

MapReduce and PageRank

Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer i and produces the list of pairs (p,i) such that p is a prime divisor of i . For example, $\text{map}(12) = [(2,12),(3,12)]$.

The reduce function is addition. That is, $\text{reduce}(p,[i_1,i_2,\dots,i_k])$ is $(p,i_1+i_2+\dots+i_k)$.

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

map function:

$\text{map}(15) = [(3, 15), (5, 15)]$

$\text{map}(21) = [(3, 21), (7, 21)]$

$\text{map}(24) = [(2, 24), (3, 24)]$

$\text{map}(30) = [(2, 30), (3, 30), (5, 30)]$

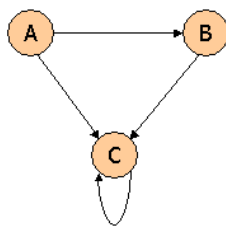
$\text{map}(49) = [(7, 49)]$

reduce function:

$\text{reduce}(2, 54)$, $\text{reduce}(3,90)$, $\text{reduce}(5,45)$, $\text{reduce}(7, 70)$

Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a β of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks a , b , and c of the three pages A, B, and C, respectively.

Value of a, b, or c as we iterate are: a

All PageRank is multiplied by .7 before distribution, and .3 is then added to each new PageRank.

$$a = \beta(0) + (1 - \beta) \implies 0.3$$

$$b = \beta(a/2) + (1 - \beta) \implies 0.7(a/2) + 0.3$$

$$c = \beta(a/2 + b + c) + (1 - \beta) \implies 0.7(a/2 + b + c) + 0.3$$

So $a = 0.3$

$$b = 0.7(0.3/2) + 0.3 \implies 0.405$$

$$c = 0.7(0.555 + c) + 0.3 \implies c = 2.295$$

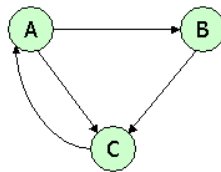
Each two variables

$$a + b = 0.705$$

$$b + c = 2.7$$

$$a + c = 2.595$$

Question 3:



Suppose we compute PageRank with $\beta = 0.85$. Write the equations for the PageRanks a , b , and c of the three pages A, B, and C, respectively.

Given $\beta = 0.85$

$$a = \beta * c + (1 - \beta)1/3$$

$$b = \beta * a/2 + (1 - \beta)1/3$$

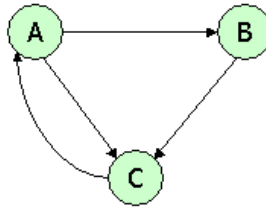
$$c = \beta * ((a/2) + b) + (1 - \beta)1/3$$

$$a = 0.85 * c + (1 - 0.85)1/3 \implies a = 0.85c + 0.05$$

$$b = 0.85 * 0.5 * a + 0.05, b = 0.425a + 0.05$$

$$c = 0.85 * [0.5 * a + b] + 0.05, c = 0.425a + 0.85b + 0.05$$

Question 4:



Assuming no "taxation,"
compute the PageRanks a , b ,
and c of the three pages A, B,
and C, using iteration, starting
with the "0th" iteration where

all three pages have rank $a = b = c = 1$. Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit.

$$a = c, b = a/2, c = a/2 + b$$

At 0th iteration: $a = 1, b = 1; c = 1$

At 1st iteration: $a = c = 1, b = 1/2, c = 1 + 1/2 \Rightarrow 3/2$

At 2nd iteration: $a = c = 3/2, b = a/2 \Rightarrow 1/2, c = 1/2 + 1/2 \Rightarrow 1$

At 3rd iteration: $a = c = 1, b = a/2 = 3/2 * 2 \Rightarrow 3/4, c = 3/4 + 1/2 \Rightarrow 5/4$

At 4th iteration: $a = c = 5/4, b = a/2 = 1/2, c = 5/4$

At 5th iteration: $a = 5/4, b = 5/8, c = 9/8$