Jet impact on surfaces

Fluids Mechanics

Laboratory session 1

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1 Introduction

This session's purpose was to measure the force exerted by a water jet onto three different geometries: planar, hemispherical and inclined surfaces. Afterwards, a theoretical analysis of the problem must be carried out in order to compare these results to the experimental ones.

2 Demonstration of the Theoretical Expressions

In order to demonstrate and make a model of what is happening in the lab in a theoretical way, a conservation laws problem can be carried out.

Here is a sketch of the problem:

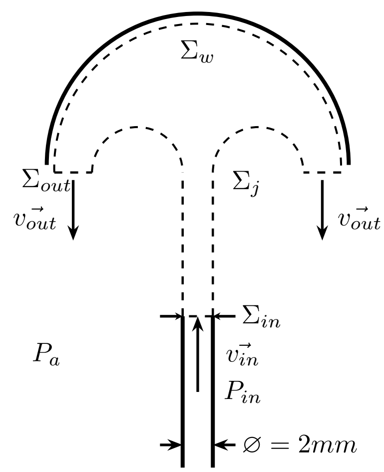

Figure 1: Example for the hemispherical surface.

The first thing that has to be noticed is the conditions from which the problem departs. Thus, the problem can be considered to be inviscid ($\cancel{\overline{\overline{\tau}}'} = 0$), incompressible ($\rho = const$) and steady ($\frac{d}{dt}m = 0$).

Therefore, Bernoulli's principle can be applied in order to compare the flow of water at the entrance and at the exit, as it fulfills the previous conditions.

$$\cancel{P_{in}} + \frac{1}{\cancel{2}}\cancel{\rho}\vec{v}_{in} + \cancel{\rho}U = \cancel{P_a} + \frac{1}{\cancel{2}}\cancel{\rho}\vec{v}_{out} + \cancel{\rho U} \qquad (P_{in} = P_a \;\; and \;\; \vec{g} \;\; negligible) \tag{1}$$

$$\vec{v}_{in} = \vec{v}_{out} \tag{2}$$

Now, applying the continuity equation and knowing that the problem is steady:

$$\cancel{\frac{d}{dt}\int_{V_c(t)}\rho\, dV} = -\int_{\Sigma_c(t)}\rho(\vec{v}-\cancel{\vec{v}_c})\cdot\vec{n}\, d\sigma = 0 \quad (\Sigma_{in}\ \checkmark, \Sigma_{out}\ \checkmark, \cancel{\Sigma_j}:\ (\vec{v}\perp\vec{n}), \cancel{\Sigma_w}:\ (\vec{v}=\vec{v}_c=0)) \tag{3}$$

$$-\rho\vec{v}_{in}A_{in} + \rho\vec{v}_{out}A_{out} = 0 \tag{4}$$

$$A_{in} = A_{out} \tag{5}$$

Finally, applying the momentum equation we will be able to get the components of the force.

$$\cancel{\frac{d}{dt}\int_{V_c(t)}\rho\vec{v}\, dV} + \int_{\Sigma_c(t)}\rho\vec{v}(\vec{v}-\cancel{\vec{v}_c})\cdot\vec{n}\, d\sigma = -\int_{\Sigma_c(t)}p\vec{n}\, d\sigma + \int_{\Sigma_c(t)}\bar{\tau}\cdot\vec{n}\, d\sigma + \cancel{\int_{V_c(t)}\rho\vec{f}_m\, dV}. \tag{6}$$

- $\int_{\Sigma_c(t)}\rho\vec{v}(\vec{v}-\cancel{\vec{v}_c})\cdot\vec{n}\, d\sigma \quad (\Sigma_{in}\ \checkmark, \Sigma_{out}\ \checkmark, \cancel{\Sigma_j}:\ (\vec{v}\perp\vec{n}), \cancel{\Sigma_w}:\ (\vec{v}=\vec{v}_c=0)).$
- $-\int_{\Sigma_c(t)}p\vec{n}\, d\sigma \quad (\Sigma_{in}\ \checkmark, \Sigma_{out}\ \checkmark, \Sigma_j\ \checkmark, \Sigma_w\ \checkmark).$
- $\int_{\Sigma_c(t)}\bar{\tau}\cdot\vec{n}\, d\sigma \quad (\cancel{\Sigma_{in}}\Sigma_{in}\ and\ \cancel{\Sigma_{out}}\Sigma_{out}:\ Uniform,\ \cancel{\Sigma_j}\ and\ \cancel{\Sigma_w}:\ Inviscid)$
- $\int_{V_c(t)}\rho\vec{f}_m\, dV \quad (\vec{j}$ negligible)

Thus, the force is going to be:

$$\vec{F} = -\rho\vec{v}_{in}^2A_{in}\vec{j} + \rho\vec{v}_{out}^2A_{out}(\cos(\alpha)\vec{i} - \sin(\alpha)\vec{j}) \tag{7}$$

Taking only its y-component:

$$y:\quad mg = \vec{F} = \rho\vec{v}_{in}^2A(1+\sin(\alpha)) \tag{8}$$

3 Description of the experimental method.

In order to carry on the experiment successfully, it is important to correctly understand the functioning of the different materials used. First of all, a hydraulic bench will be needed as it will enable what is necessary to produce and measure a constant flow rate of water towards the surface we are studying in each case (flat, oblique or hemispherical). Apart from that, we will use a measuring device that will be placed on top of the bench. It consists of a transparent cylindrical tank with a cover to which there is a vertical bar attached that crosses it. On the lowest part of the bar (which will be inside the tank) we will place the different surfaces to test, which will become the impact position where the water flow rate will hit the surfaces. On the external part of the bar there will be a surface where we will add the weights (we will start each case with 50 g and finish when we reach 500 g). It is important to take into consideration that if the bar starts having difficulties to move (upwards or downwards) oil can be applied to it to ease its displacement.

To start the experiment, we will open the lid of the tank in order to locate on the bar the surface we want to test. Once it is located in its correct position, we will put the cover on the tank (it is important to make sure that it is correctly closed). After that, the first weight will be located on the surface and we can turn on the pump of the hydraulic bench. A flow rate of water will come from the tank base and will hit the surface. By increasing the rate gently we will reach a moment when the surface starts moving upwards, this will mean that equilibrium has been broken so it's the moment to write down the data of both, the mass that has been placed on top of the tank and the outflow rate of the nozzle. The same will be made for the rest of the experiment but adding mass and changing the surface when necessary.

4 Tables of experimental data.

Planar surface

	m (g)	Q (l/h)	W (N)	$\frac{1}{2}\rho Q^2/A$ (N)	F (N)
1	50	1050	0.490	0.846	1.692
2	100	1220	0.981	1.142	2.284
3	150	1360	1.471	1.419	2.838
4	200	1500	1.962	1.727	3.454
5	250	1600	2.452	1.964	3.928
6	300	1700	2.943	2.218	4.436
7	350	1800	3.433	2.486	4.972
8	400	1860	3.924	2.655	5.310
9	450	1960	4.414	2.948	5.869
10	500	2000	4.905	3.070	6.140

Oblique surface

	m (g)	Q (l/h)	W (N)	$\frac{1}{2}\rho Q^2/A$ (N)	F (N)
1	50	900	0.490	0.6217	1.865
2	100	1000	0.981	0.7675	2.303
3	150	1100	1.471	0.9287	2.786
4	200	1240	1.962	1.1801	3.540
5	250	1310	2.452	1.3171	3.951
6	300	1420	2.943	1.5476	4.643
7	350	1500	3.433	1.7269	5.181
8	400	1600	3.924	1.9649	5.895
9	450	1680	4.414	2.1673	6.502
10	500	1720	4.905	2.2706	6.812

Hemispherical surface

	m (g)	Q (l/h)	W (N)	$\frac{1}{2}\rho Q^2/A$ (N)	F (N)
1	50	750	0.490	0.431	1.724
2	100	900	0.981	0.621	2.484
3	150	960	1.471	0.707	2.828
4	200	1060	1.962	0.862	3.448
5	250	1150	2.452	1.015	4.060
6	300	1230	2.943	1.161	4.644
7	350	1300	3.433	1.297	5.188
8	400	1370	3.924	1.440	5.760
9	450	1430	4.414	1.569	6.276
10	500	1500	4.905	1.726	6.904

5 Graph of the experimental data of $F(N)$ versus $Q(L/h)$.

In these three graphs (each one of them for a different surface) it can be seen the plot of the experimental data of F vs Q which has been represented with unconnected points. Moreover the continuous line describes the aproximation of the points which are the theoretical prediction of the results in each case.

The following graph enables us to compare both the experimental and theoretical data of F as a function of Q.

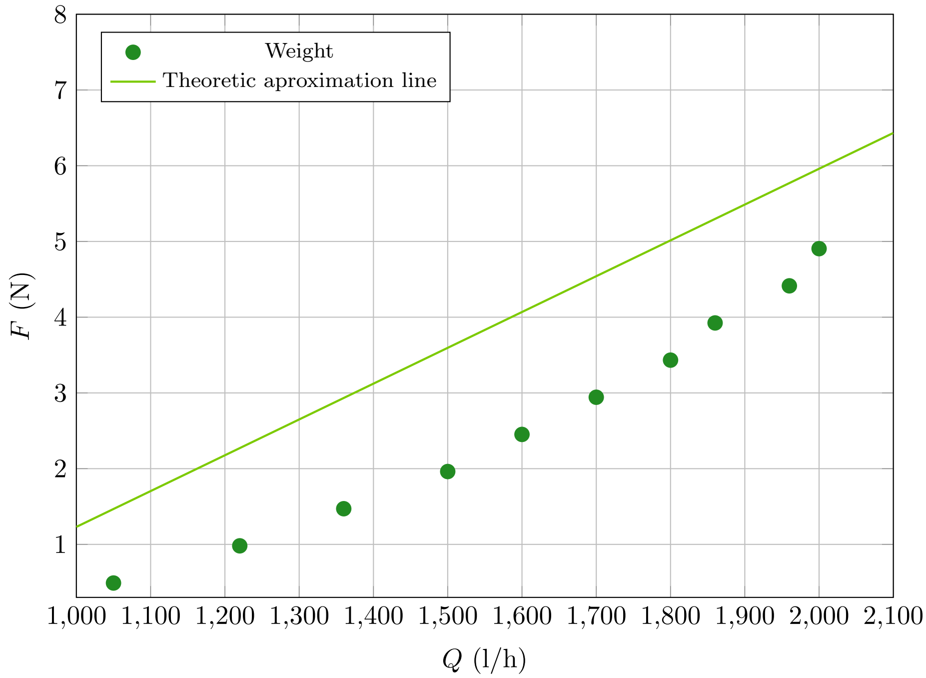

Figure 2: Planar plot F versus Q: Dots (experimental data) and tendency line (theorical data).

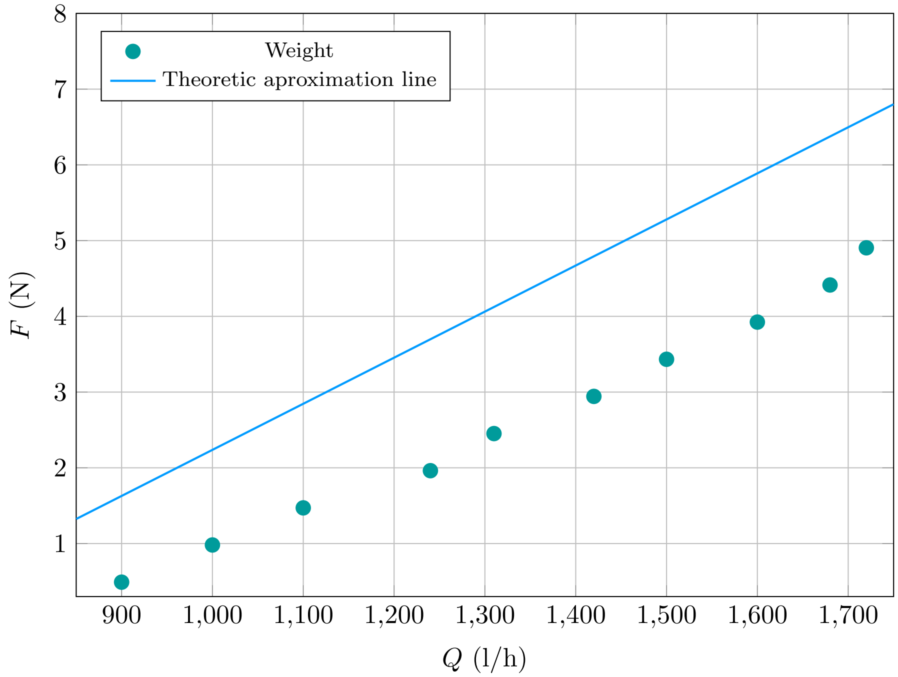

Figure 3: Oblique plot F versus Q: Dots (experimental data) and tendency line (theorical data).

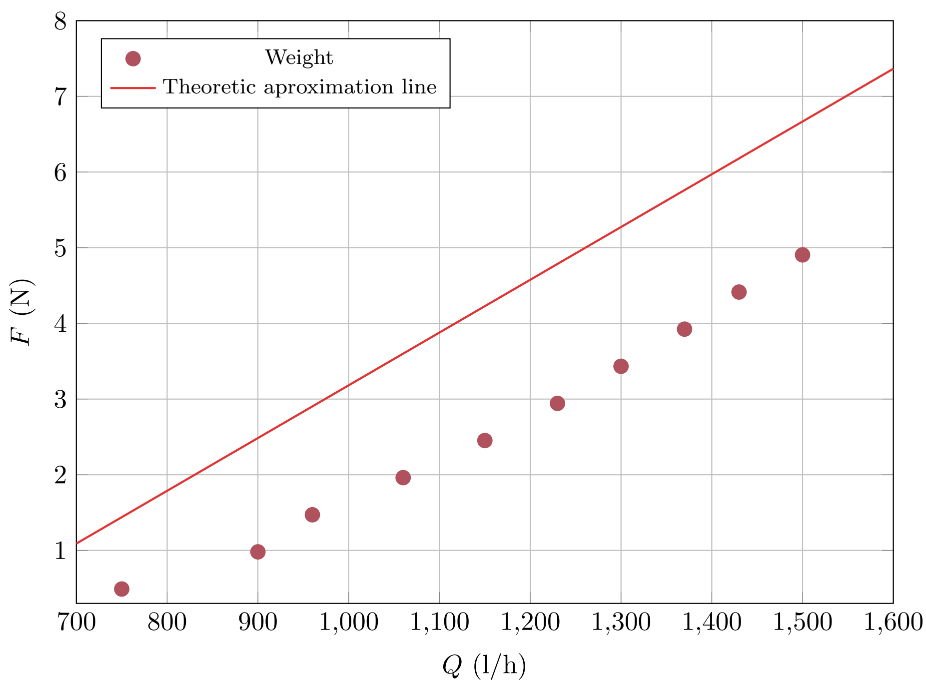

Figure 4: Hemispherical plot F versus Q: Dots (experimental data) and tendency line (theorical data).

6 Single figure, comparison of the experimental and theoretical data of $F(N)$ as a function of $Q(L/h)$ for the three tested surfaces.

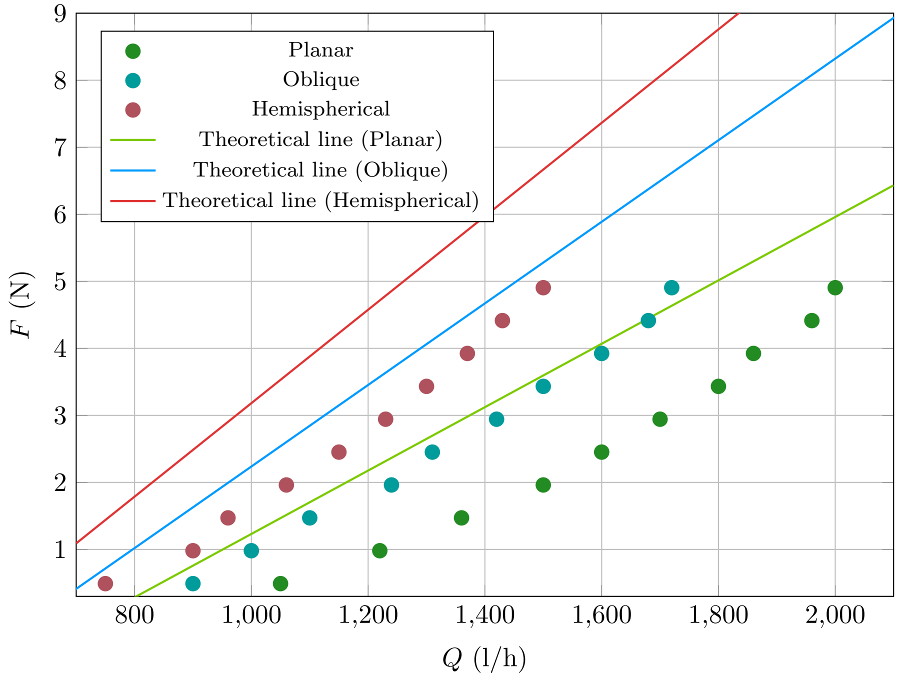

Figure 5: Plot F versus Q of the three surfaces.

7 Calculation of C_d for each obstacle.

The drag coefficient C_d is defined as the dimensionless number given by:

$$C_d = \frac{F}{\frac{1}{2}\rho v^2 A} \tag{9}$$

where:

- F is the drag force, in this case the weight W
- ρ is the fluid density,
- v is the velocity of the fluid,
- A is the cross-sectional area of the jet.

The average velocity v of the jet at the nozzle exit can be expressed as:

$$v = \frac{Q}{A} \tag{10}$$

where A is the cross-sectional area of the jet, given by:

$$A = \frac{\pi d^2}{4} \tag{11}$$

After impacting the surface, the water flow is ejected with a mean velocity $v_s = v$, tangential to the surface, forming a round water sheet with a cross-sectional area $A_s = A$.

Neglecting both viscous and gravitational effects, we consider the characteristic Reynolds number defined as:

$$Re_d = \frac{\rho v d}{\mu} \gg 1 \tag{12}$$

where μ is the viscosity of water.

Planar surface				
	W (N)	Q^2 ($\mathrm{m}^6/\mathrm{s}^2$)	C_d	Re_d
1	0.490	8.507×10^{-8}	0.579	46.42
2	0.981	1.148×10^{-7}	0.859	53.94
3	1.471	1.427×10^{-7}	1.036	60.14
4	1.962	1.736×10^{-7}	1.136	66.31
5	2.452	1.975×10^{-7}	1.248	70.72
6	2.943	2.229×10^{-7}	1.327	75.17
7	3.433	2.500×10^{-7}	1.380	79.58
8	3.924	2.679×10^{-7}	1.473	82.22
9	4.414	2.964×10^{-7}	1.497	86.64
10	4.905	3.086×10^{-7}	1.598	88.42

Oblique surface				
	W (N)	Q^2 ($\mathrm{m}^6/\mathrm{s}^2$)	C_d	Re_d
1	0.490	6.250×10^{-8}	0.788	39.72
2	0.981	7.716×10^{-8}	1.278	44.17
3	1.471	9.336×10^{-8}	1.584	48.61
4	1.962	1.186×10^{-7}	1.663	54.72
5	2.452	1.324×10^{-7}	1.862	57.78
6	2.943	1.556×10^{-7}	1.901	62.78
7	3.433	1.736×10^{-7}	1.988	66.39
8	3.924	1.975×10^{-7}	1.997	70.83
9	4.414	2.177×10^{-7}	2.038	74.17
10	4.905	2.283×10^{-7}	2.160	76.11

Hemispherical surface				
	W (N)	Q^2 ($\mathrm{m}^6/\mathrm{s}^2$)	C_d	Re_d
1	0.490	4.340×10^{-8}	1.135	33.06
2	0.981	6.256×10^{-8}	1.576	39.72
3	1.471	7.112×10^{-8}	2.079	42.50
4	1.962	8.738×10^{-8}	2.257	46.94
5	2.452	1.028×10^{-7}	2.398	50.83
6	2.943	1.227×10^{-7}	2.411	54.44
7	3.433	1.308×10^{-7}	2.639	57.50
8	3.924	1.450×10^{-7}	2.721	60.56
9	4.414	1.580×10^{-7}	2.809	63.33
10	4.905	1.714×10^{-7}	2.877	66.39

8 Values of C_d against R_{ed}.

In these three graphs we have plotted the values of Cd and Red that we have obtained for each of the surfaces that were tested.

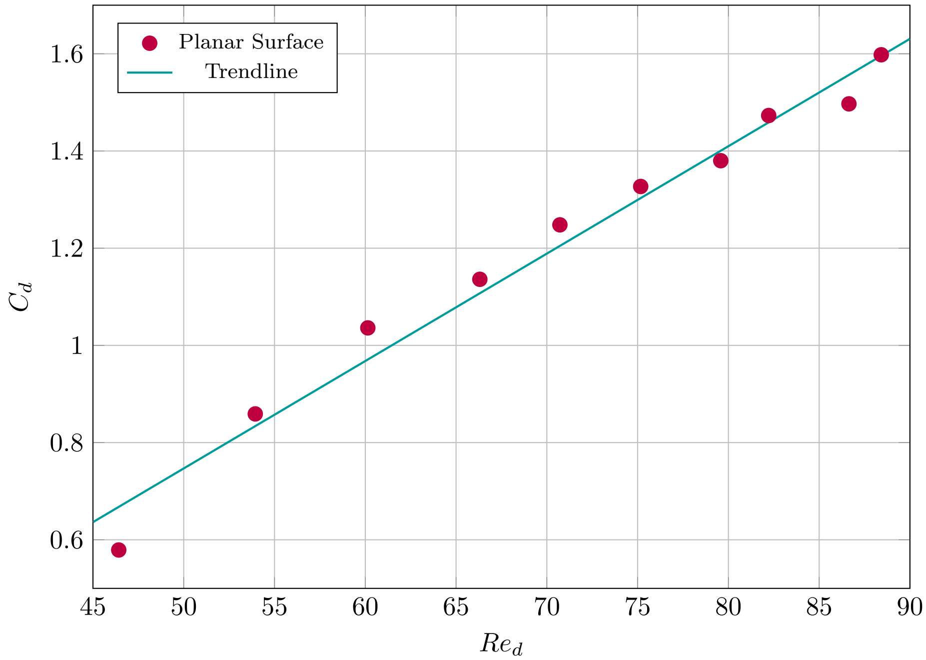

Figure 6: Plot of C_d versus Re_d for the planar surface.

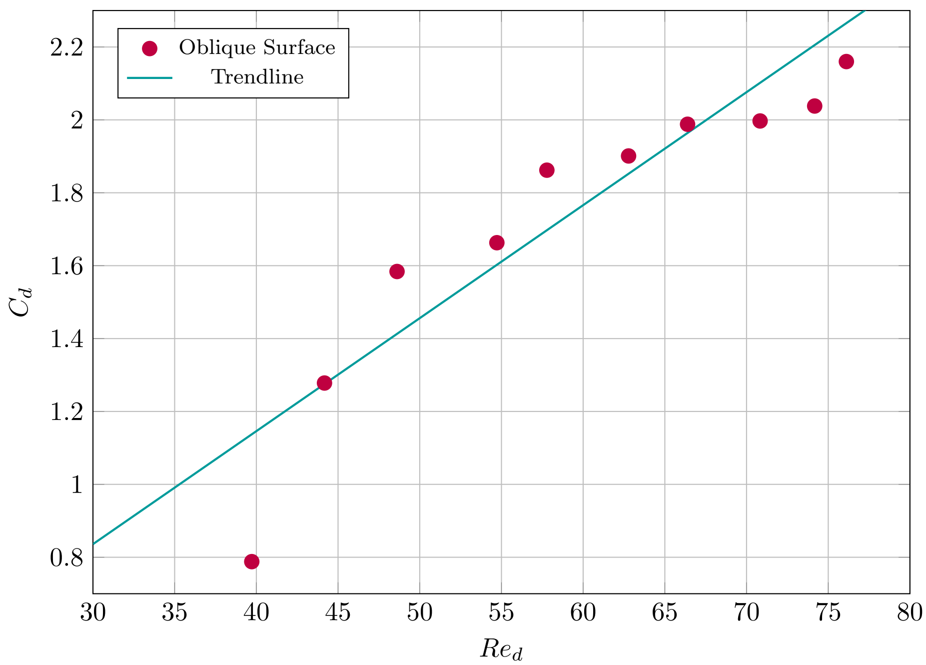

Figure 7: Plot of C_d versus Re_d for the oblique surface.

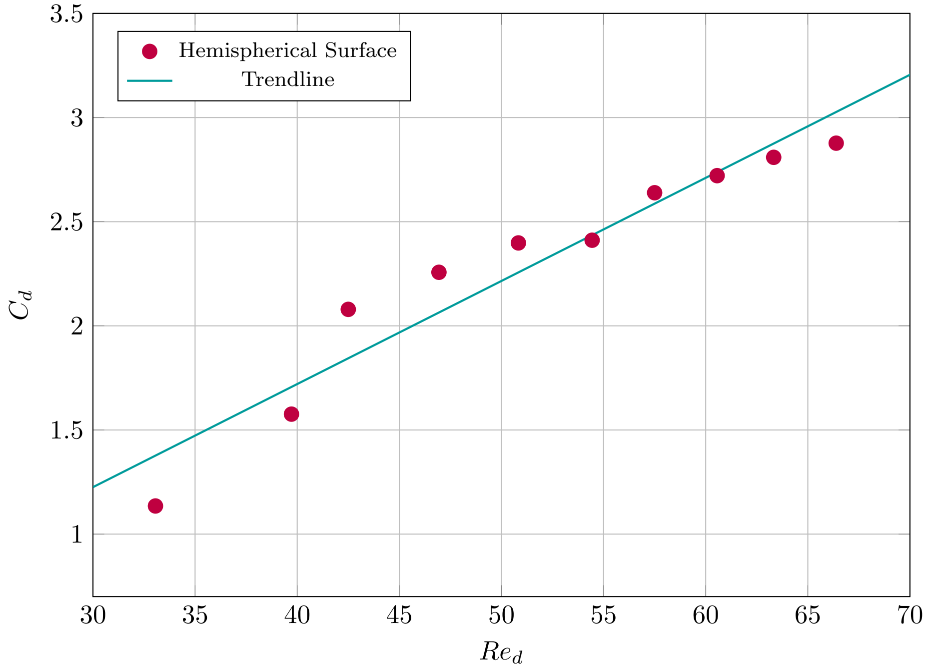

Figure 8: Plot of C_d versus Re_d for the hemispherical surface.

9 Comments and conclusions.

The main objective of this laboratory session is to compare the experimental values of the force exerted by the water flow injected to the three different surfaces tested (flat, oblique and hemispherical) with its theoretical values reached from the reasoning and evaluation of the problem. By plotting the force F vs the water flow rate Q for both experimental and theoretical cases we have reached several conclusions:

First of all, in the graphs of F vs Q for each geometry it can be seen that our experimental data is far below the ideal theoretical tendency line, even though it follows it in a parallel way. This tells us that our experiment has suffered a depression in its efficiency.

Furthermore, when seeing the graph that compares the three different geometries, it is noticeable that the hemispherical geometry is the most efficient of all of them, followed by the oblique geometry, to finish with the planar geometry as the least efficient of them. This is due to the fact that the hemispherical geometry needs less water flow rate (Q) to reach the same value of F, compared to the others.

Apart from that, from the plot of the Drag coefficient C_d against the Reynolds number R_{ed} we have been able to see that among each of the surfaces, under the same conditions of Q (flux of fluid) and R_{ed}, which measures the degree of turbulence that the fluid has at that moment, the one that exerts the most drag is the hemispherical surface, followed by the oblique and finishing with the planar surface.

This makes sense, as the hemispherical surface was the one that broke the equilibrium before, requiring a lower amount of Q compared to the others. This is because the fluid was able to exert more force on the surface rather than in the planar or in the oblique, where the water just collided with the surface in an inefficient way. It is also worth noting that water in this case always acted as a laminar flux fluid.

All in all, we truly believe that this laboratory session has helped us to gain a better understanding of how the force produced by a fluid can variate depending on the surface it is directed towards. Moreover we have been able to discover and understand devices we had never used before such as the hydraulic bench which has been quite rewarding. Finally, by the analysis of the plots we have been capable of seeing the big differences that may exist between the experimental results and the ones theoretically obtained, reaching the conclusion that any small error or lack of precision when taking the measures can lead to great difference in results.