Simulation test scenarios – TB

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Contents

1 Procedures

1.1 To buy an ask order `BUY`

Provided that the asked price r_A is lower than r_S, the price registered in the source wallet w_S, the `BUY` procedure has five steps:

1. `BUY_1`: the balance in w_S is converted to source coin: $b_S \times r_S$;
2. `BUY_2`: That balance is then converted to target, according to the asked rate
$$\frac{b_S \times r_S}{r_A};$$
3. `BUY_3`: the purchase, or the amount to be bought, is the minimum among what can is in the wallet and what is being sold:
$$p_T = min\left(\frac{b_S \times r_S}{r_A}, a_T\right);$$
4. `BUY_4`: the purchased amount is subtracted from the balance in the source wallet w_S:
$$a'_S = a_S - p_T\frac{r_A}{r_S};$$
The ratio r_A/r_S takes into account that the source coin in the w_S corresponds to a higher valued target coin.
5. `BUY_5`: the purchased amount is added to the balance in the target wallet w_T:
$$a'_T = a_T + P_T.$$

OBS: In a real exchange situation, the p_T amount is reserved in the source wallet and is only added to the target wallet when a real sale happens: there can be a situation when oner or more faster buyers get all the ask offer, or part of it. In that case, the reserved amount (or part of it) can be returned to the normal operation of the source wallet.

1.2 To sell for a bid order `SELL`

Provided that the asked price r_B is higher than r_T, the price registered in the target wallet w_T, the `SELL` procedure has three steps:

- `SELL_1`: the transfer value, or the amount to be sold, is the minimum among what can is in the target wallet and what the bid offer wants to buy:
$$t_T = min(a_T, b_T);$$
- `SELL_2`: the transfer amount is subtracted from the balance in the target wallet w_T:
$$a'_T = a_T - t_T;$$
- `SELL_3`: the purchased amount is added to the balance in the source wallet w_S:
$$a'_S = a_S + t_T.$$

OBS: In a real exchange situation, the t_T amount is reserved in the target wallet and is only added to the source wallet when a real sale happens: there can be a situation when one or more faster sellers get all the bid offer, or part of it. In that case, the reserved amount (or part of it) can be returned to the normal operation of the source wallet.

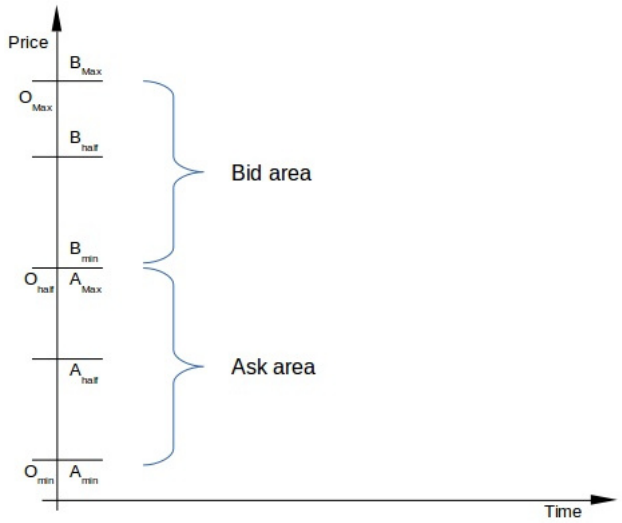

Figure 1: Graphical description of scenario *Simple1*

2 Simple 1: everything constant

2.1 Conditions

- All orders in the range $[O_{min}, O_{Max}]$;
- Middle order at $O_{half} = (O_{min} + O_{Max})/2$;
- All orders have amount a;
- Ask orders belong to $[A_{min}, A_{Max}]$, where $A_{min} = O_{min}$, and $A_{Max} = O_{half}$.
  In this scenario, all ask orders have price $r_A = A_{half} = (A_{min} + A_{Max})/2$.
- Bid orders belong to $[B_{min}, B_{Max}]$, where $B_{min} = O_{half}$, and $B_{Max} = O_{Max}$.
  In this scenario, all bid orders have price $r_B = B_{half} = (B_{min} + B_{Max})/2$.
- There is a wallet for target coin, and and another for source coin.

  Both wallets are considered in terms of the target coin, even though the source coin wallet in reality has source coin values. That eases the calculations, and is in line with the common practice in the exchanges, where ask and bid orders are given in terms of the target coin only.

  The values in the source coin are associated with a conversion rate for the target coin, what permits that;
- The wallet for *target* coin w_T has an initial amount $a_t = a$, and a constant rate

$$r_T = A_{half} = r_A;$$

- The wallet for *source* coin w_S has an initial amount $a_S = a$, and a constant rate

$$r_S = B_{half} = r_B;$$

- The simulation starts with an ask order;
- It is succeeded by a bid order, and then by an ask order – and so on indefinitely: an ask then a bid, then an ask...

2.2 Alternative 2.1 – 1st Ask

Here follow the orders:

2.2.1 Ask order # 1

When the 1st **ask** order arrives with rate A_{half}, and an amount a, the *source* wallet has an amount a, and a buy price of $r_S = B_{half}$. Since the buy rate r_S is above the ask rate A_{half}, the target coin will be bought.

- In the `BUY_1` (step 1): the balance is converted to source coin: $b_S \times r_S = a \times r_S$;
- Following the `BUY_2` step (item 2): the balance in source coin is converted to target coin using the r_A rate:
$$\frac{a \times r_S}{r_A} = a\frac{r_S}{r_A}$$
- Following step `BUY_3` (item 3): Since $r_S > r_A$, the expression from `BUY_1` is larger than a, the amount in the ask order. So, the value to be bought is simply a;
- Following step `BUY_4` (item 4): Since a was used, it is subtracted from the previous balance in the wallet w_S:
$$a\frac{r_S}{r_A} - a = a\left(\frac{r_S}{r_A} - \frac{r_A}{r_A}\right) = a\frac{r_S - r_A}{r_A}.$$
- Following step `BUY_5` (item 5): the purchased amount a is added to the balance in the target wallet w_T. Since it was a it becomes
$$a'_T = a_T + P_T = a + a = 2a.$$

2.2.2 Bid order # 1

Now comes a **bid** order that wants to buy an amount a of **target** coin, at a rate B_{half}. Since the sale rate $r_T = A_{half}$ is below the bid rate, the target coin will be sold. That is: the target will be sold by a price higher than it was bought.

- In the `SELL_1` (step 1.2): The bid offers will buy a target coins. The target coin has $2a$ – so the minimum is a;
- Following the step `SELL_2` (step 1.2), the minimum computed in the previous item will be subtracted from the target wallet w_s:
$$a'_T = 2a - a = a.$$
- according to `SELL_3` (item 1.2): the amount transfered from the sale is added to the balance in the source wallet w_S:

$$\begin{aligned} a'_S = a_S + t_T &= a\frac{r_S - r_A}{r_A} + a \\ &= a\frac{r_S - r_A}{r_A} + a\frac{r_A}{r_A} \\ &= a\left(\frac{r_S - r_A + r_A}{r_A}\right) \\ &= a\frac{r_S}{r_A} \\ &= a\frac{r_B}{r_A}. \end{aligned}$$

Since $r_S = r_B > r_A$, the balance in the source wallet is larger than a.

2.2.3 Ask order # 2

Then comes another **ask** order, and it has price A_{half}, and an amount a. Since the buy price $r_S = B_{half}$ is higher than A_{half}, the target coin will be bought.

- Following step `BUY_1` (item 1): converting the balance w_S to source coin:

$$a\frac{r_B}{r_A} \times r_S = \frac{r_B^2}{r_A},$$

since $r_S = r_B$;

- Following step `BUY_2` (item 2):

$$a\frac{r_B}{r_A} \times \frac{1}{r_A} = a\frac{r_B^2}{r_A^2};$$

- Following step `BUY_3` (item 3): since $\frac{r_B^2}{r_A^2} > 1$, the minimum between the amount in the ask offer and what is in the source wallet w_S is a;

- Following step `BUY_4` (item 4):

$$a'_S = a\frac{r_B^2}{r_A^2} - a = a\frac{r_B^2 - r_A^2}{r_A^2}.$$

- Following step `BUY_5` (item 5):

$$a'_T = a + a = 2a.$$

2.2.4 Bid order # 2

Now comes a **bid** order that wants to buy an amount a of **target** coin, at a rate $r_B = B_{half}$. Since the sale rate $r_T = A_{half}$ is below the bid rate, the target coin will be sold. That is: the target will be sold by a price higher than it was bought.

- In the `SELL_1` (step 1.2):
- Following the step `SELL_2` (step 1.2),
- according to `SELL_3` (item 1.2):