Machine Learning in Graphics and Vision

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July 19, 2018

University of Tübingen
MPI for Intelligent Systems
Autonomous Vision Group

Discussion of Lecture Evaluation

Overview

Structured Prediction I

- Graphical Models: Factor Graphs
- Inference: Belief Propagation

Structured Prediction II

- Stereo & Multi-view Reconstruction
- Optical Flow Estimation

Structured Prediction III

- Parameter Estimation
- Deep Structured Models

Disclaimer:

There are many structured prediction problems ...

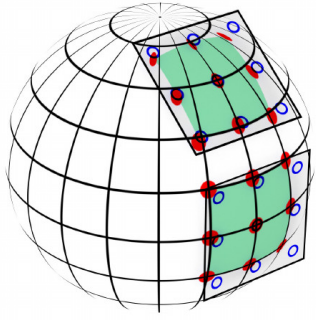
... we can not discuss all of them today

SphereNet: Learning Spherical Representations for Detection

SphereNet: Learning Spherical Representations for Detection

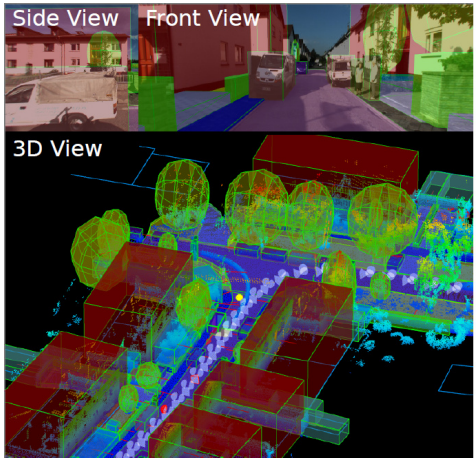
360° Cameras

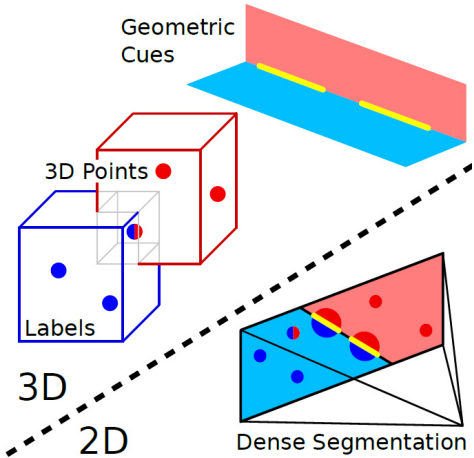
360° Image



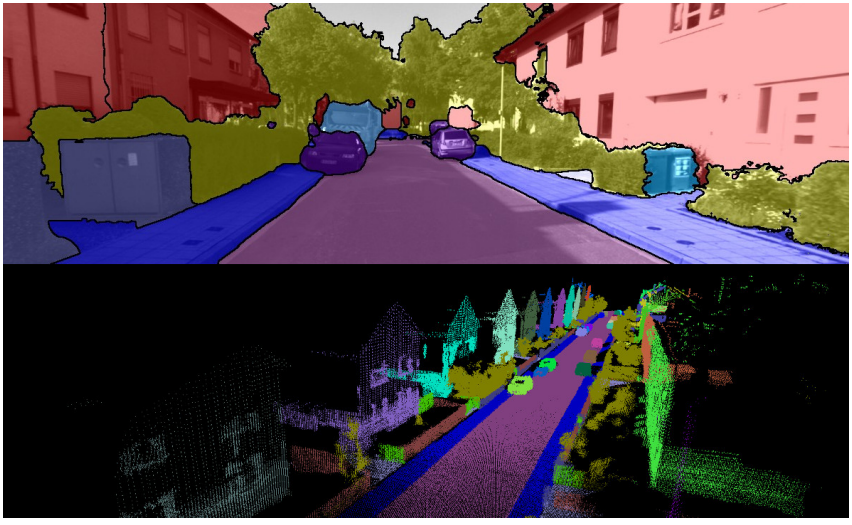
SphereNet Kernel

3D to 2D Semantic Label Transfer



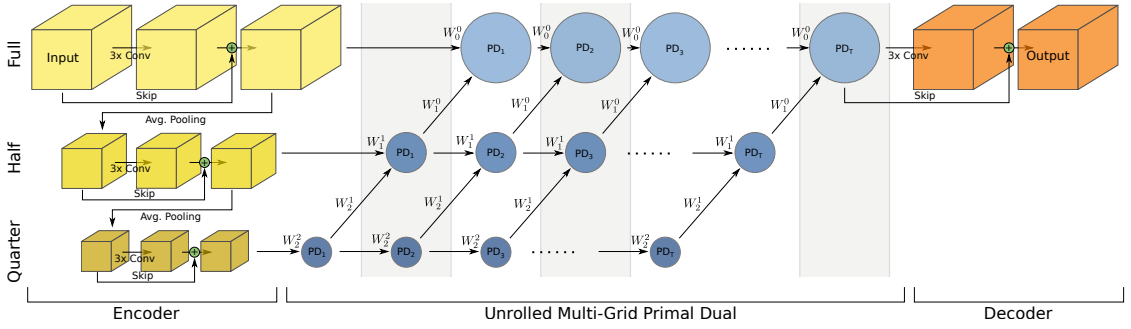


3D to 2D Semantic Label Transfer



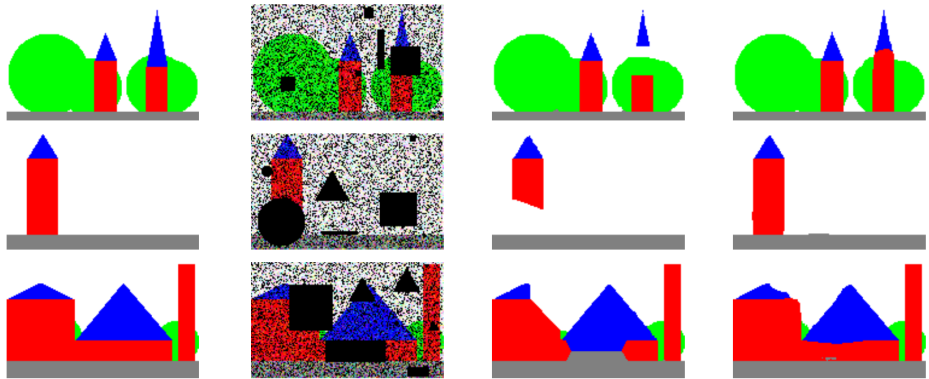
Xie, Kiefel, Sun & Geiger, CVPR 2016

Learning Priors for Semantic 3D Reconstruction

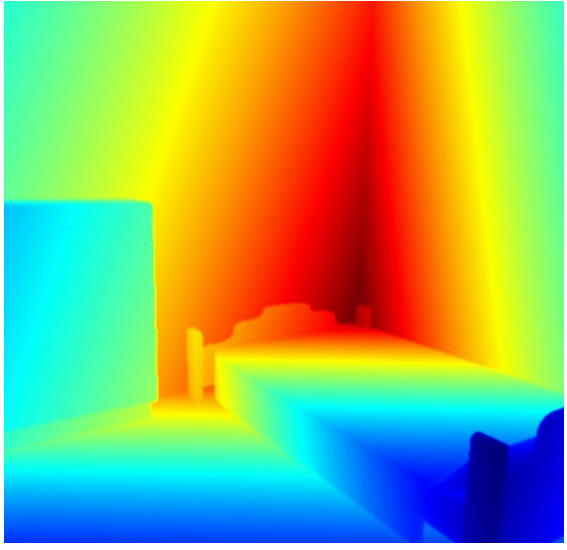


$$\underset{u}{\text{minimize}} \quad \int_{\Omega} \left(\|Wu\|_2 + fu \right) d\mathbf{x} \quad \text{subject to} \quad \forall \mathbf{x} \in \Omega : \sum_{\ell} u_{\ell}(\mathbf{x}) = 1$$

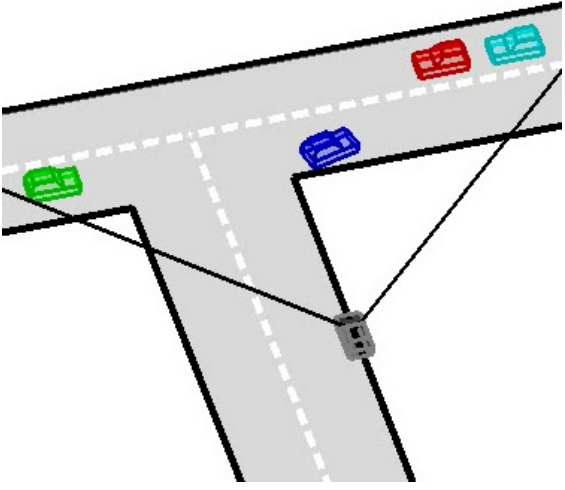
Learning Priors for Semantic 3D Reconstruction



Indoor Scene Understanding

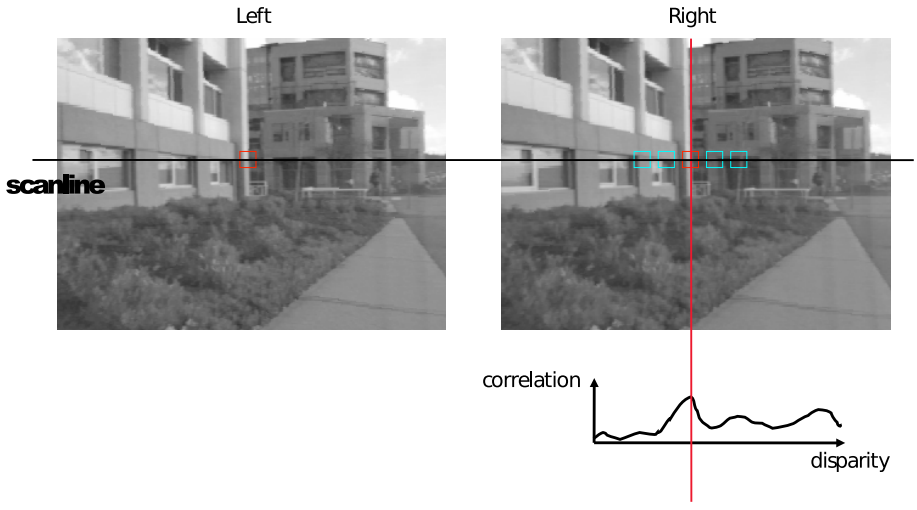


Traffic Scene Understanding

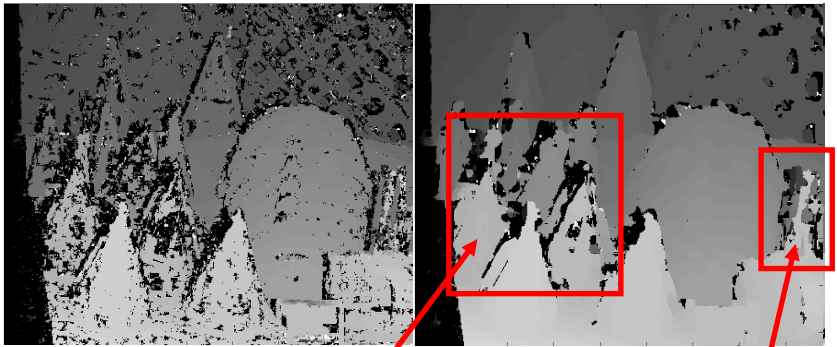


Stereo Reconstruction

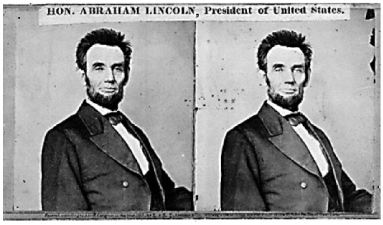
Block Matching



Block Matching – Window Size



Block Matching – Failure Cases



Textureless surfaces

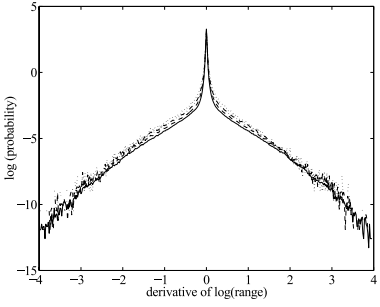
Occlusions, repetition

Non-Lambertian surfaces, specularities

Spatial Regularization

How does the real world look like?

- Leverage real-world statistics
- E.g.: Brown range image database [Mumford et al.]



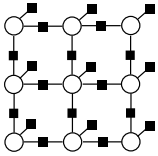
Fast Approximate Energy Minimization via Graph Cuts

[Boykov, Veksler & Zabih, PAMI 1999]

Stereo MRF

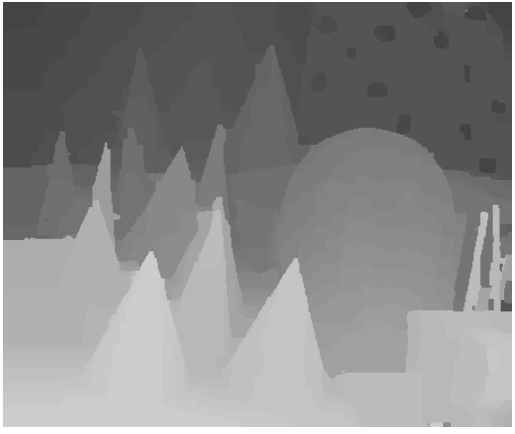
- Specify a loopy Markov Random Field (MRF) on a grid and solve for the whole disparity map $\mathbf{D}$ at once. MAP solution = minimum of energy.

$$p(\mathbf{D}) \propto \exp\left\{ -\sum_i \psi_{data}(d_i) - \lambda \sum_{i \sim j} \psi_{smooth}(d_i, d_j) \right\}$$

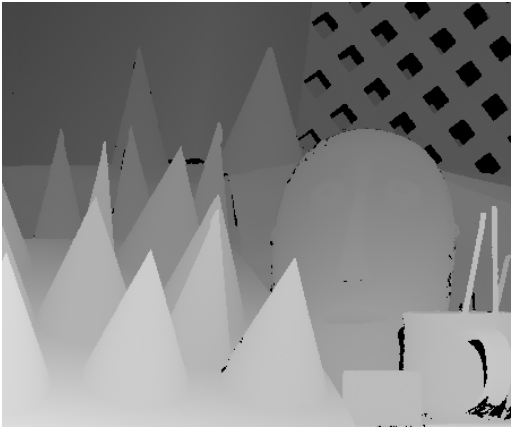


- $i \sim j$: neighboring pixels on a 4-connected grid
- Unary terms: Matching cost $\psi_{data}(d)$
- Pairwise terms: Smoothness between adjacent pixels, e.g.:
  - Potts: $\psi_{smooth}(d, d') = [d \neq d']$
  - Truncated l_1: $\psi_{smooth}(d, d') = \min(|d - d'|, \tau)$
- Solve MRF approximately using BP / graph cuts

Stereo MRF – Results



Inference Results



Ground Truth

Displets: Resolving Stereo Ambiguities using Object Knowledge

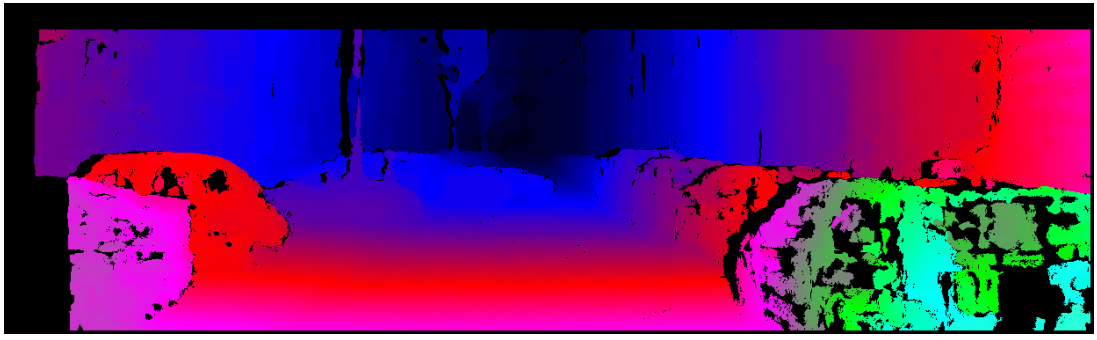
[Güney & Geiger, CVPR 2015]

Integrating Object Knowledge

$$E(D) = \underbrace{\sum_i \psi_i^{\mathcal{A}}(d_i)}_{\text{Appearance}} + \lambda \underbrace{\sum_{i \sim j} \psi_{ij}^{\mathcal{S}}(d_i, d_j)}_{\text{Smoothness}}$$

[Boykov, PAMI 1999]

Integrating Object Knowledge



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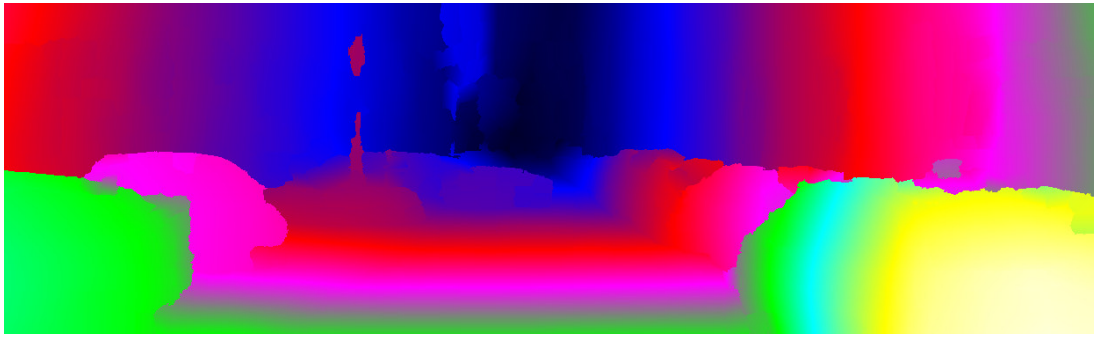
[Boykov, PAMI 1999]

Integrating Object Knowledge

$$E(D,O) = \underbrace{\sum_i \psi_i^{\mathcal{A}}(d_i)}_{\text{Appearance}} + \lambda_{\mathcal{S}} \underbrace{\sum_{i \sim j} \psi_{ij}^{\mathcal{S}}(d_i, d_j)}_{\text{Smoothness}} + \lambda_{\mathcal{O}} \underbrace{\sum_k \psi_k^{\mathcal{O}}(o_k)}_{\text{Object Semantics}} + \lambda_{\mathcal{C}} \underbrace{\sum_k \sum_i \psi_{ki}^{\mathcal{C}}(o_k, d_i)}_{\text{3D Consistency}}$$

[Güney & Geiger, CVPR 2015]

Integrating Object Knowledge



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[Güney & Geiger, CVPR 2015]

Stereo Reconstruction Summary

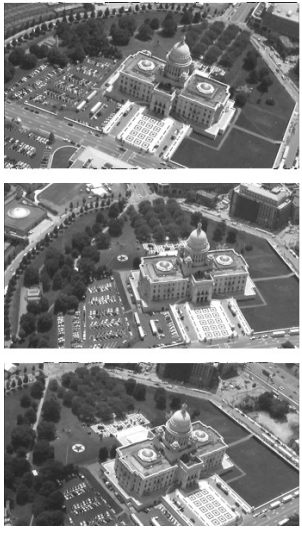
- Block matching suffers from ambiguities
- Choosing window size is problematic (tradeoff)
- Incorporating smoothness constraints can resolve some of the ambiguities and allows for choosing small windows (no bleeding artifacts)
- Can be formulated as inference in a discrete MRF
- MAP solution can be obtained using belief propagation / graph cuts
- Integrating recognition cues can further regularize the problem

Multi-View Reconstruction

Towards Probabilistic Reconstruction

[Ulusoy, Geiger & Black, 3DV 2015]

Volumetric 3D Reconstruction from Multiple Views



Related Work

Online Algorithms:

- [Bonet & Viola, ICCV 1999]
- [Agrawal & Davis, CVPR 2001]
- [Pollard & Mundy, CVPR 2007]

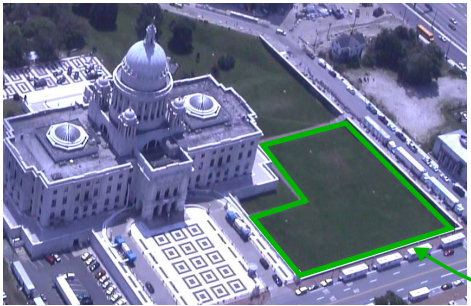
Ray Potentials:

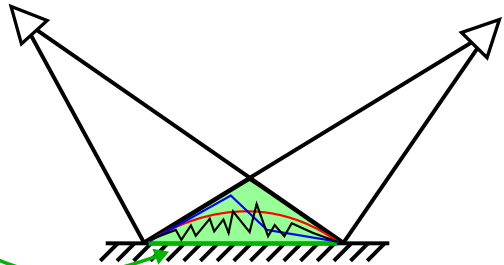
- [Gargallo, Sturm & Pujades, ACCV 2007]
- [Liu & Cooper, PAMI 2014]
- [Savinov, Ladicky, Häne & Pollefeys, CVPR 2015]

Here:

- Joint estimation of voxel occupancy and appearance
- Marginal inference algorithm $\Rightarrow$ Bayes optimal predictions

Reconstruction Ambiguities

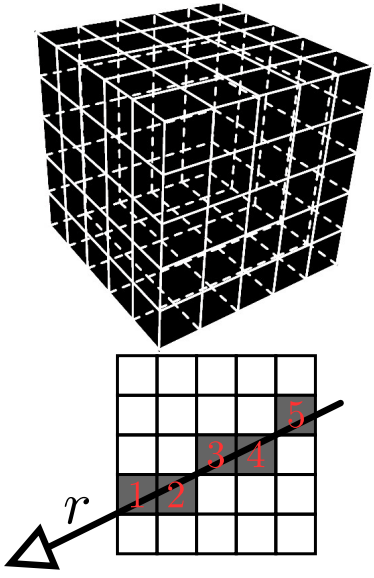




- Image-based 3D reconstruction is a highly **ill-posed** problem
  ⇒ Coping with and exposing **uncertainty** is essential

Can we formulate 3D reconstruction in a probabilistic way?

Representation



- **Voxel occupancy:**

$$o_i = \begin{cases} 1 & \text{if voxel } i \text{ is occupied} \\ 0 & \text{if voxel } i \text{ is empty} \end{cases}$$

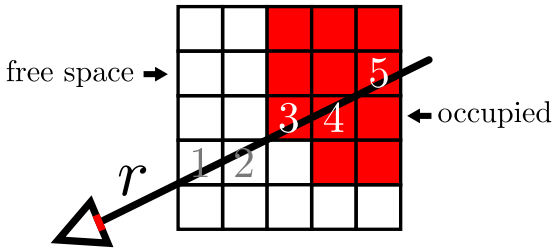
- **Voxel appearance:**

$$a_i \in \mathbb{R}$$

- **Shorthand notation:**

$$\mathbf{o}_r = \{o_1^r, \ldots, o_{N_r}^r\}$$
$$\mathbf{a}_r = \{a_1^r, \ldots, a_{N_r}^r\}$$

Image Formation Process



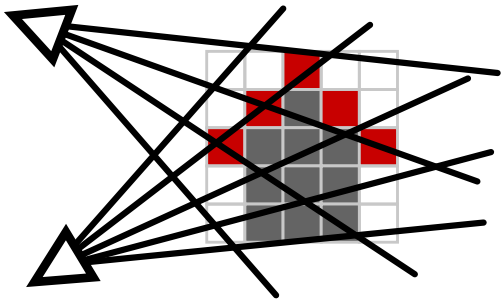
$$I_r = \sum_{i=1}^{N} o_i \prod_{j<i} (1 - o_j)\, a_i$$

- I_r: intensity at pixel r o_i: occupancy of voxel i a_i: appearance of voxel i

Probabilistic Model

Joint Distribution:

$$p(\mathbf{o}, \mathbf{a}) = \frac{1}{Z} \prod_{v \in \mathcal{V}} \underbrace{\varphi_v(o_v)}_{\text{unary}} \prod_{r \in \mathcal{R}} \underbrace{\psi_r(\mathbf{o}_r, \mathbf{a}_r)}_{\text{ray}}$$



Probabilistic Model

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Unary Potentials:

$$\varphi_v(o_v) = \gamma^{o_v} (1 - \gamma)^{1 - o_v}$$

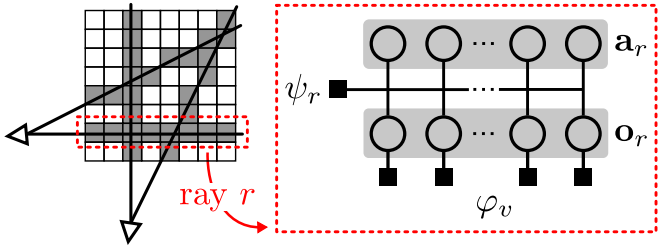
- Most voxels are empty $\Rightarrow \gamma < 0.5$

Probabilistic Model

Joint Distribution:

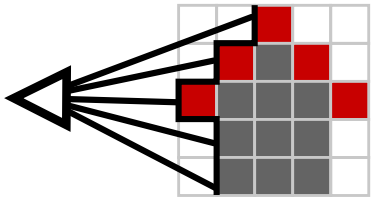
$$p(\mathbf{o}, \mathbf{a}) = \frac{1}{Z} \prod_{v \in \mathcal{V}} \underbrace{\varphi_v(o_v)}_{\text{unary}} \prod_{r \in \mathcal{R}} \underbrace{\psi_r(\mathbf{o}_r, \mathbf{a}_r)}_{\text{ray}}$$

Ray Potentials:

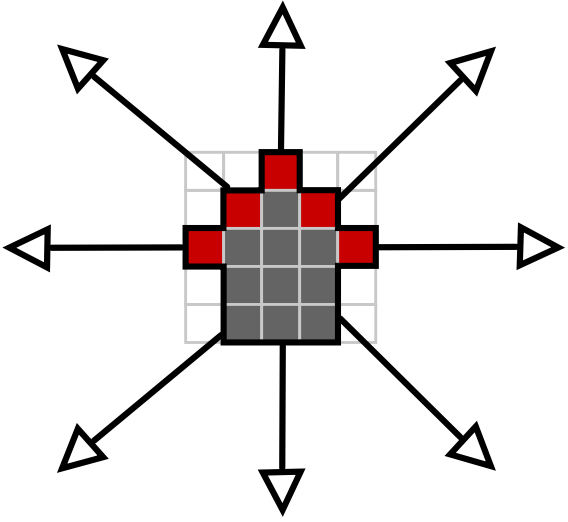


$$\psi_r(\mathbf{o}_r, \mathbf{a}_r) = \sum_{i=1}^{N_r} o_i^r \prod_{j<i} (1 - o_j^r) \underbrace{\mathcal{N}(a_i^r | I_r, \sigma)}_{\text{Gaussian Noise}}$$

3D Reconstruction

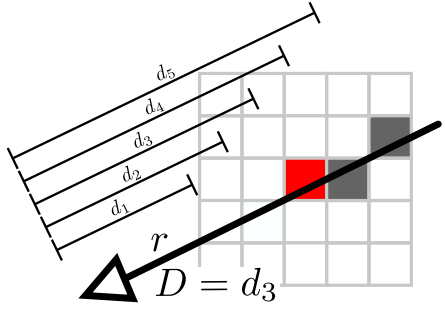


3D Reconstruction



Bayes Optimal Depth Estimation

- Consider a single ray r in space
- Let d_k be the distance from the camera to voxel k along ray r
- Depth $D \in \{d_1, \ldots, d_N\}$: distance to closest occupied voxel



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- Consider a single ray r in space
- Let d_k be the distance from the camera to voxel k along ray r
- Depth $D \in \{d_1, \ldots, d_N\}$: distance to closest occupied voxel
- Optimal depth estimate:

$$
\begin{aligned}
D^* &= \operatorname*{argmin}_{D'} \text{Risk}(D') \\
&= \operatorname*{argmin}_{D'} \mathbb{E}_{p(D)}[\,\Delta(D, D')\,] \\
&= \begin{cases} \text{mean}\,(\,p(D)\,) & \text{if } \Delta(D, D') = (D - D')^2 \\ \text{median}\,(\,p(D)\,) & \text{if } \Delta(D, D') = |D - D'| \end{cases}
\end{aligned}
$$

- Requires depth distribution $p(D)$!

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) = \underbrace{\sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} p(\mathbf{o}, \mathbf{a})}_{= p(o_1, \ldots, o_k)}$$

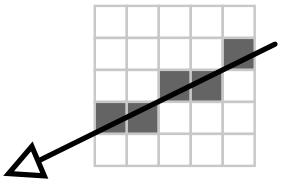
Depth Distribution for Single Ray

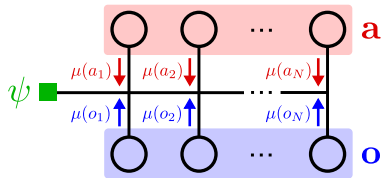
Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \psi(\mathbf{o}, \mathbf{a}) \prod_i \mu(o_i) \prod_i \mu(a_i)$$





Marginal = Product of Factor & Incoming Messages

Depth Distribution for Single Ray

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$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \psi(\mathbf{o}, \mathbf{a}) \prod_i \mu(o_i) \prod_i \mu(a_i)$$

$$\psi(\mathbf{o}, \mathbf{a}) = \underbrace{\sum_i o_i \prod_{j<i} (1 - o_j) \mathcal{N}(a_i | I, \sigma)}_{= \mathcal{N}(a_k | I, \sigma)}$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \mathcal{N}(a_k | I, \sigma) \prod_i \mu(o_i) \prod_i \mu(a_i)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}} \mathcal{N}(a_k | I, \sigma) \prod_{i>k} \mu(o_i) \prod_{i} \mu(a_i) \times \prod_{i \leq k} \mu(o_i)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}_{\neq k}} \prod_{i>k} \mu(o_i) \prod_{i \neq k} \mu(a_i)$$

$$\times \prod_{i \leq k} \mu(o_i) \int_{a_k} \mathcal{N}(a_k | I, \sigma) \mu(a_k)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

Then:

$$p(D = d_k) \propto \underbrace{\sum_{\mathbf{o}_{>k}} \int_{\mathbf{a}_{\neq k}} \prod_{i>k} \mu(o_i) \prod_{i \neq k} \mu(a_i)}_{=1}$$
$$\times \prod_{i \leq k} \mu(o_i) \int_{a_k} \mathcal{N}(a_k | I, \sigma) \mu(a_k)$$

Depth Distribution for Single Ray

Let:

$$o_1 = 0 \quad \cdots \quad o_{k-1} = 0 \quad o_k = 1$$

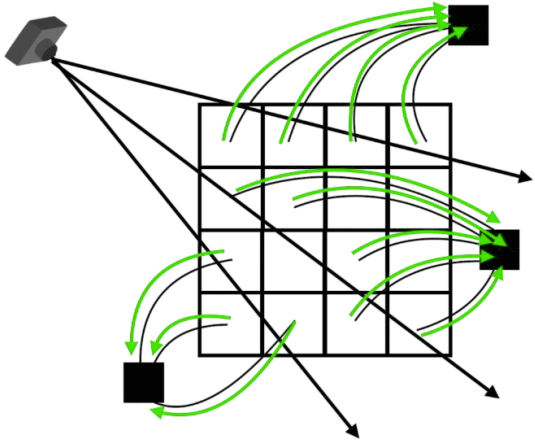
Then:

$$p(D = d_k) \propto \prod_{i \leq k} \mu(o_i) \int_{a_k} \mathcal{N}(a_k | I, \sigma) \mu(a_k)$$

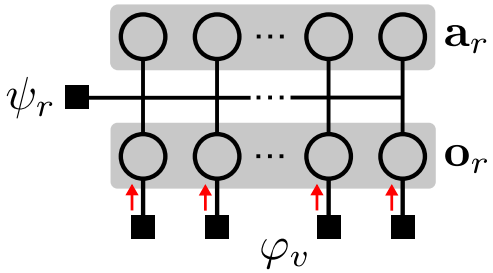
Intuition: Depth $D = d_k \Leftrightarrow$ Voxel k is occupied and visible and explains the observed pixel value.

- How can we obtain $\mu(o_i)$ and $\mu(a_k)$?

Message Passing

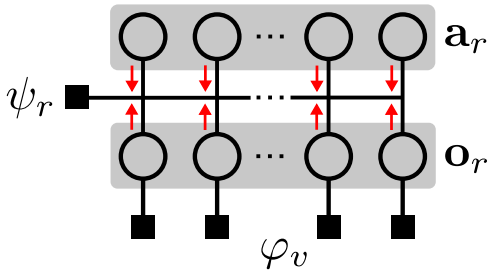


Message Passing



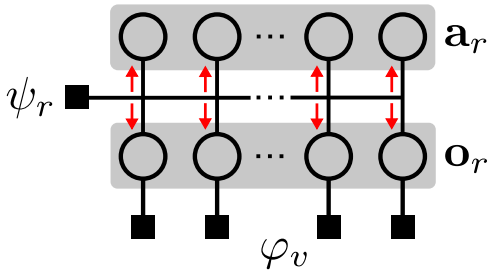
$$\mu_{\varphi_v \to o_v}(o_v) = \varphi_v(o_v)$$

Message Passing



$$\mu_{x \to \psi_r}(x) = \prod_{f \in \mathcal{F} \setminus \psi_r} \mu_{f \to x}(x)$$

Message Passing



$$\mu_{\psi_r \to x}(x) = ?$$

Ray-to-Occupancy Message

$$\mu_{\psi_r \to o_i}(o_i = 1) = \sum_{\mathbf{o}_{j \neq i}} \int_{\mathbf{a}} \psi_r(\mathbf{o}, \mathbf{a}) \prod_{j \neq i} \mu(o_j) \prod_j \mu(a_j)$$

$$\vdots$$

$$= \prod_{k=1}^{i-1} \mu(o_k = 0) \int_{a_i} \nu(a_i)\mu(a_i)$$

$$+ \sum_{j=1}^{i-1} \mu(o_j = 1) \prod_{k=1}^{j-1} \mu(o_k = 0) \int_{a_j} \nu(a_j)\mu(a_j)$$

Photoconsistent voxels behind free voxels are *likely* occupied.

Voxels behind photo-consistent voxels *may be* occupied.

Ray-to-Occupancy Message

$$\mu_{\psi_r \to o_i}(o_i = 1) = \sum_{\mathbf{o}_{j \neq i}} \int_{\mathbf{a}} \psi_r(\mathbf{o}, \mathbf{a}) \prod_{j \neq i} \mu(o_j) \prod_j \mu(a_j)$$

$$\vdots$$

$$= \prod_{k=1}^{i-1} \mu(o_k = 0) \int_{a_i} \nu(a_i)\mu(a_i)$$

$$+ \sum_{j=1}^{i-1} \mu(o_j = 1) \prod_{k=1}^{j-1} \mu(o_k = 0) \int_{a_j} \nu(a_j)\mu(a_j)$$

- Can be computed in linear time (see paper for derivation)
- Similar calculation for $\mu_{\psi_r \to o_i}(o_i = 0)$

Ray-to-Appearance Message

$$\mu_{\psi_r \to a_i}(a_i) = \underbrace{\sum_{j \neq i} \mu(o_j = 1) \prod_{k<j} \mu(o_k = 0) \int_{a_j} \nu(a_j)\mu(a_j)}_{\text{Constant}}$$

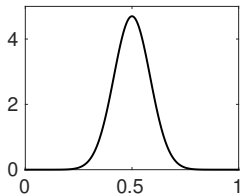
$$+ \underbrace{\mu(o_i = 1) \prod_{k<i} \mu(o_k = 0)}_{\text{Weight}} \times \underbrace{\nu(a_i)}_{\text{Gaussian}}$$

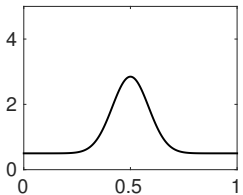
- Can also be computed in linear time (see paper for derivation)
- Constant: measures how well voxels $j \neq i$ explain evidence
- Weight: measures how likely voxel i is the first occupied voxel
- Gaussian: measures photoconsistency of a_i wrt. the image

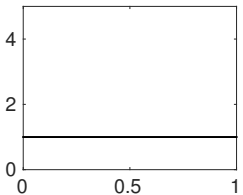
Ray-to-Appearance Message

$$\mu_{\psi_r \to a_i}(a_i) = \underbrace{\sum_{j \neq i} \mu(o_j = 1) \prod_{k<j} \mu(o_k = 0) \int_{a_j} \nu(a_j)\mu(a_j)}_{\text{Constant}}$$

$$+ \underbrace{\mu(o_i = 1) \prod_{k<i} \mu(o_k = 0)}_{\text{Weight}} \times \underbrace{\nu(a_i)}_{\text{Gaussian}}$$







Ray-to-Appearance Message

$$\mu_{\psi_r \to a_i}(a_i) = \underbrace{\sum_{j \neq i} \mu(o_j = 1) \prod_{k<j} \mu(o_k = 0) \int_{a_j} \nu(a_j)\mu(a_j)}_{\text{Constant}}$$

$$+ \underbrace{\mu(o_i = 1) \prod_{k<i} \mu(o_k = 0)}_{\text{Weight}} \times \underbrace{\nu(a_i)}_{\text{Gaussian}}$$

- **Integrals:** Monte Carlo approximation
- **Messages:** Mixture-of-Gaussian representation:

$$\mu_{a_v \to \psi_r}(a_v) = \prod_{r' \in \mathcal{R}_v \setminus r} \mu_{\psi_{r'} \to a_v}(a_v)$$

Inference

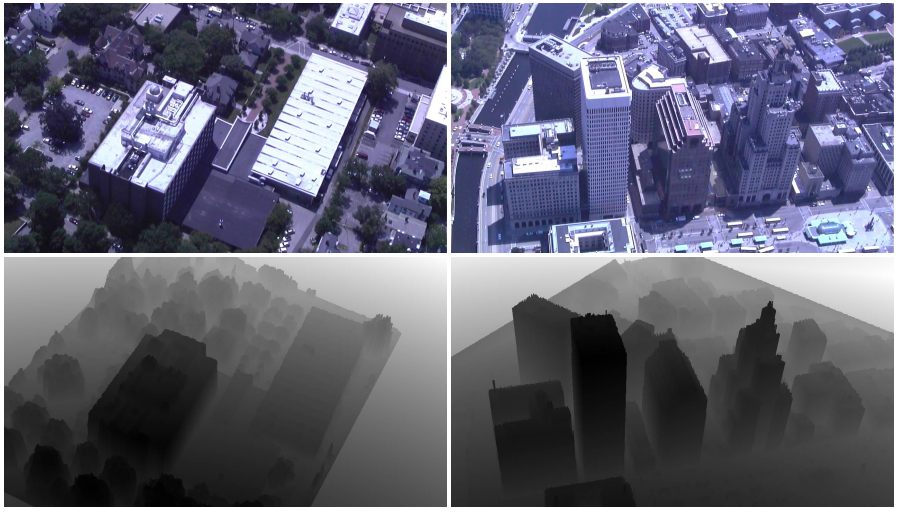
Challenges:

1. MRF comprises discrete and continuous variables
2. Ray potentials are high-order
3. Each pixel defines a factor

Our Approach:

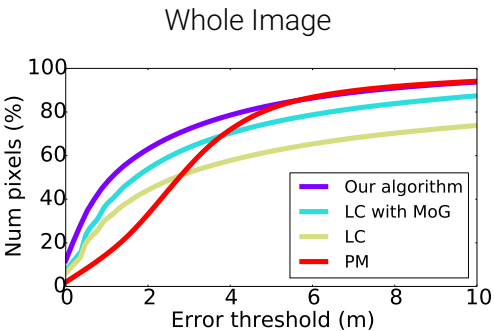
1. Approximate continuous belief propagation
   $\Rightarrow$ Update MoG's via importance sampling
2. Messages can be calculated in linear time
   $\Rightarrow$ Exact (but technical) derivation of messages
3. Octree implementation & GPGPU parallelization

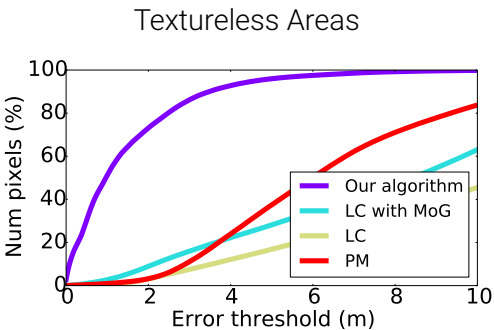
Experimental Results



[Restrepo *et al*., ISPRS 2014]

Quantitative Results

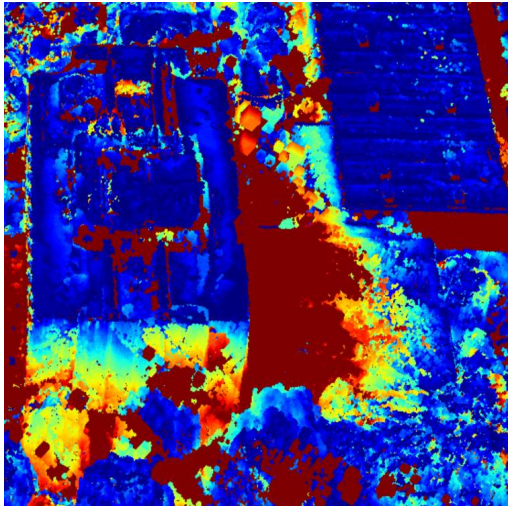




Qualitative Results

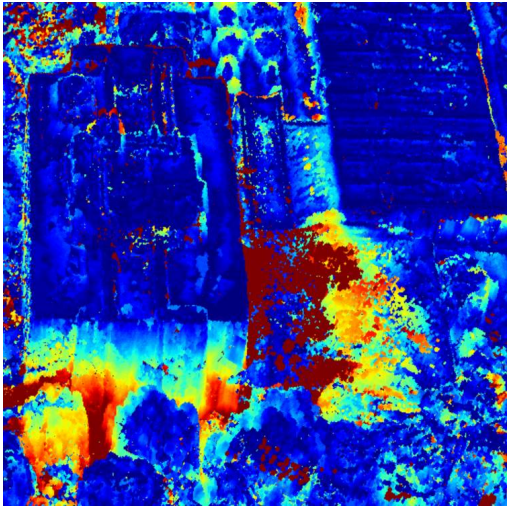
Input Image

Qualitative Results



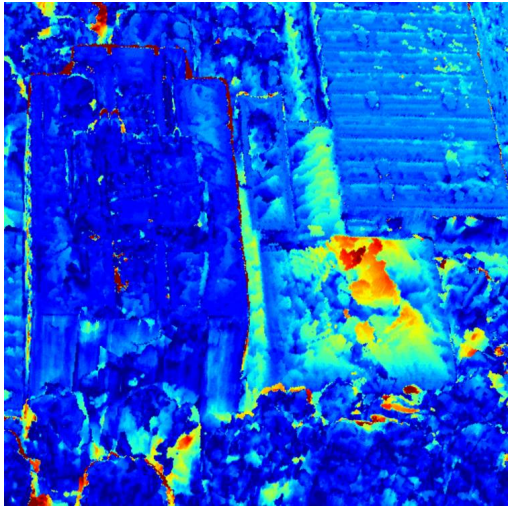
Error Map (Liu & Cooper)

Qualitative Results



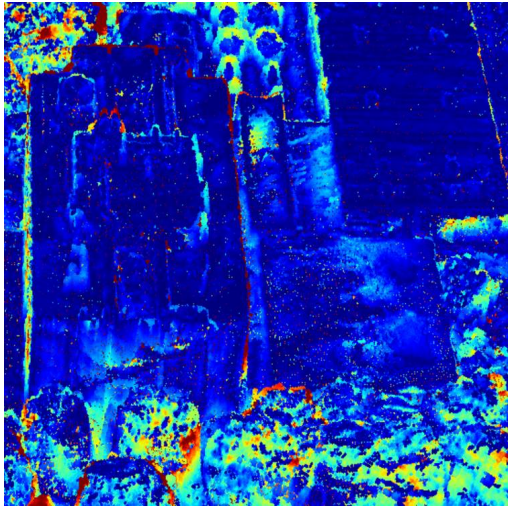
Error Map (Liu & Cooper with MoG)

Qualitative Results



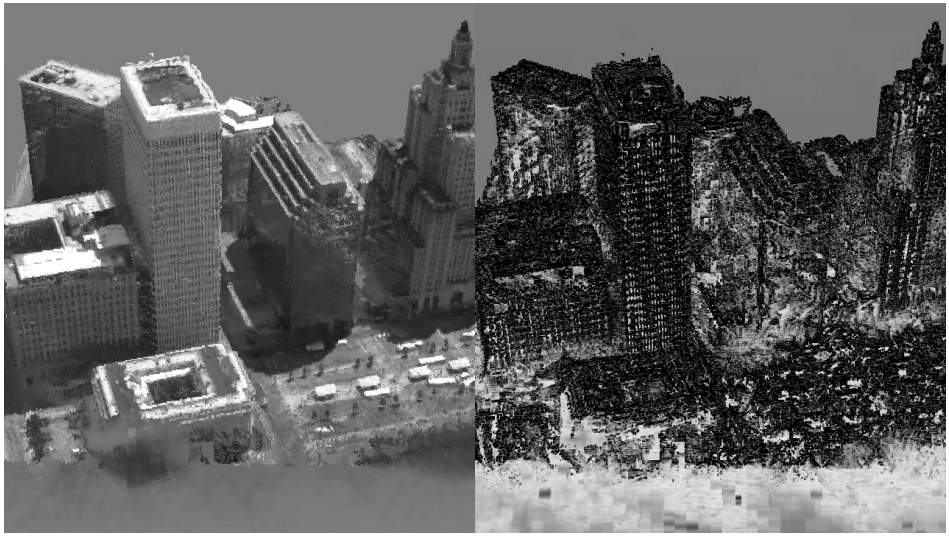
Error Map (Pollard & Mundy)

Qualitative Results

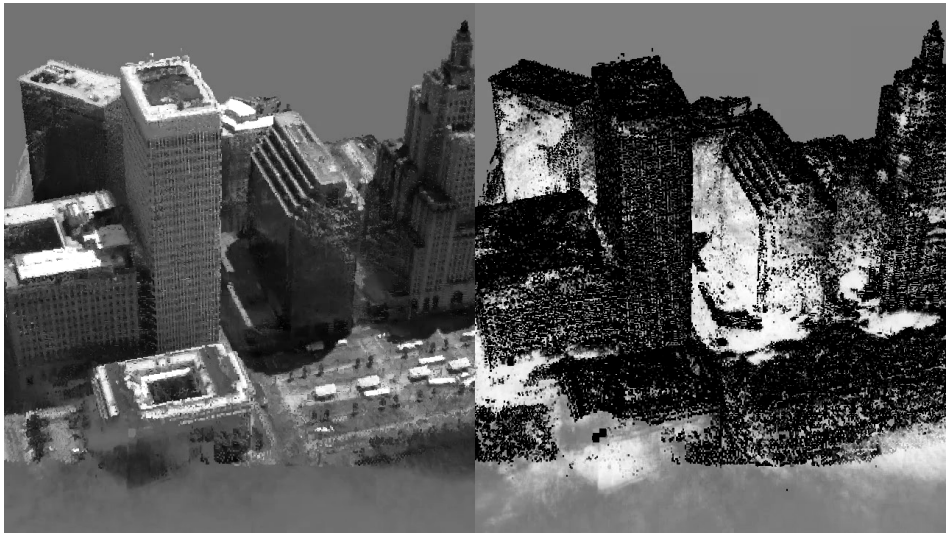


Error Map (Our Approach)

Qualitative Results: Pollard & Mundy

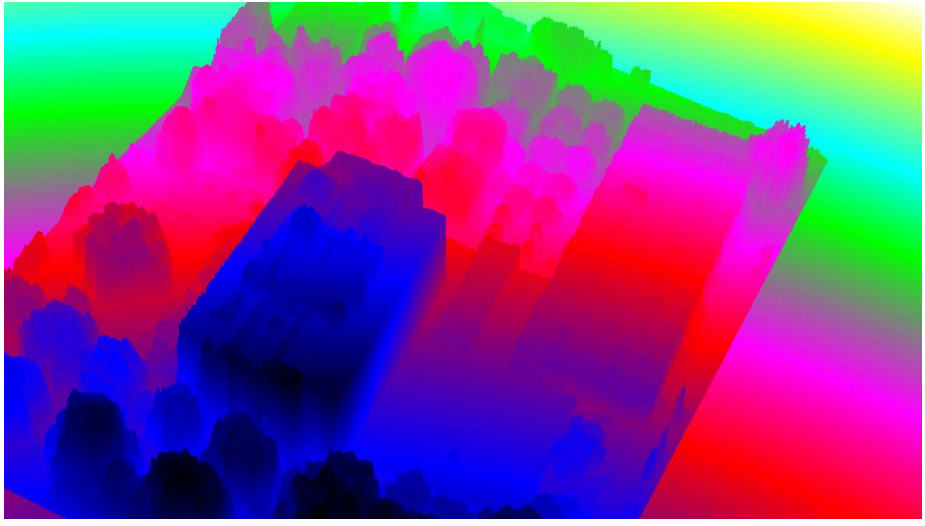


Qualitative Results: Our Results



Qualitative Results: Our Results

Code Available on GitHub

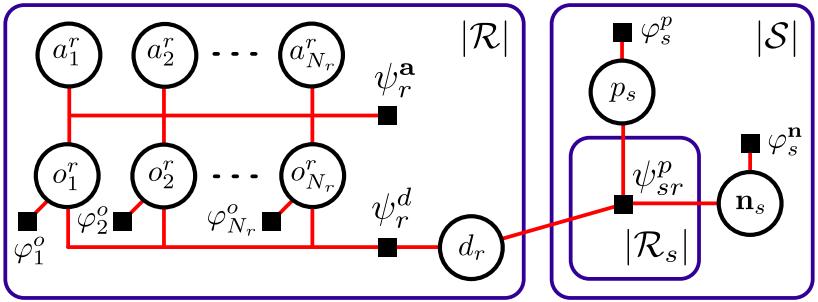


www.github.com/aliosmanulusoy

Patches, Planes and Probabilities: A non-local Prior for 3D Reconstruction

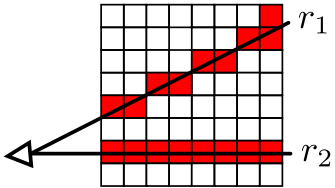
[Ulusoy, Black & Geiger, CVPR 2016]

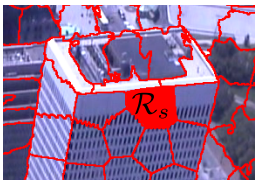
Probabilistic Model



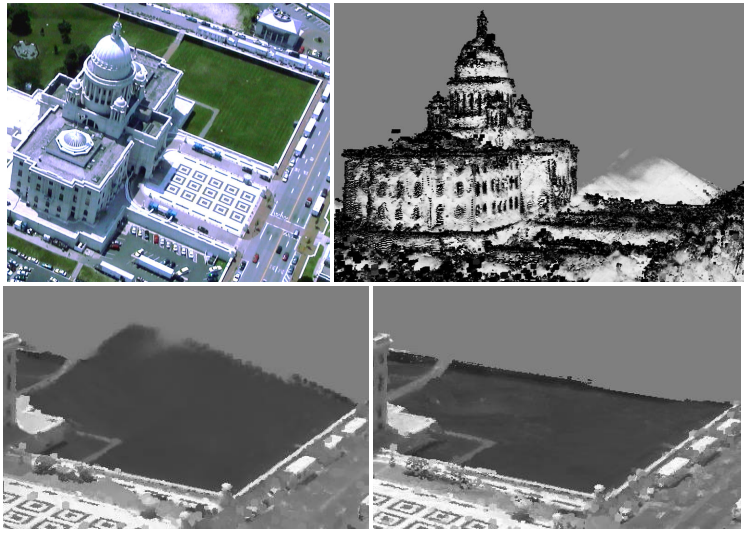
Ray Potentials & Occupancy Potentials

Patch Prior





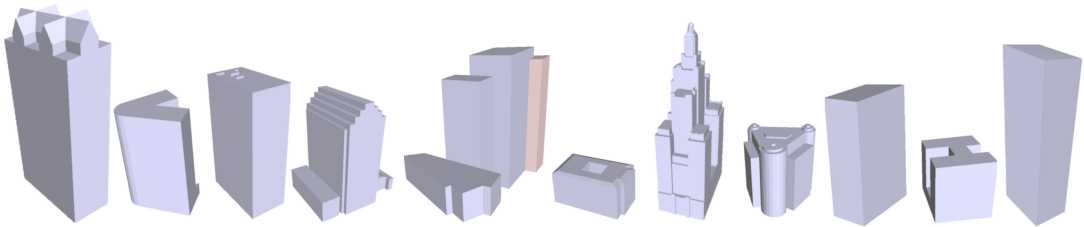
Results



Semantic Multi-view Stereo: Jointly Estimating Objects and Voxels

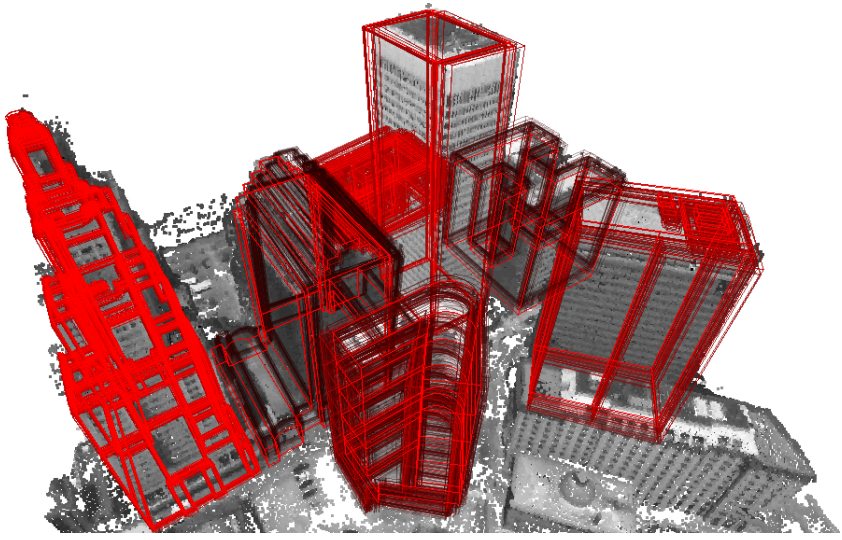
[Ulusoy, Black & Geiger, CVPR 2017]

3D Shape Priors

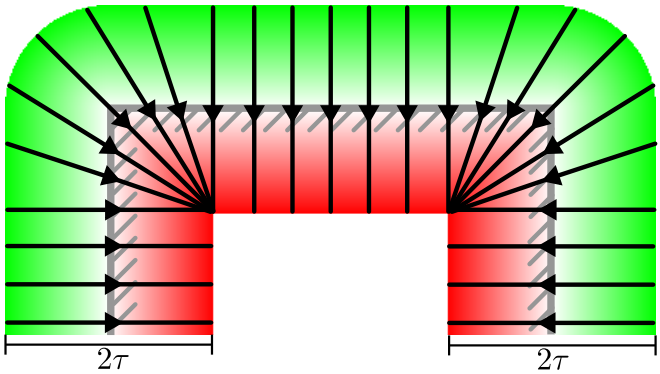


- For many scenes exclusive prior knowledge is available!
  - GPS tags can be used to retrieve 3D Warehouse models
  - 3D models of IKEA furniture for indoor scenes
- Challenges:
  - Often only coarse and inaccurate models
  - Unknown orientation and approximate location
  - Occlusions and object size

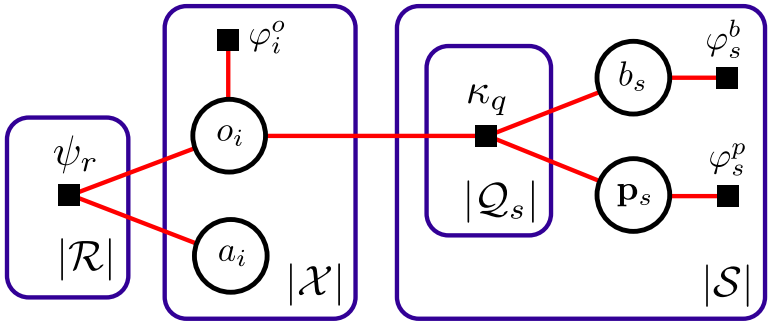
Probabilistic Model Fitting and 3D Reconstruction



Raylet Potentials



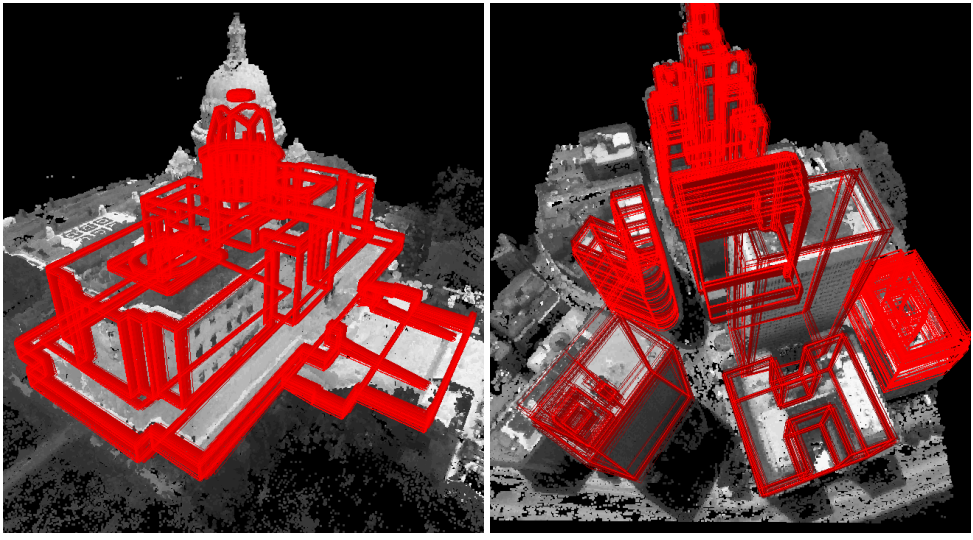
Probabilistic Model



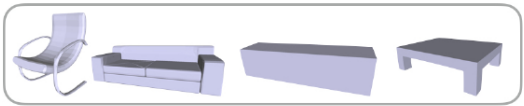
o_i/a_i: occupancy/appearance $b_s/\mathbf{p}_s$: shape model presence/pose

$\mathcal{R}$: #rays $\mathcal{X}$: #voxels $\mathcal{Q}_s$: #raylets $\mathcal{S}$: #shape models

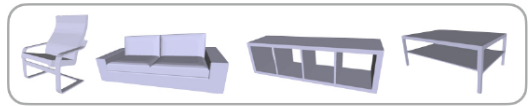
Results: Outdoor



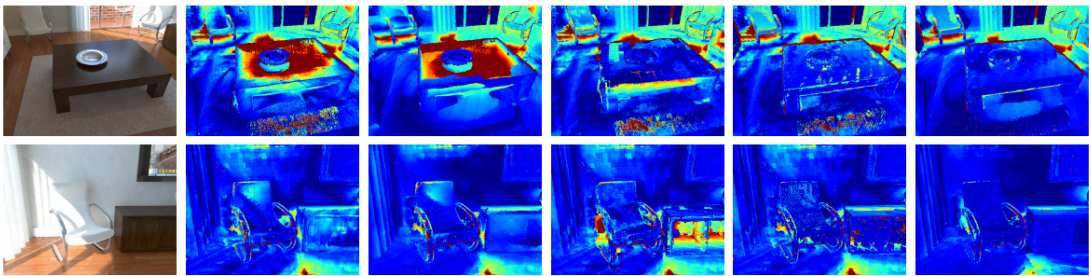
Results: Indoor



(a) Correct shape models for LIVINGROOM.



(b) Approximate shape models from IKEA [23].



(c) Ref image (d) No prior (e) Planarity prior (f) IKEA prior (g) Correct prior (h) Object+Planarity

Multi-View Reconstruction Summary

Pros:

- Probabilistic formulation is tractable as ray factors decompose
- Non-local constraints via joint inference in 2D and 3D
- CAD priors can help disambiguate textureless regions
- Using octrees reconstruction up to 1024^3 voxels possible

Cons:

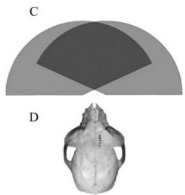
- Only approximate inference possible (highly loopy)
- Relatively slow: several minutes per scene on a GPU
- Appearance term too simplistic and not robust
- Resolution limited to discretization into voxels (as opposed to meshes)

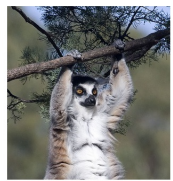
Optical Flow

Stereo vs. Optical Flow

Stereo

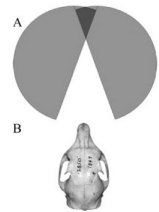
- 2 images at same time
- Only camera motion
- 1D estimation problem
- Monkeys



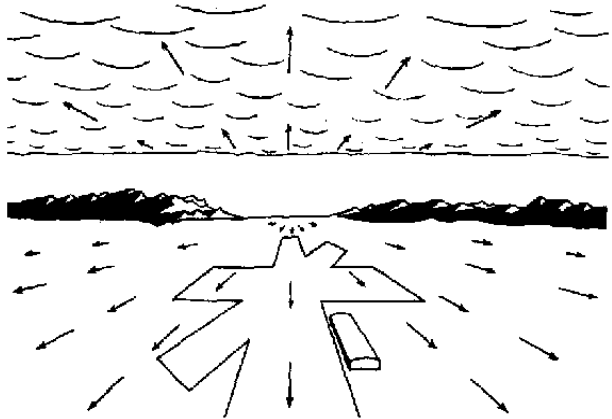


Optical Flow

- 2 images at 2 time steps
- Camera and object motion
- 2D estimation problem
- Squirrels

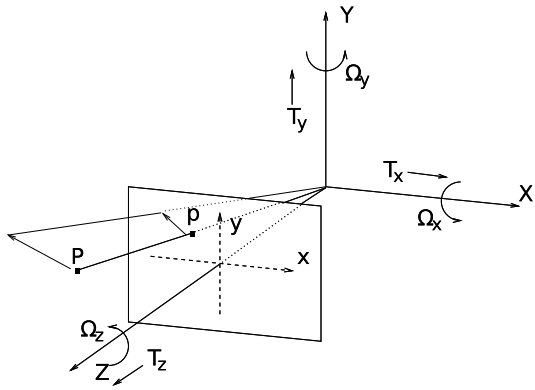


Optical Flow



[J. J. Gibson, 1950: The Ecological Approach to Visual Perception]

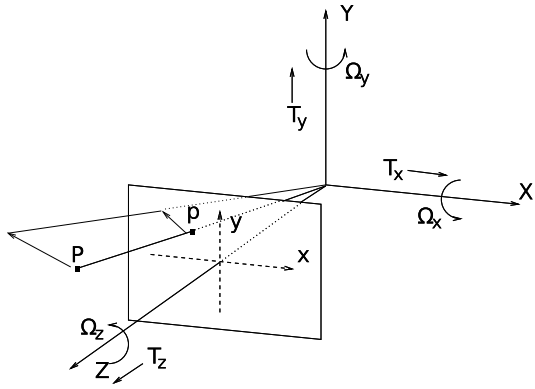
Optical Flow



Motion field:

- 2D motion field representing the **projection of the 3D motion** of points in the scene onto the image plane
- Can be the result of camera motion or object motion (or both)!

Optical Flow



Optical flow:

- 2D velocity field describing the **apparent motion** in the image (*i.e.*, the displacement of pixels looking “similar”)
- Optical flow $\neq$ motion field! Why?

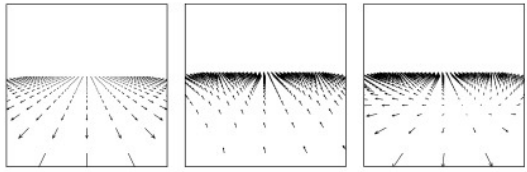
Thought Experiment

- Lambertian ball
  rotating in 3D
- What does the 2D
  motion field look like?
- What does the 2D
  optical flow field look like?

Thought Experiment

- Stationary specular ball
  moving light source
- What does the 2D
  motion field look like?
- What does the 2D
  optical flow field look like?

Optical Flow Field



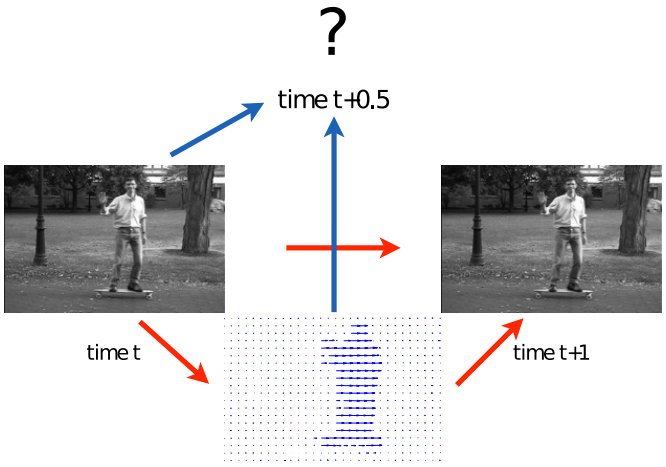
Optical flow fields tell us something (maybe ambiguous) about:

- The **3D structure** of the world
- The **motion of objects** in the viewing area
- The **motion of the observer** (if any)

In contrast to stereo:

- No epipolar geometry $\Rightarrow$ 2D estimation problem!

Applications: Video Interpolation / Frame Rate Adaption

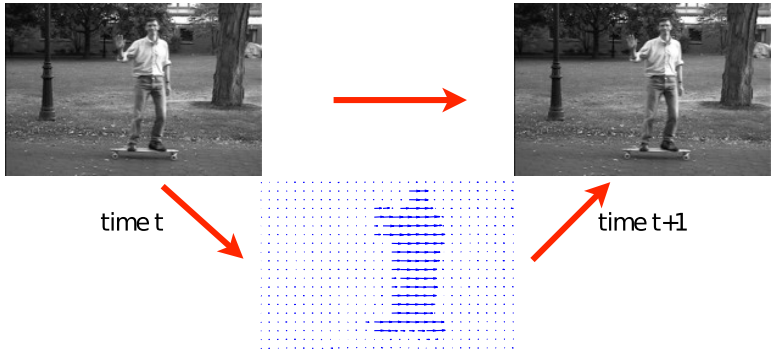


- If we know the image motion we can compute images at intermediate frames

Applications: Video Interpolation / Frame Rate Adaption

- If we know the image motion we can compute images at intermediate frames

Applications: Video Compression

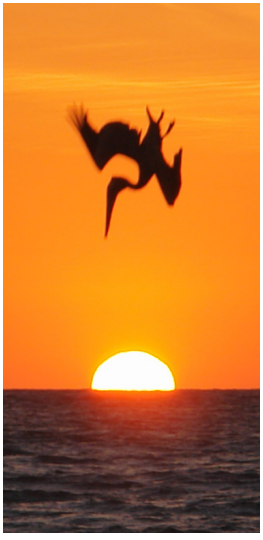


- To compress an image sequence, we can predict new frames using the optical flow field and only store how to “fix” the prediction
- Flow fields are smooth, thus easier to compress/store than images!

The Northern Gannet

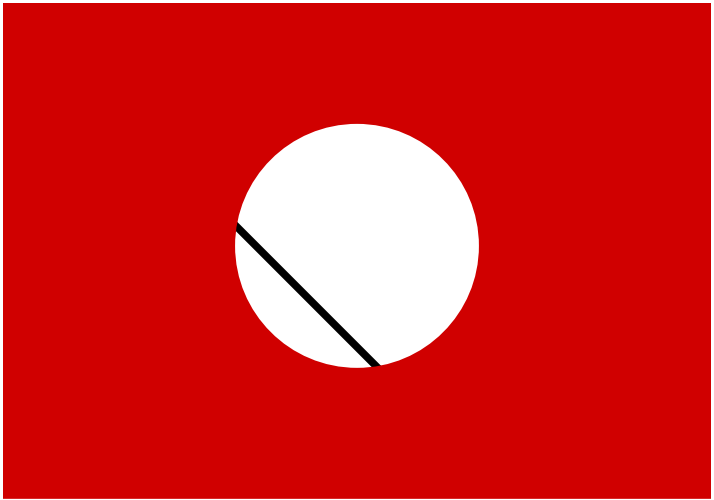
The Northern Gannet

The Northern Gannet



Aperture Problem

In which direction does the line move?



Aperture Problem

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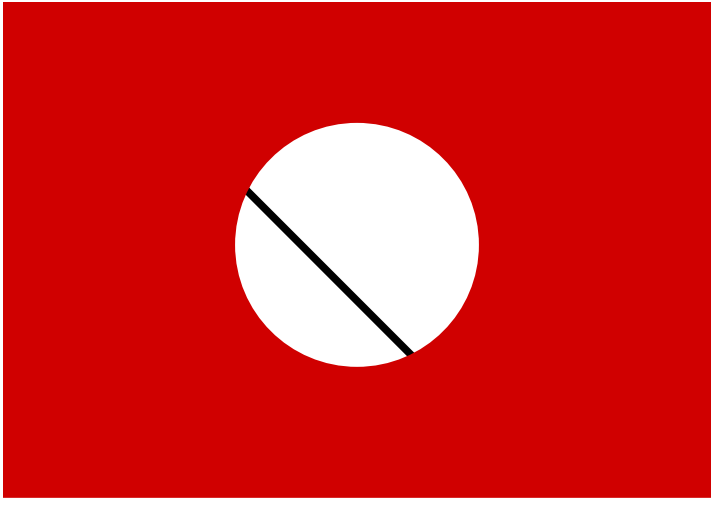
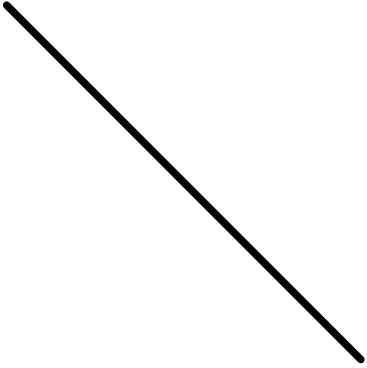


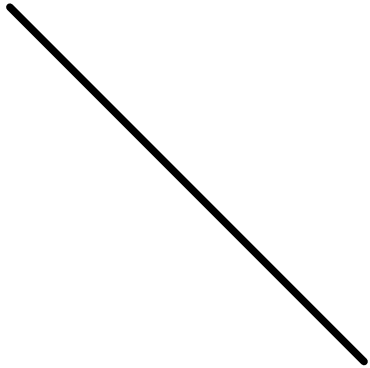
Aperture Problem

Now the full picture …



Aperture Problem

Now the full picture …



Aperture Problem

- Barber Pole: What is the motion field? What is the optic flow field?

Determining Optical Flow

[Horn & Schunck, Artificial Intelligence 1981]

Horn-Schunck Optical Flow

Horn-Schunck Model:

- Consider the image I as a function of continuous variables x, y, t
- Consider $u(x, y)$ and $v(x, y)$ as continuous flow fields
- Goal: Minimizing the following energy functional

$$
\begin{aligned}
E(u, v) &= \iint \underbrace{\left(I(x + u(x, y), y + v(x, y), t + 1) - I(x, y, t)\right)^2}_{\text{quadratic penalty for brightness change}} \\
&+ \lambda \cdot \underbrace{\left(\|\nabla u(x, y)\|^2 + \|\nabla v(x, y)\|^2\right)}_{\text{quadratic penalty for flow change}} \, dx \, dy
\end{aligned}
$$

with regularization parameter λ and $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)$.

Horn-Schunck Optical Flow

$$\begin{aligned} E(u,v) &= \iint \left(I(x+u(x,y),y+v(x,y),t+1) - I(x,y,t)\right)^2 \\ &+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2\right) dx\, dy \end{aligned}$$

- Minimizing this directly is a hard problem because the energy is highly non-convex and has many local optima
- Solution: linearize the brightness constancy assumption

$$\begin{aligned} E(u,v) &= \iint \left(I_x(x,y,t)u(x,y) + I_y(x,y,t)v(x,y) + I_t(x,y,t)\right)^2 \\ &+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2\right) dx\, dy \end{aligned}$$

Horn-Schunck Optical Flow

$$
\begin{aligned}
E(u,v) &= \iint \left(I_x(x,y,t)u(x,y) + I_y(x,y,t)v(x,y) + I_t(x,y,t)\right)^2 \\
&+ \lambda \cdot \left(\|\nabla u(x,y)\|^2 + \|\nabla v(x,y)\|^2\right) dx\, dy
\end{aligned}
$$

- Imposes quadratic penalty on optical flow constraint and gradient of flow field
- This energy is convex and thus has a unique optimum
- The flow can be estimated by discretizing it spatially
  and performing gradient descent on the discretized objective:

$$
\begin{aligned}
E(\mathbf{U},\mathbf{V}) &= \sum_{x,y} \left(I_x(x,y)\, u_{x,y} + I_y(x,y)\, v_{x,y} + I_t(x,y)\right)^2 \\
&+ \lambda \cdot \big((u_{x,y} - u_{x+1,y})^2 + (u_{x,y} - u_{x,y+1})^2 + \\
&\qquad (v_{x,y} - v_{x+1,y})^2 + (v_{x,y} - v_{x,y+1})^2\big)
\end{aligned}
$$

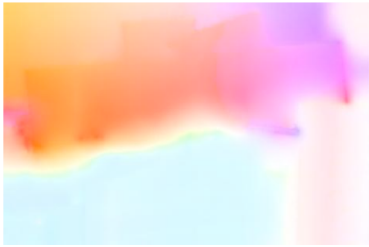
Horn-Schunck Optical Flow

Discretized Objective:

$$
\begin{aligned}
E(\mathbf{U},\mathbf{V}) &= \sum_{x,y} \left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)^2 \\
&+ \lambda \cdot \big((u_{x,y} - u_{x+1,y})^2 + (u_{x,y} - u_{x,y+1})^2 + \\
&\qquad (v_{x,y} - v_{x+1,y})^2 + (v_{x,y} - v_{x,y+1})^2\big)
\end{aligned}
$$

- Differentiate wrt. $\mathbf{U}, \mathbf{V}$ and set the gradient to 0
- Results in a huge (but sparse) linear system
- Can be solved using standard techniques (*e.g.*, Gauss-Seidel, SOR)
- What would happen for $\lambda = 0$?
- However: Linearization works only for small motions!
  - Coarse-to-fine estimation & warping

Results of Horn & Schunck



Horn & Schunck Optical Flow

- Our HS results are quite a bit better than the results using LK
- However, the flow is very smooth, *i.e.*, to overcome ambituities we need to set λ to a high value which oversmooths flow discontinuities
- Why?
- We use a quadratic penalty for penalizing changes in the flow

A Framework for Robust Estimation of Optical Flow

[Black & Anandan, ICCV 1993]

Probabilistic Interpretation

Optimization problem can be interpreted as MAP inference in MRF:

$$p(x) = \frac{1}{Z} \exp\{-E(x)\}$$

Gibbs energy: $E(\mathbf{U}, \mathbf{V}) = \sum_{x,y} \left(I_x(x,y)\, u_{x,y} + I_y(x,y)\, v_{x,y} + I_t(x,y)\right)^2$

$$+ \quad \lambda \cdot \big((u_{x,y} - u_{x+1,y})^2 + (u_{x,y} - u_{x,y+1})^2 + (v_{x,y} - v_{x+1,y})^2 + (v_{x,y} - v_{x,y+1})^2\big)$$

Gibbs distribution (prior and likelihood):

$$p(\mathbf{U}, \mathbf{V}) \propto \prod_{x,y} \exp\left\{-\left(I_x(x,y)\, u_{x,y} + I_y(x,y)\, v_{x,y} + I_t(x,y)\right)^2\right\}$$

$$\times \quad \exp\left\{-\lambda(u_{x,y} - u_{x+1,y})^2\right\} \times \exp\left\{-\lambda(u_{x,y} - u_{x,y+1})^2\right\}$$

$$\times \quad \exp\left\{-\lambda(v_{x,y} - v_{x+1,y})^2\right\} \times \exp\left\{-\lambda(v_{x,y} - v_{x,y+1})^2\right\}$$

Robust Regularization

- Quadratic penalties translate to Gaussian distributions for prior and likelihood:

$$\begin{aligned} p(\mathbf{U},\mathbf{V}) \quad &\propto \quad \prod_{x,y} \exp\left\{-\left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)^2\right\} \\ &\times \quad \exp\left\{-\lambda(u_{x,y} - u_{x+1,y})^2\right\} \times \exp\left\{-\lambda(u_{x,y} - u_{x,y+1})^2\right\} \\ &\times \quad \exp\left\{-\lambda(v_{x,y} - v_{x+1,y})^2\right\} \times \exp\left\{-\lambda(v_{x,y} - v_{x,y+1})^2\right\} \end{aligned}$$

- Both assumptions are invalid in practice (occlusions/discontinuities)
- Formulation with robust data term and smoothness penalties:

$$\begin{aligned} p(\mathbf{U},\mathbf{V}) \quad &\propto \quad \prod_{x,y} \exp\left\{-\rho_D\left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)\right\} \\ &\times \quad \exp\left\{-\lambda\,\rho_S(u_{x,y} - u_{x+1,y})\right\} \times \exp\left\{-\lambda\,\rho_S(u_{x,y} - u_{x,y+1})\right\} \\ &\times \quad \exp\left\{-\lambda\,\rho_S(v_{x,y} - v_{x+1,y})\right\} \times \exp\left\{-\lambda\,\rho_S(v_{x,y} - v_{x,y+1})\right\} \end{aligned}$$

Robust Regularization

Formulation with robust data term and smoothness penalties:

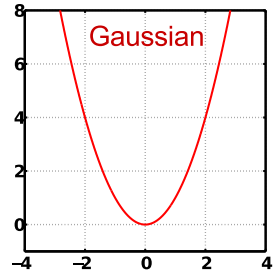
$$\begin{aligned} p(\mathbf{U},\mathbf{V}) \quad &\propto \quad \prod_{x,y} \exp\left\{-\rho_D\left(I_x(x,y)\,u_{x,y} + I_y(x,y)\,v_{x,y} + I_t(x,y)\right)\right\} \\ &\times \quad \exp\left\{-\lambda\,\rho_S(u_{x,y} - u_{x+1,y})\right\} \times \exp\left\{-\lambda\,\rho_S(u_{x,y} - u_{x,y+1})\right\} \\ &\times \quad \exp\left\{-\lambda\,\rho_S(v_{x,y} - v_{x+1,y})\right\} \times \exp\left\{-\lambda\,\rho_S(v_{x,y} - v_{x,y+1})\right\} \end{aligned}$$

- But how to choose $\rho_D(\cdot)$ and $\rho_S(\cdot)$?
- We want a prior that allows for discontinuities in the optical flow field and a likelihood that allows for occlusions in the photoconsistency term
- Thus, we need something more heavy-tailed than a Gaussian distribution,*e.g.*, a Student-t distribution (Lorentzian penalty):

$$p(x) \propto \left(1 + \frac{x^2}{2\sigma^2}\right)^{-\alpha} \quad \Rightarrow \quad \rho(x) = ? - \log\left(p(x)\right) = \alpha \log\left(1 + \frac{x^2}{2\sigma^2}\right)$$

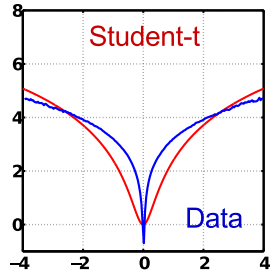
Robust Regularization

- Student-t distribution / Lorentzian penalty:



$$p(x) \propto \left(1 + \frac{x^2}{2\sigma^2}\right)^{-\alpha}$$

negative
log-density
(i.e. energy)



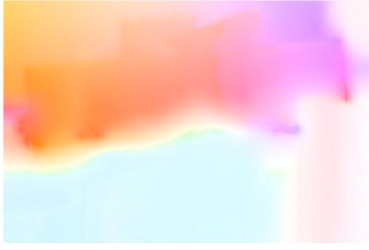
$$\rho(x) = \alpha \log\left(1 + \frac{x^2}{2\sigma^2}\right)$$

- Has been proposed in [Black-Anandan, ICCV 1993]
- How to estimate the parameters? Learn them from data!

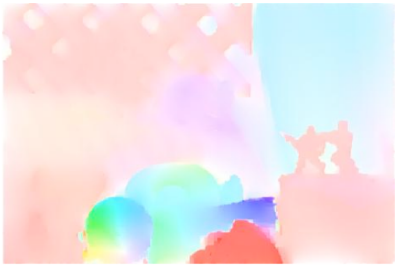
Robust Regularization

- How to obtain optical flow ground truth data?
- There is no sensor which can measure optical flow directly
- Synthesize optical flow fields using
  - A set of natural geometries (*e.g.*, Brown range database)
  - A set of natural camera motions

Result of [Horn-Schunck, AI 1981]



Results of [Black-Anandan, ICCV 1993]



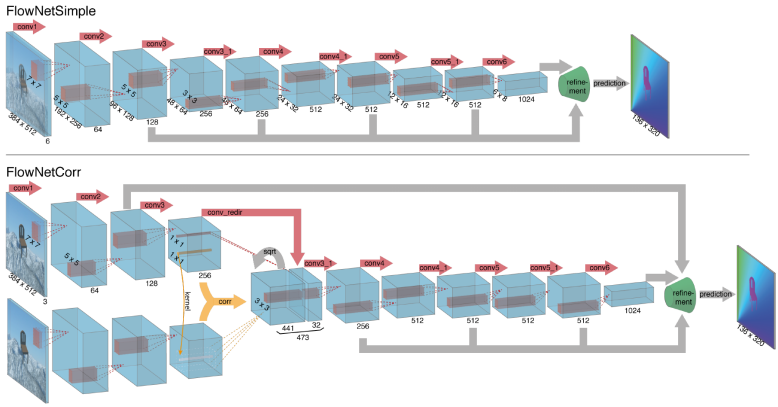
Results of [Sun *et al.*, CVPR 2010]

Deep Learning for Optical Flow Estimation

FlowNet: Learning Optical Flow with Convolutional Networks

[Dosovitskiy *et al.*, ICCV 2015]

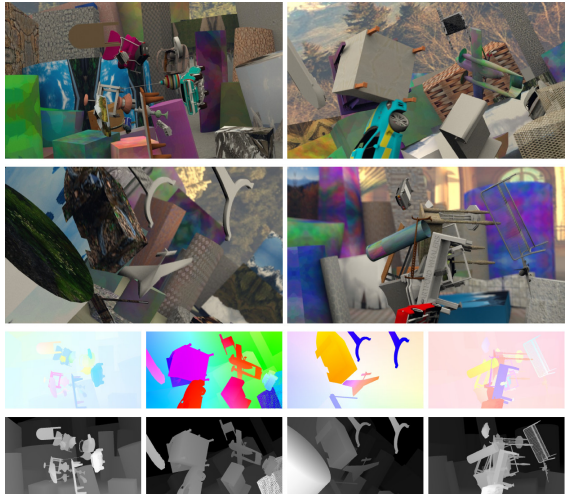
FlowNet



[Dosovitskiy *et al*., ICCV 2015]

- Encoder: Convolution with stride, decoder: Upconvolution, skip-connections
- Multi-scale loss (EPE in pixels), curriculum learning, synth. training data

FlowNet – Synthetic Datasets



Flying Things

FlowNet – Synthetic Datasets

Monkaa

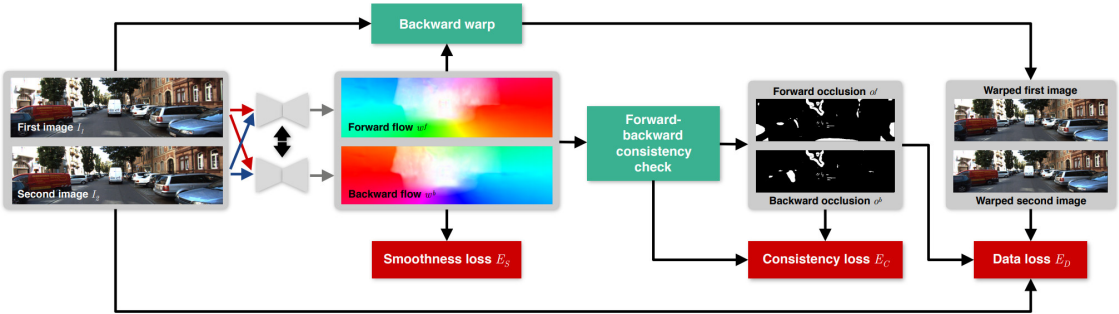
FlowNet – Synthetic Datasets

MPI Sintel

UnFlow: Unsupervised Learning of Optical Flow with a Bidirectional Census Loss

[Meister, Hur & Roth, AAAI 2018]

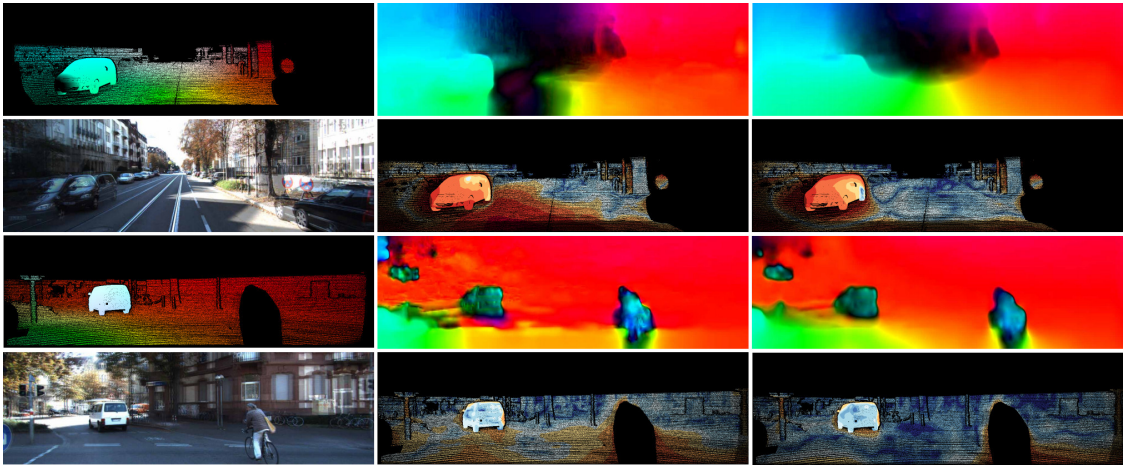
UnFlow: Unsupervised Optical Flow



[Meister *et al.*, AAAI 2018]

- Learn optical flow without supervision by warping images into the other frame
- Recycles photometric/smoothness terms as loss functions for NN training

UnFlow: Unsupervised Optical Flow – Results



Optical Flow Summary

- Classical OF approaches state-of-the-art until 2016
- DL based methods on par or better since 2017
- But require
  - Big models
  - Enormous amount of (synthetic) training data
  - GPU compute time
  - Sophisticated curriculum learning schedules
- Classical methods still best on related scene flow task
- Top performing DL methods borrow many elements from classical methods (*e.g.*, warping, cost volume, coarse-to-fine estimation, loss functions)

Questions?