

1 Summation of Series

$$\begin{aligned}\sum_{x=0}^n x &= \frac{n(n+1)}{2} \\ \sum_{x=0}^n x^2 &= \frac{n(n+1)(2n+1)}{6} \\ \sum_{x=0}^n x^3 &= \frac{n^2(n+1)^2}{4}\end{aligned}$$

2 Complex Numbers

1) Translation

$$w = z + a + bi : \text{translation by } \begin{pmatrix} a \\ b \end{pmatrix}$$

2) Enlargement

$$w = kz : \text{enlargement by a scale factor } k$$

3) Enlargement followed by translation

$$w = kz + a + bi : \text{enlargement by a scale factor } k \text{ followed by a translation by } \begin{pmatrix} a \\ b \end{pmatrix}$$

2.1 Transformations

2.1.1 Example 1

Find the transformation $w = \frac{1}{z}, z \neq 0$, find the locus of w when z lies on the line with equation $y = 2x + 1$

$$x + yi = \frac{1}{u + vi} = \frac{u - vi}{u^2 + v^2} = \frac{u}{u^2 + v^2} + \frac{-v}{u^2 + v^2}i$$

3 Differentiation

3.1 First order differentiation

$$f(x) \frac{dy}{dx} + f'(x)y = \frac{d(f(x)y)}{dx}$$

Integration factor: $\boxed{e^{\int p dx}}$

$$\frac{dy}{dx} + py = Q \Rightarrow \frac{d(\boxed{e^{\int p dx}} y)}{dx} = \boxed{e^{\int p dx}} Q$$

3.2 Appendix: Formulas of Integration and Differentiation

$$\int \frac{f'(x)}{f(x)} dx = \ln(f(x))$$

$$\int \tan x \sec x dx = \sec x + C$$

$$\int \cot x \csc x dx = -\csc x + C$$

$$\int \sec x dx = \ln(\sec x + \tan x) + C$$

$$\int \csc x dx = -(\ln \csc x + \cot x) + C$$