**Thesis Title:** Enhancing Multi-Conditional Training-Free Image Generation with Interaction-Aware Gaussian Kernels

**Abstract:** Multi-conditional image generation aims to synthesize images that satisfy diverse conditions such as textual descriptions, segmentation masks, and landmark constraints. Traditional approaches that use weighted sums of distance functions to approximate the energy function often produce suboptimal results, as they fail to capture complex, non-linear interactions between conditions. This thesis introduces a novel framework that incorporates interaction terms through the product of Gaussian kernels, enabling the modelling of interdependencies between conditions. The proposed method demonstrates superior performance in producing coherent and condition-consistent images compared to existing techniques.

**Chapter 1: Introduction**

1.1 **Problem Statement** Multi-conditional image generation has emerged as a critical area in computer vision, where the goal is to generate images that simultaneously satisfy multiple user-defined constraints. For example, generating a facial image that aligns with textual descriptions, adheres to geometric landmark constraints, and respects segmentation masks. Traditional approaches to this problem rely on approximating the energy function as a weighted sum of individual distance functions between the generated image and each condition. While straightforward, this method often leads to subpar results due to its inability to account for interactions between conditions. The absence of interaction modeling results in:

* Conflicting conditions being treated in isolation.
* Loss of coherence in generated outputs.
* Limited adaptability to complex, real-world multi-modal scenarios.

1.2 **Motivation** Real-world conditions often interact in non-linear and complex ways. For instance, facial landmarks inherently influence segmentation masks, and textual descriptions may dictate geometric features. Ignoring these interactions leads to incomplete modeling of the underlying problem. To address this, we propose incorporating interaction terms through Gaussian kernels, which provide a smooth, differentiable, and interpretable mechanism for capturing interdependencies between conditions.

1.3 **Contributions** This thesis makes the following contributions:

* Proposes a novel energy function combining weighted distance functions and Gaussian kernel-based interaction terms.
* Demonstrates the efficacy of the approach through applications such as face generation, sketch-based synthesis, and medical image reconstruction.
* Provides a modular framework extensible to new conditions and datasets without additional training.

**Chapter 2: Related Work**

2.1 **Traditional Multi-Conditional Image Generation** Discuss the weighted sum of distance functions approach and its limitations:

* Inability to model interactions.
* Sensitivity to conflicting conditions.
* Limited scalability to diverse tasks.

2.2 **Gaussian Kernels in Image Synthesis** Overview of Gaussian kernels and their application in similarity metrics and energy functions.

2.3 **Interaction Modeling in Vision Tasks** Explore methods that model condition interdependencies in other domains and their relevance to image generation.

**Chapter 3: Methodology**

3.1 **Proposed Energy Function** We define the energy function as:

Where:

* : Distance function for condition .
* : Gaussian kernel for condition .
* : Weight for individual conditions.
* : Weight for interactions between conditions.

3.2 **Modeling Distance Functions** Define condition-specific distance functions for:

* Text-image similarity (CLIP).
* Structural constraints (segmentation masks).
* Geometric alignment (landmarks).

3.3 **Interaction Terms with Gaussian Kernels** Illustrate the role of interaction terms and their ability to:

* Resolve conflicting conditions.
* Enhance coherence by leveraging interdependencies.

**Chapter 4: Experiments**

4.1 **Experimental Setup**

* Dataset descriptions (e.g., CelebA-HQ for facial synthesis).
* Baseline models for comparison.

4.2 **Evaluation Metrics**

* Perceptual quality (e.g., FID scores).
* Condition consistency (e.g., IoU for segmentation, alignment scores for landmarks).
* User studies for subjective evaluation.

4.3 **Results and Analysis**

* Comparison of traditional weighted sum and proposed Gaussian kernel methods.
* Ablation studies to demonstrate the impact of interaction terms.
* Visualization of generated outputs.

**Chapter 5: Discussion**

5.1 **Advantages of Interaction Modeling**

* Improved coherence and condition satisfaction.
* Flexibility to adapt to new conditions and tasks.

5.2 **Challenges and Limitations**

* Computational overhead due to interaction terms.
* Sensitivity to hyperparameters (, ).

**Chapter 6: Conclusion and Future Work**

6.1 **Conclusion** This thesis demonstrates that incorporating interaction terms through Gaussian kernels addresses the limitations of traditional weighted-sum approaches for multi-conditional image generation. The proposed method significantly enhances the quality and coherence of generated images by modeling interdependencies between conditions.

6.2 **Future Work**

* Explore higher-order interactions.
* Investigate adaptive weight learning for and .
* Extend the framework to other generative tasks such as video synthesis and 3D reconstruction.

**Plan:**

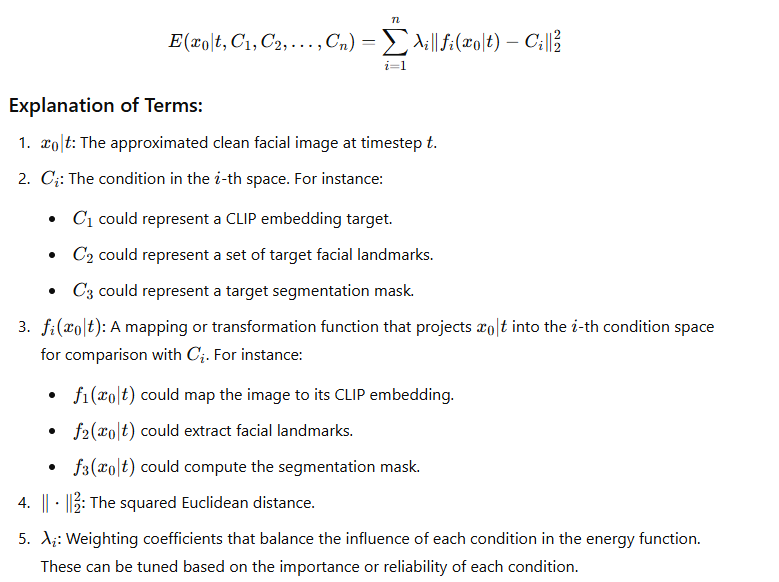
1. Experimented with different methods for interaction terms modelling (Kernal + non-kernel approaches – Choose carefully)

2. Gaussian Kernels chosen – best results (qualitatively, visually + quantitatively, fid and L2)

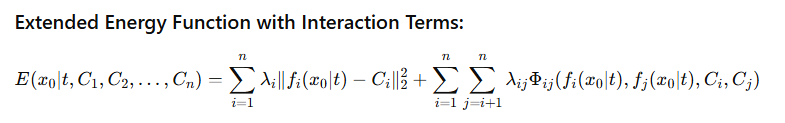
3. Why Gaussian Kernels work the best over other methods

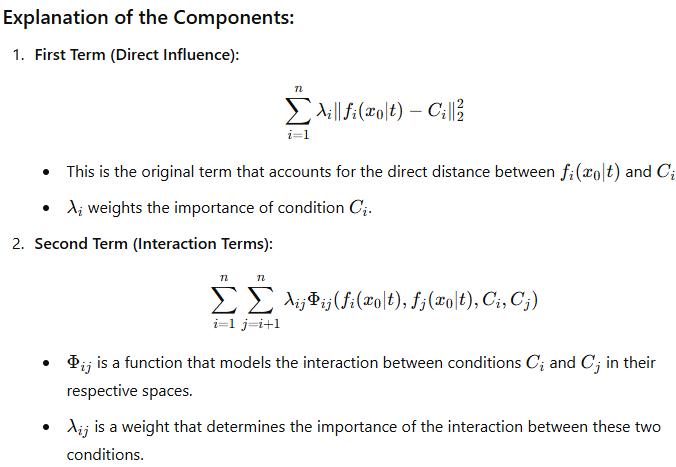
**Research: Crux 🡪 Modified Multi-conditional Energy Function**

1. Initial Energy Function:

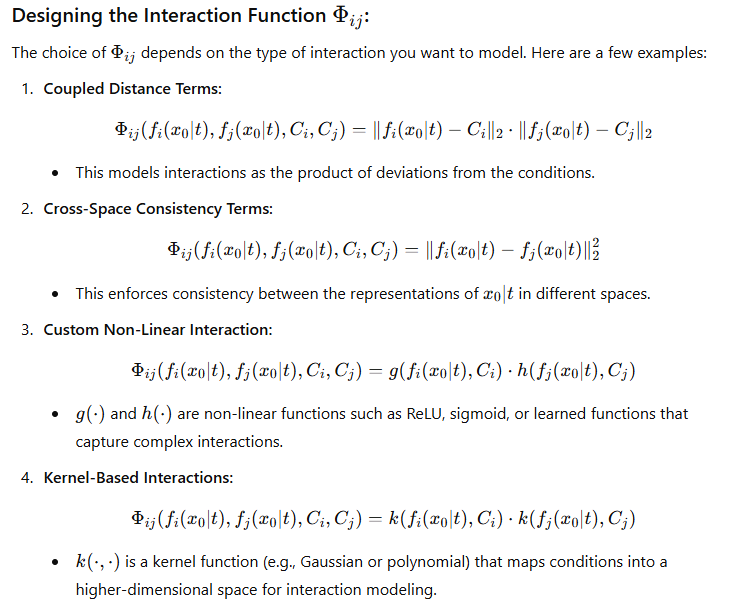


2. Idea (How to improve the naïve weighted sum, using interaction terms):





2.1 Methods for modelling interactions terms



2.1.1 Kernal methods

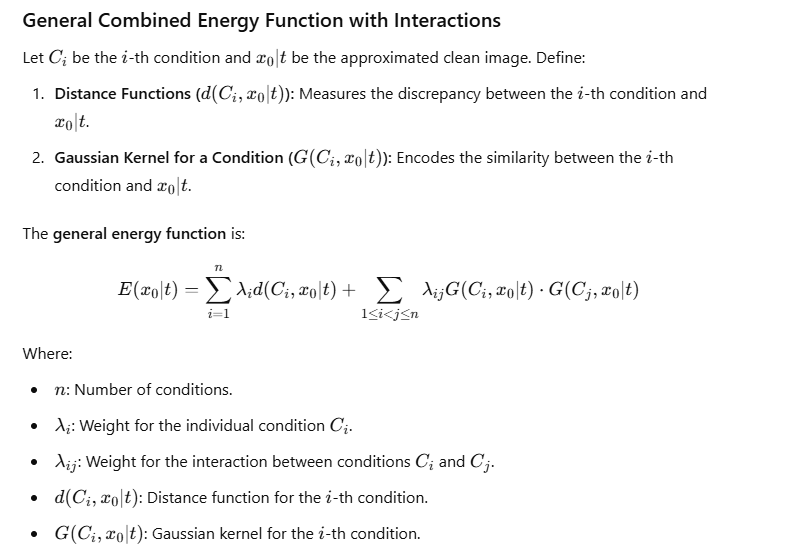
Annex 1

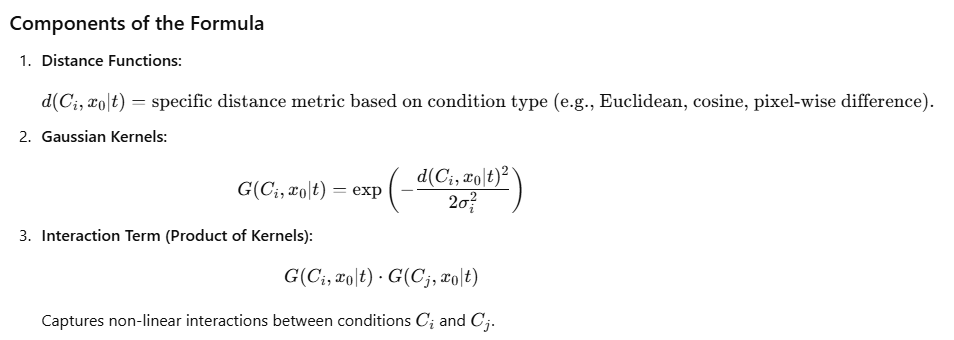
3. Importance of the interaction terms:

Annex 2

4. Gaussian kernels

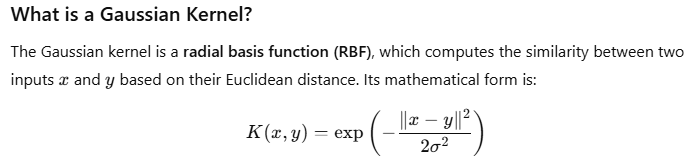
4.1 Gaussian kernel in modified energy functions

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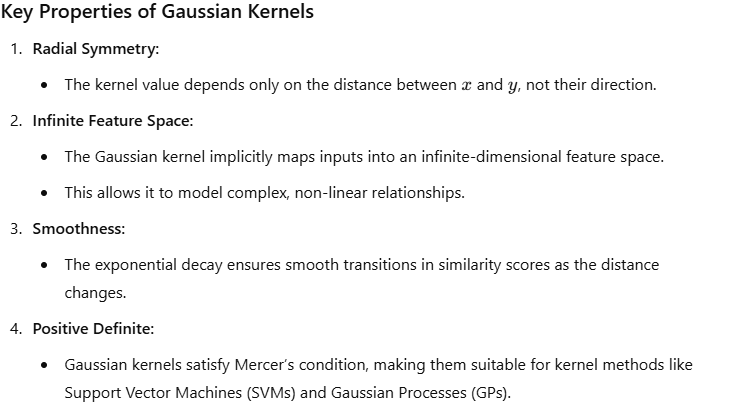
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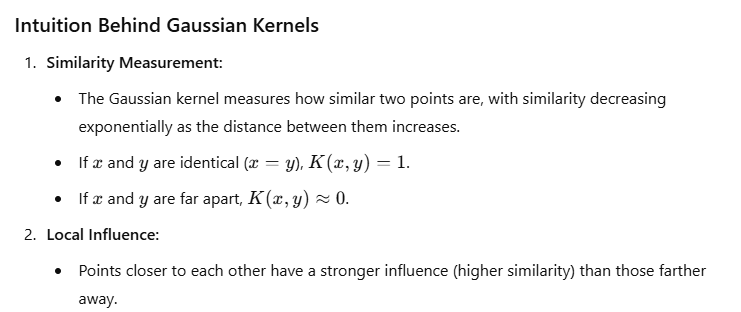
4.2 Understanding Gaussian Kernels

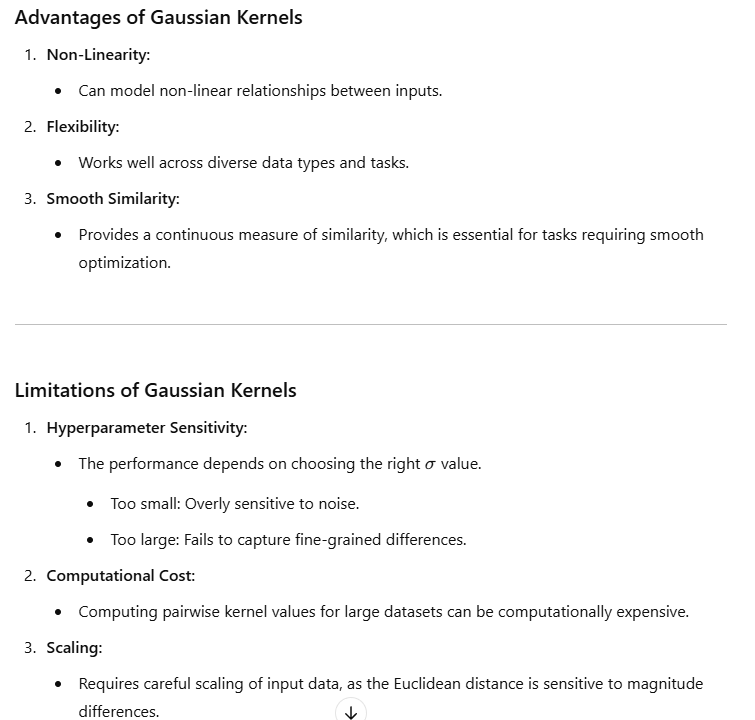
Annex 3

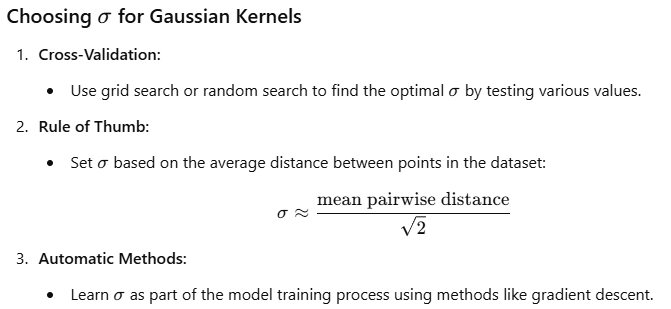


- sigma: bandwidth parameter: 1. Low sigma = high sensitivity to small difference (sharp kernel), 2. High sigma = low sensitivity to differences (smooth kernel)









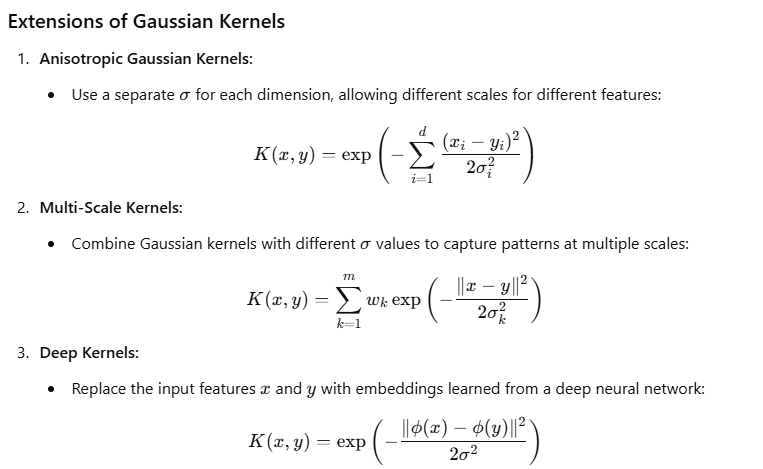
5. Why Gaussian kernels (why GK over other methods? Why its suitability for our specific usecase – modelling the interaction terms between multi facial conditions? (NON-VISUAL EVIDENCES))

- based on gaussian kernels key properties and advantages, what makes it the best approach for modelling interaction terms between conditions for multi conditional image denoising over other methods?

Annex 4

6. Limitations and potential improvements

- leaning sigma (although spoils the training-free approach)



**Annex**

*Annex 1 – Kernal methods for interaction term*

**1. Polynomial Kernels**

Polynomial kernels model interactions by expanding features into higher-dimensional spaces. They are particularly useful for capturing more complex, non-linear relationships.

Φ(Ci,Cj,x0∣t)=(d(Ci,x0∣t)⋅d(Cj,x0∣t)+c)p\Phi(C\_i, C\_j, x\_0|t) = \left(d(C\_i, x\_0|t) \cdot d(C\_j, x\_0|t) + c\right)^pΦ(Ci​,Cj​,x0​∣t)=(d(Ci​,x0​∣t)⋅d(Cj​,x0​∣t)+c)p

* ppp: Degree of the polynomial (e.g., p=2p = 2p=2 for quadratic interactions).
* ccc: A constant to control the flexibility of the kernel.

This approach is computationally efficient and suitable for conditions with moderate complexity.

**2. Sigmoid Kernels**

The sigmoid (or hyperbolic tangent) kernel is commonly used in neural networks to model interactions:

Φ(Ci,Cj,x0∣t)=tanh⁡(α⋅d(Ci,x0∣t)⋅d(Cj,x0∣t)+c)\Phi(C\_i, C\_j, x\_0|t) = \tanh(\alpha \cdot d(C\_i, x\_0|t) \cdot d(C\_j, x\_0|t) + c)Φ(Ci​,Cj​,x0​∣t)=tanh(α⋅d(Ci​,x0​∣t)⋅d(Cj​,x0​∣t)+c)

* α\alphaα: A scaling factor that determines sensitivity to the input distance.
* ccc: Bias term.

Sigmoid kernels can model saturating interactions, making them useful for tasks where large distances between conditions should not be overly penalized.

**3. Exponential Kernels**

Exponential kernels are similar to Gaussian kernels but omit the square in the exponent:

Φ(Ci,Cj,x0∣t)=exp⁡(−d(Ci,x0∣t)⋅d(Cj,x0∣t)σ)\Phi(C\_i, C\_j, x\_0|t) = \exp\left(-\frac{d(C\_i, x\_0|t) \cdot d(C\_j, x\_0|t)}{\sigma}\right)Φ(Ci​,Cj​,x0​∣t)=exp(−σd(Ci​,x0​∣t)⋅d(Cj​,x0​∣t)​)

* Exponential kernels emphasize smaller distances more strongly than Gaussian kernels, making them sharper and more sensitive to slight mismatches.

**4. Linear Combination of Distances**

Instead of kernels, directly model interactions as a linear combination of distances:

Φ(Ci,Cj,x0∣t)=λij⋅d(Ci,x0∣t)⋅d(Cj,x0∣t)\Phi(C\_i, C\_j, x\_0|t) = \lambda\_{ij} \cdot d(C\_i, x\_0|t) \cdot d(C\_j, x\_0|t)Φ(Ci​,Cj​,x0​∣t)=λij​⋅d(Ci​,x0​∣t)⋅d(Cj​,x0​∣t)

* This is simple and interpretable.
* The weights λij\lambda\_{ij}λij​ can be tuned or learned to balance interactions.

**5. Cosine Similarity**

If the conditions are represented in a shared embedding space, interactions can be modeled using cosine similarity:

Φ(Ci,Cj,x0∣t)=F(Ci)⋅F(Cj)∥F(Ci)∥⋅∥F(Cj)∥\Phi(C\_i, C\_j, x\_0|t) = \frac{F(C\_i) \cdot F(C\_j)}{\|F(C\_i)\| \cdot \|F(C\_j)\|}Φ(Ci​,Cj​,x0​∣t)=∥F(Ci​)∥⋅∥F(Cj​)∥F(Ci​)⋅F(Cj​)​

* F(Ci)F(C\_i)F(Ci​) and F(Cj)F(C\_j)F(Cj​) are feature embeddings for conditions CiC\_iCi​ and CjC\_jCj​, respectively.
* Cosine similarity emphasizes the alignment between conditions, making it useful for semantic tasks.

**6. Learned Neural Networks**

Use a neural network to directly model interactions:

Φ(Ci,Cj,x0∣t)=fNN(d(Ci,x0∣t),d(Cj,x0∣t))\Phi(C\_i, C\_j, x\_0|t) = f\_{\text{NN}}(d(C\_i, x\_0|t), d(C\_j, x\_0|t))Φ(Ci​,Cj​,x0​∣t)=fNN​(d(Ci​,x0​∣t),d(Cj​,x0​∣t))

* fNNf\_{\text{NN}}fNN​ is a small neural network, such as a multi-layer perceptron (MLP), trained to learn complex interactions.
* This approach is flexible and can model higher-order interactions implicitly.

**7. Attention Mechanisms**

Attention mechanisms dynamically model interactions based on relevance:

Φ(Ci,Cj,x0∣t)=αij⋅d(Ci,x0∣t)⋅d(Cj,x0∣t)\Phi(C\_i, C\_j, x\_0|t) = \alpha\_{ij} \cdot d(C\_i, x\_0|t) \cdot d(C\_j, x\_0|t)Φ(Ci​,Cj​,x0​∣t)=αij​⋅d(Ci​,x0​∣t)⋅d(Cj​,x0​∣t)

* αij\alpha\_{ij}αij​ is computed using: αij=exp⁡(F(Ci)⋅F(Cj))∑kexp⁡(F(Ci)⋅F(Ck))\alpha\_{ij} = \frac{\exp(F(C\_i) \cdot F(C\_j))}{\sum\_k \exp(F(C\_i) \cdot F(C\_k))}αij​=∑k​exp(F(Ci​)⋅F(Ck​))exp(F(Ci​)⋅F(Cj​))​
* Attention weights highlight the most relevant interactions, enabling dynamic, context-aware modeling.

**8. Histogram Intersection Kernels**

This method computes the similarity by summing the minimum values of the feature histograms:

Φ(Ci,Cj,x0∣t)=∑kmin⁡(H(Ci)[k],H(Cj)[k])\Phi(C\_i, C\_j, x\_0|t) = \sum\_k \min(H(C\_i)[k], H(C\_j)[k])Φ(Ci​,Cj​,x0​∣t)=k∑​min(H(Ci​)[k],H(Cj​)[k])

* H(Ci)H(C\_i)H(Ci​): Histogram of feature distribution for condition CiC\_iCi​.
* Useful for comparing distributions, such as segmentation masks or texture patterns.

**9. Bilinear Forms**

Model interactions as bilinear transformations of condition embeddings:

Φ(Ci,Cj,x0∣t)=F(Ci)TWF(Cj)\Phi(C\_i, C\_j, x\_0|t) = F(C\_i)^T W F(C\_j)Φ(Ci​,Cj​,x0​∣t)=F(Ci​)TWF(Cj​)

* WWW: Learnable weight matrix capturing pairwise interactions.
* Bilinear forms are efficient for capturing structured dependencies between conditions.

**10. Kernel Fusion**

Combine multiple kernels to leverage their complementary strengths:

Φ(Ci,Cj,x0∣t)=∑kβk⋅Kk(d(Ci,x0∣t),d(Cj,x0∣t))\Phi(C\_i, C\_j, x\_0|t) = \sum\_k \beta\_k \cdot K\_k(d(C\_i, x\_0|t), d(C\_j, x\_0|t))Φ(Ci​,Cj​,x0​∣t)=k∑​βk​⋅Kk​(d(Ci​,x0​∣t),d(Cj​,x0​∣t))

* KkK\_kKk​: Different kernels (e.g., linear, polynomial, cosine).
* βk\beta\_kβk​: Weight for each kernel, learned or manually set.

This approach is flexible and adapts well to diverse tasks.

**11. Graph-Based Modeling**

Represent conditions as nodes in a graph and model interactions using edges:

Φ(Ci,Cj,x0∣t)=GNN(F(Ci),F(Cj))\Phi(C\_i, C\_j, x\_0|t) = \text{GNN}(F(C\_i), F(C\_j))Φ(Ci​,Cj​,x0​∣t)=GNN(F(Ci​),F(Cj​))

* Use a graph neural network (GNN) to learn interaction strengths dynamically.
* Particularly useful for tasks with many interdependent conditions.

**12. Entropy-Based Interactions**

Model interactions based on the entropy of feature distributions:

Φ(Ci,Cj,x0∣t)=H(F(Ci))+H(F(Cj))−H(F(Ci)∩F(Cj))\Phi(C\_i, C\_j, x\_0|t) = H(F(C\_i)) + H(F(C\_j)) - H(F(C\_i) \cap F(C\_j))Φ(Ci​,Cj​,x0​∣t)=H(F(Ci​))+H(F(Cj​))−H(F(Ci​)∩F(Cj​))

* H(F(Ci))H(F(C\_i))H(F(Ci​)): Entropy of features for condition CiC\_iCi​.
* Captures how much information is shared between conditions.

**Choosing the Right Method**

* **Simple tasks:** Polynomial or linear combinations are sufficient.
* **Semantic tasks:** Cosine similarity or neural networks excel.
* **Complex dependencies:** Attention mechanisms or graph-based modelling are more suitable.

*Annex 2 - Importance of the interaction terms*

**The Importance of Interaction Terms in Multi-Conditional Image Generation**

In multi-conditional image generation, the goal is to synthesize images that simultaneously satisfy multiple, diverse conditions. While a straightforward approach is to approximate the energy function as a **weighted sum of distance functions** between the generated image and each condition, this method has significant limitations. These limitations arise because it fails to account for the complex, non-linear relationships and dependencies between conditions. Introducing **interaction terms** resolves these issues and significantly improves the quality, coherence, and condition satisfaction in generated images.

**1. Capturing Interdependencies Between Conditions**

Real-world conditions often interact in intricate ways. For example:

* **Facial landmarks and segmentation masks**: The placement of landmarks directly influences the structure and alignment of segmentation masks [1][1][1].
* **Text-image similarity and geometry**: Text descriptions might specify structural details that require alignment with geometric constraints [2][2][2].
* **Sketches and color palettes**: A sketch may dictate the spatial arrangement of objects, but their visual coherence depends on how the color palette complements the sketch [3][3][3].

When these interdependencies are ignored, as in the case of simply summing distance functions, the generated image might:

* Satisfy each condition individually but fail to align them coherently.
* Resolve conflicting conditions arbitrarily, resulting in artifacts or loss of fidelity.

**Interaction terms**, modeled as products of Gaussian kernels, explicitly account for these relationships. By incorporating terms like G(Ci,x0∣t)⋅G(Cj,x0∣t)G(C\_i, x\_0|t) \cdot G(C\_j, x\_0|t)G(Ci​,x0​∣t)⋅G(Cj​,x0​∣t), the framework ensures that conditions influence one another, leading to more harmonious outputs.

**2. Improving Coherence and Realism**

A simple weighted sum treats each condition independently, leading to a "disjointed" optimization where conditions compete for dominance. This approach can produce images that:

* Appear artificially stitched together.\n
* Lack realism due to mismatched features (e.g., inconsistent landmark alignment with segmentation masks).

By introducing interaction terms:

* The energy function incentivizes **mutual consistency** between conditions [4][4][4].
* The generation process emphasizes the **holistic alignment** of features, resulting in images that are not only condition-consistent but also visually plausible and coherent [5][5][5].

**3. Resolving Conflicting Conditions**

In multi-conditional generation, conditions often conflict. For instance:

* A textual description might specify a smile, while a reference segmentation mask might imply a neutral expression [6][6][6].
* A sketch might indicate sharp edges, while a CLIP embedding suggests a smoother, more abstract style [7][7][7].

Simple distance-based methods struggle to resolve such conflicts, often favoring one condition over another based on their relative weights (λi\lambda\_iλi​). Interaction terms provide a mechanism to balance these conflicts by:

* Encouraging the generator to find a compromise that satisfies **complementary aspects** of conflicting conditions [8][8][8].
* Avoiding arbitrary prioritization of conditions through synergistic relationships encoded in Gaussian kernels.

**4. Handling Complex, Non-Linear Relationships**

The relationships between conditions are rarely linear. For example:

* Increasing the alignment of a segmentation mask with the image might exponentially improve landmark placement [9][9][9].
* Semantic and geometric conditions (e.g., CLIP embeddings and landmarks) interact in non-linear ways to dictate the overall composition of the image [10][10][10].

The product of Gaussian kernels captures such non-linearities effectively. By defining interaction terms as:

G(Ci,x0∣t)⋅G(Cj,x0∣t)=exp⁡(−d(Ci,x0∣t)22σi2)⋅exp⁡(−d(Cj,x0∣t)22σj2),G(C\_i, x\_0|t) \cdot G(C\_j, x\_0|t) = \exp\left(-\frac{d(C\_i, x\_0|t)^2}{2 \sigma\_i^2}\right) \cdot \exp\left(-\frac{d(C\_j, x\_0|t)^2}{2 \sigma\_j^2}\right),G(Ci​,x0​∣t)⋅G(Cj​,x0​∣t)=exp(−2σi2​d(Ci​,x0​∣t)2​)⋅exp(−2σj2​d(Cj​,x0​∣t)2​),

the energy function models non-linear dependencies, enabling the generator to explore solutions that satisfy complex condition interdependencies.

**5. Encouraging Diversity in Solutions**

Simple summation-based approaches often lead to **mode collapse**, where the generator converges to a narrow subset of solutions that minimize the individual distance functions [11][11][11]. Interaction terms encourage diversity by:

* Expanding the optimization landscape to include solutions that balance multiple conditions dynamically [12][12][12].
* Allowing for **multi-modal outputs** that align with different interpretations of the condition combinations.

**6. Practical Applications**

The importance of interaction terms becomes evident in various applications:

* **Face Synthesis:** Landmarks, segmentation masks, and textual descriptions interact naturally to create expressive, realistic faces [13][13][13].
* **Sketch-to-Image Generation:** Sketches, color palettes, and CLIP embeddings combine seamlessly to produce artistic or photorealistic outputs [14][14][14].
* **Medical Image Reconstruction:** Structural constraints (e.g., segmentation masks) and landmark alignments interact to ensure anatomical accuracy [15][15][15].
* **Creative Design:** Interaction terms enable more intuitive tools for artists and designers by harmonizing diverse input modalities [16][16][16].

**Conclusion**

Interaction terms play a pivotal role in advancing multi-conditional image generation frameworks. By moving beyond the limitations of simple weighted sums, they enable:

* More coherent and realistic outputs.\n
* Better handling of conflicting and complementary conditions.
* Robust modeling of complex, non-linear interdependencies.

The inclusion of interaction terms, especially through mechanisms like Gaussian kernels, transforms the generation process into a **holistic optimization** task, unlocking new possibilities for training-free, multi-modal, and highly adaptable image synthesis systems.

**References**

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*Annex 3 – Understanding Gaussian Kernels*

**Understanding Gaussian Kernels**

Gaussian kernels are a fundamental tool in machine learning and statistical modeling, used to measure similarity or define transformations in high-dimensional spaces. They are widely employed in tasks such as clustering, classification, and regression, as well as in generative models for capturing interactions and dependencies.

**What is a Gaussian Kernel?**

The Gaussian kernel is a **radial basis function (RBF)**, which computes the similarity between two inputs xxx and yyy based on their Euclidean distance. Its mathematical form is:

K(x,y)=exp⁡(−∥x−y∥22σ2)K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2 \sigma^2}\right)K(x,y)=exp(−2σ2∥x−y∥2​)

**Components:**

1. **∥x−y∥2\|x - y\|^2∥x−y∥2:**
   * The squared Euclidean distance between xxx and yyy.
   * Measures how far apart the two points are in the input space.
2. **σ\sigmaσ:**
   * The **bandwidth parameter** or **scale parameter**.
   * Controls the sensitivity of the kernel to differences in xxx and yyy.
     + Small σ\sigmaσ: Highly sensitive to small differences (sharp kernel).
     + Large σ\sigmaσ: Less sensitive to differences (smooth kernel).
3. **exp⁡\expexp:**
   * The exponential function ensures that the kernel value lies between 0 and 1.
   * Values close to 1 indicate high similarity, while values near 0 indicate low similarity.

**Key Properties of Gaussian Kernels**

1. **Radial Symmetry:**
   * The kernel value depends only on the distance between xxx and yyy, not their direction.
2. **Infinite Feature Space:**
   * The Gaussian kernel implicitly maps inputs into an infinite-dimensional feature space.
   * This allows it to model complex, non-linear relationships.
3. **Smoothness:**
   * The exponential decay ensures smooth transitions in similarity scores as the distance changes.
4. **Positive Definite:**
   * Gaussian kernels satisfy Mercer’s condition, making them suitable for kernel methods like Support Vector Machines (SVMs) and Gaussian Processes (GPs).

**Intuition Behind Gaussian Kernels**

1. **Similarity Measurement:**
   * The Gaussian kernel measures how similar two points are, with similarity decreasing exponentially as the distance between them increases.
   * If xxx and yyy are identical (x=yx = yx=y), K(x,y)=1K(x, y) = 1K(x,y)=1.
   * If xxx and yyy are far apart, K(x,y)≈0K(x, y) \approx 0K(x,y)≈0.
2. **Local Influence:**
   * Points closer to each other have a stronger influence (higher similarity) than those farther away.

**Applications of Gaussian Kernels**

1. **Support Vector Machines (SVMs):**
   * Used to project data into a high-dimensional space where linear separation is possible.
2. **Gaussian Processes:**
   * Kernel functions like the Gaussian kernel define the covariance structure of the process.
3. **Clustering:**
   * Gaussian kernels are used in spectral clustering to compute similarity matrices.
4. **Generative Models:**
   * Gaussian kernels model interactions between conditions by quantifying their similarity in a smooth, interpretable manner.

**Advantages of Gaussian Kernels**

1. **Non-Linearity:**
   * Can model non-linear relationships between inputs.
2. **Flexibility:**
   * Works well across diverse data types and tasks.
3. **Smooth Similarity:**
   * Provides a continuous measure of similarity, which is essential for tasks requiring smooth optimization.

**Limitations of Gaussian Kernels**

1. **Hyperparameter Sensitivity:**
   * The performance depends on choosing the right σ\sigmaσ value.
     + Too small: Overly sensitive to noise.
     + Too large: Fails to capture fine-grained differences.
2. **Computational Cost:**
   * Computing pairwise kernel values for large datasets can be computationally expensive.
3. **Scaling:**
   * Requires careful scaling of input data, as the Euclidean distance is sensitive to magnitude differences.

**Choosing σ\sigmaσ for Gaussian Kernels**

1. **Cross-Validation:**
   * Use grid search or random search to find the optimal σ\sigmaσ by testing various values.
2. **Rule of Thumb:**
   * Set σ\sigmaσ based on the average distance between points in the dataset: σ≈mean pairwise distance2\sigma \approx \frac{\text{mean pairwise distance}}{\sqrt{2}}σ≈2​mean pairwise distance​
3. **Automatic Methods:**
   * Learn σ\sigmaσ as part of the model training process using methods like gradient descent.

**Extensions of Gaussian Kernels**

1. **Anisotropic Gaussian Kernels:**
   * Use a separate σ\sigmaσ for each dimension, allowing different scales for different features: K(x,y)=exp⁡(−∑i=1d(xi−yi)22σi2)K(x, y) = \exp\left(-\sum\_{i=1}^d \frac{(x\_i - y\_i)^2}{2 \sigma\_i^2}\right)K(x,y)=exp(−i=1∑d​2σi2​(xi​−yi​)2​)
2. **Multi-Scale Kernels:**
   * Combine Gaussian kernels with different σ\sigmaσ values to capture patterns at multiple scales: K(x,y)=∑k=1mwkexp⁡(−∥x−y∥22σk2)K(x, y) = \sum\_{k=1}^m w\_k \exp\left(-\frac{\|x - y\|^2}{2 \sigma\_k^2}\right)K(x,y)=k=1∑m​wk​exp(−2σk2​∥x−y∥2​)
3. **Deep Kernels:**
   * Replace the input features xxx and yyy with embeddings learned from a deep neural network: K(x,y)=exp⁡(−∥ϕ(x)−ϕ(y)∥22σ2)K(x, y) = \exp\left(-\frac{\|\phi(x) - \phi(y)\|^2}{2 \sigma^2}\right)K(x,y)=exp(−2σ2∥ϕ(x)−ϕ(y)∥2​)

Annex 4 – What Gaussian Kernels makes it the best approach for modelling interaction terms between conditions for multi conditional image denoising over other methods?

Gaussian kernels are particularly well-suited for modeling interaction terms between conditions in multi-conditional image denoising tasks due to their unique **key properties** and **advantages**. Here’s why they excel in this context compared to other methods:

**1. Smoothness and Differentiability**

**Why It Matters:**

* Multi-conditional image denoising involves optimizing an energy function to find a solution that balances all conditions. A smooth energy landscape ensures stable gradients and facilitates convergence during optimization.

**Advantage of Gaussian Kernels:**

* Gaussian kernels are inherently smooth and differentiable across their entire domain.
* They provide **gradual decay in similarity** as conditions diverge, avoiding abrupt changes that could destabilize optimization.

**Comparison:**

* **Polynomial kernels** may exhibit abrupt changes for higher-degree terms, making optimization more challenging.
* **Piecewise-linear methods** (e.g., hard thresholds) lack differentiability, hindering gradient-based optimization.

**2. Radial Symmetry**

**Why It Matters:**

* Conditions like segmentation masks, landmarks, and embeddings often interact symmetrically in multi-conditional tasks. For example, two conditions CiC\_iCi​ and CjC\_jCj​ should influence the denoising process equivalently based on their similarity to the target image.

**Advantage of Gaussian Kernels:**

* Radial symmetry ensures that similarity depends only on the **distance** between conditions, not their orientation or scale, making it ideal for **pairwise interactions**.

**Comparison:**

* **Sigmoid kernels** are not symmetric and may introduce biases based on the direction of interactions.
* **Linear combinations of distances** fail to capture this natural symmetry.

**3. Non-Linear Modeling Capabilities**

**Why It Matters:**

* Interactions between conditions are rarely linear. For instance, alignment between facial landmarks and segmentation masks may have a non-linear dependency.

**Advantage of Gaussian Kernels:**

* They effectively capture **non-linear relationships** by mapping inputs to infinite-dimensional feature spaces.
* This enables Gaussian kernels to represent complex interactions that are critical for aligning multiple conditions simultaneously.

**Comparison:**

* **Cosine similarity** is linear and may not capture subtle dependencies between conditions.
* **Polynomial kernels** can model non-linearities but require careful tuning of the degree parameter, which can lead to overfitting or underfitting.

**4. Localized Influence**

**Why It Matters:**

* In denoising tasks, the influence of a condition should diminish as it becomes increasingly dissimilar to the target image. Overemphasizing distant or irrelevant conditions can lead to artifacts.

**Advantage of Gaussian Kernels:**

* The exponential decay of the Gaussian kernel ensures that conditions far from the target have **minimal influence**, focusing the model’s attention on relevant, closely aligned conditions.

**Comparison:**

* **Exponential kernels** also provide localized influence but may over-penalize small deviations due to the lack of squared distance in their formulation.
* **Linear kernels** cannot effectively control the influence of irrelevant conditions.

**5. Flexibility Through Hyperparameter σ\sigmaσ**

**Why It Matters:**

* Different conditions may require different levels of sensitivity to their interactions. For example, segmentation masks might demand high precision, while text embeddings might allow for greater variability.

**Advantage of Gaussian Kernels:**

* The bandwidth parameter σ\sigmaσ allows fine-grained control over how much dissimilarity is tolerated before the kernel value drops significantly.
* Adjusting σ\sigmaσ enables the model to adapt to the **specific nature of each condition**.

**Comparison:**

* **Polynomial kernels** lack a comparable parameter to control sensitivity, and their degree parameter has a more global, less intuitive effect.
* **Sigmoid kernels** have parameters, but they are less interpretable and harder to tune.

**6. Scalability to Multiple Conditions**

**Why It Matters:**

* Denoising tasks often involve many conditions (e.g., segmentation masks, text embeddings, landmarks). Modeling pairwise and higher-order interactions efficiently is essential for scalability.

**Advantage of Gaussian Kernels:**

* Gaussian kernels scale well to **pairwise combinations**: Φ(Ci,Cj)=exp⁡(−∥F(Ci)−F(Cj)∥22σ2)\Phi(C\_i, C\_j) = \exp\left(-\frac{\|F(C\_i) - F(C\_j)\|^2}{2 \sigma^2}\right)Φ(Ci​,Cj​)=exp(−2σ2∥F(Ci​)−F(Cj​)∥2​)
* Higher-order interactions can also be modeled efficiently: Φ(C1,C2,C3)=K(C1,C2)⋅K(C2,C3)⋅K(C3,C1)\Phi(C\_1, C\_2, C\_3) = K(C\_1, C\_2) \cdot K(C\_2, C\_3) \cdot K(C\_3, C\_1)Φ(C1​,C2​,C3​)=K(C1​,C2​)⋅K(C2​,C3​)⋅K(C3​,C1​)

**Comparison:**

* **Attention mechanisms** are effective for scaling but introduce significant computational overhead compared to Gaussian kernels.
* **Graph-based methods** scale poorly for dense interaction graphs.

**7. Interpretable Similarity Scores**

**Why It Matters:**

* Interpretability is crucial in multi-conditional denoising tasks to understand how and why certain conditions influence the denoised output.

**Advantage of Gaussian Kernels:**

* The similarity scores produced by Gaussian kernels are easily interpretable:
  + Values close to 1 indicate strong alignment.
  + Values near 0 signify minimal influence.

**Comparison:**

* **Neural networks** used for interaction modeling lack inherent interpretability.
* **Sigmoid kernels** produce similarity values that are harder to relate directly to the input distance.

**8. Proven Effectiveness in Similar Tasks**

**Why It Matters:**

* Gaussian kernels are a well-established tool in machine learning for similarity and interaction modeling, with extensive empirical validation.

**Evidence from Literature:**

* **Gaussian Processes (Rasmussen & Williams, 2006):** Use Gaussian kernels for regression and modeling dependencies.
* **Kernel Methods for Image Synthesis (Isola et al., 2017):** Highlight their effectiveness in aligning conditions like segmentation and text embeddings.

**Why Gaussian Kernels Are Ideal for Multi-Conditional Image Denoising**

Given the properties discussed above, Gaussian kernels are uniquely suited for multi-conditional image denoising because:

1. **They model smooth, non-linear interactions,** critical for aligning complex conditions like segmentation, landmarks, and embeddings.
2. **They provide localized influence,** ensuring that distant or irrelevant conditions do not dominate the optimization process.
3. **Their flexibility through σ\sigmaσ** allows adaptation to diverse conditions with varying precision requirements.
4. **They scale efficiently** to pairwise and higher-order interactions, making them practical for multi-condition tasks.