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A novel Improved Model for Failure Modes and Effects Analysis based on Fuzzy DBSCAN clustering and Z-numbers

***Abstract*-Failure Modes and Effects Analysis (FMEA) is a methodical, pro-active method applied to evaluate processes, designs, or products for the identification of critical failures and thus analyze their effects for safety measures. Such a vital task requires an effective and practical procedure. The traditional method has some inherent defects thus rendering it insufficient for applications in real-life scenarios. Several rank-based models have been developed to improve FMEA but they focus on prioritizing all failure modes (FMs). The article presents a new FMEA model using Z-numbers and fuzzy extension of density-based spatial clustering of applications with noise (Fuzzy DBSCAN) algorithm for the classification of the FMs into different risk classes. The Z-number incorporates both the evaluation and the degree of confidence of the FMEA expert team members in the risk evaluation process. The underlying probability distribution that connects the two components of a Z-number, is also taken into account for the determination of the distance/dissimilarity between two fuzzy numbers. A fuzzy extension to the DBSCAN algorithm is then adopted for improved clustering based on distances between the failure modes. An illustrative case study of a direct reduced iron (DRI) plant, in the production of steel, is provided to support the rationality of the newly introduced FMEA model.**

***Keywords*-Failure Modes and Effect Analysis (FMEA), Fuzzy DBSCAN clustering, Z-numbers, Hellinger distance, Center of gravity (COG) distance, Vertex distance, non-linear optimization, risk classification**

I. Introduction:

In today's industrial world, there are many complex systems that may face critical failures due to problems in some part. With the Industrial Revolution 4.0, more products and services are coming up and failures may lead to high costs and fatal accidents, thus jeopardizing the safety of workers [1]. Failure Modes and Effects Analysis (FMEA) is a methodical, pro-active method for evaluating a process for identification of its failure points and assessment of their comparative effects to prioritize preventive measures to be taken. FMEA includes review of the Failure modes (where and how something could go wrong?), causes of failure and their consequences.

FMEA is used to evaluate for potential failures and taking preventive measures instead of correcting adverse events following failures. This may reduce risk of injury to both workers and consumers depending on the field of application.

In traditional FMEA, failure modes (FMs) are sorted with respect to their risk priority numbers (RPN). $RPN = O \times S \times D$, where, occurrence$(O)$, severity$(S)$, and detection$(D)$ are the scores of the risk factors, within some fixed range. Accordingly, corrective measures are taken to prevent those FMs. The main demerits [2] of this method are listed below:

(1) Precise values for the evaluation of risk are difficult to be determined, especially in uncertain situations.
(2) The RPN values can be obtained using any scores for O, S, and D. They are meaningless according to the measurement theory.

(3) Only 3 risk factors, viz. Occurrence, Severity, and Detectability are considered thus ignoring the impact of other factors.

It is difficult to confer crisp numerical values to risk factors considering the complexity and uncertainty of the problem [3]. Also, the reliability of every expert varies based on their expertise. Hence, each expert does not have the same confidence in their respective evaluations. To incorporate these aspects, Z-number [4] which are composed of restriction and reliability measures, are very suitable.

Previously, many methods were proposed to improve the traditional FMEA but most of these focus on risk ranking for all FMs. This is both unnecessary and costly as only the FMs with the highest risks are a serious concern. A better approach would be to simply cluster the FMs into risk classes [5], [6]. There exist several clustering algorithms such as k-means [7], hierarchical, spectral, and density-based spatial clustering of applications with noise (DBSCAN)[8]. Among these, DBSCAN has some typical features [9]: It can effectively identify noise, can cluster arbitrarily-shaped clusters, and does not require specification of the number of clusters beforehand. But it needs input parameters $minPts$(minimum cardinality of the local neighborhood to generate a cluster) and $epsilon$(local neighborhood for considering $minPts$) and the clustering depends heavily on the choice of these parameters.

Fuzzy DBSCAN [10] goes a step ahead of DBSCAN and enables uncertainty in classifying data points into crisp clusters. It introduces fuzziness such that a point can belong to one or more clusters with different memberships. Also, introducing fuzzy clusters has many benefits, for example, in a single execution of the clustering, we can sum up many different executions of DBSCAN. As such, it also encapsulates clusters of varying densities. It allows for a simple investigation of the spatial distribution while evading the precise setting of the DBSCAN parameters.

Based on the aforementioned deliberations, the article puts forward a novel FMEA approach, which considers Z-numbers and Fuzzy DBSCAN for the differentiation of FMs into risk classes. Z-numbers give justice to representing the risk assessment of FMEA experts in uncertain conditions and fuzzy set theory is used to aggregate the assessments of all the experts. The Fuzzy DBSCAN algorithm is further adjusted to support the clustering of identified FMs. The remnants of this article are arranged into the following Sections: Section II deals with fundamental definitions and concepts of Z-numbers are established. Section III explains a refined FMEA method using Z-numbers and Fuzzy DBSCAN clustering. Section IV presents a case study of direct reduced iron (DRI) plant, from data acquisition to the results of the new FMEA applied to its hazards. Section V is a comparative analysis between different methods of clustering. Section VI concludes the article.

II. PRELIMINARIES:

Definition 1: [4] A Z-number is comprised of an ordered pair (A, B), and is useful in expressing an uncertain variable X defined on $\mathbb{R}$. A acts as a fuzzy restriction on the possible values of X. Whereas B is the reliability measure restricting the probability measure of A. The components of a Z-number can be depicted using triangular fuzzy numbers (TFNs) or trapezoidal fuzzy numbers (TrFNs). The definition of a Z-number on X is:

$$Z(A,B) = \{(x, A(x), B(x)) | x \in X\} \quad (1)$$

Here, the restriction(A) and the reliability(B) measures are intertwined together. Separate calculation of these leads to loss of information. The hidden probability distribution p_X is a bond linking A and B, and it cannot be uniquely determined. The only way it can be estimated is by subjecting to some restrictions of the known μ_A , μ_B and the probability measure.

Definition 2: [4], [11] Consider a discrete random variable X having pdf $p(x)$ and $Z = (A, B)$, where A is a discrete fuzzy restriction on the range of X and B is the fuzzy restriction on the probability measure of A. Also, and μ_A is the membership function of A and $X = \{x_1, x_2, x_3, \dots\}$. Therefore, defining the probability that X is A, i.e., $P(X\ is\ A)$ as equal to B, we can express it arithmetically as $P(X\ is\ A) = \sum_{i=1}^{n} \mu_A(x_i)p(x_i) = B$.

III. Methodology

This section presents a novel FMEA model using Z-numbers and the Fuzzy DBSCAN clustering for evaluation and classification of Failure Modes (FMs) based on their risks.

For this problem, an FMEA team of k experts $Exp_h(h = 1,2, \dots, k)$ evaluate n FMs, $FM_i(i = 1,2, \dots, n)$ while considering L risk factors $RF_j(j = 1,2, \dots, L)$. Let $M_h = (z_{ij}^h)_{n \times L}$ be the scaled risk assessment matrix given by the h^{th} expert, where $z_{ij}^h = Z(A_{ij}^h, B_{ij}^h)$ is the Z-number corresponding to the linguistic variables $S(A) = \{l_0, l_1, l_2, \dots, l_{2x}\}$ and $S(B) = \{s_0, s_1, s_2, \dots, s_{2y}\}$. Here, A is a TrFN and B is a TFN.

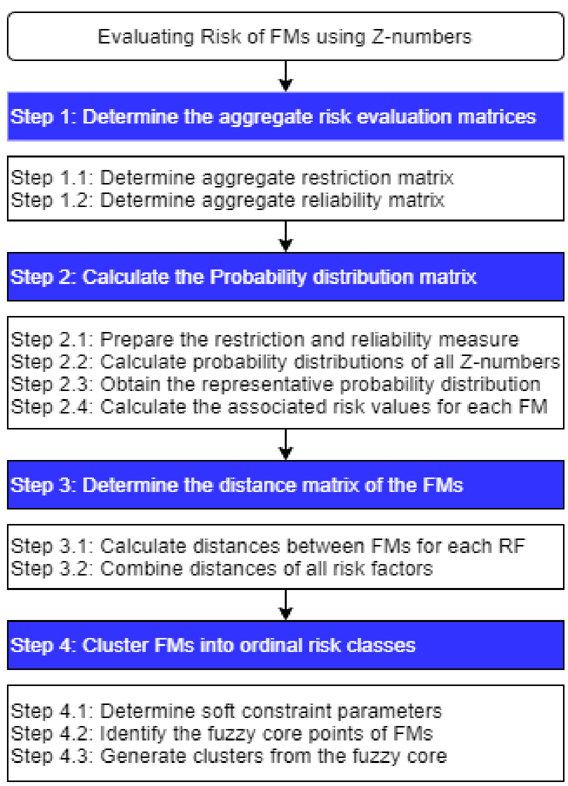

Fig. 1. Methodology of the novel FMEA model

As depicted in Fig. 1, the phases of the FMEA method are explained.

$\boldsymbol{Step\ 1}$: Determine the aggregate risk evaluation matrices.

$Step$ 1.1: Determine aggregate restriction matrix

Given, $A_{ij}^{h} = (a_1^h, a_2^h, a_3^h, a_4^h)$ for $h = 1,2,\ldots,k$. Let

$$a_1^* = average\left(a_1^h\right), a_2^* = average\left(a_2^h\right), a_3^* = average\left(a_3^h\right), a_4^* = average\left(a_4^h\right); h = 1,\ldots,k. \tag{2}$$

Then the aggregate restriction measure would be, $A_{ij}^{*} = (a_1^*, a_2^*, a_3^*, a_4^*)$. Thus, aggregate restriction matrix $aggrA = (A_{ij}^*)_{n\times L}$. Now combining L risk factors to arrive at a single restriction component for each FM.

$$ra = \left(average(A_{ij}^*);\ j = 1,\ldots,L\right)_{n\times 4} \tag{2a}$$

$Step$ 1.2: Determine the aggregate reliability matrix

Given, $B_{ij}^{h} = (b_1^h, b_2^h, b_3^h)$ for $h = 1,2,\ldots,k$. Let

$$b_1^* = average\left(b_1^h\right), b_2^* = average\left(b_2^h\right), b_3^* = average\left(b_3^h\right);\ h = 1,2,\ldots,k. \tag{3}$$

Then the aggregate reliability measure would be, $B_{ij}^{*} = (b_1^*, b_2^*, b_3^*)$. Thus, aggregate reliability matrix $aggrB = (B_{ij}^*)_{n\times L}$. Now combining L risk factors to arrive at a single reliability component for each FM.

$$rb = \left(average(B_{ij}^*);\ j = 1,\ldots,L\right)_{n\times 3} \tag{3a}$$

$\boldsymbol{Step\ 2}$: Calculate the probability distribution matrix.

Do the following for each Z-number, represented by the aggregate risk evaluation matrices, $aggrA$ and $aggrB$, obtained in Step 1.

$Step$ 2.1: Prepare the restriction and reliability measures.

Convert the 4-point Restriction measures, $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)$ and the 3-point Reliability measures, $B = (\beta_1, \beta_2, \beta_3)$, into 7-point measures –

$$A^* = (\alpha_1, (\alpha_1 + \alpha_2)/2, \alpha_2, (\alpha_2 + \alpha_3)/2, \alpha_3, (\alpha_3 + \alpha_4)/2, \alpha_4) \tag{4}$$

$$B^* = (\beta_1, (2\times\beta_1 + \beta_2)/3, (\beta_1 + 2\times\beta_2)/3, \beta_2, (2\times\beta_2 + \beta_3)/3, (\beta_2 + 2\times\beta_3)/3, \beta_3) \tag{5}$$

$Step$ 2.2: Calculate probability distributions of all Z-numbers.

For $Z\ (A^*, B^*)$, which represents FM_i and RF_j, formulate the non-linear programming problem [12] –

Objective function: $$min\ f = -\sum_{t=1}^{7} p(x_t)\log\big(p(x_t)\big)\,; x_t \in A^* \tag{6}$$

Constraints:
$$\sum_{t=1}^{7} p(x_t) = 1 \tag{7}$$
$$\sum_{t=1}^{7} x_t p(x_t) = \frac{\sum_{t=1}^{7} x_t \mu^A(x_t)}{\sum_{t=1}^{7} \mu^A(x_t)} \text{ and} \tag{8}$$
$$\sum_{t=1}^{7} p(x_t)\mu^A(x_t) = B_m^* \tag{9}$$

Bounds: $$p(x_t)\ \geq 0\ \ \forall\ t = 1,2,\dots,7 \tag{10}$$

Run the optimization problem for B_m^*, m = 1, 2, ..., 7 to obtain 7 probability distributions corresponding to, RF_j of FM_i - $p_{ij}^m = (p_1^m, p_2^m, p_3^m, p_4^m, p_5^m, p_6^m, p_7^m)\quad m = 1, 2, \dots, 7\ \ where,$ $p_t = p(x_t)$

$Step$ 2.3: Obtain representative probability distribution

$$rpd_{ij} = \sum_{m=1}^{7} \frac{p_{ij}^m}{7};\ \ i = 1,2,\dots,n \text{ and } j = 1,2,\dots,L \tag{11}$$

At the end of this step, we have matrices $aggrA, aggrB$ and $RPD = (rpd_{ij})_{n\times L}$ representing aggregate restriction measure(A), reliability measure(B) and probability distribution(P) respectively. Similarly, using 7-point measures of $ra(say\ RA)$ according to (4) and de-fuzzified values of $rb(defuzz(rb))$ we calculate probability distributions $rp = (rp_{i1}, rp_{i2}, rp_{i3}, rp_{i4}, rp_{i5}, rp_{i6}, rp_{i7})_{n\times 7}$ according to (6)-(11).

$Step$ 2.4: Calculate the associated risk values for each FM.

First, we calculate the expectation of ra: $xp_i = \sum_{j=1}^{7} RA_{ij} \times rp_{ij}$. The final risk value is calculated by combining xp and the de-fuzzified values of ra and rb as follows:

$$rv = 0.5 * (0.5 * (defuzz(ra) + xp) + defuzz(rb)) \tag{12}$$

(Where, *defuzz* uses the gravity centroid criterion for defuzzification of each fuzzy number)

$\boldsymbol{Step}$ **3**: Determine the distance matrix of the FMs

$Step$ 3.1: Calculate distances between FMs for each RF.

For j^{th} risk factor RF$_j$, the distance between Z_{1j} and Z_{2j} is calculated as follows -
Using the matrices $aggrA$ and $aggrB$, obtained in Step 2, we have,

$$Z_{rj} = Z\left(A_{rj}, B_{rj}\right), where, A_{rj} = \left(a_{rj_1}, a_{rj_2}, a_{rj_3}, a_{rj_4}\right) \text{ and } B_{rj} = \left(b_{rj_1}, b_{rj_2}, b_{rj_3}\right) \forall\, r = 1,2, \dots, n$$

(1) Distance between restriction measures is defined as the COG distance [13] between them:

$$d_{Aj}\left(A_{rj}, A_{sj}\right) = \frac{\sqrt{(\pi_{A_{rj}} - \pi_{A_{sj}})^2 + (\rho_{A_{rj}} - \rho_{A_{sj}})^2}}{\sqrt{1.25}} \quad \forall\ r, s = 1,2, \dots, n \tag{13}$$

where, $\pi_{A_{qj}}$ and $\rho_{A_{qj}}$ determine the COG of the TrFN A_{qj} with the assumption that height of the TrFN $\omega_{A_{qj}} = 1\ \forall\, q = 1,2, \dots, n$. They are defined as:

$$\rho_{A_{qj}} = \begin{cases} \dfrac{\dfrac{(a_{qj_3} - a_{qj_2})}{(a_{qj_4} - a_{qj_1})} + 2}{6}, & if\ a_{qj_4} \neq a_{qj_1} \\ \dfrac{1}{2}, & if\ a_{qj_4} = a_{qj_1} \end{cases} \tag{14}$$

$$\pi_{A_{qj}} = \frac{\rho_{A_{qj}} \times \left(a_{qj_3} + a_{qj_2}\right) + (a_{qj_4} + a_{qj_1}) \times (1 - \rho_{A_{qj}})}{2} \tag{15}$$

(2) Distance between reliability measures is defined by the Vertex Method [14] :

$$d_{Bj}\left(B_{rj}, B_{sj}\right) = \sqrt{\frac{(b_{rj_1} - b_{sj_1})^2 + (b_{rj_2} - b_{sj_2})^2 + (b_{rj_3} - b_{sj_3})^2}{3}} \tag{16}$$

(3) Distance between probability distributions is defined as the Hellinger Distance between them:

$$d_{Pj}\left(P_{rj}, P_{sj}\right) = \frac{\left\|\sqrt{P_{rj}} - \sqrt{P_{sj}}\right\|}{\sqrt{2}} \tag{17}$$

Finally, combining these 3 distances into a single metric:

$$d_j\left(Z_{rj}, Z_{sj}\right) = 0.5 * (0.5 * \left(d_{Aj}\left(A_{rj}, A_{sj}\right) + d_{Pj}\left(P_{rj}, P_{sj}\right)\right) + d_{Bj}(B_{rj}, B_{sj})) \tag{18}$$

$Step$ 3.2: Combine distances of all risk factors

$$d_{rs} = average\left(d_j\left(Z_{rj}, Z_{sj}\right)\right)\ \forall\ r, s = 1,2, \dots, n \tag{19}$$

Finally, $DistanceMatrix = (d_{rs})_{n \times n}$ containing distances between each pair of FMs is obtained.

$\boldsymbol{Step\ 4}$: Cluster FMs into ordinal risk classes.

Now that we have obtained the $DistanceMatrix$, proceed to cluster the FMs using Fuzzy DBSCAN [10] algorithm.

$Step$ 4.1: Determine the soft constraint parameters.

- A soft constraint ($\varepsilon_{min} \leq \varepsilon_{max}$) on the $\mathbb{R}^+$, describes a fuzzy sphere around a point with radius between the constraints. A point can belong to the fuzzy neighborhood of one or more points to a degree in (0,1].
- Another soft constraint ($MPts_{min} \leq MPts_{max}$) on $\mathbb{N}$, describes a fuzzy locally dense area. It specifies the approximate least cardinality of the spherical set(with radius ε_{max}) around a point, to initiate the formation of the fuzzy core.

ε_{min} and ε_{max} can be estimated using the k-distance graph [8]. The distance of k nearest neighbors of every FM is determined from the $DistanceMatrix$. After sorting these distances in an ascending order, the k-distance graphs for $k = 3,4,5, \ldots$ are plotted. Two threshold points with abrupt changes can be found and the k-distance corresponding to these points are the values of ε_{min} and ε_{max}.

For simplicity, $MPts_{min}$ and $MPts_{max}$, are taken to be the respective k values for which ε_{min} and ε_{max} were calculated. $MPts_{min}$ should not be too low as it would generate large number of clusters. Similarly, if $MPts_{max}$ is very large, then many FMs would be classified as noise.

Step 4.2: Identify the fuzzy core points of FMs.

(1) Local density $dens(p)$ of a point p is measured by aggregating the membership $\mu_{dist}(p, p_i)$ of all the points in the local neighborhood $neigh(p, \varepsilon_{max})$. It is defined as:

$$dens(p) = \sum_{p_i \in neigh(p,\varepsilon_{max})} \mu_{dist}(p, p_i), \qquad where,$$

$$\mu_{dist}(p, p_i) = \begin{cases} 1, & if\ \ d(p, p_i) \leq \varepsilon_{min} \\ \dfrac{\varepsilon_{max} - \|p - p_i\|}{\varepsilon_{max} - \varepsilon_{min}}, & if\ \varepsilon_{min} < \ d(p, p_i) < \varepsilon_{max} \\ 0, & if\ \ d(p, p_i) \geq \varepsilon_{max} \end{cases} \tag{20}$$

$$\text{and } neigh(p, \varepsilon_{max}) = \{p_i \ \text{ s.t. } \ d(p, p_i) < \varepsilon_{max}\}$$

$$*\ d(p, p_i) \text{ can be obtained from } DistanceMatrix$$

(2) Membership degree of a point z in the fuzzy core of a cluster is defined as:

$$\mu_{minP}(\hat{n}) = \begin{cases} 1, & if\ \hat{n} \geq MPts_{max} \\ \dfrac{\hat{n} - MPts_{min}}{MPts_{max} - MPts_{min}}, & if\ MPts_{min} < \hat{n} < MPts_{max} \\ 0, & if\ \hat{n} \leq MPts_{min} \end{cases} \quad (21)$$
$$where\ \hat{n} = dens(z)$$

If $dens(z) > MPts_{min}$, then z is in the fuzzy core of some cluster with a membership degree $fuzzycore(z) = \mu_{minP}(dens(z))$. If $dens(z) \leq MPtsMin$, then z might be a border or a noise point. In this manner we can arrive at the set of fuzzy core points.

Step 4.3: Generate clusters from the fuzzy core.

If $\mu_{dist}(z_i, z_j) > 0$ where z_i and z_j are fuzzy core points, i.e., $z_j \in neigh(z_i, \varepsilon_{max})$ then they generate a cluster c. Membership of z_i in cluster c is $fuzzycore_c(z_i) = fuzzycore(z_i)$ and that of z_j is $fuzzycore_c(z_j) = fuzzycore(z_j)$

Any z belonging to cluster c, is a fuzzy border point of that cluster, if $\mu_{minP}\big(dens(z)\big) = 0$ and $\mu_{dist}(z, z_i) > 0$. It belongs to the cluster c with membership defined as:

$$\mu_{b(z)} = min_{z_i \in neighfcore(z)}(\min\ (fuzzycore_c(z_i), \mu_{dist}(z, z_i)))$$
$$where, neighfcore(z) = \{z_j\ \ \text{s.t.}\ \ fuzzycore_c(z_j) > 0\ and\ \mu_{dist}(z, z_j) > 0\} \quad (22)$$

Each such cluster is comprised of FMs along with adjacent risk values. The FMs not belonging to any cluster are classified as noise points. The high-risk clusters can be tagged based on the associated risk values of the FMs, obtained in (12). Issues such as too many noise points or disorder in noise points can be addressed by tuning the soft-constrained parameters. If the problem persists, it may indicate discrepancies in risk evaluation by the experts.

IV. Case Study

This section describes an experimental risk analysis conducted on a direct reduced iron (DRI) plant [15] to signify the feasibility and potential of the proposed FMEA model.

$A. Background$

Today the iron ore-based steel production amounts to greater than $2/3^{rds}$ of the total steel manufacturing in the world. The crude steel industry produces intermediate crude forms such as billets, slabs, or plates which are mechanically converted into bars, sheets, wires, structural elements, or rails. One of the important components of steel production is the direct reduced iron (DRI) plant. Its product, Direct Reduced Iron (DRI) is a solid metallic iron produced through direct reduction (removal of oxygen) of high-grade iron ore in its solid-state without melting like in a blast furnace. DRI has iron content similar to pig iron, making it an excellent feedstock for electric furnaces used by mini-mills.

Critical equipment in this plant consists of conveyor belts, shaft furnaces, rotary coolers, hydraulic systems, and others. It is thus associated with various safety hazards like fire, radiation,

electrocution, and other accidents. Hence, risk identification and analysis are critical for adopting preventive methods and safety.
The hazards to the plant are recognized as listed in Table I. We consider 5 risk factors, namely, Occurrence(O), Severity(S), Detectability(D), Maintenance Cost(C) and Profit(P).

$B. Implementation$

The Hazards are assessed by a team of experts in various domains of the DRI. For the risk evaluation, appropriate linguistic term sets are chosen as given below:

$S(A) = \{l_0 = Very\ Poor, l_1 = Poor, l_2 = Medium\ Poor, l_3 = Medium, l_4 = Medium\ Good, l_5 = Good, l_6 = Very\ Good\}$;

$S(B) = \{s_0 = Very\ Low, s_1 = Low, s_2 = Medium\ Low, s_3 = Medium, s_4 = Medium\ High, s_5 = High, s_6 = Very\ High\}$;

Thus, risk evaluations of 5 experts of the team are obtained as shown in Table II. We use simple linguistic scales (given in Table III) defined discretely for each linguistic term of both sets. The risk evaluations in Table II, are scaled to fuzzy numbers for further calculations. The restriction measure(A) is scaled to TrFN and the reliability measure(B) to TFN. We obtain 5 matrices composed of TrFNs and TFNs for each of the 5 risk factors.

Now we proceed with the implementation of the steps of the proposed FMEA approach in the direct reduced iron (DRI) plant.

Step 1: The aggregate risk evaluation matrices $aggrA = (A^*_{ij})_{46\times5}$ and $aggrB = (B^*_{ij})_{46\times5}$ are computed according to (2), (3) and are given in Table IV. Further, the restriction and reliability components corresponding to the 5 risk factors are combined according to (2a) and (3a) to get ra and rb. An example for Hazard H_1,

$$ra = (0.58, 0.63, 0.68, 0.704, 0.728, 0.766, 0.804) \text{ and } rb = (0.54, 0.732, 0.872)$$

Step 2: The restriction $aggrA$ and reliability $aggrB$ measures are prepared as the 7-point measures ($matrix\ A\ and\ B$) for the optimization problem using (4), (5) and are given in Table V(A) and V(B), respectively. The non-linear programming problem is formulated as in (6)-(10) and solved for each Z-number $Z(A, B)$, where, A and B are the 7-point measures, as in Table V(A) and V(B). In our case, the membership function of 7-point restriction measure is fixed for all A as $\mu_A = (0, 0.3333, 0.6667, 1, 0.6667, 0.3333, 0)$. An example of the above:

We compute representative probability distribution for risk factor O of Hazard H_1. The Z-number representing it is- $Z(A^*, B^*)$;

$A^* = (0.62, 0.67, 0.72, 0.77, 0.82, 0.984, 0.86)$, $\quad B^* = (0.7, 0.78, 0.86, 0.94, 0.96, 0.98, 1)$

Solving the optimization problem mentioned in $Step$ 2.2, with $B^*_1 = 0.7$ and $initial\ guess = [0.14286,\ 0.14286,\ 0.14286,\ 0.14286,\ 0.14286,\ 0.14286,\ 0.14286]$, we get, $\mathrm{P}_1 = [0, 0, 0.364, 0.339, 0.297, 0, 0]$

Similarly solving for other values of B_m^* $m = 2,3, \dots ,7$, we get, $\mathcal{P}_2 = [0, 0, 0.284, 0.499, 0.217, 0, 0]$, $\mathcal{P}_3 = [0, 0, 0.203, 0.66, 0.137, 0, 0]$, $\mathcal{P}_4 = [0, 0, 0.123, 0.82, 0.057, 0, 0]$, $\mathcal{P}_5 = [0, 0, 0.093, 0.88, 0.027, 0, 0]$ $\mathcal{P}_6 = [0,0, 0.067, 0.933, 0, 0, 0]$ and $\mathcal{P}_7 = [0, 0, 0.075, 0.925, 0, 0, 0]$

Thus, representative probability distribution (11), for risk factor O of Hazard H_1 is:

$$rpd = [0, 0, 0.17, 0.72, 0.1, 0, 0]$$

Repeating this for every risk factor of each hazard, we obtain the representative probability distribution matrix $RPD = (rpd_{ij})_{46\times5}$ given in Table VI. Similarly, the 7-point measure of ra (4) and de-fuzzified value of rb is used to calculate the probability distributions rp for calculation of associated risk values. An example for Hazard H_1-

Using, $RA = (0.58, 0.63, 0.68, 0.704, 0.728, 0.766, 0.804)$ and $defuzz(rb) = 0.833$, and solving the optimization problem (6)-(10), we get, $rp = [0,0,0.278,0.499,0.223,0,0]$. Calculating the expectation of ra given rp, we get, $xp = 0.703$. Finally, using (12) and $defuzz(ra) = 0.697$ we arrive at the risk value of hazard H_1, $rv = 0.5 * (0.5 * (0.697 + 0.703) + 0.833) = 0.767$

Step 3: The distance matrix $DistanceMatrix = (d_{rs})_{46\times46}$ is calculated according to (13)-(19) and given in Table VII. For example, distance between 2 Z-numbers is calculated as given below:

Let, $Z_1 = Z(A_1, B_1)$ and $Z_2 = Z(A_2, B_2)$, where, $A_1 = (0.64,0.74,0.8,0.86), B_1 = (0.62,0.78,0.88)$ and $A_2 = (0.48,0.58,0.66,0.72), B_2 = (0.58,0.74,0.84)$. Their representative probability distributions are obtained as, $P_1 = (0,0,0.393,0.308,0.279,0.02,0)$ and $P_2 = (0,0,0.437,0.219,0.292,0.052,0)$. Calculating the distances between restriction, reliability and probability components using (13), (16) and (17), we get, $d_A(A_1, A_2) = 0.1347, d_B(B_1, B_2) = 0.04$ and $d_P(P_1, P_2) = 0.0897$

Finally, using (18), the distance between Z_1 and Z_2 is $d = 0.0761$

Step 4: Plotting the 3-distance(Fig. 2(a)) and 5-distance graphs(Fig. 2(b)), the 2 threshold points $\varepsilon_{min} = 0.033$ and $\varepsilon_{max} = 0.052$, are observed to have abrupt changes. Using these, the fuzzy core points are determined and clusters are generated. Since there high number of noise points, the soft constraints are tuned to $\varepsilon_{min} = 0.037$ and $\varepsilon_{max} = 0.057$ with $MPts_{min} = 3$ and $MPts_{max} = 5$. Finally, clustering of the 46 hazards is done using the Fuzzy DBSCAN algorithm, given in Section III, equation (20)-(22). The results are put forth in Table VIII and visualized in Fig. 3. As displayed in Table VIII, the most serious hazards to the direct reduced iron (DRI) plant are H_1, H_6, H_7, H_{10}, H_{11}, H_{12}, H_{13}, H_{14}, H_{15}, H_{16}, H_{17}, H_{18}, H_{19}, H_{20}, H_{26}, H_{27}, H_{29}, H_{30}, H_{37}, H_{38}, H_{39}, H_{40}, H_{41}, H_{42}, H_{43}, H_{44}, H_{45}. Correspondingly, they must be highly emphasized for effective remedial actions.

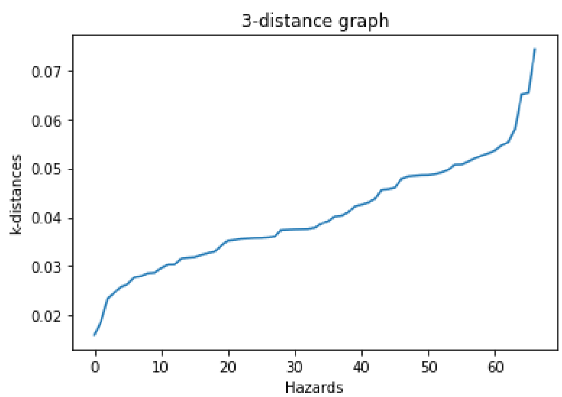

Fig. 2(a). Three-distance graph

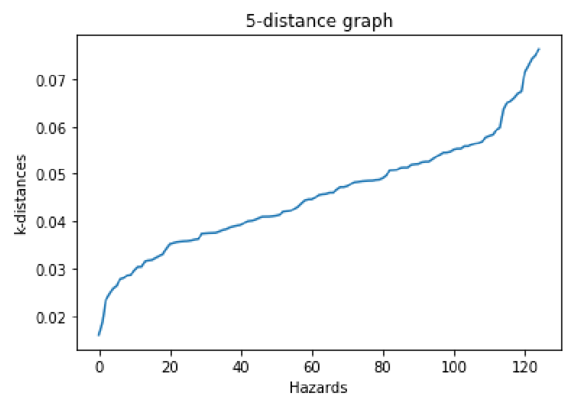

Fig. 2(b). Five-distance graph

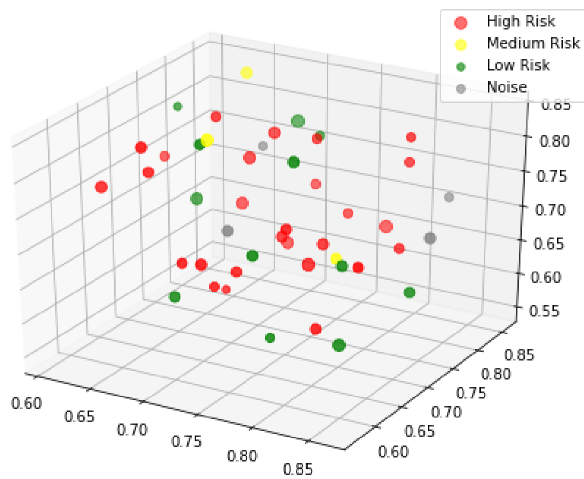

Fig. 3. Cluster results of Hazards

TABLE I

HAZARDS OF THE DIRECT REDUCED IRON (DRI) PLANT

SR No.	Hazards	Section
H1	Fire Hazard	Material Handling
H2	Nip point Hazard	
H3	Belt sway	
H4	CO leakage	Charge hopper
H5	Falling of pellets	
H6	Shell crack and corroded structure	
H7	Heat	Reduction shaft furnace and cooler for gas-based DR plant
H8	Radiation	
H9	Confined space entry	
H10	Gas leakage	
H11	Shell crack and corroded structure	
H12	Slip, trip and fall hazard	
H13	Material falls from height	
H14	Explosion	
H15	Gas leakage	Process and cooling gas
H16	Confined space entry	
H17	Shell crack and corroded structure	
H18	Slip, trip and fall hazard	
H19	Steam	Hot DRI Briquette machine/cold DRI
H20	Hot water	
H21	Dust	
H22	Hot product	
H23	Fumes	
H24	Hot fines	
H25	Fire Hazard	
H26	Spillage of Hot DRI	Hot DRI handing & storage system
H27	Explosion	
H28	Exposure to high temperature	
H29	Oxygen deficient zone	
H30	Rotating parts (Rollers & chain guide)	
H31	CO leakage	Process gas compressors, cooling gas compressor, seal gas compressors, etc.
H32	Work at height	
H33	Noise	
H34	Heat	Process gas system including process gas heater/reformer/recuperator etc.
H35	Gas leakage	
H36	Fire Hazard	
H37	Mechanical failure	
H38	CO leakage	CO2 removal plant
H39	Exposure to chemicals	
H40	Dust	Dedusting system
H41	Slippery floor	
H42	Fall from height	
H43	Fire Hazard	DRI Storage
H44	Electric shock flash over	Electrical panels
H45	Electric shock	Transformer room
H46	Oil leakage	

Table II

Expert Evaluations

Hazards	Experts		E1	E2	E3	E4	E5
H1	O	A	VG	VG	MG	VG	MP
		B	VH	H	H	VH	H
	S	A	G	VG	M	VG	MG
		B	H	MH	H	H	H
	D	A	VG	G	G	G	MG
		B	VH	H	H	H	H
	C	A	MG	G	P	G	P
		B	MH	MH	H	H	H
	P	A	MG	G	G	G	P
		B	M	M	H	VH	H
H2	O	A	MP	VG	MG	VG	P
		B	ML	VH	H	VH	M
	S	A	M	G	M	VG	M
		B	M	H	H	H	M
	D	A	MG	MG	G	G	VP
		B	MH	H	H	H	M
	C	A	G	G	P	MG	P
		B	MH	H	H	H	M
	P	A	G	G	G	G	P
		B	MH	M	H	VH	M
H3	O	A	MP	VG	G	VG	G
		B	ML	VH	MH	VH	H
	S	A	M	P	G	VG	MP
		B	M	H	M	H	H
	D	A	MG	G	G	G	MG
		B	MH	H	H	H	H
	C	A	G	MG	P	G	P
		B	H	MH	H	H	M
	P	A	MG	MG	G	G	P
		B	M	M	H	VH	M
…	…	…	…	…	…	…	…
H44	O	A	VG	VG	MP	VG	MP
		B	VH	VH	H	VH	H
	S	A	G	VG	MP	VG	MG
		B	H	H	H	H	H
	D	A	VG	G	G	G	MG
		B	VH	H	H	H	H
	C	A	P	MG	P	G	P
		B	H	MH	H	H	M
	P	A	P	MG	G	G	P
		B	H	M	H	H	M
H45	O	A	VG	G	G	VG	MP
		B	VH	MH	MH	VH	H
	S	A	G	VG	G	VG	MG
		B	H	MH	M	H	H
	D	A	VG	MG	G	G	MG
		B	VH	H	H	H	H
	C	A	P	G	P	G	P
		B	H	MH	H	H	H
	P	A	P	G	G	G	P
		B	M	MH	H	H	H
H46	O	A	MG	MG	P	VG	G
		B		VH	M	VH	H
	S	A	G	G	G	VG	MP
		B		H	MH	H	H
	D	A	G	G	MG	G	MG
		B		H	MH	H	H
	C	A	P	G	MP	G	P
		B	M	H	M	H	M
	P	A	P	MG	G	G	P
		B	M	M	H	H	M

Table III

Linguistic Scale

$\mathbf{S(A)}$	(VP)	(P)	(MP)	(M)	(MG)	(G)	(VG)
TrFNs	(0,0,0.1,0.2)	(0.1,0.2,0.2,0.3)	(0.2,0.3,0.4,0.5)	(0.4,0.5,0.5,0.6)	(0.5,0.6,0.7,0.8)	(0.7,0.8,0.8,0.9)	(0.8,0.9,1.0,1.0)
$\mathbf{S(B)}$	(VL)	(L)	(ML)	(M)	(MH)	(H)	(VH)
TFNs	(0,0,0.1)	(0,0.1,0.3)	(0.1,0.3,0.5)	(0.3,0.5,0.7)	(0.5,0.7,0.9)	(0.7,0.9,1)	(0.9,1,1)

Table IV

Aggregate Matrix

	O		S		…	P	
	A	**B**	**A**	**B**	**…**	**A**	**B**
H1	0.62 0.72 0.82 0.86	0.78 0.94 1	0.52 0.62 0.68 0.76	0.7 0.9 1	…	0.54 0.64 0.66 0.76	0.54 0.74 0.88
H2	0.64 0.74 0.8 0.86	0.66 0.86 0.98	0.68 0.78 0.82 0.9	0.74 0.92 1	…	0.66 0.76 0.84 0.9	0.58 0.76 0.88
H3	0.68 0.78 0.82 0.9	0.74 0.92 1	0.38 0.48 0.52 0.62	0.54 0.74 0.9	…	0.64 0.74 0.8 0.86	0.66 0.86 0.98
H4	0.42 0.52 0.54 0.64	0.62 0.82 0.96	0.58 0.68 0.74 0.84	0.58 0.76 0.88	…	0.64 0.74 0.8 0.88	0.74 0.92 1
H5	0.54 0.64 0.66 0.76	0.58 0.76 0.88	0.66 0.76 0.84 0.9	0.74 0.9 0.98	…	0.36 0.46 0.48 0.58	0.46 0.66 0.82
H6	0.48 0.58 0.66 72	0.58 0.74 0.92	0.52 0.62 0.68 0.76	0.66 0.86 0.98	…	0.54 0.64 0.66 0.76	0.58 0.78 0.92
H7	0.54 0.64 0.66 0.74	0.58 0.74 0.92	0.64 0.74 0.8 0.88	0.74 0.92 1	…	0.52 0.62 0.68 0.74	0.62 0.8 0.92
…	…	…	…	…	…	…	…
H40	0.5 0.6 0.64 0.74	0.58 0.76 0.88	0.6 0.7 0.78 0.84	0.78 0.94 1	…	0.34 0.44 0.44 0.54	0.66 0.86 0.98
H41	0.62 0.72 0.76 0.82	0.62 0.78 0.88	0.6 0.7 0.78 0.86	0.62 0.82 0.94	…	0.46 0.56 0.56 0.66	0.58 0.78 0.92
H42	0.64 0.74 0.8 0.86	0.62 0.82 0.96	0.7 0.8 0.86 0.92	0.74 0.92 1	…	0.52 0.62 0.68 0.76	0.62 0.78 0.88
H43	0.64 0.74 0.8 0.88	0.7 0.88 0.98	0.3 0.4 0.42 0.52	0.66 0.86 0.98	…	0.62 0.72 0.76 0.84	0.58 0.78 0.92
H44	0.48 0.58 0.6 0.7	0.46 0.66 0.84	0.42 0.52 0.54 0.64	0.54 0.74 0.88	…	0.62 0.72 0.76 0.86	0.58 0.78 0.92
H45	0.66 0.76 0.78 0.88	0.66 0.84 0.96	0.64 0.74 0.8 0.86	0.7 0.86 0.94	…	0.36 0.46 0.48 0.58	0.46 0.66 0.82
H46	0.68 0.78 0.88 0.92	0.78 0.94 1	0.58 0.68 0.74 0.82	0.66 0.86 0.98	…	0.42 0.52 0.54 0.64	0.46 0.66 0.82

Table V(A)

7-Point Restriction Measures

(Only 3 Risk Factors Shown)

	O	S	D
H1	(0.62, 0.67, 0.72, 0.77, 0.82, 0.984, 0.86)	(0.52, 0.57, 0.62, 0.65, 0.68, 0.72, 0.76)	(0.68, 0.73, 0.78, 0.8, 0.82, 0.86, 0.9)
H2	(0.64, 0.69, 0.74, 0.77, 0.8, 0.83, 0.86)	(0.68, 0.73, 0.78, 0.8, 0.82, 0.86, 0.9)	(0.34, 0.39, 0.44, 0.44, 0.44, 0.49, 0.54)
H3	(0.68, 0.673, 0.78, 0.8, 0.82, 0.86, 0.9)	(0.38, 0.43, 0.48, 0.5, 0.52, 0.57, 062)	(0.46, 0.51, 0.56, 0.56, 0.56, 0.61, 0.66)
H4	(0.42, 0.47, 0.52, 0.53, 0.54, 0.59, 0.64)	(0.58, 0.63, 0.68, 0.71, 0.74, 0.79, 0.84)	(0.54, 0.59, 0.64, 0.68, 0.72, 0.75, 0.78)
H5	(0.54, 0.59, 0.64, 0.65, 0.66, 0.71, 0.76)	(0.66, 0.71, 0.76, 0.8, 0.84, 0.87, 0.9)	(0.58, 0.63, 0.68, 0.71, 0.74, 0.78, 0.82)
H6	(0.48, 0.53, 0.58, 0.62, 0.66, 0.69, 0.72)	(0.52, 0.57, 0.62, 0.65, 0.68, 0.72, 0.76)	(0.64, 0.69, 0.74, 0.77, 0.8, 0.84, 0.88)
H7	(0.54, 0.59, 0.64, 0.65, 0.66, 0.7, 0.74)	(0.64, 0.69, 0.74, 0.77, 0.8, 0.84, 0.88)	(0.34, 0.39, 0.44, 0.44, 0.44, 0.49, 0.54)
…	…	…	…
H40	(0.5, 0.55, 0.6, 0.62, 0.64, 0.69, 0.74)	(0.6, 0.65, 0.7, 0.74, 0.78, 0.81, 0.84)	(0.54, 0.59, 0.64, 0.68, 0.72, 0.76, 0.8)
H41	(0.62, 0.67, 0.72, 0.74, 0.76, 0.79, 0.82)	(0.6, 0.65, 0.7, 0.74, 0.78, 0.82, 0.86)	(0.64, 0.69, 0.74, 0.77, 0.8, 0.84, 0.88)
H42	(0.64, 0.69, 0.74, 0.77, 0.8, 0.83, 0.86)	(0.7, 0.75, 0.8, 0.83, 0.86, 0.89, 0.92)	(0.34, 0.39, 0.44, 0.44, 0.44, 0.49, 0.54)
H43	(0.64, 0.69, 0.74, 0.77, 0.8, 0.84, 0.88)	(0.3, 0.35, 0.4, 0.41, 0.42, 0.47, 0.52)	(0.54, 0.59, 0.64, 0.65, 0.66, 0.71, 0.76)
H44	(0.48, 0.53, 0.58, 0.59, 0.6, 0.65, 0.7)	(0.42, 0.47, 0.52, 0.53, 0.54, 0.59, 0.64)	(0.58, 0.63, 0.68, 0.71, 0.74, 0.77, 0.8)
H45	(0.66, 0.71, 0.76, 0.77, 0.78, 0.83, 0.88)	(0.64, 0.69, 0.74, 0.77, 0.8, 0.83, 0.86)	(0.52, 0.57, 0.62, 0.65, 0.68, 0.72, 0.76)
H46	(0.68, 0.73, 0.78, 0.83, 0.88, 0.9, 0.92)	(0.58, 0.63, 0.68, 0.71, 0.74, 0.78, 0.82)	(0.6, 0.64, 0.68, 0.71, 0.74, 0.77, 0.8)

Table V(B)
7-Point Reliability Measures
(Only 3 Risk Factors Shown)

	O	S	D
H1	(0.7, 0.78, 0.86, 0.94, 0.96, 0.98, 1)	(0.7,0.767, 0.833, 0.9, 0.933, 0.967, 1)	(0.7, 0.773, 0.847, 0.92, 0.947, 0.973, 1)
H2	(0.5, 0.62, 0.74, 0.86, 0.907, 0.953, 1)	(0.7,0.773, 0.847, 0.92, 0.947, 0.973, 1)	(0.3, 0.433, 0.567, 0.7, 0.8, 0.9, 1)
H3	(0.7, 0.773, 0.847, 0.92, 0.947, 0.973, 1)	(0.3,0.447, 0.593, 0.74, 0.827, 0.913, 1)	(0.3, 0.433, 0.567, 0.7, 0.8, 0.9, 1)
H4	(0.5, 0.607, 0.713, 0.82, 0.88, 0.94, 1)	(0.3,0.453, 0.607, 0.76, 0.84, 0.92, 1)	(0.7, 0.787, 0.873, 0.96, 0.973, 0.987, 1)
H5	(0.3, 0.453, 0.607, 0.76, 0.84, 0.92, 1)	(0.5,0.633, 0.767, 0.9, 0.933, 0.967, 1)	(0.7, 0.767, 0.833, 0.9, 0.933, 0.967, 1)
H6	(0.1, 0.313, 0.527, 0.74, 0.827, 0.913, 1)	(0.5,0.62, 0.74, 0.86, 0.907, 0.953, 1)	(0.7, 0.773, 0.847, 0.92, 0.947, 0.973, 1)
H7	(0.3, 0.447, 0.593, 0.74, 0.827, 0.913, 1)	(0.7,0.773, 0.847, 0.92, 0.947, 0.973, 1)	(0.3, 0.473, 0.647, 0.82, 0.88, 0.94, 1)
…	…	…	…
H39	(0.3, 0.447, 0.593, 0.74, 0.827, 0.913, 1)	(0.1,0.3, 0.5, 0.7, 0.8, 0.9, 1)	(0.7, 0.787, 0.873, 0.96, 0.973, 0.987, 1)
H40	(0.3, 0.453, 0.607, 0.76, 0.84, 0.92, 1)	(0.7,0.78, 0.86, 0.94, 0.96, 0.98, 1)	(0.7, 0.767, 0.833, 0.9, 0.933, 0.967, 1)
H41	(0.3, 0.46, 0.62, 0.78, 0.853, 0.927, 1)	(0.3,0.473, 0.647, 0.82, 0.88, 0.94, 1)	(0.5, 0.627, 0.753, 0.88, 0.92, 0.96, 1)
H42	(0.5, 0.607, 0.713, 0.82, 0.88, 0.94, 1)	(0.7,0.773, 0.847, 0.92, 0.947, 0.973, 1)	(0.3, 0.473, 0.647, 0.82, 0.88, 0.94, 1)
H43	(0.5, 0.627, 0.753, 0.88, 0.92, 0.96, 1)	(0.5,0.62, 0.74, 0.86, 0.907, 0.953, 1)	(0.3, 0.473, 0.647, 0.82, 0.88, 0.94, 1)
H44	(0.3, 0.42, 0.54, 0.66, 0.773, 0.887, 1)	(0.3,0.447, 0.593, 0.74, 0.827, 0.913, 1)	(0.3, 0.487, 0.673, 0.86, 0.907, 0.953, 1)
H45	(0.5, 0.613, 0.727, 0.84, 0.893, 0.947, 1)	(0.3,0.487, 0.673, 0.86, 0.907, 0.953, 1)	(0.3, 0.447, 0.593, 0.74, 0.827, 0.913, 1)
H46	(0.7, 0.78, 0.86, 0.94, 0.96, 0.98, 1)	(0.5,0.62, 0.74, 0.86, 0.907, 0.953, 1)	(0.3, 0.48, 0.66, 0.84, 0.893, 0.947, 1)

Table VI
Representative Probability Distribution

	O	S	D	…	P
H1	(0, 0, 0.17, 0.72, 0.1, 0, 0)	(0, 0, 0.21, 0.61, 0.18, 0, 0)	(0, 0, 0.19, 0.67, 0.14, 0, 0)	…	(0, 0.04, 0.34, 0.25, 0.34, 0.04, 0)
H2	(0, 0, 0.26, 0.52, 0.21, 0, 0)	(0, 0, 0.19, 0.67, 0.14, 0, 0)	(0, 0.06, 0.34, 0.19, 0.34, 0.06, 0)	…	(0, 0, 0.41, 0.28, 0.27, 0.05, 0)
H3	(0, 0, 0.19, 0.67, 0.14, 0, 0)	(0, 0.04, 0.33, 0.27, 0.33, 0.04, 0)	(0, 0.06, 0.34, 0.19, 0.34, 0.06, 0)	…	(0, 0, 0.26, 0.52, 0.21, 0, 0)
H4	(0, 0.01, 0.28, 0.43, 0.28, 0.01, 0)	(0, 0.02, 0.34, 0.28, 0.34, 0.02, 0)	(0, 0, 0.12, 0.8, 0.08, 0, 0)	…	(0, 0, 0.18, 0.67, 0.15, 0, 0)
H5	(0, 0.02, 0.34, 0.28, 0.34, 0.02, 0)	(0, 0, 0.21, 0.63, 0.16, 0, 0)	(0, 0, 0.21, 0.61, 0.18, 0, 0)	…	(0, 0.09, 0.34, 0.13, 0.34, 0.09, 0)
H6	(0, 0, 0.44, 0.22, 0.29, 0.05, 0)	(0, 0, 0.26, 0.51, 0.23, 0, 0)	(0, 0, 0.18, 0.67, 0.15, 0, 0)	…	(0, 0.02, 0.31, 0.33, 0.31, 0.02, 0)
…	…	…	…	…	…
H40	(0, 0.02, 0.34, 0.28, 0.34, 0.02, 0)	(0, 0, 0.16, 0.73, 0.11, 0, 0)	(0, 0, 0.2, 0.61, 0.18, 0, 0)	…	(0, 0, 0.24, 0.51, 0.24, 0, 0)
H41	(0, 0, 0.42, 0.31, 0.26, 0.02, 0)	(0, 0, 0.32, 0.41, 0.25, 0.02, 0)	(0, 0, 0.23, 0.57, 0.2, 0, 0)	…	(0, 0.02, 0.31, 0.33, 0.31, 0.02, 0)
H42	(0, 0, 0.33, 0.43, 0.22, 0.02, 0)	(0, 0, 0.19, 0.67, 0.13, 0, 0)	(0, 0.01, 0.28, 0.41, 0.28, 0.01, 0)	…	(0, 0, 0.38, 0.31, 0.29, 0.02, 0)
H43	(0, 0, 0.23, 0.57, 0.2, 0, 0)	(0, 0, 0.24, 0.51, 0.24, 0, 0)	(0, 0.01, 0.28, 0.41, 0.28, 0.01, 0)	…	(0, 0, 0.4, 0.33, 0.22, 0.04, 0)
H44	(0, 0.09, 0.34, 0.15, 0.34, 0.09, 0)	(0, 0.04, 0.34, 0.25, 0.34, 0.04, 0)	(0, 0, 0.28, 0.51, 0.21, 0, 0)	…	(0, 0.02, 0.31, 0.33, 0.31, 0.02, 0)
H45	(0, 0, 0.26, 0.47, 0.26, 0, 0)	(0, 0, 0.28, 0.51, 0.21, 0, 0)	(0, 0.01, 0.43, 0.25, 0.25, 0.07, 0)	…	(0, 0.09, 0.34, 0.13, 0.34, 0.09, 0)
H46	(0, 0, 0.17, 0.72, 0.1, 0, 0)	(0, 0, 0.26, 0.51, 0.23, 0, 0)	(0, 0, 0.29, 0.45, 0.26, 0, 0)	…	(0, 0.09, 0.34, 0.13, 0.34, 0.09, 0)

TABLE VII

DISTANCE MATRIX

	H1	H2	H3	H4	H5	H6	H7	…	H40	H41	H42	H43	H44	H45	H46
H1	0	0.13	0.1	0.14	0.09	0.04	0.09	…	0.06	0.11	0.04	0.07	0.04	0.06	0.14
H2	0.13	0	0.06	0.07	0.06	0.12	0.12	…	0.12	0.1	0.14	0.16	0.13	0.12	0.08
H3	0.1	0.06	0	0.09	0.05	0.08	0.09	…	0.08	0.06	0.11	0.11	0.11	0.09	0.07
H4	0.14	0.07	0.09	0	0.1	0.15	0.13	…	0.15	0.1	0.15	0.17	0.16	0.13	0.08
H5	0.09	0.06	0.05	0.1	0	0.07	0.08	…	0.07	0.06	0.1	0.12	0.11	0.1	0.08
H6	0.04	0.12	0.08	0.15	0.07	0	0.07	…	0.03	0.09	0.06	0.06	0.05	0.07	0.12
H7	0.09	0.12	0.09	0.13	0.08	0.07	0	…	0.05	0.07	0.09	0.08	0.09	0.1	0.1
H8	0.12	0.05	0.07	0.05	0.06	0.12	0.11	…	0.12	0.08	0.12	0.14	0.13	0.11	0.07
H9	0.11	0.12	0.1	0.09	0.09	0.12	0.1	…	0.11	0.09	0.11	0.13	0.12	0.1	0.09
H10	0.03	0.13	0.11	0.14	0.12	0.05	0.09	…	0.08	0.1	0.07	0.05	0.04	0.09	0.14
H11	0.05	0.13	0.11	0.14	0.11	0.07	0.08	…	0.09	0.11	0.07	0.08	0.07	0.08	0.14
H12	0.05	0.13	0.1	0.15	0.09	0.06	0.1	…	0.08	0.11	0.05	0.09	0.07	0.04	0.14
H13	0.04	0.13	0.11	0.15	0.12	0.05	0.09	…	0.08	0.1	0.08	0.04	0.03	0.1	0.14
H14	0.07	0.12	0.1	0.13	0.11	0.09	0.11	…	0.1	0.11	0.08	0.1	0.09	0.05	0.13
H15	0.04	0.1	0.09	0.13	0.07	0.05	0.05	…	0.04	0.08	0.04	0.08	0.05	0.06	0.11
H16	0.04	0.14	0.12	0.15	0.12	0.06	0.1	…	0.07	0.11	0.06	0.05	0.02	0.08	0.15
H17	0.06	0.11	0.08	0.13	0.1	0.07	0.08	…	0.08	0.08	0.09	0.05	0.06	0.07	0.1
H18	0.04	0.13	0.11	0.16	0.1	0.06	0.11	…	0.07	0.09	0.04	0.07	0.04	0.04	0.13
H19	0.06	0.12	0.09	0.13	0.09	0.05	0.06	…	0.05	0.05	0.07	0.05	0.06	0.07	0.09
H20	0.04	0.14	0.11	0.17	0.09	0.04	0.09	…	0.04	0.09	0.06	0.04	0.03	0.08	0.13
H21	0.19	0.07	0.1	0.08	0.11	0.17	0.15	…	0.15	0.09	0.18	0.16	0.17	0.17	0.08
H22	0.15	0.07	0.08	0.03	0.08	0.14	0.12	…	0.13	0.08	0.15	0.15	0.15	0.14	0.07
H23	0.09	0.07	0.05	0.09	0.07	0.09	0.09	…	0.09	0.06	0.08	0.12	0.09	0.06	0.09
H24	0.09	0.06	0.04	0.09	0.06	0.1	0.11	…	0.09	0.09	0.08	0.14	0.11	0.07	0.09
H25	0.1	0.08	0.04	0.11	0.06	0.08	0.09	…	0.07	0.06	0.1	0.12	0.12	0.09	0.08
H26	0.04	0.12	0.1	0.12	0.1	0.06	0.06	…	0.07	0.09	0.05	0.06	0.05	0.07	0.12
H27	0.05	0.14	0.12	0.15	0.1	0.07	0.1	…	0.07	0.11	0.05	0.07	0.05	0.08	0.15
H28	0.09	0.06	0.05	0.09	0.08	0.09	0.09	…	0.08	0.07	0.08	0.11	0.08	0.07	0.09
H29	0.03	0.13	0.11	0.15	0.1	0.06	0.09	…	0.06	0.11	0.02	0.08	0.06	0.04	0.14
H30	0.07	0.07	0.06	0.1	0.04	0.07	0.08	…	0.06	0.05	0.07	0.11	0.08	0.07	0.09
H31	0.1	0.08	0.06	0.1	0.07	0.11	0.13	…	0.11	0.09	0.08	0.15	0.12	0.07	0.11
H32	0.19	0.08	0.13	0.06	0.14	0.19	0.17	…	0.19	0.14	0.2	0.2	0.19	0.18	0.1
H33	0.09	0.06	0.06	0.07	0.04	0.1	0.08	…	0.09	0.06	0.1	0.11	0.09	0.09	0.07
H34	0.1	0.08	0.08	0.07	0.06	0.12	0.1	…	0.11	0.06	0.1	0.12	0.11	0.1	0.08
H35	0.16	0.08	0.1	0.02	0.11	0.16	0.14	…	0.16	0.11	0.15	0.17	0.16	0.13	0.08
H36	0.11	0.06	0.07	0.07	0.05	0.11	0.12	…	0.1	0.1	0.11	0.14	0.12	0.09	0.08
H37	0.06	0.12	0.1	0.12	0.1	0.05	0.07	…	0.06	0.07	0.08	0.06	0.06	0.08	0.11
H38	0.08	0.12	0.1	0.09	0.1	0.09	0.1	…	0.1	0.09	0.09	0.1	0.09	0.09	0.09
H39	0.1	0.1	0.07	0.1	0.07	0.09	0.06	…	0.06	0.03	0.09	0.08	0.09	0.09	0.05
H40	0.06	0.12	0.08	0.15	0.07	0.03	0.05	…	0	0.07	0.06	0.05	0.06	0.07	0.1
H41	0.11	0.1	0.06	0.1	0.06	0.09	0.07	…	0.07	0	0.1	0.08	0.1	0.1	0.05
H42	0.04	0.14	0.11	0.15	0.1	0.06	0.09	…	0.06	0.1	0	0.09	0.06	0.04	0.13
H43	0.07	0.16	0.11	0.17	0.12	0.06	0.08	…	0.05	0.08	0.09	0	0.04	0.1	0.11
H44	0.04	0.13	0.11	0.16	0.11	0.05	0.09	…	0.06	0.1	0.06	0.04	0	0.08	0.13
H45	0.06	0.12	0.09	0.13	0.1	0.07	0.1	…	0.07	0.1	0.04	0.1	0.08	0	0.12
H46	0.14	0.08	0.07	0.08	0.08	0.12	0.1	…	0.1	0.05	0.13	0.11	0.13	0.12	0

TABLE VIII

CLUSTER RESULTS OF HAZARDS

Clusters	Hazards
High Risk	H_1, H_6, H_7, H_{10}, H_{11}, H_{12}, H_{13}, H_{14}, H_{15}, H_{16}, H_{17}, H_{18}, H_{19}, H_{20}, H_{26}, H_{27}, H_{29}, H_{30}, H_{37}, H_{38}, H_{39}, H_{40}, H_{41}, H_{42}, H_{43}, H_{44}, H_{45}
Medium Risk	H_4, H_{22}, H_{35}
Low Risk	H_2, H_3, H_5, H_8, H_{23}, H_{24}, H_{25}, H_{28}, H_{31}, H_{33}, H_{34}, H_{36}
Noise	H_9, H_{21}, H_{32}, H_{46}

V. COMPARATIVE ANALYSIS

In order to signify the efficacy of the posited FMEA method, it is collated with the traditional RPN procedure and the DBSCAN [16] clustering method. In the case of RPN, the risk rankings of the 46 Hazards whereas, the high-risk hazards identified by the proposed method and the DBSCAN clustering algorithm are plotted in Fig. 4.

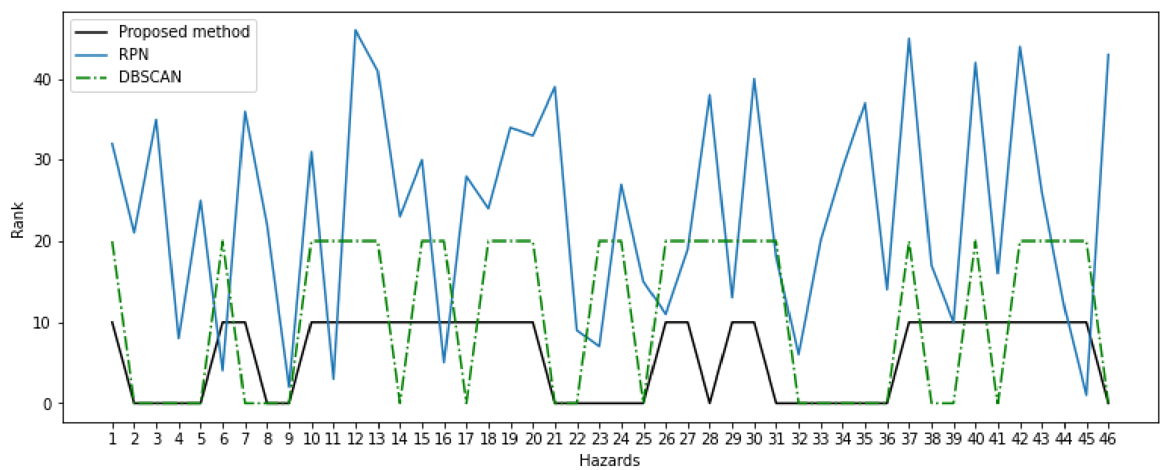

Fig. 4. Ranks (RPN) and cluster outcome by distinct algorithms.

It can be seen clearly that the most critical hazards identified by the RPN method are inconsistent with the proposed FMEA method. By the RPN method the hazards $H_7, H_{10}, H_{12}, H_{13}, H_{19}, H_{20}, H_{30}, H_{37}, H_{40}$ and H_{42}, are not identified as high-risk, whereas, $H_2, H_4, H_9, H_{22}, H_{23}, H_{25}$ and H_{35} are wrongly identified as high-risk. The difference can be attributed to the demerits of the traditional approach, as given in Section I.

Next, the DBSCAN clustering did not identify $H_{17}, H_{14}, H_{38}, H_{39}$ and H_{41} as high-risk but clustered H_{23}, H_{24}, H_{31} as high-risk as opposed to Fuzzy DBSCAN. This can be attributed to the crispness of the parameters in DBSCAN which may fail in detecting clusters with heterogeneous densities which may not account for the uncertainty in the FMEA data.

VI. Conclusion

To summarize, this article introduced us to a novel FMEA approach by integrating Z-numbers and the Fuzzy DBSCAN algorithm to cluster hazards in an uncertain environment. While the Z-numbers were used to incorporate the uncertainty of risk evaluation by experts, the Fuzzy DBSCAN algorithm was used to capture clusters of varying densities. A case study of a direct reduced iron (DRI) plant was examined. The results indicate that the proposed method is capable of determining the most serious hazards in a realistic manner.

The facets that can be enhanced in the future are mentioned as follows. Though Fuzzy DBSCAN overcomes the crispness of DBSCAN, allowing to control distinct fuzzification characteristics of the clusters, it still suffers from its inherent demerits such as determination of parameters and noise-cluster confusion. Secondly, the reliability of the individual expert evaluation can be incorporated in terms of their knowledge, expertise, experience, and unbiasedness, using appropriate metrics. Hence, considering the economic impact and more importantly the safety concerns of industrial workers, it is vital to develop improved FMEA methods.

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