# **Dimensionality Reduction**

Question 1: Note: In this question, all columns will be written in their transposed form, as rows, to make the typography simpler. Matrix M has three rows and three columns, and the columns form an orthonormal basis. One of the columns is [2/7,3/7,6/7], and another is [6/7,2/7,-3/7]. Let the third column be [x,y,z]. Since the length of the vector [x,y,z] must be 1, there is a constraint that  $x^2+y^2+z^2=1$ . However, there are other constraints, and these other constraints can be used to deduce facts about the ratios among x, y, and z. Compute these ratios.

#### Sol:

Let C1 be 
$$[2/7,3/7,6/7]$$
, C2 be  $[6/7,2/7,-3/7]$  and C3 be  $[x, y, z]$   
The dot product of any two columns must be zero.  
C1.C2 =  $(2/7*6/7) + (3/7*2/7) + (6/7*-3/7) = 0$   
C2.C3 =  $(6/7*x) + (2/7*y) + (-3/7*z) = 0 \rightarrow 6x + 2y - 3z = 0 - Eq 1$   
C3.C1 =  $(x*2/7) + (y*3/7) + (z*6/7) = 0 \rightarrow 2x + 3y + 6z = 0 - Eq 2$   
 $2*Eq 1 + Eq 2 \rightarrow 12x + 4y - 6z + 2x + 3y + 6z = 0 \rightarrow 14x + 7y = 0 \rightarrow y = -2x$   
 $3*Eq 2 - Eq 1 \rightarrow 6x + 9y + 18z - 6x - 2y + 3z = 0 \rightarrow 7y + 21z = 0 \rightarrow y = -3z$ 

## Question 2: Find the eigenvalues and eigenvectors of the following matrix:



You should assume the first component of an eigenvector is 1. Then, find out One eigenvalue and One eigenvector.

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---> Let the given matrix be 
$$A = 2.3$$
 and the eigen vector be of the form 1  $3.10$  e  $Ax = \lambda x \Rightarrow 2.3$  \*  $1 = \lambda * 1 \rightarrow 2 + 3e = \lambda$  and  $3 + 10e = \lambda e \rightarrow 3 + 10e = (2 + 3e)e$   $3.10$  e e  $3.10$  e e  $3.10$  e e  $3.10$  e e  $3.10$  The eigen vectors are 1 and 1  $3.10$  The eigen values are  $2 + 3e = \lambda \rightarrow \lambda = 2 + 3*3 = 11$  and  $\lambda = 2 + 3*(-1/3) = 1$ 

Question 3: Suppose [1,3,4,5,7] is an eigenvector of some matrix. What is the unit eigenvector in the

same direction? Find out the components of the unit eigenvector.

## Sol:

Given the eigen vector of some matrix be M = [1,3,4,5,7]

To get the unit eigen vector of given matrix, we need to divide each component by square root of sum of squares in the same direction.

Sum of squares = 12 + 32 + 42 + 52 + 72 = 100 and its square root is 10

Unit Eigen Vector = [1/10,3/10,4/10,5/10,7/10]

Question 4: Suppose we have three points in a two dimensional space: (1,1), (2,2), and (3,4). We want to perform PCA on these points, so we construct a 2-by-2 matrix, call it N, whose eigenvectors are the directions that best represent these three points. Construct the matrix N and identify, its elements.

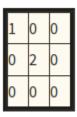
#### Sol:

The given three points in a 2- D space are (1,1), (2,2), and (3,4).

We should construct a matrix whose rows correspond to points and columns correspond to dimensions of the space.

Then the matrix will be M = 11

**Question 5: Consider the diagonal matrix M =** 



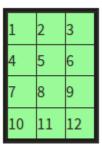
## Compute its Moore-Penrose pseudoinverse.

#### Sol:

Compute its Moore-Penrose pseudoinverse.

Moore-Penrose pseudoinverse means the matrix having diagonal elements replaced by 1 and divided by corresponding elements of given matrix and the other elements will be zero. Moore-Penrose pseudoinverse of given matrix is 1 0 0

Question 6: When we perform a CUR dcomposition of a matrix, we select rows and columns by using a particular probability distribution for the rows and another for the columns. Here is a matrix that we wish to decompose:



## Calculate the probability distribution for the rows.

Sol:

Probability with which we choose now = ( sum of squares of elements in the rows )/(sum of squares of elements in the matrix )

Sum of squares of elements in the matrix = 12\*13\*25/6 = 3900/6 = 650

$$P(R1) = (12 + 22 + 32)/650 = 14/650 = 0.02$$

$$P(R2) = (42 + 52 + 62) / 650 = 77 / 650 = 0.12$$

$$P(R3) = (72 + 82 + 92)/650 = 194/650 = 0.298$$

$$P(R4) = (102 + 112 + 122)/650 = 365/650 = 0.56$$