1 Summation of Series

$$\sum_{x=0}^{n} x = \frac{n(n+1)}{2}$$
$$\sum_{x=0}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{x=0}^{n} x^3 = \frac{n^2(n+1)^2}{4}$$

2 Complex Numbers

1) Translation

   $w = z + a + bi$: translation by $\begin{pmatrix} a \\ b \end{pmatrix}$

2) Enlargement

   $w = kz$: enlargement by a scale factor k

3) Enlargement followed by translation

   $w = kz + a + bi$: enlargement by a scale factor k followed by a translation by $\begin{pmatrix} a \\ b \end{pmatrix}$

2.1 Transformations

2.1.1 Example 1

Find the transformation $w = \frac{1}{z}, z! = 0$, find the locus of w when z lies on the line with equation $y = 2x + 1$

$$x + yi = \frac{1}{u + vi} = \frac{u - vi}{u^2 + v^2} = \frac{u}{u^2 + v^2} + \frac{-v}{u^2 + v^2} i$$

3 Differentiation

3.1 First order differentiation

$$f(x)\frac{dy}{dx} + f'(x)y = \frac{d(f(x)y)}{dx}$$

Integration factor: $\boxed{e^{\int p dx}}$

$$\frac{dy}{dx} + py = Q \rightrightarrows \frac{d(\boxed{e^{\int p dx}}y)}{dx} = \boxed{e^{\int p dx}}Q$$

3.2 Second order differentiation

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = 0$$

3.2.1 Auxiliary equation

$$am^2 + bm + c = 0$$

If $\Delta > 0$, it has two distinct roots α, β. General solution:

$$y = Ae^{\alpha x} + Be^{\beta x}$$

If $\Delta = 0$, it has two repeated roots. General solution:

$$y = (A + Bx)e^{\alpha x}$$

If $\Delta < 0$, it has two complex roots, $p + qi$ and $p - qi$. General solution:

$$y = e^{px}(A \cos qx + B \sin qx)$$

3.2.2 Example for finding a general solution for Second order differentiation

$$\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 0$$
$$a = 1, b = 5, c = 6$$
$$m^2 + 5m + 6 = 0$$
$$m = -2 or m = -3$$
$$y = Ae^{-2x} + Be^{-3x}$$

3.2.3 Complementary functions

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

Solution: $y = complementary\ function + particular\ integral$

Particular integral is the general form of $f(x)$.

3.2.4 Complementary functions example

$$\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 12y = 36x$$

Step 1. State CF and PI
CF: $y = Ae^{2x} + Be^{6x}$
PI: $y = \lambda x + \mu$

Step 2. Differentiate PI
Obtain:

$$\frac{dy}{dx} = \lambda$$
$$\frac{d^2y}{dx^2} = 0$$

Step 3. Substitute $\frac{d^2y}{dx^2}$, $\frac{dy}{dx}$, y into the differentiation equation.
Then find λ and μ.

3.3 Appendix: Particular Integrals

$f(x)$	Particular integral
k	λ
$ax + b$	$\lambda x + \mu$
$ax^2 + bx + c$	$\lambda x^2 + \mu x + \gamma$
ae^{kx}	λe^{kx}
$a \sin kx$ $a \sin kx$ $a \sin kx + b \cos kx$	$\lambda \sin kx + \mu \cos kx$

3.4 Appendix: Formulas of Integration and Differentiation

$$\int \frac{f'(x)}{f(x)} dx = \ln(f(x))$$

$$\int \tan x \sin x dx = \sec x + C$$

$$\int \cot x \csc x dx = -\csc x + C$$

$$\int \sec x dx = \ln\left(\sec x + \tan x\right) + C$$

$$\int \csc x dx = -(\ln \csc x + \cot x) + C$$

$$\int \csc x dx = -(\ln \csc x + \cot x) + C$$