solution02

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1 Question 1 Bayesian Probility

$A : Strain = New,$
$B : Test = Positive,$
$C : Test = false\ Positive\ for\ the\ previous,$
$P(A) = 0.01,$
$P(B) = 0.99,$
$P(C) = 0.02,$
$P(B) = P(B) \cdot P(A) + P(B) \cdot P(C),$

$$\begin{aligned} P(A|B) &= \frac{P(B|A) \cdot P(B)}{P(B)} \\ &= \frac{0.01 \cdot 0.99}{0.99 \cdot 0.01 + 0.02 \cdot 0.99} \\ &= \frac{1}{3} \end{aligned}$$

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2 Question 2 The new Normal

Recall that the normal distribution $N(\mu, \delta^2)$

2.1 a)

$\because X \backsim N(0,1),$
than, we know that

$$\begin{aligned} E(Y) &= E(aX + b) \\ &= aE(X) + b \\ &= b \end{aligned}$$

and

$$\begin{aligned} Var(Y) &= Var(aE(X) + b) \\ &= a^2 Var(X) \\ &= a^2 \end{aligned}$$

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2.2 b)

Let x be any real number. We will first compute $F_Y(y) = P(Y \leq x)$. Since $Y = aX + b$, we know that

$$\begin{aligned} F(y) &= P(Y \leq y) \\ &= P(zX + b \leq y) \\ &= P(X \leq \frac{y-b}{a}) \\ &= \Phi(\frac{y-b}{b}) \end{aligned}$$

So $F_Y(y) = \Phi(\frac{y-b}{a})$.
Differentiating this with respect to y, we find

$$\begin{aligned} f_Y(y) &= \frac{1}{a}\Phi(\frac{y-b}{a}) \\ &= \frac{1}{a}\phi(\frac{y-b}{a}) \\ &= \frac{1}{\sqrt{2\pi}a} exp-(\frac{(y-b)^2}{2a^2}) \end{aligned}$$

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2.3 c)

from b), we knows that, $f_Y(y)$ is pdf of $N(b, a^2)$ distribution.

2.4 d)

$\because E(Y) = b$ and $Var(Y) = a^2$,
than, we know that

$$\begin{aligned} E(Z) &= E(cY + d) \\ &= cE(Y) + d \\ &= cb + d \end{aligned}$$

and

$$\begin{aligned} Var(Z) &= Var(cY + d) \\ &= c^2 Var(Y) \\ &= c^2 a^2 \end{aligned}$$

$\therefore Z \backsim N(cb + d, c^2 a^2)$.

3 Question 3 Don't be late

3.1 a)

The joint probability density function is $f(a,b) = \frac{1}{3600}$,
and

$$\begin{aligned} F(a,b) &= \int_0^a \int_0^b f(s,t) ds dt \\ &= \frac{ab}{3600} \end{aligned}$$

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3.2 b)

$\because$ A and B are independent uniformly distributed random variables on $[0,60]$.
then

$$P(B \geq 30) = \frac{1}{2}$$

3.3 c)

A: Alice arrives between 12:15 and 12:30,
B: Bob arrives after 12:45

3.3.1 i)

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B) \\ &= \frac{1}{4} \cdot \frac{1}{4} \\ &= 0.0625 \end{aligned}$$

3.3.2 ii)

3.4 d)

A: The time Alice arrives at t_A,
B: the time Bob arrives at most five minutes after Alice(or before her) t_B,
So $t_A - 5 \leq t_B$ or $t_B \leq t_A + 5$,

$$P(t_a - 5 \leq t_B \leq t_A + 5) = \frac{1}{3600}(3600 - \frac{55^2}{2})$$

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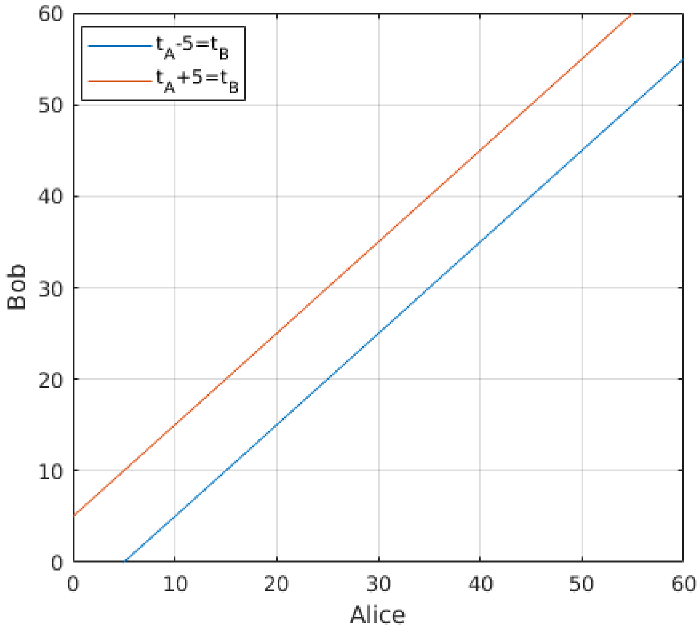

3.5 e)

C: Alice and Bob arrive within 20 minutes of each other.
So

$$P(C) = \frac{3600 - 40^2}{3600} = \frac{5}{9}$$

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