PROBLEM 3 **Himani Patel**
SOEN 6011 **40071101**
Function 9 : $f(x, y) = x^y$ Date: 19/07/2019

1 Introduction

There are various ways to implement power function such as using iterative method , Recursive method , Taylor's Series, and so on.
The Taylor series for the exponential function e^x at a = 0 is represented as,

$\sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

as function is $f(x) = x^y$ is also represented as $(y \ln x)$ Hence, by substitute $(y \ln x)$ as x in the above series,

$x = e^{y \ln x}$

Therefore , the above series can also be represented as,

$e^x = e^x \ln a = 1 + \frac{(y \ln x)}{1!} + \frac{(y \ln x)^2}{2!} + \frac{(y \ln x)^3}{3!} + ...$

In order to find the value of the above series , we have to calculate the value of $(y \ln x)$.

For that, we will use the Taylor's series for the natural logarithm which is represented as,

$\ln x = 2[(\frac{x-1}{x+1}) + \frac{1}{3}(\frac{x-1}{x+1})^3 + \frac{1}{5}(\frac{x-1}{x+1})^5 + ...]$

In general, $\ln x = 2 \sum_{k=1}^{\infty} \frac{1}{2k-1} (\frac{x-1}{x+1})^{2k-1}$

Algorithm 1 Calculate Power Function

Require: value: For $x > 0$, $y \in R$; $For\ x = 0$, $y \geq 0$; $For\ x < 0$, $y \in Q$.
Ensure: $result = x^y$

procedure CalculateFactorial($number$)
    $factorial \leftarrow 1$
    $factorial \leftarrow number * $ CalculateFactorial($number - 1$)
    return $factorial$ ▷ It returns the factorial of the givem number
end procedure

procedure CalculatePowerForInteger(x , y)
    $power \leftarrow 1$
    for $i \leftarrow 1, y$ **do**
        $power \leftarrow power * x$
    end for
    return $power$ ▷ It returns the x raised to the power of y
end procedure

procedure CalculateNaturalLog($number$)
    $ans \leftarrow 0$
    $base \leftarrow (number - 1)/(number + 1)$
    for $i \leftarrow 1, 10$ **do**
        $exponent \leftarrow (2 * i) - 1$
        $ans \leftarrow ans + (1/exponent) *$ CalculatePowerForInteger($base, exponent$)
    end for
    return $2 * ans$ ▷ It returns natural logarithm for the given number.
end procedure

procedure CalculatePowerForReal(x,y)
    $answer \leftarrow 0$
    $logValue \leftarrow$ CalculateNaturalLog(x)
    for $i \leftarrow 1, 10$ **do**
        $numerator \leftarrow$ CalculatePowerForInteger($(y * logValue), i$)
        $denominator \leftarrow$ CalculateFactorial(i)
        $answer \leftarrow answer + (numerator/denominator)$
    end for
    return $answer$ ▷ Returns the Final Answer of x^y
end procedure

Algorithm 2 Calculate Power Function

Require: value: For $x > 0$, $y \in R$; $For\ x = 0$, $y \geq 0$; $For\ x < 0$, $y \in Q$.
Ensure: $result = x^y$

$nthRoot \leftarrow 1$
$exponent \leftarrow 1$

procedure CALCULATERATIONAL(x , y)
    $SValue \leftarrow (String)number$
    $SLength \leftarrow length\ of\ SValue$
    $index \leftarrow index\ of\ decimal\ point$
    $digits \leftarrow SLength - 1 - index$
    for $i \leftarrow 0, digits$ **do**
        $d \leftarrow d * 10$
        $r \leftarrow nthRoot * 10$
    end for
    $exponent \leftarrow d$
    $nthRoot \leftarrow r$

    procedure CALCULATEPOWER(x , y)
        $answer \leftarrow 1$
        $base \leftarrow 0$
        if $n\%1 \leftarrow 0$ **then**
            $base \leftarrow x$
            $exponent \leftarrow y$
        else
            CALCULATERATIONAL(y)
            $base \leftarrow nthRoot\ of\ x$
        end if
        if $(exponent < 0)$ **then**
            $exponent \leftarrow absoluteValue(exponent)$
        end if
        for $i \leftarrow 0, exponent$ **do**
            $answer \leftarrow answer * base$
        end for
        if $y < 0$ **then**
            $answer \leftarrow 1/answer$
        end if
        return $answer$ ▷ Returns the Final Answer of x^y

2 Advantages Disadvantages for Algorithm 1

2.1 Advantages

- It gives the incredibly accurate answer depending on the number of iterations used in the taylor series.
- Taylor series method is well-established series.Hence, it is easy to understand and simple to implement.

2.2 Disadvantages

- Time to execute is depend on the number of iterations used.
- Truncation error tends to grow rapidly away from expansion point.

3 Advantages and Disadvantages for Algorithm2

3.1 Advantages

- It is quite faster compared to the second approach.

3.2 Disadvantages

- Finding of nth root of value is quite difficult.
- Multiple conditions need to consider.
- Need to handle the case separately when the exponent is the fractional number.