

## 1 Centre of mass

### 1.1 General centre of mass

Shape  $A(x_1, y_1)$  weighted  $w_1$ ,  $B(x_2, y_2)$  weighted  $w_2$ ,  $C(x_3, y_3)$  weighted  $w_3$ , centre of mass  $G(\bar{x}, \bar{y})$ :

$$w_1 \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + w_2 \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} + w_3 \begin{pmatrix} x_3 \\ y_3 \end{pmatrix} = (w_1 + w_2 + w_3) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

### 1.2 Centre of mass of uniform triangular lamina

$$G \left( \frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

### 1.3 Centre of mass of uniform sector of a circle

Make the origin at the centre of the circle, make x-axis crossing the symmetry of the sector, then

$$distance = \frac{2r \sin \alpha}{3\alpha}$$

$$G \left( 0, \frac{2r \sin \alpha}{3\alpha} \right)$$

### 1.4 Combined centre of mass of two laminae

Shape  $A$ ,  $G_A(x_a, y_a)$  area  $A_a$ ;

Shape  $B$ ,  $G_B(x_b, y_b)$  area  $A_b$ .

Combined centre of mass:

$$A_a \begin{pmatrix} x_a \\ y_a \end{pmatrix} + A_b \begin{pmatrix} x_b \\ y_b \end{pmatrix} = (A_a + A_b) \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

### 1.5 Centre of mass of an arc of a circle

Make the origin at the centre of the circle, make x-axis crossing the symmetry of the sector, then

$$distance = \frac{r \sin \alpha}{\alpha}$$

### 1.6 Centre of mass of a frame

Consider all lines as a dedicated shape, use the general method to calculate its centre then use centre point to represent the shape then apply the general method again.

## 2 Further Center of Mass

### 2.1 Center of mass of a lamina

$$\bar{x} = \frac{\int_a^b xy \, dx}{\int_a^b y \, dx}$$

$$\bar{y} = \frac{\int_a^b \frac{1}{2} y^2 \, dx}{\int_a^b y \, dx}$$

$$M = \int_a^b \rho y \, dx$$

$$M\bar{x} = \int_a^b \rho xy \, dx$$

$$M\bar{y} = \int_a^b \frac{1}{2} \rho y^2 \, dx$$

### 3 Momentum

#### 3.1 Coefficients of restitution

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

When  $e = 1$ , perfect elastic

#### 3.2 Collisions with wall

$$e = \frac{v}{u}$$

### 4 Stiffness coefficient

#### 4.1 Force by extended string

$$F = \frac{\lambda x}{L}$$

#### 4.2 Energy stored in string

$$W = \frac{\lambda x^2}{2L}$$