# 100 APTITUDE SHORTCUTS FOR PLACEMENTS

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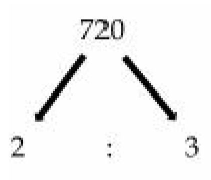
## Shortcut # 1 - Ratio and Proportion

**Splitting a number in the given ratio.**

1. Divide 720 in the ratio 2: 3.




Solution 1:
Total parts = 2 + 3 = 5
First number = (2/5) 720 = 288
Second number = (3/5) 720 = 432

Solution 2:
2x + 3x = 720; 5x = 720; x = 144
2x = 2(144) = 288; 3x = 3(144) = 432

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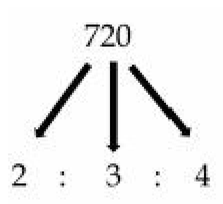
2. Divide 720 in the ratio 2: 3: 4.




Solution 1:
Total parts = 2 + 3 + 4 = 9
First number = (2/9) 720 = 160; Second number = (3/9) 720 = 240; Third number = (4/9)720 = 320

Solution 2:
2x + 3x + 4x= 720; 9x = 720; x = 80
2x = 2(80) = 160; 3x = 3(80) = 240; 4x = 4(80) = 320

Shortcut # 2 – Ratio and Proportion

**Direct Proportion.**

**When two parameters are in direct proportion if one parameter increases the other one will also increase and if one parameter decreases the other one will also decrease.**

**If the parameters A and B are in direct proportion they will satisfy the below equation.**

$$\frac{A_1}{A_2} = \frac{B_1}{B_2}$$

*Question:*

Price of diamond is directly proportional to its weight. If 2 grams of diamond costs $ 45,000 then what is the price of a diamond that weighs 6 grams?

*Answer:*

Weight of diamond in first case, $A_1$ = 2
Weight of diamond in second case, $A_2$ = 6
Price of diamond in first case, $B_1$ = 45000
Price of diamond in second case, $B_2$ =?

According to the equation,

$2/6 = 45000/B_2$

$B_2 = 45000 \text{ x } (6/2)$

$B_2 = 135{,}000$

Shortcut # 3 – Ratio and Proportion

## Inverse Proportion.

**When two parameters are in inverse proportion if one parameter increases the other one will decrease and if one parameter decreases the other one will decrease.**

**If the parameters A and B are in inverse proportion they will satisfy the below equation.**

$$\frac{A_1}{A_2} = \frac{B_2}{B_1}$$

*Question:*

Mileage and Engine capacity are inversely proportional. A bike with 100cc engine capacity gives a mileage of 80 km. What will be the mileage given by a bike with 150cc engine capacity?

*Answer:*

| | | |
|---|---|---|
| Engine capacity of first bike, | $A_1$ | = 100 |
| Engine capacity of second bike, | $A_2$ | = 150 |
| Mileage of first bike, | $B_1$ | = 80 |
| Mileage of second bike, | $B_2$ | =? |

According to the equation,

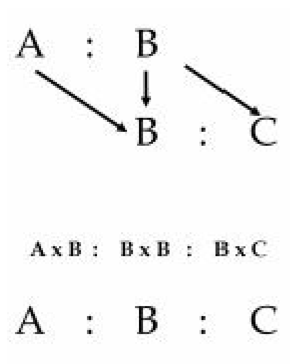
$100/150 = B_2/80$

$B_2 = 100(80)/150$

$B_2 = 53.33$ km

## Shortcut # 4 – Ratio and Proportion

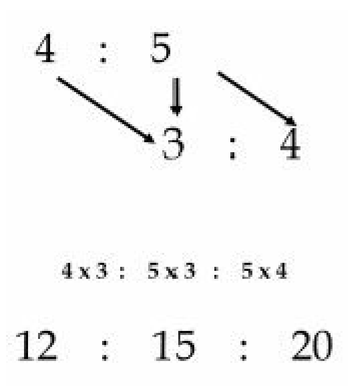
**Finding A: C from A: B and B: C**




*Question:*

The ratio between salary of A and B is 4: 5. The ratio between salary of B and C is 3: 4. Find the salary of C if A is earning $3600.

*Answer:*



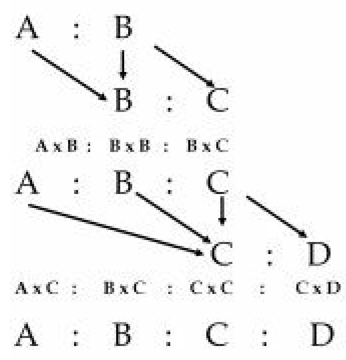

A: C = 12: 20 = 3: 5

A/C = 3/5

3600/C = 3/5;   C = 6000

Shortcut # 5 – Ratio and Proportion

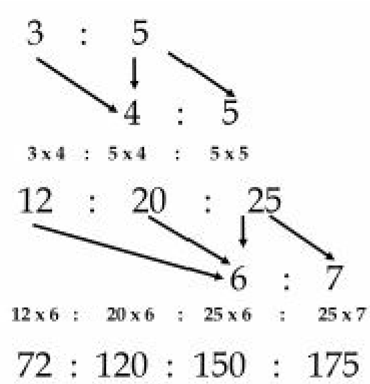
**Finding A: D from A: B, B: C and C: D**




*Question:*

Ratio between salary of A and B is 3: 5, B and C is 4: 5, C and D is 6: 7. If the salary of A is $ 7200, find the salary of D.

*Answer:*




A: D = 72: 175

7200/D = 72/175

D = 17500

Shortcut # 6 – Ratio and Proportion

**Usage of Common Factor x**

*Question:*

Two numbers are in the ratio 4: 5. Sum of their squares is 1025. Find the numbers.

*Answer:*

Substitute the common factor x to the ratio 4: 5

Assume the actual numbers as 4x and 5x

Given,

$(4x)^2 + (5x)^2 = 1025$

$16x^2 + 25x^2 = 1025$; $41x^2 = 1025$

$x^2 = 25$; $x = 5$

Substitute x = 5 in 4x and 5x to find the numbers

The numbers are, 20 and 25.

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*Question:*

Two numbers are in the ratio 3: 2. Cube of their difference is 125. Find the numbers.

*Answer:*

$(3x – 2x)^3 = 125$

$x^3 = 125$

$x = 5$

The numbers are 15 and 10.

Shortcut # 7 – Ratio and Proportion

**Finding number of Coins in a bag.**

Quantity ratio ----- $Q_1 : Q_2 : Q_3$

Value ratio ----- $V_1 : V_2 : V_3$

$(Q_1 \times V_1) + (Q_2 \times V_2) + (Q_3 \times V_3) = T$

Common factor X = Total Amount/T

Quantity of each coin = $Q_1X$, $Q_2X$, $Q_3X$

*Question:*

A bag contains 5 paise, 10 paise and 20 paise coins in the ratio 2:4:5. Total amount in the bag is Rs. 4.50. How many coins are there in 20 paise?

*Answer:*

T = (2 x 5) + (4 x 10) + (5 x 20)

= 150 paise

= Rs. 1.50

X = 4.50/1.50

X = 3

Quantity of 20 paise coins

= 5 x 3

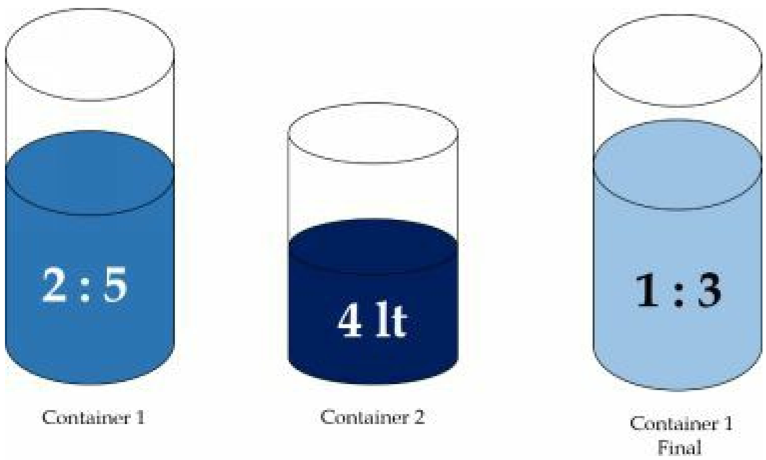
= 15 coins.

Shortcut # 8 – Ratio and Proportion

**Adding and removing quantities.**

*Question:*

Container 1 has milk and water in the ratio 2 : 5. After adding 4 liters of pure water from container 2, the ratio between milk and water in container 1 became 1 : 3. Find the quantity of milk in the container.




*Answer:*

Let us assume the actual quantity of milk and water as 2x and 5x.

New quantity of water = 5x + 4

2x/(5x + 4) = 1/3

6x = 5x + 4

x = 4

Quantity of milk = 2x

= 2(4)

= 8 liters.

## Shortcut # 9 – Partnership

**Finding ratio of the profit share.**

Income (or) Profit share from a business is determined using two parameters – Investment and Duration.

Profit share is directly proportional to the duration of investment and the amount invested.

$P_1 : P_2 : P_3$ = Ratio between profits

$T_1, T_2, T_3$ = Respective time of investment

$I_1, I_2, I_3$ = Respective investment

*Question:*

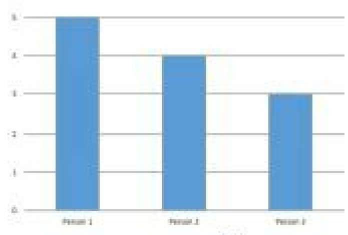
A started a business investing Rs. 5000. After 1 year B joined him investing Rs. 6000. After another year C joined them investing Rs. 9000. What is the ratio between their profit shares at the end of 3 years?

*Answer:*

$P_1 : P_2 : P_3 = 5000 \text{ x } 3 : 6000 \text{ x } 2 : 9000 \text{ x } 1$

$P_1 : P_2 : P_3 = 15000 : 12000 \text{ x } 9000$

$P_1 : P_2 : P_3 = 5 : 4 : 3$



## Shortcut # 10 – Mixtures and Allegation

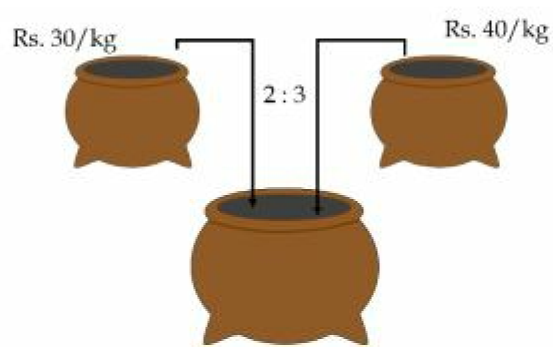
**Finding average price of a mixture.**

$$A_p = \frac{A_1N_1 + A_2N_2}{N_1 + N_2}$$

$A_p$ = Average price of the mixture.
$A_1$ = Price of the first variety.
$A_2$ = Price of the second variety.
$N_1$ = Quantity of the first variety.
$N_2$ = quantity of the second variety.

*Question:*

Two varieties of rice with prices Rs. 30 per kg and Rs. 40 per kg are mixed in the ratio 2 : 3. Find the mean price.




*Answer:*

$N_1 = 2$; $N_2 = 3$

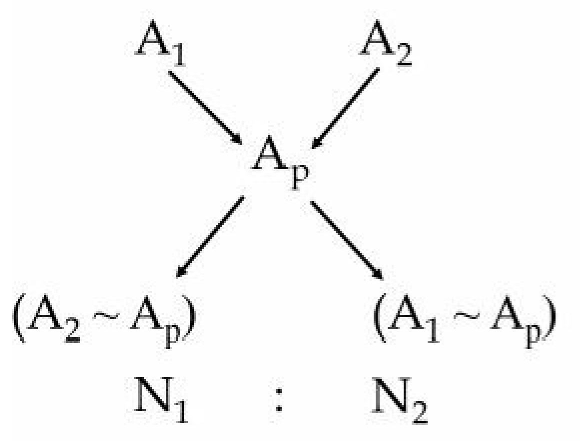
$_{A1} = 30$; $A_2 = 40$

Substitute the values in the above equation.

Mean price = Rs. 36

Shortcut # 11 – Mixtures and Allegation

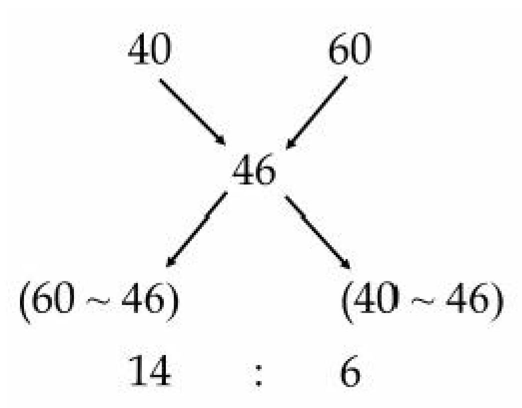
**Allegation Rule.**




*Question:*

In what ratio two varieties of rice worth Rs. 40 per kg and Rs. 60 per kg should be mixed to give a variety worth Rs. 46 per kg?

*Answer:*




The two mixtures have to be mixed in the ratio 7 : 3 to get Rs. 46 mixture.

Shortcut # 12 – Mixtures and Allegation

**Successive removal and replacement type.**

$$F = I[1 - (R/I)]^N$$

I – Initial quantity; R – Removal quantity;

F – Final quantity; N – Number of processes.

*Question:*

A container has 1000 liters of wine. 100 liters of wine is drawn from the container and filled with water. This process is done two more times. What is the ratio between wine and water in the final mixture?


*Answer:*

Number of draws n = 3

FQ = 1000 $[1-(100/1000)]^3 = 1000(0.9)^3 = 729$

Present quantity of wine = 729

Present quantity of water = 1000 – 729 = 271

Ratio between wine and water = 729 : 271

(Or)

1000 – 100 = 900

900 – 90 = 810

810 – 81 = 729

729 = final quantity of wine.

## Shortcut # 13 – Average

**Common increase or decrease for all the elements.**

**If al the elements in a series are increased or decreased or multiplied or divided by a certain number, the old average should also be added or subtracted or multiplied or divided respectively to get the new average.**

*Question:*

Average age of a family of four members is 34. What is the average age of the family after 4 years?

*Answer:*

Since every ones age is increased by four in 4 years, the average will also increase by 4

The new average = 34 + 4 = 38

*Proof:*

Total age of the family before 4 years = 34 x 4 = 136

Total increased age for four members = 4 x 4 = 16

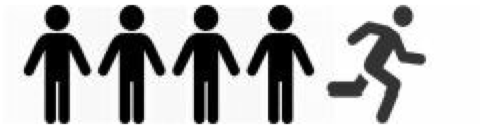
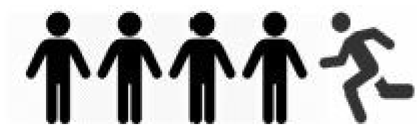
New total age = 136 + 16 = 152

New average
= 152/4
= 38

Shortcut # 14 – Average

**Adding or removing or replacing an element.**

$$A_1N_1 \sim A_2N_2$$

*Question 1:*

Average age of 5 students was 18. After adding teacher's age the average became 20. What is the age of the teacher?

Answer = (18 x 5) ~ (20 x 6) = 30

*Question 2:*

Average age of 5 students and a teacher was 21. After removing the teacher's age the average became 18. What is the age of the teacher?

Answer = (21 x 6) ~ (18 x 5) = 36

*Question 3:*

Average weight of 40 bags is 20kg. After removing 1 bag the average became 19.5. What is the weight of removed bag?

Answer = (20 x 40) ~ (19.5 x 39) = 39.5

*Question 4:*

Average weight of 40 bags is 20kg. After replacing a bag by another bag the average became 39. What is the difference between the weights of those two bags?

Answer = (20 x 40) ~ (20 x 39) = 20

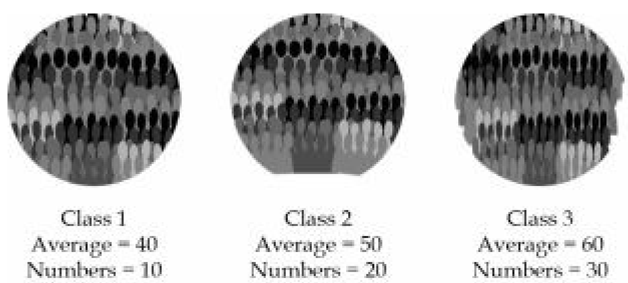
Shortcut # 15 – Average

**Weighted Average**

$$A_o = \frac{A_1N_1 + A_2N_2 + \ldots + A_nN_n}{N_1 + N_2 + \ldots + N_n}$$

*Question:*

Average marks scored by students in three classes are 40, 50, and 60. If there are 10, 20 and 30 students in the classes respectively, find the overall average of the three classes.




*Answer:*

$A_1 = 40$; $A_2 = 50$; $A_3 = 60$

$N_1 = 10$; $N_2 = 20$; $N_3 = 30$

Substitute in the above formula. We get,

=[(40x10) + (50x20) + (60x30)]/[10+20+30]

=[400 + 1000 + 1800]/60

= 53.33

Shortcut # 16 - Average

**Finding the middle subject mark, if the middle subject overlaps.**

$$M = A_1N_1 + A_2N_2 - A_oN_o$$

*Question:*

Average marks scored by Sai in 11 subjects is 60. Average marks scored by her in first 6 subjects is 50 and average marks scored by her in last 6 subjects is 62. Find the mark scored by Sai in 6$^{th}$ subject.

*Answer:*

$A_o = 60$; $N_o = 11$

$A_1 = 50$; $N_1 = 6$

$A_2 = 62$; $N_2 = 6$

Substituting the values in the formula, we get

M = 50x6 + 62x6 – 60x11

= 300 + 372 – 660

= 12

Marks scored by Sai in 6$^{th}$ subject = 12

Shortcut # 17 - Average

**Finding the middle subject mark if the middle subject is left out.**

$$M = A_oN_o - A_1N_1 - A_2N_2$$

*Question:*

The average marks scored by Shradha in 9 subjects is 75. The average marks in first 4 subjects is 69 and average marks in last 4 subjects is 78. Find the marks scored by her in $5^{th}$ subject.

*Answer:*

$A_o = 75$; $N_o = 9$

$A_1 = 69$; $N_1 = 4$

$A_2 = 78$; $N_1 = 4$

Substituting the above values in the formula, we get

$M = 75x9 – 69x4 – 78x4$

$= 675 – 276 – 312$

$= 87$

Marks scored by Sradha in $5^{th}$ subject = 87

## Shortcut # 18 – Percentage

**Percentage Calculation**

$$\% = \frac{\text{Value to be compared}}{\text{Value to which it has to be compared}} \times 100$$

$V_c$ = Value to be compared

$V_b$ = Value to which it has to be compared, base value.

*Question:*

What is the percentage of marks scored by Sai in final exam, if she has scored 1102 out of 1200?

*Answer:*

$V_c$ = 1102

$V_b$ = 1200

Percentage = (1102/1200) x 100

= 91.83%

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*Question:*

Out of 65000 population, men are 39000. What is the percentage of women?

*Answer:*
Women = 65000 – 39000 = 26000
Percentage of women = (26000/65000) x 100
= (2/5) 100 = 40 %

Shortcut # 19 – Percentage

**Percentage increase or decrease.**

$$\text{Percentage change} = \frac{\text{From} \sim \text{To}}{\text{From}} \times 100$$

From – the value from which the change is happening
To – the value to which the change has happened.

If **From** value is less than **To** value, it is % increase

If **From** value is more than **To** value, it is % decrease.

*Question:*

Sradha's monthly salary is Rs. 40000. If an increment of Rs. 5000 is provided, what is the percentage increase in her salary?

*Answer:*

From = 40000; To = 45000.

Substituting the values in the above formula, we get

% increase = [(45000 – 40000)/40000] x 100

= (5000/40000)x 100

= 12.5 % increase

Shortcut # 20 – Percentage

**Net change in percentage when two changes are made.**

$$\text{Net percentage change} = \pm a \pm b + \frac{(\pm a) \times (\pm b)}{100}$$

a = first percentage change

b = second percentage change

*Question:*

Salary of Sai was Rs. 20000. If the salary is decreased by 10 percent and then increased by 30 percent, what is the new salary of Sai?

*Answer:*

a = -10

b = 30

Net change = -10 + 30 + [(-10)(30)/100]

= 20 – 3 = +17% (positive value = % increase)

New salary of Sai

= 20000 + (17/100)20000

= 23400

Note:
If the net change value is negative, it means percentage decrease.

Shortcut # 21 – Percentage

**Final value after multiple percentage changes.**

$$\text{Final Value} = \left[\frac{100 \pm a}{100}\right] \times \text{Initial Value}$$

$$\text{Final Value} = \left[\frac{100 \pm a}{100} \times \frac{100 \pm b}{100}\right] \times \text{Initial Value}$$

$$\text{Final Value} = \left[\frac{100 \pm a}{100} \times \frac{100 \pm b}{100} \times \frac{100 \pm c}{100}\right] \times \text{Initial Value}$$

(a, b and c) are the percentage changes.
The formula can be expanded or reduce according to the number of changes given in the question.
Substitute '+' for percentage increase.
Substitute '-'for percentage decrease.

*Question:*

Salary of Shradha is Rs. 30000. It is increased by 10%, then decreased by 20% and again increased by 30%. What is her final salary?

*Answer:*

a = +10; b = -20; c = +30

Substitute the values in the above formula, we get

Final Value

= (11/10)(8/10)(13/10) x 30000

= 34320

Shortcut # 22 – Percentage

**Net percentage change when (a %) increase and (a %) decrease.**

$$\text{Net percentage change} = \frac{a^2}{100} \text{ \% decrease}$$

*Question:*

Price of a watch was Rs. 1000. The price is increased by 10% and then decreased by 10%. What is the price of the watch now?

*Answer:*

a = 10

Net change in percentage = $10^2/100$ = 1% decrease.

New price of watch = 1000 – (1/100)1000

= 1000 – 10

= 990

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*Question:*

Price of a product increased by 20% and the sales was reduced by 20%. By what percentage the income has reduced?

*Answer:*

a = 20%

Percentage reduction in income

= $20^2/100$

= 4%

Shortcut # 23 – Percentage

**Steady percentage increase or decrease.**

$$\text{Final Value} = \left[\frac{100 \pm a}{100}\right]^n \times \text{Initial Value}$$

a – steady rate of percentage increase or decrease.
Substitute '+' for percentage increase.
Substitute '-'for percentage decrease.
n – Number of times the percentage change occurs.

*Question:*

The population of a city is 200000. The population increases by 10% every year. What will be the population of the city after 3 years?

*Answer:*

a = + 10

Initial Value = 200000

Substitute the values in the above equation, we get

Final Value = $200000[1.1]^3$

Final Value = 266200

Population of the city after three years

= 266200

## Shortcut # 24 – Profit, Loss and Discount

**Profit or Loss Percentage.**

$$\text{Profit or Loss Percentage} = \frac{S.P \sim C.P}{C.P} \times 100$$

S.P – Selling Price.

C.P – Cost Price.

*Question:*

A bike worth Rs. 40000 is sold for Rs. 46000. What is the profit percentage?

*Answer:*

S.P = 46000;  C.P = 40000

Substitute the values in the above equation, we get

P% = [(46000-40000)/40000]x100  = 15% profit

*Question:*

A mobile phone worth Rs. 9900 is sold for Rs. 9000. What is the percentage loss?

*Answer:*

S.P = 9000;  C.P = 9900

Substitute the values in the above equation, we get

L% = [(9900-9000)/9900] x 100;  L% = 9.09% loss

Shortcut # 25 – Profit, Loss and Discount

**Net Profit or Loss Percentage when two changes are made.**

$$\text{Net Profit or Loss \%} = \pm a \pm b + \frac{(\pm a) \times (\pm b)}{100}$$

a = percentage profit or loss in the first case.

b = percentage profit or loss in the second case.

Substitute '+' for profit percentage.
Substitute '-' for loss percentage.

*Question:*
A product worth $ 3000 is sold for 10% profit and then for 20% loss. What is the overall profit or loss percentage in the sale?

*Answer:*
Net change = 10 – 20 + [(10)(-20)/100]

= -12

The value obtained is negative.

This indicates that a loss is incurred in the sale.

12% loss.

Shortcut # 26 – Profit, Loss and Discount

**Finding selling price after multiple changes.**

$$S.P = \left[\frac{100 \pm a}{100}\right] x\ C.P$$

$$S.P = \left[\frac{100 \pm a}{100} \ x\ \frac{100 \pm b}{100}\right] x\ C.P$$

$$S.P = \left[\frac{100 \pm a}{100} \ x\ \frac{100 \pm b}{100} \ x\ \frac{100 \pm c}{100}\right] x\ C.P$$

(a, b and c) are the profit or loss percentages.
The formula can be expanded or reduce according to the number of changes given in the question.
Substitute '+' for profit percentage.
Substitute '-' for loss percentage.
*Question:*

A watch worth Rs. 5000 is sold by A to B at 10% loss. B sold it to C at 30% loss. C sold it to D at 40% profit. What is the price at which D bought the watch?

*Answer:*

a = -10; b = -30; c = 40

C.P = 5000

Substitute the values in the above equation, we get

S.P = (9/10)(7/10)(14/10) x 5000

S.P = 4410

Shortcut # 27 – Profit, Loss and Discount

**Steady Profit or Loss.**

$$\text{Selling Price} = \left[\frac{100 \pm a}{100}\right]^n \times \text{Cost Price}$$

a – steady rate of profit or loss percentage.
Substitute '+' for profit percentage.
Substitute '-' for loss percentage.
n – Number of times the percentage change occurs.

*Question:*

The price of a laptop decreases by 20% every year. It the laptop was bought for Rs. 45000, at what price will it be sold after 2 years?

*Answer:*

a = -20

C.P = 45000

Substitute the values in the above equation, we get

S.P = $45000[0.8]^2$

S.P = 28800

Selling price after two years

= Rs. 28800

# Shortcut # 28 – Profit, Loss and Discount

Net gain or loss percentage when (a %) profit and (a %) loss occurs.

$$\text{Net Loss Percentage} = \frac{a^2}{100} \% \text{ loss}$$

a – percent loss and percentage gain.

*Question:*

Two laptops were sold for the same selling price. The first one is sold at 50% profit and the second one is sold at 50% loss. Due to this sale what is the profit or loss percentage incurred by the seller?

*Answer:*
Assume that the selling price of each laptop = Rs. 150
Cost price of first laptop:
150 = (150/100)CP
$C.P_1 = 100$
Cost price of second laptop:
150 = (50/100)CP
$C.P_2 = 300$
Total cost price = 100 + 300 = 400
Total selling price = 150 + 150 = 300
Loss % = (100/400)100 = 25%
(Or)
$(a^2/100)\%$ loss = $(50^2/100)\%$loss = 25% loss.

Note:
Shortcut applicable only when selling price of both products are equal.

Shortcut # 29 – Profit, Loss and Discount

**Discount, Marked Price and Selling Price.**

$$S.P = \left[\frac{100 \pm d}{100}\right] \times M.P$$

d – Discount percentage

M.P = Marked price or Market price of the product

*Question:*

A product is marked Rs. 450 by the seller. If he sells at a price of Rs. 330, what is the discount provided in percentage?

*Answer:*

M.P = 450

S.P = 330

Substitute the values in the above equation, we get

D% = [(450-330)/450]x100

D% = 26.67%

Discount provided in percentage = 26.67%

## Shortcut # 30 – Problems on ages

**One variable linear equation.**

*Question:*

Age of father is twice that of son at present. After 5 years, sum of their ages will be 100. What is the present age of father?

*Answer:*

Assume age of son = S

Age of father = 2S

Age of father after 5 years = 2S + 5

Age of son after 5 years = S + 5

Given,

(2S + 5) + (S + 5) = 100

3S + 10 = 100

S = 30

Present age of father = 2S = 60

Shortcut # 31 – Problems on ages

**One variable quadratic equation.**

*Question:*

Sum of the ages of father and son is 45. Product of their ages is 350. What is the difference between their ages?

*Answer:*

Age of father = F; Age of son = S

F + S = 45; F = 45 – S --- Equ (1)

F x S = 350 --- Equ (2)

Sub Equ (1) in (2)

$(45 - S)S = 350$

$45S - S^2 = 350$

$S^2 - 45S + 350 = 0$

By solving this quadratic equation, we get

S = 10, 35. (Neglect 35 since son's age cannot be greater than father's.)

From (1) Father's age = 35

Difference between their ages = 35 -10 = 25

Formula to find roots of a quadratic equation:

A – Coefficient of $x^2$; b - coefficient of x; c – constant

Shortcut # 32 – Problems on ages

**Simultaneous equations.**

*Question:*

Sum of ages of father and son at present is 40. After 5 years the ratio between their ages become 7 : 3. Find the present ages of father and son.

*Answer:*

Assume present age of father = F

Assume present age of son = S

Given, F + S = 40 --- Equ (1)

Father's age after 5 years = F + 5

Son's age after 5 years = S + 5

Given, (F + 5)/(S + 5) = 7/3, which gives

3F + 15 = 7S + 35

3F – 7S = 20 --- Equ (2)

By solving Equ (1) and (2), we get

Age of son, S = 10

Age of father, F = 30

Shortcut # 33 – Problems on ages

**Solution using common factor ‘x’.**

$$A - C = d \left\{ \begin{matrix} A & : & B \\ C & : & D \end{matrix} \right\} B - D = d$$

$$\text{Common factor 'x'} = \frac{\text{Difference in years}}{d}$$

*Question:*
Age of MS and VK were in the ratio 4 : 3 five years back. Their ages will be in the ratio 5 : 4 in two years. Find their present ages.

*Answer:*
Difference between the two compared years = 7.
Difference in ratio = 1

x = 7/1 = 7

Age of MS and VK 5 years ago = 4x and 3x

4x = 4(7) = 28

3x = 3(7) = 21

Age of MS at present = 28 + 5 = 34

Age of VK at present = 21 + 5 = 26

Note:
This method is applicable only when ‘d’ is equal on both sides.
The ratios can be modified to get ‘d’ same in both sides.

## Shortcut # 34 – Time, Speed and Distance

**Relative Speed – Same direction**

When two objects are moving we have to take relative speed for the calculation.

Relative speed of two objects moving in same direction

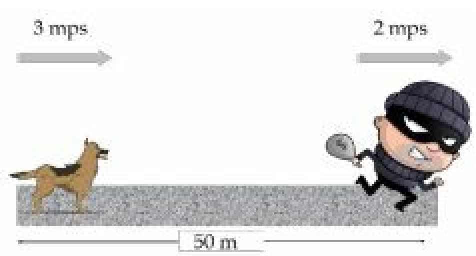
$= S_1 \sim S_2$

Relative speed of two objects moving in opposite direction

$= S_1 + S_2$

$$\text{Time taken} = \frac{\text{Distance}}{\text{Relative Speed}}$$

*Question:*




A thief standing 50 away from a dog starts running away from it at a speed of 2 m/s. The dog suddenly starts chasing him at a speed of 3m/s. How long will the dog take to catch the thief?

*Answer:*

D = 50m; $S_1 = 3$; $S_2 = 2$

Time taken, T = 50/(3-2) = 50 seconds

Distance travelled by the thief = Speed x Time to catch

= 2 x 50 = 100 m

Distance travelled by the dog = 3 x 50 = 150 m

Shortcut # 35 – Time, Speed and Distance

## Relative Speed – Opposite direction

When two objects are moving we have to take relative speed for the calculation.

Relative speed of two objects moving in same direction

$= S_1 \sim S_2$

Relative speed of two objects moving in opposite direction

$= S_1 + S_2$

$$\text{Time taken} = \frac{\text{Distance}}{\text{Relative Speed}}$$

*Question:*

Two friends start from their home 300 m apart and walk towards each other at a speed of 2mps and 3mps. How long (in minutes) will they take to meet each other?


*Answer:*

D = 300m

$S_1$ = 2mps;      $S_2$ = 3mps

T = 3000/(2+3) = 60 seconds

Time taken in minutes = 60/60 = 1 minute.

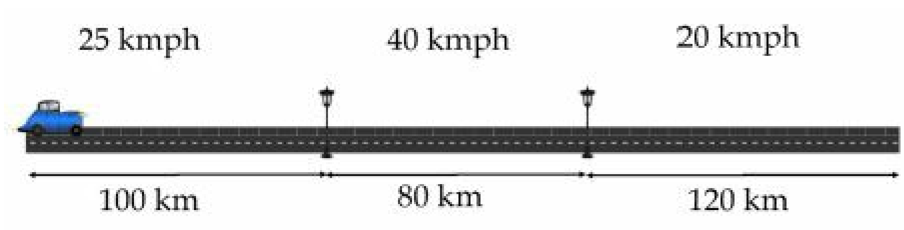
Shortcut # 36 – Time, Speed and Distance

**Average Speed – Different distance and time**

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}} = \frac{D_1 + D_2 + \ldots\ldots + D_n}{\frac{D_1}{S_1} + \frac{D_2}{S_2} + \ldots + \frac{D_n}{S_n}}$$

*Question:*

A man travels from a point to other. The first 100 km he travels at a speed of 25 kmph. The next 80 km he travels at a speed of 40 kmph. The last 120 km he travels at a speed of 20 kmph. What is the average speed of his journey?




*Answer:*

$$= \frac{100 + 80 + 120}{\frac{100}{25} + \frac{80}{40} + \frac{120}{20}}$$

= 300/12

= 25 kmph

Shortcut # 37 – Time, Speed and Distance

**Average Speed – Distance is same.**

General formula when distance is same in all the cases.

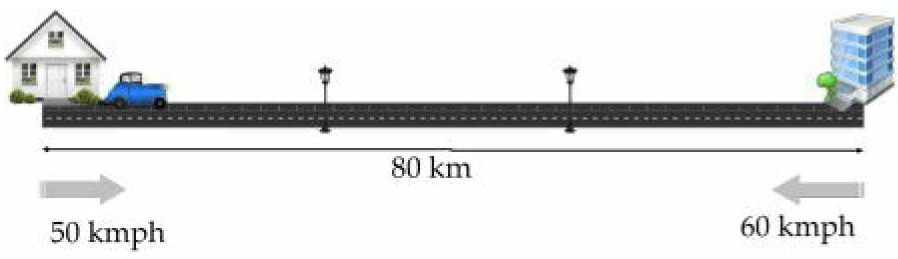
$$\frac{n}{\text{Average Speed}} = \frac{1}{S_1} + \frac{1}{S_2} + \dots + \frac{1}{S_n}$$

Formula for 2 cases with same distance.

$$\frac{2}{\text{Average Speed}} = \frac{1}{S_1} + \frac{1}{S_2} \quad \text{(Or)} \quad \text{Average Speed} = \frac{2\,S_1\,S_2}{S_1 + S_2}$$

*Question:*

Sai travels from her house to office at a speed of 50kmph. She suddenly returns home at a speed of 60kmph. What is her average speed?




*Answer:*

$S_1 = 50$; $S_2 = 60$

$S_a = 2(50 \times 60)/(50+60)$

$= 6000/110$ $= 54.54$ kmph

Note: Distance value is unnecessary.

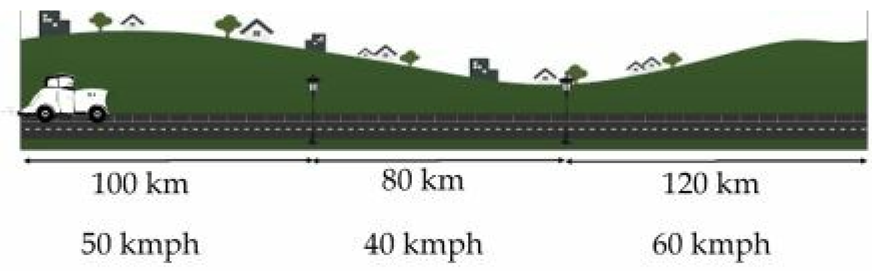
Shortcut # 38 – Time, Speed and Distance

**Average Speed – Time taken is same in all the cases.**

$$\text{Average Speed} = \frac{S_1 + S_2 + S_3}{3}$$

*Question:*

A man travels from a point to other. The first 100 km he travels at a speed of 50 kmph. The next 80 km he travels at a speed of 40 kmph. The last 120 km he travels at a speed of 60 kmph. What is the average speed of his journey?




*Answer:*

Time taken in the first case = 100/50 = 2 hours
Time taken in the second case = 80/40 = 2 hours
Time taken in the third case = 120/60 = 2 hours

Time taken is same in all the cases. So, average speed can be calculated by directly taking the average of the speeds in each case.

Average speed = (50 + 40 + 60)/3
= 50 kmph

Shortcut # 39 – Time, Speed and Distance

**Circular race – Time in which the runners meet at the starting point**

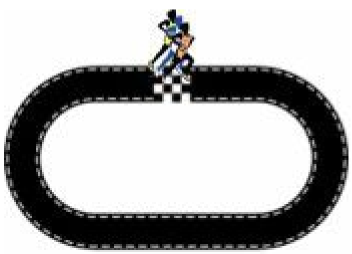
$$\text{LCM}\ (T_1, T_2, \ldots T_n) = \text{LCM}\ \left(\frac{D}{S_1}, \frac{D}{S_2}, \ldots, \frac{D}{S_n}\right)$$

D – Circumference of the race track.

$S_1, S_2, \ldots, S_n$ – Speed of the individual runners.

$T_1, T_2, \ldots, T_n$ – Time taken by the respective runners to complete one round the track.

*Question:*



Three persons participate in a race on a circular track of length 400m. They can run at a speed of 2mps, 4mps and 5mps respectively. How long will they take to meet in the starting point for the first time?

*Answer:*

Time taken by each person to complete one circle

Person 1 = 400/2 = 200 seconds = $T_1$

Person 2 = 400/4 = 100 seconds = $T_2$

Person 3 = 400/5 = 80 seconds = $T_3$

LCM($T_1$, $T_2$, $T_3$) = 400 seconds

Time taken by them to meet at starting point for the first time = 400 seconds.

## Shortcut # 40 – Trains

**Train crossing a stationary object with negligible width.**

$$\text{Time taken to cross} = \frac{L_t}{S_t}$$

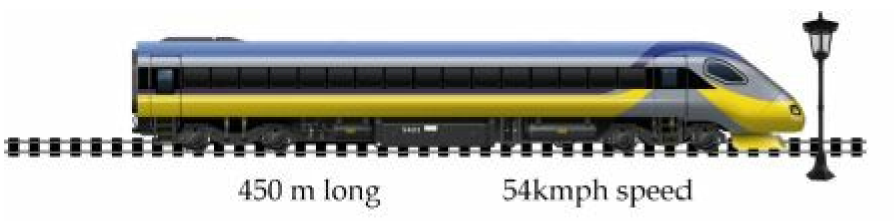
$L_t$ – Length of the train

$S_t$ – Speed of the train

| Conversion from Kmph to mps <\|> kmph x (5/18) = mps | | | | | | |
|---|---|---|---|---|---|---|
| Kmph | 18 | 36 | 54 | 72 | 90 | 108 |
| mps | 5 | 10 | 15 | 20 | 25 | 30 |

*Question:*

A train of length 450m is travelling at a speed of 54kmph. How long will it take to cross a pole?




*Answer:*

Speed of train $S_T$ in m/s = 54x(5/18)

= 15mps

Length of train = $L_T$ = 450m

Time taken to cross = 450/15

= 30 seconds

Shortcut # 41 – Trains

**Train crossing a stationary object with considerable length.**

$$\text{Time taken to cross} = \frac{L_t + L_p}{S_t}$$

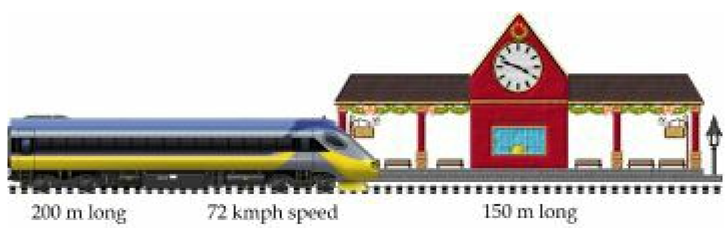
$L_t$ – Length of the train; $L_p$ – Length of the platform

$S_t$ – Speed of the train

| Conversion from Kmph to mps <\|> kmph x (5/18) = mps | | | | | | |
|---|---|---|---|---|---|---|
| Kmph | 18 | 36 | 54 | 72 | 90 | 108 |
| mps | 5 | 10 | 15 | 20 | 25 | 30 |

*Question:*

A train of length 200m crosses a platform of length 150m at a speed of 72kmph. How long will the train take to cross the platform?




*Answer:*

$L_t$ = 200m

$L_p$ = 150m

$S_t$ = 72x(5/18) = 20mps

Time taken to cross the platform = (200+150)/20

= 17.5 seconds

Shortcut # 42 – Trains

**Two trains crossing in the opposite directions.**

$$\text{Time taken to cross} = \frac{L_1 + L_2}{S_1 + S_2}$$

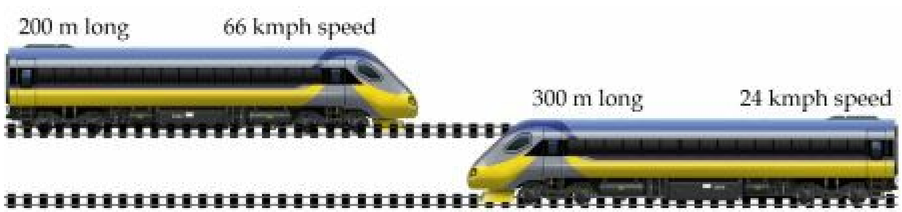
$L_1$ – Length of the train 1; $L_2$ – Length of train 2

$S_1$ – Speed of the train 1; $S_2$ – Speed of train 2

| Conversion from Kmph to mps <\|> kmph x (5/18) = mps | | | | | | |
|---|---|---|---|---|---|---|
| Kmph | 18 | 36 | 54 | 72 | 90 | 108 |
| mps | 5 | 10 | 15 | 20 | 25 | 30 |

*Question:*

Two trains A and B of length 200m and 300 are travelling at a speed of 54kmph and 36kmph respectively in opposite directions. How long will they take to cross each other?




*Answer:*

$L_1 = 200$; $L_2 = 300$

$S_1 + S_2 = 66 + 24 = 90$ kmph = 25 mps.

Substitute the values in the above equation, we get

T = (200+300)/(25)

= 20 seconds

Shortcut # 43 – Trains

**Train crossing another train from the same direction.**

$$\text{Time taken to cross} = \frac{L_1 + L_2}{S_1 \sim S_2}$$

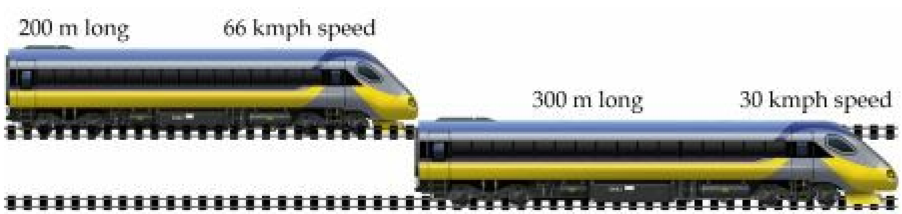
$L_1$ – Length of the train 1; $L_2$ – Length of train 2

$S_1$ – Speed of the train 1; $S_2$ – Speed of train 2

| Conversion from Kmph to mps <\|> kmph x (5/18) = mps | | | | | | |
|---|---|---|---|---|---|---|
| Kmph | 18 | 36 | 54 | 72 | 90 | 108 |
| mps | 5 | 10 | 15 | 20 | 25 | 30 |

*Question:*

Two trains A and B of length 200m and 300 are travelling at a speed of 66 kmph and 30 kmph respectively in same directions. How long will they take to cross each other?




*Answer:*

$L_1 = 200$; $L_2 = 300$

$S_1 \sim S_2 = 66 – 30 = 36$ kmph = 10 mps

Substitute the values in the above equation, we get

T = (200+300)/(10)

= 50 seconds

Shortcut # 44 – Trains

**Train crossing a man sitting on another train travelling in opposite direction.**

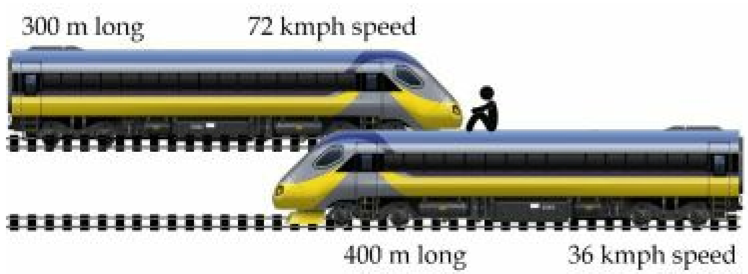
$$\text{Time taken to cross} = \frac{L_1}{S_1 + S_2}$$

$L_1$ – Length of the train which is crossing the man

$S_1$ – Speed of the train 1; $S_2$ – Speed of train 2

*Question:*

Two trains A and B of length 300m and 400 are travelling at a speed of 72kmph and 36kmph respectively in opposite directions. How long will train A take to cross a man travelling in train B?




*Answer:*

$L_1$ = 300, (Length of train crossing the man)

$S_1 + S_2$ = 72 + 36 = 108 kmph = 30 mps

Substitute the values in the above equation, we get

T = (300)/(30)

= 10 seconds

Shortcut # 45 – Trains

**Train crossing a man sitting on another train travelling in same direction.**

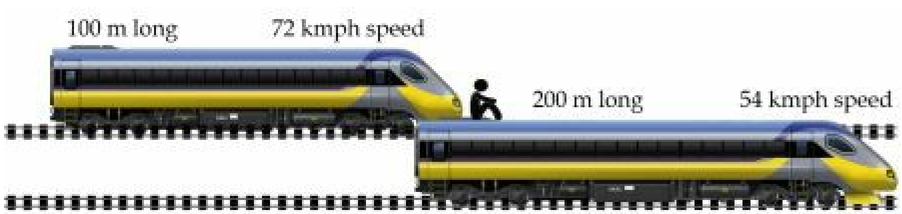
$$\text{Time taken to cross} = \frac{L_1}{S_1 \sim S_2}$$

$L_1$ – Length of the train which is crossing the man

$S_1$ – Speed of the train 1; $S_2$ – Speed of train 2

*Question:*

Two trains A and B of length 100m and 200 are travelling at a speed of 72kmph and 54kmph respectively in same directions. How long will train A take to cross a man travelling in train B?




*Answer:*

$L_T$ = 100, (Length of train crossing the man)

$S_1$ = 72kmph = 72(5/18) mps = 20mps

$S_2$ = speed of the man travelling in train B

$S_2$ = 54kmph = 36(5/18) mps = 10mps

Substitute the values in the above formula, we get

T = (100)/(20-10)

= 10 seconds

## Shortcut # 46 – Boats

**Time taken to row upstream.**

$$\text{Time taken to row upstream} = \frac{D_{us}}{S_b - S_r}$$

$D_{us}$ – Distance travelled upstream.

$S_b$ – Speed of the boat in still water.

$S_r$ – Speed of the river flow (or) stream (or) current.

*Question:*

The speed of boat in still water is 40kmph. How long will the boat take to row 175 km upstream in a river of speed 5kmph?

*Answer:*

D = 175km

$S_b$ = 40kmph, (speed of boat in still water)

$S_r$ = 5kmph, (speed of river water)

$S_b - S_r$ = 35kmph, (Upstream speed of boat)

Substitute the values in the above formula, we get

T = 175/35

= 5 hours

Shortcut # 47 – Boats

**Time taken to row downstream.**

$$\text{Time taken to row downstream} = \frac{D_{ds}}{S_b + S_r}$$

$D_{ds}$ – Distance travelled downstream.

$S_b$ – Speed of the boat in still water.

$S_r$ – Speed of the river flow (or) stream (or) current.

*Question:*

The speed of boat in still water is 30kmph. How long will the boat take to row 96 km downstreamin a river of speed 2kmph?

*Answer:*

D = 96km

$S_b$ = 30kmph, (speed of boat in still water)

$S_r$ = 2kmph, (speed of river water)

$S_b + S_r$ = 32kmph, (Downstream speed of boat)

Substitute the values in the above formula, we get

T = 96/32

= 3 hours

Shortcut # 48 – Boats

**Finding speed of the boat when upstream and downstream speed are known.**

$$\text{Speed of the boat, } S_b = \frac{S_{ds} + S_{us}}{2}$$

$S_b$ – Speed of the boat in still water.

$S_{us}$ - Upstream speed.

$S_{ds}$ – Downstream speed.

*Question:*

Time taken by a boat to row 40km upstream in 4 hours. It took 2 hours to return back to the starting point. Find the speed of the boat.


*Answer:*

Step 1: Find the speed upstream and downstream.

$S_{us} = 40/4 = 10\text{kmph}$

$S_{ds} = 40/2 = 20\text{kmph}$

Step 2: Find the speed of boat.

$S_b = (10 + 20)/2 = 15\text{kmph}$

Shortcut # 49 – Boats

**Finding speed of the river when upstream and downstream speed are known.**

$$\text{Speed of the boat, } S_b = \frac{S_{ds} - S_{us}}{2}$$

$S_b$ – Speed of the boat in still water.

$S_{us}$ - Upstream speed.

$S_{ds}$ – Downstream speed.

*Question:*

Time taken by a boat to row 40km upstream in 4 hours. It took 2 hours to return back to the starting point. Find the speed the river.


*Answer:*

Step 1: Find the speed upstream and downstream

$S_{us}$ = 40/4 = 10kmph

$S_{ds}$ = 40/2 = 20kmph

Step 2: Find the speed of the river

$S_r$ = (20 – 10)/2 = 5kmph

## Shortcut # 50 – Time and Work

**Work is directly proportional to time.**

$$\frac{W_1}{W_2} = \frac{T_1}{T_2}$$

$W_1$ – Work done in first case.
$T_1$ – Time taken in first case.
$W_2$ – Work done in second case.
$T_2$ – Time taken in second case.

*Question:*

It took 40 days to build 10 buildings. How long will it take to build 50 buildings using same number of employees?


*Answer:*

$W_1 = 10$; $W_2 = 50$

$T_1 = 40$; $T_2 = ?$

Substitute the values in the above formula, we get

$10/50 = 40/T_2$

$T_2 = 200$ days

Shortcut # 51 – Time and Work

**Work is directly proportional to Resource.**

$$\frac{W_1}{W_2} = \frac{R_1}{R_2}$$

*Question:*

40 men are required to complete 20 buildings. How many men are required to build 50 buildings in the same time?

*Answer:*

$R_1 = 40$; $R_2 = ?$

$W_1 = 20$; $W_2 = 50$

Substitute the values in the above formula, we get

$40/R_2 = 20/50$

$R_2 = 100$

Shortcut # 52 – Time and Work

**Time is inversely proportional to Resource.**

$$\frac{T_1}{T_2} = \frac{R_2}{R_1}$$

*Question:*

If 20 men can do the work in 40 days, how many men are required to do the same work in 10 days?

*Answer:*

$R_1 = 20$; $R_2 = ?$

$T_1 = 40$; $T_2 = 10$

Substitute the values in the above formula, we get

$20/R_2 = 10/40$

$R_2 = 80$

Shortcut # 53 – Time and Work

**Comparison of Time, Work and Resource.**

$$\frac{W_1}{W_2} = \frac{R_1T_1}{R_2T_2}$$

*Question:*

If 10 men can cut 40 trees in 4 days, how many trees can be cut by 40 men 6 days?

*Answer:*

$R_1 = 10$; $R_2 = 40$

$T_1 = 4$; $T_2 = 6$

$W_1 = 40$; $W_2 = ?$

Substitute the values in the above formula, we get

$40/W_2 = (10 \text{ x } 4)/(40 \text{ x}6)$

$W_2 = 240$ trees

Shortcut # 54 – Time and Work

**When the number of working hours per day is included in the question.**

$$\frac{W_1}{W_2} = \frac{R_1T_1H_1}{R_2T_2H_2}$$

*Question:*

30 men working 6 hours day for 50 days can make 2000 toys. How many hours per day should 40 men work for 60 days to make 3000 toys?

*Answer:*

$R_1 = 30$; $R_2 = 40$, (Resources in each case)

$T_1 = 50$; $T_2 = 60$, (Days in each case)

$W_1 = 2000$; $W_2 = 3000$, (Work in each case)

$H_1 = 6$; $H_2 = ?$, (Hours per day in each case)

Substitute the values in the above formula, we get

$2000/3000 = (30 \text{ x } 50 \text{ x } 6)/(40 \text{ x } 60 \text{ x } H_2)$

$H_2 = 5(5/8)$ hours = 5 hours 32 minutes 30 seconds

Shortcut # 55 – Time and Work

**Two resources working together.**

$$T = \frac{AB}{A + B} \quad \text{or} \quad \frac{1}{T} = \frac{1}{A} + \frac{1}{B}$$

*Question:*

A can complete a work in 40 days. B can complete the same work in 60 days. How long will they take to complete the work if they are working together?


*Answer:*

A = 40

B = 60

Substitute the values in the above formula, we get

Time taken together = (40 x 60)/(40 + 60)

= 24 days

Shortcut # 56 – Time and Work

**Three resources working together.**

$$T = \frac{ABC}{AB + BC + AC} \quad \text{or} \quad \frac{1}{T} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C}$$

*Question:*

A can do a work in 40 days, B in 50 days and C in 60 days. If they work together, how long will they take to complete the work?


*Answer:*

A = 40

B = 50

C = 60

Substitute the values in the above formula, we get

Time taken together = (40 x 50 x 60)/(40 + 50 + 60)

= 16.21 days

Shortcut # 57 – Time and Work

**Work done.**

Work done per day = 1/Time taken to do complete work

Work done = Number of days worked x Work done per day

*Question:*

A can do a piece of work in 40 days. What is the fraction of work done by him in 25 days?

*Answer:*

Per day work of A = 1/40

Number of days for which A worked = 25

Work done = (1/40) x 25

= $\frac{5}{8}$

---

*Question:*

A can do a piece of work in 30 days. What is the fraction of work done by him in 45 days?

*Answer:*

Per day work of A = 1/30

Number of days for which A worked = 45

Work done = (1/30) x 45

= $1\frac{1}{2}$

Shortcut # 58 – Time and Work

**Remaining work.**

Remaining Work = 1 - Work Done

*Question:*

A can do a work in 30 days, B in 40 days. A works for 9 days and the remaining work is done by B. What is the fraction of work done by B?

*Answer:*

Remaining work for B = 1 – Work done by A

Work done by A = (1/30) x 9 = 3/10

Remaining work for B = 1 – (3/10) = 7/10

---

*Question:*

A can do a work in 40 days, B in 60 days. A works for 24 days and the remaining work is done by B. What is the fraction of work done by B?

*Answer:*

Remaining work for B = 1 – Work done by A

Work done by A = (1/40) x 24 = 6/10

Remaining work for B = 1 – (6/10) = 4/10

= 2/5

Shortcut # 59 – Time and Work

**Time taken to do the remaining work.**

Time taken to do remaining Work

= Remaining work x Time taken to do full work

*Question:*

A can do a work in 30 days, B in 40 days. A works for 9 days and the remaining work is done by B. What is the total number of days to complete the work?

*Answer:*

Time taken to complete the work

= Remaining work x time to finish full work

Remaining work for B = 1 – Work done by A

Work done by A = (1/30) x 9 = 3/10

Remaining work for B = 1 – 3/10 = 7/10

Time taken by B to complete the remaining work

= 7/10 x 40

= 28 days

Total time taken
= 28 + 9
= 37 days

Shortcut # 60 – Time and Wok

**Salary ratio of resources working together.**

Salary of a resource is directly proportional to the per day work.

$$S_A : S_B = \frac{1}{A} : \frac{1}{B}$$

$$S_A : S_B : S_C = \frac{1}{A} : \frac{1}{B} : \frac{1}{C}$$

*Question:*

A can do a work in 20 days and B can do the same work in 40 days. They both work together and get a combined salary of Rs. 3000. What is the salary of A?

*Answer:*

$S_A$ = Salary of A; $S_B$ = Salary of B

A = 20; B =40

$S_A : S_B = 40 : 20 = 2 : 1$

$S_A = [2/(2+1)] \times 3000 = 2000$

---

*Question:*

A can do a work in 30 days B in 40 days and C in 50 days. If they all work together, what will the ratio between their salaries?

*Answer:*

A = 30; B = 40; C = 50

$S_A : S_B : S_C = (1/30) : (1/40) : (1/50) = 20 : 15 : 12$

## Shortcut # 61 – Pipes and Cisterns

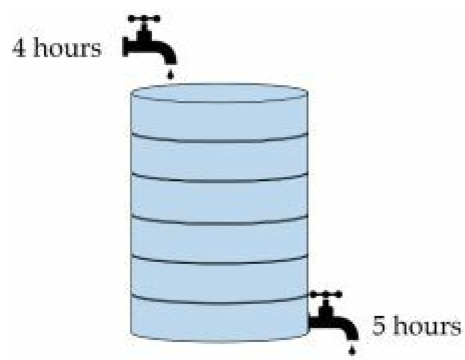
**One pipe fills the tank and the other empties the tank.**

$$\frac{1}{T} = \frac{1}{A} - \frac{1}{B}$$

The pipe which empties the tank is doing negative work. So we have to assign '-'symbol for the work done by that pipe.
T – Total time taken to fill the tank.

*Question:*

Pipe A can fill a tank in 4 hours and pipe B can empty the tank in 5 hours. If both pipes are opened together, how long will they take to fill a complete tank?




*Answer:*

A = 4; B = 5

Substitute the values in the above equation, we get

Time taken to fill the tank = (4 x 5)/(-4 + 5)

= 20 hours

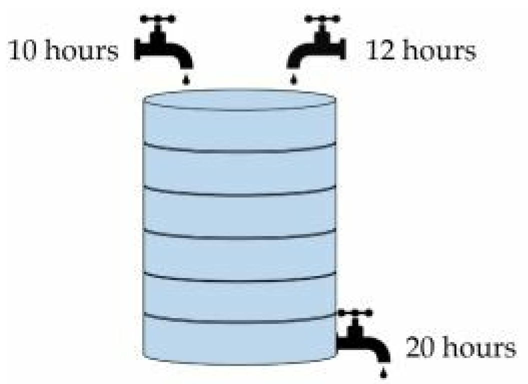
Shortcut # 62 – Pipes and Cisterns

**Two pipes fill a tank and another pipes empties the tank.**

$$\frac{1}{T} = \frac{1}{A} + \frac{1}{B} - \frac{1}{C}$$

*Question:*

Pipes A and B can fill a tank in 10 hours and 12 hours respectively. Pipe C can empty a tank in 20 hours. If all the pipes are opened together, how long will they take to fill a complete tank?




*Answer:*

A = 10; B = 12; C = 20

Substitute the values in the above equation, we get

Time taken to fill the tank

= (10 x 12 x 20 )/[(-10 x 12) + (12 x 20) + (10 x 20)]

= 7. 5 hours = 7 hours 30 minutes

## Shortcut # 63 – Interest – Simple and Compound

### Simple Interest

$$\text{Simple Interest} = \frac{PNR}{100}$$

P = Principle amount invested or borrowed

R = Rate of interest per term

N = Number of terms

Term = duration for which interest is calculated

*Question:*

A man invested Rs. 10000 at 4% per annum simple interest.

1. What is the interest he will get at the end of 3 years?
2. What is the total amount earned after 3 years?

*Answer:*

P = 10000; N = 3; R = 4

Substitute the values in the above equation, we get

SI = 10000 x 3 x 4/100

= 1200

**Amount = Principle + Interest**

Amount = 10000 + 1200 = 11200

Shortcut # 64 – Interest – Simple and Compound

**Compound Interest.**

$$\text{Amount after } n^{th} \text{ term} = \frac{100 + R}{100} \text{ Amount after } (n-1)^{th} \text{ term}$$

*Question:*

What is the compound interest obtained for Rs. 16000 at 2% per annum for three years?

*Answer:*

Amount at the end of 1st year = $\frac{102}{100}$ x 16000 = 16320

Amount at the end of 2nd year = $\frac{102}{100}$ x 16320 = 16646.4

Amount at the end of 3rd year = $\frac{102}{100}$ x 16646.4 = 16979.328

Interest obtained at the end of three years:

Interest = Amount – Principle

Interest = 16979.328 – 16000 = 979.328

Note:

In simple interest, the interest is calculated only for the principle. In compound interest, the interest is calculated for the principle as well as the interest obtained till the last term.

## Shortcut # 65 – Number Systems

**Divisibility.**

| Divisible by | Rule |
|---|---|
| 2 | Last digit of a number should be 0, 2, 4, 6 or 8 |
| 3 | Sum of the digits should be divisible by 3 |
| 4 | Last two digits of a number should be divisible by 4 |
| 5 | Last digit of a number should be 0 or 5 |
| 6 | Number should be divisible by 2 and 3 |
| 8 | Last three digits of a number should be divisible by 8 |
| 9 | Sum of the digits should be divisible by 9 |
| 10 | Last digit of a number should be 0 |
| 11 | Difference between sum of digits in odd places and sum of digits in even places should be o or 11 |

Divisibility rule for composite numbers:

If a number is a multiple of 'x' and 'y', then it is divisible by LCM of (x, y). 30 is the LCM of (2, 3 and 5). So all the multiples of 30 are divisible by 2, 3 and 5.

*Question:*

Which of the following numbers is divisible by both 5 and 9?

1115, 11115, 111115, 1111115

*Answer:*

11115 – It is divisible by 5 and 9.

Shortcut # 66 – Number Systems

**Finding largest four digit number that is divisible by certain numbers.**

Step 1: LCM of the divisors
Step 2: Remainder of 10000/LCM
Step 3: 10000 – Remainder = Answer

*Question:*

Find the largest number that leaves

1. Remainder 0 when divided by 12, 15 and 20.
2. Remainder 2 when divided by 12, 15 and 20.
3. Remainder 9, 12 and 17 when divided by 12, 15 and 20 respectively.

*Answer:*

1. Remainder 0:

LCM(12, 15, 20) = 60
Remainder(10000/60) = 40
10000 – 40 = 9960.
The largest number that leaves remainder 0 = 9960

2. Remainder 2:

9960 + 2 = 9962 is the largest 4 digit number that leaves remainder 2 when divided by 12, 15 and 20.

3. Remainder 9, 12 and 17:

Difference between divisor and remainder = 3.
9960 – 3 = 9957 is the largest 4 digit number that leaves remainder 9, 12 and 17 when divided by 12, 15 and 20.

Shortcut # 67 – Number Systems

## Finding unit digit of a number with certain power.

The expression will be in the form $X^Y = Z$

| Unit digit of X | Unit digit of Z |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 5 | 5 |
| 6 | 6 |

Power value → Unit place of Z

| Unit place of X | Odd | Even |
|---|---|---|
| 4 | 4 | 6 |
| 9 | 9 | 1 |

Remainder when power value is divided by 4 → Unit place of Z

| Unit place of X | 1 | 2 | 3 | 0 |
|---|---|---|---|---|
| 2 | 2 | 4 | 8 | 6 |
| 3 | 3 | 9 | 7 | 1 |
| 7 | 7 | 9 | 3 | 1 |
| 8 | 8 | 4 | 2 | 6 |

Shortcut # 68 – Number Systems
Remainder Concept.

$$\text{Remainder}\left(\frac{A \times B \times C}{K}\right) = R\left(\frac{A}{K}\right) \times R\left(\frac{B}{K}\right) \times R\left(\frac{C}{K}\right)$$

This concept is also applicable if the numbers in the numerator are added or subtracted.

*Question:*

Find the remainder when 212 x 313 x 414 is divided by 5.

*Answer:*

Using the above concept we can find the remainder for each term and then we can multiply.

R(212/5) = 2

R(313/5) = 3

R(414/5) = 4

2 x 3 x 4 = 24

The value we got is still greater than 5, so again divide it and find the remainder.

R(24/5) = 4

The remainder of the expression is = 4

## Shortcut # 69 – HCF and LCM

**Finding both HCF and LCM using single L-Division.**

While doing the L-Division method, initially divide only using the prime numbers which are common factors for all the numbers, then divide using the prime numbers which are not common.

*Question:*

Find LCM, and HCF for the numbers 240, 450 and 570.

| | |
|---|---|
| ***2*** | 240, 450, 570 |
| ***5*** | 120, 225, 285 |
| ***3*** | 24, 45, 57 |
| 19 | 8, 15, 19 |
| 5 | 8, 15, 1 |
| 3 | 8, 3, 1 |
| 2 | 8, 1, 1 |
| 2 | 4, 1 , 1 |
| 2 | 2, 1, 1 |
| | 1, 1, 1 |

The common factors for the three numbers are 2, 3 and 5.
Variation between common factors and the others is shown by using different font.

Multiply only the common factors to get the HCF

HCF = 2 x 3 x 5 = 30

Multiply all the factors to get LCM

LCM = 2 x 3 x 5 x 19 x 5 x 3 x 2 x 2 x 2

$= 2^4 \times 3^2 \times 5^2 \times 19^1$

= 68400

Shortcut # 70 – HCF and LCM

## Application problem using HCF.

There will be different sets of elements with different quantities in each. They have to be arranged based on the following conditions.

1. Every group should have equal number of elements.
2. Every group should have same kind of elements.

Under these conditions one must find out:

1. The maximum number of elements per group.
2. The minimum number of groups required.

**HCF of the numbers of elements in all the groups will give the maximum number of elements per group.**

**Total number of elements divided by HCF will give the minimum number of groups required.**

*Question:*

There are 40 apples and 32 oranges. They have to be packed in boxes such that each box will have one kind of fruit and every box will have equal number of fruits. Under this condition, what is the:

1. Maximum number of fruits per box?
2. Minimum number of boxes required?

*Answer:*

Maximum number of fruits per box = HCF(40, 32) = 8

Minimum number of boxes required = (40 + 32)/8 = 9

Shortcut # 71 – HCF and LCM
**Application problem using HCF.**

**For three or more numbers, the HCF of the difference between the successive numbers arranged in ascending order is the largest number that can divide the given numbers and leave same remainder.**

*Question:*

What is the largest number that can divide 212, 254, 310 and 338 and leave the same remainder?

*Answer:*
The difference between the numbers = 42, 56 and 28.
HCF (42, 56, 28) = 14
Remainder (212/14) = 2
Remainder (254/14) = 2
Remainder (310/14) = 2
Remainder (338/14) = 2
14 is the largest number that can divide the given numbers and leave the same remainder.

---

*Question:*

What is the largest number that can divide 34, 52 and 88 and leave the same remainder?

*Answer:*
Difference between the numbers = 18 and 36
HCF (18, 36) = 18

18 is the largest number that can divide 34, 52 and 88 and leave same remainder.

Shortcut # 72 – HCF and LCM

**Application problem using LCM.**

**When different events occur in different intervals of their own and if all the events are started at the same time, at some point of time all the events will again occur together. To find the duration between two common occurrences, LCM of the intervals of each event has to be calculated.**

*Question:*

There are three different bells. The first bell rings every 4 hour, the second bell rings every 6 hours and the third bell rings every 15 hours. If all the three bells are rung at same time, how long will it take for them to ring together again at the same time?

*Answer:*

Take LCM of 4, 6, 15

LCM (4,6,15) = 60

All the three bells will ring after 60 hours.

---

*Question:*

There are three different alarms. The first alarm rings every 2 minutes, the second alarm rings every 3 minutes and the third alarm rings every 5 minutes. If all the three alarms are rung at 6 am together, at what time the three alarms will ring together for the next time?

Answer:
LCM (2, 3, 5) = 30. After 30 minutes = 06:30 am.

# Shortcut # 73 – Heights and Distances

**Solution using tan ϴ .**

| ϴ | 0 | 30 | 45 | 60 | 90 |
|---|---|---|---|---|---|
| tan | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ |

*Question:*

Jack Sparrow from a boat saw a lighthouse of height 100m at an angle of elevation, $60^o$. What is the distance between Jack Sparrow and the tower?


*Answer:*

tan ϴ = $\frac{\text{opposite side}}{\text{adjacent side}}$

tan $60^o$ = $\frac{100}{x}$ = $\sqrt{3} = \frac{100}{x}$ = x = 57.7 m

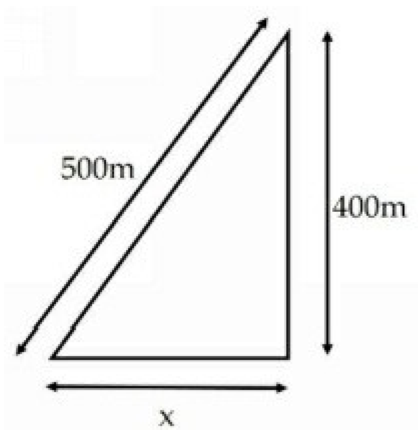
Shortcut # 74 – Heights and Distances

**Solution using Pythagoras theorem.**

$$\text{Hypotenuse}^2 = \text{Oppsoite}^2 + \text{Adjacent}^2$$

*Question:*

A man looks at the top of a tower which is 400m height. The minimum distance between him and top of the tower is 500m. What is the distance between him and the base of the tower?




*Answer:*

$hyp^2 = opp^2 + adj^2$

$500^2 = 400^2 + x^2$

$x^2 = 90000$

$x = 300$ m

## Shortcut # 75 – Arithmetic Progression.

**Finding $n^{th}$ term of an Arithmetic Progression.**

$$t_n = a + (n - 1)d$$

Parameters:

a - first term of the series.

d - common difference between successive terms. $a_2 - a_1$

n - the number position of the term.

$t_n$ - the term that has to found out.

*Question:*

What is the $100^{th}$ term of the series?

1. 2, 7, 12, 17, 22, …
2. 501, 497, 493, 489, 485, …

*Answer:*

1. a = 2; d = 5; n = 100

   $t_n = 2 + (100 – 1)5$

   $t_n = 2 + 495 = 497$

   $t_{100} = 497$

2. a = 501; d = - 4; n = 100

   $t_n = 501 + (100 – 1)(- 4)$

   $t_n = 105$

   $t_{100} = 105$

Shortcut # 76 – Arithmetic Progression

**Number of terms in an Arithmetic Progression.**

$$n = \frac{(l - a)}{d} + 1$$

$l$ - Last term of the series.

*Question:*

Find the number of terms in the series:

1. 2, 7, 12, 17, 22, … 382
2. 501, 497, 493, 489, 485, … 201

*Answer:*

1. $n = \frac{(382 - 2)}{5} + 1$

   $n = 77$

2. $n = \frac{(201 - 501)}{-4} + 1$

   $n = 76$

Shortcut # 77 – Arithmetic Progression

**Sum of the terms in an Arithmetic Progression.**

$$S_n = (a + l)\frac{n}{2}$$

l - Last term of the arithmetic series.

*Question:*

Find the sum of the series:

1. 2, 7, 12, 17, 22, … 497
2. 501, 497, 493, 489, 485, … 105

*Answer:*

1. Number of terms, n = $\frac{(497 - 2)}{5}$ + 1

n = 100

Sum = (2 + 497)(100/2)

S = 24950

3. Number of terms, n = $\frac{(105 - 501)}{-4}$ + 1

n = 100

Sum = (501 + 105)(100/2)

S = 30300

Shortcut # 78 – Geometric Progression

**Finding nth term of a Geometric Progression.**

$$t_n = ar^{n-1}$$

$t_n$ - $n^{th}$ term.
a - first term of the series.
r - common ratio between the successive terms. ($a_2/a_1$)
n - number position of the term in the series.

*Question:*

Find $8^{th}$ term of the series 3, 6, 12, …

*Answer:*
n = 8; a = 3; r = (6/3) = 2
$t_8 = 3(2)^{8-1}$
$t_8 = 3(128) = 384$

---

*Question:*

Find $6^{th}$ term of the series 400, 200, 100, …

*Answer:*
n = 6; a = 400; r = (200/400) = 01/2
$t_6 = 400(1/2)^{6-1}$
$t_6 = 400(1/32) = 12.5$

Shortcut # 79 – Geometric Progression

**Sum of the terms of a finite series where r > 1.**

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$S_n$ - Sum of n terms of the series.

*Question:*

Find the sum of the series 3, 6, 12, 24, … up to 10 terms.

*Answer:*

a = 3

n = 10

r = (6/3) = 2; here r > 1.

Sn = $\frac{3(2^{10} - 1)}{2 - 1}$ = 3069

---

*Question:*

Find the sum of the series 10, 30, 90, 270, … up to 8 terms.

*Answer:*

a = 10

n = 8

r = (30/10) = 3; here r > 1.

Sn = $\frac{10(3^{8} - 1)}{3 - 1}$ = 32800

Shortcut # 80 – Geometric Progression

**Sum of the terms of a finite series where r < 1.**

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

*Question:*

Find the sum of the series 600, 300, 150, … up to 6 terms.

*Answer:*

a = 600
n = 6
r = (300/600) = 1/2; here r < 1

$$Sn = \frac{600(1 - \frac{1}{2}^{6})}{1 - \frac{1}{2}} = 1181.25$$

---

*Question:*

Find the sum of the series 128, 64, 32, … up to 10 terms.

*Answer:*

a = 128; n = 10; r = 64/128 = ½ (r < 1)

$$Sn = \frac{128(1 - \frac{1}{2}^{10})}{1 - \frac{1}{2}} = 255.75$$

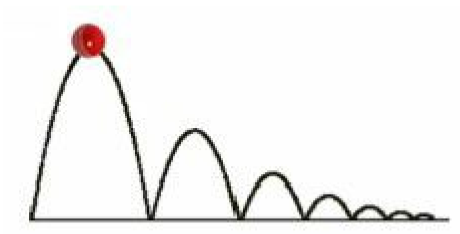
Shortcut # 81 – Geometric Progression

**Sum of the terms of an infinite series where r < 1.**

$$S_n = \frac{a}{1 - r}$$

*Question:*

A ball is thrown up in the air for 400 m. every time the ball hits the ground it will bounce back one third of the height it went the last time. The ball repeats this bounce as long as it rests in the ground. What is the total distance covered by the ball?



*Answer:*

Distance travelled in the first throw = 450 + 450 = 900.
(450 + 450 is because the total distance is both up and down distance covered).

Distance covered in the first bounce = (1/3)900 = 300
Distance covered in second bounce = (1/3)300 = 100
The series is 900, 300, 100, … infinite terms.
Sum = 900/[1 – (1/3)]
Sum = 1350 m

## Shortcut # 82 – Calendar

**Finding day of a date using a reference day.**

*Question:*

Find the day of birth of MS Dhoni (7 July, 1981), if 31 Dec 1999 is Friday

Answer:

Step 1: Number of days after 7/7/1981 in 1981 = 177

Step 2: Number of years between 1981 and 1999 = 17

Step 3: Number of leap years between 1981 and 1999 = 4

Step 4: Number of days in 1999 till Dec 31 = 365

Step 5: 177 + 17 + 4 + 365 = 563

Step 6: Remainder of (563/7) = 3

Our reference day is Friday and the remainder is 3.

Step 7: 3 days before Friday is Tuesday.

MSD was born on Tuesday.

**Note:**
If there are no years in between the day in the question and reference day, step 2 and step 3 will be = 0
If a date in the future is asked, remainder number of days after the reference day will be the answer.
If remainder is 0, the day to be found out is the same day as the reference day.

## Shortcut # 83 – Clocks

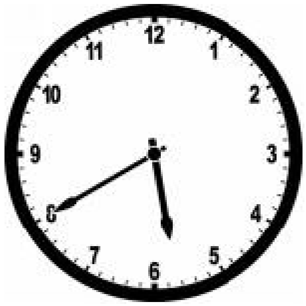
**Angle between the hour hand and the minute hand.**

$$\text{Angle} = \left(30H + \frac{M}{2}\right) \sim 6M$$

H - Hour value
M - Minute value

*Question:*

What is the angle between minute hand and hour hand at 05:40?




*Answer:*

H = 5
M = 40
Angle = [30(5) + (40/2)] ~ 6(40)
= 170 ~ 240
= 70`

Note:
If the value is above 180`, subtract it from 360 to get refract angle.

Shortcut # 84 – Clocks

**Time at which the two hands overlap.**

$$\text{Time} = \frac{12}{11}\,5H$$

H - Least hour between 'H' and 'H+1' hour

*Question:*

At what time between 3 o' clock and 4 o' clock the two hands of the clock will overlap?

*Answer:*

Time = [12/11][5(3)]
= 180/11 minutes past 3 'o' clock
= 16.36 minutes past 3 'o' clock
= 16 minutes 22 seconds past 3
= 03 : 16 : 22

---

*Question:*

At what time between 10 o' clock and 11 o' clock the two hands of the clock will overlap?

*Answer:*

Time = [12/11][5(10)]
= 600/11 minutes past 10 'o' clock
= 54.54 minutes past 10 'o' clock
= 54 minutes 32 seconds past 10
= 10 : 54 : 32

Shortcut # 85 – Clocks

**Time at which the two hands face opposite directions.**

Case 1: H < 6

$$\text{Time} = \frac{12}{11}[5H + 30]$$

*Question:*

At what time between 1 o' clock and 2 o' clock the two hands will be facing opposite direction?

*Answer:*

Time = 12/11[5(1) + (180/6)]

= 12/11[5(1) + 30]

= 420/11 minutes past 1 'o' clock

= 38.18 minutes past 1 'o' clock

= 38 minutes 11 seconds past 1

= 01 : 38 : 11

---

Case 2: H > 6

$$\text{Time} = \frac{12}{11}[5H - 30]$$

*Question:*

At what time between 9 o' clock and 10 o' clock the two hands will be facing opposite directions?

Answer:

Time = 12/11[5(9) - (180/6)]

= 12/11[5(9) - 30]

= 180/11 minutes past 9 'o' clock

= 16.36 minutes past 9 'o' clock

= 16 minutes 22 seconds past 9

= 09 : 16 : 22

Shortcut # 86 – Clocks

**Time at which the two hands will be certain angle apart.**

$$\text{Time} = \frac{12}{11}\left[5H \pm \frac{\theta}{6}\right]$$

There are two different time at which the hands will be certain degrees apart. $T_1$ and $T_2$.

*Question:*

At what time between 4 o' clock and 5 o' clock the two hands of the clock will be 60 degrees apart?

*Answer:*

$T_1$ = 12/11[5(4) + (60/6)]

= 360/11 minutes past 4 'o' clock

= 32.72 minutes past 4 'o' clock

= 32 minutes 44 seconds past 4

= 04 : 32 : 44

$T_2$ = 12/11[5(4) – (60/6)]

= 120/11 minutes past 4 'o' clock

= 10.9 minutes past 4 'o' clock

= 10 minutes 54 seconds past 4

= 04 : 10 : 54

Shortcut # 87 – Clocks

**Time at which the two hands will be certain minute spaces apart.**

$$\text{Time} = \frac{12}{11}[5H \pm M]$$

There are two different time at which the hands will be certain degrees apart. $T_1$ and $T_2$.

*Question:*

At what time between 7 o' clock and 8 o' clock the two hands of the clock will be 15 minutes apart?

*Answer:*

$T_1$ = 12/11[5(7) + 15]

= 600/11 minutes past 7 'o' clock

= 54.54 minutes past 7 'o' clock

= 54 minutes 33 seconds past 7

= 07 : 54 : 33

$T_2$ = 12/11[5(7) – 15]

= 240/11 minutes past 7 'o' clock

= 21.81 minutes past 7 'o' clock

= 21 minutes 48 seconds past 7

= 07 : 21 : 48

## Shortcut # 88 – Permutation Combination and Probability

**Dependent and Independent Events.**

Dependent events – Multiplication; Key word – AND
Independent events - Addition; Key word – OR

*Question:*

There 3 busses from city A to city B and there are 5 busses from city B to city C. In how many ways a person can travel from city A to C through B?

*Answer:*

Choosing a bus from city B depends on choosing a bus from city A.

Number of ways = 3 x 5 = 15

---

*Question:*

There 3 busses from city A to city B and there are 5 busses from city A to city C. In how many ways a person can travel from city A to C or B?

*Answer:*

Choosing a bus to city B or C are not dependent.

Number of ways = 3 + 5 = 8

Shortcut # 89 - Permutation Combination and Probability

**Permutation – Arrangement with repetition.**

## $n^n$

*Question:*

In how many ways the letters of the word "ORANGE" can be arranged with repetition?

*Answer:*

n = 6   (n = total number of elements)

Since all the elements are taken,

Number of arrangements = $6^6$

---

## $n^r$

*Question:*

In how many ways three letters from the word "ORANGE" can be arranged with repetition?

*Answer:*

n = 6; r = 3    (r = number of elements taken for arrangement)

Number of arrangements = $6^3$ = 216

Shortcut # 90 - Permutation Combination and Probability

**Permutation – Arrangement without repetition.**

$$\mathbf{n!}$$

*Question:*

In how many ways the letters of the word "MANGO" can be arranged without repetition?

*Answer:*

n = 5

Since all the elements are taken for arrangement,

Number of elements = n! = 5! = 120

---

$$ {}^{n}P_{r} = \frac{n!}{(n-r)!} $$

*Question:*

In how many ways any three letters of the word 'MANGO" can be arranged without repetition?

*Answer:*

n= 5; r = 3

${}^{n}P_{r}$ = 5!/(5 – 3)! = 120/2 = 60 ways

Shortcut # 91 - Permutation Combination and Probability

**Permutation – Elements occurring together.**

If two elements occur together

$$2!(n - 1)!$$

If three elements occurtogther

$$3!(n - 2)!$$

If four elements occur together

$$4!(n - 3)!$$

and so on

*Question:*

In how many ways letters of the word "ORANGE" arranged so that the vowels will always occur together?

*Answer:*

$n = 6$

Three letters 'O, A and E' should occur together.

Since three letters occur together

$3! \times (6 - 2)! = 6 \times 24 = 144$

**Note:**

If there are 4 letters occurring together, the answer is

$4!(6 - 3)!$

If there are 5 letters occurring together, the answer is

$5!(6 - 4)!$

Shortcut # 92 - Permutation Combination and Probability

**Permutation – When similar kind of elements occur.**

$$\frac{n!}{a! \times b!}$$

*Question:*

In how many ways the letters of the word "ENVIRONMENT" can be arranged?

*Answer:*

n = 11
Let, a = 2 (E is repeated twice)
Let, b = 3 (N is repeated thrice)
Number of arrangements = 11!/(2! x 3!)
= 277200

---

*Question:*

In how many ways the letters of the word "PIIZZZAAAA" can be rearranged?

*Answer:*

n = 10
a = 2 (I is repeated twice)
b = 3 (Z is repeated thrice)
c = 4 (A is repeated four times)
Number of arrangements = 10!/(2! x 3! x 4!)
= 12600

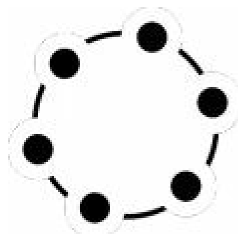
Shortcut # 93 - Permutation Combination and Probability

**Permutation – Circular arrangement.**

# (n – 1)!

*Question:*

In how many ways 6 persons can be arranged in a circle?



*Answer:*

n = 6

Number of arrangements = (6 – 1)! = 5! = 120

---

*Question:*

In how many ways 8 persons can be arranged in a circle?

*Answer:*

n = 8

Number of arrangements = (8 – 1)! = 7! = 5040

Shortcut # 94 - Permutation Combination and Probability

**Permutation – Elements occurring together in a circle.**

If two elements occur together

**2!(n – 2)!**

If three elements occur together

**3!(n – 3)!**

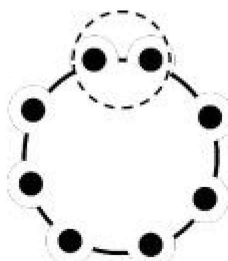
If four elements occur together

**4!(n – 4)!**

and so on

*Question:*

In how many ways 8 persons can be seated around a circular table with two persons always sitting together?



*Answer:*

2! x (8 – 2)! = 2 x 720 = 1440

**Note:**

If three persons are sitting together, the answer is

3! x (8 – 3)!

If four persons are sitting together, the answer is

4! x (8 – 4)!

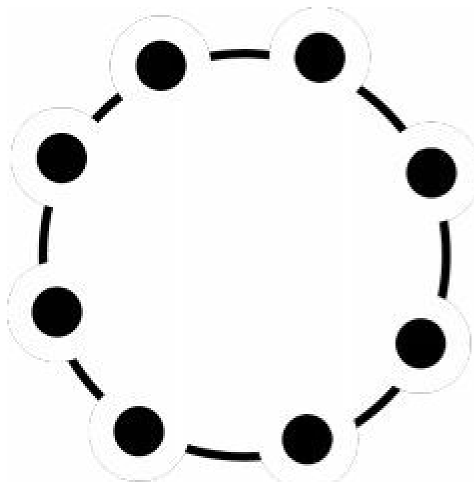
Shortcut # 95 - Permutation Combination and Probability

**Permutation – Arrangement of Necklace.**

# $(n - 1)!/2$

*Question:*

In how many ways a necklace with 8 different colored beads can be arranged?



*Answer:*

$n = 8$

Number of arrangements = $(8 - 1)!/2 = 2520$

Shortcut # 96 - Permutation Combination and Probability

**Combination**

$$^{n}C_{r} = \frac{n!}{r!(n-r)!}$$

*Question:*

In how many ways 2 shirts and 3 pants can be selected from 5 shirts and 7 pants?

*Answer:*

Selecting 2 shirts out of 5 = $^{5}C_{2}$ = 5!/[2!(5 – 2)!] = 10

Selecting 3 shirts out of 7 = $^{7}C_{3}$ = 7!/[3!(7 – 3)!] = 35

Since the two events are dependent,

Total ways of selecting = 10 x 35 = 350

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*Question:*

In how many ways 2 shirts or 3 pants can be selected from 5 shirts and 7 pants?

*Answer:*

Selecting 2 shirts out of 5 = $^{5}C_{2}$ = 5!/[2!(5 – 2)!] = 10

Selecting 3 shirts out of 7 = $^{7}C_{3}$ = 7!/[3!(7 – 3)!] = 35

Since the two events are independent,

Total ways of selecting = 10 + 35 = 45

## Shortcut # 97 - Permutation Combination and Probability

**Probability.**

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

*Question:*

What is the probability of selecting 2 spades from a pack of 52 cards?

*Answer:*

Total number of ways of selecting 2 cards from 52 = ${}^{52}C_2 = 1275$

Total ways of selecting 2 spades from 13 spades = ${}^{13}C_2 = 78$

Probability = 78/1275

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*Question:*

What is the probability of selecting 2 students from a class of 10 boys and 8 girls, where both the selected students are boys?

*Answer:*

Total number of ways of selecting 2 students from 18

= ${}^{18}C_2 = 153$

Total ways of selecting 2 boys from 10 boys

= ${}^{10}C_2 = 45$

Probability = 45/153

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

*Question:*

What is the probability of getting 4 heads when 7 coins are tossed?

*Answer:*

Total number of results = 2 x 2 x 2 x 2 x 2 x 2 x2 = $2^8$ = 128

When the required results need 4 heads, the remaining three will be tails.
The results will be the different arrangement of the following:

" H H H H T T T "

The number of ways in which the above can be arranged is
= 7!/(4! x 3!)

= 5040/144

= 35

Probability = 35/128

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

*Question:*

What is the probability of getting a sum which is equal to prime number from the results obtained by throwing two dice with six faces numbered 1 to 6 on the faces?

*Answer:*

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 |

The above table shows the different sums obtained from different results.
Total number of results = 36.
Expecting result = 15. (Highlighted with the shade).

Probability = 15/36

$$\text{Probability} = \frac{\text{Expecting number of results}}{\text{Total number of results}}$$

*Question*:

What is the probability of selecting 4 balls from a bag that has 5 red balls and 6 black balls where the selection has at least 1 red ball?

*Answer:*

The different cases are:

1 red and 3 black $= {}^5C_1 \times {}^6C_3 = 100$

2 red and 2 black $= {}^5C_2 \times {}^6C_2 = 150$

3 red and 1 black $= {}^5C_3 \times {}^6C_1 = 60$

4 red $= {}^5C_4 = 5$

Number of ways of selecting 4 balls with at least 1 red ball:
$= 100 + 150 + 60 + 5 = 315$

Total ways of selecting 4 balls without any condition:
$= {}^{11}C_4 = 330$
Probability = 315/330

**Alternate method:**

Probability = 1 – probability of selecting no red balls
= 1 – probability of selecting 4 blackballs

Probability $= 1 - ({}^6C_4/{}^{11}C_4)$
= 1 – (15/330)
= 315/330