023_Folien

2018 年 11 月 23 日

```
In [1]: from matplotlib import pyplot as plt
        import numpy as np
        %matplotlib inline
```

1 Beschreibende Statistik

1.1 Übersicht

LagemaSS

Streuung

Graphiken

Mehrdimensionale Daten

Abhängigkeit

- Korrelation
- Regression
  - linear
  - polynomial

1.2 Mehrdimensionale Daten - Regression

Korrelation

- Voraussetzung: Zusammenhang von zusammengehörigen Daten ist nachgewiesen
- Korrelationskoeffizient $\neq 0$

Beispiele:

- Strom und Spannung: festes Verhältnis gemäSS Ohmschen Gesetzes
- Luftdruck und Höhe über Meer: Barometrische Höhenformel

1.3 Frage: Wie *stark* hängt y von x ab?

Beispiel: Fahrstrecke x und Fahrzeit t

- Zusammenhang: je mehr Zeit t, desto weiter die Strecke s
  - linear $x \sim t$
  - Proportionalitätsfaktor "*Geschwindigkeit v*": $x = v \cdot t$
- Für $x = 10$km benötigen Sie $t = 12$Minuten

In [4]:
```
'''Python as a pocket calculator: calculate car speed'''
xmin = 12                      # time in minutes
xh = xmin/60                   # time in hours
y = 10                         # distance in km
v = y / xh                     # speed in km/h
print('Going {} km in {} mins, your speed is {} km/h'.format(y, xmin, v))
```

```
Going 10 km in 12 mins, your speed is 50.0 km/h
```

Anwendung Bei linearer Abhängigkeit $x = v \cdot t$ erlaubt der Proportionalitätsfaktor Vorhersagen

Beispielsweise: wie lang ist die zurückgelegte Strecke in 1 Sekunde?

$$x = v \cdot t = \frac{10\text{km}}{10\text{min}} \cdot 1s = \frac{10000\text{m}}{10 \cdot 60\text{s}} \cdot 1s = 16.7m$$

Ergebnis: In der Reaktionszeit von 1s legt ein Auto im Stadtverkehr 17m zurück

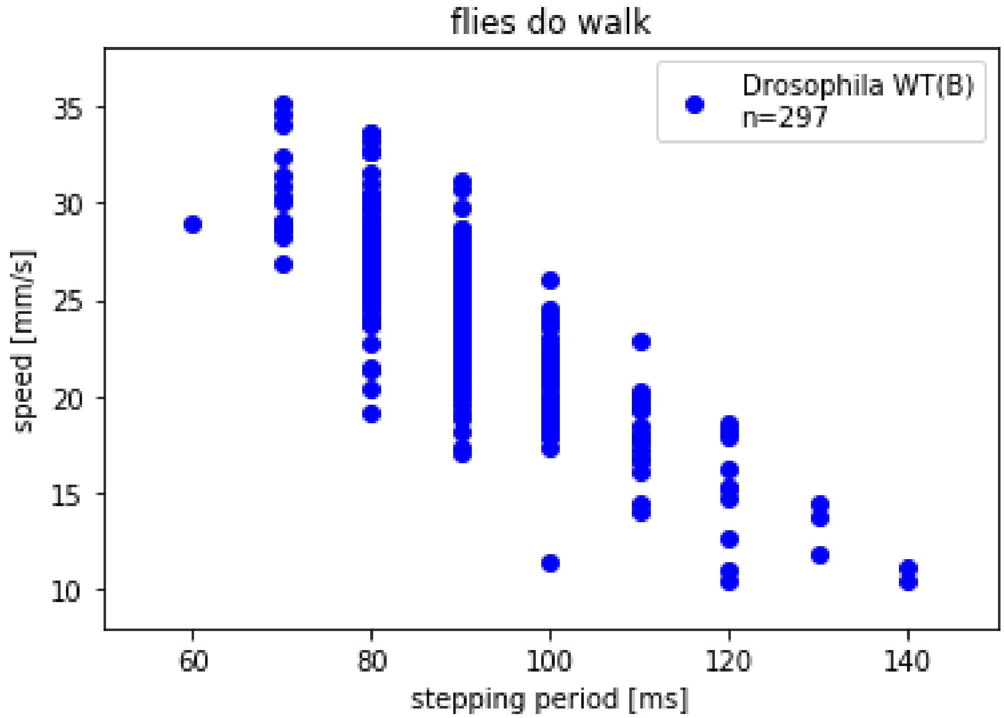
1.3.1 Mehrere Messungen

Mehrere (n) Messungen y_i zu verschiedenen Werten der x-Variable, hier Schrittperiodendauer (Zeiten, Längen, Alter, Höhe, ...)

In [13]:
```
'''read data: speed vs stepping period of walking fly Drosophila
   two columns space-separated: period [ms], speed [mm/s] '''
data = np.genfromtxt('data/Drosophila_geschw-periode.dat').T #ranspose
print('data is of size {}, a {} of {} of {}'
      .format(data.shape, type(data), type(data[0]), type(data[0, 1])))
print('period {} ...'.format(data[0, :5]))    # 1d-array of periods
print('speed {} ...'.format(data[1, :5]))     # 1d-array of speed
```

```
data is of size (2, 297), a <class 'numpy.ndarray'> of <class 'numpy.ndarray'> of <class 'numpy.float64'>
period [60. 70. 70. 70. 70.] ...
speed [29.  34.  34.6 30.3 26.9] ...
```

In [14]:
```
'''show the data'''
plt.plot(data[0], data[1], 'bo', label='Drosophila WT(B)\nn={}'.format(len(data[0])))
plt.axis((50, 150, 8, 38))
plt.title('flies do walk')
plt.xlabel('stepping period [ms]')
plt.ylabel('speed [mm/s]')
plt.legend(loc='upper right');
```



1.3.2 Korrelation?

```
In [15]: '''correlation between speed and period?'''
         r_xy = np.corrcoef(data)[0, 1]
         print('correlation coefficient r_xy = {:.3f}'.format(r_xy))
```

```
correlation coefficient r_xy = -0.819
```

1.3.3 Regression?

lineare Abhängigkeit - mehrere x-Werte (Periodendauern) - mehrere Messungen (Geschwindigkeit)

Ausgleichsgerade

1.3.4 Methode der *Kleinsten Quadratischen Abweichung*

(C. F. Gauss 1809, A. M. Legendre 1805)

$$\underset{a,b}{\operatorname{argmin}} \sum_i (y_i - f_{a,b}(x_i))^2$$

des linearen Zusammenhangs

$$y = f_{a,b}(x) = a \cdot x + b$$

führt zu

$$a = \frac{\sum_i (y_i - \bar{y}) x_i}{\sum_i (x_i - \bar{x}) x_i}$$

$$b = \bar{y} - a \cdot \bar{x}$$

Die Steigung oder lineare Abhängigkeit a wird auch **Regressionskoeefizient** genannt und entspricht

$$a = r_{XY} \cdot \frac{\sigma_Y}{\sigma_X}$$

Beweise: [ÜA]

$$\begin{aligned} a &= \frac{\sum_i (y_i - \bar{y}) x_i}{\sum_i (x_i - \bar{x}) x_i} \\ &= r_{XY} \frac{\sigma_Y}{\sigma_X} \\ &= \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \cdot \frac{\sigma_Y}{\sigma_X} \\ &= \end{aligned}$$

Im Allgemeinen

$$r_{XY} = r_{YX}$$

$$a_{XY} \neq a_{YX}$$

$$a_{XY} \neq \frac{1}{a_{YX}}$$

Später mehr…

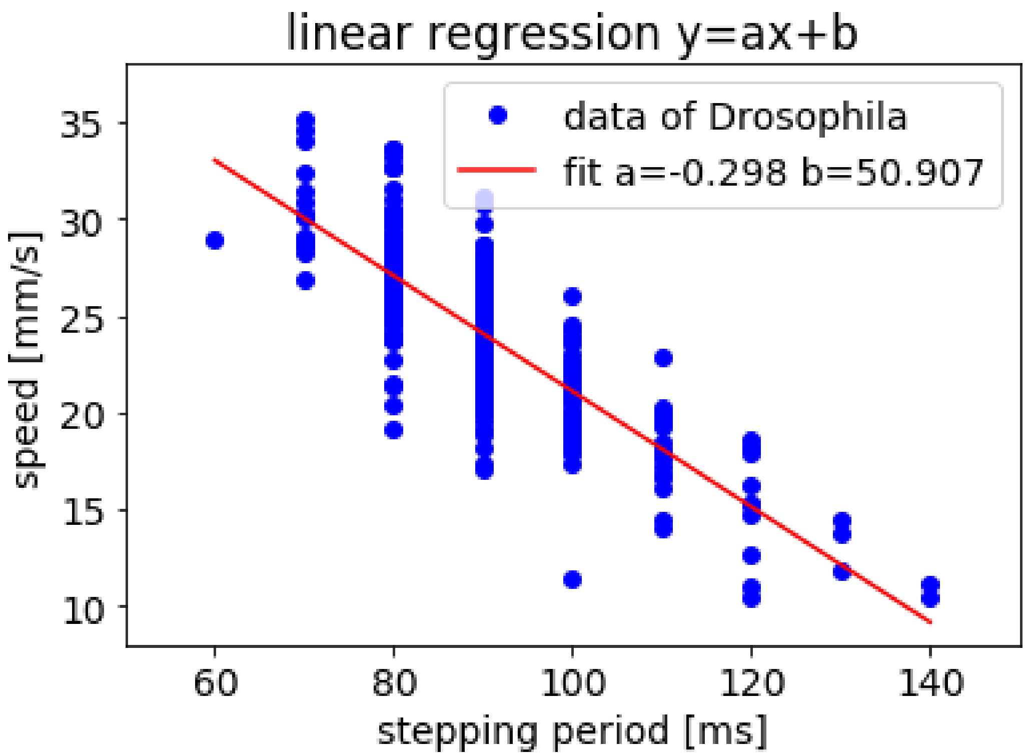
In [7]:
```
'''linear function y=f(x)=a*x+b
   input:  x: scalar or np.array (x values)
           a: scalar (regression parameter)
           b: scalar (offset parameter)
   output: same type and size as x            '''
def fct_y( x, a, b ):
    return a*x + b

''' make handy names for data: '''
x = data[0]     # the period durations
y = data[1]     # the measured walking speed

'''calculate linear regression according to formula above'''
a = ((y - y.mean())*x).sum() / ((x - x.mean())*x).sum()
b = y.mean() - a*x.mean()
print('manually fitted:  y = {:.3f}*x + {:.3f}'.format(a, b))
```

```
manually fitted:  y = -0.298*x + 50.907
```

In [8]:
```
'''show the fit result'''
plt.plot(x, y, 'bo', label='data of Drosophila')         # raw data as dots
plt.plot(x, fct_y(x, a, b), 'r-', label='fit a={:.3f} b={:.3f}'.format(a, b))
plt.title('linear regression y=ax+b')                     # fitted line
plt.axis((50, 150, 8, 38))
plt.xlabel('stepping period [ms]')
plt.ylabel('speed [mm/s]')
plt.legend(loc='upper right');
```



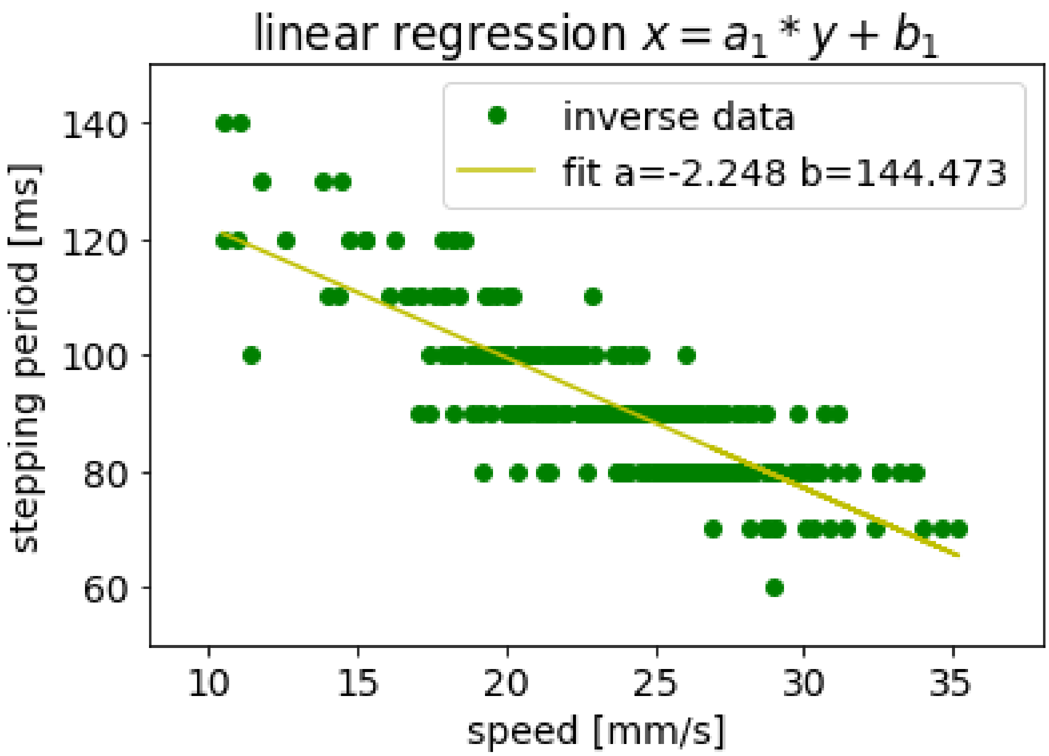
1.3.5 fit with scipy.stats

```
In [9]: '''fit it with scipy directly'''
        from scipy import stats          # contains also fitting routines,
        fit = stats.linregress(x, y)     #      e.g. linear regression fit
        print('fitted by scipy:  y = {:.3f}*x + {:.3f}'.format(fit.slope, fit.intercept))
```

```
fitted by scipy:  y = -0.298*x + 50.907
```

```
In [11]: '''fit the reverse values x(y)'''
         fit_r = stats.linregress(y, x)                # linear regression of reverse data
         print('linear fit:  x = {:.3f}*y + {:.3f}'.format(fit_r.slope, fit_r.intercept))
         plt.plot(y, x, 'go', label='inverse data')
         plt.plot(y, fct_y(y, fit_r.slope, fit_r.intercept), 'y-',
                  label='fit a={:.3f} b={:.3f}'.format(fit_r.slope, fit_r.intercept))
         plt.title('linear regression $x=a_1*y+b_1$') # yes, matplotlib speaks LaTeX
         plt.axis((8, 38, 50, 150))
         plt.ylabel('stepping period [ms]')
         plt.xlabel('speed [mm/s]')
         plt.legend(loc='upper right');
```

```
linear fit:  x = -2.248*y + 144.473
```




```
In [12]: ''' Correlation coefficient Pearson r, see 022 lecture'''
         def ccpearson( a, b ):
             return np.corrcoef(a, b)[0][1]      # side element of correlation matrix

         '''compare fits of data versus reverse-data'''
         print('slope {:.3f} * inverse slope {:.3f} = {:.3f} does not equal 1.000'
               .format(fit.slope, fit_r.slope, fit.slope*fit_r.slope))
         print('but correlation coefficients match: {:.5f} = {:.5f}'
               .format(ccpearson(x, y), ccpearson(y, x)))
```

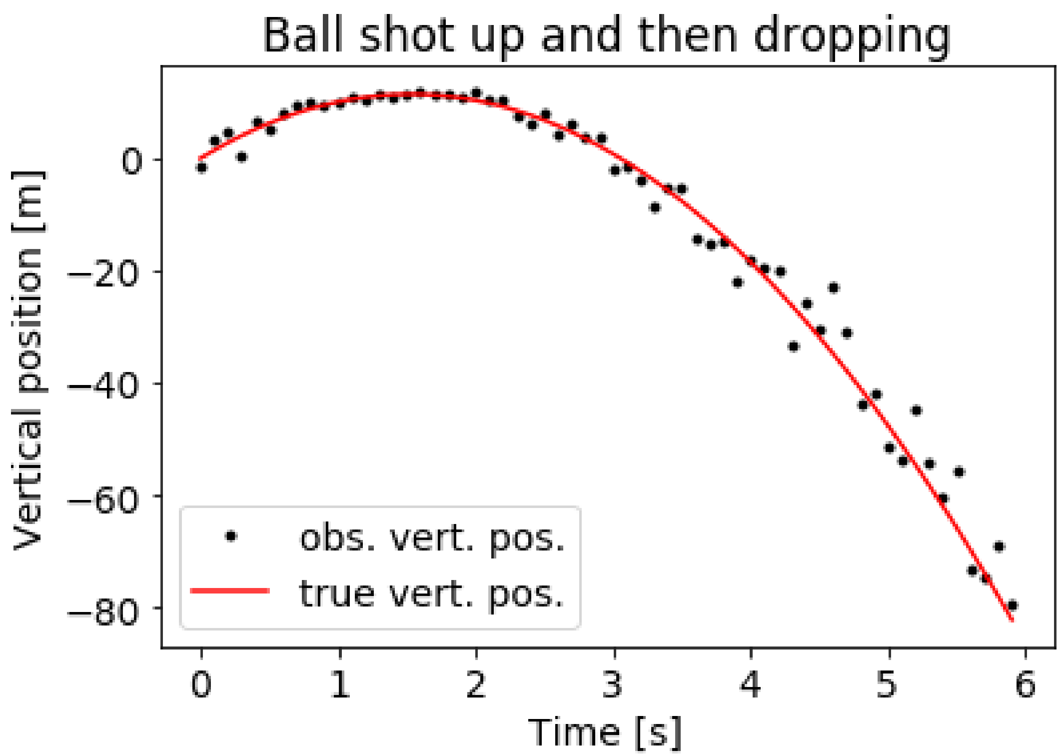
```
slope -0.298 * inverse slope -2.248 = 0.670 does not equal 1.000
but correlation coefficients match: -0.81869 = -0.81869
```

1.4 Nicht-lineares Beispiel: Wurfparabel

```
In [13]: np.random.seed(98765)
         deltaT = 0.1                         # time steps in seconds
         period = 6                           # duration of flight in sec
         g = 9.81                             # gravity constant
         ae = 0.15                            # speed dependent noise amplitude
         be = 0.05                            # constant noise amplitude
         v0 = 15                              # speed at start in m/s
         numOfPoints = int(period/deltaT)     # measured points
         t_a  = np.zeros(numOfPoints)         # time value[]
```

```
    v_a  = np.zeros(numOfPoints)       # vertical speed[]
    yt_a = np.zeros(numOfPoints)       # y theoretically[]
    y_a  = np.zeros(numOfPoints)       # y result: with noise[]
    for it in range(numOfPoints):      # calculate
        t = it*deltaT                  # time now
        t_a[it] = t                    # store
        v = v0-g*t                     # speed reduced by gravity
        v_a[it] = v                    # store
        yt = -g*t**2/2+v0*t            # squared by gravitational acceleration
        yt_a[it] = yt                  # store
        error = np.random.normal(loc=0, scale=ae*abs(v)+be)
        y_a[it] = yt+error             # add noise
```

```
In [14]: plt.title('Ball shot up and then dropping')
         plt.ylabel('Vertical position [m]')
         plt.xlabel('Time [s]')
         plt.plot(t_a, y_a, 'k.', label='obs. vert. pos.')  # observed points with noise
         plt.plot(t_a, yt_a, 'r-', label='true vert. pos.') # theoretical position
         plt.legend(loc="lower left");
```



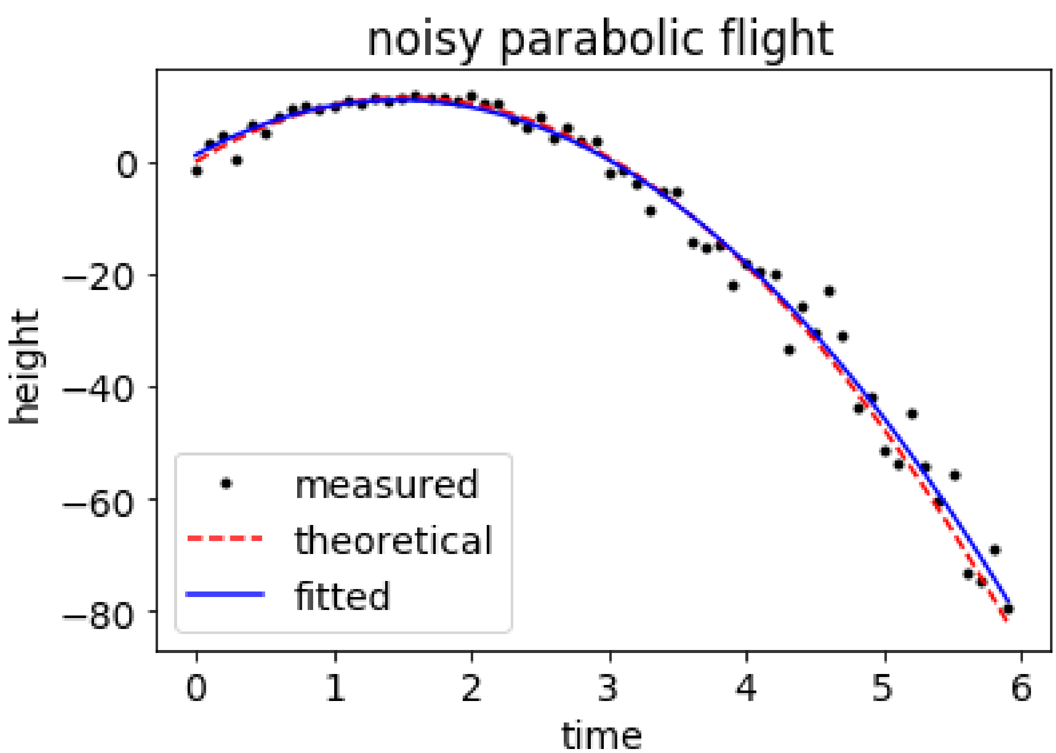

```
In [15]: '''How to fit a polynomial dependency (example: squared)?'''
         np.polyfit?
```

```
In [16]: '''use polyfit (square----v) to extract parabolic parameters'''
         pp = np.polyfit(t_a, y_a, 2)        # polynomparameters from fit of degree 2
         print('fitted polynom has parameters pp={}'.format(pp))
```

```
fitted polynom has parameters pp=[ -4.54965459  13.37918126   1.2131438 ]
```

```
In [17]: '''compare measured and fitted values (with randomly drawn distribution)'''
         plt.plot(t_a, y_a, 'k.', label='measured')
         plt.plot(t_a, yt_a, 'r--', label='theoretical')
         py = np.poly1d(pp)            # define polynomial function with fitted parameters
         plt.plot(t_a, py(t_a), 'b-', label='fitted')# show fitted locations versus time
         plt.legend(loc='lower left')
         plt.title("noisy parabolic flight")
         plt.xlabel('time')
         plt.ylabel('height')
         print('gravitational constant = {:6.3f} m/s^2'.format(-2*pp[0])) # from y ~ -g*t^2/2
         print('starting speed was v0  = {:6.3f} m/s'.format(pp[1]))      # derivative at t=0
```

```
gravitational constant =  9.099 m/s^2
starting speed was v0  = 13.379 m/s
```



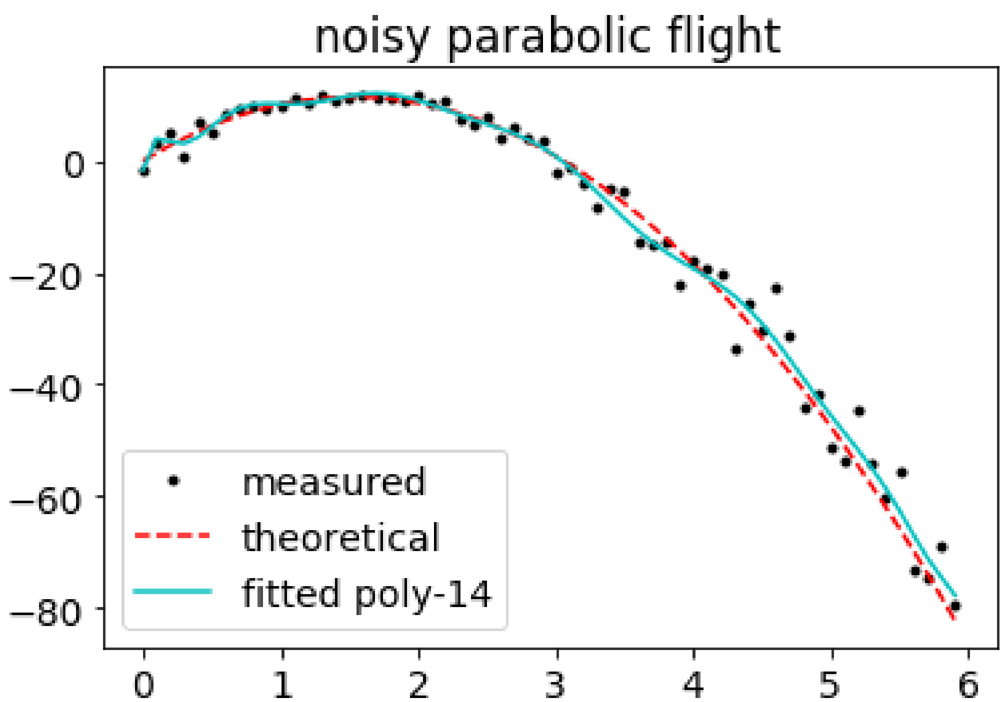
Siehe auch

- numpy.polyfit
- scipy.optimize.curve_fit

```
In [18]: p14 = np.polyfit(t_a, y_a, 14)        # polynomparameters from fit of degree 14 (?)
         print('fitted polynom of degree {} has parameters p14=\n{}'.format(len(p14)-1, p14))
         plt.plot(t_a, y_a, 'k.', label='measured')
```

```
    plt.plot(t_a, yt_a, 'r--', label='theoretical')
    py = np.poly1d(p14)                  # define polynomial function with parameters
    plt.plot(t_a, py(t_a), 'c-', label='fitted poly-14')
    plt.legend(loc='lower left')
    plt.title("noisy parabolic flight");
```

```
fitted polynom of degree 14 has parameters p14=
[ -1.71045973e-03   7.25881832e-02  -1.37704393e+00   1.54074336e+01
  -1.12969223e+02   5.70392805e+02  -2.02844834e+03   5.10788156e+03
  -9.02800340e+03   1.09398432e+04  -8.70967673e+03   4.23721214e+03
  -1.11180856e+03   1.33379787e+02  -1.67218166e+00]
```




```
In [19]: '''compare polynomial coefficients and residual error'''
    pp14, residual14, rk, sv, rc = np.polyfit(t_a, y_a, 14, full=True)
    pp2, residual2, rk, sv, rc = np.polyfit(t_a, y_a, 2, full=True)
    print('polynomial models with {} / {} parameters decreases residual error from {:.1f} to {:.1f}'.
          format(len(pp2), len(pp14), residual2[0], residual14[0]))
    for ppx in [pp2, pp14]:
        l = len(ppx)
        print('last 3 parameters of {:2d}-parameter model: {:8.2f}, {:8.2f}, {:8.2f}'.
              format(l, ppx[l-3], ppx[l-2], ppx[l-1]))
```

```
polynomial models with 3 / 15 parameters decreases residual error from 668.9 to 581.9
last 3 parameters of  3-parameter model:    -4.55,    13.38,     1.21
last 3 parameters of 15-parameter model: -1111.81,   133.38,    -1.67
```

Ergebnis:

- obwohl der Restfehler weniger wird
  - für das 14-parametrige Modell
  - gegenüber dem (korrekten) parabolischen
- ist das Ergebnis zweifelhaft
  - Ein Ball steigt nicht wieder
  - der quadratische Koeffizient ist unsinnig hoch

2 Fragen?

3 Zusammenfassung beschreibende Statistik

- Datenreduktion
- KenngröSSen herausarbeiten
  - Mittelwert
  - Standardabweichung
  - Korrelationskoeffizient
- Struktur in den Daten erkennen
  - Form (Normalverteilung, Gleichverteilung, ...)
- Anschauliche Darstellung
  - Box-Plot
  - Fehlerbalken
  - Histogramm
  - 2D-Histogramm
- Vergleichbarkeit unter ähnlichen Bedingungen
- Abhängigkeit mehrerer Variabler
  - Korrelation
  - Regression

4 Fragen an die Statistik

- Was ist die *wahre* GröSSe bei mehreren Messungen/Daten?
- Wie genau ist das erlangte Wissen?
- Was sind Ausreisser
- Effekt zwischen unterschiedlichen Versuchsbedingungen?
  - andere Wert?
  - andere Verteilung?
  - Zufall?
- Abhängigkeit wenn nichtlinear?

5 Ausblick

- Wahrscheinlichkeitstheoretische Modelle
  - Entspricht das Histogramm einer bekannten Verteilung?
  - Wie kommen die Daten zustande?
- SchlieSSende Statistik arbeitet mit Wahrscheinlichkeiten
  - Gibt es grundlegende Parameter der Daten?

6 Links

- Matplotlib Graphikgalerie http://matplotlib.org/gallery.html
- Pandas Graphiken: http://pandas.pydata.org/pandas-docs/stable/visualization.html

7 Fragen?