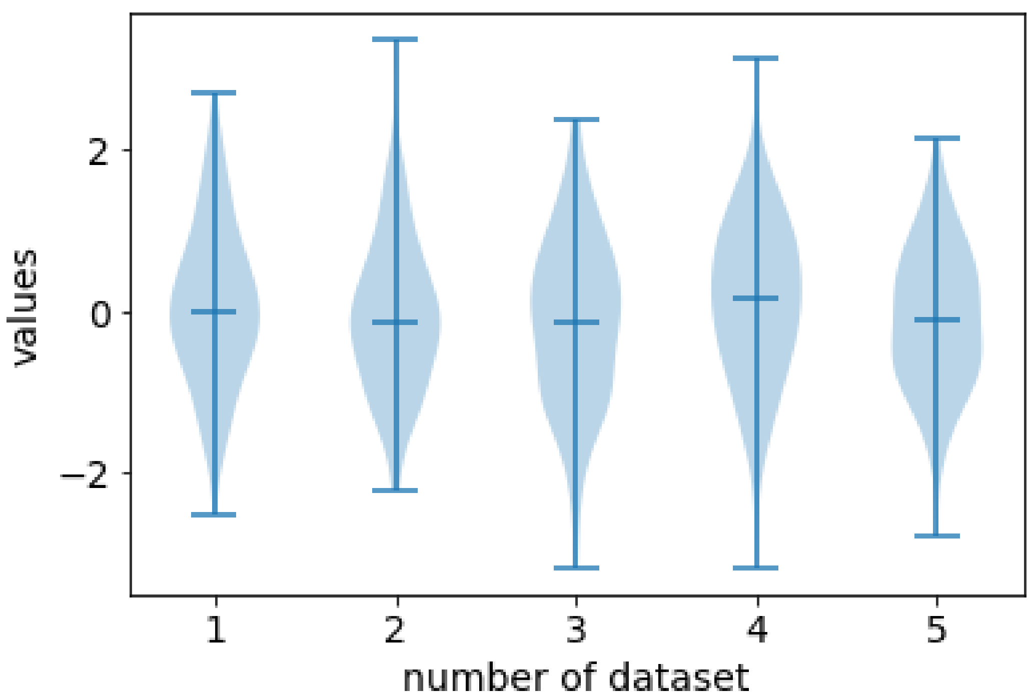
8.3 Violinen-Plot

```
In [21]: vp = plt.violinplot(xmany, showmedians=True)
         vp['bodies'][0].set_color('blue')
         plt.ylabel('values')
```

```
plt.xlabel('rel. frequency')
plt.xticks([]);                    # off, numbers of no meaning here
```




```
In [22]: xx = np.random.normal(size=(5, 100))           # draw 5x100 values into 2D vector
         plt.violinplot([x for x in xx], showmedians=True)
         plt.ylabel('values')
         plt.xlabel('number of dataset');
```



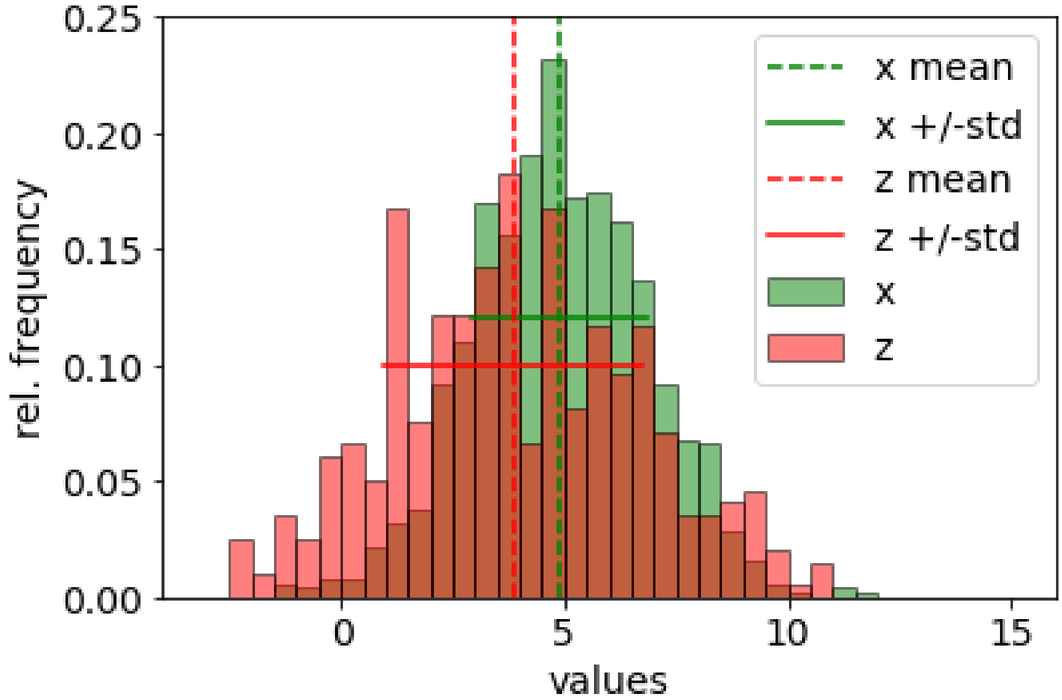
9 Fragen?

10 Vergleich zweier Stichproben

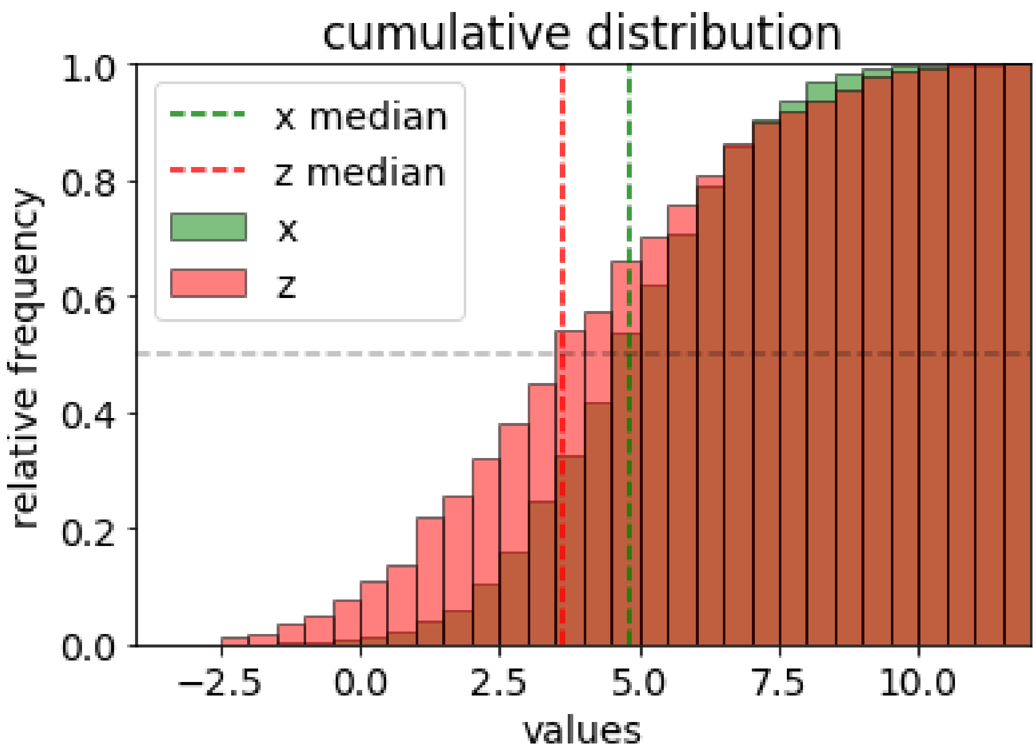
- Kennwerte
- Histogramm-Stapel
- kumulierte Verteilung

In [23]:
```
'''compare characteristic numbers / histograms of two distributions'''
bins = np.linspace(-3, 12, 31)                          # half steps
xmean = xmany.mean()
zmean = zmany.mean()
xstd = np.std(xmany)
zstd = np.std(zmany)
plt.axis([-4, 16, 0.0, 0.25])
print('x mean = {:.2f}  std = {:.2f}'.format(xmean, xstd))
plt.hist(xmany, bins=bins, color='green', density=True, edgecolor='black',
         label='x', alpha=.5)    # 1st histogram x, shine through
print('z mean = {:.2f}  std = {:.2f}'.format(zmean, zstd))
plt.hist(zmany, bins=bins, color='red', density=True, edgecolor='black',
         label='z', alpha=.5)    # 2nd histogram z, shine through
plt.plot(2*[xmean], [0, .25], 'g--', label='x mean')
plt.plot([xmean-xstd, xmean+xstd], [.12, .12], 'g-', label='x +/-std')
plt.plot(2*[zmean], [0, .25], 'r--', label='z mean')
plt.plot([zmean-zstd, zmean+zstd], [.1, .1], 'r-', label='z +/-std')
plt.xlabel('values')
plt.ylabel('rel. frequency')
plt.legend();
```

```
x mean = 4.90  std = 1.99
z mean = 3.84  std = 2.91
```



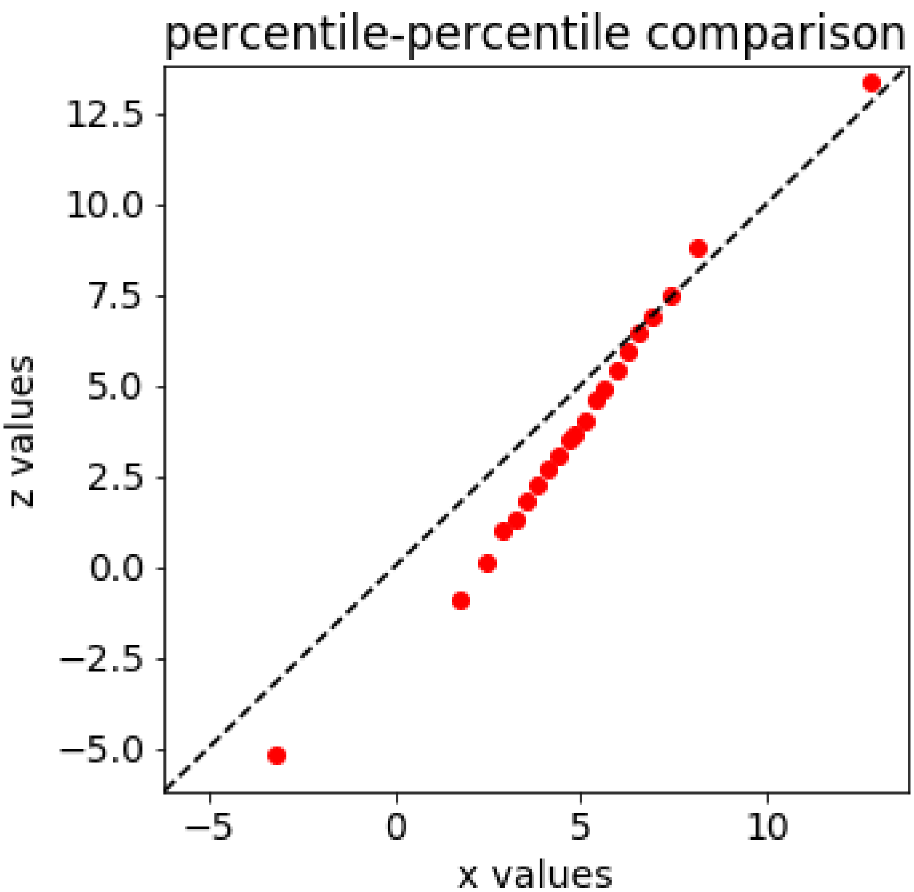

```
In [24]: '''two distributions
            culmulative & normalized for better comparison
         '''
         xmedn = np.median(xmany)
         zmedn = np.median(zmany)
         print('x median = {:.2f}  z median = {:.2f}'.format(xmedn, zmedn))
         plt.hist(xmany, bins=bins, color='green', density=True, edgecolor='black',
                  cumulative=True,
                  label='x', alpha=.5)   # 1st histogram x, cumulative density
         plt.hist(zmany, bins=bins, color='red', density=True, edgecolor='black',
                  cumulative=True,
                  label='z', alpha=.5)   # 2nd histogram z, cumulative density
         plt.title('cumulative distribution')
         plt.xlabel('values')
         plt.ylabel('relative frequency')
         plt.plot(2*[xmedn], [0, 1.0], 'g--', label='x median')
         plt.plot(2*[zmedn], [0, 1.0], 'r--', label='z median')
         plt.plot([-4, 12], 2*[0.5], 'k--', alpha=.3)
         plt.axis([-4, 12, 0., 1.])
         plt.legend(loc='upper left');

x median = 4.83  z median = 3.65
```



10.1 QQ-Plot

Gegenüberstellung der **Q**uantile

In [26]:
```
'''compare two distributions by percentiles'''
percentiles = np.linspace(0, 100, 20+1)     # 5% steps
percx = np.percentile(xmany, percentiles)
percz = np.percentile(zmany, percentiles)
fig = plt.figure(figsize=(5, 5))
plt.plot(percx, percz, 'ro')
diag = (np.min((xmany.min(), zmany.min()))-1, np.max((xmany.max(), xmany.max()))+1)
plt.plot(diag, diag, 'k--')
plt.xlabel('x values')
plt.ylabel('z values')
plt.xlim(diag)
plt.ylim(diag)
plt.title('percentile-percentile comparison');
```



11 Wertebereiche

- Nominal
  - Bsp: Münze: Kopf/Zahl, gezogene Farbe
- ordinale Daten
  - Bsp: Schadensklasse, Zufriedenheit “sehr, ja, nein, garnicht”
  - Kategorien
- Diskrete Daten
  - Bsp: gewürfelte Zahl, Alter
- Kontinuerliche Daten
  - Bsp: Temperatur, Alter,
  - prozentuale Werte

11.0.1 Weitere Unterscheidung für nominale und ordinale Daten:

- dichotom/binär/binomial

- polytom/multinomial

Mittels Klasseneinteilung diskretisieren und in Histogramm darstellen.

12 Zusammenfassung Kennzahlen

12.0.1 Lageparameter

- Median
- Modus
- Arithmetisches Mittel
- Harmonisches Mittel, ...
- ⇒ (wichtigste) erste Kennzahl

12.0.2 Streuung, Breite der Verteilung:

- Wertebereich Min-Max
- Standardabweichung
- Quartile
- Perzentile
- ⇒ weitere Kennzahl(en) beschreiben die Streuung

12.0.3 Ausblick: Form der Verteilung

```
In [27]: '''Python reports some main parameters'''
         # import scipy.stats
         print('mean = {:.2f}  median = {:.2f}'
               .format(xmany.mean(), np.median(xmany)))
         print('standard deviation = {:.2f}'
               .format(xmany.std()))
         print('quartile range = {:.2f} to {:.2f}'
               .format(np.percentile(xmany, 25), np.percentile(xmany, 75) ))
         print('1% - 99% range = {:.2f} to {:.2f}'
               .format(np.percentile(xmany,  1), np.percentile(xmany, 99) ))
```

```
mean = 4.90  median = 4.83
standard deviation = 1.99
quartile range = 3.53 to 6.27
1% - 99% range = 0.05 to 9.41
```

13 Zusammenfassung Graphik

- Mittelwert und Streuung - “Fehlerbalken-Diagramm”

- Boxplot
- Violinenplot
- Histogramm
- kumulierte Verteilungsfunktion

14 Ausblick

- mehrdimensionale Daten
- SchlieSSende Statistik arbeitet mit Wahrscheinlichkeiten
- Stochastische Modelle

15 Fragen?