Further Maths

10. September 2020

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1 Summation of Series

$$\sum_{x=0}^{n} x = \frac{n(n+1)}{2}$$

$$\sum_{x=0}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{x=0}^{n} x^3 = \frac{n^2(n+1)^2}{4}$$

$$\frac{d\left(e^{(\sqrt{p}dx)y}\right)}{dx} = e^{\int pdx}Q$$

2 Complex Numbers

1) Translation

   $w = z + a + bi$: translation by $\begin{pmatrix} a \\ b \end{pmatrix}$

2) Enlargement

   $w = kz$: enlargement by a scale factor k

3) Enlargement followed by translation

   $w = kz + a + bi$: enlargement by a scale factor k followed by a translation by $\begin{pmatrix} a \\ b \end{pmatrix}$

2.0.1 Example 1

Find the transformation $w = \frac{1}{z}, z! = 0$, find the locus of w when z lies on the line with equation $y = 2x + 1$

$$x + yi = \frac{1}{u + vi} = \frac{u - vi}{u^2 + v^2} = \frac{u}{u^2 + v^2} + \frac{-v}{u^2 + v^2}i$$

3 Differentiation

3.1 First order differentiation

$$f(x)\frac{dy}{dx} + f'(x)y = \frac{d(f(x)y)}{dx}$$

$$\int \frac{f'(x)}{f(x)}dx = \ln(f(x))$$

Integration factor: $\boxed{e^{\int pdx}}$

$$\frac{dy}{dx} + py = Q \rightrightarrows \frac{d(\boxed{e^{\int pdx}}y)}{dx} = \boxed{e^{\int pdx}}Q$$

3.2 Appendix: Formulas of Integration and Differentiation

$$\int \tan x \sin x dx = \sec x + C$$

$$\int \cot x \csc x dx = -\csc x + C$$

$$\int \sec x dx = \ln x + \tan x + C$$

$$\int \csc x dx = -\ln(\csc x + \cot x) + C$$