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Parameter Estimation

Sol^n) Given sample $(X_1 \rightarrow X_n)$ from normal Dist.

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \, e^{\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)}$$

Taking log of fx^n

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2}\ln(2\pi\sigma^2) \right)$$

To find MLES, differentiating the fx^n with respect to θ_1, θ_2

$$\frac{\delta}{\delta\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{x_i - \mu}{2\sigma^2} \right) = 0$$

$$\sum_{i=1}^{n} x_i - n\mu = 0$$

$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$$

or

$$\theta_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

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Q2. $\frac{\delta}{\delta\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n}\left(-\frac{(x_i-\theta_1)}{2\theta_2^2} + \frac{1}{2\theta_2}\right) = 0$

$$\sum_{i=1}^{n}\left(\frac{x_i-\theta_1}{\theta_2^2}\right)^2 - \frac{n}{\theta_2} = 0$$

$$\Rightarrow \frac{\theta_2^2}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\theta_1)^2$$

Sol^n MLE of parameter θ for Binomial Dist. $B(m,\theta)$ when $m \longrightarrow$ known the integer

$$L(\theta) = \prod_{i=1}^{n}\binom{m}{x_i}\theta^{x_i}(1-\theta)^{m-x_i}$$

Taking natural log

$$\ln(L(\theta)) = \sum_{i=1}^{n}\left|\ln\binom{m}{x_i} + x_i(\ln(\theta)) + (n-x_i)\ln(1-\theta)\right.$$

$$\frac{\partial}{\partial\theta}\ln(L(\theta)) = \sum_{i=1}^{n}\left(x_i - \frac{m-x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n}\frac{x_1}{\theta} = \sum_{i=1}^{n}\frac{m-x_1}{1-\theta}$$

$$\Rightarrow \sum_{i=1}^{n} x_i (1-\theta) = \sum_{i=1}^{n} (m - x_i)\theta$$

$$\theta \sum_{i=1}^{n} x_i = m \sum_{i=1}^{n} \theta$$

$$\boxed{\theta = \frac{1}{m} \sum_{i=1}^{n} x_i}$$

MLE of θ = Sample of mean of observato