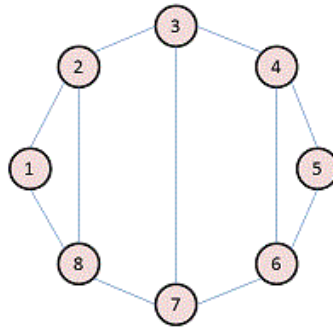


# Communities

## Question 1:

For the following graph:



Write the adjacency matrix  $A$ , the degree matrix  $D$ , and the Laplacian matrix  $L$ . For each, find the sum of all entries and the number of nonzero entries.

Adjacent matrix:

	1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0	1
2	1	0	1	0	0	0	0	1
3	0	1	0	1	0	0	1	0
4	0	0	1	0	1	1	0	0
5	0	0	0	1	1	0	0	0
6	0	0	0	1	1	0	1	0
7	0	0	1	0	0	1	0	1
8	1	1	0	0	0	0	1	0

Number of non-zero entries = 22, Sum of all elements = 22

Degree Matrix:

	1	2	3	4	5	6	7	8
1	2	0	0	0	0	0	0	0
2	0	3	0	0	0	0	0	0
3	0	0	3	0	0	0	0	0

4	0	0	0	3	0	0	0	0
5	0	0	0	0	2	0	0	0
6	0	0	0	0	0	3	0	0
7	0	0	0	0	0	0	3	0
8	0	0	0	0	0	0	0	3

Number of non- zero entries = 8, Sum of all entries = 8

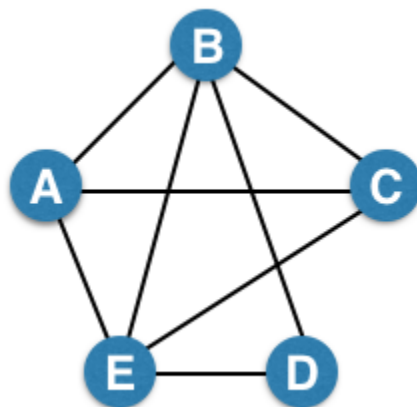
Laplacian matrix( $L=D-A$ ):

	1	2	3	4	5	6	7	8
1	2	-1	0	0	0	0	0	-1
2	-1	3	-1	0	0	0	0	-1
3	0	-1	3	-1	0	0	-1	0
4	0	0	-1	3	-1	-1	0	0
5	0	0	0	-1	2	-1	0	0
6	0	0	0	-1	-1	3	-1	0
7	0	0	-1	0	0	-1	3	-1
8	-1	-1	0	0	0	0	-1	3

Number of non-zero entries = 30, Sum of all entries = 0

### Question 2:

Consider the following undirected graph (i.e., edges may be considered bidirectional):



Run the "trawling" algorithm for finding dense communities on this graph and find all complete bipartite subgraphs of types  $K_{3,2}$  and  $K_{2,2}$ . Note: In the case of  $K_{2,2}$ , we consider  $\{\{W, X\}, \{Y, Z\}\}$  and  $\{\{Y, Z\}, \{W, X\}\}$  to be identical.

From the graph

$A=\{B, C, E\}, B=\{A, C, D, E\}, C=\{A, B, E\}, D=\{B, E\}, E=\{A, B, C, D\}$

So, B and E have support more than 3 Therefore, Bipartite subgraph of  $K_{3,2}$

1)A            B

C            E

D

$A=\{B,E\}$   $C=\{B,E\}$   $D=\{B,E\}$

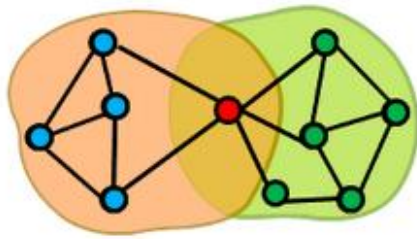
Bipartite subgraph of  $K_{2,2}$

1)A            C	2)B            A	3)B            A
B            E	E            D	E            C
$A=\{C,E\}$ $B=\{C,E\}$	$B=\{A,D\}$ $E=\{A,D\}$	$B=\{A,C\}$ $E=\{A,C\}$

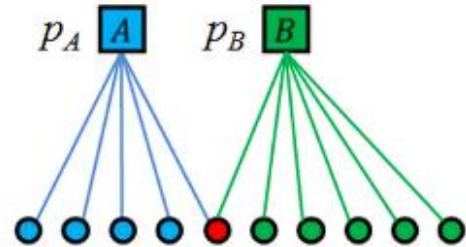
4)B            A	5)B            C
C            E	E            D
$B=\{A,E\}$ $C=\{A,E\}$	$B=\{C,D\}$ $E=\{C,D\}$

**Question 3:**

We fit AGM to the network on the left, and found the parameters on the right:



Network



Learned AGM parameters

Find the optimal values for  $p_A$  and  $p_B$ .

$$p_A = \frac{\text{no. of edges in the network}}{\text{total possible number of edges}} = \frac{7}{5 \times 4} = \frac{7}{20} = 0.35$$

$$p_B = \frac{\text{no. of edges in the network}}{\text{total possible number of edges}} = \frac{8}{6 \times 5} = \frac{8}{30} = \frac{4}{15} \approx 0.267$$