Efficiently Answering Probabilistic Threshold Top-k Queries on Uncertain Data

Technical Report # 88, School of Computing Science, Simon Fraser University

Ming Hua Jian Pei
Simon Fraser University
{mhua, jpei}@cs.sfu.ca

Wenjie Zhang Xuemin Lin
The University of New South Wales
{zhangw, lxue}@cse.unsw.edu.au

June 27, 2007

Abstract

Uncertain data is inherent in a few important applications such as environmental surveillance and mobile object tracking. Top-k queries (or as known as ranking queries) are often natural and useful in analyzing uncertain data in those applications. In this paper, we study the problem of answering probabilistic threshold top-k queries on uncertain data, which computes uncertain records taking a probability of at least p to be in the top-k list where p is a user specified probability threshold. We present an efficient exact algorithm and a fast sampling algorithm. An empirical study using real and synthetic data sets verifies the effectiveness of probabilistic threshold top-k queries and the efficiency of our methods.

1 Introduction

In a few emerging important applications such as environmental surveillance using large scale sensor networks, uncertainty is inherent in data due to various factors like incompleteness of data, limitations of equipment, and delay or loss in data transfer. In those applications, top-k queries (or as known as ranking queries) are often natural and useful in analyzing uncertain data.

Example 1 (Motivation) Sensors are often used to detect presence of endangered, threatened, or special concern risk categories of animals in remote or preserved regions. Due to limitations of sensors, detections cannot be accurate all the time. Instead, detection confidence is often estimated. Table 1 lists a set of synthesized records of presence of an endangered species of animals detected by sensors. Once a sensor detects a suspect of presence, it records the duration that the suspect stays in the detection range of the sensor.

In some locations where the targets are active, multiple sensors are deployed to improve the detection quality. Two sensors in the same location (e.g., $S206$ and $S231$, as well as $S063$ and $S732$

RID	Loc.	Time	Sensor-id	Duration	Conf.
$R1$	A	6/2/06 2:14	$S101$	25 min	0.3
$R2$	B	7/3/06 4:07	$S206$	21 min	0.4
$R3$	B	7/3/06 4:09	$S231$	13 min	0.5
$R4$	A	4/12/06 20:32	$S101$	12 min	1
$R5$	E	3/13/06 22:31	$S063$	17 min	0.8
$R6$	E	3/13/06 22:28	$S732$	11 min	0.2

Table 1: Panda counting records.

Possible world	Probability	Top-2 on Duration
$W1 = \{R1, R2, R4, R5\}$	0.096	$R1, R2$
$W2 = \{R1, R2, R4, R6\}$	0.024	$R1, R2$
$W3 = \{R1, R3, R4, R5\}$	0.12	$R1, R5$
$W4 = \{R1, R3, R4, R6\}$	0.03	$R1, R3$
$W5 = \{R1, R4, R5\}$	0.024	$R1, R5$
$W6 = \{R1, R4, R6\}$	0.006	$R1, R4$
$W7 = \{R2, R4, R5\}$	0.224	$R2, R5$
$W8 = \{R2, R4, R6\}$	0.056	$R2, R4$
$W9 = \{R3, R4, R5\}$	0.28	$R5, R3$
$W10 = \{R3, R4, R6\}$	0.07	$R3, R4$
$W11 = \{R4, R5\}$	0.056	$R5, R4$
$W12 = \{R4, R6\}$	0.014	$R4, R6$

Table 2: The possible worlds of Table 1.

RID	$R1$	$R2$	$R3$	$R4$	$R5$	$R6$
Probability	0.3	0.4	0.38	0.202	0.704	0.014

Table 3: The top-2 probability values of records in Table 1.

in Table 1) may detect the presence of a suspect at the (approximately) same time, such as records $R2$ and $R3$, as well as $R5$ and $R6$. In such a case, if the durations detected by the multiple sensors are inconsistent, at most one sensor can be correct.

The uncertain data in Table 1 carries the possible world semantics [1, 11, 15]. The data can be viewed as the summary of a set of possible worlds. The possible worlds are governed by some underlying generation rules which constrain the presence of tuple instances. In Table 1, the fact that $R2$ and $R3$ cannot be true at the same time can be captured by a generation rule $R2 \oplus R3$. Another generation rule is $R5 \oplus R6$. Table 2 shows all possible worlds and their existence probability values.

Top-k queries can be used to analyze uncertain data. For example, a scientist may be interested in the top-2 longest durations that a suspect stays in a location at a time. In different possible worlds the answers to this question are different. The third column of Table 2 lists the top-2 records in all possible worlds according to the duration attribute.

It is interesting to examine the probability that a tuple is in the top-2 lists of all possible worlds, as shown in Table 3. The method to calculate the probability values will be discussed in Section 2. To answer the top-2 query on the uncertain data, it is helpful to find the tuples whose probability values in the top-2 lists are at least p, where p is a user-specified probability threshold. In this example, if $p = 0.35$, then $\{R2, R3, R5\}$ should be returned. ■

Example 1 demonstrates the probabilistic threshold top-k queries which will be studied in this paper. Generally, different from the top-k queries on certain data, probabilistic top-k queries on uncertain data raise a few interesting challenges that will be addressed in this paper.

Challenge 1: What does a probabilistic top-k query mean? A top-k query on certain data returns the best k results in the ranking function. However, top-k queries on uncertain data may be formulated differently to address different application interests.

For example, in [17], an *uncertain top-k query* (*U-TopK query* for short) returns a list of k records (i.e., a record vector of length k) that has the highest probability to be the top-k list in all possible worlds. In Table 1, $\langle R5, R3\rangle$ should be returned. An *uncertain k ranks query* (*U-KRanks query* for short) returns a list of k records such that the i-th record has the highest probability to be the i-th best record in all possible worlds. In the case of Table 1, $\langle R5, R5\rangle$ should be returned since $R5$ has the highest probability to be ranked first in all possible worlds, and also has the highest probability to be ranked second in all possible worlds. Simultaneously, in [14], given a query Q, the top-k records that have the highest probabilities to satisfy Q in all possible worlds are extracted. For example, on Table 1, if the query is to find a record where a suspect stays for more than 12 minutes, then, the top-2 records are $R3$ and $R5$. Since the ranking is on the probability to satisfy the query, $R1$ is not selected in the top-2 answers though it has a longer duration than both $R3$ and $R5$.

Our contribution. We address an application scenario other than the recent proposals in [17, 14]. Given a probability threshold p, a *probabilistic threshold top-k query* in our model, as elaborated in Example 1, finds the set of records where each takes a probability of at least p to be in the top-k lists in the possible worlds. The new type of queries can find tuples like $R2$ in Example 1 which has a high probability to appear in the top-k lists, but may not be in the top vector of tuples or be the top in some positions.

Challenge 2: How can a probabilistic threshold top-k query be answered efficiently? A naïve method can examine the top-k list in every possible world, derive the top-k probability for each uncertain record, and select the ones passing the threshold. However, it can be very costly on a large data set where the number of possible worlds can be huge.

Can the methods for U-Top$k$ and U-kRanks queries [17] be extended to answer probabilistic threshold top-k queries? The algorithms in [17] scan the tuples in the ranking descending order and materialize all the possible states based on the tuples seen so far. A state is a set of possible worlds which share the top-i tuples and the rest tuples are unknown. The number of states needs to be maintained is exponential in the number of tuples searched. In U-Top$k$ query evaluation, the scan stops when there is a top-k tuple vector whose probability is greater than the upper bound of all other candidates. In U-kRanks query answering, the scan stops when the current answer for each rank is higher than the upper bound of the rest tuples.

The above algorithms cannot be extended to answer probabilistic threshold top-k queries efficiently because both U-Top$k$ queries and U-kRanks queries are "rank sensitive", which require materializing all the possible states. On the other hand, a probabilistic threshold top-k query finds the tuples with high probability to be in the top-k lists regardless of their exact ranks. Thus, there is no need to maintain as many possible states as in U-Top$k$ query and U-kRanks query answering. More efficient algorithms are feasible for answering probabilistic threshold top-k queries.

Our contribution. We develop efficient methods to tackle the problem. First, we give an exact algorithm which exploits a few interesting techniques to avoid unfolding all possible worlds. It finds the exact top-k probability for each tuple by scanning the sorted list of all tuples only once. The rule-tuple compression technique is proposed to handle generation rules. Several pruning techniques are proposed to further improve the efficiency using the probability threshold. Second, we devise a sampling method which can quickly compute an approximation with quality guarantee to the answer set by drawing a small sample of the uncertain database.

The rest of the paper is organized as follows. In Section 2, we formulate probabilistic threshold top-k queries. We review the related work in Section 3. We develop an exact algorithm in Section 4, and devise a sampling method in Section 5. A systematic experimental evaluation is reported in Section 6. The paper is concluded in Section 7.

2 Probabilistic Threshold Top-k Queries

We consider uncertain data in the *possible worlds* semantics model [1, 11, 15], which is also adopted by some recent studies on uncertain data processing, such as [17, 4, 13].

Generally, an *uncertain table* T contains a set of (uncertain) tuples, where each tuple $t \in T$ is associated with a *membership probability* value $Pr(t) > 0$. When there is no confusion, we also call an uncertain table simply a table.

A *generation rule* on a table T specifies a set of exclusive tuples in the form of $R : t_{r_1} \oplus \cdots \oplus t_{r_m}$ where $t_{r_i} \in T$ $(1 \leq i \leq m)$ and $\sum_{i=1}^{m} Pr(t_{r_i}) \leq 1$. The rule R constrains that, among all tuples $t_{r_1}, \ldots, t_{r_m}$ involved in the rule, at most one tuple can appear in a possible world.

As [17, 4], we assume that each tuple is involved in at most one generation rule. For a tuple t not involved in any generation rule, we can make up a trivial rule $R_t : t$. Therefore, conceptually, an uncertain table T comes with a set of generation rules $\mathcal{R}_T$ such that each tuple is involved in one and only one generation rule in $\mathcal{R}_T$. We write $t \in R$ if tuple t is involved in rule R. The *probability* of a rule is the sum of the membership probability values of all tuples involved in the rule, denoted by $Pr(R) = \sum_{t \in R} Pr(t)$.

The *length* of a rule is the number of tuples involved in the rule, denoted by $|R| = |\{t | t \in R\}|$. A generation rule R is a *singleton rule* if $|R| = 1$. R is a *multi-tuple rule* if $|R| > 1$. A tuple is *dependent* if it is involved in a multi-tuple rule, otherwise, it is *independent*.

For a subset of tuples $S \subseteq T$ and a generation rule R, we denote the tuples involved in R and appearing in S as $R \cap S$. A *possible world* W is a subset of T such that for each generation rule $R \in \mathcal{R}_T$, $|R \cap W| = 1$ if $Pr(R) = 1$, and $|R \cap W| \leq 1$ if $Pr(R) < 1$. We denote by $\mathcal{W}$ the set of all possible worlds.

Clearly, for an uncertain table T with a set of generation rules $\mathcal{R}_T$, the number of all possible worlds is $|\mathcal{W}| = \prod_{R \in \mathcal{R}_T, Pr(R)=1} |R| \prod_{R \in \mathcal{R}_T, Pr(R)<1} (|R| + 1)$. The number of possible worlds on a large table can be huge.

Each possible world is associated with an *existence probability* $Pr(W)$ that the possible world happens. Following with the basic probability principles, we have

$$Pr(W) = \prod_{R \in \mathcal{R}_T, |R \cap W|=1} Pr(R \cap W) \prod_{R \in \mathcal{R}_T, R \cap W = \emptyset} (1 - Pr(R)) \tag{1}$$

Apparently, for a possible world W, $Pr(W) > 0$. Moreover, $\sum_{W \in \mathcal{W}} Pr(W) = 1$.

A *top-k query* $Q^k(P, f)$ contains a *predicate* P, a *ranking function* f, and an integer $k > 0$. When Q is applied on a set of certain tuples, the tuples satisfying predicate P are ranked according to ranking function f, and the top-k tuples are returned. For tuples t_1, t_2, $t_1 \preceq_f t_2$ if t_1 is ranked higher than or equal to t_2 according to ranking function f. $\preceq_f$, called the *ranking order*, is a total order on all tuples.

Since a possible world W is a set of tuples, a top-k query Q can be applied to W directly. We denote by $Q^k(W)$ the top-k tuples returned by a top-k query Q on a possible world W. $Q^k(W)$ contains k tuples.

A *probabilistic threshold top-k query* (*PT-k* query for short) on an uncertain table T consists of a top-k query Q and a *probability threshold* p ($0 < p \leq 1$). For each possible world W, Q is applied and a set of k tuples $Q^k(W)$ is returned. For a tuple $t \in T$, the *top-k probability* of t is the probability that t is in $Q^k(W)$ in all $W \in \mathcal{W}$, that is,

$$Pr^k_{Q,T}(t) = \sum_{W \in \mathcal{W}, t \in Q^k(W)} Pr(W) \tag{2}$$

When Q and T are clear from context, we often write $Pr^k_{Q,T}(t)$ as $Pr^k(t)$ for the interest of simplicity.

The *answer set* to a PT-k query is the set of all tuples whose top-k probability values are at least p. That is, $Answer(Q, p, T) = \{t | t \in T, Pr^k_Q(t) \geq p\}$. We are interested in how to compute efficiently the answer set for a PT-k query on an uncertain table.

A naïve method to answer a PT-k query is to enumerate all possible worlds and apply the query to each possible world. Then, we can compute the top-k probability of each tuple and select the tuples passing the probability threshold. Unfortunately, the naïve method is inefficient since, as discussed before, there can be a huge number of possible worlds on an uncertain table. In [8], Dalvi and Suciu showed that even the problem of enumerating all possible worlds with different top-k lists is $\#P$-

Complete. Therefore, enumerating all possible worlds is too costly on large uncertain data sets. That motivates our exploration of efficient algorithms which avoid searching all possible worlds.

3 Related Work

To the best of our knowledge, [17] by Soliman et al. is the existing work most related to our study, which considers ranking queries as we do here. However, as discussed in Section 1, our study and [17] assume different semantics of ranking queries. In Section 6.1, we will further compare PT-k queries, U-TopK queries and R-KRanks queries using a real data set. Our study is also related to [14], which considers arbitrary SQL queries and the ranking is on the probability that a tuple satisfies the query instead of using a ranking function. [14] and our study address essentially different queries and applications.

Probabilistic threshold top-k queries are substantially different from the existing work on both query type and semantics. To the best of our knowledge, we are the first to address this kind of queries.

Our study is also generally related to the previous work on modeling and querying uncertain data, which has been a fast growing research direction [12, 9, 15, 3].

The working model for uncertain data proposed in [15] describes the existence probability of a tuple in an uncertain data set and the constraints (i.e., exclusiveness) on the uncertain tuples, which is essentially the uncertain model adopted in this study.

Cheng et al. [5] provided a general classification of probabilistic queries and evaluation algorithms over uncertain data sets. Different from the query answering in traditional data sets, a probabilistic quality estimate was proposed to evaluate the quality of results in probabilistic query answering. Dalvi and Suciu al. [7] proposed an efficient algorithm to evaluate arbitrary SQL queries on probabilistic databases and ranked the results by their probability. Later, they showed in [8] that the complexity of evaluating conjunctive queries on a probabilistic database is either PTIME or $\#P$-complete.

Cheng et al. [6] modeled uncertain data as a set of ranges and the associated probability density functions. Probabilistic threshold queries were proposed on the above model, which return the results whose confidence is higher than a user defined threshold. An R-tree based index structure was proposed to help answer such queries. Tao et al. [18] proposed a U-tree index to facilitate probabilistic range queries on uncertain objects represented by multi-dimensional probability density functions. Singh et al. [16] extended the inverted index and signature tree to index uncertain categorical data, where the attribute value of each tuple was represented by a set of alternatives.

4 An Exact Algorithm

In this section, we first observe a useful property of top-k probability. Then, we propose an exact algorithm to compute top-k probability efficiently.

Hereafter, by default we consider a top-k query $Q^k(P, f)$ on an uncertain table T. $P(T) = \{t | t \in T \wedge P(t) = true\}$ is the set of tuples satisfying the query predicate. $P(T)$ is also an uncertain table where each tuple in $P(T)$ carries the same membership probability as in T. Moreover, a generation rule R in T is projected to $P(T)$ by removing all tuples from R that are not in $P(T)$. Then, the problem of answering the PT-k query is to find the tuples in $P(T)$ whose top-k probability values pass the probability threshold.

4.1 The Dominant Set Property

$P(T)$ contains all tuples satisfying the query, as well as the membership probabilities and the generation rules. Removing tuples not in $P(T)$ does not affect the answer to a top-k query. Therefore, $Answer(Q, p, T) = Answer(Q, p, P(T))$. We only need to consider $P(T)$ in answering a top-k query.

For a tuple $t \in P(T)$ and a possible world W such that $t \in W$, whether $t \in Q^k(W)$ depends only on how many other tuples in $P(T)$ ranked higher than t appear in W. Technically, for a tuple $t \in P(T)$, the *dominant set* of t is the subset of tuples in $P(T)$ that are ranked higher than t, i.e., $S_t = \{t' | t' \in P(T) \wedge t' \prec_f t\}$.

Theorem 1 (The dominant set property) *For a tuple* $t \in T$, $Pr^k_{Q,T}(t) = Pr^k_{Q,S_t}(t)$.
Proof sketch. The theorem follows with Equations 1 and 2, and the assumption that each tuple is involved in only one generation rule. ■

The dominant set property is very useful in answering PT-k queries. The PT-k query answering algorithm developed in the rest of this section scans the tuples in $P(T)$ in the ranking order, and derives the top-k probability of a tuple t based on the tuples preceding t in the ranking order. Generation rules involving multiple tuples are handled by the rule-tuple compression technique. The probability threshold is used to prune tuples whose top-k probability values fail the threshold.

4.2 The Basic Case

We first consider the basic case where all tuples are independent. Let $L = t_1 \cdots t_n$ be the list of all tuples in table $P(T)$ in the ranking order. Then, in a possible world W, a tuple $t_i \in W$ ($1 \leq i \leq n$) is ranked at the j-th ($j > 0$) position if and only if exactly $(j-1)$ tuples in the dominant set $S_{t_i} = \{t_1, \ldots, t_{i-1}\}$ also appear in W.

The *position probability* $Pr(t_i, j)$ is the probability that tuple t_i is ranked at the j-th position in possible worlds. Moreover, the *subset probability* $Pr(S_{t_i}, j)$ is the probability that j tuples in S_{t_i} appear in possible worlds.

Trivially, we have $Pr(\emptyset, 0) = 1$ and $Pr(\emptyset, j) = 0$ for $0 < j \leq n$. Then,

$$Pr(t_i, j) = Pr(t_i)Pr(S_{t_{i-1}}, j-1) \quad (3)$$

TID	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9
Prob.	0.7	0.2	1	0.3	0.5	0.8	0.1	0.8	0.1

Table 4: The ranked list of uncertain tuples.

Apparently, the top-k probability of t_i is given by

$$Pr^k(t_i) = \sum_{j=1}^{k} Pr(t_i, j) = Pr(t_i) \sum_{j=1}^{k} Pr(S_{t_{i-1}}, j-1) \tag{4}$$

Particularly, when $i \leq k$, we have $Pr^k(t_i) = Pr(t_i)$.

Theorem 2 *In the basic case, for* $1 \leq i, j \leq |T|$,

$$Pr(S_{t_i}, 0) = Pr(S_{t_{i-1}}, 0)(1 - Pr(t_i)) = \prod_{j=1}^{i}(1 - Pr(t_i))$$

$$Pr(S_{t_i}, j) = Pr(S_{t_{i-1}}, j-1)Pr(t_i) + Pr(S_{t_{i-1}}, j)(1 - Pr(t_i))$$

Proof sketch. All tuples are independent. The theorem follows with the basic probability principles. ■

Theorem 2 can be used to compute the top-k probability values efficiently, as illustrated in the following example.

Example 2 (Top-k Probability – the basic case) Consider a top-k query $Q^3(true, f)$. Suppose f ranks the tuples in an uncertain table T into a list L shown in Table 4. Apparently, $S_{t_i} = L[1..i-1]$. In this example, we assume all tuples are independent.

To compute the top-3 probability of each tuple, we first initialize $Pr(\emptyset, 0) = 1$, $Pr(\emptyset, 1) = 0$ and $Pr(\emptyset, 2) = 0$. Then, we scan the ranked list.

For t_j $(1 \leq j \leq 3)$, $Pr^3(t_j) = Pr(t_j)$. Thus, $Pr^3(t_1) = 0.7$, $Pr^3(t_2) = 0.2$, and $Pr^3(t_3) = 1$.

To compute $Pr^3(t_4)$, we first compute $Pr(S_{t_3}, 0) = 0$, $Pr(S_{t_3}, 1) = 0.24$, and $Pr(S_{t_3}, 2) = 0.62$ using Theorem 2. Then, following Equation 4, we have $Pr^3(t_4) = Pr(t_4)(Pr(S_{t_3}, 0) + Pr(S_{t_3}, 1) + Pr(S_{t_3}, 2)) = 0.258$. ■

4.3 Handling Generation Rules

In the basic case, Theorem 2 can be used to compute the top-k probability values for all tuples in time $O(kn)$, where n is the number of tuples in the uncertain table. However, a general case may contain some multi-tuple generation rules.

Suppose tuples $t_1, \ldots, t_n \in P(T)$ are in the ranking order. For a tuple t_i, two situations due to the presence of multi-tuple generation rules complicate the computation. First, there may be a rule R such that some tuples involved in R are ranked higher than t_i. Those tuples cannot appear together in

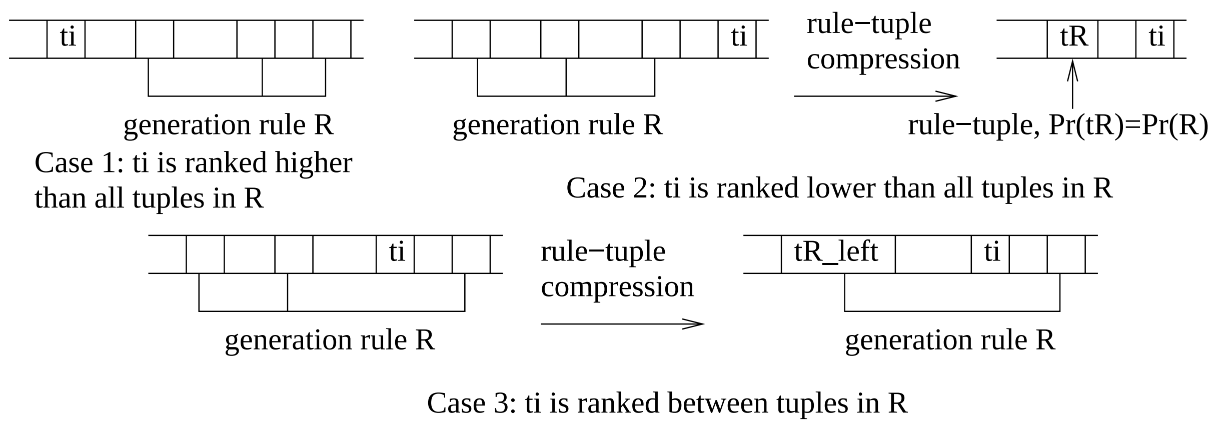


Figure 1: Computing $Pr^k(t_i)$ for one tuple t_i.

a possible world. To compute subset probability $Pr(S_{t_i}, j)$ correctly, we need to make sure that rule R is respected. Second, t_i itself may be involved in a generation rule R, and cannot appear together in a possible world with any other tuples involved in the same rule. In both cases, some tuples in S_{t_i} are dependent and thus Theorem 2 cannot be applied directly. Can dependent tuples in S_{t_i} be transformed to independent ones so that Theorem 2 can still be used?

4.3.1 Rule-Tuple Compression

Consider $P(T) = t_1 \cdots t_n$ in the ranking order, i.e., $t_i \preceq_f t_j$ for $i < j$. Let us compute $Pr^k(t_i)$ for a tuple $t_i \in P(T)$. A multi-tuple generation rule $R : t_{r_1} \oplus \cdots \oplus t_{r_m}$ $1 \leq r_1 < \cdots < r_m \leq n$ can be handled in one of the following cases (see Figure 1 for illustration.)

Case 1: $t_i \preceq_f t_{r_1}$, i.e., t_i is ranked higher than or equal to all tuples in R. According to Theorem 1, R can be ignored.

Case 2: $t_{r_m} \prec_f t_i$, i.e., t_i is ranked lower than all tuples in R. R is called *completed* with respect to t_i. At most one tuple in R can appear in a possible world. According to Theorem 1, we can combine all tuples in R into a *rule-tuple* t_R with membership probability $Pr(R)$.

Corollary 1 (Rule-tuple compression) *For a tuple $t \in P(T)$ and a multi-tuple rule R, if $\forall t' \in R$, $t' \prec_f t$, then $Pr^k_{Q,T}(t) = Pr^k_{Q,T(R)}(t)$ where $T(R) = (T - \{t | t \in R\}) \cup \{t_R\}$, tuple t_R takes any value such that $t_R \prec_f t$, $Pr(t_R) = Pr(R)$, and other generation rules in T remain the same in $T(R)$.* ■

Case 3: $t_{r_1} \prec_f t_i \preceq_f t_{r_m}$, i.e., t_i is ranked in between tuples in R. R is called *open* with respect to t_i. Among the tuples in R ranked better than t_i, let $t_{r_{m_0}} \in R$ be the lowest ranked tuple i.e., $r_{m_0} = \max_{l=1}^{m}\{r_l < i\}$. The tuples involved in R can be divided into two parts: $R_{left} = \{t_{r_1}, \ldots, t_{r_{m_0}}\}$ and $R_{right} = \{t_{r_{m_0}+1}, \ldots, t_{r_m}\}$. $Pr^k(t_i)$ is affected by tuples in R_{left} only and not by those in R_{right}.

Two subcases may arise. First, if $t_i \notin R$, similar to Case 2, we can compress all tuples in R_{left} into a rule-tuple $t_{r_1,\ldots,r_{m_0}}$ where membership probability $Pr(t_{r_1,\ldots,r_{m_0}}) = \sum_{j=1}^{m_0} Pr(t_{r_j})$, and compute

$Pr^k(t_i)$ using Corollary 1.

Second, $t_i \in R$, i.e., $t_i = t_{r_{m_0}+1}$. In a possible world where t_i appears, any tuples in R cannot appear. Thus, to determine $Pr^k(t_i)$, according to Theorem 1, we only need to consider the tuples ranked higher than t_i and not in R, i.e., $S_{t_i} - \{t'|t' \in R\}$.

Corollary 2 (Tuple in rule) *For a tuple $t \in R$ such that $P(t) = true$ and $|R| > 1$, $Pr^k_{Q,T}(t) = Pr^k_{Q,T'}(t)$ where uncertain table $T' = (S_{t_i} - \{t'|t' \in R\}) \cup \{t\}$.* ∎

For a tuple t and its dominant set S_t, we can check t against the multi-tuple rules one by one. Each multi-tuple rule can be handled by one of the above three cases, and dependent tuples in S_t can be either compressed into some rule-tuples or removed due to the involvement in the same rule as t. After the rule-tuple compression, the resulted set is called the *compressed dominant set* of t, denoted by $T(t)$. Based on the above discussion, for a tuple $t \in P(t)$, all tuples in $T(t) \cup \{t\}$ are independent, $Pr^k_{Q,T}(t) = Pr^k_{Q,T(t)\cup\{t\}}(t)$. We can apply Theorem 2 to calculate $Pr^k(t)$ by scanning $T(t)$ once.

Example 3 (Top-k probability – rule-tuple compression) Consider the uncertain table and the top-k query in Example 2 again. Now, suppose we have two multi-tuple generation rules: $R_1 = t_2 \oplus t_4 \oplus t_9$ and $R_2 = t_5 \oplus t_7$. Let us consider how to compute $Pr^3(t_6)$ and $Pr^3(t_7)$.

Tuple t_6 is ranked between tuples in R_1, but $t_6 \notin R_1$. The first subcase of Case 3 should be applied. Thus, we compress $R_{1left} = \{t_2, t_4\}$ into a rule-tuple $t_{2,4}$ with membership probability $Pr(t_{2,4}) = Pr(t_2) + Pr(t_4) = 0.5$. Similarly, t_6 is also ranked between tuples in R_2 and $t_6 \notin R_2$, but $R_{2left} = \{t_5\}$. The compression does not remove any tuple. After the compression, we compute $Pr^3(t_6)$ using $T(t_6) = \{t_1, t_{2,4}, t_3, t_5\}$. The tuples in $T(t_6) \cup \{t_6\}$ are independent. We apply Theorem 2 as illustrated in Example 2 to obtain $Pr^3(t_6) = 0.32$.

Since $t_7 \in R_2$, the tuples in R_2 except for t_7 itself should be removed. Thus, we have $T(t_7) = \{t_1, t_{2,4}, t_3, t_6\}$. We get $Pr^3(t_6) = 0.025$ on $T(t_7) \cup \{t_7\}$. ∎

We can sort all tuples in $P(T)$ into a sorted list L in the ranking order. For each tuple t_i, by one scan of the tuples in L before t_i, we obtain the compressed dominant set $T(t_i)$ where all tuples are independent. Then, we can compute $Pr^k(t_i)$ on $T(t_i) \cup \{t_i\}$ using Theorem 2. In this way, the top-k probability for all tuples can be computed in $O(n^2)$ time where n is the number of tuples in the uncertain table.

4.3.2 Scan Reduction by Prefix Sharing

Example 4 (Example 3 revisited) In Example 3, t_6 and t_7 are neighbors in the sorted list L. However, since t_5 is in $T(t_6)$ but not in $T(t_7)$, the subset probability values $Pr(T(t_6), j)$ cannot be used in calculating $Pr^k(t_7)$.

However, $T(t_6)$ and $T(t_7)$ share the subset $\{t_1, t_{2,4}, t_3\}$. The subset probability values $Pr(S_{t_3}, j)$ computed using $T(t_6)$ can be reused for $T(t_7)$. In other words, by sharing the prefix between the two compressed dominant sets, we can reduce the cost of computing subset probability, which is the major cost in top-k probability computation. ∎

Equation 4 indicates that to compute $Pr^k(t_i)$ using subset probability $Pr(S_{t_{i-1}}, j)$, the order of tuples in $S_{t_{i-1}}$ does not matter. This gives us the space to order the tuples in compressed dominant sets of different tuples so that the prefixes and the corresponding subset probability values can be shared as much as possible.

How can we achieve good sharing? Consider the list $L = t_1 \cdots t_n$ of all tuples in $P(T)$ and a tuple t_i in L. Two observations may help the reordering.

First, for a tuple t that is independent or is a rule-tuple of a completed rule with respect to t_i (Case 2 in Section 4.3.1), t is in $T(t')$ for any tuple $t' \succ_f t_i$. Thus, t should be ordered before any rule-tuple of a rule open with respect to t_i (Case 3 in Section 4.3.1).

Second, there can be multiple rules open with respect to t_i. Each such a rule R_j has a rule-tuple $t_{R_{j_{left}}}$, which will be combined with the next tuple $t' \in R_j$ to update the rule-tuple. Thus, if t' is close to t_i, $t_{R_{j_{left}}}$ should be ordered close to the rear so that the rule-tuple compression affects the shared prefix as less as possible. In other words, those rule-tuples of rules open with respect to t_i should be ordered in their next tuple indices descending order.

An *aggressive method* to reorder the tuples is to always put all independent tuples and rule-tuples of completed rules before rule-tuples of open rules, and order rule-tuples of open rules according to their next tuples in the rules. On the other hand, a *lazy method* always reuses the maximum common prefix in $T(t_{i-1})$, and reorders only the tuples not in the common prefix using the above two observations.

Example 5 (Reordering) Figure 2 shows a list of tuples in an uncertain table T in the ranking order with respect to a top-k query. There are two multi-tuple rules $R_1 : t_1 \oplus t_2 \oplus t_8 \oplus t_{11}$ and $R_2 : t_4 \oplus t_5 \oplus t_{10}$. The compressed dominant sets of tuples in the orders made by the aggressive method and the lazy method are also listed in the figure.

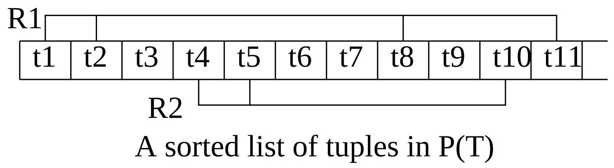
The two methods order the compressed dominant sets in the same way for t_1, t_2 and t_3.

For t_4, the aggressive method orders t_3, an independent tuple, before $t_{1,2}$, the rule-tuple for rule R_1 which is open with respect to t_4. The subset probability values computed in $T(t_3)$ cannot be reused.

On the other hand, the lazy method reuses the prefix $t_{1,2}$ from $T(t_3)$, and appends t_3 after $t_{1,2}$. All subset probability values computed in $T(t_3)$ can be reused.

The lazy method reorders only tuples not in the maximum common prefix. For example, for t_8, $t_{1,2}$ is not in $T(t_8)$ and thus no subset probability values computed in $T(t_7)$ by the lazy method can be reused. Then, the lazy method reorders the tuples in $T(t_8)$ by putting the independent tuples before the rule-tuples of open rules. ∎

For two consecutive tuples t_i and t_{i+1} in the sorted list L of all tuples in $P(T)$, let $L(t_i)$ and $L(t_{i+1})$ be the sorted lists of the tuples in $T(t_i)$ and $T(t_{i+1})$ given by a reordering method such as the aggressive method and the lazy method. Let $Prefix(L(t_i), L(t_{i+1}))$ be the longest common prefix



Tuple	Aggressive reordering	Lazy reordering
t_1	$\emptyset$	$\emptyset$
t_2	$\emptyset$	$\emptyset$
t_3	$t_{1,2}$	$t_{1,2}$
t_4	$t_3t_{1,2}$	$t_{1,2}t_3$
t_5	$t_3t_{1,2}$	$t_{1,2}t_3$
t_6	$t_3t_{4,5}t_{1,2}$	$t_{1,2}t_3t_{4,5}$
t_7	$t_3t_6t_{4,5}t_{1,2}$	$t_{1,2}t_3t_{4,5}t_6$
t_8	$t_3t_6t_7t_{4,5}$	$t_3t_6t_7t_{4,5}$
t_9	$t_3t_6t_7t_{1,2,8}t_{4,5}$	$t_3t_6t_7t_{4,5}t_{1,2,8}$
t_{10}	$t_3t_6t_7t_9t_{1,2,8}$	$t_3t_6t_7t_9t_{1,2,8}$
t_{11}	$t_3t_6t_7t_9t_{4,5,10}$	$t_3t_6t_7t_9t_{4,5,10}$

Figure 2: Comparison of two reordering methods.

between $L(t_i)$ and $L(t_{i+1})$. Then, the total number of subset probability values needed to be calculated is

$$Cost = \sum_{i=1}^{n-1} (|L(t_{i+1})| - |Prefix(L(t_i), L(t_{i+1}))|) \tag{5}$$

In Example 5, $Cost_{aggressive} = 15$ and $Cost_{lazy} = 12$. We can show that the lazy method is always better than the aggressive method. Limited by space, we omit the details here. In our experiments, the lazy method shows a great advantage against the aggressive method.

4.4 Pruning Techniques

So far, we implicitly have a requirement: all tuples satisfying the predicate in the PT-k query, i.e., $P(T)$, should be sorted in the ranking order. However, a PT-k query is interested in only those tuples whose top-k probability values pass the probability threshold. Can we avoid retrieving or checking all tuples satisfying the query predicates?

Some existing methods such as the well known TA algorithm [10] can retrieve in batch tuples satisfying the predicate in the ranking order. Using such a method, we can retrieve tuples in $P(T)$ progressively. Now, the problem becomes how we can use the tuples seen so far to prune some tuples ranked lower in the ranking order.

Hereafter, by default we consider a PT-k query using probability threshold p. We give three pruning rules which can determine that some tuples not checked yet cannot pass the probability threshold. The tuple retrieval method (e.g., an adaption of the TA algorithm [10]) uses the pruning rules in the retrieval. Once it can determine all remaining tuples in $P(T)$ fail the probability threshold, the retrieval can stop.

Please note that we still have to retrieve a tuple t failing the probability threshold if some tuples ranked lower than t may satisfy the threshold, since t may be in the compressed dominant sets of those promising tuples.

Theorem 3 (Pruning by membership probability) $Pr^k(t) \leq Pr(t)$. *Moreover, for an independent tuple* t, *if* $Pr^k(t) < p$, *then (1) for any independent tuple* t' *such that* $t \preceq_f t'$ *and* $Pr(t') \leq Pr(t)$, $Pr^k(t') < p$; *and (2) for any multi-tuple rule* R *such that* t *is ranked higher than all tuples in* R *and* $Pr(R) \leq Pr(t)$, $Pr^k(t'') < p$ *for any* $t'' \in R$. ∎

To use Theorem 3, we maintain the largest membership probability p_{member} of all independent tuples and rule-tuples for completed rules checked so far whose top-k probability values fail the probability threshold. All tuples identified by the above pruning rule should be marked failed.

A tuple involved in a multi-tuple rule may be pruned using the other tuples in the same rule.

Theorem 4 (Pruning by tuples in the same rule) *For tuples* t *and* t' *involved in the same multi-tuple rule* R, *if* $t \preceq_f t'$, $Pr(t) \geq Pr(t')$, *and* $Pr^k(t) < p$, *then* $Pr^k(t') < p$. ∎

Based on the above pruning rule, for each rule R open with respect to the current tuple, we maintain the largest membership probability of the tuples seen so far in R whose top-k probability values fail the threshold. Any tuples in R that have not been seen should be tested against this largest membership probability.

Our last pruning rule is based on the observation that the sum of the top-k probability values of all tuples is exactly k. That is $\sum_{t \in T} Pr^k(t) = k$.

Theorem 5 (Pruning by total top-k probability) *Let* A *be a set of tuples whose top-*k *probability values pass the probability threshold* p. *If* $\sum_{t \in A} Pr^k(t) > k - p$, *then for every tuple* $t' \notin A$, $Pr^k(t') < p$. ∎

In summary, the exact algorithm for PT-k query answering is shown in Figure 3. The complexity of the algorithm can be analyzed as follows. For a multi-tuple rule $R : t_{r_1} \oplus \cdots \oplus t_{r_m}$ where $t_{r_1}, \ldots, t_{r_m}$ are in the ranking order, let $span(R) = r_m - r_1$. When tuple t_{r_l} $(1 < l \leq m)$ is processed, we need to remove rule-tuple $t_{r_1,\ldots,r_{l-1}}$, and compute the subset probability values of the updated compressed dominant sets. When the next tuple not involved in R is processed, $t_{r_1,\ldots,r_{l-1}}$ and t_{r_l} are combined. Thus, in the worst case, each multi-tuple rule causes the computation of $O(2k \cdot span(R))$ subset probability values. Moreover, in the worst case where each tuple $P(T)$ passes the probability threshold, all tuples in $P(T)$ have to be read at least once. The time complexity of the whole algorithm is $O(kn + k \sum_{R \in \mathcal{R}_T} span(R))$.

```
Input: an uncertain table $T$, a set of generation rules $\mathcal{R}_T$, a top-$k$ query $Q^k(P,f)$, and a probability
    threshold $p$;
Output: $Answer(Q,p,T)$;
Method:
1:  retrieve tuples in $P(T)$ in the ranking order one by one,
    for each $t_i \in P(T)$ do
2:      compute $T(t_i)$ by rule-tuple compression;
3:      compute subset probability values and $Pr^k(t_i)$;
4:      if $Pr^k(t_i) \geq p$ then output $t_i$;
5:      check whether $t_i$ can be used to prune future tuples;
6:      if all remaining tuples in $P(T)$ fail the probability threshold then exit;
    end for
```

Figure 3: The exact algorithm.

As indicated by our experimental results, in practice the three pruning rules are effective. Often, only a very small portion of the tuples in $P(T)$ are retrieved and checked before the exact answer to a PT-k query is obtained.

5 A Sampling-based Method

In some situations, a user may want to trade off the accuracy of answers against the efficiency. In this section, we present a simple yet effective sampling-based method.

For a tuple t, let X_t be a random variable as an indicator to the event that t is ranked top-k in possible worlds. $X_t = 1$ if t is ranked in the top-k list, and $X_t = 0$ otherwise. Apparently, the top-k probability of t is the expectation of X_t, i.e., $\Pr^k(t) = E[X_t]$. Our objective is to draw a set of samples S of possible worlds, and compute the mean of X_t in S, namely $E_S[X_t]$, as the approximation of $E[X_t]$.

We use uniform sampling with replacement. For table $T = \{t_1, \ldots, t_n\}$ and the set of generation rules $\mathcal{R}_T$, a sample unit (also as known as an observation) is a possible world. We generate the sample units under the distribution of T: to pick a sample unit s, we scan T once. An independent tuple t_i is included in s with probability $Pr(t_i)$. For tuples in a multi-tuple generation rule $R : t_{r_1} \oplus \cdots \oplus t_{r_m}$, s takes a probability of $Pr(R)$ to include one tuple involved in R into s. If s takes a tuple in R, then tuple t_{r_l} $(1 \leq l \leq m)$ takes the probability of $\frac{Pr(t_{r_l})}{Pr(R)}$ to be chosen. s can contain at most 1 tuple from any generation rule.

Once a sample unit s is generated, we compute the top-k tuples in s. For each tuple t in the top-k list, $X_t = 1$. The indicators for other tuples are set to 0.

The above sample generation process can be repeated so that a sample S is obtained. Then, $E_S[X_t]$ can be used to approximate $E[X_t]$. When the sample size is large enough, the approximation quality

Rank	1	2	3	4	5	6	7	8	9	10
Tuple	$R1$	$R2$	$R3$	$R5$	$R6$	$R9$	$R9$	$R11$	$R11$	$R18$
Probability at this rank	0.8	0.64	0.512	0.348	0.328	0.258	0.224	0.234	0.158	0.163

Table 5: The answers to the *U-KRanks* query.

Tuple	$R1$	$R2$	$R3$	$R4$	$R5$	$R6$	$R7$	$R8$	$R9$	$R10$	$R11$	$R14$	$R18$
Drifted days	435.8	341.7	335.7	323.9	284.7	266.8	259.5	240.4	233.6	233.3	232.6	230.9	229.3
Membership prob.	0.8	0.8	0.8	0.6	0.8	0.8	0.4	0.15	0.8	0.7	0.8	0.6	0.8
Top-10 prob.	0.8	0.8	0.8	0.6	0.8	0.8	0.4	0.15	0.8	0.7	0.79	0.52	0.359

Table 6: Some tuples in the IIP Iceberg Sightings Database 2006.

can be guaranteed.

Theorem 6 (Sample size) *For any δ ($0 < \delta < 1$), ϵ ($\epsilon > 0$), and a sample S of possible worlds, if $|S| \geq \frac{3 \ln \frac{2}{\delta}}{\epsilon^2}$ then for any tuple t $\Pr\{|E_S[X_t] - E[X_t]| > \epsilon E[X_t]\} \leq \delta$.*

Proof sketch. The theorem follows with the well known Chernoff-Hoeffding bound [2]. ∎

In practice, the sampling method can be improved in the following two ways.

First, we can sort all tuples in $P(T)$ in the ranking order into a sorted list L. The first k tuples in a sample unit are the top-k answers in the unit. Thus, when generating a sample unit, instead of scanning the whole table T, we only need to scan L from the beginning and generate the tuples in the sample as described before. However, once the sample unit has k tuples, the generation of this unit can stop. This idea reduces the cost of generating sample units, but does not affect the quality of the sample. To see the effect, consider the situation where all tuples are independent. If the average membership probability is μ, the expected number of tuples we need to scan to generate a sample unit is $\lceil \frac{k}{\mu} \rceil$, which can be much smaller than $|P(T)|$ when k is small. In the worst case, we only need to read the whole list L.

Second, in practice, the actual approximation quality may converge well before the sample size reaches the bound given in Theorem 6. Therefore, *progressive sampling* can be adopted: we generate sample units one by one and compute the estimated top-k probability of tuples after each unit is drawn. For given parameters $d > 0$ and $\phi > 0$, the sampling process stops if in the last d sample units the change of the estimated X_t for any tuple t is smaller than ϕ.

As shown in our experiments, equipped with the above two techniques, the sampling method is efficient.

6 Experimental Results

We conducted a systematic empirical study using a real data set and some synthetic data sets on a PC computer with a 3.0 GHz Pentium 4 CPU, 1.0 GB main memory, and a 160 GB hard disk, running the Microsoft Windows XP Professional Edition operating system. Our algorithms were implemented in Microsoft Visual C++ V6.0.

6.1 Results on IIP Iceberg Database

We use the International Ice Patrol (IIP) Iceberg Sightings Database (http://nsidc.org/data/g00807.html) to examine the effectiveness of top-k queries on uncertain data in real applications. The International Ice Patrol (IIP) Iceberg Sightings Database collects information on iceberg activity in the North Atlantic. The mission is to monitor iceberg danger near the Grand Banks of Newfoundland by sighting icebergs (primarily through airborne Coast Guard reconnaissance missions and information from radar and satellites), plotting and predicting iceberg drift, and broadcasting all known ice to prevent icebergs threatening.

In the database, each sighting record contains the sighting date, sighting location (latitude and longitude), number of days drifted, etc. Among them, number of days drifted is derived from the computational model of the IIP, which is crucial in determining the status of icebergs. It is interesting to find the icebergs drifting for a long period.

However, each sighting record in the database is associated with a confidence level according to the source of sighting, including: R/V (radar and visual), VIS (visual only), RAD(radar only), SAT-L(low earth orbit satellite), SAT-M (medium earth orbit satellite) and SAT-H (high earth orbit satellite). In order to quantify the confidence, we assign confidence values 0.8, 0.7, 0.6, 0.5, 0.4 and 0.3 to the above six confidence levels, respectively.

Moreover, generation rules are defined in the following way. For the sightings with the same time stamp, if the sighting locations are very close – differences in latitude and longitude are both smaller than 0.01 (i.e.,0.02 miles), they are considered referring to the same iceberg, and only one of the sightings is correct. All tuples involved in such a sighting form a multi-tuple rule. For a rule $R : t_{r_1} \oplus \cdots \oplus t_{r_m}$, $Pr(R)$ is set to the maximum confidence among the membership probability values of tuples in the rule. Then, the membership probability of a tuple is adjusted to $Pr(t_{r_l}) = \frac{conf(t_{r_l})}{\sum_{1 \leq i \leq m} conf(t_{r_i})} Pr(R)$ $(1 \leq l \leq m)$, where $conf(t_{r_l})$ is the confidence of t_{r_l}. After the above preprocessing, the database contains $4,231$ tuples and 825 multi-tuple rules. The number of tuples involved in a rule varies from 2 to 10. We name the tuples in the number of drifted days descending order. That is, tuple $R1$ has the largest value and $R2$ has the second largest value on the attribute.

We applied *PT-k query*, *U-TopK query* and *U-KRanks query* on the database by setting $k = 10$ and $p = 0.5$. The ranking order is the number of drifted days descending order. The PT-k query returns a set of 10 objects $\{R1, R2, R3, R4, R5, R6, R9, R10, R11, R14\}$. The U-TopK query returns a vector $\langle R1, R2, R3, R4, R5, R6, R7, R9, R10, R11\rangle$ with probability 0.0299. The U-KRanks query

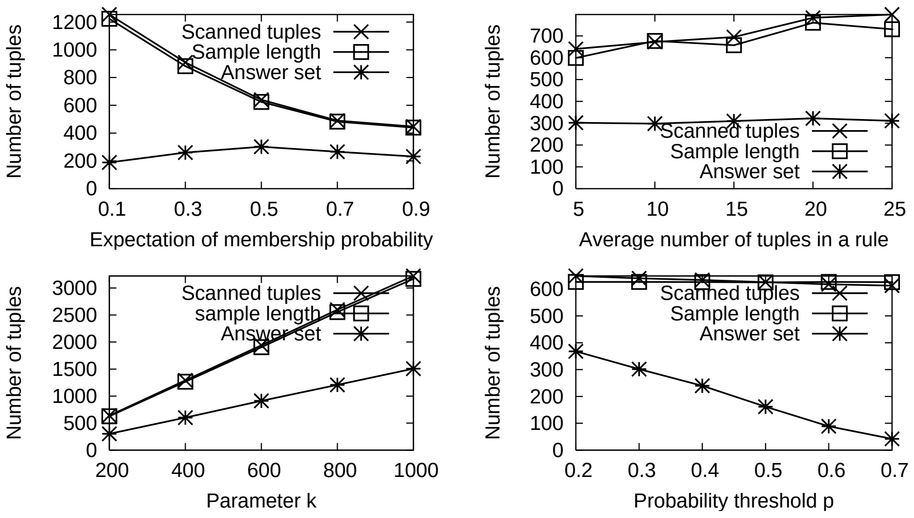


Figure 4: Scan depth (each test data set contains 20, 000 tuples and 2, 000 generation rules).

returns 10 tuples as shown in Table 5. The probability values of the tuples at the corresponding ranks are also shown in the table. To understand the answers, in Table 6 we also list the membership probability values and the top-10 probability values of some tuples including those returned by those queries.

All tuples with top-10 probability at least 0.5 are returned by the PT-k query. The top-10 probability of $R14$ is higher than $R7$, but $R7$ is included in the answer of the U-TopK query and $R14$ is missing. Moreover, the presence probability of the top-10 list returned by the U-TopK query is quite low. Although it is the most probable top-10 tuple list, the low presence probability limits its usefulness and interestingness.

$R10$ and $R14$, whose top-10 probability values are high, are missing in the results of the U-KRanks query, since none of the them is the most probable at any rank. On the other hand, $R18$ is returned by the U-KRanks query at the 10-th position, though its top-10 probability is far lower than $R10$ and $R14$. Moreover, $R9$ and $R11$ each occupies two positions in the answer of the U-KRanks query.

We can see from the results that, *PT-k query* captures some important tuples missed by *U-TopK query* and *U-KRanks query*. This example clearly shows the differences among the three types of top-k queries on uncertain data.

6.2 Results on Synthetic Data Sets

To evaluate the query answering quality and the scalability of our algorithms, we generate various synthetic data sets. The membership probability values of independent tuples and multi-tuple generation rules follow the normal distribution $N(\mu_{P_t}, \sigma_{P_t})$ and $N(\mu_{P_R}, \sigma_{P_R})$, respectively. The rule complexity, i.e., the number of tuples involved in a rule, follows the normal distribution $N(\mu_{|R|}, \sigma_{|R|})$.

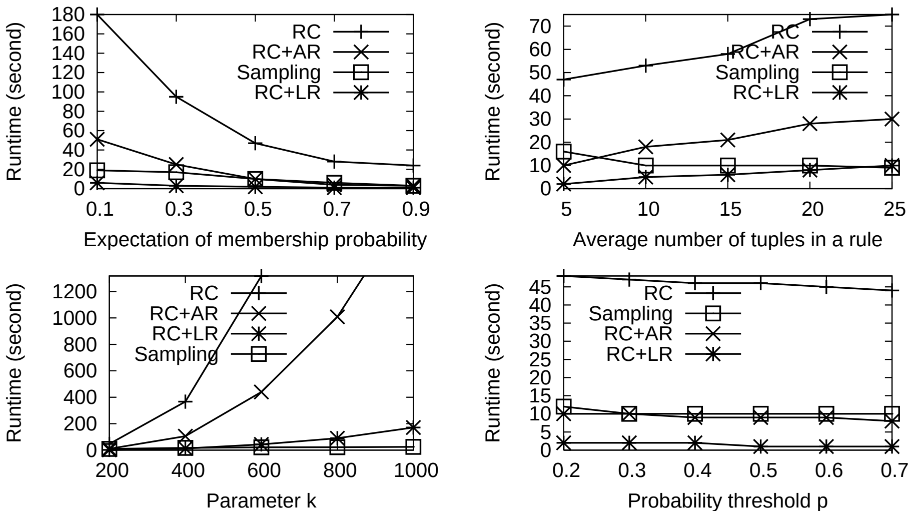


Figure 5: Efficiency (same settings as in Figure 4).

By default, a synthetic data set contains $20,000$ tuples and $2,000$ multi-tuple generation rules. The number of tuples involved in each multi-tuple generation rule follows the normal distribution $N(5,2)$. The probability values of independent tuples and multi-tuple generation rules follow the normal distribution $N(0.5,0.2)$ and $N(0.7,0.2)$, respectively. We test the probabilistic threshold top-k queries with $k = 200$ and $p = 0.3$.

We make all tuples in synthetic data sets satisfy the predicates in the top-k queries. Since ranking queries are extensively supported by modern database management system, we treat the generation of ranked list of uncertain tuples as a black box, and test our algorithms on top of the ranked list.

We compare the exact algorithm and the sampling method. For the exact algorithm, we compare three versions: RC (rule-tuple compression only), RC+AR (RC with aggressive reordering), and RC+LR (RC with lazy reordering). The sampling method includes the two improvements described in Section 5.

First, we test the number of tuples scanned by the methods (Figure 4). We count the number of distinct tuples read by the exact algorithm and the *sample length* as the average number of tuples read by the sampling algorithm to generate a sample unit. For reference, we also plot the number of tuples in the answer set, i.e., the tuples satisfying the probabilistic threshold top-k queries.

When the expected membership probability is high, the tuples at the beginning of the ranked list are more likely to appear, which reduce the probability of the lower ranked tuples to be ranked in top-k lists in possible worlds. In the extreme case, if the membership probability of each tuple is very close to 1, then very likely we can prune all the tuples after the first k tuples are scanned. The number of tuples satisfying the query is maximized when the expectation of membership probability is 0.5 where the uncertainty of the data set is maximized according to information theory and thus the opportunities of being ranked top-k are shared by more tuples.

When the rule complexity increases, more tuples are involved in a rule. The probability of those tuples decreases, and thus more tuples need to be scanned to answer the query. Both the scan depth and the answer set increase linearly when k increases. The size of the answer set drops dramatically as the probability threshold p increases. However, the number of tuples scanned decreases slower in the same situation. As discussed in Section 4.4, a tuple t failing the probability threshold still has to be retrieved if some tuples ranked lower than t may satisfy the threshold.

Figure 4 verifies the effectiveness of the pruning techniques discussed in Section 4.4. With the pruning techniques, the exact algorithm only accesses a small portion of the tuples in the data set. Interestingly, the average sample length is very close to the number of scanned tuples in the exact algorithm, which verifies the effectiveness of our pruning techniques.

Figure 5 compares the runtime of the three versions of the exact algorithm and the sampling algorithm with respect to the four aspects tested in Figure 4. We also count the number of times in the three versions of the exact algorithm that subset probability values are computed. The trends are exactly the same as their runtime. Limited by space, we omit the figures here. The results confirm that the rule-tuple compression technique and the reordering techniques speed up the exact algorithm substantially. Lazy reordering always outperforms aggressive reordering substantially.

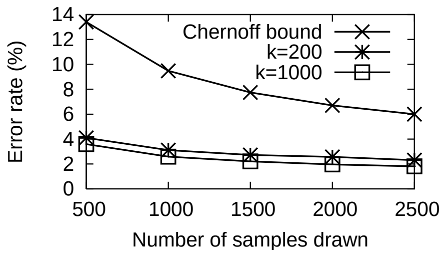
Compared to the exact algorithms, the sampling method is generally more stable in runtime. Interestingly, the exact algorithm (RC+LR) and the sampling algorithm each has its edge. For example, when k is small, the exact algorithm is faster. The sampling method becomes the winner with large k value. As k increases, more tuples need to be scanned in the exact algorithm, and those tuples may be revisited in subset probability computation. But the only overhead in the sampling method is to scan more tuples when generating a sample unit, which is linear in k. This justifies the need for both the exact algorithm and the sampling algorithm.

Figure 6 tests the average error rate of the top-k probability approximation as well as the precision and the recall of answering PT-k queries using the sampling method. Suppose the top-k probability of tuple t is $Pr^k(t)$, and the top-k probability estimated by the sampling method is $\widehat{Pr}^k(t)$, the average error rate is Error rate $= \frac{\sum_{Pr^k(t)>p} |Pr^k(t) - \widehat{Pr}^k(t)|/Pr^k(t)}{|\{t|Pr^k(t)>p\}|}$. For reference, we also plot the error bound calculated from the Chernoff-Hoeffding bound [2] given the sample size. We can clearly see that the error rate of the sampling method is much better than the theoretical bound by Chernoff-Hoeffding bound. Moreover, with a larger k value, we need more samples to achieve the same quality.

Both the precision and the recall of the sampling method are higher than 97% when $k = 200$ or $k = 1000$. This further demonstrates the effectiveness of the sampling method.

Last, Figure 7 shows the scalability of the two algorithms with respect to both the number of tuples in the database and the number of multi-tuple generation rules. To test the scalability with respect to the number of tuples, we vary the number of tuples from $20,000$ to $100,000$, and set the number of multi-tuple rules to 10% of the number of tuples. We set $k = 200$ and $p = 0.3$.

The runtime increases very mildly when the database size increases. Due to the pruning rules and the improvement on extracting sample units, the scan depth, i.e., the number of tuples read, in both



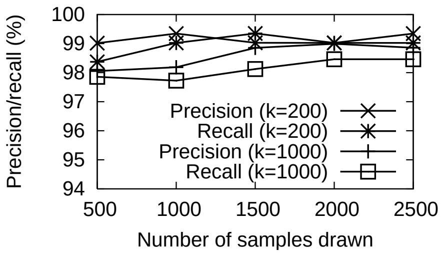
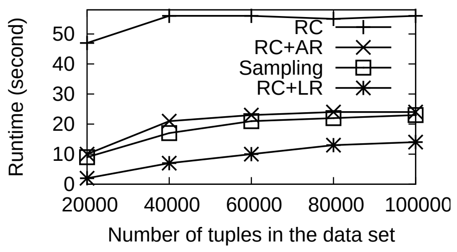


Figure 6: The approximation quality of the sampling-based method.



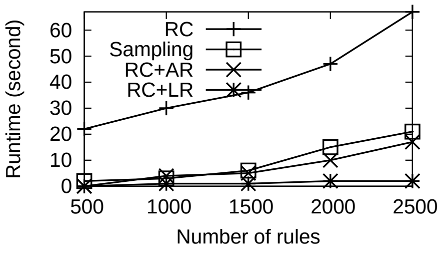


Figure 7: Scalability.

the exact algorithm and the sampling algorithm mainly depends on k and is insensible to the total number of tuples in the data set.

To test the scalability with respect to the number of rules, we fix the number of tuples to $20,000$, and vary the number of rules from 500 to $2,500$. The runtime of both algorithms increases since more rules lead to smaller tuple probability and more back and forth scans of tuples in the span of rules. However, the reordering technique can handle the rule complexity nicely, and make RC+AR and RC+LR scalable.

7 Conclusions

In this paper, we studied the novel probabilistic threshold top-k queries on uncertain data, which are different from the recent proposals in semantics. An exact algorithm and a sampling method were developed and examined empirically. The results show that they are efficient and each of them has its unique edge.

It is interesting to extend our study to handle different kinds of ranking and preference queries on uncertain data. For example, it is interesting to study the top-k skyline queries on uncertain data.

References

[1] S. Abiteboul, P. Kanellakis, and G. Grahne. On the representation and querying of sets of possible worlds. In *Proceedings of the 1987 ACM SIGMOD international conference on Management of*

data (SIGMOD'87), pages 34–48, New York, NY, USA, 1987. ACM Press.

[2] D. Angluin and L. G. Valiant. Fast probabilistic algorithms for hamiltonian circuits and matchings. In *Proceedings of the ninth annual ACM symposium on Theory of computing (STOC'77)*, pages 30–41, New York, NY, USA, 1977. ACM Press.

[3] L. Antova, C. Koch, and D. Olteanu. 10^{10^6} worlds and beyond: Efficient representation and processing of incomplete information. In *Proceedings of the 2007 IEEE 23rd International Conference on Data Engineering (ICDE'07)*, April 2007.

[4] O. Benjelloun, A. D. Sarma, A. Halevy, and J. Widom. Uldbs: databases with uncertainty and lineage. In *VLDB'2006: Proceedings of the 32nd international conference on Very large data bases*, pages 953–964. VLDB Endowment, 2006.

[5] R. Cheng, D. V. Kalashnikov, and S. Prabhakar. Evaluating probabilistic queries over imprecise data. In *Proceedings of the 2003 ACM SIGMOD international conference on Management of data (SIGMOD'03)*, pages 551–562, New York, NY, USA, 2003. ACM Press.

[6] R. Cheng, Y. Xia, S. Prabhakar, R. Shah, and J. S. Vitter. Efficient indexing methods for probabilistic threshold queries over uncertain data. In *VLDB*, pages 876–887, 2004.

[7] N. N. Dalvi and D. Suciu. Efficient query evaluation on probabilistic databases. In *Proceedings of the Thirtieth International Conference on Very Large Data Bases*, pages 864–875, Toronto, Canada, 2004.

[8] Nilesh Dalvi and Dan Suciu. The dichotomy of conjunctive queries on probabilistic structures. In *Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems (PODS'07)*, pages 293–302, New York, NY, USA, 2007. ACM Press.

[9] Debabrata Dey and Sumit Sarkar. A probabilistic relational model and algebra. *ACM Trans. Database Syst.*, 21(3):339–369, 1996.

[10] Ronald Fagin, Amnon Lotem, and Moni Naor. Optimal aggregation algorithms for middleware. In *PODS '01: Proceedings of the twentieth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*, pages 102–113. ACM Press, 2001.

[11] Tomasz Imielinski and Jr. Witold Lipski. Incomplete information in relational databases. *Journal of ACM*, 31(4):761–791, 1984.

[12] Suk Kyoon Lee. Imprecise and uncertain information in databases: An evidential approach. In *Proceedings of the Eighth International Conference on Data Engineering (ICDE'92)*, pages 614–621, Washington, DC, USA, 1992. IEEE Computer Society.

[13] J. Pei, B. Jiang, X. Lin, and Y. Yuan. Probabilistic skylines on uncertain data. In *Proceedings of the 33rd International Conference on Very Large Data Bases (VLDB'07)*, Viena, Austria, September 2007.

[14] C. Ré, N. Dalvi, and D. Suciu. Efficient top-k query evaluation on probabilistic data. In *Proceedings of the 23nd International Conference on Data Engineering (ICDE'07)*, Istanbul, Turkey, April 2007. IEEE.

[15] A. D. Sarma, O. Benjelloun, A. Halevy, and J. Widom. Working models for uncertain data. In *ICDE '06: Proceedings of the 22nd International Conference on Data Engineering (ICDE'06)*, page 7, Washington, DC, USA, 2006. IEEE Computer Society.

[16] S. Singh, C. Mayfield, S. Prabhakar, R. Shah, and S. Hambrusch. Indexing uncertain categorical data. In *Proceedings of the 2007 IEEE 23rd International Conference on Data Engineering (ICDE'07)*, April 2007.

[17] M. A. Soliman, I. F. Ilyas, and K. C.-C. Chang. Top-k query processing in uncertain databases. In *Proceedings of the 23nd International Conference on Data Engineering (ICDE'07)*, Istanbul, Turkey, April 2007. IEEE.

[18] Y. Tao, R. Cheng, X. Xiao, W. K. Ngai, B. Kao, and S. Prabhakar. Indexing multi-dimensional uncertain data with arbitrary probability density functions. In *Proceedings of the 31st international conference on Very large data bases (VLDB'05)*, pages 922–933. VLDB Endowment, 2005.