CS771 Assignment 1

Deep Learners

Task 1

We are given some 32×1 challenge vectors $\tilde{\mathbf{c}}$.

We have 2 linear models namely $(\vec{u}, p)$ and $(\vec{v}, q)$ for working and the reference PUF respectively:

$$\Delta_\omega = \vec{u}^\top \cdot \vec{x} + p$$
$$\Delta_r = \vec{v}^\top \cdot \vec{x} + q$$

where

$$\vec{x_i} = d_i \cdot d_{i+1} \cdot d_{i+2} \cdot \ldots \cdot d_{32}$$

and,

$$d_i = 1 - 2 \cdot c_i$$

.

Now,

$$\Delta_\omega - \Delta_r = (\vec{u} - \vec{v})^\top \cdot \vec{x} + (p - q)$$

$$\text{Let } \vec{z} = (\vec{u} - \vec{v}) \text{ and } t = p - q.$$

Now, where τ is the secret threshold, we get the following:

$$\begin{aligned} |\Delta_\omega - \Delta_r|^2 - \tau^2 &= \left|\vec{z}^\top \cdot \vec{x} + t\right|^2 - \tau^2 \\ &= \left(\vec{z}^\top \cdot \vec{x}\right)^2 + 2t\left(\vec{z}^\top \cdot \vec{x}\right) + t^2 - \tau^2 \\ &= \sum {z_i}^2 {x_i}^2 + 2\sum\sum_{i \neq j} z_i z_j x_i x_j + 2t\sum z_i x_i + t^2 - \tau^2 \end{aligned}$$

Since $c_i = 0$ or $c_i = 1$, we have:

$$d_i = \pm 1 \quad \implies \quad x_i = \pm 1 \quad \implies \quad {x_i}^2 = 1$$

Hence,

$$|\Delta_\omega - \Delta_r|^2 - \tau^2 = 2\sum\sum_{i \neq j} z_i z_j x_i x_j + 2t\sum z_i x_i + \sum {z_i}^2 + t^2 - \tau^2$$

Note that,

$$\frac{1 + \text{sign}\left(|\Delta_\omega - \Delta_r|^2 - \tau^2\right)}{2} = r$$

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We have to create a model (W, b) such that,

$$\frac{1 + \text{sign}\left(\vec{W}^\top \cdot \phi(\vec{c}) + b\right)}{2} = r$$

Comparing both equations we get:

$$W = \begin{bmatrix} 2z_1z_2 \\ 2z_1z_3 \\ \vdots \\ \vdots \\ 2z_{31}z_{32} \\ 2tz_1 \\ 2tz_2 \\ \vdots \\ 2tz_{32} \end{bmatrix}$$

$$\phi(\vec{c}) = \begin{bmatrix} x_1x_2 \\ x_1x_3 \\ \vdots \\ \vdots \\ \vdots \\ x_{31}x_{32} \\ x_1 \\ x_2 \\ \vdots \\ x_{32} \end{bmatrix}$$

and

$$b = \sum {z_i}^2 + t^2 - \tau^2$$

where $x_i = d_i \cdot d_{i+1} \cdot d_{i+2} \cdot \ldots \cdot d_{32}$ and $d_i = 1 - 2 \cdot c_i$.

Now the dimensions of $\phi(c)$:

$$\text{Number of terms } x_ix_j = \frac{32 \times 31}{2} = 496$$
$$\text{Number of terms } x_i = 32$$
$$\text{Number of dimensions } D = 528$$

Task 3

Report outcomes of experiments with both the `sklearn.svm.LinearSVC` and `sklearn.linear_model.LogisticRegression` methods when used to learn the linear model. In particular, report how various hyperparameters affected training time and test accuracy using tables and/or charts.

1. Changing the loss hyperparameter in `LinearSVC` (hinge vs squared hinge)
2. Setting C in `LinearSVC` and `LogisticRegression` to high/low/medium values
3. Changing `tol` in `LinearSVC` and `LogisticRegression` to high/low/medium values
4. Changing the penalty (regularization) hyperparameter in `LinearSVC` and `LogisticRegression` (`l2` vs `l1`)

Experimental Results

LinearSVC

Table 1: Effect of changing the loss hyperparameter

Loss	Training Time (s)	Test Accuracy (%)
Hinge	11s	98.925
Squared Hinge	11s	99.2

Table 2: Effect of setting C

C Value	Training Time (s)	Test Accuracy (%)
High	11s	98.975
Medium	11s	99.2625
Low	4s	98.225

Table 3: Effect of setting tol

tol Value	Training Time (s)	Test Accuracy (%)
High	0.8s	93.0375
Medium	12s	98.00
Low	12s	99.075

Table 4: Effect of changing the penalty

penalty	Training Time (s)	Test Accuracy (%)
l1	120s	99.175
l2	3s	99.1875

LogisticRegression

Table 5: Effect of setting C

C Value	Training Time (s)	Test Accuracy (%)
High	2s	99.2125
Medium	0.8s	98.9375
Low	0.6s	95.775

Table 6: Effect of setting tol

tol Value	Training Time (s)	Test Accuracy (%)
High	0.2s	87.3875
Medium	0.3s	98.9125
Low	1s	98.9375

Table 7: Effect of changing the penalty

penalty	Training Time (s)	Test Accuracy (%)
l1	37s	98.9
l2	1s	98.9375