Deep Neural Networks

Assignment 2

Assignment due by: 15.05.2018, Discussions on: 29.05.2018

Question 1 Bayesian Probability (5 Points)

A test has been developed for a new strain of a certain disease. About one in a hundred people displaying symptoms for the disease carry the new strain which is resistant to the previous treatment. This test has been shown to identify the new strain 99 % of the time if it is present, but also give false positives for the previous version 2 % of the time. Since the new treatment is more expensive, say whether this test is enough to decide if the new treatment needs to be used, i.e.compute $P(Strain = New|Test = Positive)$

Question 2 The new Normal (2+3+1+1=7 Points)

Recall that the normal distribution $\mathcal{N}(\mu, \sigma^2)$ has probability density function

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

The standard normal distribution $\mathcal{N}(0, 1)$ has mean 0 (by symmetry), variance 1, and probability density function $\phi(x)$ given by setting $\mu = 0$ and $\sigma = 1$ above. The cumulative density function is denoted $\Phi(x)$ and does not have a nice formula. In this problem, we'll show that scaling and shifting a normal random variable gives a normal random variable. Suppose $X \sim \mathcal{N}(0, 1)$ and $Y = aX + b$.

(a) Compute the mean $\mathrm{E}[Y]$ and variance $\mathrm{Var}(Y)$ of Y.

(b) Express the cumulative density function $F_Y(y)$ of Y in terms of Φ and then use the chain rule to find the probability density function $f_Y(y)$ of Y.

(c) Use (b) to show that Y follows the $\mathcal{N}(b, a^2)$ distribution.

(d) Suppose now that $Z = cY + d$, show that $Z \sim \mathcal{N}(cb + d, a^2c^2)$.

Question 3 Don't be late (1+1+2+2+2=8 Points)

Alice and Bob are trying to meet for lunch and both will arrive, independently of each other, uniformly and at random between noon and 1pm. Let A and B be the number of minutes after noon at which Alice and Bob arrive, respectively. Then A and B are independent uniformly distributed random variables on $[0, 60]$.

(a) Find the joint probability density function $f(a, b)$ and joint cumulative density function $F(a, b)$.

(b) Find the probability that Bob arrives after $12:30$.

(c) Find the probability that Bob arrives after $12:45$ and Alice arrives between $12:15$ and $12:30$ in two ways:

  (i) By using the fact that A and B are independent.

(ii) By shading the corresponding area of the square $[0, 60] \times [0, 60]$ and showing that the proportion of shaded area equals the probability.

(d) Find the probability that Bob arrives at most five minutes after Alice (or before her). (Hint: use method (ii) from part (c).)

(e) Now suppose that Alice and Bob are both rather impatient and will leave if they have to wait more than 20 minutes for the other to arrive. What is the probability that Alice and Bob will have lunch together?

Hint: For parts (d-e) you might find it easiest to find the fraction of the square $[0, 60] \times [0, 60]$ filled by the event.