CEN 103: Numerical Methods and Computer Programming

BTech 1st year, Civil Engineering **Autumn 2021-22**

Practical 10: Numerical solution of nonlinear equations

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P4. Write a code to find root of x^3-2x-5=0 using Newton Raphson method. The iterations should be performed till the error is less than 1e-3 or number of iterations is more than 1000.
Using the same code can you solve P1? For the initial guess 1.1, what is the root given by your code?

Solution
the code is

```
#include <iostream>
using namespace std;
double Function(double x)
{
    return (x*x*x - 2*x - 5) ;
}
double Function_derivative(double x)
{
    return (3*x*x -2) ;
}
void new_Raphson(double x, double r)
{
    int i = 0;
    double a;

    do
    {
        a = x - (Function(x) / Function_derivative(x));
        x = a;
        i++;
} while (((Function(a)>= r)||(Function(a)<= -r))&&(i<1000));
 cout <<"The approximated root of equation is "<<a<< endl;
}
int main()
{
    double initial, range;
    cout << "Enter the initial guess " << endl;
    cin >> initial;
    cout << "Enter the permissible error " << endl;
    cin >> range;
    new_Raphson(initial   , range);
     return 0;
}
```

Output

Enter the initial guess
2

Enter the permissible error
0.001
The approximated root of equation is 2.09457

Same code for 1st problem

Enter the initial guess
1.1
Enter the permissible error
0.001
The approximated root of equation is -1.8955

CEN-103

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P1. $f(x) = x - 2\sin x$

(a) $f'(x) = 1 - 2\cos x$

$x_0 = 1.1$ $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$\Rightarrow 1.1 - \left(\frac{1.1 - 2\sin(1.1)}{1 - 2\cos(1.1)}\right)$

$x_1 = 1.1 - \left(\frac{-0.682415}{0.092807}\right)$

$x_1 = 8.45299$

$x_2 = 8.45299 - \left(\frac{8.45299 - 2\sin(8.45299)}{1 - 2\cos(8.45299)}\right)$

$x_2 = 5.25641$

$x_3 = 5.25641 - \left(\frac{5.25641 - 2\sin(5.25641)}{1 - 2\cos(5.25641)}\right)$

$x_3 = 203.384.$

$x_4 = 203.384 - \left(\frac{203.384 - 2\sin(203.384)}{1 - 2\cos(203.384)}\right)$

$x_4 = 118.019$

$$x_5 = 118.019 - \left(\frac{118.019 - 2\sin(118.019)}{1 - 2\cos(118.019)} \right)$$

$x_5 = -87.4709$

$$x_6 = -87.4709 - \left(\frac{-87.4709 - 2\sin(-87.4709)}{1 - 2\cos(-87.4709)} \right)$$

$x_6 = \cancel{128}\ -203.636$

After 6 iterations we got value of $x_6 = -203.636$

b) for $x_0 = 1.5$

$$x_1 = 1.5 - \left(\frac{1.5 - 2\sin(1.5)}{1 - 2\cos(1.5)} \right)$$

$x_1 = 2.07656$

$$x_2 = 2.07656 - \left(\frac{2.07656 - 2\sin(2.07656)}{1 - 2\cos(2.07656)} \right)$$

$x_2 = 1.91051$

$$x_3 = 1.91051 - \left(\frac{1.91051 - 2\sin(1.91051)}{1 - 2\cos(1.91051)} \right)$$

$x_3 = 1.89562$

$$x_4 = 1.89562 - \left(\frac{1.89562 - 2\sin(1.89562)}{1 - 2\cos(1.89562)}\right)$$

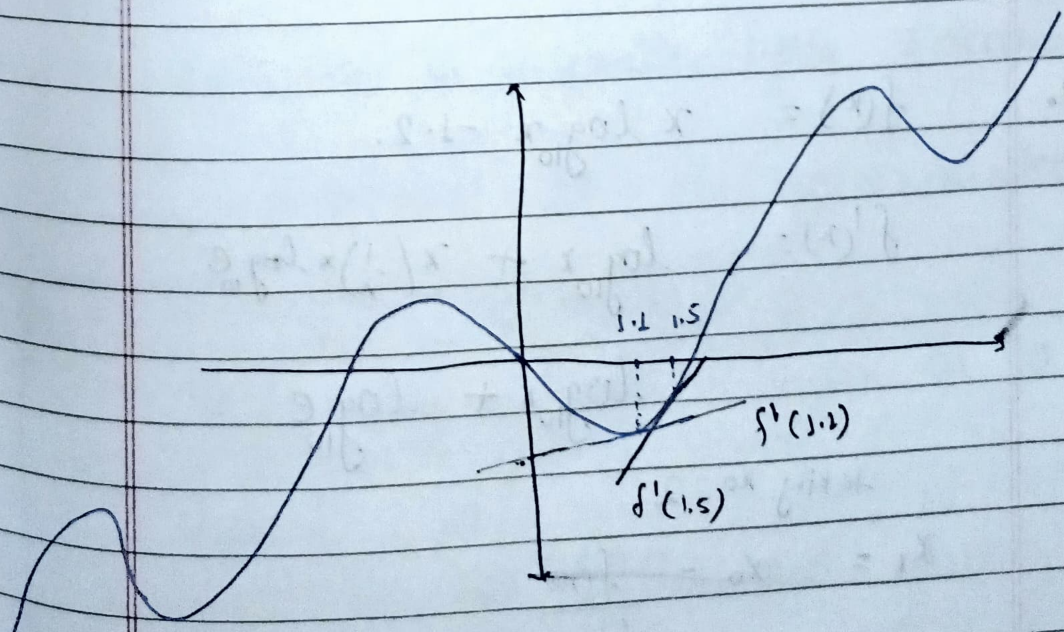
$x_4 = 1.89549$

$$x_5 = 1.89549 - \left(\frac{1.89549 - 2\sin(1.89549)}{1 - 2\cos(1.89549)}\right)$$

$x_5 = 1.8954942$

So the value is matching upto 5 decimal places.

(c). The graph of Given f^n is.



the $f'(1.1) = 0.0928.$

$f'(1.5) = 0.8585$

As at 1.1 $f'(1.1)$ is tending to 0 making slope parallel to x axis and that's why it intersect very far away.

In 1.5 $f'(1.5)$ is 0.8585 that cuts the x axis near by.

So the drawback is if the slope of f^n becomes zero at some point it will create the tangent line parallel to x axis and it will never intersect axis and we don't get the root.

P 2.

$$f(x) = x \log_{10} x - 1.2.$$

$$f'(x) = \log_{10} x + x\left(\frac{1}{x}\right) \times \log_{10} e$$

$$= \log_{10} x + \log_{10} e$$

taking $x_0 = 2$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 2 - \left(\frac{2 \log_{10}(2) - 1.2}{\log_{10} 2 + 0.434294}\right)$$

$x_1 = 2.81316$

$$x_2 = 2.81316 - \left(\frac{(2.81)\log_{10}(2.81316) - 1.2}{\log_{10}(2.81316) - 0.4342944}\right)$$

$x_2 = 2.74111$

$$x_3 = 2.74111 - \left(\frac{(2.74)\log_{10}(2.74111) - 1.2}{\log_{10}(2.74111) + 0.4342944}\right)$$

$x_3 = 2.74065.$

$$x_4 = 2.74065 - \left(\frac{(2.74065)\log_{10}(2.74065) - 1.2)}{\log_{10}(2.74065) + 0.4342944}\right)$$

$x_4 = 2.7406531$

So x_4 is similar upto 5 decimal places

2.74065 is the approximated root.

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Ans 3 $f(x) = \cos x - x e^{x}$

$f(0) = 1$

$f(1) = -2.177979$

$f(0.6) = -0.267936$

Let $x_{-1} = 0$ $f(0) = 1$

$x_0 = 0.6$ $f(x_0) = -0.267936$

1st Approx

$x_1 = 0.6 - (0.267936)\left(\dfrac{0 - 0.6}{1 - (-0.267936}\right)$

$x_1 \Rightarrow 0.47321$

$f(x_1) \Rightarrow \cos(0.47321) - (0.47321)e^{(0.47321)}$

$f(0.47321) = 0.130542$

2nd Approx

$x_2 = 0.47321 - (0.130542)\left(\dfrac{0.6 - 0.47321}{-0.267 - 0.130}\right)$

$x_2 = 0.514747$

3rd Approximation

$f(x_2) = 0.009135$

$x_3 = 0.514747 - \dfrac{(0.009135)(0.47321 - 0.514747)}{(0.130542 - 0.009135)}$

$x_3 = 0.517872.$

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4th Approx

$f(x_3) = -0.000349687$

$$x_4 = 0.517872 - (-0.000349)\frac{(0.514747 - 0.517872)}{(0.009135 - (-0.000349687))}$$

$x_4 = 0.517757$

$f(x_4) = 8.8265 \times 10^{-7}$ So

The Root of eqn is 0.517757

Using Regula falsi Method.

$f(x) = \cos x - x e^x$.

$x_0 = 0$ $x_1 = 0.6$

$f(x_0)\, f(x_1) < 0$

first approximation.

$$x_2 = 0 - 1\left(\frac{0.6 - 0}{0.2679 - 1}\right)$$

$x_2 = 0.4732$

$f(x_2)$ 0.130571

$f(x_0)\, f(x_2) < 0$

$$x_3 = 0.6 - (-0.267936)\left(\frac{0.6 - 0.4732}{-0.26793 - 0.130571}\right)$$

$x_3 = 0.51472$

$f(x_3)$ 0.009165

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$f(x_1)\, f(x_3) < 0$

So we take $f(x_1)$ & (x_3).

$x_3 = (0.51472)$ $f(x_3) = ~~0.931~~ 0.0052165$

$$x_4 = 0.6 - \frac{(-0.267936)(0.6 - 0.51472)}{(-0.267936 - 0.009316)}$$

$x_4 = 0.5175837$

$f(x_4) = -0.003051$

$f(x_3)(x_4) < 0$

$$x_5 = (0.51472) - \frac{(0.00931)(0.51472 - 0.5175)}{(0.0052165 - (-0.003051))}$$

$x_5 = 0.517758.$

$f(x_5) = 0.00052.$

So Approximated Root is 0.517758

So We can see Regula falsi is a bit slow as after 5 approximation we only get error of 10^{-4} approximation 10^{-4} while in Secant Method in 4 approximation we got accuracy of ~~10~~ 10^{-7}.