STAT2401 Analysis of Experiments – Test 1

Solutions

Sweetness of Orange Juice. The quality of the orange juice produced by a manufacturer (e.g., Minute Maid, Tropicana) is constantly monitored. There are numerous sensory and chemical components that combine to make the best tasting orange juice. For example, one manufacturer has developed a quantitative index of the "sweetness" (`Sweetness`) of orange juice. (The higher the index, the sweeter the juice.) Is there a relationship between the sweetness index and a chemical measure such as the amount of water-soluble pectin (`Pectin`). (parts per million) in the orange juice? Data collected on these two variables for 24 production runs at a juice manufacturing plant. Suppose a manufacturer wants to use simple linear regression to predict the sweetness (`Sweetness`) from the amount of pectin (`Pectin`), that is .

$$\texttt{Sweetness} = \beta_0 + \beta_1 \texttt{Pectin} + \epsilon$$

where ϵ is normally distributed with mean 0 and variance σ^2.

We have the following `R` output

```
Call:
lm(formula = Sweetness ~ Pectin)
Residuals:
    Min      1Q  Median     3Q    Max
 -51.43  -26.25   -8.38  29.84  88.99
Coefficients:
              Estimate  Std. Error  t value    Pr(>|t|)
 (Intercept)  -52.1506     47.6769  -1.0938     0.2859
      Pectin    1.6262      0.1536  10.5900  4.211e-10   ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 36.5121 on 22 degrees of freedom
Multiple R-squared: 0.8360, Adjusted R-squared: 0.8285
F-statistic: 112.1485 on 1 and 22 DF,  p-value: 4.211e-10
```

Answer the following questions:

Question 1. [1 Mark]

The estimate of β_0 is

(a) `1.6262`,

(b) `0.1536`,

(c) `-52.1506`, or ✓

(d) `47.6769`.

0

Question 2. [1 Mark]

The estimate of β_1 is

(a) `0.1536`,
(b) `1.6262`, ✓
(c) `-52.1506`, or
(d) `47.6769`.

Question 3. [1 Mark]

The fitted model is given by

(a) $\widehat{\texttt{Sweetness}} = \texttt{1.6262} + \texttt{-52.1506} \times \texttt{Pectin}$,
(b) $\widehat{\texttt{Sweetness}} = \texttt{-52.1506} + \texttt{1.6262} \times \texttt{Pectin}$, ✓
(c) $\widehat{\texttt{Sweetness}} = \texttt{47.6769} + \texttt{1.6262} \times \texttt{Pectin}$, or
(d) $\widehat{\texttt{Sweetness}} = \texttt{-52.1506} + \texttt{0.1536} \times \texttt{Pectin}$.

Question 4. [1 Mark]

The estimate of $\sqrt{\sigma^2}$ is

(a) `0.8360`,
(b) `0.4180`,
(c) `36.5121`, or ✓
(d) `112.1485`.

Question 5. [1 Mark]

The estimate of $\mathrm{SE}(\widehat{\beta}_0)$ is

(a) $\texttt{0.1536}^2 = \texttt{0.0236}$,
(b) `47.6769`, ✓
(c) `0.1536`, or
(d) $\texttt{47.6769}^2 = \texttt{2273.0868}$.

Question 6. [1 Mark]

The estimate of $\mathrm{Var}(\widehat{\beta}_0)$ is

(a) $\texttt{0.1536}^2 = \texttt{0.0236}$,
(b) $(\texttt{-52.1506})^2 = \texttt{2719.6851}$,
(c) $\texttt{1.6262}^2 = \texttt{2.6445}$, or
(d) $\texttt{47.6769}^2 = \texttt{2273.0868}$. ✓

Question 7. [1 Mark]

The estimate of $\mathrm{SE}(\widehat{\beta}_1)$ is

(a) $\texttt{47.6769}$,
(b) $\texttt{47.6769}^2 = \texttt{2273.0868}$,
(c) $\texttt{0.1536}^2 = \texttt{0.0236}$, or
(d) $\texttt{0.1536}$. ✓

Question 8. [1 Mark]

The estimate of $\mathrm{Var}(\widehat{\beta}_1)$ is

(a) $\texttt{47.6769}^2 = \texttt{2273.0868}$,
(b) $\texttt{0.1536}^2 = \texttt{0.0236}$, ✓
(c) $(\texttt{-52.1506})^2 = \texttt{2719.6851}$, or
(d) $\texttt{1.6262}^2 = \texttt{2.6445}$.

Question 9. [1 Mark]

The t-statistic for the test $\mathrm{H}_0 : \beta_0 = 1$ vs $\mathrm{H}_1 : \beta_0 \neq 1$ is given by

(a) $\frac{\texttt{-52.1506-1}}{\texttt{47.6769}}$, ✓
(b) $\frac{\texttt{-52.1506}}{\texttt{47.6769}}$,
(c) $\frac{\texttt{1.6262}}{\texttt{0.1536}}$, or
(d) $\frac{\texttt{1.6262-1}}{\texttt{0.1536}}$.

Question 10. [1 Mark]

The t-statistic for the test $\mathrm{H}_0 : \beta_1 = 1$ vs $\mathrm{H}_1 : \beta_1 \neq 1$ is given by

(a) $\frac{\texttt{-52.1506-1}}{\texttt{47.6769}}$,
(b) $\frac{\texttt{1.6262-1}}{\texttt{0.1536}}$, ✓
(c) $\frac{\texttt{-52.1506}}{\texttt{47.6769}}$, or
(d) $\frac{\texttt{1.6262}}{\texttt{0.1536}}$.

Question 11. [1 Mark]

The p-value for the test $\mathrm{H}_0 : \beta_1 = 0$ vs $\mathrm{H}_1 : \beta_1 \neq 0$ is given by

(a) `4.211e-10`, ✓
(b) `10.5900`,
(c) `0.2859`, or
(d) `-1.0938`.

Question 12. [1 Mark]

The p-value for the test $\mathrm{H}_0 : \beta_0 = 0$ vs $\mathrm{H}_1 : \beta_0 \neq 0$ is given by

(a) `47.6769`,
(b) `10.5900`,
(c) `4.211e-10`, or
(d) `0.2859`. ✓

Question 13. [1 Mark]

For the test $\mathrm{H}_0 : \beta_0 = 0$ vs $\mathrm{H}_1 : \beta_0 \neq 0$, at the $(\alpha =)5\%$ significance level, we

(a) reject H_0 because the p-value $< 0.05 = \alpha$,
(b) do not reject H_0 because the p-value $> 0.025 = \alpha/2$,
(c) do not reject H_0 because the p-value $> 0.05 = \alpha$, or ✓
(d) reject H_0 because the p-value $< 0.025 = \alpha/2$.

Question 14. [1 Mark]

For the test $\mathrm{H}_0 : \beta_1 = 0$ vs $\mathrm{H}_1 : \beta_1 \neq 0$, at the $(\alpha =)5\%$ significance level, we

(a) do not reject H_0 because the p-value $> 0.025 = \alpha/2$,
(b) reject H_0 because the p-value $< 0.025 = \alpha/2$,
(c) do not reject H_0 because the p-value $> 0.05 = \alpha$, or
(d) reject H_0 because the p-value $< 0.05 = \alpha$. ✓

Question 15. [1 Mark]

For the linear regression, we are interested to see whether there is linear relationship between the variables `Pectin` and `Sweetness`, so we are interested in the test:

(a) $\mathrm{H}_0 : \beta_1 = 0$ vs $\mathrm{H}_1 : \beta_1 \neq 0$, ✓
(b) $\mathrm{H}_0 : \beta_1 = \beta_0$ vs $\mathrm{H}_1 : \beta_1 \neq \beta_0$,
(c) $\mathrm{H}_0 : \sigma^2 = 0$ vs $\mathrm{H}_1 : \sigma^2 \neq 0$, or
(d) $\mathrm{H}_0 : \beta_0 = 0$ vs $\mathrm{H}_1 : \beta_0 \neq 0$.

Note that `qt(1-0.05/2,22)=2.0739`

Question 16. [1 Mark]

The 95% CI for β_1 is given by

(a) $(\texttt{-52.1506} - \texttt{2.0739} \times \texttt{0.1536}, \texttt{-52.1506} + \texttt{2.0739} \times \texttt{0.1536}) = (\texttt{-52.4692}, \texttt{-51.8320})$,
(b) $(\texttt{1.6262} - \texttt{2.0739} \times \texttt{47.6769}, \texttt{1.6262} + \texttt{2.0739} \times \texttt{47.6769}) = (\texttt{-97.2509}, \texttt{100.5033})$,
(c) $(\texttt{-52.1506} - \texttt{2.0739} \times \texttt{47.6769}, \texttt{-52.1506} + \texttt{2.0739} \times \texttt{47.6769}) = (\texttt{-151.0277}, \texttt{46.7265})$, or
(d) $(\texttt{1.6262} - \texttt{2.0739} \times \texttt{0.1536}, \texttt{1.6262} + \texttt{2.0739} \times \texttt{0.1536}) = (\texttt{1.3076}, \texttt{1.9448})$. ✓

Question 17. [1 Mark]

The 95% CI for β_0 is given by

(a) $(\texttt{-52.1506} - \texttt{2.0739} \times \texttt{47.6769}, \texttt{-52.1506} + \texttt{2.0739} \times \texttt{47.6769}) = (\texttt{-151.0277}, \texttt{46.7265})$, ✓
(b) $(\texttt{-52.1506} - \texttt{2.0739} \times \texttt{0.1536}, \texttt{-52.1506} + \texttt{2.0739} \times \texttt{0.1536}) = (\texttt{-52.4692}, \texttt{-51.8320})$,
(c) $(\texttt{1.6262} - \texttt{2.0739} \times \texttt{47.6769}, \texttt{1.6262} + \texttt{2.0739} \times \texttt{47.6769}) = (\texttt{-97.2509}, \texttt{100.5033})$, or
(d) $(\texttt{1.6262} - \texttt{2.0739} \times \texttt{0.1536}, \texttt{1.6262} + \texttt{2.0739} \times \texttt{0.1536}) = (\texttt{1.3076}, \texttt{1.9448})$.

Now, we donote $(\widehat{y}_1, \ldots, \widehat{y}_n)$ be the fitted values, $(y_1, \ldots, y_n)$ be the observed values of **`Sweetness`**, $(x_1, \ldots, x_n)$ be the observed values of `Pectin`. Let $e_i = y_i - \widehat{y}_i$ for $i = 1, \ldots, n$. So the residuals are $(e_1, \ldots, e_n) = (y_1 - \widehat{y}_1, \ldots, y_n - \widehat{y}_n)$. Here we also take $\overline{x} = \frac{1}{n}\sum_{i=1}^n x_i$ and $\overline{y} = \frac{1}{n}\sum_{i=1}^n y_i$. We are also given $\overline{x} = 306.6667$.

Question 18. [1 Mark]

The Sum of Squares $\sum_{i=1}^n (x_i - \overline{x})^2$ is then given by

(a) $0.1536^2 \times 36.5121^2 = 31.4526$,

(b) $10.5900^2 \times 36.5121^2 = 149508.3831$,

(c) $\frac{36.5121^2}{10.5900^2} = 11.8873$, or

(d) $\frac{36.5121^2}{0.1536^2} = 56505.5613$. ✓

Question 19. [1 Mark]

We call $\sum_{i=1}^n e_i^2$ Residual Sum of Squares and it has the value

(a) $36.5121^2 \times 24 = 31995.2027$,

(b) $36.5121 \times 24 = 876.2904$,

(c) $36.5121 \times 22 = 803.2662$, or

(d) $36.5121^2 \times 22 = 29328.9358$. ✓

Question 20. [1 Mark]

The term $\sum_{i=1}^n (y_i - \overline{y})^2 - \widehat{\beta}_1^2 \sum_{i=1}^n (x_i - \overline{x})^2$ is

(a) Total Sum of Squares,

(b) Residual Standard Error,

(c) Regression Sum of Squares, or

(d) Residual Sum of Squares. ✓

Question 21. [1 Mark]

Let r be the sample correlation coefficient between **`Pectin`** and **`Sweetness`**, so $|r|$ is given by

(a) 0.4180,

(b) $\sqrt{0.4180} = 0.6465$,

(c) 0.8360, or

(d) $\sqrt{0.8360} = 0.9143$. ✓

0

Question 22. [1 Mark]

The F-statistic at the bottom of the table is calculated for the test

(a) $\mathrm{H}_0 : \beta_0 = \beta_1 = 0$ vs $\mathrm{H}_1 : \beta_0 \neq 0$ or $\beta_1 \neq 0$,
(b) $\mathrm{H}_0 : \beta_0 - \beta_1 = 0$ vs $\mathrm{H}_1 : \beta_0 - \beta_1 \neq 0$,
(c) $\mathrm{H}_0 : \beta_0 = 0$ vs $\mathrm{H}_1 : \beta_0 \neq 0$, or
(d) $\mathrm{H}_0 : \beta_1 = 0$ vs $\mathrm{H}_1 : \beta_1 \neq 0$. ✓

Question 23. [1 Mark]

The term $\widehat{\beta}_1^2 \sum_{i=1}^n (x_i - \overline{x})^2 + \sum_{i=1}^n (y_i - \widehat{y}_i)^2$ is

(a) Residual Standard Error,
(b) Residual Sum of Squares,
(c) Total Sum of Squares, or ✓
(d) Regression Sum of Squares.

Question 24. [1 Mark]

Regression Sum of Squares is also given by

(a) $\frac{\texttt{36.5121}^2}{\texttt{0.1536}^2} = \texttt{56505.5613}$,
(b) $\frac{\texttt{36.5121}^2}{\texttt{10.5900}^2} = \texttt{11.8873}$,
(c) $\texttt{0.1536}^2 \times \texttt{36.5121}^2 = \texttt{31.4526}$, or
(d) $\texttt{10.5900}^2 \times \texttt{36.5121}^2 = \texttt{149508.3831}$. ✓

Question 25. [1 Mark]

The sample average of `Sweetness` $\overline{y}$ is also given by

(a) $(\texttt{-52.1506}) + \texttt{1.6262} \times \texttt{306.6667} = \texttt{446.5508}$, ✓
(b) $(\texttt{-52.1506}) + \texttt{306.6667} = \texttt{254.5161}$,
(c) $\texttt{1.6262} \times \texttt{306.6667} = \texttt{498.7014}$, or
(d) $\texttt{1.6262} + (\texttt{-52.1506}) \times \texttt{306.6667} = \texttt{-15991.2262}$.

Question 26. [1 Mark]

The R^2 is also given by

(a) $\widehat{\beta}_1^2 \frac{\sum_{i=1}^n (x_i - \overline{x})^2}{\sum_{i=1}^n (y_i - \overline{y})^2}$, ✓

(b) $\widehat{\beta}_0^2 \frac{\sum_{i=1}^n (x_i - \overline{x})^2}{\sum_{i=1}^n (y_i - \widehat{y}_i)^2}$,

(c) $\widehat{\beta}_0^2 \frac{\sum_{i=1}^n (x_i - \overline{x})^2}{\sum_{i=1}^n (y_i - \overline{y})^2}$, or

(d) $\widehat{\beta}_1^2 \frac{\sum_{i=1}^n (x_i - \overline{x})^2}{\sum_{i=1}^n (y_i - \widehat{y}_i)^2}$.

Question 27. [1 Mark]

The correlation coefficient r between `Pectin` and `Sweetness` is given by

(a) $\frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2 \sum_{i=1}^n (y_i - \widehat{y}_i)^2}}$,

(b) $\frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \widehat{y}_i)}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2 \sum_{i=1}^n (y_i - \overline{y})^2}}$,

(c) $\frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \overline{y})}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2 \sum_{i=1}^n (y_i - \overline{y})^2}}$, or ✓

(d) $\frac{\sum_{i=1}^n (x_i - \overline{x})(y_i - \widehat{y}_i)}{\sqrt{\sum_{i=1}^n (x_i - \overline{x})^2 \sum_{i=1}^n (y_i - \widehat{y}_i)^2}}$.

Question 28. [1 Mark]

The correlation coefficient r between `Pectin` and `Sweetness`, it is also true that

(a) $r = \sqrt{R^2}$,

(b) $R^2 = r$,

(c) $r^2 = \sqrt{R^2}$, or

(d) $R^2 = r^2$. ✓

Question 29. [1 Mark]

Total Sum of Squares $\sum_{i=1}^n (y_i - \overline{y})^2$ is also given by

(a) $R^2 \times \sum_{i=1}^n (y_i - \widehat{y}_i)^2$,

(b) $R^2 \times \sum_{i=1}^n (y_i - \overline{y})^2$,

(c) $\frac{\sum_{i=1}^n (y_i - \widehat{y}_i)^2}{R^2}$, or

(d) $\frac{\sum_{i=1}^n (\widehat{y}_i - \overline{y})^2}{R^2}$. ✓

Question 30. [1 Mark]

Regression Sum of Squares $\sum_{i=1}^{n}(\widehat{y}_i - \overline{y})^2$ is also given by

(a) $R^2 \times \sum_{i=1}^{n}(y_i - \widehat{y}_i)^2$,

(b) $R^2 \times \sum_{i=1}^{n}(y_i - \overline{y})^2$, ✓

(c) $\frac{\sum_{i=1}^{n}(\widehat{y}_i - \overline{y})^2}{R^2}$, or

(d) $\frac{\sum_{i=1}^{n}(y_i - \widehat{y}_i)^2}{R^2}$.

0

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Answer

1	Ⓐ	Ⓑ	●	Ⓓ	**11**	●	Ⓑ	Ⓒ	Ⓓ	**21**	Ⓐ	Ⓑ	Ⓒ	●
2	Ⓐ	●	Ⓒ	Ⓓ	**12**	Ⓐ	Ⓑ	Ⓒ	●	**22**	Ⓐ	Ⓑ	Ⓒ	●
3	Ⓐ	●	Ⓒ	Ⓓ	**13**	Ⓐ	Ⓑ	●	Ⓓ	**23**	Ⓐ	Ⓑ	●	Ⓓ
4	Ⓐ	Ⓑ	●	Ⓓ	**14**	Ⓐ	Ⓑ	Ⓒ	●	**24**	Ⓐ	Ⓑ	Ⓒ	●
5	Ⓐ	●	Ⓒ	Ⓓ	**15**	●	Ⓑ	Ⓒ	Ⓓ	**25**	●	Ⓑ	Ⓒ	Ⓓ
6	Ⓐ	Ⓑ	Ⓒ	●	**16**	Ⓐ	Ⓑ	Ⓒ	●	**26**	●	Ⓑ	Ⓒ	Ⓓ
7	Ⓐ	Ⓑ	Ⓒ	●	**17**	●	Ⓑ	Ⓒ	Ⓓ	**27**	Ⓐ	Ⓑ	●	Ⓓ
8	Ⓐ	●	Ⓒ	Ⓓ	**18**	Ⓐ	Ⓑ	Ⓒ	●	**28**	Ⓐ	Ⓑ	Ⓒ	●
9	●	Ⓑ	Ⓒ	Ⓓ	**19**	Ⓐ	Ⓑ	Ⓒ	●	**29**	Ⓐ	Ⓑ	Ⓒ	●
10	Ⓐ	●	Ⓒ	Ⓓ	**20**	Ⓐ	Ⓑ	Ⓒ	●	**30**	Ⓐ	●	Ⓒ	Ⓓ