

1 Introduction and Scope

Public transportation is essential for sustainable urban growth, and the Société de transport de Montréal (STM) is in the forefront of efforts to improve the quality and accessibility of transit services in Montreal. Achieving STM's lofty social sustainability goals requires a thorough understanding of the linked elements that affect the transportation system. Achieving these objectives requires a deep understanding of the complex interdependencies within the transit system and the dynamic variables that influence ridership levels and service quality levels.

Our project “A Systems Dynamics Model to Improve Service and Maximize Benefits of Public Transit for STM” focuses on building a system dynamics model in AnyLogic software that creates a channel between Ridership levels and Service Quality to increase service capabilities and the number of trips of the transport network - therefore maximizing its societal benefits.

1. System Dynamics

System dynamics is a method for analysing the nonlinear behaviour of complex systems over time by utilizing stocks, flows, internal feedback loops, table functions, and time delays. It helps a person, or an organization understand the changes in the behaviour of a given system over time. The stochastic dynamics of a system is one of the most complex forms of modeling to explain the relationships between components and the attributes of the concerned system. Stocks refer to the amount or quantity that is available at any point, like ridership level, service quality, etc while flow refers to the rate at which these stocks are being built up or depleted. Positive feedback maintains changes in the system, amplifying other changes, whereas negative feedback cancels changes within the system, resulting in relative system interaction that may be modeled and examined to better understand the system. Positive feedback is carried by reinforcing loops and negative feedback are conveyed by balancing loops.

2. Objective

The objective of this system dynamics simulation model is to explore and understand the interdependencies among various social sustainability indicators within STM Montreal's public transit system. This model aims to illuminate how different factors such as ridership levels (stock value) and service quality (stock value) influence each other and collectively impact the social sustainability goals of the transit system. The goal is to develop a comprehensive understanding of the dynamic relationships and leverage this knowledge to inform policy decisions, improve service quality, and enhance the overall effectiveness and sustainability of the public transit system. By simulating complex interactions, the model seeks to provide valuable insights into how changes in one aspect (parameter or combination of

parameters) of the system can propagate through and affect others. By parameterizing the model with collected social data from STM and calibrating it to reflect real-world dynamics, the simulation will serve as a decision-support tool.

Consider STM Montreal's transit system as a dynamic entity. There are several moving elements in it, and changes to one can have an impact on the system. For instance, more individuals may choose to utilize the transportation system if the quality of the services increases. This can be controlled by relationships with the overall accessibility for the customers, the sense of security felt by customers in the overall transportation system etc. The increase in ridership level is another factor which brings in more revenue and is also considered as a stock value which are impacted by various parameters. Conversely, if there are frequent service disruptions, people might stop using the system, leading to less revenue and further declines in service quality.

By simulating the interactions within the STM, the model aims to:

- a) Explore the Interdependencies: Understand the relationships among key sustainability indicators and their impact on ridership levels.
- b) Identify Critical Variables: Determine the relative importance of different variables in achieving the desired outcomes.
- c) Simulate System Interactions: Analyse how changes in one part of the system influence other components and overall performance.
- d) Optimize Ridership Levels: Identify strategies to attract and retain more transit users by improving service quality and accessibility.
- e) Enhance Service Quality: Develop actionable insights for maintaining and improving service reliability, coverage, and overall user experience.
- f) Support Decision-Making: Provide STM management with a powerful tool to test different scenarios and make data-driven decisions to enhance public transit services.

3. Scope

The scope of the project focuses on examining ridership levels and service quality as primary flows, with the aim of improving the number of trips (ridership level) and the service quality, both treated as stocks within the system dynamics model. This approach highlights the crucial relationship between these two factors and how they influence the overall performance and sustainability of the public transit system. By concentrating on these elements, the project aims to identify and analyse the key drivers that can enhance service quality and, in turn, attract more riders to use public transit, thereby maximizing the benefits and efficiency of the system.

To achieve this, the model will gather and utilize data related to social sustainability indicators of the STM which influences the inflow and outflow of the stocks from the year 2006 to 2014 such as the population

growth rate, fare incentive, service accessibility rate, service degradation and even the degree of satisfaction level of the STM users. To describe the dynamics of the system, the model is then calibrated after setting up the individual parameters. Once the dynamic variables have been set up, the analysis will reveal the best ways to increase ridership and service quality, offering practical recommendations for enhancing the social sustainability of STM Montreal's public transit system. Additionally, rigorous validation and verification methods will be employed to ensure the reliability and accuracy of the model.

By examining historical data from 2006 to 2014, the model will analyse various social sustainability indicators, such as population growth rate, fare incentives, service accessibility, service degradation, user satisfaction levels etc. It is anticipated that the analysis would produce useful suggestions for STM Montréal, with an emphasis on tactics to raise ridership and service quality in a sustainable manner. These suggestions might include pricing incentives to draw in new passengers, training initiatives to boost consumer satisfaction and security, and focused infrastructure expenditures to increase service accessibility and dependability. Furthermore, the simulation might identify locations where service degradation happens, enabling preventive measures to mitigate these problems. The results will be dependable due to the model's thorough validation and verification, providing a solid foundation for strategic planning. The project's goal is to improve STM Montreal's public transportation system's social sustainability by making sure it efficiently serves users' demands and encourages increased utilization and long-term viability.

2 Description of the system

The model is built using AnyLogic, a robust platform for simulating complex systems. It incorporates historical data from 2006 to 2014 and leverages stock and flow diagrams to represent accumulations and rates of change. The system built here mainly focuses on the service quality and ridership level (stocks) and the interdependencies of parameters and dynamic variables representing indicators to explore and understand the intricate dynamics of its public transit system.

The system can be modeled as two diagrams: one that considers ridership level and the variables related to it, and another that considers service quality and its variables. The two models are interconnected through causal relationships. For instance, the Service Quality Score influences the Ridership Level through the Service Quality Overall Performance Score in the Ridership Inflow. Similarly, the Total Ridership affects the Service Quality Score through the SQ Improvement Inflow.

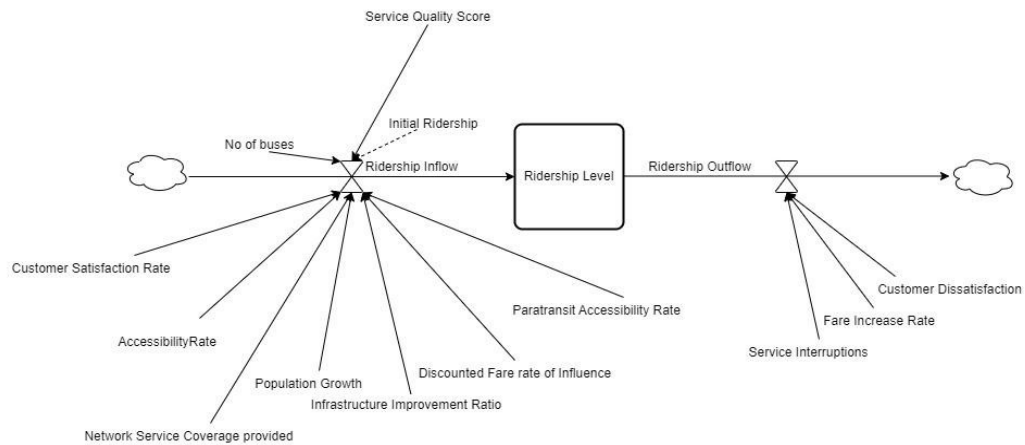


Figure 1 Ridership Stock Definition

The first diagram focuses on the Ridership Level, representing the number of people using the transit service. This stock is affected by Ridership Inflow and Ridership Outflow. The Ridership Inflow is influenced by various factors, including the Service Quality Score, which indicates that higher service quality attracts more riders. Initial Ridership sets the baseline for the number of users. The Number of Buses impacts ridership by determining the service's capacity to accommodate passengers. Customer Satisfaction Rate, Accessibility Rate, and Population Growth are significant contributors, with satisfied customers, better accessibility, and a growing population leading to increased ridership. Infrastructure Improvement Ratio and Network Service Coverage Provided ensure that the transit system is efficient and widely accessible, further boosting ridership. Additionally, Discounted Fare Rate of Influence and Paratransit Accessibility Rate play essential roles in making the service more attractive and accessible to a broader range of users.

On the contrary, Ridership Outflow refers to the elements that cause ridership to decrease. Customer dissatisfaction is a significant factor as dissatisfied clients are less likely to utilize the service. Due to increased expenses, fare increases could deter passengers. Service Interruptions cause significant disruptions, prompting riders to seek alternative transportation methods.

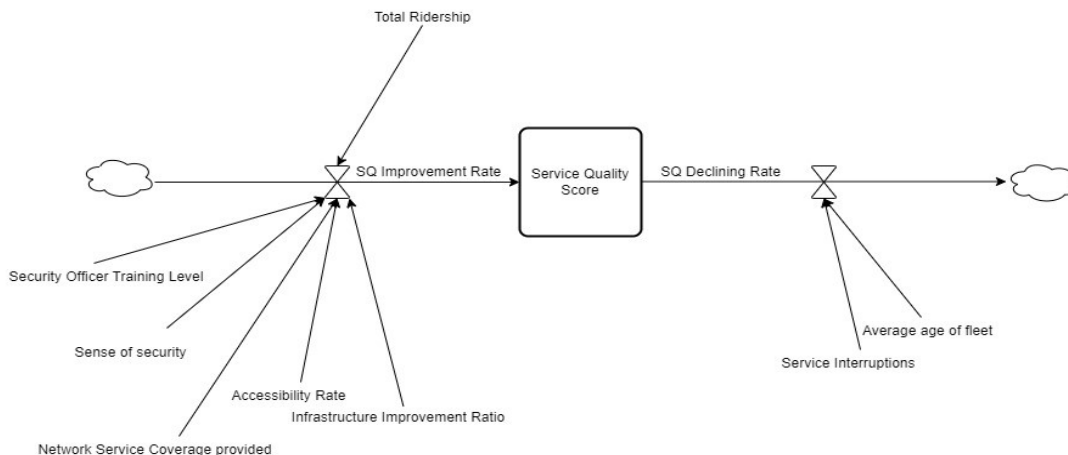


Figure 2 Service Quality Stock Definition

The Service Quality Score (SQS), a crucial indicator for evaluating the level of services provided, is the focal point of the second diagram. The SQ Declining Rate and the SQ Improvement Rate are the two main flows that impact the SQS. Total Ridership, which measures the number of passengers and their effect on perceived service quality, is one of the parameters that contribute to the SQ Improvement rate. Security Officer Training Level and Sense of Security are also crucial, as higher levels of training and security enhance the overall service quality. Accessibility Rate, Infrastructure Improvement Ratio, and Network Service Coverage Provided contribute significantly to improving service quality by ensuring that the service is accessible, the infrastructure is continuously upgraded, and the network coverage is extensive.

Conversely, the SQ Declining Inflow comprises factors that negatively impact the Service Quality Score. The Average Age of Fleet is a notable factor, as older vehicles can decrease service reliability and quality. Additionally, Service Interruptions play a detrimental role, with frequent disruptions causing customer dissatisfaction and lowering the perceived quality of the service.

The boundaries of this AnyLogic model are defined by the factors influencing service quality and ridership levels in the transit system. The values of the parameters are defined exclusively from the dataset that is found through research the STM annual activity reports and sustainability reports from the year 2006 to 2014. These historical data will be pictured and standardised throughout the model's representation and simulation. The system is scoped to consider both positive and negative influences on service quality and ridership from additional data gathered from other sources.

1. Causal loops

Causal loops are feedback loops where the cause-and-effect connections create a closed chain that returns to the initial variable. These loops, show how various system components interact and impact one another over time and are essential to comprehending the behaviour of complex systems. In the context of influencers leading to ridership levels, the variable service quality performance score attracts more riders (total ridership), contributing further to positive perceptions of the service. This amplification of the changes is made by reinforcing loops. This decrease in ridership can result in reduced revenues, limiting the ability to maintain or improve service quality, thus creating a balancing loop that stabilizes or reduces ridership over time. The causal relationships and its dependencies will be explained in detail during the modeling phase of the process

3 Modeling and Simulation Methodology

By defining the parameters and understanding their interactions, STM Montreal can use the system dynamics model to simulate various scenarios and make informed decisions. The primary objective of the modeling and simulation of the transit system is to explore the interdependencies among various sustainability indicators that influence the ridership level. The simulation aims to provide insights into how different factors contribute to or hinder the performance of the transit system. The expected outcomes include a comprehensive understanding of the causal relationships between service quality and ridership levels, identification of leverage points for improvement, and strategic recommendations for optimizing the transit system to maximize its benefits for public users. Additionally, the simulation is expected to predict the behaviour of the transit system during the years from 2006 to 2014. A validation and verification experiment is conducted to determine how the actual values created by the model deviates with a predicted value using machine learning algorithms. The equations of flow rate is determined using regression analysis and a follow up ridge regression analysis which implies that the model will be executed using a linear model and will be altered further by a ridge regression equation developed by linking the dependent variable and independent variables of ridership and service quality (inflows and outflows).

The model represents data (parameters) which is fetched from the sustainability reports of the STM and other annual reports and are interconnected with the help of links. The data will be mapped to create equations that represents dynamic values which changes over time. These dynamic values are the dependencies in the system that are connected to flow rates of ridership and service quality level. For example, the accessibility rate is a combination of the infrastructure accessibility and vehicle accessibility which are further divided into accessibility of the dwellings and accessibility of the metro and buses respectively by the customers. These dynamic variables influence a flow rate which is fed into an accumulator that increases or decreases over time. Time variation of the accumulation can be mapped to see the relationship among the variables and thereby, helping in influencing decisions to be made. The causal loop diagrams identify the feedback nature of the system.

System Dynamics (SD) was selected as the simulation methodology for this modeling due to its ability to capture the complex, feedback-driven nature of the transit system. It is particularly well-suited for understanding how different variables interact over time and how changes in one part of the system can affect the whole. This methodology allows for the creation of causal loop diagrams and stock-and-flow structures that clearly illustrate the reinforcing and balancing feedback loops within the transit system.

Using system dynamics, the model can effectively capture the interdependencies among service quality, ridership levels, and other influencing factors. System Dynamics is ideal for modeling continuous processes and understanding the long-term behaviour of systems, making it a perfect fit for exploring how improvements in service quality can lead to increased ridership and vice versa. This methodology also supports scenario analysis, enabling the prediction of system behaviour under various future conditions and the comparison of different strategic alternatives. Correlation analysis and regression analysis is used to determine the strength and direction of dependencies among different variables.

Other simulation techniques, such as Agent-Based Modeling (ABM) and Discrete-Event Simulation (DES), were considered but deemed less appropriate for this study. ABM focuses on the interactions of individual agents and their behaviours, which is more suitable for systems where individual actions and decisions are critical to the overall system dynamics. However, for a public transit system, the primary interest lies in the aggregate behaviour of the system rather than individual passenger actions. DES, on the other hand is specific on the operational aspects of the transit system, it is less effective in capturing the continuous feedback loops and long-term dynamics that are central to understanding and improving service quality and ridership levels.

The abstraction level of the model is high, focusing on the systemic interactions and aggregate effects rather than detailed operational specifics. This level of abstraction allows for a clearer understanding of the broad trends and patterns in the transit system, making it easier to identify key leverage points for intervention. The goal is to maximize the benefits of public transit by ensuring it meets social sustainability objectives, such as accessibility, affordability, and reliability. The simulation seeks to offer an improved comprehension of the ways in which different components interact and impact the overall performance of the transportation system by examining the system's behaviour under diverse inputs. While taking STM into account, system dynamics can simulate different scenarios and predict future behaviour under various conditions, which provides valuable insights that can guide strategic decision-making.

4 Modeling process

The model's task is to create an analysis of the service quality and ridership numbers in a public transportation system. The model seeks to identify critical points of influence and interdependencies between different elements that impact ridership and service quality. Transit authorities can enhance the overall performance and sustainability of STM by implementing strategic interventions based on their understanding of these dynamics. This involves balancing investment in infrastructure, security, accessibility, and fare policies to achieve a high-quality service that attracts and retains riders.

The model should answer several critical questions, including:

1. What parameters should be taken into consideration before building the model? (brainstorming and data collection)
2. Which parameters are the right ones to be connected to the dynamic variables that are dependent on the flow rate of ridership as well as service quality? (developing equations)
3. What are the most effective strategies for increasing ridership while maintaining or improving service quality which can be used as an end goal for developing the system? (developing causal loop diagram)
4. How do changes in service quality influence the other stock – Ridership Level? (connecting influencing factors to test the model)
5. What are the potential impacts of variables such as fare increase, accessibility rate etc. on dynamic variables? (Creating functions to call in various values of parameters from the dataset)
6. How can the transit system optimize its resources to achieve long-term sustainability goals? (optimisation and prospects)
7. What values can be altered to observe various model modifications, how is the procedure verified to see accurate values, and how do the models respond to various values? (Validation and verification)

System Dynamics (SD) modeling brings several unique advantages to solving this problem. It is particularly effective in capturing the complex feedback loops and time delays inherent in public transit systems. Unlike other modeling approaches, System Dynamics allows for a holistic view of the system, showing how different variables interact over time. This approach can reveal unintended consequences of certain policies and highlight long-term trends that might not be immediately apparent. The primary decision makers in the transit system include transit authorities, policymakers, and government officials responsible for public transportation. These stakeholders are tasked with making decisions about investments in infrastructure, security measures, fare policies, and overall service management.

Interactions in the System Dynamics model occur through feedback loops that connect stocks, flows, variables, and parameters. For example, improvements in infrastructure (a variable) can lead to an increase in the Service Quality Score (a stock), which in turn can attract more riders (affecting the ridership flow). These interactions are governed by mathematical equations that capture the relationships between different elements of the system, allowing the model to simulate how changes in one part of the system affect the whole. The data for the model can come from various sources, including historical records of ridership levels from sustainability reports, and financial records such as annual reports.

Additional data might be sourced from government statistics on population growth and economic conditions as well as other websites.

1. Step-by-Step Explanation of the System Dynamics Model

a. Problem Definition

The first step in developing a System Dynamics (SD) model is to clearly define the problem and the objectives of the modeling effort. In this context, the problem is the need to optimize service quality and ridership levels in a public transit system. The objectives include understanding the interdependencies among various factors influencing these metrics, identifying leverage points for improvement, and providing strategic recommendations for decision-making. This step is crucial as it sets the scope and direction of the entire modeling process.

b. System Conceptualisation

In the system conceptualization phase, a conceptual model is developed, outlining the key components of the system and how they interact. This involves identifying the main stocks (such as Service Quality Score and Ridership Level), flows (rates of change in these stocks), variables (factors influencing the stocks and flows), and parameters (constants that define the system's behaviour). For example, in our transit system model, the Service Quality Score might be influenced by variables like the security officer training level and the infrastructure improvement ratio. Conceptualizing these elements helps in visualizing the system's structure and dynamics, laying the foundation for the formal model.

c. Model Formulation

Model formulation involves translating the conceptual model into a formal system dynamics model using mathematical equations. This step requires defining the relationships between stocks, flows, variables, and parameters quantitatively. For instance, the flow into the Service Quality Score might be modeled as a function of the total ridership and the security officer training level. Similarly, the ridership inflow could depend on factors like customer satisfaction rate and accessibility rate. Formulating these relationships mathematically allows for the precise simulation of how changes in one part of the system affect the whole.

d. Data Collection

Data collection is essential to populate the model with accurate and relevant information. This involves gathering historical data on ridership levels, service quality indicators, infrastructure improvements, and other variables. For example, data on the average age of the fleet and the frequency of service interruptions can be collected from reports available about the STM such as sustainability reports as well as other information available about the STM in the internet. This step ensures that the model is grounded

and can produce reliable results. The quality and accuracy of the data directly impact the model's validity and usefulness.

e. Simulation and Testing

Once the model is formulated and populated with data, the next step is to run simulations to test its behaviour under different scenarios. This involves using the model to simulate various conditions and observe how the system responds. For the simulation, the model must initially respond to data collected through assumptions and regression analysis. Then, the model should slowly adapt to equations that are more streamlined and highly correlated.

For example, the model can simulate the impact of increasing the number of buses on ridership levels. Testing the model helps identify any inconsistencies or errors in the assumptions and equations, allowing for refinements to improve accuracy. This step is crucial for ensuring that the model behaves as expected and produces meaningful insights.

f. Verification and validation

Verification in the context of the System Dynamics model for the public transit system involves ensuring that the model accurately represents the conceptual framework and mathematical relationships as intended. This process includes checking the model's structure, equations, and data inputs for consistency and correctness. One key aspect of verification is conducting thorough reviews of the model's code and logic to ensure that all feedback loops, stocks, flows, variables, and parameters are correctly implemented. For example, we would verify that the equations governing the influence of infrastructure improvements on the Ridership level accurately reflect the intended relationships. This step is crucial to build confidence that the model is correctly constructed and ready for further analysis.

Validation involves comparing the model's outputs with real-world data to ensure that it accurately represents the behaviour of the actual transit system. This process includes back testing the model by using historical data to see if it can correctly predict past behaviour. Several machine learning algorithms can be used in this case. For instance, we could validate the model by comparing its predictions for ridership levels and service quality scores over the past five years with actual historical data. Sensitivity analysis is another critical part of validation, where we test how sensitive the model's outputs are to changes in key parameters. If the model's predictions align closely with observed data and show realistic responses to changes in parameters, it indicates that the model is valid.

Implementation and Monitoring

The final step involves implementing the recommended system variables and run the system producing corresponding outcomes. The model continues to be a valuable tool in this phase, as it can simulate

ongoing adjustments and refinements based on real-time data and feedback. For example, if a process to improve infrastructure is monitored and tested, the model can be used to find its impact and relationship on service quality levels and ridership levels, make further adjustments and give optimised results as needed. Continuous monitoring and adaptation ensure that the system remains optimized and responsive to changing conditions.

2. Entities with behaviour

Entities such as stocks, flows, variables, and parameters are essential in each step of the modeling process because they represent the fundamental components of the system being studied.

- a. **Stocks:** Represent the state of the system at any given time. These include the Service Quality Score and Ridership Level, representing the accumulated state of service quality and the number of riders, respectively.
- b. **Flows:** Indicate the rates of change in stocks. These are the rates at which service quality and ridership levels change over time, influenced by various factors such as infrastructure improvements, security enhancements, fare changes etc. (influenced by dynamic variables)
- c. **Dynamic Variables:** Influence the flows and stocks by representing dynamic characteristics. These include factors like accessibility rates, infrastructure improvement ratios, and customer satisfaction rates. These are equations developed and designed among the interconnection of various parameters available in the system.
- d. **Parameters:** These are constants in the model that influence the behaviour of variables and flows, such as the initial ridership level, security officer training level etc. Advanced Java functions are used in the construction of objects and events to allow for the switching of data values during year updation process when the model is running.

3. Additional Operations after running the model

The system dynamics model suits characteristic behaviours of calling data parameters every year by building an event function from the agent palette. This keeps on rotating the values every year to identify relationships and dynamic behaviour. The system dynamics model can be integrated with agents to create behavioural patterns and interact with the model during system run. Additionally, state transitions can help the system to switch between states and change dependent variable values to create new interdependencies. State charts can be mentioned as a control that switches between states when a message, timeout etc. is triggered.

5 Simulation System Implementation

The model built in AnyLogic depicts a comprehensive simulation of a public transit system designed to explore the dynamics between service quality and ridership levels. The model incorporates various factors influencing these two central metrics, using a system dynamics approach to capture the feedback loops and interactions within the system.

For building the model, different parameters are analysed. Incorporating various parameters involve a rigorous task of testing the values that help the parameters to create dynamic variables which further create linkages between flowrates.

1. Building the initial model

The first step in creating the model is to analyse the inflows and outflows of the two stocks that are taken into account – Ridership Level and Service Quality Score.

- a. Ridership level measures the number of passengers utilizing the transit system.
- b. Service Quality Score represents the overall quality of the transit service, encapsulating various factors that contribute to passenger satisfaction and service reliability.

New model is created after clicking the new model from the toolbar. The model time unit is set as years. A graphical editor appears where the model to be created will be added on. The left side of the graphical editor incorporates various tools for creating different types of models such as agent based and even system dynamics. Appropriate number of stocks and flows are dragged and dropped into the graphical editor.

The initial values for both the stocks are set into 0 while two flow rates are created into and out of the accumulated stocks to represent the flow.

- A. Inflow rate of Ridership: This flow increases the Ridership Level, representing factors that attract more passengers to the transit system.
- B. Outflow rate of Ridership: This flow decreases the Ridership Level, representing factors that deter passengers from using the transit system.
- C. SQ Improvement Rate: This flow increases the Service Quality Score, representing improvements made to the transit system.
- D. SQ Declining Rate: This flow decreases the Service Quality Score, representing factors that degrade service quality.

Once the stocks and flows are defined, we can look at the factors influencing the stock and flow. The properties section suggests that the rate of flow increases or decreases the stock unit with respect to time. Then, different parameters influencing this rate must be set up.

The data that is available from sources like sustainability reports and are also gathered from external sources for the years under study – 2006 to 2014. Few data that are brainstormed to be in the system include:

- a. Overall Population and its growth rate over the selected years.
- b. Customer Feedback: Sustainability reports indicating regular passenger surveys which can be useful in finding out where there are deficiencies and what adjustments need to be made to better accommodate the riders.
- c. Accessibility Enhancements: Addressing the transportation needs of the challenged or disabled passengers (paratransit customers) as well as the availability of infrastructure and vehicles for all customers that enables everyone in the society to use the transit systems.
- d. Equipment Failure: The failure of cars, structures or tools can hamper the delivery of services and cause discomfort to passengers. It can be developed as the relative ages of the overall infrastructure accessible to customers. Rate of failure data can also be used (MTBF).
- e. Security level: This includes how well a passenger feel safe using the infrastructure of the STM.
- f. Fare policies and procedures: How pricing during the 9-year period reduce or increase the impact of making decisions among the customers whether to use STM or not. This can be either calculated by knowing the increase rate of fare or the frequency or change in discounts applied to trips and free trips given as a total rate.
- g. Trips from paratransit customers are also considered important since paratransit customers make a good portion of STM usage and keeping customer satisfaction among them can be considered a social objective.
- h. The overall capital invested on the infrastructure can be taken in a ratio of the total goods and services acquired.

2. Defining the variables that influences other variables and parameters

With the data available, different parameters can be set up: These are:

1. Population growth rate: The overall population growth rate over the years.
2. Total Number of Paratransit Trips: Overall number of trips travelled by people as the sum of trips by people with motor disability, visual impairment, psychological disability and Intellectual disability.

3. Total Number of paratransit customers: Overall number of paratransit customers using the STM over the years 2006-2014 as a cumulative of the number of customers with motor disability, visual impairment, psychological disability and Intellectual disability.

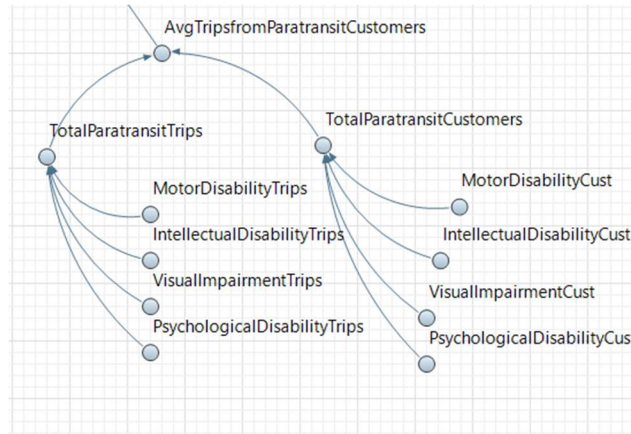


Figure 3 Trips from Paratransit Customers

4. Security Officer Training Level as a parameter: Measures the training quality of security personnel (constant value).
5. Number of buses available with the STM.
6. Sense of security in both metro and buses.
7. Total acquisition of goods and services as an expenditure on operational goods and services.
8. Total Number of Buses in Fleet: Indicates the capacity of the bus service. (bus fleet composition)
 - a) Percentage of Buses with Bicycle Racks: Enhances accessibility and attractiveness of the service to cyclists.
 - b) Percentage of Articulated Buses: Articulated buses can carry more passengers, affecting capacity and ridership.
 - c) Percentage of Buses with Front Ramp: Improves accessibility for passengers with disabilities.
9. Operational metrics in terms of reliability (overall reliability of the bus service and the average delay of trips for buses).
10. Dwelling Proximity
 - a. Percentage of Dwellings within 500 m – Morning Rush Hour, Day, Evening, Saturday, Sunday: Indicates the accessibility of transit services to residential areas during different times.
 - b. Percentage of Dwellings within 1,000 m – Night: Indicates the accessibility during nighttime.
11. Metro Station Accessibility
 - a. Percentage of Metro Stations with Elevators: Important for accessibility for disabled passengers.

- b. Percentage of Metro Stations with Escalators: Enhances convenience and accessibility for all passengers.

12. Percentage of Reduced Fare-Trips: The proportion of trips made using discounted fares. This impacts overall ridership and fare revenue.

13. Initial Ridership as a parameter: Indicates the value of ridership as a constant from the first year. (2006)

14. Total Ridership: A rate of total trips in millions of trips influencing the discounted percentage of reduced fare.

15. Metro Service Provided and Surface Network Service Provided: Measures of service coverage and frequency.

16. Customer Satisfaction and Dissatisfaction: Passenger satisfaction levels from the data given.

17. Number of Free Trips: Total number of trips taken for free.

18. Rate of increase in the fare over the years.

These values influence each other to create dynamic variables which are connected to the flowrate to represent interdependencies.

3. Dynamic Variables (higher order)

To define variables, equations and logical connections are required. For example, a percentage value should only be added to a percentage value. Different variables and their corresponding equations found from the data are:

- a. Sense of Security: Passengers' perceived safety while using the transit system, influencing their satisfaction and service quality score. It is a cumulative of sense of security of metro and sense of security of the buses.

$$\text{Sense of Security} = \text{Sense of security, Bus} + \text{Sense of Security, Metro}$$

- b. Accessibility Rate: The ease of access to transit services for all users. This can be a cumulative value of the total accessibility of both infrastructure and vehicles.

$$\text{Accessibility Rate} = \text{Infrastructure Accessibility} + \text{Availability of Accessible Vehicle (buses and metro)}$$

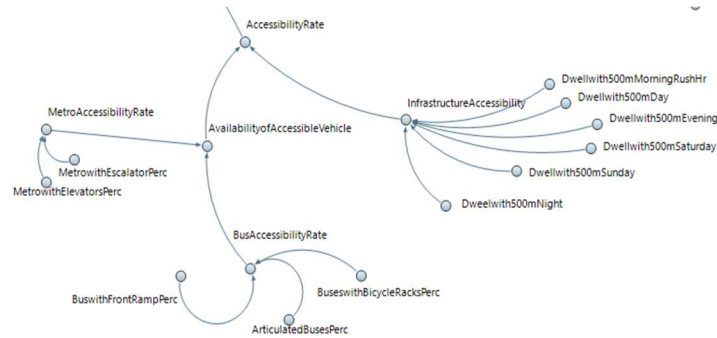


Figure 4 Accessibility Rate

- c. Network Service Coverage Provided: The extent of the transit network's reach, increasing convenience.

Network Service Coverage Provided = Service provided by metro network + service provided by buses

- d. Average Age of Fleet: The average age of buses and metro cars.
- e. Infrastructure Improvement Ratio: The rate of upgrades and maintenance of transit infrastructure, directly improving service quality and ridership levels.

Infrastructure Improvement Ratio = Total capital investment/Total value of goods and services which are acquired over the years

- f. Service Interruptions: The frequency and severity of disruptions in service.
- Expressed as a percentage of ratio of Mean distance between failure for buses and average of trip delays.

- g. Discounted Fare Rate of Influence: The impact of fare discounts on attracting more passengers to use the transit system.

Discounted fare rate = $(1.8 \times \text{Reduced Fare Number}) + (19 \times \text{Number of Free Trips})$

Where reduced fare number = Total ridership x Reduced fare percentage.

- h. Paratransit Accessibility Rate: The level of accessibility of transit services for paratransit users, influencing their ridership levels. It is calculated as the ratio of total number of paratransit trips to the total number of paratransit customers.

4. Causal loops

The model created will be individually connected to each rate of flow as explained in the figure below. The main agenda of the model creation is to create a causal relationship between ridership and service quality (as a performance score) to understand the behaviour of both with respect to each other and create an interdependency among the variables connected to it. Improved service quality leads to increased ridership, which generates more revenue and supports further enhancements in service quality.

The loops are created after connecting the dynamic variables to the inflows and outflows respectively. Then, the two stocks are connected to create an overall performance score of service level. The accumulated service quality score and the total number of trips accumulated in the ridership level is connected to the inflows of each other respectively to understand the relationship between the two.

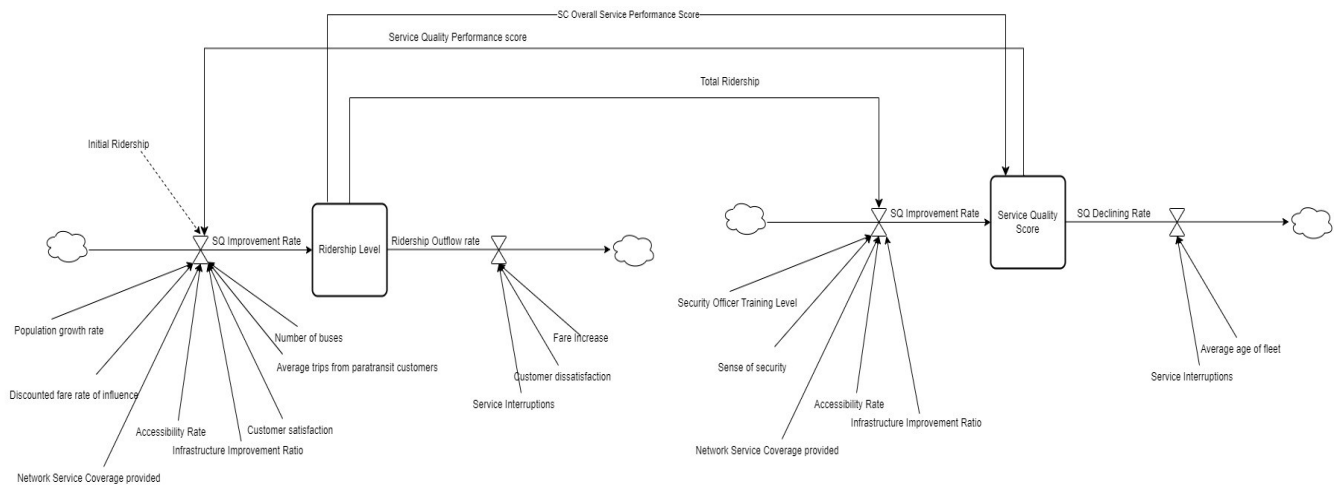


Figure 5 Model mock-up

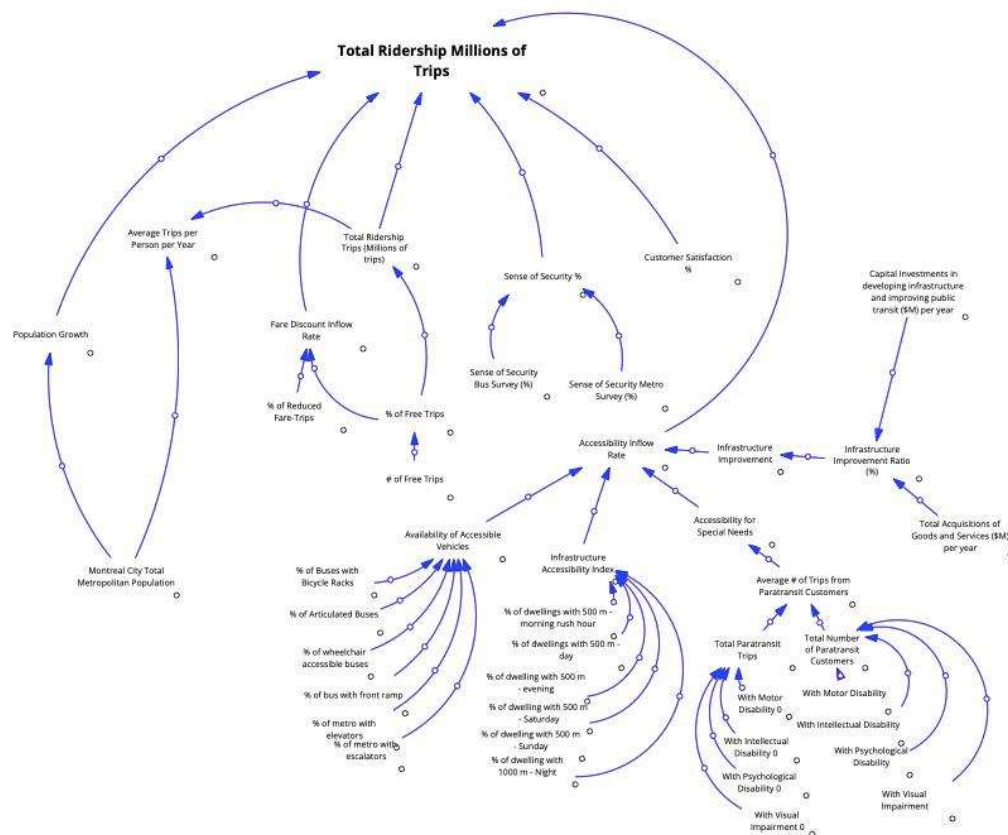


Figure 6 Causal loop diagram

5. Implementation

After determining the causal loop diagram and calibration of the equations of dynamic variables, the next step is the implementation of the model. This can be done by two phases. Firstly, we used linear regression to test the values and corresponding interactions.

Using linear regression, the tested Inflow rate and outflow rate of ridership level and service quality level are mentioned below:

Ridership Increase Rate = (Population growth x -56470000) + (Network Service Coverage Provided x 1806.5578) + (Infrastructure Improvement Ratio x -21750000) + Initial Ridership + (Customer Satisfaction Rate x 219200000) + (Accessibility Rate x -114900000) + (Discounted Fare Rate of Influence x -0.1916) + (SI Service Quality Score x 47690000) + (Number of Buses x -143700) + (Average Trips from Paratransit Customers x -374600) + 525400000

Ridership Decrease Rate = (Customer Dissatisfaction Rate x -107800000) + (Fare Increase x 351400000) + (Service interruptions x -381000000) + 399100000

Service Quality Improvement Rate = (Sense of Security x 13.5264) + (Accessibility Rate x 0.4941) + Security Officer Training Level + (Network Service Coverage Provided x 0.000005797) + (Infrastructure Improvement Ratio x -0.5282) + (SI Ridership Trips x 0.0000000007788) + -13.8509

Service Quality Declining Rate = (Average Age of Fleet x 0.1310) + (Service Interruptions x 32.1900) + -6.0412

Then, another algorithm (Ridge regression) is used to test it again.

Using ridge regression, the tested Inflow rate and outflow rate of ridership level and service quality level are mentioned below:

Ridership Increase Rate = (Population growth x -2961082.903265464) + (Network Service Coverage Provided x 3964623.043734427) + (Infrastructure Improvement Ratio x 3043965.9452915816) + Initial Ridership + (Customer Satisfaction Rate x 285177.6661953458) + (Accessibility Rate x 1902041.2240823647) + (Discounted Fare Rate of Influence x 4925824.7209089445) + (SI Service Quality Score x 2662485.2886600737) + (Number of Buses x 2123833.910029024) + (Average Trips from Para transit Customers x -1147113.1980347312) + 393148208.636197

Ridership Decrease Rate = (Customer Dissatisfaction Rate x -4760698.105800799) + (Fare Increase x 10348287.315796513) + (Service interruptions x -3125059.3947411813) + 391786237.4749965

$(\text{SenseofSecurity} \times 0.07882208967083519) + (\text{Accessibility Rate} \times 0.08737008149756291) + (\text{Network Service Coverage Provided} \times 0.05903564460948984) + (\text{Infrastructure Improvement Ratio} \times 0.05104743950421533) + (\text{SI Ridership Trips} \times 0.045355233539337125) + 0.3921025675527193 + \text{Security Officer Training Level}$

$\text{Service Quality Declining Rate} = (\text{Average Age of Fleet} \times 0.1310) + (\text{Service Interruptions} \times 32.1900) + -6.0412$

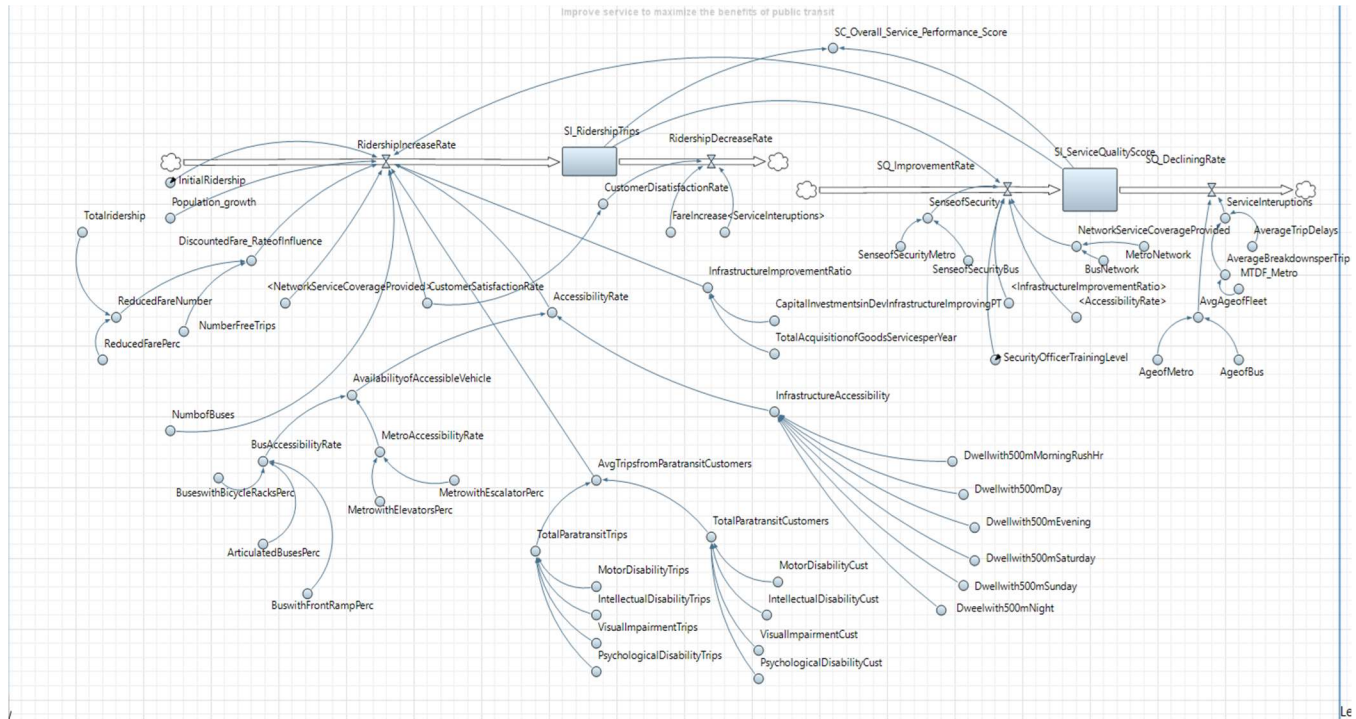


Figure 7 Created Model

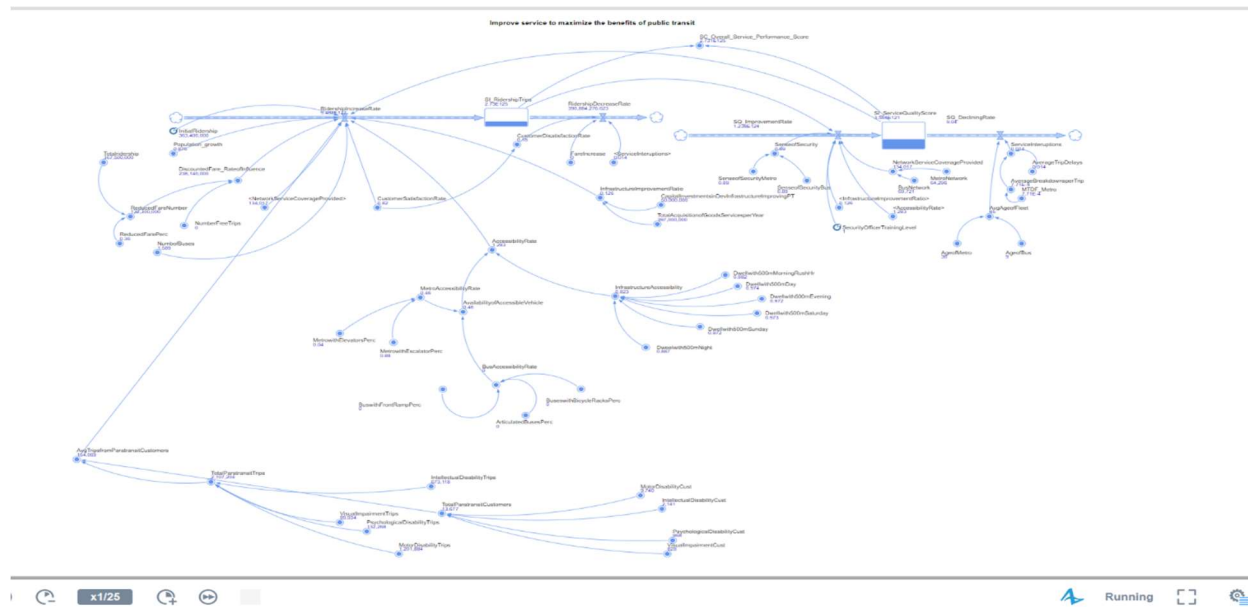


Figure 8 Model under Simulation

Testing with multiple algorithms ensures that your findings are robust and not specific to a single method. This can increase confidence in the model's reliability and validity during the latter stages.

6. Calling 'Event'

We developed a code that is designed to dynamically update and retrieve various transportation-related statistics for a given year, initially set to 2006. It uses a helper method, `getValueFromTable`, to access values from pre-defined tables corresponding to different variables (e.g., population growth, customer satisfaction, total ridership) based on the current year. Each dynamic variable has a dedicated method that invokes this helper method to fetch the relevant value. The `updateYear` method increments the `current_year` and traces/logs updated values for all dynamic variables, thus simulating a year-by-year update of these statistics. The main function demonstrates this by incrementing the year, retrieving the updated values, and logging them for verification, thereby providing a comprehensive overview of the changes in the transportation system metrics annually. This approach ensures that the variables are kept current and reflect the latest available data for any given year in the simulation.

6 Validation and Verification

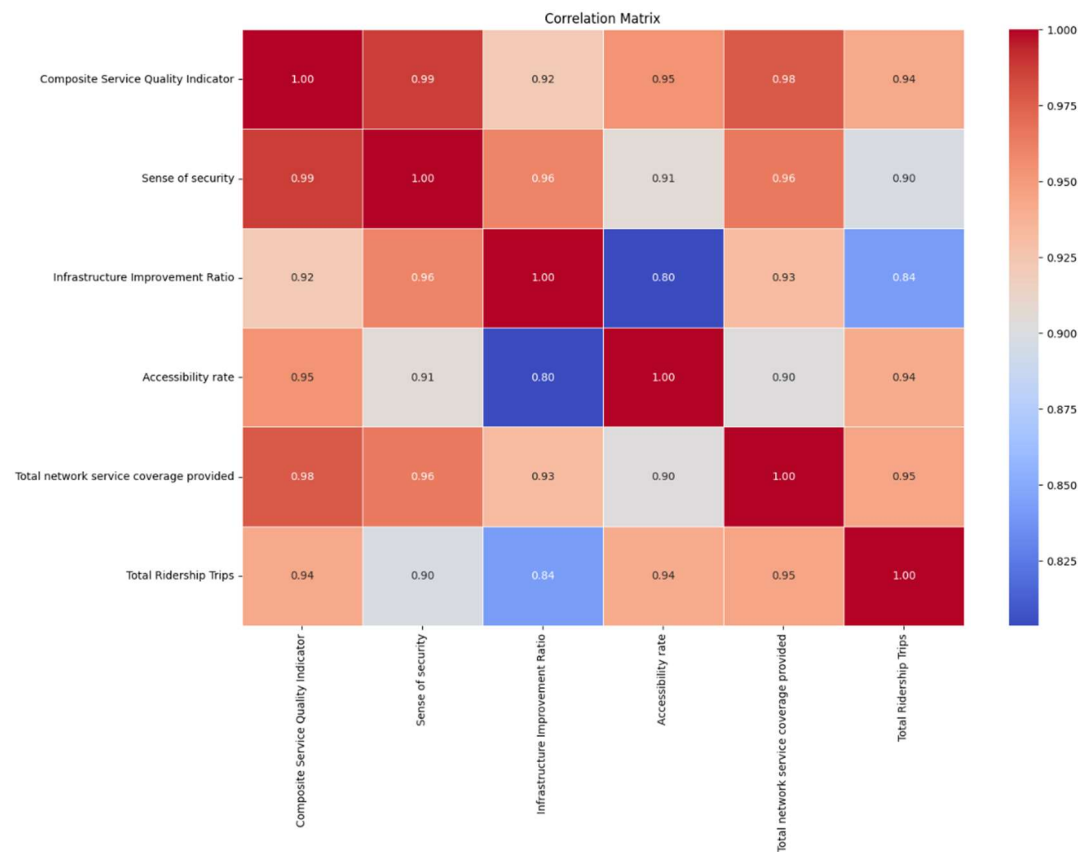
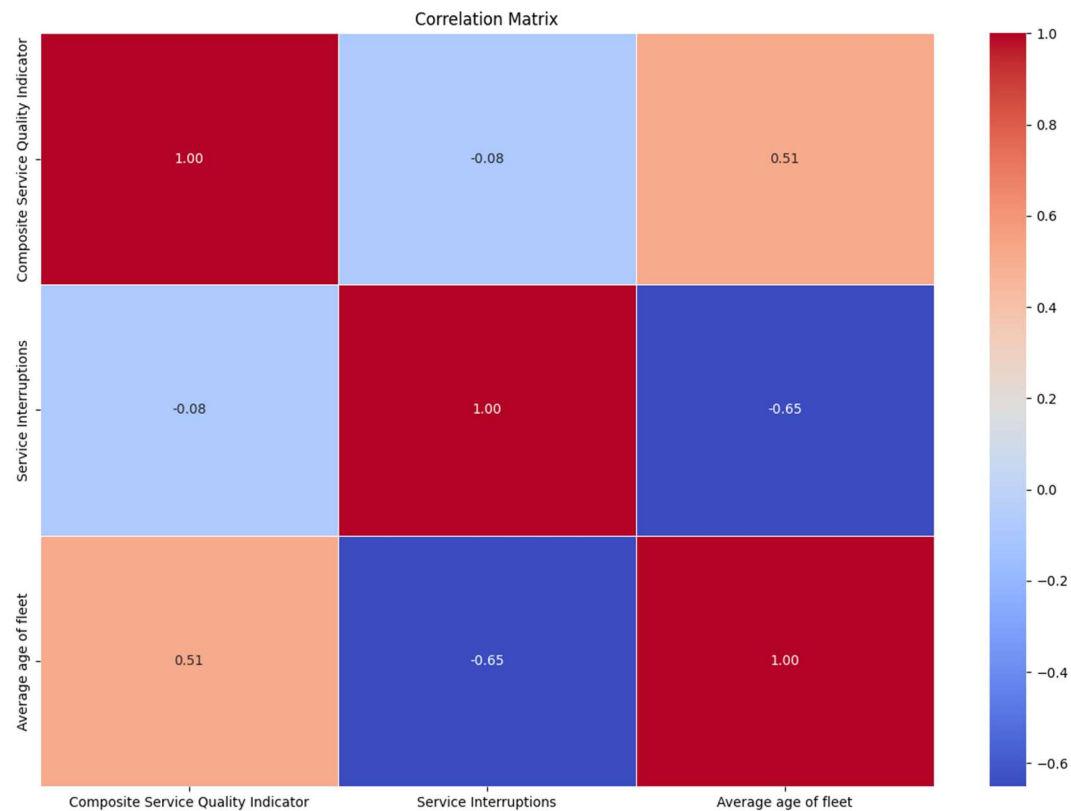
- a. Verification Process:** Verification ensures the model accurately reflects the intended relationships between variables in your system dynamics model. Different verification methodologies adopted during the project includes the following:

1. Model Inspection: In the data cleaning process, we properly prepared the dataset to ensure accuracy and completeness for the AnyLogic model. We began by identifying and handling missing values through various methods. For minor gaps in the data, we applied linear interpolation to estimate the missing values based on the surrounding data points. In cases where entire sections of data were missing, we used imputation techniques, such as replacing missing values with the mean or median of the available data to maintain consistency. Additionally, we averaged out fluctuating data points to smooth out anomalies and ensure a more reliable dataset. These steps ensured that the dataset was comprehensive and accurate, enabling robust and precise modeling in AnyLogic. We meticulously reviewed the dataset for any erroneous data entries, such as impossible values or inconsistent data points, and corrected them by cross-referencing with original data sources or using domain knowledge.

We conducted a comprehensive documentation review process, which involved systematically gathering and analysing relevant information from various sources. We reviewed past studies, reports, and academic papers that provided insights into the dynamics of public transit systems, including factors influencing service quality and ridership levels. Sources of data included websites, Sustainability reports and annual reports of STM, journals that helps in understanding parametric relationships and additional data gathered through other sources.

2. Structural Verification: We created relationships between variables in the causal loop diagram and the system dynamics model equations. To ensure the logic in building interactions in the causal loop diagram, we made equations through regression analysis so that each outflow and inflows match the intended representation of the system. The model simulated follow logical behaviour and represents the intended function. To understand the dependencies between different data variables, we conducted correlation and regression analysis to quantify the strength and direction of relationships.

A. Correlation Analysis: Different variables are correlated, and values are interpreted for both outflows and inflows of Ridership Levels and Service Quality Levels.



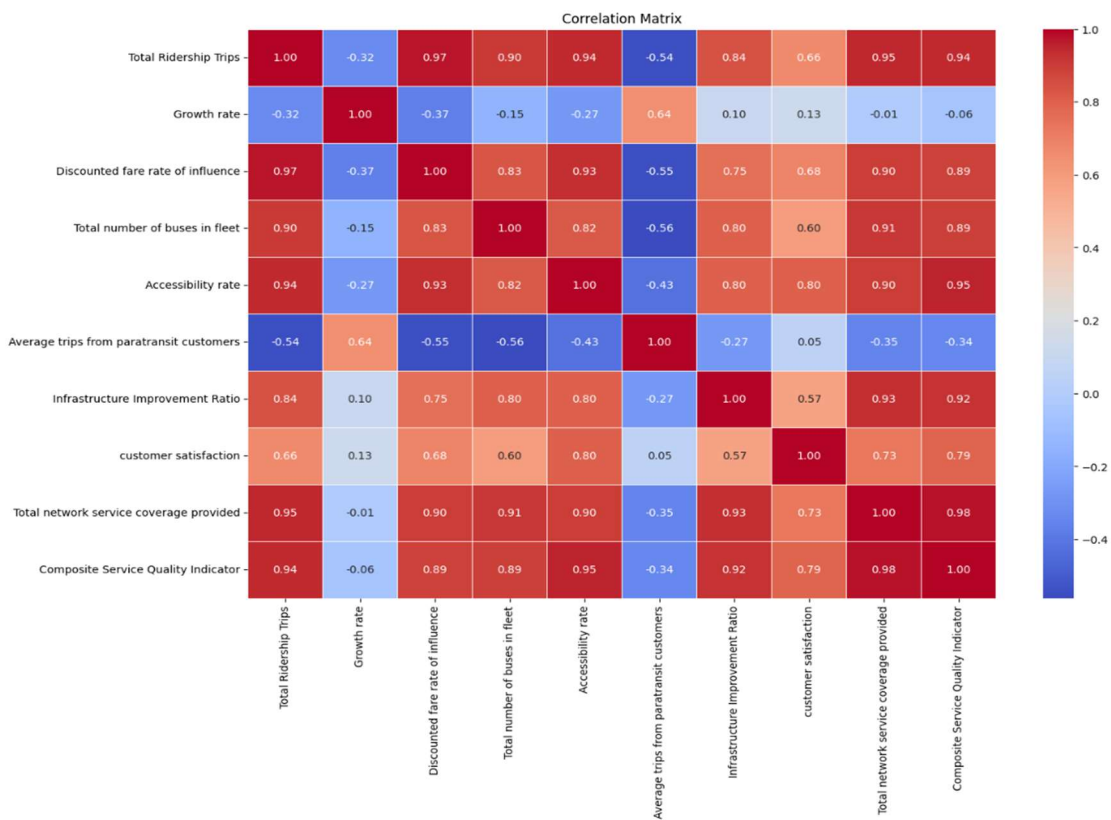
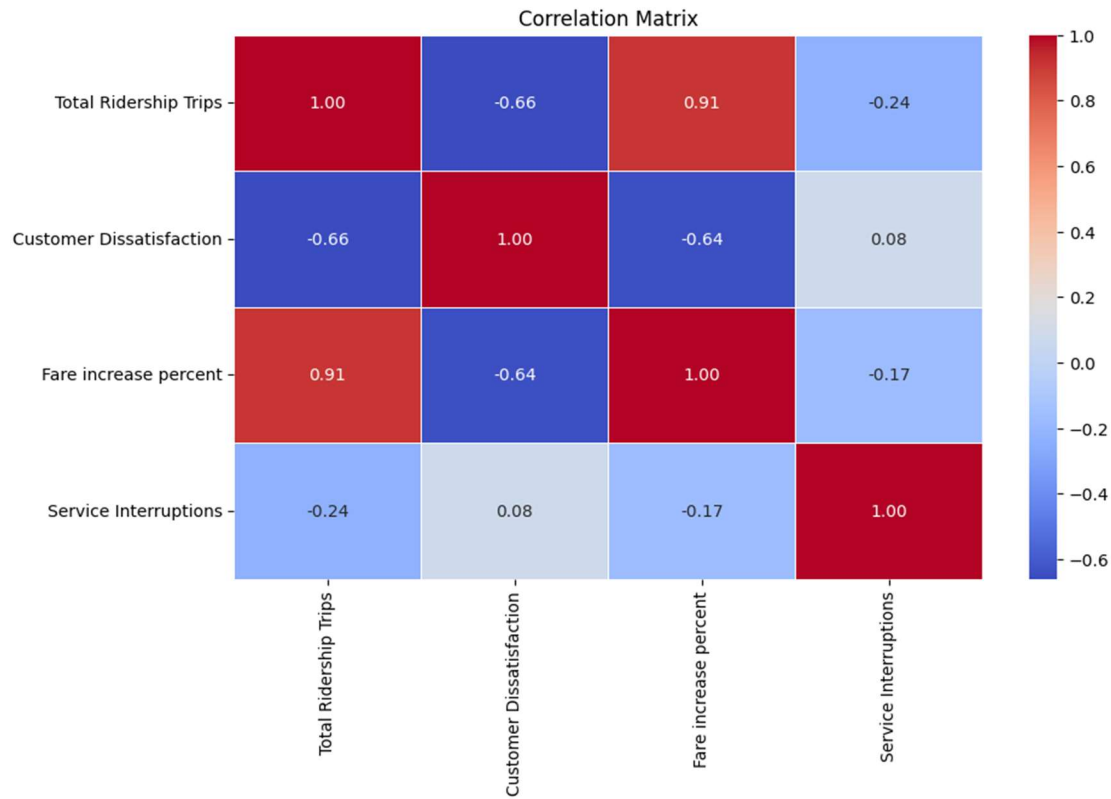


Figure 9 Correlation Matrices

The correlation matrices provide insights into the relationships between various factors affecting service quality and ridership in a transit system. The first matrix shows a moderate positive correlation (0.51) between the Composite Service Quality Indicator and the average age of the fleet, and a strong negative correlation (-0.65) between service interruptions and the average age of the fleet, indicating that newer fleets enhance service quality and reduce interruptions. The second matrix highlights very high positive correlations between the Composite Service Quality Indicator and factors such as sense of security (0.99), network service coverage (0.98), and infrastructure improvement ratio (0.92), emphasizing the importance of these factors in enhancing service quality. The third matrix reveals that total ridership trips are strongly negatively correlated (-0.66) with customer dissatisfaction and strongly positively correlated (0.91) with fare increase percent, indicating that higher ridership is associated with lower dissatisfaction and higher fares. The fourth matrix shows that total ridership trips are strongly positively correlated with discounted fare rate of influence (0.97), accessibility rate (0.94), and total network service coverage provided (0.95), demonstrating that fare discounts, accessibility, and extensive service coverage significantly drive ridership. These matrices collectively highlight critical areas for strategic improvement to enhance both service quality and ridership in the transit system.

B. Regression Analysis

Along with correlation, linear regression analysis and ridge regression analysis is performed and the following results were interpreted.

OLS Regression Results						
Dep. Variable:	Total Ridership Trips	R-squared:	1.000			
Model:	OLS	Adj. R-squared:	-inf			
Method:	Least Squares	F-statistic:	nan			
Date:	Tue, 25 Jun 2024	Prob (F-statistic):	nan			
Time:	05:09:43	Log-Likelihood:	-38.267			
No. Observations:	9	AIC:	94.53			
Df Residuals:	0	BIC:	96.31			
Df Model:	8					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
const	5.254e+08	inf	0	nan	nan	nan
Growth rate	-5.647e+07	inf	-0	nan	nan	nan
Discounted fare rate of influence	-0.1916	inf	-0	nan	nan	nan
Total number of buses in fleet	-1.437e+05	inf	-0	nan	nan	nan
Accessibility rate	-1.149e+08	inf	-0	nan	nan	nan
Average trips from paratransit customers	-3.746e+05	inf	-0	nan	nan	nan
Infrastructure Improvement Ratio	-2.175e+07	inf	-0	nan	nan	nan
customer satisfaction	2.192e+08	inf	0	nan	nan	nan
Total network service coverage provided	1806.5578	inf	0	nan	nan	nan
Composite Service Quality Indicator	4.769e+07	inf	0	nan	nan	nan

OLS Regression Results						
Dep. Variable:	Total Ridership Trips	R-squared:	0.850			
Model:	OLS	Adj. R-squared:	0.760			
Method:	Least Squares	F-statistic:	9.438			
Date:	Wed, 26 Jun 2024	Prob (F-statistic):	0.0168			
Time:	06:24:36	Log-Likelihood:	-155.31			
No. Observations:	9	AIC:	318.6			
Df Residuals:	5	BIC:	319.4			
Df Model:	3					
Covariance Type:	nonrobust					
	coef	std err	t	P> t	[0.025	0.975]
const	3.991e+08	3.19e+07	12.508	0.000	3.17e+08	4.81e+08
Customer Dissatisfaction	-1.078e+08	1.87e+08	-0.578	0.588	-5.87e+08	3.72e+08
Fare increase percent	3.514e+08	9.87e+07	3.561	0.016	9.78e+07	6.05e+08
Service Interruptions	-3.81e+08	7.72e+08	-0.493	0.643	-2.37e+09	1.6e+09

OLS Regression Results

Dep. Variable:Composite Service Quality Indicator

R-squared:0.999

Model:OLS

Adj. R-squared:0.997

Method:Least Squares

F-statistic:595.0

Date:Tue, 25 Jun 2024

Prob (F-statistic):0.000108

Time:05:51:53

Log-Likelihood:28.465

No. Observations:9

AIC:-44.93

Df Residuals:3

BIC:-43.75

Df Model:5

Covariance Type:nonrobust

coefstd errtP>|t|[0.0250.975]

const-13.85092.217-6.2480.008-20.905-6.796

Sense of security13.52642.6415.1220.0145.12121.931

Infrastructure Improvement Ratio-0.52820.277-1.9090.152-1.4090.352

Accessibility rate0.49410.2022.4440.092-0.1491.138

Total network service coverage provided5.797e-062.53e-062.2910.106-2.26e-061.38e-05

Total Ridership Trips7.788e-101.43e-090.5460.623-3.76e-095.32e-09

OLS Regression Results

Dep. Variable:Composite Service Quality Indicator

R-squared:0.374

Model:OLS

Adj. R-squared:0.166

Method:Least Squares

F-statistic:1.794

Date:Wed, 26 Jun 2024

Prob (F-statistic):0.245

Time:06:28:10

Log-Likelihood:-0.47758

No. Observations:9

AIC:6.955

Df Residuals:6

BIC:7.547

Df Model:2

Covariance Type:nonrobust

coefstd errtP>|t|[0.0250.975]

const-6.04123.469-1.7420.132-14.5292.447

Service Interruptions32.190030.8921.0420.338-43.400107.780

Average age of fleet0.13100.0701.8790.109-0.0400.302

Figure 10 Linear Regression results

Mean Squared Error: 1917932756374.437
R-squared: 0.9968047767490639
Intercept: 393148208.636197
Coefficient for Growth rate: -2961082.903265464
Coefficient for Discounted fare rate of influence: 4925824.7209089445
Coefficient for Total number of buses in fleet: 2123833.910029024
Coefficient for Accessibility rate: 1902041.2240823647
Coefficient for Average trips from paratransit customers: -1147113.1980347312
Coefficient for Infrastructure Improvement Ratio: 3043965.9452915816
Coefficient for customer satisfaction: 285177.6661953458
Coefficient for Total network service coverage provided: 3964623.043734427
Coefficient for Composite Service Quality Indicator: 2662485.2886600737

Mean Squared Error: 158067782606815.03
R-squared: 0.7366634192306288
Intercept: 391786237.4749965
Coefficient for Customer Dissatisfaction: -4760698.105800799
Coefficient for Fare increase percent: 10348287.315796513
Coefficient for Service Interruptions: -3125059.3947411813

Mean Squared Error: 0.003956080757542365
R-squared: 0.9774177748077048
Intercept: 0.3921025675527193
Coefficient for Sense of security: 0.07882208967083519
Coefficient for Infrastructure Improvement Ratio: 0.05104743950421533
Coefficient for Accessibility rate: 0.08737008149756291
Coefficient for Total network service coverage provided: 0.05903564460948984
Coefficient for Total Ridership Trips: 0.045355233539337125

```

Mean Squared Error: 0.13760391346167514
R-squared: 0.21452499289648852
Intercept: 0.4380536437130083
Coefficient for Service Interruptions: 0.11082642771960971
Coefficient for Average age of fleet: 0.1911197342938665

```

Figure 11 Ridge Regression results

3. Dimensional Verification: During the modeling process, we undertook a thorough dimensional verification to ensure consistency in the units of measurement for all variables and parameters. Each variable, such as the Ridership Level and Population, was meticulously checked to confirm that the units were consistent and logical. For example, the ridership level was measured in terms of the "number of riders," while population figures were expressed in "number of people." We ensured that other dynamic variables like the Percentage of Buses with Bicycle Racks or the Accessibility Rate had units that were consistent with their descriptions, such as percentages or ratios. This static verification process was critical to prevent any unit mismatches that could lead to erroneous calculations and misleading simulation results.

We ran simulations and check if the resulting values for stocks and flows fall within realistic ranges and interpreted the result.

b. Validation

- 1. Baseline Testing:** This involved collecting and analysing historical data on key metrics such as service quality score, ridership levels, frequency of service interruptions, customer satisfaction rates, and operational efficiency. We used this data to run initial simulations in the AnyLogic model, capturing the existing dynamics and interactions between various system components. By comparing these baseline results with future simulation outcomes, we were able to identify the impact of different variables and interventions on the overall system performance, thereby validating the model's accuracy and ensuring it accurately reflected real-world conditions.
- 2. Sensitivity Analysis:** The outliers of the parameter initial ridership ranges from 545100000 trips to 181700000 trips. The variation of this value from it's maximum to minimum range produces different results to the model. The model can further be tested by changing the maximum level of security officer training level to its minimum realistic value of 20 percent and the results can be interpreted.
- 3. Predictive Analysis:** This is the important part of validation, and we focused more on this area where the model's predictive accuracy is evaluated by comparing its predictions with actual data.

For this process, we split the STM dataset used for simulation into train and test set (train-test split). To calibrate the model, the first five years of historical data, or the training set, will be reviewed. In order to

fit the historical data as closely as possible, parameters must be adjusted. Using the calibrated parameters, the model will run at the conclusion of the training phase and forecast the following four years (validation period). During the validation phase, the actual data from the STM Montreal dataset will be compared with the predictions of the model. 4 machine learning algorithms, Linear Regression, SVR, Random Forest and Ridge regression is used to compare the results and the best model is selected. Both the ridership value and the composite service quality value which is a cumulative sum of positive and negative weights of its inflows are predicted using the algorithms.

```
Predicted values and errors for the test data:
Linear Regression Predictions: [ 0.73542126 -0.1366888  0.63611742 -0.16059699]
Linear Regression Error: 0.024317115738519208
Ridge Regression Predictions: [ 0.76661471 -0.14089135  0.59571248 -0.19588179]
Ridge Regression Error: 0.052088551138390574
SVR Predictions: [0.53683413 0.44960652 0.53039889 0.45649831]
SVR Error: 0.3623574888996221
Random Forest Predictions: [0.62083766 0.25286714 0.60407931 0.24004019]
Random Forest Error: 0.21963712760979653
```

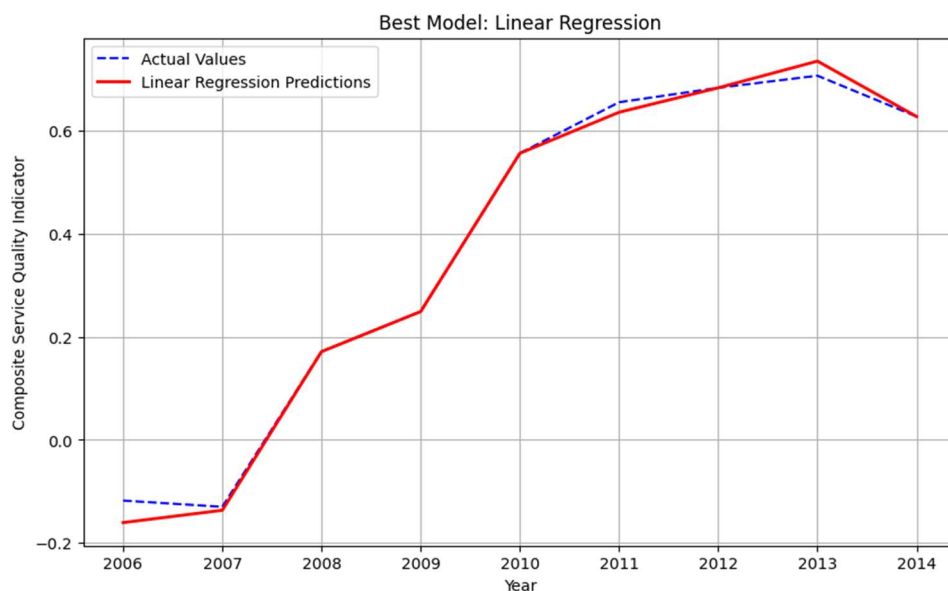


Figure 12 Linear Regression Prediction for Composite Service Quality Model

```
Predicted values and errors for the test data:
Linear Regression Predictions: [4.17090219e+08 3.77376338e+08 4.05654175e+08 3.74559336e+08]
Linear Regression Error: 5620017.024271101
Ridge Regression Predictions: [4.15890033e+08 3.76046519e+08 4.05020635e+08 3.73936980e+08]
Ridge Regression Error: 4978525.385139182
SVR Predictions: [3.88600001e+08 3.88600000e+08 3.88600001e+08 3.88600000e+08]
SVR Error: 22599999.557607055
Random Forest Predictions: [4.08959e+08 3.86739e+08 4.04642e+08 3.86525e+08]
Random Forest Error: 12515750.0
```

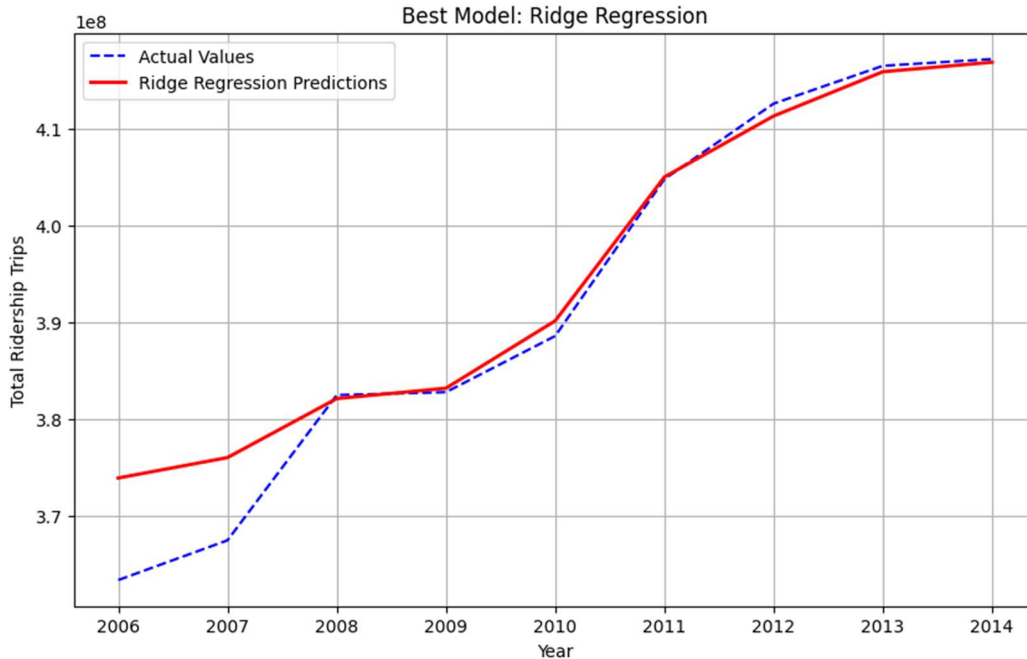


Figure 13 Ridge Regression Prediction for Ridership Level

7 Possible extensions

The further development of the model might include the following

- The system dynamics model for ridership and service quality offers a powerful foundation for understanding public transit systems. However, its potential extends far beyond its current capabilities. By incorporating new features and functionalities, future iterations of the model can provide even more comprehensive and insightful analyses, enabling data-driven decision making for optimized and sustainable public transit systems.
- The current model excels at identifying the factors influencing ridership and service quality. But future advancements can delve deeper, exploring the intricate relationships between these factors. Techniques like sensitivity analysis can be employed to understand how seemingly small changes in specific parameters (e.g., security budget increase, minor fare reduction) can have cascading effects on ridership and service quality. This granular analysis empowers public transit authorities (PTAs) like STM to prioritize investments strategically and allocate resources for maximum system improvement. For instance, the model might reveal that a seemingly cost-effective measure like reducing security personnel might lead to a decline in perceived safety, ultimately deterring ridership and negating any cost savings. Conversely, the model could identify a sweet spot where a targeted increase in security training significantly improves the service quality score without incurring excessive costs.

- c. The model can be further enhanced by incorporating state transformations triggered by parameter changes. For example, the switch function could be used to model the impact of interventions like increased security training. By defining different states for security training levels (e.g., basic, advanced, ongoing), the model can dynamically adjust the service quality score based on the current training state. This allows for a more nuanced understanding of how interventions translate into tangible improvements for riders. Additionally, future models can expand their scope by incorporating environmental factors like population density, walkability, and urban design. These factors significantly influence travel behaviour and public transit usage. For instance, the model could be designed to account for how higher population density areas might see increased ridership due to proximity to transit stops, while lower density areas might require alternative solutions like feeder buses or improved cycling infrastructure. Furthermore, factors like air quality and carbon emissions can be linked to social parameters like public perception. By including these environmental considerations, the model can paint a more holistic picture of how public transit systems interact with the broader urban ecosystem, allowing PTAs to make strategic decisions that not only improve ridership but also contribute to environmental sustainability goals.
- d. The current model focuses on core factors like security and accessibility. However, future iterations can integrate a wider range of variables to create a richer representation of the public transit system. This could include staff training levels, public perception of service, marketing effectiveness, and employee attrition rates. By considering how these factors interact with each other and influence ridership, PTAs can gain a deeper understanding of the human element within the system. For example, the model could be designed to analyze how increased staff training on customer service skills translates into higher passenger satisfaction, ultimately leading to increased ridership and fare revenue. Similarly, the model could explore the impact of marketing campaigns on public perception, potentially revealing the effectiveness of targeted social media campaigns versus traditional advertising methods in attracting new riders. By incorporating these human factors, the model can move beyond purely technical considerations and provide insights into the social aspects of public transit, empowering PTAs to make data-driven decisions regarding staff development, marketing strategies, and employee retention programs that not only optimize the system but also foster a more positive and welcoming environment for riders.
- e. The most transformative advancement lies in incorporating agent-based modeling (ABM) within the system dynamics framework. ABM allows for the creation of individual agents within the model, such as simulated commuters. These agents can have their own unique characteristics, travel preferences, decision-making processes, and responses to system changes. By analysing the collective behavior of these agents, the model can predict how ridership patterns might shift as a result of new policies,

infrastructure improvements, or even changes in weather conditions. Imagine a scenario where the model simulates the behaviour of thousands of virtual commuters as a new bus rapid transit line is introduced. The ABM component could analyse how individual agents adjust their travel routes based on factors like travel time, convenience, and cost, ultimately revealing the predicted impact of the new line on overall ridership and traffic congestion. This level of behavioural analysis empowers PTAs to anticipate the impact of interventions before they are implemented, allowing them to make informed decisions that lead to a more efficient and user-friendly public transit system that caters to the diverse needs of its ridership. Furthermore, ABM can be used to explore the long-term effects of system changes, helping PTAs understand how ridership patterns might evolve over time and identify potential bottlenecks or areas for further improvement.

- f. Public transit systems are complex ecosystems with diverse stakeholders. Future advancements in system dynamics modeling can embrace a more collaborative approach, allowing for the integration of data and insights from various stakeholders. For instance, PTAs could work with urban planning departments to incorporate planned development projects into the model, enabling them to anticipate the impact of these developments on public transit demand and adjust service levels accordingly. Additionally, PTAs could involve advocacy groups representing riders with disabilities or special needs in the modeling process. These groups could provide valuable data on accessibility challenges and potential improvements, allowing the model to be refined to better reflect the needs of all users. By fostering a collaborative modeling approach, PTAs can leverage the collective intelligence of various stakeholders to create a more comprehensive and inclusive model that informs data-driven decisions for a public transit system that is truly optimized for the benefit of all.

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